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Buying Ecological Services: Fragmented Reserves, Core and Periphery National Park Structure, and the Agricultural Extensification Debate

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ABSTRACT. Growing demand for cropland products has placed intense pressure on the ability of land resources to support nature, straining public budgets to purchase environmental goods. Fixing overall agricultural output, two environmental policy options are whether to a) promote more agricultural extensification and nature friendly farming practices or b) produce intensively on some land and leave the rest wild. Microeconomic models of the topic have not accounted for widely recognized spatial externalities regarding fragmented reserves. This article does so, using Wirtinger's inequality to also identify a third policy possibility. This is that ecological services can follow a smoothly varying spatial path chararacterized by harmonic functions. We use the results to rationalize the core and periphery National Park structure put in place around the world, i.e., versions of our third policy possibility have been implemented.

KEY WORDS. Environmental policy; Land use; National Park management; Spatial externalities; Wirtinger's inequality.

JEL CLASSIFICATION. H40, Q28, D62.

1. Introduction. Rising real incomes, an expanding global population and biofuel policies have increased demand for land under crops over the first decade of the 21st century (Mitchell [2008]). With a marked expansion of crop land and higher land rental rates, governments are posed with the problem of how best to provide a wide variety of environmental services when the price of obtaining these has increased.[1] A widely noted feature of these services is that spatial contiguity matters, and so the spatial arrangement of targeted land matters.

Services at issue include carbon sequestration where output is a global public good and spatial externalities are clearly secondary. They also include green space for outdoor amenities, flora and fauna habitat, as well as riparian buffer zones. For riparian buffer zones to encourage nature, reduce erosion, and filter chemical runoff, spatial effects likely involve local substitution. This is because service provision by neighboring lands may be almost as effective. Substitution effects are also likely when woodland is intended to control rainwater flow and so prevent flooding. In other cases local complementarity is likely, as with contiguity in scenery or with existence value for an unspoiled ecosystem. Spatial fragmentation is especially harmful for larger plant and animal species, where a reduction in their presence can greatly alter the presence, extent, and behavior of other organisms, including the risk of species invasion, see for example Terborgh *et al.* [2001] or Damschen *et al.* [2006].

In the presence of such spatial spillovers, there has been much debate on how to construct public policy to most efficiently provide ecological public goods. One way of doing so is to buy down farmer profit opportunities through requiring investments that increase environmental outputs or through purchasing restrictions on agricultural practices. In a widely cited paper, Green *et al.* [2005] developed on a suggestion in Waggoner [1995] that it may be optimal to load the buy downs on one set of acres, concentrating agricultural production on the remaining acres. This is the intensification option.

[1] For example, Iowa row cropland cash rent increased by about 35% between 2006 and 2009

Or it may be optimal to spread the buy downs across all acres so that all acres provide roughly similar bundles of agricultural production and environmental services. This is the extensification option. Which is optimal, they suggest, depends on the relationship between agricultural losses and environmental services. If the marginal reduction in environmental services due to an increase in agricultural outputs declines with an increase in these outputs (is convex) then intensification is the better policy. If the relation is concave then extensification is the better policy. Their model does not account for spatial spillovers. The intensification policy suggestion is controversial, see Jordan *et al.* [2007], DeFries *et al.* [2007], or Ceotto [2008]. See Tichit *et al.* [2007], Scherr and McNeeley [2008], or Fischer *et al.* [2008] for elaboration on how ecosystem and socioeconomic context matters in this debate.

In practice, both intensification and extensification policies have been enacted in the past. The Conservation Reserve Program, which buys cropland out of production for periods of a decade or more, has been the primary policy instrument in the United States with 2009 budget of about $2 billion. Forestry schemes in the European Union have sought to convert farmland into deciduous and native woodland. Extensification-type agro-environmental policies have been more common in the European Union. Regulation 1257/1997 schemes support such activities as hedgerow restoration, maximum stocking rates, input use reductions, use of native species, production of organic crops, as well as rotation, green manure and fallowing practices where Donald and Evans [2006] provide a review. These schemes cover about 20 percent of EU farmland at an annual cost of about €3.5 billion (Whitfield [2006]).

The effectiveness of agro-environmental schemes has been brought into question by Kleijn *et al.* [2001] and others, especially with respect to a lack of regard for scientific evaluation (Whitfield [2006]). Many schemes may have had little or even ecologically adverse effects, a belief that may have led to cuts in funding for such EU programs. Regarding evaluation, Wätzold

(Edwards, Smith and Johanns [2009]).

and Schwerdtner [2005] point to a dearth of expertise at the interface of ecology and economics among those designing and implementing schemes.

Even at the level of academic evaluation, we note in particular that spatial interactions seldom enter assessment of such schemes despite widespread agreement that land contiguity and fragmentation are salient in any ecosystem. Recognizing the very decentralized organization of conservation endeavors and with an eye toward strategic public policy, Albers, Ando, and Chen [2008] consider spatial interactions between public land in conservation and private land trusts. In Polasky *et al.* [2008] and papers cited therein, models with land contiguity effects have been developed to answer questions about how to arrange conservation and economic activities. The methods are, however, intended to provide practical assistance to the land manager and to draw lessons from case study applications. The methods are not as well suited to addressing analytic questions on land management.

A literature exists on providing incentives for the positive externalities generated by land use contiguities (Smith and Shogren [2002], Parkhurst *et al.* [2002], Lewis and Plantinga [2007], Tanaka [2008], Drechsler and Wätzold [2009]). The idea is to give an additional payment (agglomeration bonus) to those who sign their land up for an environmental program if a neighbor also does so or is in the process of doing so. In this way, the positive externality is at least partly endogenized.[2] As is typical of situations where positive externalities exist, many Nash equilibria are possible. Unlike much of the literature, the agglomeration bonus approach is concerned with feasible solutions rather than first-best solutions. Effective implementation would require adequate prior communication in order to ensure profitable coordination by landowners making the signup decision.

The intent of this article is to go some way toward including spatial effects in economic

[2] Contexts exist where the potential may be large. These include the fragmented Atlantic forest region of South coastal Bahai, Brazil, where low profit farmed land intersperses forest (Chomitz *et al.* [2006]). A further example is agricultural practice modification to support adequate

analysis of the land use extensification debate. We provide a simple model of a circular ecosystem that allows for spatially complementary spillovers. The cycle topology has a long pedigree in economic analysis. Salop [1979], Vogel [2008], Hennessy and Lapan [2009] and others have studied pricing, and sometimes location, strategic decisions among firms discretely located on a circle and selling to consumers continuously located on the circle. Krugman, [1993], Picard and Tabuchi [2009] and others have considered agglomeration motives for economic activity on a circle. Choudhary and Orszag [2008], and also Corazzini and Gianazza [2008], have found the approach useful when seeking to understand local externalities.

Under general conditions we show that omission of spatially complementary spillovers may tilt policy toward extensification when a more intensive policy approach might be optimal. Spatial complementarities penalize spatial disruptions in land use. As such they resemble a concavity, and might be seen as promoting extensification. Spatial complementarities also encourage the spatial concentration of land use to exploit complementarities, and so might be seen as a convexity that promotes intensification. As we shall show in Property 2, spatial externalities should be seen as neither a concavity nor a convexity. This leads us to our main theoretical point. This is that accommodating spatial spillovers in environmental services can admit a third alternative, not mentioned in the debate to this point. For identical farms located around a disc, this is that the first-best environmental buy down policy is to manage these spatial effects by smoothly varying the provision of environmental services as one moves across space.

Notice that while agglomeration bonuses do seek to account for spatial externalities in incentives, that literature has fixed the nature of program possibilities between enrolling or not enrolling in a single program. The third alternative we propose allows the program to vary with spatial location. We believe our approach is unique in suggesting that spatial externalities should motivate a spatially conditioned first-best program design. We show that optimum program buy

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meadowland for the butterfly *Maculinea teleius* in Germany (Drechsler *et al.* [2007]).

down levels would follow a linear combination of a sine and cosine function around the circle. A land strip topology is also considered, leading to a markedly different optimal landscape design.

A practical interpretation of our findings is to approximate the optimal program by zones that moderate spatial variation in land use. In the United States, for example, many of the larger National Parks are surrounded by National Forests and other land use designations that restrict the nature and extent of human activities. Similar models are in place for National Parks throughout the world. In our penultimate section, we point to the structure of land use restrictions on National Parks to argue that our third alternative has in fact been implemented. We conclude with a brief discussion.

2. Model. There are N profit maximizing farms of equal size located on a disc, called the region, with unit radius. Apart from location, these farms are identical. Land quality, resource availabilities, and prices are the same, while all farms have cheese wedge shape with angle $360/N$ degrees and area π/N. For agricultural output q, each farm has cost function

$$C(q) = \begin{cases} cq, & q \in [0, \hat{q}]; \\ \infty, & q > \hat{q}. \end{cases} \tag{1}$$

Here, $c > 0$ and $\hat{q} > 0$ while the market price for output is $P > c$ so that all farms produce at $\hat{q}$ absent a policy intervention. The government seeks to buy down production in order to provide environmental services. It is prepared to pay for a production buy down of R output in total, or an average of $\bar{r} = R/N \in (0, \hat{q})$ per farm. Given (1), the cost of this buy down is $(P-c)R$ and does not depend on which farms participate. Producers are indifferent concerning the size of buy down it chooses at auction or is allocated.

