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Koji Miyawaki
National Institute for Environmental Studies
Yasuhiro Omori
University of Tokyo
Akira Hibiki
National Institute for Environmental Studies

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Koji Miyawaki[a*] Yasuhiro Omori[b] Akira Hibiki[c]

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[a] National Institute for Environmental Studies, Ibaraki, 305-8506, Japan (miyawaki.koji@nies.go.jp).

[b] Faculty of Economics, University of Tokyo, Tokyo, 113-0033, Japan (omori@e.u-tokyo.ac.jp).

[c] National Institute for Environmental Studies, Ibaraki, 305-8506, Japan (hibiki@nies.go.jp).

Abstract

The discrete/continuous choice approach is often used to analyze the demand for public utility services under block rate pricing, which is a nonlinear price system. Although a consumer's budget set is convex under increasing block rate pricing, a consumer's budget set is nonconvex under decreasing block rate pricing as is the case with the gas supply in Japan and the United Kingdom. The nonlinearity problem, which has not been examined in previous studies, arises under nonconvex budget sets in which the indirect utility function corresponding to the demand function becomes highly nonlinear. To address this problem, this article proposes a feasible, efficient method of demand on the nonconvex budget set and implements a case study using household-level data on Japanese residential gas consumption. The advantages of our method are as follows: (i) the construction of an efficient Markov chain Monte Carlo algorithm with an efficient blanket based on the Hermite-Hadamard integral inequality and the power-mean

*Corresponding author: Tel:+81-29-850-2314, E-mail:miyawaki.koji@nies.go.jp

inequality, (ii) the explicit consideration of the (highly nonlinear) separability condition, which often makes numerical likelihood maximization difficult, and (iii) the introduction of normal disturbance into the discrete/continuous choice model.

Key words: Residential gas demand, Nonconvex budget set, Discrete/Continuous choice approach, Bayesian analysis, Hermite-Hadamard integral inequality.
JEL classification: C11, C24, D12.

1 Introduction

The discrete/continuous choice approach is often used to analyze the demand for public utility services under block rate pricing (e.g., Hausman, Kinnucan, and McFaddden (1979); Hewitt and Hanemann (1995)), which is a nonlinear price system.[1] There are two types of block rate pricing: increasing and decreasing block rate pricing. Under increasing block rate pricing, the unit prices increase with the quantity consumed, whereas they decline under decreasing block rate pricing. For example, residential water is often supplied under increasing block rate pricing in Japan. On the other hand, residential gas is widely supplied under decreasing block rate pricing in Japan and the United Kingdom. Other services, such as the mobile phone service (the personal handy-phone system) in Japan and some of the residential electricity services in the United States, also employ this price system. This type of price schedule is likely to be employed partly because the production cost is decreasing in scale and partly because this system is considered to encourage a larger amount of consumption. Chapter 7 of Train (1991) provides a brief microeconomic analysis of block rate pricing.

Under increasing block rate pricing, a consumer's budget set is convex. However, under decreasing block rate pricing, as with the gas supply in Japan and the United Kingdom, the consumer's budget set is nonconvex (see Figure 1(b) on page 6). Nonconvex budget sets

[1] This approach has also been used to examine a wide range of topics including housing (Lee and Trost (1978)), transportation (de Jong (1990); West (2004)), and labor supply (Burtless and Hausman (1978); Burtless and Moffitt (1985)).

also arise when a good is supplied with a fixed cost. For example, de Jong (1990) analyzed the joint choice of car ownership and mileage. Because there is a fixed cost associated with owing a car, the consumer's budget set becomes nonconvex.

We face the following problem under the nonconvex budget set, though we do not under the convex budget set, that the consumer's utility maximization problem derives its corresponding demand function involving the comparison of indirect utilities (see, e.g., Moffitt (1986)). The demand function and the indirect utility are related to each other via Roy's identity. Because of this identity, even if we assume a simple form of the demand function, its corresponding indirect utility function becomes highly nonlinear.

To avoid this nonlinearity, Blomquist and Newey (2002) proposed the use of a nonparametric approach. They analyzed the effect of tax reform in Sweden on working hours for married or cohabiting men from 20 to 60 years of age and estimated the labor supply function as a nonparametric function of the entire budget set. Though their approach is free of the nonlinearity caused by Roy's identity and of model misspecifications and distributional errors, it ignores foundational aspects of the theory like Roy's identity. Thus, this article considers a parametric model of demand on the nonconvex budget set to appropriately address Roy's identity.

The methodology employed in most of the previous literature (e.g., Burtless and Hausman (1978); Hausman (1980); Burtless and Moffitt (1985)) has used the so-called discrete/continuous choice approach to derive the parametric models and analyze the effect of block rate pricing involving a two-block decreasing block rate pricing using the maximum likelihood method.[2] However, two-block rate pricing is too simple for use in the analysis of real data such as Japanese residential gas data, where the number of blocks is much greater than two. (Indeed, the number of blocks is three to six depending on the gas company.) If the block structure was

[2] Recently, Szabó (2009) proposed the maximum likelihood estimation method for general block rate pricing where the linear demand function is assumed. Szabó (2009) imposed a condition that the direct utility function is quasiconcave. This condition aims to guarantee that the underlying preference relation be strictly convex. However, as stated in Hurwicz and Uzawa (1971), two more conditions (the nonnegative demand condition and the separability condition) are required for the underlying preference relation to be strictly convex. These additional conditions often make it difficult to numerically maximize the likelihood function.

simplified to mimic two-block rate pricing, the estimates of the demand function as used for policy-making would be biased. Thus, we consider general multiple-block decreasing block rate pricing as a type II Tobit model subject to many nonlinear constraints (see Chapter 10 of Amemiya (1985) for the Tobit classification) and propose a Bayesian estimation method using a Markov chain Monte Carlo (MCMC) simulator with an efficient blanket.

Because the resulting statistical model includes many nonlinear constraints on model parameters (the comparison of indirect utilities and the separability condition, which will be explained in the next paragraph), the support of the full conditional distribution for elasticity parameters becomes analytically intractable. One possible solution to this problem is rejection sampling. However, using a simple envelope function (or a simple blanket) for the support is extremely inefficient because the acceptance rate of the proposed samples is extremely low (see Section 3.3). Thus, this article develops an efficient blanket using two properties of convex functions: the Hermite-Hadamard integral inequality and the power-mean inequality.

Our approach also has another particular advantage. The previous studies employing maximum likelihood estimation do not to explicitly consider the separability condition, though this condition is necessary for the demand model under block rate pricing with more than two blocks.[3] In contrast, under multiple-block decreasing block rate pricing, our statistical model includes the separability condition, which is highly nonlinear, to properly estimate the model parameters. Because of this condition, it is often difficult to numerically maximize the likelihood, and we need to pursue the Bayesian approach, using the MCMC simulator to estimate the model parameters.

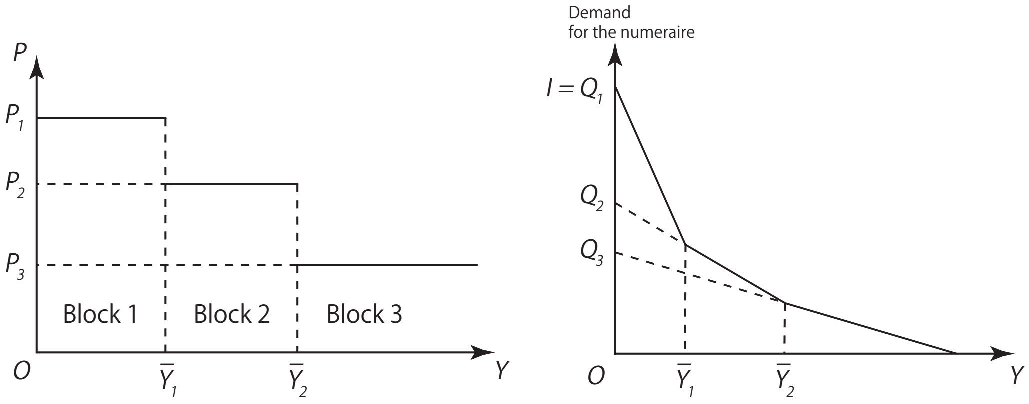
Finally, we would like to note that our proposed method has an advantage over the other type of discrete/continuous choice analysis used in the context of the multinomial choice model, as in Dubin and McFadden (1984). The resulting statistical model is the same as that for demand on the nonconvex budget set. Dubin and McFadden (1984) analyzed the

[3] Miyawaki, Omori, and Hibiki (2010) dealt with this issue in the context of increasing block rate pricing. Under increasing block rate pricing, this condition is a set of linear constraints on elasticity parameters.

joint choice of electric appliances and electricity demand using this approach and estimated the model parameters based on a combination of the maximum likelihood and the conditional expectation correction method. Their statistical model is simplified by introducing the logit error into the choice of electric appliance portfolios. However, such a specification implies the independence of irrelevant alternatives. The subsequent literature addresses this problem in two ways: by using the nested logit model (e.g., Goldberg (1998)) or by linearizing the nonlinear indirect utilities (e.g., Bernard, Bolduc, and Bélanger (1996)). Carpio, Wohlgenant, and Safley (2008) used a different method and applied it to the estimation of the demand for pick-your-own versus preharvested strawberries with normal error. However, their statistical model is a binary choice model: thus, they do not consider the separability condition. Thus, this article is the first study to propose a multinomial choice model based on the discrete/continuous choice approach with normal disturbance.