For $n \in \{0, 1, \ldots, N-1\} \equiv \Omega$ as the set of farms enumerated clockwise around the disc, the government seeks understanding on how best to allocate R over these farms where each farm is allocated $r_n \in [0, \hat{q}]$. Environmental benefits are given by $B[\bar{r}, v(r), \rho(r)] : \bar{\mathbb{R}}_+^2 \times \mathbb{R} \to \bar{\mathbb{R}}_+$ where

$v(r) = N^{-1}\sum_{n\in\Omega}(r_n - \overline{r})^2$ is the variance of the coordinates drawn from simplex $S \equiv \{r : r \in [0,\widehat{q}]^N, \sum_{n\in\Omega} r_n = R\}$ with $[0,\widehat{q}]^N \equiv [0,\widehat{q}] \times ... \times [0,\widehat{q}]$. Variance is assumed to have domain $[0, v^{\max}]$, $v^{\max} > 0$, where v is maximized on S whenever all but one acre is either producing $\widehat{q}$ or 0. Benefits depend on $\overline{r}$ with $\partial B(\cdot)/\partial\overline{r} = B_{\overline{r}}(\cdot) > 0$. They also depend on v with $\partial B(\cdot)/\partial v = B_v(\cdot)$ of unassigned sign.

All other arguments fixed, including the as yet unexplained index ρ, if $B_v(\cdot) < 0$ on its domain then benefits would be maximized when $v = 0$ and $r_n = \overline{r}\ \forall n \in \Omega$. This would occur under the extensification policy in Green *et al.* [2005].[3] This variance-is-bad situation could arise if there were low-hanging fruit to be had from low intrusion agro-environmental schemes, perhaps requiring slightly wider hedgerows or the use of conservation tillage in the presence of high cultivation costs. All other arguments fixed, if $B_v(\cdot) > 0$ on its domain then benefits would be maximized under $\widehat{q}$-or-0, i.e., all-or-nothing, allocations. This is the other policy arrived at in Green *et al.* [2005], intensification. This situation could reflect threshold effects whereby a key top predator, the wolf, tiger or lynx, will not be tolerated in a neighborhood if even minimal livestock farming occurs in an area.

The third statistic entering the benefits function is spatial index $\rho(r)$. This index accounts for local spatial spillovers in environmental policy and may be viewed as an index of cohesion, i.e., an inverse index of fragmentation. It is obtained by viewing the production reduction r_n on the n^{th} farm as providing environmental benefit $A(r_{[n-1]}, r_n, r_{[n+1]}) = 0.5(r_{[n-1]} - \overline{r})(r_n - \overline{r}) + 0.5(r_n - \overline{r})(r_{[n+1]} - \overline{r})$, $n \in \Omega$, where

[3] Lewis, Plantinga and Wu [2009] model local non-convexities due to edge effects and show that these non-convexities can motivate programs that incentivize land clusters dedicated to wildlife.

(2)

$$r_{[n+1]} = \begin{cases} r_{n+1}, & n \in \Omega, \quad n \neq N-1; \\ r_0, & n = N-1; \end{cases}$$

$$r_{[n-1]} = \begin{cases} r_{n-1}, & n \in \Omega, \quad n \neq 0; \\ r_{N-1}, & n = 0. \end{cases}$$

The conditions in (2) are just the modular arithmetic maps required on the disc to ensure that the *0*th and $N-1$ th farms are neighbors. The derivative sign of $\partial^2 A(\cdot)/\partial r_{[n-1]}\partial r_n = \partial^2 A(\cdot)/\partial r_n \partial r_{[n+1]} = 0.5 > 0$ is intended to capture farm-level contiguity effects in the provision of environmental services. There is local complementarity so that marginal environmental benefits on a given farm increase with an increase in production buy down on neighboring farms.

Index ρ is obtained from averaging $A(r_{[n-1]}, r_n, r_{[n+1]})$ over the set:

(3) $$\rho(r) = N^{-1}\sum_{n\in\Omega} A(r_{[n-1]}, r_n, r_{[n+1]}) = N^{-1}\sum_{n\in\Omega}(r_{[n-1]} - \overline{r})(r_n - \overline{r}).$$

Note first that $\rho(r)$ is the lag 1 spatial covariance. The index can have a positive or negative value, and has upper bound $v(r)$. We adopt the index for two reasons. First-order autocovariance is a widely used modeling approach to characterizing ‘nearness’ effects so that technical machinery is well developed to work with it, and indeed with generalizations to higher order autocovariance structures (Hoel, Port and Stone [1972]). A feature of the statistic that many would find agreeable is the way that first-order spatial covariance allows for gradual decay in effect as sites become more removed in spatial distance. The second motive for adopting the index is that the econometrics of spatial autocorrelation are well-developed (Anselin [1988]) so that the index lends itself to empirical analysis.

To observe how the index captures cohesion, let $N = 6$ and $R = 3$ while assuming that the r_n must take integer values 0 or 1. Then there are only three distinct arrangements of the reductions. These are I) $\{0,0,0,1,1,1\}$ with $\rho(r) = 1/3$, II) $\{0,0,1,0,1,1\}$ with $\rho(r) = -1/3$, and III) $\{0,1,0,1,0,1\}$ with $\rho(r) = -1$. We hold that $B_\rho(\cdot) > 0$ so as to capture concerns about the loss in environmental services arising from fragmentation.

We seek to understand the nature of

(4) $$\arg\max_{r\in S} B\left[\overline{r}, v(r), \rho(r)\right].$$

Letting $1_N = (1,1, \dots ,1)^t$ be the vector of 1s in $\mathbb{R}^N$, with superscript t as the transpose operation, policy options will be characterized as follows:

DEFINITION 1: *Extensification is said to be optimal whenever* $\arg\max_{r\in S} B(\cdot) = \overline{r}1_N$. *Partial intensification is said to be optimal whenever* $\arg\max_{r\in S} B(\cdot)$ *has one or more ordinates with values 0 or* $\widehat{q}$.

Intuitively, our interest in partial intensification arises from the fact that if $B_\rho(\cdot) > 0$ then a landscape arrangement involving $r_n = 0$ and $r_{[n+1]} = \widehat{q}$ or involving $r_n = \widehat{q}$ and $r_{[n+1]} = 0$ will suffer a benefits penalty. If we do not restrict the size of $B_\rho(\cdot)$, and we do not, the penalty may be large. We will develop on this point in some detail.

3. Index properties. Note first that $\rho(r)$ is possessed of strong symmetry properties. In particular for $r = (r_0, r_1, r_2, \dots , r_{N-2}, r_{N-1})^t$, write $L^1(r) = (r_1, r_2, r_3, \dots , r_{N-1}, r_0)^t$ and in general write $L^i(r) = (r_{[i]}, r_{[i+1]}, r_{[i+2]}, \dots , r_{[i-2]}, r_{[i-1]})^t$ where modular arithmetic has been applied to the subscripts. We say function $f(x)$ is invariant under rotation if $f(x) \equiv f[L^i(x)]$ for all integers i. Also, write $M(r) = (r_0, r_{N-1}, r_{N-2}, \dots , r_2, r_1)^t$ as the vector reflection operation and we assert that a function $f(x)$ is invariant under reflection if $f(x) \equiv f[M(x)]$. Finally with $\circ$ as the composition operation, function $f(x)$ is said to be invariant under reflection and rotation composition if $f(x) \equiv f[M \circ L^i(x)] \equiv f[L^i \circ M(x)]$ for all integers i.

PROPERTY 1: *Index* $\rho(r)$ *is invariant under* A) *rotation,* B) *reflection, and consequently* C) *reflection and rotation composition.*

As will also be the case for other results that are not immediate, demonstration of Property 1 is provided in the Appendix. We turn next to one possibility for gleaning inferences from these symmetries. Uniform curvature in conjunction with symmetry provides the potential for exploitable structure on level sets.[4] We will see next that any such opportunities will be qualified. Although qualified, we will show later that such opportunities do exist. The Hessian for the index is given by

$$\begin{pmatrix} 0 & 1 & 0 & \cdots & 0 & 1 \\ 1 & 0 & 1 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ 1 & 0 & 0 & \cdots & 1 & 0 \end{pmatrix}. \tag{5}$$

This is a circulant matrix in that the second row is obtained from applying a rotation operation on the first row, the third row is obtained from applying the same operation on the second row and so on. The eigenvalues of circulant matrices are highly structured, allowing us to obtain:

PROPERTY 2: *The index is neither concave nor convex. The eigenvalues of its Hessian matrix are* $\lambda_n = 2\cos[2\pi(n-1)/N] \in [-2,2]$, $n \in \Omega$. *They are negative whenever* $N+4 < 4n < 3N+4$, *zero whenever* $n = (N+4)/4$ *or* $n = (3N+4)/4$, *and positive otherwise.*

For $N = 6$, then $2\cos[2\pi(n-1)/N] \in \{-2,-1,1,2\}$ where some roots are repeated. For $N = 9$, then $2\cos[2\pi(n-1)/N] \in \{-1.879,-1,0.347,1.532,2\}$. While 2 is always an eigenvalue, -2 is an eigenvalue only when N is even. All other eigenvalues arise twice whenever they arise at all.