Therefore, in presenting a parametric model for demand on the nonconvex budget set, this article proposes the use of Bayesian analysis to make the following contributions: (i) the construction of an efficient MCMC algorithm with an efficient blanket based on the Hermite-Hadamard integral inequality and the power-mean inequality: (ii) the explicit consideration of the (highly nonlinear) separability condition, which often makes numerical likelihood maximization difficult: and (iii) the introduction of normal disturbance into the discrete/continuous choice model.

Using the proposed method, we analyze the residential gas demand function and evaluate the effect of price schedule changes. We do not consider the substitution between residential gas and electricity because our main interest is the demand function on the nonconvex budget set. This article is organized as follows. Section 2 describes the demand function based on the discrete/continuous choice approach, introduces two stochastic terms, and derives the corresponding likelihood function with the separability condition. Section 3 discusses the Bayesian approach and its MCMC algorithm with an efficient blanket. We also evaluate the adequacy of the proposed blankets. Section 4 estimates the Japanese residential gas demand



(a) Price schedule. (b) Budget line.

Figure 1: A three-block decreasing block rate pricing.

function and evaluates the effect of price schedule changes. Section 5 concludes the study.

2 Demand function under decreasing block rate pricing

2.1 Discrete/Continuous choice approach

Decreasing block rate pricing is characterized by four nonnegative variables: the number of blocks (K), the unit prices ($P_{k+1} < P_k$ for $k = 1, \ldots, K-1$), the upper limits ($\bar{Y}_k$ for $k = 1, \ldots, K-1$), and the fixed service fee as the fixed cost (FC). Figure 1(a) illustrates an example of three-block decreasing block rate pricing. The residential gas services in Japan and the United Kingdom are often provided under such a price schedule (see also Section 4 for the case of Japan).

The demand function under decreasing block rate pricing is derived using the so-called discrete/continuous choice approach (see, e.g., Moffitt (1986)). This approach is a two-step procedure used to solve the utility maximization problem under block rate pricing.

Suppose there are two goods: a good under decreasing block rate pricing and the numeraire good. The demand for the former is denoted by Y. Let $Q_k = I - FC - \sum_{j=1}^{k-1}(P_j - P_{j+1})\bar{Y}_j$ $(k = 1, \ldots, K)$ be the virtual income for the k-th block, where I is the total income (see Figure 1(b)). We note that $Q_{k+1} < Q_k$ for $k = 1, \ldots, K-1$. Then, under decreasing block rate pricing, the discrete/continuous choice approach is described as follows.

Step 1. For $k = 1, \dots, K$, maximize the utility under the uniform price system, where a consumer faces the single unit price P_k and its corresponding virtual income Q_k. As the solution and maximum, we obtain Y_k and V_k ($k = 1, \dots, K$), respectively.

Step 2. Find the block k such that $V_k = \max_j V_j$. Then, Y_k is the optimal demand.

In Step 1, both the price and the virtual income are given as constants. Because V_k is the maximized utility conditional on block choice, it is called the conditional indirect utility. By following the above two steps, we obtain the demand function under decreasing block rate pricing:

$$Y = Y_k, \quad V_k = \max_j V_j. \tag{1}$$

Roy's identity connects Y_k and V_k. This article first assumes both P_K and Q_K to be positive. Then, $P_k > 0$ and $Q_k > 0$ for all k. Next, we assume Y_k to be linear in logarithm, that is,

$$\ln Y_k = \beta_1 \ln P_k + \beta_2 \ln Q_k. \tag{2}$$

The log-linear function is popular in the analysis of demand under block rate pricing, because β_1 and β_2 can be directly interpreted as price and (virtual) income elasticity, respectively, conditional on block choice (see, e.g., Hewitt and Hanemann (1995); Olmstead, Hanemann, and Stavins (2007)). Furthermore, when we impose the conditions stated in Hurwicz and Uzawa (1971), the underlying preference relation satisfies strict convexity. After specifying the conditional demand function, Roy's identity implies

$$V_k = -\frac{P_k^{1+\beta_1}}{1+\beta_1} + \frac{Q_k^{1-\beta_2}}{1-\beta_2}, \tag{3}$$

where $\beta_1 \neq -1$ and $\beta_2 \neq 1$, as derived in Burtless and Hausman (1978). Plugging equations

(2) and (3) into equation (1), we have the demand function under decreasing block rate pricing based on the discrete/continuous choice approach.[4]

We note that this theoretical framework does not exclude cases in which multiple blocks are simultaneously optimal. Such a case is excluded by introducing a continuous random disturbance into the consumer's heterogeneity in preferences. Subsection 2.3 describes its specification.

Remark 1. Hanemann (1984) proposed two other demand functions that are less popular in the literature: the linear expenditure system (LES) model and the price independent generalized log-linear (PIGLOG) model.

2.2 Compensating variation

Because the demand function includes the (conditional) indirect utility, we can evaluate the effect of the price schedule changes on welfare using the compensating variation (see Chapter 3 of Mas-Colell, Whinston, and Green (1995) for a general discussion of the compensating variation). Let $P = \{\{P_k, \bar{Y}_k\}_{k=1}^{K-1}, P_K, FC\}$ and $P' = \{\{P'_k, \bar{Y}'_k\}_{k=1}^{K'-1}, P'_{K'}, FC'\}$ denote the current and the suppositional price schedule, respectively. Then, by solving

$$V = (\text{the right hand side of equation (3) evaluated with } P'), \tag{4}$$

for I, where V is a certain utility level, we obtain the expenditure function at the certain utility level under the suppositional price schedule, which is given by

$$E_{k'}(P', V) = \left[(1-\beta_2)\left\{V + \frac{(P'_{k'})^{1+\beta_1}}{1+\beta_1}\right\}\right]^{1/(1-\beta_2)} + FC' + \sum_{j=1}^{k'-1}\left(P'_j - P'_{j+1}\right)\bar{Y}'_j, \tag{5}$$

[4] As pointed out in Hausman (1985), the approach that involves deciding the demand function first and deriving its corresponding indirect utility function has two advantages: (i) we can flexibly choose the functional form of the demand function based on the empirical data set, and (ii) the stochastic specification becomes convenient.

where $k' = \text{argmax}_j V'_j$ and V'_j is the indirect utility conditional on the j-th block under P' (see Hausman (1981) for the case in which there is a single unit price). Then, welfare can be quantitatively evaluated using the so-called compensating variation, which is derived as

$$CV = I - E_{k'}(P', V_k), \tag{6}$$

where $k = \text{argmax}_j V_j$ and V_j is the j-th conditional indirect utility under P. By definition, CV is the difference between the current income and the income required to attain the current utility level under the suppositional price system. The amount of positive (negative) difference can be interpreted as the degree of improvement (decline) in consumer welfare under the suppositional price schedule. When we assume P' to be the uniform price system, that is, $P' = \{P^*, FC^*\}$, we have

$$E_{k'}(P', V) = \left[(1-\beta_2)\left\{V + \frac{(P^*)^{1+\beta_1}}{1+\beta_1}\right\}\right]^{1/(1-\beta_2)} + FC^*. \tag{7}$$

The conditional indirect utility under P is given by equation (3). Then, the compensating variation is calculated as

$$CV = I - \left[(1-\beta_2)\left\{\frac{(P^*)^{1+\beta_1} - P_k^{1+\beta_1}}{1+\beta_1} + \frac{Q_k^{1-\beta_2}}{1-\beta_2}\right\}\right]^{1/(1-\beta_2)} - FC^*. \tag{8}$$

In subsection 4.3, we will conduct the welfare analysis based on the compensating variation using the empirical data.

Remark 2. Another welfare measure is equivalent variation, which is given by

$$EV = E_k\left(P, V'_{k'}\right) - I.$$

Because both EV and CV show similar patterns with our empirical data set, the discussion and the results of EV are suppressed.

2.3 Type II Tobit model with a nonlinear indirect utility comparison

This subsection describes a statistical model that is a nonlinear type II Tobit model based on the theoretical framework with equations (1)-(3). There are n consumers. Let subscript i denote the consumer i ($i = 1,\dots,n$) and let $(y_i, y_{ik}, p_{ik}, q_{ik}) = (\log Y_i, \log Y_{ik}, \log P_{ik}, \log Q_{ik})$.

Then, the statistical model for the demand function under decreasing block rate pricing is given by

$$y_i = y_{is_i^*} + w_i^* + u_i, \quad u_i \sim \text{i.i.d. } N\left(0, \sigma_u^2\right), \tag{9}$$

where

$$y_{is_i^*} = \beta_1 p_{is_i^*} + \beta_2 q_{is_i^*},^{5} \tag{10}$$

$$w_i^* = \boldsymbol{z}_i' \boldsymbol{\delta} + v_i, \quad v_i \sim \text{i.i.d. } N\left(0, \sigma_v^2\right), \tag{11}$$

$$s_i^* = k, \quad \text{if } w_i^* \in R_{ik} = \left\{ w_i^* \mid V_{ik} > V_{ij} \text{ for } k \neq j \right\} \text{ and } k = 1,\dots,K_i, \tag{12}$$

$$V_{ik} = -\exp\left(w_i^*\right) \frac{P_{ik}^{1+\beta_1}}{1+\beta_1} + \frac{Q_{ik}^{1-\beta_2}}{1-\beta_2}, \tag{13}$$

$\beta_1 \neq -1$, and $\beta_2 \neq 1$.