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[4] For example, when $f(x): \mathbb{R} \to \mathbb{R}$ is convex then $\sum_{n\in\Omega} f(x_n)$ is permutation symmetric such that any transfer $\varepsilon > 0$ to increase x_i by ε and decrease x_j by ε increases the value of the sum whenever $x_j \le x_i$.

The eigenvalue symmetries together with Property 1 suggest that symmetric structure will be important to the analysis. Property 2 also suggests that trigonometric, or harmonic, analysis should prove useful in understanding how best to allocate buy downs. Throughout the analysis we will give content to these speculations.

4. Optimization problem. For our first result we fix the values in vector r, but allow them to be arranged at will across the farms. Let $T(\hat{r})$ be the set of all $N!$ rearrangements of vector $r = \hat{r} \in \mathbb{R}^N$ where, for convenience in presentation only, we assume the r_n are distinct.

PROPOSITION 1: *Let $r_{(n-1)}$ be the n^th^ least value of r. When N is*

A) *even then the arrangement $r \in T(\hat{r})$ maximizing $\rho(r)$ is some rotation $L^i(r^{*,ev})$ of*

(6) $$r^{*,ev} = \begin{pmatrix} \hat{r}_{(0)} & \hat{r}_{(1)} & \hat{r}_{(3)} & \cdots & \hat{r}_{(N-3)} & \hat{r}_{(N-1)} & \hat{r}_{(N-2)} & \cdots & \hat{r}_{(4)} & \hat{r}_{(2)} \end{pmatrix}^t,$$

i an integer, or some rotation of reflection $M(r^{*,ev})$ where the largest value $\hat{r}_{(N-1)}$ in $r^{*,ev}$ is at farm $n = N/2$ in Ω.

B) *odd, then the maximizing arrangement is some rotation $L^i(r^{*,od})$ of*

(7) $$r^{*,od} = \begin{pmatrix} \hat{r}_{(0)} & \hat{r}_{(1)} & \hat{r}_{(3)} & \cdots & \hat{r}_{(N-2)} & \hat{r}_{(N-1)} & \hat{r}_{(N-3)} & \cdots & \hat{r}_{(4)} & \hat{r}_{(2)} \end{pmatrix}^t,$$

or some rotation of the reflection $M(r^{*,od})$ where the largest value $\hat{r}_{(N-1)}$ in $r^{*,od}$ is at farm n $= (N+1)/2$ in Ω.

COROLLARY 1.1: *Solutions to $\arg\max_{r \in S} B(\cdot)$ have the spatial arrangement laid out in Proposition 1.*

This corollary tells us that even if the policy maker is restricted to rearrangements of some $r = \hat{r} \in S$ then the optimal solution will be highly symmetric. The optimal vector must be at least weakly decreasing over half the disc and weakly increasing over the other half. Notice too that the solutions are entirely ordinal, rank order is all that matters.

We turn now to providing further rationalization of the cohesion index. In the next result, we do not confine attention to the set $r \in T(\hat{r})$, but rather let the values be arbitrary on simplex S.

PROPOSITION 2: *Consider a transfer* $(\tilde{r}_i, \tilde{r}_j) = (r_i + \varepsilon, r_j - \varepsilon)$, $\varepsilon > 0$ *but infinitesimally small, while* $\tilde{r}_n = r_n$ *for all* $n \in \Omega$ *other than i and j. Then* $\rho(\tilde{r}) \geq \rho(r)$ *iff* $r_{[i-1]} + r_{[i+1]} \geq r_{[j-1]} + r_{[j+1]}$ *while* $v(\tilde{r}) \geq v(r)$ *iff* $r_i \geq r_j$.

COROLLARY 2.1: *Environmental services increase under the transfer considered in Proposition 2*

A) *whenever i)* $B_v(\cdot) \geq 0$ *and ii) both of* $r_{[i-1]} + r_{[i+1]} \geq r_{[j-1]} + r_{[j+1]}$ *and* $r_i \geq r_j$ *apply, while they decrease whenever both inequalities in ii) are reversed.*

B) *whenever i)* $B_v(\cdot) \leq 0$ *and ii) both of* $r_{[i-1]} + r_{[i+1]} \geq r_{[j-1]} + r_{[j+1]}$ *and* $r_i \leq r_j$ *apply, while they decrease whenever both inequalities in ii) are reversed.*

In particular, suppose $\varepsilon = r_j - r_i$. Then $(\tilde{r}_i, \tilde{r}_j) = (r_j, r_i)$ so that switching the locations of r_i and r_j when both $r_i \leq r_j$ and $r_{[i-1]} + r_{[i+1]} \geq r_{[j-1]} + r_{[j+1]}$ apply increases the value of the cohesion index while holding vector mean and variance fixed, implying an increase in environmental benefits. We also note in passing that $r_{[i-1]} + r_{[i+1]} \geq r_{[j-1]} + r_{[j+1]}$ reduces to one of $r_{[i-1]} \geq r_{[j+1]}$ or $r_{[i+1]} \geq r_{[j-1]}$ when the farms are not adjacent but do have a common neighbor. General condition $r_{[i-1]} + r_{[i+1]} \geq r_{[j-1]} + r_{[j+1]}$ shows how the index captures the idea of cohesion. Regardless of the r_i, the cohesion index increases upon a small buy down transfer from j^{th} farm to i^{th} farm if and only if the farms flanking the latter have an average buy down that is higher than those flanking r_j.

Our findings above point to considerable structure on what increases environmental services.

Both propositions 1 and 2 suggest some sort of buy down agglomeration on a segment of the circle might be best. Concentration is an issue elsewhere in economics, as with income inequality and market power. There, such statistics as Gini coefficients and variance have been found to be relevant where well-known references are Atkinson [1970] and Bergstrom and Varian [1985]. But these statistics do not account for spatial effects, and so are inappropriate in our context.

One way of allowing for spatial effects, and also providing opportunities for econometric study, is to present the chosen buy downs in spectral form. In our case, an additional advantage is the suitability of harmonic analysis for the circular topology. With $\Omega^0 = \{1, 2, \ \dots \ , N-1\}$, specify the spectral sum

(8) $$r_n - \bar{r} = \sum_{k \in \Omega^0} a_k \cos(2\pi nk / N) + \sum_{k \in \Omega^0} b_k \sin(2\pi nk / N).$$

Since $2N-2$ orthogonal functions have been used to fit $N \geq 2$ buy down values, this together with some constant a_0 provides an exact representation of the parameters.

PROPOSITION 3: *Let* r *be represented by a spectral sum with parameters* $\{a_0 \bigcup (a_k, b_k), k \in \Omega^0\}$. *Then* $\rho(r) = 0.5\sum_{k \in \Omega^0} (a_k^2 + b_k^2)\cos(2\pi k / N)$ *and* $v(r) = 0.5\sum_{k \in \Omega^0} (a_k^2 + b_k^2)$.

Notice next that $\cos(2\pi n / N)$ is reflexively symmetric around π or 180^0 in $n \in \Omega^0$. Thus, $\cos(2\pi / N) \equiv \cos(2\pi(N-1)/N)$, $\cos(4\pi / N) \equiv \cos(2\pi(N-2)/N)$ and so on. For even N there will be a middle entrant in Ω^0, namely $n = N/2$. For it, $\cos(2\pi n / N) = -1$. For N odd, Ω^0 will not have a middle entrant. But break the set in two with $\Omega^{0,low} = \{1, 2, \ \dots \ , (N-1)/2\}$ and $\Omega^{0,high}$ $= \{(N+1)/2, (N+3)/2, \ \dots \ , N-1\}$. The intent of what is to follow is to provide exact conditions under which the mean and variance of production buy downs are held fixed, but the cohesion index increases.