In this statistical model, there are three components in addition to the theoretical framework with equations (1)-(3). The first component is w_i^*, which represents the consumer's heterogeneity in preferences. We introduce a hierarchical structure into the heterogeneity and assume it to be linear in the d-dimensional covariate vector $\boldsymbol{z}_i$ with its corresponding coefficient vector $\boldsymbol{\delta}$. The disturbance v_i of the heterogeneity is normally distributed with a mean of 0 and a variance of σ_v^2. Then, the indirect utility conditional on the block choice is derived from the sum of y_{ik} and w_i^* using Roy's identity.

Consequently, the comparison of conditional indirect utilities is solved with respect to

[5] Because of the log-linear function in (10), we require $P_{iK_i} > 0$ and $Q_{iK_i} > 0$ for all i. In our empirical data set, there are no households whose $Q_{iK_i} \leq 0$.

heterogeneity. The resulting interval is called the heterogeneity interval and is denoted by R_{ik}. The explicit formula for the heterogeneity interval is given in Appendix A.1. To be rigorous, this interval must be $\bar{R}_{ik} = \{w_i^* \mid V_{ik} = \max_j V_{ij}\}$, where a tie among the conditional indirect utilities is allowed. Clearly, $R_{ik} \subseteq \bar{R}_{ik}$. However, because the set, $V_{ik} = V_{ij}$ $(j \neq k)$, has a probability of zero in our statistical model, we replace $\bar{R}_{ik}$ with R_{ik}. This zero probability implies that the statistical model excludes the multiple optima. The reason is as follows. Conditional on β_1 and β_2, the condition $V_{ik} = V_{ij}$ leads to the condition that w_i^* must equal to a certain real value, $\ln E_{kj}$, which is derived in Appendix A.1. Because w_i^* is a continuous random variable, this condition has a zero probability.

The second component is the state variable, s_i^*, and we can use the data augmentation method to estimate the model parameters (see Tanner and Wong (1987) for more information on this method). The s_i^* is a discrete latent variable that takes one of the values from 1 to K_i and indicates the optimal block for the i-th consumer.

The third component is an error u_i for demand that follows a normal distribution with a mean of 0 and a variance of σ_u^2. This term is assumed to be independent of v_i. As discussed in Hausman (1985) and Moffitt (1986), u_i represents an optimization error by the consumer and a misspecification error by the statistician.

2.4 Likelihood function subject to many nonlinear constraints

The likelihood function augmented by the latent variables is given by

$$\begin{aligned} f\left(y_i, s_i^*, w_i^* \mid \boldsymbol{\beta}, \boldsymbol{\delta}, \sigma_u^2, \sigma_v^2\right) \\ \propto (\sigma_u \sigma_v)^{-1} \exp\left[-\frac{1}{2}\left\{\sigma_u^{-2}\left(y_i - y_{is_i^*} - w_i^*\right)^2 + \sigma_v^{-2}\left(w_i^* - \boldsymbol{z}_i'\boldsymbol{\delta}\right)^2\right\}\right] I\left(w_i^* \in R_{is_i^*}\right) \\ \times \prod_{k=2}^{K_i - 1} I(RL_{ik} \leq RU_{ik}), \quad (14) \end{aligned}$$

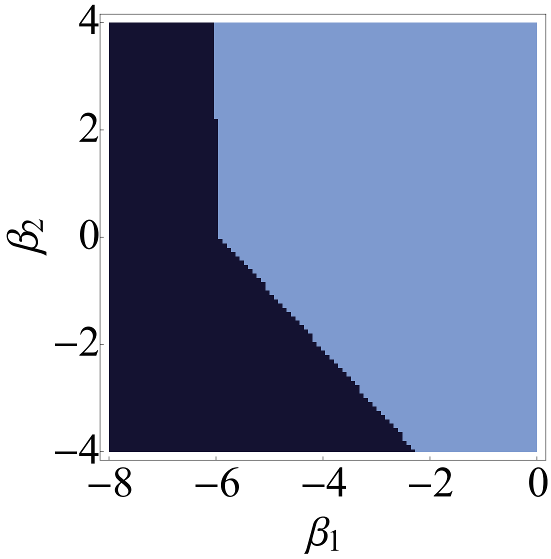


Figure 2: Region implied by the separability condition.

where $\boldsymbol{\beta} \equiv (\beta_1,\beta_2)'$ and $I(A)$ is the indicator function: $I(A) = 1$ if A is true and $I(A) = 0$ otherwise. RL_{ik} and RU_{ik} are the respective lower and upper limits of the heterogeneity interval R_{ik}, and their definitions are given in equation (31) in Appendix A.1. Because we take a Bayesian approach as described later and treat $\boldsymbol{\beta}$ as a continuous random vector, the conditions $\beta_1 \neq -1$ and $\beta_2 \neq 1$ are omitted hereafter.

The last term, the product of the $K_i - 2$ indicator functions, is the condition that the heterogeneity intervals are separable, that is, $R_{ik} \neq \emptyset$ (for all k). We call this condition the separability condition. This condition is a set of nonlinear constraints on β_1 and β_2, and the number of nonlinear constraints increases as the number of observations and blocks grows. Because of this condition, it is often difficult to numerically maximize the likelihood.

Figure 2 is included to show how the separability condition restricts (β_1,β_2) by using the empirical data set. Because the separability condition is analytically intractable, each point is checked whether it satisfies the condition to draw this figure. The light blue area is the area in which the separability condition holds, whereas the deep blue area is the area in which it does not. We can see that the separability condition simulated by the empirical data set imposes nonlinear (piecewise-linear) constraints on (β_1,β_2).

In general, when we analyze the multinomial choice model, such a condition is always required so that every choice is separable. Similarly, Miyawaki et al. (2010) analyzed the demand model under increasing block rate pricing, which is another multinomial choice model, and explicitly considered the requirement that the choice intervals be separable. In this case, the separability condition is a set of linear constraints on elasticity parameters. Furthermore, the separability condition is one of the sufficient conditions to make the underlying preference relation strictly convex (see Hurwicz and Uzawa (1971)). The separability condition is illustrated in the next subsection.

We refer to the identification problem of two errors: u_i for the observed demand and v_i for heterogeneity. They cannot be fully identified unless there is additional information through the prior distribution about these errors because there is only one equation for them: $y_i = y_{is_i^*} + w_i^* + u_i$.

3 Efficient MCMC simulator based on two inequalities

3.1 Prior-Posterior analysis

This article assumes the following proper prior distributions.

$$\begin{aligned} \beta_j \mid \sigma_u^2 &\sim TN_{B_j}\left(\mu_{\beta_j,0}, \sigma_u^2\sigma_{\beta_j,0}^2\right), \ (j=1,2), \quad & \sigma_u^2 &\sim IG\left(\frac{n_{u,0}}{2}, \frac{S_{u,0}}{2}\right), \\ \boldsymbol{\delta} \mid \sigma_v^2 &\sim N_d\left(\boldsymbol{\mu}_{\boldsymbol{\delta},0}, \sigma_v^2\boldsymbol{\Sigma}_{\boldsymbol{\delta},0}\right), & \sigma_v^2 &\sim IG\left(\frac{n_{v,0}}{2}, \frac{S_{v,0}}{2}\right). \end{aligned} \tag{15}$$

Conditional on σ_u^2, β_j follows the truncated normal distribution with mean $\mu_{\beta_j,0}$, variance $\sigma_u^2\sigma_{\beta_j,0}^2$, and support $B_j = [l_j, m_j]$ $(j = 1,2)$. Conditional on σ_v^2, $\boldsymbol{\delta}$ follows the d-dimensional multivariate normal distribution with mean vector $\boldsymbol{\mu}_{\boldsymbol{\delta},0}$ and covariance matrix $\sigma_v^2\boldsymbol{\Sigma}_{\boldsymbol{\delta},0}$. The variance parameter σ_j follows the inverse gamma distribution with parameters $n_{j,0}/2$ and $S_{j,0}/2$ $(j = u, v)$. Its mean and variance are $S_{j,0}/(n_{j,0} - 2)$ for $n_{j,0} > 2$ and $2S_{j,0}^2/\{(n_{j,0} - 2)^2(n_{j,0} - 4)\}$ for $n_{j,0} > 4$, respectively. The support of β_j $(j = 1,2)$ reflects our prior knowledge. To elicit the prior distribution, one can make use of knowledge based on demand theory or utilize the estimates obtained from a similar population (see Subsection 4.2).

Let $\pi(\boldsymbol{\beta},\boldsymbol{\delta},\sigma_u^2,\sigma_v^2)$ be the prior density function of $(\boldsymbol{\beta},\boldsymbol{\delta},\sigma_u^2,\sigma_v^2)$. Then, it is straightforward to derive the posterior density function, which is given by

$$\pi\left(\boldsymbol{\beta},\boldsymbol{\delta},\sigma_u^2,\sigma_v^2,\boldsymbol{s}^*,\boldsymbol{w}^*\mid\boldsymbol{y}\right)\propto\pi\left(\boldsymbol{\beta},\boldsymbol{\delta},\sigma_u^2,\sigma_v^2\right)$$
$$\times(\sigma_u\sigma_v)^{-n}\exp\left[-\frac{1}{2}\left\{\sigma_u^{-2}(\boldsymbol{y}-\boldsymbol{y}_{s^*}-\boldsymbol{w}^*)'(\boldsymbol{y}-\boldsymbol{y}_{s^*}-\boldsymbol{w}^*)+\sigma_v^{-2}(\boldsymbol{w}^*-\boldsymbol{Z\delta})'(\boldsymbol{w}^*-\boldsymbol{Z\delta})\right\}\right]$$
$$\times\prod_{i=1}^{n}\left\{I\left(w_i^*\in R_{is_i^*}\right)\prod_{k=2}^{K_i-1}I(RL_{ik}\le RU_{ik})\right\},\quad(16)$$

where $\boldsymbol{y}=(y_1,y_2,\dots,y_n)'$, $\boldsymbol{y}_{s^*}=(y_{1s_1^*},y_{2s_2^*},\dots,y_{ns_n^*})'$, $\boldsymbol{s}^*=(s_1^*,s_2^*,\dots,s_n^*)'$, $\boldsymbol{w}^*=(w_1^*,w_2^*,\dots,w_n^*)'$, and $\boldsymbol{Z}=(\boldsymbol{z}_1,\boldsymbol{z}_2,\dots,\boldsymbol{z}_n)'$.

To draw samples of model parameters from this posterior density function, we use the standard Gibbs sampler, the details of which are given in Appendix A.2 and the next subsection.

3.2 Sampling β_1 with an efficient blanket

The full conditional distribution of β_1 is the truncated normal distribution, $TN_{C_1}(\mu_{\beta_1,1},\sigma_u^2\sigma_{\beta_1,1}^2)$, where

$$\sigma_{\beta_1,1}^{-2}=\sigma_{\beta_1,0}^{-2}+\sum_{i=1}^{n}\left(p_{is_i^*}\right)^2,\quad(17)$$

$$\mu_{\beta_1,1}=\sigma_{\beta_1,1}^{2}\left[\sigma_{\beta_1,0}^{-2}\mu_{\beta_1,0}+\sum_{i=1}^{n}p_{is_i^*}\left(y_i-\beta_2q_{is_i^*}-w_i^*\right)\right],\quad(18)$$

$$C_1=\left[\bigcap_{i=1}^{n}\bigcap_{j=1,j\neq s_i^*}^{K_i}\left\{\beta_1\mid V_{ik}>V_{ij}\right\}\right]\cap\left[\bigcap_{i=1}^{n}\bigcap_{k=2}^{K_i-1}\{\beta_1\mid RL_{ik}\le RU_{ik}\}\right]\cap[l_1,m_1].\quad(19)$$

Because C_1 is difficult to evaluate analytically, we use rejection sampling. However, as revealed in the next subsection, a simple blanket, the envelope function in rejection sampling, is not efficient in the sense that the acceptance rate of the proposed candidate is extremely low. Therefore, we closely approximate C_1 by $\tilde{C}_1$, which is derived by using two properties of convex functions (the Hermite-Hadamard integral inequality and the power-mean inequality), thus improving our sampling efficiency.

First, without loss of generality, we assume that the support of the prior for β_1 is $B_1 = [l_1, 0]$. Then, we decompose C_1 into a set of larger sets and approximate them to obtain $\tilde{C}_1$. More precisely,

$$C_1 \subset \bigcap_{i=1}^{n} \bigcap_{j=1, j \neq s_i^*}^{K_i} C_{s_i^* j}^{1i} \subset \bigcap_{i=1}^{n} \bigcap_{j=1, j \neq s_i^*}^{K_i} \tilde{C}_{s_i^* j}^{1i} \equiv \tilde{C}_1, \tag{20}$$

where $C_{kj}^{1i} = \{\beta_1 \mid V_{ik} > V_{ij}\} \cap [l_1, 0]$. Third, we construct the interval $\tilde{C}_{kj}^{1i}$ ($\supset C_{kj}^{1i}$) using the following three steps.

Step 1. Apply the Hermite-Hadamard integral inequality. The Hermite-Hadamard integral inequality[6]and $\beta_1 \in [l_1, 0]$ imply

$$\int_{P_{ij}}^{P_{ik}} x^{\beta_1} dx \geq \begin{cases} \left(P_{ik} - P_{ij}\right)\left(\frac{P_{ik}+P_{ij}}{2}\right)^{\beta_1}, & \text{if } k < j, \\ \left(P_{ik} - P_{ij}\right)\frac{P_{ik}^{\beta_1}+P_{ij}^{\beta_1}}{2}, & \text{if } k > j. \end{cases} \tag{21}$$

Using this inequality, we have

$$V_{ik} > V_{ij} \Longleftrightarrow a_1 > \int_{P_{ij}}^{P_{ik}} x^{\beta_1} dx \Longrightarrow a_1 > \text{(the right hand side of equation (21))}, \tag{22}$$

where $a_1 = \exp(-w_i^*)(1-\beta_2)^{-1}(Q_{ik}^{1-\beta_2} - Q_{ij}^{1-\beta_2})$.

[6]Let $f : [a,b] \to \mathbb{R}$ be a convex function. Then,

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)\,dx \leq \frac{f(a)+f(b)}{2}. \tag{23}$$

See, for example, Niculescu and Persson (2003) for a proof. Niculescu and Persson (2003) also noted that the first (or last) inequality can define the convex function itself.

Step 2. Apply the power-mean inequality. The power-mean inequality and $\beta_1 \in [l_1, 0]$ imply

$$\left(\frac{P_{ik}^{l_1}+P_{ij}^{l_1}}{2}\right)^{1/l_1} < \left(\frac{P_{ik}^{\beta_1}+P_{ij}^{\beta_1}}{2}\right)^{1/\beta_1} \Longleftrightarrow \frac{P_{ik}^{\beta_1}+P_{ij}^{\beta_1}}{2} < \left(\frac{P_{ik}^{l_1}+P_{ij}^{l_1}}{2}\right)^{\beta_1/l_1}.^{7} \tag{24}$$

Step 3. Combine the above two-step results. By combining equations (22) and (24), and by rearranging these inequalities for β_1, we derive the closely approximated interval $\tilde{C}_{kj}^{1i} = \tilde{C}_{kj}^{\star 1i} \cap [l_1, 0]$, where

$$\tilde{C}_{kj}^{\star 1i} = \begin{cases} (-\infty, b_1/\bar{p}(1)), & \text{if } k < j \text{ and } \bar{p}(1) > 0, \\ (-\infty, \infty), & \text{if } k < j \text{ and } \bar{p}(1) = 0, \\ (b_1/\bar{p}(1), \infty), & \text{if } k < j \text{ and } \bar{p}(1) < 0, \\ (b_1/\bar{p}(l_1), \infty), & \text{if } k > j \text{ and } \bar{p}(l_1) > 0, \\ (-\infty, \infty), & \text{if } k > j \text{ and } \bar{p}(l_1) = 0, \\ (-\infty, b_1/\bar{p}(l_1),), & \text{if } k > j \text{ and } \bar{p}(l_1) < 0, \end{cases} \tag{25}$$

$b_1 = \log(a_1/(P_{ik} - P_{ij}))$[8], and $\bar{p}(x) = x^{-1}\log\{(P_{ik}^x + P_{ij}^x)/2\}$ $(x = 1, l_1)$. By construction, $C_{kj}^{1i} \subset \tilde{C}_{kj}^{1i}$. If $P_{iK_i} > 1$ is assumed, we have $\bar{p}(1) > \bar{p}(l_1) > 0$, which simplifies the above expression.

Finally, by using this interval $\tilde{C}_{kj}^{1i}$, we approximate C_1 by $\tilde{C}_1 = \cap_{i=1}^{n} \cap_{j=1, j \neq s_i^*}^{K_i} \tilde{C}_{s_i^* j}^{1i}$ as mentioned above. Figure 3 illustrates the relationships among C_1, $\tilde{C}_1$, and B_1.

With $\tilde{C}_1$, the sampling procedure for β_1 is implemented using the following two steps.

Step a. Generate β_1' from the uniform distribution on $\tilde{C}_1$ until it is in C_1.

[7]See, for example, Chapter 2 of Hardy, Littlewood, and Pólya (1952) for a proof of the power-mean inequality. This equivalence also uses the fact that $f(x) = x^{\beta_1}$ $(\beta_1 \in [l_1, 0])$ is decreasing as $x(> 0)$ increases.

[8]Because $a_1 \gtrless 0$ for all $k \lessgtr j$, $a_1/(P_{ik} - P_{ij}) > 0$ for any k and j $(k \neq j)$.