We will first develop what we call summary coefficients. These account for the degrees of freedom that the spectral sum allows in fitting the buy down parameters. Drawing from both sets

$\Omega^{0,low}$ and $\Omega^{0,high}$ so that cosine evaluations are the same, write the first summary coefficient as $c_1 = a_1^2 + b_1^2 + a_{N-1}^2 + b_{N-1}^2$ and in general the m^{th} coefficient as $c_m = a_m^2 + b_m^2 + a_{N-m}^2 + b_{N-m}^2 \ \forall m \in \{1,2,\ \ldots\ ,(N-1)/2\}$. For even N, place middle entrant $n = N/2$ in $\Omega^{0,low}$ so that $\Omega^{0,low} = \{1,2,\ \ldots\ ,N/2\}$ and $\Omega^{0,high} = \{(N+2)/2,(N+4)/2,\ \ldots\ ,N-1\}$. In this case, and as before, write the m^{th} summary coefficient as $c_m = a_m^2 + b_m^2 + a_{N-m}^2 + b_{N-m}^2 \ \forall m \in \{1,2,\ \ldots\ ,(N-2)/2\}$. The only difference when compared with N odd is that $c_{N/2} = a_{N/2}^2 + b_{N/2}^2$.

We are interested in the partial sums of these coefficients. Formally, define

$$
\begin{aligned}
& c_m = \begin{cases} a_m^2 + b_m^2 + a_{N-m}^2 + b_{N-m}^2 & \forall m \in \{1,2,\ \ldots\ ,(N-1)/2\}, \qquad N \text{ odd;} \\ a_m^2 + b_m^2 + a_{N-m}^2 + b_{N-m}^2 & \forall m \in \{1,2,\ \ldots\ ,(N-2)/2\}, \qquad N \text{ even;} \\ a_{N/2}^2 + b_{N/2}^2, & m = N/2, \qquad N \text{ even;} \end{cases} \\
& \Omega^{0,low} = \begin{cases} \{1,2,\ \ldots\ ,(N-1)/2\} & N \text{ odd;} \\ \{1,2,\ \ldots\ ,N/2\} & N \text{ even;} \end{cases} \qquad (9) \\
& C_k = \sum_{i=1}^{k} c_i, \quad k \in \Omega^{0,low}.
\end{aligned}
$$

Bearing in mind that a_0 determines the value of $\bar{r}$, we have

PROPOSITION 4: *Let spectral representations of r' and r'' be given by $\{a_0' \cup (a_k', b_k'), k \in \Omega^0\}$ and $\{a_0'' \cup (a_k'', b_k''), k \in \Omega^0\}$, respectively. Let the summary representations be $\{a_0' \cup c_k', k \in \Omega^{0,low}\}$ and $\{a_0'' \cup c_k'', k \in \Omega^{0,low}\}$, respectively. Then*

A) $\bar{r}$ *and* $v(r)$ *do not change whenever*

$$
C_k'' \geq C_k' \quad \forall k \in \Omega^{0,low} \quad \text{with} \quad \begin{cases} C_{(N-1)/2}'' = C_{(N-1)/2}' & N \text{ odd;} \\ C_{N/2}'' = C_{N/2}' & N \text{ even.} \end{cases} \qquad (10)
$$

B) $\rho(r'') \geq \rho(r')$ *under condition set (10).*

C) *Let* $c_m'' = c_m' \ \forall m \in \Omega^{0,low}, m \notin \{j,k\}$ *with* $j,k \in \Omega^{0,low}$ *and* $j < k$. *If* $c_j'' = c_j' + \delta$ *and* $c_k'' = c_k' - \delta$ *with* $\delta \geq 0$ *then* $\rho(r'') \geq \rho(r')$.

COROLLARY 4.1: *Environmental services are larger under* r'' *than under* r' *where these vectors are comparable in the sense of (10) above.*

COROLLARY 4.2: *Among the set of spectral representations given by*

(11) $$\left\{a_0 \bigcup (a_k, b_k), k \in \Omega^0 : \sum_{k\in\Omega^0}\left(a_k^2 + b_k^2\right) = \Gamma\right\}, \qquad \Gamma \text{ a constant,}$$

the value of $\rho(r)$ *is maximized whenever* $a_1^2 + b_1^2 = \Gamma$, *requiring* $(a_k, b_k) = (0,0)\ \forall k \in \Omega^0, k \neq 1$.

Part A) of the proposition assures that only $\rho(r)$ is affected. Part B) then leads to Corollary 4.1 in light of the assumption $B_\rho(\cdot) > 0$. Part C) gives deeper insight into Proposition 3. The loading of variance onto low frequency harmonics, or $\cos(2\pi k / N)$ where *k* is low, can be seen as ensuring a more coherent or smoother manner of variability and so a larger cohesion index. At the limit we obtain Corollary 4.2.

Corollary 4.2 is very important. It demonstrates part of a discrete version of Wirtinger's inequality, where such demonstration was first provided by different means in Fan, Taussky and Todd [1955]. In our setting this inequality asserts that if $r_{[-1]} = r_{N-1}$ and $\sum_{n=0}^{N-1}(r_n - \overline{r}) = 0$ then $\sum_{n=0}^{N-1}(r_n - r_{n-1})^2 \geq 4\sin^2(\pi / N)\sum_{n=0}^{N-1}(r_n - \overline{r})^2$. The inequality is satisfied as an equality if and only if the r_n follow

(12) $$r_n = \overline{r} + K_1\cos(2\pi n / N) + K_2\sin(2\pi n / N),$$

where K_1 and K_2 are free parameters subject to $v = N^{-1}\sum_{n=0}^{N-1}(r_n - \overline{r})^2$. The sequence defined by (12) has period 2π, i.e., one must travel around the entire disc before the sequential buy-down pattern repeats.

Since we also know from Proposition 3 that $v(r) = 0.5K_1^2 + 0.5K_2^2$, we may write

$K_2 = \pm\sqrt{2v - K_1^2}$. Upon re-labeling $K = K_1$, (12) may be written as

(13) $$r_n = \overline{r} + K\cos(2\pi n / N) \pm \left(2v - K^2\right)^{0.5} \sin(2\pi n / N),$$

With $v^+ = 0.5\min[\overline{r}^2, (\hat{q} - \overline{r})^2]$, relation (13) leads us to

PROPOSITION 5: A) *Solution (13) is interior on simplex* S *with variance* $v(r)$ *whenever*

(14) $$v(r) < v^+;$$

B) *In that case,*

(15) $$\rho(r) = \cos(2\pi / N)v(r).$$

In some ways, equation (13) should not be all that surprising given the undulation attribute of the solution to Proposition 1. There of course we confined attention to arrangements of a given vector.[5] Of interest to us is when the path given in (13) emerges as a solution.

DEFINITION 2: *Any solution satisfying (13) is said to be harmonic.*

Now insert (15) into the benefit function to obtain

(16) $$H(\overline{r}, v) \overset{\text{defn}}{=} B\left(\overline{r}, v, \cos(2\pi / N)v\right).$$

This function is sufficient to describe the optimal solution whenever $v(r) < v^+$.

PROPOSITION 6*: If*

A) $H_v(\overline{r}, v) < 0 \; \forall v \in [0, v^{\max}]$ *then extensification is optimal.*

B) $H_v(\overline{r}, v) > 0 \; \forall v \in [0, v^+]$ *then (at least) partial intensification is optimal.*

C) $H(\overline{r}, v)$ *is quasiconcave on* $v \in [0, v^{\max}]$ *with an interior maximum on* $v \in [0, v^+]$, *then harmonic solution (13) is optimum.*

---

[5] Non-spatial models where the idea that identical firms should be treated asymmetrically has arisen include works by Salant and Shaffer [1999] and Long and Soubeyran [2001].

D) $H(\overline{r},v)$ *is quasiconvex on* $v \in [0,v^{\max}]$ *with* $H(\overline{r},0) < H(\overline{r},v^{+})$ *, then (at least) partial intensification is optimal.*

To obtain a better sense of Part C), consider the additively separable form

$$B\left(\overline{r},v,\cos(2\pi/N)v\right) = B^1(\overline{r}) + B^2(v) + B^3(\rho), \tag{17}$$

where all of $B_v^2(v) < 0$, $B_\rho^3(\rho) > 0$, and $B_{vv}^2(v) + \cos^2(2\pi/N)B_{\rho\rho}^3(\rho) < 0$ apply on $v \in [0,v^{\max}]$:

COROLLARY 6.1*: For objective function (17), any interior solution* v^* *to*

$$H_v(\overline{r},v) = B_v^2(v) + \cos(2\pi/N)B_\rho^3\left[\cos(2\pi/N)v\right] = 0 \tag{18}$$

is the unique harmonic solution while if $H_v(\overline{r},v)\big|_{v=0} < 0$ *then extensification is optimal and if* $H_v(\overline{r},v)\big|_{v=v^{+}} > 0$ *then at least partial intensification is optimal.*

Notice that the solution to (18) would have $v^* = 0$ were the spatial spillovers ignored. As $\cos(2\pi/N) > 0$ when $N > 4$ and as $B_\rho^3(\rho) > 0$, inclusion of spatial effects increase optimal variance. Ignoring spatial complementarities may tilt the identified optimum toward a lower variance, or more extensive, policy choice. This has policy implications in light of *i*) the tendency to ignore these effects in policy assessments, and *ii*) previously mentioned concerns with the effectiveness of implemented agro-environmental schemes.

5. Between city and wilderness; Accounting for heterogeneities. Finally we ask how the production buy downs should be arranged under an alternative topographical setting. In order to demonstrate the general applicability of the approach, we will also allow for heterogeneities in this alternative model. So as to be explicit, suppose that the optimization problem is to choose $r \in S$ to maximize

(19) $$\sum_{n\in\{1,2,\,\dots\,,N-2\}} \alpha_n r_n - 0.5\chi \sum_{n\in\Omega} r_n^2 + \tau \sum_{n\in\Omega} r_n r_{n+1}, \qquad \chi > 0, \qquad \tau > 0,$$
$$\text{subject to} \quad r_0 = \hat{r}_0, \quad r_{N-1} = \hat{r}_{N-1};$$

where $\chi - 2\tau > 0$ is assumed to ensure concavity.

Here the boundary values are to capture external effects arising from, say, *i*) a city located on a circle so that no environmental services are provided and $\hat{r}_0 = \hat{r}_{N-1} = 0$, or *ii*) a city at one end of a line, $\hat{r}_0 = 0$, and a National Park at the other end so that $\hat{r}_{N-1}$ is large. Parameters α_n are intended to capture farm-specific benefit heterogeneities due to locational idiosyncracies, perhaps arising from geographic features such as rivers, wetlands, or geological formations. These parameters are net of the shadow cost of raising taxes so no explicit constraint on the sum of buy downs has been included.

The assumption is made that (19) is concave in r.[6] So were $\alpha_n = \alpha \; \forall n \in \Omega$ on the disc topology studied to this point then extensification would be optimal. Consequently, there should be a tendency for the buy downs to be similar across farms in this setting too. This we call the cohesion force, and the resulting levels of r_n will depend on the opportunity cost of tax dollars. A simple calculation shows that setting $r_n = \alpha/(\chi - 2\tau) \; \forall n \in \{1,2,\,\dots\,,N-2\}$ is optimal whenever $\alpha_n = \alpha$ for all *n* and $\hat{r}_0 = \hat{r}_{N-1} = \alpha/(\chi - 2\tau)$. However, there is a second force at play. Low $\hat{r}_0$ and $\hat{r}_{N-1}$ values should tend to depress the values for r_n near to the boundaries when compared with farms near the center of the line. This we call the boundary action force or the edge effect.

The first-order optimality conditions are:[7]

---

[6] In general, concavity is not assured. It does apply whenever the value of τ / χ is sufficiently small.

[7] We have used terminology that may bring physical problems to mind. System (20) is an example of a Sturm-Liouville difference equation. Sturm-Liouville systems are widely encountered in physics when modeling a variety of phenomena from sound waves to quantum mechanics.

$$
(20)\qquad
\begin{aligned}
&r_1: && \alpha_1-\chi r_1+\tau\hat{r}_0+\tau r_2=0;\\
&r_n: && \alpha_n-\chi r_n+\tau r_{n-1}+\tau r_{n+1}=0, \qquad n\in\{2,3,\ldots,N-4,N-3\};\\
&r_{N-2}: && \alpha_{N-2}-\chi r_{N-2}+\tau r_{N-3}+\tau\hat{r}_{N-1}=0.
\end{aligned}
$$

Thus, asymmetries arise due to imposed boundary values. The system may be written as

$$
(21)\qquad
\begin{pmatrix}
1+2\xi & -\xi & 0 & \cdots & 0 & 0\\
-\xi & 1+2\xi & -\xi & \cdots & 0 & 0\\
0 & -\xi & 1+2\xi & \cdots & 0 & 0\\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\
0 & 0 & 0 & \cdots & 1+2\xi & -\xi\\
0 & 0 & 0 & \cdots & -\xi & 1+2\xi
\end{pmatrix}
\begin{pmatrix} r_1\\ r_2\\ r_3\\ \vdots\\ r_{N-3}\\ r_{N-2}\end{pmatrix}
=\frac{1}{\kappa}
\begin{pmatrix} \alpha_1+\tau\hat{r}_0\\ \alpha_2\\ \alpha_3\\ \vdots\\ \alpha_{N-3}\\ \alpha_{N-2}+\tau\hat{r}_{N-1}\end{pmatrix};
$$

where $\kappa=\chi-2\tau>0$ and $\xi=\tau/\kappa>0$. Notice that the roots ϕ_l and ϕ_h of the characteristic equation for system (21), $\xi x^2-(1+2\xi)x+\xi=0$, are

$$
(22)\qquad \phi_l=1+\frac{1-\Delta}{2\xi}\in(0,1);\qquad \phi_h\equiv\phi_l^{-1}=1+\frac{1+\Delta}{2\xi}=\phi_l+\frac{\Delta}{\xi}>1;\qquad \Delta=\sqrt{1+4\xi}\in(1,1+2\xi);
$$

and are both positive. We have

PROPOSITION 7: A) *With* $\Psi=1/[\Delta(\phi_h^{N-1}-\phi_l^{N-1})]$, *the inverse matrix* P *of the* $(N-2)\times(N-2)$ *matrix in (21) has row i, column j entries*

$$
(23)\qquad p_{i,j}=\begin{cases}\left(\phi_h^i-\phi_l^i\right)\left(\phi_h^{N-1}\phi_l^j-\phi_h^j\phi_l^{N-1}\right)\Psi, & i\le j;\\ \left(\phi_h^j-\phi_l^j\right)\left(\phi_h^{N-1}\phi_l^i-\phi_h^i\phi_l^{N-1}\right)\Psi, & i>j.\end{cases}
$$

B) *The entries are all strictly positive.*

C) *The entries are all symmetric in that* $p_{i,j}=p_{j,i}\ \forall i,j\in\{1,2,\ldots,N-2\}$, *and furthermore, centro-symmetric in that* $p_{i,j}=p_{N-1-i,N-1-j}=p_{N-1-j,N-1-i}\ \forall i,j\in\{1,2,\ldots,N-2\}$.

D) *The entries satisfy the unimodality properties* $p_{i-1,j}\le p_{i,j}\ \forall i\le j$, $p_{i,j}\ge p_{i+1,j}\ \forall i\ge j$, $p_{i,j-1}\ge p_{i,j}\ \forall i\le j$, *and* $p_{i,j+1}\ge p_{i,j}\ \forall i\ge j$. *This means that* $p_{1,j}$ *is decreasing in* $j, \forall j\in\{1,\ldots,N-2\}$, *while* $p_{i,N-2}$ *is increasing in* $i, \forall i\in\{1,\ldots,N-2\}$.

E) *If* N *is odd then* $p_{(N-1)/2,k+(N-1)/2} = p_{(N-1)/2,-k+(N-1)/2} = p_{k+(N-1)/2,(N-1)/2} = p_{-k+(N-1)/2,(N-1)/2}$ $\forall k \in \{1, \ldots, (N-3)/2\}$.

F) *If* N *is odd then* $p_{i,i}$ *increases on* $i \in \{1,2, \ldots, (N-1)/2\}$ *to attain maximum value at* $i = (N-1)/2$ *and decreases on* $i \in \{(N-1)/2, (N+1)/2, \ldots, N-2\}$. *If* N *is even then* $p_{i,i}$ *increases on* $i \in \{1,2, \ldots, (N-2)/2\}$ *to attain maximum value when at* $i \in \{(N-2)/2, N/2\}$ *and decreases on* $i \in \{N/2, (N+2)/2, \ldots, N-2\}$.

G) *For* N *odd then* $p_{i,1} + p_{i,N-2} < p_{i-1,1} + p_{i-1,N-2}$ *on* $i \in \{2,3, \ldots, (N-1)/2\}$ *and* $p_{i,1} + p_{i,N-2} > p_{i-1,1} + p_{i-1,N-2}$ *on* $i \in \{(N+1)/2, (N+3)/2, \ldots, (N-2)\}$. *For* N *even then* $p_{i,1} + p_{i,N-2} < p_{i-1,1} + p_{i-1,N-2}$ *on* $i \in \{2,3, \ldots, (N-2)/2\}$ *and* $p_{i,1} + p_{i,N-2} > p_{i-1,1} + p_{i-1,N-2}$ *on* $i \in \{(N+2)/2, \ldots, (N-2)\}$, *with* $p_{i',1} + p_{i',N-2} = p_{i'-1,1} + p_{i'-1,N-2}$ *at* $i' = N/2$.

H) *With* $P_i = \sum_{j=1}^{N-2} p_{i,j}$, *then* $P_i = P_{N-2-i}$. *If* N *is odd then* P_i *increases on* $i \in \{1,2, \ldots, (N-1)/2\}$ *to attain maximum value at* $i = (N-1)/2$ *and decreases on* $i \in \{(N-1)/2, (N+1)/2, \ldots, N-2\}$. *If* N *is even then* P_i *increases on* $i \in \{1,2, \ldots, (N-2)/2\}$ *to attain maximum value when at* $i \in \{(N-2)/2, N/2\}$ *and decreases on* $i \in \{N/2, (N+2)/2, \ldots, N-2\}$.