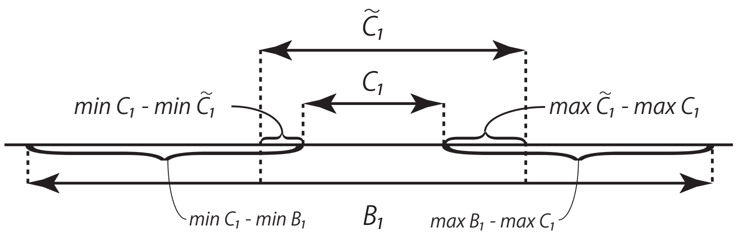


Figure 3: Relationships among C_1, $\tilde{C}_1$, and B_1.

Step b. Accept β_1' with the acceptance probability $\alpha(\beta_1, \beta_1')$; otherwise, retain β_1, where

$$\alpha\left(\beta_1, \beta_1'\right) = \min\left[1, \frac{\phi\left\{\left(\beta_1' - \mu_{\beta_1,1}\right)\sigma_u^{-1}\sigma_{\beta_1,1}^{-1}\right\}}{\phi\left\{\left(\beta_1 - \mu_{\beta_1,1}\right)\sigma_u^{-1}\sigma_{\beta_1,1}^{-1}\right\}}\right], \tag{26}$$

and $\phi(\cdot)$ is the probability density function of the standard normal distribution.

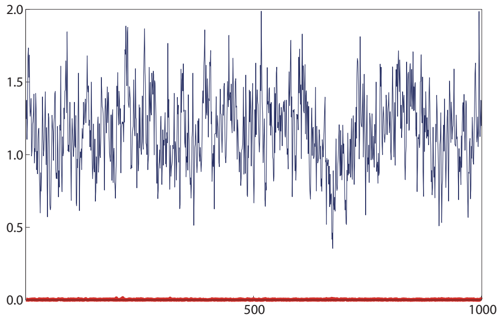
The sampling of β_2 is conducted in a similar manner. See Appendix A.2 for its full conditional distribution and Appendix A.3 for the derivation of its efficient blanket.

Joint sampling for (β_1, β_2) is an alternative sampling algorithm. While using this strategy could improve the sampling efficiency, its efficient two-dimensional blanket is difficult to construct. One of the simplest blankets is $B_1 \times B_2$, which is the support of the joint prior distribution of (β_1, β_2). As we see in Figure 4 and Table 1, however, this blanket is extremely inefficient with respect to the empirical data set.

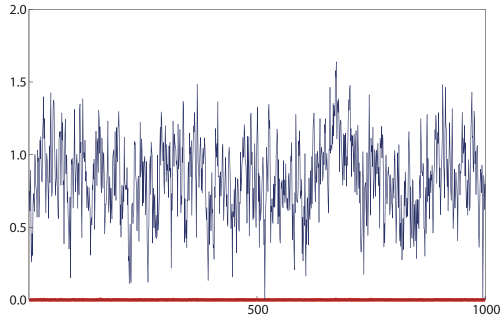
3.3 Adequacy of the efficient blankets

In this subsection, we evaluate the adequacy of the efficient blanket in two respects by using the Japanese residential gas demand data. The first measure is the absolute differences, $\max \tilde{C}_j - \max C_j$ and $\min C_j - \min \tilde{C}_j$ $(j = 1, 2)$, and the second measure is the adequacy ratio, $|C_j|/|\tilde{C}_j|$ $(j = 1, 2)$, where $|A|$ is the area of the set A. Figure 3 is helpful in that it clarifies what these measures mean.

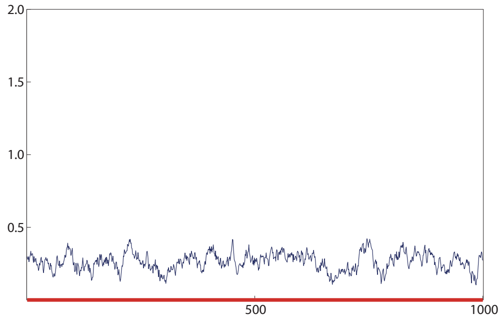
Because C_j is analytically intractable, we calculate these measures via simulation. During each step in the MCMC iterations (Appendix A.2), we obtain the approximated interval,



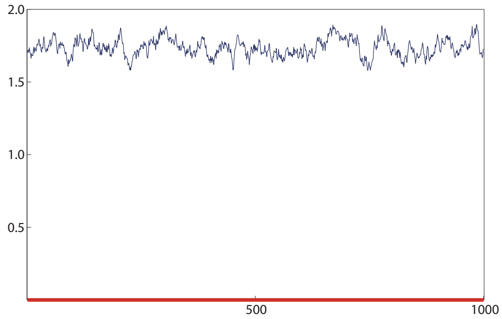
(a) $\min C_1 - \min \tilde{C}_1$ (red) and $\min C_1 - \min B_1$ (blue).



(b) $\max \tilde{C}_1 - \max C_1$ (red) and $\max B_1 - \max C_1$ (blue).



(c) $\min C_2 - \min \tilde{C}_2$ (red) and $\min C_2 - \min B_2$ (blue).



(d) $\max \tilde{C}_2 - \max C_2$ (red) and $\max B_2 - \max C_2$ (blue).

Figure 4: Absolute differences.

Table 1: Adequacy ratios

Coefficient	$\lvert C_j\rvert/\lvert\tilde{C}_j\rvert = r_1$	$\lvert C_j\rvert/\lvert B_j\rvert = r_2$	*Efficiency ratio* (r_1/r_2)
β_1	.67 (.21)	.0037 (.0026)	181
β_2	1.00 (.00)	.0004 (.0003)	2,500

* Standard deviations in parentheses.

$\tilde{C}_j$. Then, we compute 1,001 equispaced samples in this approximated interval and determine whether they belong to C_j. Among the samples that are in C_j, we obtain the maximum and the minimum to calculate the absolute differences. Furthermore, the ratio of the number of samples that belong to C_j to the number of those that do not is the adequacy ratio conditional on model parameters. These conditional adequacy ratios are averaged to calculate the adequacy ratio after the MCMC iterations are complete.

We calculate these two measures using the empirical data set. The results are shown in Figure 4 and given in Table 1. Figure 4 presents time series plots of absolute differences. The red lines represent time series plots of absolute differences that calculated from our efficient blankets, whereas the blue lines are those obtained using the simple method, where $\tilde{C}_j$ is replaced by B_j. The red lines are very close to the horizontal lines at zero, which implies that the proposed efficient blankets are sufficiently close to the true sets. Table 1 indicates the adequacy ratios in the first two columns and the efficiency ratio, the ratio of two adequacy ratios, in the third column. Although the adequacy ratios of the efficient blankets differ with respect to their parameters, they are much (about 200 to 2,500 times) higher than those of the simple blanket B_j. Therefore, based on the empirical data set, our proposed method well approximates the true regions for both β_1 and β_2.

4 Empirical analysis and policy evaluation of residential gas demand

4.1 Data description

This subsection describes the data to be used for the empirical study in the next subsection. We conducted an online survey on the Internet from June 2006 to May 2008 that was designed to analyze the water and energy consumption and the garbage emission behavior of Japanese households. The population of this survey was comprised of the households living in the Tokyo and Chiba prefectures. There were about 8.4 million households as of January 2007. Among them, $47,239$ individuals were registered to the survey company, INTAGE Inc. (http://www.intage.co.jp/english/). Out of $47,239$ individuals, $1,687$ individuals were randomly selected. Then, out of $1,687$ individuals, $1,250$ participated in our survey. They were asked for household attributes such as annual income, the number of members in the household, and so on in June 2006 and April 2007. They were also asked to record their water and energy consumptions and the garbage emission behavior every month.

For the empirical study, we used the attribute data in June 2006 and the gas consumption

Table 2: Independent variables used in the gas demand function

Coefficient	*Variable*	*Attribute*
β_1	$(p_{i1},\ldots,p_{iK_i})$	log of monthly unit prices of gas (log ¥50 /m^3)
β_2	$(q_{i1},\ldots,q_{iK_i})$	log of monthly virtual incomes (log ¥50)
δ_1	z_{i1}	the constant
δ_2	z_{i2}	the number of members in a household (person)
δ_3	z_{i3}	the number of rooms in a home (room)
δ_4	z_{i4}	the total floor space of a home (50m^2)

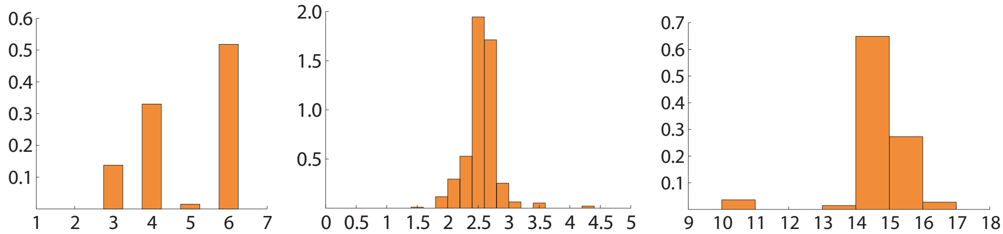
data in January 2007. The dependent variable is the amount of gas consumption ($\log \mathrm{m}^3$), which was calculated from the bill by using the corresponding gas price schedule that depends on the area in which the individuals were living. The list of independent variables and their corresponding coefficients is given in Table 2.