As an example, when $\xi = 1$ and $N = 7$ then the inverted matrix is

$$\begin{pmatrix} 0.382 & 0.146 & 0.056 & 0.021 & 0.007 \\ 0.146 & 0.437 & 0.167 & 0.062 & 0.021 \\ 0.056 & 0.167 & 0.444 & 0.167 & 0.056 \\ 0.021 & 0.062 & 0.167 & 0.437 & 0.146 \\ 0.007 & 0.021 & 0.056 & 0.146 & 0.382 \end{pmatrix}. \tag{24}$$

Positivity and symmetry are clearly satisfied while centro-symmetry can be seen from observing reflections in the off diagonal. Unimodality is confirmed by observing that the largest entry in any row is on the main diagonal while the entry values decline monotonically as one moves away from the diagonal along a row. Symmetry ensures that unimodality is true for columns as well as rows. As $N = 5$, odd, we can observe a cross pattern around the central entry $p_{3,3} =$ 0.444. So $p_{2,3} = p_{4,3} = p_{3,2} = p_{3,4} = 0.167$ while $p_{1,3} = p_{5,3} = p_{3,1} = p_{3,5} = 0.056$. As for row sums, they are $P_1 = P_5 = 0.612 \le P_2 = P_4 = 0.833 \le P_3 = 0.89$. That F) applies in this case can be seen from unimodality along the main diagonal.

We have also from the proposition that sensitivity to edge values decreases with displacement and that, all else equal, edge values can create a variety of optimal buy down arrangements.

COROLLARY 7.1: $0 < dr_{N-2} / d\hat{r}_0 < \ldots < dr_n / d\hat{r}_0 < \ldots < dr_1 / d\hat{r}_0$ *and* $dr_{N-2} / d\hat{r}_{N-1} > \ldots > dr_n / d\hat{r}_0 > \ldots$ $> dr_1 / d\hat{r}_0 > 0$.

COROLLARY 7.2: *Suppose that* $\alpha_i = \alpha \ \forall i \in \{1, 2, \ldots, N-2\}$.

A) *For* $\hat{r}_0 = \hat{r}_{N-1} > \alpha / \kappa$ *then* r_n *forms a U shape, with minimum at the middle and maxima at the edges. That is,* r_n *decreases in* n *for small n, reaches a minimum and increases in n for large n.*

B) *For* $\hat{r}_0 = \hat{r}_{N-1} < \alpha / \kappa$ *then* r_n *forms an inverted U shape, with minima at the edges and maximum at the middle. That is,* r_n *increases in* n *for small n, reaches a maximum and decreases in n for large n.*

C) *For* $\hat{r}_0 < \alpha / \kappa$ *and* $\hat{r}_{N-1} > \alpha / \kappa$ *then* r_n *is monotone increasing.*

D) *For* $\hat{r}_0 > \alpha / \kappa$ *and* $\hat{r}_{N-1} < \alpha / \kappa$ *then* r_n *is monotone decreasing.*

The proposition's economic content is best illustrated through the corollaries. Corollary 7.1 shows how the complementary spillovers diminish over space. All else equal, a nature reserve at one end of a strip should encourage relatively large buy downs over neighboring farms when compared with more distant farms. Parts A) and B) of Corollary 7.2 identify two opposing forces. For low values of $\hat{r}_0$ and $\hat{r}_{N-1}$, as when there are cities at both ends of a strip, then buy downs should rise upon moving from the edges toward the center. This is just as viscous fluids dip toward the sides of a glass. The reverse is true for high values of $\hat{r}_0$ and $\hat{r}_{N-1}$. When the edges are wilderness, then buy downs should taper off so that the most intensively farmed land lies at the center of the strip.

6. National parks. The ultimate purchase of ecological services for public use is a National Park. The management goal for most national parks is to preserve as pristine, or to restore, land such that uses inconsistent with resource preservation are prohibited. But land uses are many and so the government can attenuate property rights in many ways. For example, hunting, animal grazing, lumber extraction, and mining rights can be restricted to varying degrees. Consider the United States, where the 'National Forest' designation is consistent with these alternative uses whereas the 'National Park' designation is not. For any given land parcel, the economic cost of National Forest designation is smaller than that of National Park designation since user fees can be, and are, extracted.

It may then be instructive to study the spatial distribution of different land use designations. Nearly all flagship properties in the U.S. national parks system are surrounded by lands preserved under less restrictive designations, such as wildernesses as governed by the Wilderness Act of 1964. In these areas, human activities must be non-invasive (e.g., hunting, fishing, backpacking, birding, and horse riding), and cannot involve motorized vehicles. National forests other than wildernesses are generally less restrictive still.

The Yellowstone, Grand Teton National Park complex is completely surrounded by lands with National Wilderness and National Forest designations. It is true of Yosemite, Kings

Canyon, Sequoia, and Death Valley in California, Crater Lake in Oregon, Mount Rainier and North Cascades in Washington, Rocky Mountain National Park in Colorado, and the Grand Canyon in Arizona. Instances where the peripheral protection zone is incomplete, such as with the Everglades in Florida and the Great Smoky Mountains in Appalachia, tend to be in the east where human development had longer precedence. The general evolution of the core-periphery protection pattern first saw the National Park designation, followed by placing and then enhancing protection on the periphery.

Motivations for subsequent protections on an extensive fringe were many, and were sometimes due to growing demands for land use in the area. But there was also recognition of positive spatial spillovers from ecological services provided by the surrounding lands. The Greater Yellowstone area provides two topical examples. The largest wild bison herd in North American is located in the area. Rather than roam, they move in search of food and shelter beyond park boundaries and onto lands grazed by cattle. The large elk herds in the area also wander, and both host the infectious agent *Brucella abortus*, which can cause great harm to cattle and humans. Much effort at private, state, and federal levels has been devoted to eradicating the disease from cattle herds. As of 2009, Greater Yellowstone is the sole remaining region in the United States with brucellosis infected cattle herds. Eradication consistent with comparatively free bison and elk movement requires restricted use of lands on the park borders.

The U.S. wolf recovery program focused on three main areas; Northern gray wolves in the Northern Rockies including the Greater Yellowstone region; Mexican gray wolves in the Southwest; and red wolves in the east. The reintroductions in the Northern Rockies since the middle 1990s have proven to be very successful, with a total population of about 1,650 wolves by late 2008 extending from Wyoming and Montana through Idaho and into Oregon and Washington. The red wolf reintroduction in North Carolina's Alligator River National Wildlife Reserve has also proven successful with a population of perhaps 100 animals surviving on a remote coastal swampland patchwork of Defense Department, other Federal, and private lands.

However the Great Smoky Mountain National Park red wolf reintroduction program, commencing 1991, proved to be a failure and was discontinued in 1998. It was found that food needs forced wolf home ranges to outside park boundaries. In addition, wolves will breed with coyotes in the absence of an available mate so that viability is precluded for small populations in fragmented ranges. The Mexican gray wolf reintroduction in 1998 in the Gila National Forest region between Interstates 10, 25, 40, and 17 in Arizona and New Mexico has also had problems. The wild population in 2009 was about 50 and pack formation has proven to be difficult (Cart [2009]). Exceeding 5,000 square miles in area, deer and elk prey are in ample supply. But the region is without a significant national park while large tracts of the federal lands are leased for year-round grazing, suggesting that the spatial arrangement of ecological service buy-downs are part of the problem.

Another distinction relative to the Northern Rockies is that ranchers in the Southwest are not required to remove cattle carcasses in order to deter wolf scavenging. This land use restriction may indeed be more costly to impose in the Southwest so that willingness to pay grazing fees to the federal government would fall. Wolf predation on cattle is apparently more of a concern to ranchers in the Gila National Forest area, where wolves have been shot illegally. A 2005 compromise regulation has required the removal of wolves deemed responsible for multiple cattle deaths. Removals and relocations distort socialization and predation patterns. It should be noted that while ranchers are compensated for identified wolf kills and such kills are not large (Muhly and Musiani [2009]), the producer incurs other uncompensated costs. And if, for whatever reason, livestock producers choose to sell out privately held land then the land may assume rural residency uses. Such uses would likely further fragment species range.

The core and periphery approach has also been adopted elsewhere, even in parts of the world where human landscape interventions are of long-standing. Vanoise, in the French High Alps, has a 200 square mile central zone surrounded by a 560 square mile peripheral zone. Intended as a buffer to protect the inner core, the outer disc allows restricted land use consistent with more traditional lifestyles. The intent is to manage the periphery so as to secure positive spatial external spillovers onto the core. Other French national parks that take this approach include the Cévennes in the Massif Central as well as Les Ecrins and Mercantour, both Alpine preserves. Job [2008] tabulates the core-periphery structure of a variety of major German National Parks where the periphery is referred to as a transition zone. All fourteen have significant transition zones.[8]

7. Discussion. Working with a simple spatial model on a disc, this paper has sought to clarify some issues in agri-environmental policy. For farms identical in all ways, we find it may be optimal to treat them asymmetrically in order to avoid loss in ecosystem services due to fragmentation. In doing so, a trade off can arise if eco-service benefits on any given farm are concave. This trade off leads to the possibility of a third policy option, one not considered in the formal literature to this point. A smoothly varying buy down policy around the disc may be best, where we find a closed-form trigonometric solution for the optimal policy.