The number of households is decreased from $1,250$ to 473 for the reasons listed below.

- Dropped out of the survey before January 2007.
- Missing data concerning household attributes or gas consumption.
- Use of liquefied petroleum gas. (Its price schedule is not publicly available.)
- Consumption within the zero marginal price block.

For these 473 households, we conducted an empirical study that is presented in the next subsection. The mean, standard deviation, minimum, and maximum of the dependent variable are $3.75 \log \mathrm{m}^3$, $0.78 \log \mathrm{m}^3$, $0.053 \log \mathrm{m}^3$, and $5.70 \log \mathrm{m}^3$, respectively. All households faced decreasing block rate pricing, and their price schedules differed depending on the cities in which they live. The price structures are shown in Figure 5, wherein the relative frequency of the number of blocks, the histogram of the unit price where the gas was actually consumed, and the histogram of the fixed gas service fee are illustrated.

Because the exact annual income level is sensitive information to request, our survey divides annual income levels into eight categories: (in million yen) 0-2, 2-4, 4-6, 6-8, 8-10, 10-12, 12-15, and over 15. Then, we ask the household its income category. The monthly



(a) Number of blocks (number). (b) Unit price (¥50/m^3). (c) Fixed gas service fee (¥50).

Figure 5: Relative frequency of the number of blocks and histograms of the unit price and the fixed gas service fee in January 2007.

Table 3: Summary statistics of explanatory variables for the heterogeneity

Variable	*Unit*	*Mean*	*SD*	*Min.*	*Max.*
z_{i2} (the number of members in a household)	person	2.81	1.28	1	9
z_{i3} (the number of rooms in a home)	room	4.09	1.10	1	8
z_{i4} (the total floor space of a home)	50m^2	1.54	.74	.20	8.00

income variable to be used for the empirical study is estimated using the median of the recorded income category divided by 12. For the last category (over 15 million yen), the approximate annual income is also recorded, and we use this figure divided by 12 as the monthly income. This income variable has a mean of 9.22, a standard deviation of 0.56, a minimum of 7.42, and a maximum of 10.82 in log ¥50.

The summary statistics for the explanatory variables for heterogeneity are given in Table 3. We calculated the correlation coefficients among the explanatory variables for heterogeneity: $Corr(z_{i2}, z_{i3}) = 0.49$, $Corr(z_{i2}, z_{i4}) = 0.38$, and $Corr(z_{i3}, z_{i4}) = 0.71$. Thus, we can establish that there is a high positive correlation between the number of rooms and the total floor space, such that either of these variables could not explain the residential gas demand.

4.2 Residential gas demand function

We assume the following prior distributions.

$$\begin{aligned}
\beta_1 \mid \sigma_u^2 &\sim TN_{[-2,0]}\left(0, 100\sigma_u^2\right), \quad \sigma_u^2 \sim IG(0.01, 0.01), \\
\beta_2 \mid \sigma_u^2 &\sim TN_{[0,2]}\left(0, 100\sigma_u^2\right), \quad \sigma_v^2 \sim IG(0.01, 0.01), \\
\boldsymbol{\delta} \mid \sigma_v^2 &\sim N_4\left(\mathbf{0}, 100\sigma_v^2 \boldsymbol{I}\right),
\end{aligned} \tag{27}$$

where $\boldsymbol{I}$ is the identity matrix. The truncation interval for β_j ($j = 1, 2$) is elicited as follows. Because residential gas is one of the necessities for households, its demand is relatively inelastic with respect to price and income. Thus, we can expect the absolute values of β_1 and β_2 to be less than one. Furthermore, we assume negative price elasticity according to microeconomic demand theory (see, e.g., Mas-Colell et al. (1995)), and positive income elasticity according to the estimate taken from the Family Income and Expenditure Survey (FIES) conducted in 2008. The FIES survey is intended to analyze the Japanese households and estimated the expenditure elasticity for gas to be 0.29 (for households with more than two members) and away from zero at a 5% significant level. Thus, we assume the interval $[-2, 0]$ ($[0, 2]$) for β_1 (β_2), where -1 (1) is included to examine whether β_1 (β_2) is less than -1 (more than 1). Further analysis of our empirical data set reveals that this prior truncation area for $\boldsymbol{\beta}$ is included in the area in which the separability condition is satisfied (see Figure 2).

With these prior distributions, the MCMC simulation (Appendix A.2) was carried out to obtain 6×10^6 samples after deleting the first 6×10^5 samples. We reduced the obtained 6×10^6 samples to 10^4 samples by picking up every 600-th sample. The results are given in Table 4 and shown in Figure 6.

Each column of the table represents the parameter names, the posterior means, the posterior standard deviations, the 95% credible intervals, and the estimated inefficiency factors. The inefficiency factor is defined as $1 + 2\sum_{j=1}^{\infty} \rho(j)$, where $\rho(j)$ is the sample autocorrelation

Table 4: Gas demand function

Parameter	*Mean*	*SD*	*95% interval*		*INEF*
β_1 (price)	−.84	.26	[−1.35	−.32]	136
β_2 (income)	.26	.060	[.14	.38]	218
δ_1 (constant)	.84	.62	[− .32	2.06]	259
δ_2 (number of members)	.17	.026	[.12	.22]	11
δ_3 (number of rooms)	.18	.037	[.11	.25]	5
δ_4 (total floor space)	.038	.052	[− .067	.14]	6
σ_u (measurement error)	.55	.13	[.12	.65]	19
σ_v (heterogeneity error)	.17	.15	[.049	.58]	30

* "SD" and "INEF" denote the posterior standard deviation and the inefficiency factor, respectively.

at lag j, and is estimated using the spectral density. It can be interpreted as the ratio of the variance of the sample mean obtained by the MCMC draws to the variance of the sample mean by an uncorrelated Monte Carlo draw (see, e.g., Chib (2001)).

4.2.1 Estimates of price and income elasticities

Price and income elasticities are highly credible to be negative and positive, respectively, in the sense that their 95% credible intervals do not include zero. Furthermore, income elasticity is highly credible to be less than one. The estimated inefficiency factors of elasticity parameters (as well as that of δ_1) are much higher than other parameters. This is partly because of the tight restrictions on $\boldsymbol{\beta}$ and partly because of the high correlation between β_2 and δ_1 ($Corr(\beta_2, \delta_1) = -0.82$). The other correlation coefficients are less than 0.7 in their absolute values except for that between σ_u and σ_v ($Corr(\sigma_u, \sigma_v) = -0.93$).

We compared these estimates with those of previous studies. One of the classical studies of residential gas demand is the study by Balestra and Nerlove (1966). They analyzed the new gas demand using a dynamic model with random effects. Their data are the state-level panel data for the United States during 1950 – 62. They estimated the (long-run) price and income elasticities to be −0.63 and 0.62, respectively, when the depreciation rate for gas appliances is unconstrained. While the estimated income elasticity calculated by these

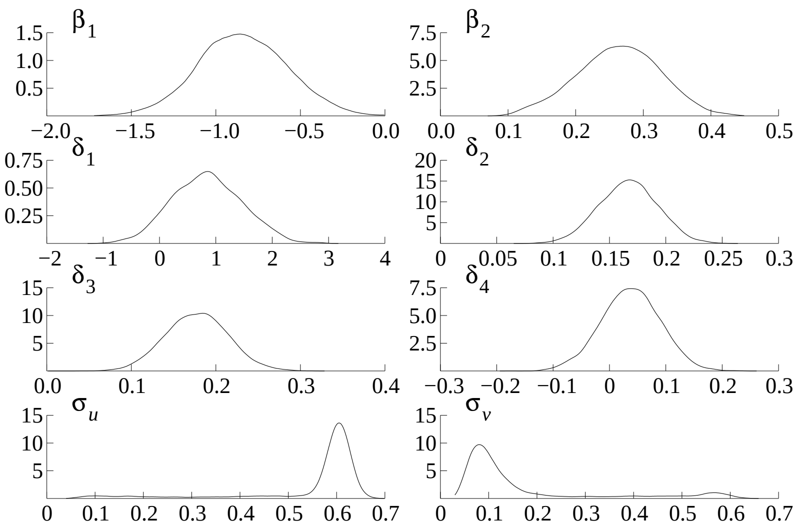


Figure 6: Marginal posterior densities.

researchers using aggregate data is larger than ours, the estimated price elasticity is similar to ours.

Bloch (1980) also investigated residential gas demand by using the household-level data. This includes gas usage data for households living in Twin Rivers, New Jersey, during the winter months (November through April) from 1971 to 1976. The explanatory variables that Bloch (1980) used are the number of heating degree days, the price of natural gas, and the consumer price index. He found that the (long-run) price elasticity is estimated to be −0.596 or −0.224 depending on the functional form of the demand function. The former estimate is similar to our results.

4.2.2 Other parameters

Among the explanatory variables for heterogeneity, the number of members in a household and the number of rooms in a home are highly credible to be positive in terms of their 95% credible intervals. These factors should have a positive relationship with gas demand through water demand for the two following reasons: (1) these two variables are also credible to be positive in the Japanese residential water demand function (see Table 4 of Miyawaki et al.

(2010)); and (2) in Japan, residential gas is mainly used for boiling water.

4.3 Policy evaluation—the effect of price schedule changes

In this subsection, we conduct a welfare analysis and evaluate the effect of price schedule changes. As the suppositional price schedules, we use the following three uniform price systems, which differ in their unit price: (unit price, fixed service fee) = (¥50/m^3, ¥725), (¥120/m^3, ¥725), and (¥250/m^3, ¥725). These unit prices are less expensive, as high as, or more expensive than the unit price that most households are actually facing. The fixed service fee is set close to the actual fee for most households.

Figure 7 shows the effect of price changes on households in terms of compensating variation. Each boxplot is the predictive distribution of the compensating variation in one thousand yen for each household. The number of households is reduced to 90 by selecting every 5-th household. Boxplots are sorted in ascending order based on the number of members in a household.

These results are consistent with what we expect based on microeconomic theory. We observe the positive (negative) compensating variation when the unit price decreases (increases). That is, the unit price decrease (increase) implies welfare improvement (decline). However, uniform pricing itself does not seem to have a noticeable influence on compensating variation (see the panel of ¥120/m^3). Furthermore, the degree of improvement (decline) is affected by explanatory variables for heterogeneity. The above panels show that the more members there are in a household, the more the compensating variation is likely to change. A similar pattern is also found with other explanatory variables for heterogeneity.

5 Concluding remarks

There are many previous studies that have used the discrete/continuous choice approach in the analysis of household behaviors under block rate pricing, transportation, housing, labor

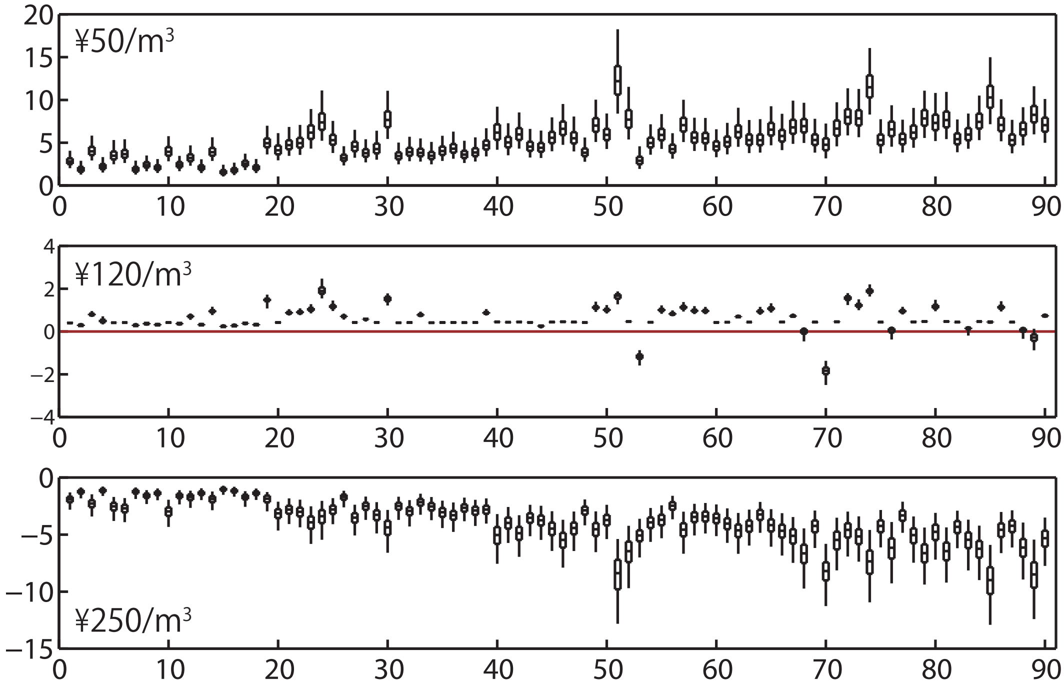


Figure 7: Boxplots of the predictive distribution of the compensating variation (¥10^3). Each box represents the range between the first and third quartiles. The upper and lower whiskers denote the 95-th and 5-th percentiles, respectively.

supply, etc. It should be noted that the indirect utility function becomes highly nonlinear, when the budget set is nonconvex, such as in the case of decreasing block rate pricing. However, previous studies (Burtless and Hausman (1978); Hausman (1980); Burtless and Moffitt (1985)) on decreasing block rate pricing do not address this problem. Blomquist and Newey (2002) proposed a nonparametric approach to address this problem, but their approach lacks the microeconomic theoretical background. This article proposes a new Bayesian estimation method for residential gas demand on the nonconvex budget set by extending the Bayesian approach taken by Miyawaki et al. (2010), which proposed a Bayesian estimation method to analyze consumer demand under increasing block rate pricing. The advantage of our method is not only that it addresses the nonlinearity problem associated with the nonconvex budget sets but also that it incorporates the (highly nonlinear) separability condition that is necessary for the demand model under multiple-block block decreasing block rate pricing and introduces normal disturbance into the multinomial choice model.

Finally, our method has the potential to estimate the multiple residential energy expenditure function. Previous studies have focused on the cross-elasticity of electricity and gas demand (see Beierlein, Dunn, and James C. McConnon (1981); Baker, Blundell, and Micklewright (1989); Lee and Singh (1994); Maddala, Trost, Li, and Joutz (1997); Vaage (2000)). However, they do not take into consideration the price structure of electricity and gas services. Japanese electricity services are provided under increasing block rate pricing where the unit price increases as the volume consumed increases. Thus, by combining the proposed method and the method of Miyawaki et al. (2010) to estimate the demand function under increasing block rate pricing, we could also construct a multivariate demand function under both increasing and decreasing block rate pricing in a natural manner and estimate the residential energy demand function using the Bayesian approach. We will leave this for our future work.

A Appendices

A.1 Heterogeneity interval

We derive the explicit bounds of the heterogeneity interval, which is given by

$$R_{ik} = \left\{ w_i^* \mid V_{ik} > V_{ij} \text{ for } j \neq k \right\} = \bigcap_{j \neq k} \left\{ w_i^* \mid V_{ik} > V_{ij} \right\}. \tag{28}$$

Let $D(x_1, x_0; \theta) = \theta^{-1}(x_1^\theta - x_0^\theta)$ ($x_0 > 0$, $x_1 > 0$, $\theta \neq 0$). Then, $D(x_1, x_0; \theta) \gtrless 0$ if $x_1 \gtrless x_0$.[9]

With this function, we solve $V_{ik} > V_{ij}$ for w_i^*.

$$V_{ik} > V_{ij} \Longleftrightarrow -\exp\left(w_i^*\right) D\left(P_{ik}, P_{ij}; 1 + \beta_1\right) > -D\left(Q_{ik}, Q_{ij}; 1 - \beta_2\right) \tag{29}$$

[9]Suppose $x_1 > x_0 > 0$. Then, because x_l^θ ($l = 0, 1$) is decreasing (increasing) with respect to x_l if $\theta < (>)0$, the numerator $x_1^\theta - x_0^\theta \lesseqgtr 0$ if $\theta \lesseqgtr 0$. Therefore, $D(x_1, x_0; \theta) > 0$ if $x_1 > x_0 > 0$. Similarly, $D(x_1, x_0; \theta) < 0$ if $x_0 > x_1 > 0$.

$$\iff \begin{cases} w_i^* < \ln E_{kj}, & \text{if } k < j, \\ w_i^* > \ln E_{kj}, & \text{if } k > j, \end{cases} \tag{30}$$

where $E_{kj} = D(Q_{ik}, Q_{ij}; 1-\beta_2)/D(P_{ik}, P_{ij}; 1+\beta_1)$. The last equivalence makes use of the property of decreasing block rate pricing: $P_{ik} \gtrless P_{ij}$ and $Q_{ik} \gtrless Q_{ij}$ if $k \lessgtr j$. Both $P_{ik} > 0$ and $Q_{ik} > 0$ for all k because we assume the log-linear function (10). Thus, $D(P_{ik}, P_{ij}; 1+\beta_1) \gtrless 0$ and $D(Q_{ik}, Q_{ij}; 1-\beta_2) \gtrless 0$ if $k \lessgtr j$.

Finally, we have

$$\begin{aligned} R_{i1} &= \left(-\infty, \min_{1<j} \ln E_{1j}\right), \\ R_{ik} &= \left(\max_{k>j} \ln E_{kj}, \min_{k<j} \ln E_{kj}\right), \quad k = 2, \ldots, K_i - 1, \\ R_{iK_i} &= \left(\max_{K_i>j} \ln E_{K_i j}, \infty\right). \end{aligned} \tag{31}$$

We note that $R_{ik} \cap R_{ij} = \emptyset$ $(k \neq j)$.

A.2 Gibbs sampler

The Gibbs sampler is implemented in seven steps.

Step 1. Set initial values to $(\boldsymbol{\beta}, \boldsymbol{\delta}, \boldsymbol{s}^*, \boldsymbol{w}^*, \sigma_u^2, \sigma_v^2)$.

Step 2. Generate β_1 given $\beta_2, \boldsymbol{s}^*, \boldsymbol{w}^*, \sigma_u^2$.

See Subsection 3.2.

Step 3. Generate β_2 given $\beta_1, \boldsymbol{s}^*, \boldsymbol{w}^*, \sigma_u^2$.