We also allow for farm-level heterogeneities in the provision of eco-services. In order to better understand the implications of topological structure, we model farm-level heterogeneities on a strip of land rather than on a closed disc. While the setting is very different, spatial spillovers lead to a preference for smooth variation in this situation too. If the boundaries are wilderness and the land between is homogeneous then the buy downs should largely be near the bounds, and will decrease steeply toward the center whenever the opportunity cost of tax funds is high. If the bounds are urban and the opportunity cost of tax funds is low then the buy downs will be at the center of the land strip.

[8] A brief web search, dated 11/13/’09 has found that some Greek, South African, Madagascar and Indian National Parks use a core and periphery management approach.

The solutions we have identified are first-best for a given technology and preference structure. We have not considered implementation, where governments face political and information constraints. The literature on constrained first-best policies in the presence of spatial externalities is sparse. We hope that our model will allow for insights in this regard, beyond providing explicit benchmark solutions to the unconstrained problem.

As to the practical relevance of our work, a review of land use restrictions around National Park systems throughout the world identifies public policy choices that effect the sort of graduated variation in land use that our model suggests as optimal. Our normative analysis can be viewed as consistent with public policy on ecological services. This may be policy as planned from the outset or as later adapted to better secure earlier or evolving ecological goals given political constraints and limited resources.

We see other applications of the general approach. One is to better understand spatial effects within city residency patterns where positive neighborhood spillovers can be seen in equilibria that involve family wealth gradients. Somewhat more abstractly parents worry about the friends children keep, perhaps in part arising from beliefs about behavioral norms, peer effects, and mutual re-enforcement. Public health studies lend credence to these concerns. For example, Christakis and Fowler [2007] have identified social effects in the propensity to become obese where these effects are not entirely explained by the endogenous formation of social ties by people of different body mass indices. If a group of individuals are arranged in a circle and each person is viewed as being friends of just the contiguous neighbors on either side, then our model may be able to say something about equilibrium behavior concerning diet, social deviancy, personal discipline, and parenting.

Appendix: Omitted Proofs

Proof of Property 1. Since $\rho(r) = N^{-1}\sum_{n\in\Omega} r_{[n-1]}r_n - \bar{r}^2$ and $\bar{r}$ is invariant to reflection, rotation or their composition, we need only consider the effect on $\theta(r)=\sum_{n\in\Omega} r_{[n-1]}r_n$. For part A), note that $\sum_{n\in\Omega} r_{[n-1]}r_n \equiv \sum_{n\in\Omega} r_{[n-1+i]}r_{[n+i]}\ \forall i\in\Omega$. Parts B) and C) follow from

$$
\text{(A1)}\qquad
\begin{aligned}
& r_0r_1 + r_1r_2 + \ldots + r_{n-1}r_n + \ldots + r_{N-2}r_{N-1} + r_{N-1}r_0 \\
& \equiv r_{N-1}r_0 + r_{N-2}r_{N-1} + \ldots + r_{N-n}r_{N-n+1} + \ldots + r_1r_2 + r_0r_1 \\
& \equiv r_{[N-1+i]}r_{[0+i]} + r_{[N-2+i]}r_{[N-1+i]} + \ldots + r_{[N-n+i]}r_{[N-n+i+1]} + \ldots + r_{[1+i]}r_{[2+i]} + r_{[0+i]}r_{[1+i]};
\end{aligned}
$$

i.e., impose reflection and composition permutations on the arguments of $\theta(r)$. ■

Proof of Property 2. Eqn. (3.1.5') on p. 68 and eqn. (3.2.6) on p. 73 in Davis [1994] provide the eigenvalue formula for circulant matrices

$$
\text{(A2)}\qquad \lambda_j = \sum_{n\in\Omega} c_n \times \left(\cos[2\pi n(j-1)/N] + \left(\sqrt{-1}\right)\sin[2\pi n(j-1)/N]\right),
$$

where $(c_0, c_1,\ \ldots\ , c_{N-1})$ is the first row of the matrix. In our case, for eqn. (5), $c_1 = c_{N-1} = 1$ and all other entries are zero so that

$$
\text{(A3)}\qquad
\begin{aligned}
& \lambda_j = \cos[2\pi(j-1)/N] + \left(\sqrt{-1}\right)\sin[2\pi(j-1)/N] \\
& + \cos[2\pi(N-1)(j-1)/N] + \left(\sqrt{-1}\right)\sin[2\pi(N-1)(j-1)/N] \\
& = 2\cos[2\pi(j-1)/N],
\end{aligned}
$$

as $\cos[2\pi(j-1)/N] \equiv \cos[2\pi(N-1)(j-1)/N]$ and $\sin[2\pi(j-1)/N] \equiv$ $-\sin[2\pi(N-1)(j-1)/N]$. The other statements follow from cosine function properties. ■

Proof of Proposition 1. Theorem 10 in Chao and Liang [1992] shows that $r^{*,ev} =$ $\min_{r\in T(\hat{r})} \sum_{n\in\Omega} f\left(\left|r_{[n+1]} - r_n\right|\right)$ when N is even and $r^{*,od} = \min_{r\in T(\hat{r})} \sum_{n\in\Omega} f\left(\left|r_{[n+1]} - r_n\right|\right)$ for $f(\cdot):$ $\mathbb{R}\to\mathbb{R}$ a convex function and $|\cdot|$ the absolute value function. Let $f(x)=x^2$ and note that $\sum_{n\in\Omega}\left(\left|r_{[n+1]} - r_n\right|\right)^2 = 2\sum_{n\in\Omega} r_n^2 - 2\sum_{n\in\Omega} r_n r_{[n+1]}$. Since $\sum_{n\in\Omega} r_n^2$ is invariant for $r\in T(\hat{r})$, it follows that these vectors must maximize $\rho(r)$ over domain $r\in T(\hat{r})$. ■

Proof of Corollary 1.1. Rearrangements of any r do not affect $\bar{r}$ or $v(r)$, while $B_{\rho}(\cdot) > 0$. So the optimal r in some set $T(\hat{r})$ will be the one maximizing $\rho(r)$. As it is true of any vector, it is true of $T\left(\arg\max_{r \in S} B(\cdot)\right)$. ■

Proof of Proposition 2. Again, we need only consider the effect on $\theta(r) = \sum_{n \in \Omega} r_{[n-1]} r_n$. There are two cases: either the ith and jth farms are adjacent or they are not. Suppose they are adjacent, or $j = [i+1]$ where symmetry ensures that consideration of $j = [i-1]$ is identical. Then

(A4)
$$\begin{aligned} \theta(\tilde{r}) &= r_0 r_1 + \ldots + r_{[i-1]}(r_i + \varepsilon) + (r_i + \varepsilon)(r_{[i+1]} - \varepsilon) + (r_{[i+1]} - \varepsilon) r_{[i+2]} + \ldots + r_{N-1} r_0 \\ &= \theta(r) + r_{[i-1]}\varepsilon - r_i \varepsilon + r_{[i+1]}\varepsilon - \varepsilon^2 - r_{[i+2]}\varepsilon, \end{aligned}$$

with ε derivative $\partial\theta(\tilde{r}) / \partial\varepsilon\big|_{\varepsilon=0} = r_{[i-1]} - r_i + r_{[i+1]} - r_{[i+2]}$ where $i = [j-1]$ and $[i+2] = [j+1]$. Suppose instead the farms are not adjacent. Then

(A5)
$$\theta(\tilde{r}) = \theta(r) + r_{[i-1]}\varepsilon + r_{[i+1]}\varepsilon - r_{[j-1]}\varepsilon - r_{[j+1]}\varepsilon,$$

with ε derivative $\partial\theta(\tilde{r}) / \partial\varepsilon\big|_{\varepsilon=0} = r_{[i-1]} + r_{[i+1]} - r_{[j-1]} - r_{[j+1]}$.

As for variance, $\bar{r}$ does not change so we need only consider the effect on the sum of squares. The expression

(A6)
$$r_0^2 + \ldots + (r_i + \varepsilon)^2 + \ldots + (r_j - \varepsilon)^2 + \ldots + r_{N-1}^2$$

has ε derivative $2r_i - 2r_j$ when $\varepsilon = 0$. ■

Proof of Corollary 2.1. This follows from the assumed sign on $B_v(\cdot)$ in addition to $B_{\rho}(\cdot) > 0$.