The full conditional distribution of β_2 is the truncated normal distribution, $TN_{C_2}(\mu_{\beta_2,2}, \sigma_u^2 \sigma_{\beta_2,2}^2)$, where

$$\sigma_{\beta_2,1}^{-2} = \sigma_{\beta_2,0}^{-2} + \sum_{i=1}^{n} \left(q_{ir_i^*}\right)^2, \tag{32}$$

$$\mu_{\beta_2,1} = \sigma_{\beta_2,1}^2 \left[\sigma_{\beta_2,0}^{-2} \mu_{\beta_2,0} + \sum_{i=1}^{n} q_{is_i^*}\left(y_i - \beta_1 p_{is_i^*} - w_i^*\right)\right], \tag{33}$$

$$C_2 = \left[\bigcap_{i=1}^{n} \bigcap_{j\neq s_i^*, j=1}^{K_i} \left\{\beta_2 \mid V_{ik} > V_{ij}\right\}\right] \cap \left[\bigcap_{i=1}^{n} \bigcap_{k=2}^{K_i-1} \{\beta_2 \mid RL_{ik} \leq RU_{ik}\}\right] \cap [l_2, m_2]. \qquad (34)$$

The rejection sampling with an efficient blanket is applied to obtain samples of β_2. The efficient blanket $\tilde{C}_2$ will be derived in the next appendix. The acceptance probability is given by

$$\alpha\left(\beta_2, \beta_2'\right) = \min\left[1, \frac{\phi\left\{\left(\beta_2' - \mu_{\beta_2,1}\right)\sigma_u^{-1}\sigma_{\beta_2,1}^{-1}\right\}}{\phi\left\{\left(\beta_2 - \mu_{\beta_2,1}\right)\sigma_u^{-1}\sigma_{\beta_2,1}^{-1}\right\}}\right]. \qquad (35)$$

Step 4. Generate $(\sigma_v^2, \boldsymbol{\delta})$ given $\boldsymbol{w}^*$.

By integrating the joint density function of $(\sigma_v^2, \boldsymbol{\delta})$ given $\boldsymbol{w}^*$ over $\boldsymbol{\delta}$, we have the full conditional distribution of σ_v^2 as the inverse gamma distribution, $IG(n_{v,1}/2, S_{v,1}/2)$, where $n_{v,1} = n_{v,0} + n$ and

$$S_{v,1} = S_{v,0} + \boldsymbol{\mu}_{\boldsymbol{\delta},0}' \boldsymbol{\Sigma}_{\boldsymbol{\delta},0}^{-1} \boldsymbol{\mu}_{\boldsymbol{\delta},0} + \boldsymbol{w}^{*\prime} \boldsymbol{w}^* - \boldsymbol{\mu}_{\boldsymbol{\delta},1}' \boldsymbol{\Sigma}_{\boldsymbol{\delta},1}^{-1} \boldsymbol{\mu}_{\boldsymbol{\delta},1}. \qquad (36)$$

Then, given σ_v^2, the full conditional distribution of $\boldsymbol{\delta}$ is the multivariate normal distribution, $N_d(\boldsymbol{\mu}_{\boldsymbol{\delta},1}, \sigma_v^2 \boldsymbol{\Sigma}_{\boldsymbol{\delta},1})$, where

$$\boldsymbol{\mu}_{\boldsymbol{\delta},1} = \boldsymbol{\Sigma}_{\boldsymbol{\delta},1}\left(\boldsymbol{\Sigma}_{\boldsymbol{\delta},0}^{-1} \boldsymbol{\mu}_{\boldsymbol{\delta},0} + \boldsymbol{Z}' \boldsymbol{w}^*\right), \quad \boldsymbol{\Sigma}_{\boldsymbol{\delta},1}^{-1} = \boldsymbol{\Sigma}_{\boldsymbol{\delta},0}^{-1} + \boldsymbol{Z}'\boldsymbol{Z}. \qquad (37)$$

Step 5. Generate $\{s_i^*, w_i^*\}_{i=1}^n$ given $\boldsymbol{\beta}, \boldsymbol{\delta}, \sigma_u^2, \sigma_v^2$.

The blocking technique is applied to draw samples of (s_i^*, w_i^*). The full conditional distribution of s_i^* is the multinomial distribution, the probability mass function of which is given by

$$\pi\left(s_i^* = s \mid \boldsymbol{\beta}, \boldsymbol{\delta}, \sigma_u^2, \sigma_v^2\right) \propto \left[\Phi\left\{\tau^{-1}\left(RU_{is} - \theta_{is}\right)\right\} - \Phi\left\{\tau^{-1}\left(RL_{is} - \theta_{is}\right)\right\}\right] \exp\left(-\frac{m_{is}}{2}\right), \qquad (38)$$

for $s = 1, \ldots, K_i$, where $\tau^2 = (\sigma_u^{-2} + \sigma_v^{-2})^{-1}$ and

$$(m_{is}, \theta_{is}) = \left(\frac{(\sigma_u \sigma_v)^{-2} \left(y_i - y_{is} - \mathbf{z}_i' \boldsymbol{\delta} \right)^2}{\sigma_u^{-2} + \sigma_v^{-2}}, \frac{\sigma_u^{-2} (y_i - y_{is}) + \sigma_v^{-2} \mathbf{z}_i' \boldsymbol{\delta}}{\sigma_u^{-2} + \sigma_v^{-2}} \right). \tag{39}$$

Given $s_i^* = s$, the full conditional distribution of w_i^* is the truncated normal distribution, $TN_{R_{is}}(\theta_{is}, \tau^2)$.

Step 6. Generate σ_u^2 given $\boldsymbol{\beta}, \boldsymbol{s}^*, \boldsymbol{w}^*$.

The full conditional distribution of σ_u^2 is the inverse gamma distribution, $IG(n_{u,1}/2, S_{u,1}/2)$, where $n_{u,1} = n_{u,0} + n + 2$ and

$$S_{u,1} = S_{u,0} + \left(\boldsymbol{\beta} - \boldsymbol{\mu}_{\boldsymbol{\beta},0} \right)' \boldsymbol{\Sigma}_{\boldsymbol{\beta},0}^{-1} \left(\boldsymbol{\beta} - \boldsymbol{\mu}_{\boldsymbol{\beta},0} \right) + (\boldsymbol{y} - \boldsymbol{y}^*)' (\boldsymbol{y} - \boldsymbol{y}^*). \tag{40}$$

Step 7. Go to Step 2.

A.3 Efficient blanket of C_2

We assume that the support of the prior distribution for β_2 is $B_2 = [0, m_2]$. Let

$$C_{kj}^{2i} = \left\{ \beta_2 \mid V_{ik} > V_{ij} \right\} \cap [0, m_2] \text{ and } a_2 = \exp(w_i^*)(1 + \beta_1)^{-1} \left(P_{ik}^{1+\beta_1} - P_{ij}^{1+\beta_1} \right). \tag{41}$$

Then, the Hermite-Hadamard integral inequality and $\beta_2 \in [0, m_2]$ derive

$$a_2 < \begin{cases} \left(Q_{ik} - Q_{ij} \right) \frac{Q_{ik}^{-\beta_2} + Q_{ij}^{-\beta_2}}{2}, & \text{if } k < j, \\ \left(Q_{ik} - Q_{ij} \right) \left(\frac{Q_{ik} + Q_{ij}}{2} \right)^{-\beta_2}, & \text{if } k > j. \end{cases} \tag{42}$$

By applying the power-mean inequality, we have $\tilde{C}_{kj}^{2i} = \tilde{C}_{kj}^{\star 2i} \cap [0, m_2]$ $(\supset C_{kj}^{2i})$, where

$$\tilde{C}_{kj}^{\star 2i} = \begin{cases} (-\infty, -b_2/\bar{q}(-m_2)), & \text{if } k < j \text{ and } \bar{q}(-m_2) > 0, \\ (-\infty, \infty), & \text{if } k < j \text{ and } \bar{q}(-m_2) = 0, \\ (-b_2/\bar{q}(-m_2), \infty), & \text{if } k < j \text{ and } \bar{q}(-m_2) < 0, \\ (-b_2/\bar{q}(1), \infty), & \text{if } k > j \text{ and } \bar{q}(1) > 0, \\ (-\infty, \infty), & \text{if } k > j \text{ and } \bar{q}(1) = 0, \\ (-\infty, -b_2/\bar{q}(1)), & \text{if } k > j \text{ and } \bar{q}(1) < 0, \end{cases} \tag{43}$$

$b_2 = \log(a_2/(Q_{ik} - Q_{ij}))$, and $\bar{q}(x) = x^{-1}\log\{(Q_{ik}^x + Q_{ij}^x)/2\}$ $(x = 1, -m_2)$. If $Q_{iK_i} > 1$ is assumed, we have $\bar{q}(1) > \bar{q}(-m_2) > 0$, which simplifies the above expression. With this closely approximated interval $\tilde{C}_{kj}^{2i}$, we have $\tilde{C}_2 = \cap_{i=1}^{n} \cap_{j=1, j\neq s_i^*}^{K_i} \tilde{C}_{s_i^* j}^{2i}$, which includes C_2.

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