■

Proof of Proposition 3. Insert (8) into (3) and expand to obtain

$$\text{(A7)}\quad \begin{aligned}\rho(r) &= N^{-1}\sum_{n\in\Omega}\Big[\sum_{k\in\Omega^0} a_k\cos(2\pi nk/N)\sum_{k\in\Omega^0} a_k\cos(2\pi(n+1)k/N)\Big]\\ &+N^{-1}\sum_{n\in\Omega}\Big[\sum_{k\in\Omega^0} a_k\cos(2\pi nk/N)\sum_{k\in\Omega^0} b_k\sin(2\pi(n+1)k/N)\Big]\\ &+N^{-1}\sum_{n\in\Omega}\Big[\sum_{k\in\Omega^0} b_k\sin(2\pi nk/N)\sum_{k\in\Omega^0} a_k\cos(2\pi(n+1)k/N)\Big]\\ &+N^{-1}\sum_{n\in\Omega}\Big[\sum_{k\in\Omega^0} b_k\sin(2\pi nk/N)\sum_{k\in\Omega^0} b_k\sin(2\pi(n+1)k/N)\Big].\end{aligned}$$

Consider each of these four right-hand terms in turn. For the first, use

$$\text{(A8)}\quad \cos(2\pi(n+1)k/N) = \cos(2\pi nk/N)\cos(2\pi k/N) - \sin(2\pi nk/N)\sin(2\pi k/N),$$

to write

$$\text{(A9)}\quad \begin{aligned}&\sum_{n\in\Omega}\Big[\sum_{k\in\Omega^0} a_k\cos(2\pi nk/N)\sum_{l\in\Omega^0} a_l\cos(2\pi(n+1)l/N)\Big]\\ &=\sum_{n\in\Omega}\Big[\sum_{k\in\Omega^0} a_k\cos(2\pi nk/N)\sum_{l\in\Omega^0} a_l\cos(2\pi nl/N)\cos(2\pi l/N)\Big]\\ &-\sum_{n\in\Omega}\Big[\sum_{k\in\Omega^0} a_k\cos(2\pi nk/N)\sum_{l\in\Omega^0} a_l\sin(2\pi nl/N)\sin(2\pi l/N)\Big].\end{aligned}$$

But multiplication of the first inner product of sums and then use of the orthogonality property

$$\text{(A10)}\quad \sum_{n\in\Omega}\cos(2nk\pi/N)\cos(2nl\pi/N) = 0 \ \text{ for } k \text{ and } l \neq k \text{ integers,}$$

on the sum over $n \in \Omega$ leads to

$$\text{(A11)}\quad \begin{aligned}&\sum_{n\in\Omega}\Big[\sum_{k\in\Omega^0} a_k\cos(2\pi nk/N)\sum_{l\in\Omega^0} a_l\cos(2\pi nl/N)\cos(2\pi l/N)\Big]\\ &=\sum_{n\in\Omega}\Big[\sum_{k\in\Omega^0} a_k^2\cos^2(2\pi nk/N)\cos(2\pi k/N)\Big]\\ &=\sum_{k\in\Omega^0} a_k^2\cos(2\pi k/N)\sum_{n\in\Omega_N}\cos^2(2\pi nk/N).\end{aligned}$$

Multiply out the remaining inner product of sums in (A9) and apply orthogonality property

$$\text{(A12)}\quad \sum_{n\in\Omega}\cos(2nk\pi/N)\sin(2nl\pi/N) = 0 \ \text{ for } k \text{ and } l \text{ integers,}$$

to confirm

$$\text{(A13)}\quad \sum_{n\in\Omega}\Big[\sum_{k\in\Omega^0} a_k\cos(2\pi nk/N)\sum_{l\in\Omega^0} a_l\sin(2\pi nl/N)\sin(2\pi l/N)\Big] = 0$$

so that

$$\text{(A14)}\quad \begin{aligned}&\sum_{n\in\Omega}\Big[\sum_{k\in\Omega^0} a_k\cos(2\pi nk/N)\sum_{l\in\Omega^0} a_l\cos(2\pi(n+1)l/N)\Big]\\ &=\sum_{k\in\Omega^0} a_k^2\cos(2\pi k/N)\sum_{n\in\Omega_N}\cos^2(2\pi nk/N) = 0.5N\sum_{k\in\Omega^0} a_k^2\cos(2\pi k/N),\end{aligned}$$

where $\sum_{n\in\Omega_N}\cos^2(2\pi nk/N)=0.5N\ \forall N\geq 3$ simplifies. With the additional use of

$$\text{(A15)}\qquad \begin{aligned}&\sin(2\pi(n+1)k/N)=\sin(2\pi nk/N)\cos(2\pi k/N)+\sin(2\pi k/N)\cos(2\pi nk/N);\\ &\sum_{n\in\Omega}\sin(2nk\pi/N)\sin(2nl\pi/N)=0 \text{ for } k \text{ and } l\neq k \text{ integers;}\\ &\sum_{n\in\Omega_N}\sin^2(2\pi nk/N)=0.5N\ \forall N\geq 3,\end{aligned}$$

it can be shown that

$$\text{(A16)}\qquad \begin{aligned}&\sum_{n\in\Omega}\left[\sum_{k\in\Omega^0}a_k\cos(2\pi nk/N)\sum_{k\in\Omega^0}b_k\sin(2\pi(n+1)k/N)\right]\\ &=0.5N\sum_{k\in\Omega^0}a_kb_k\sin(2\pi k/N);\\ &\sum_{n\in\Omega}\left[\sum_{k\in\Omega^0}b_k\sin(2\pi nk/N)\sum_{k\in\Omega^0}a_k\cos(2\pi(n+1)k/N)\right]\\ &=-0.5N\sum_{k\in\Omega^0}a_kb_k\sin(2\pi k/N);\\ &\sum_{n\in\Omega_N}\left[\sum_{k\in\Omega^0}b_k\sin(2\pi nk/N)\sum_{k\in\Omega^0}b_k\sin(2\pi(n+1)k/N)\right]\\ &=0.5N\sum_{k\in\Omega^0}b_k^2\cos(2\pi k/N).\end{aligned}$$

The sum in (A7) therefore resolves to

$$\text{(A17)}\qquad \rho(r)=0.5\sum_{k\in\Omega^0}\left(a_k^2+b_k^2\right)\cos(2\pi k/N).$$

As for variance, insert (8) into the expression for variance to obtain

$$\text{(A18)}\qquad \begin{aligned}v(r)=&N^{-1}\sum_{n\in\Omega}\left[\sum_{k\in\Omega^0}a_k\cos(2\pi nk/N)\sum_{k\in\Omega^0}a_k\cos(2\pi nk/N)\right]\\ &+N^{-1}\sum_{n\in\Omega}\left[\sum_{k\in\Omega^0}a_k\cos(2\pi nk/N)\sum_{k\in\Omega^0}b_k\sin(2\pi nk/N)\right]\\ &+N^{-1}\sum_{n\in\Omega}\left[\sum_{k\in\Omega^0}b_k\sin(2\pi nk/N)\sum_{k\in\Omega^0}a_k\cos(2\pi nk/N)\right]\\ &+N^{-1}\sum_{n\in\Omega}\left[\sum_{k\in\Omega^0}b_k\sin(2\pi nk/N)\sum_{k\in\Omega^0}b_k\sin(2\pi nk/N)\right].\end{aligned}$$

The orthogonality conditions laid out above, i.e., where the sum of a product equals 0, readily leads to $v(r)=0.5\sum_{k\in\Omega^0}(a_k^2+b_k^2)$. ■

Proof of Proposition 4. Part A): For mean, it is readily shown that $\sum_{n\in\Omega}\cos(2\pi n/N)=\sum_{n\in\Omega}\sin(2\pi n/N)=0$ so that $\sum_{n\in\Omega}r_n=N\bar{r}$ regardless of how values of the spectral sum coefficients are rearranged. On variance, $v(r)=0.5\sum_{k\in\Omega^0}(a_k^2+b_k^2)$ in Proposition 3 may be viewed in terms of partial sums. The condition that $C''_{(N-1)/2} = C'_{(N-1)/2}$ when N is odd and $C''_{N/2} = C'_{N/2}$ when N is even ensures that variance is held fixed.

Part B): Use Proposition 3 and summation by parts to write

$$\begin{aligned}
(\text{A19}) \qquad & \rho(r) = 0.5\sum_{k\in\Omega^{0,low}} c_k \cos(2\pi k/N) \\
& = 0.5c_1\cos(2\pi/N) + 0.5c_2\cos(4\pi/N) + \ \dots\ + 0.5c_{(N-1)/2}\cos(\pi(N-1)/N) \\
& = 0.5C_1\left[\cos(2\pi/N) - \cos(4\pi/N)\right] + 0.5C_2\left[\cos(4\pi/N) - \cos(6\pi/N)\right] \\
& +0.5C_3\left[\cos(6\pi/N) - \cos(8\pi/N)\right] + \ \dots\ + 0.5C_{(N-1)/2}\cos(\pi(N-1)/N),
\end{aligned}$$

when N is odd and replace the last term by $0.5C_{N/2}\cos(\pi N/N) = -0.5C_{N/2}$ when N is even. If buy down vectors r' and r'' are represented by vectors c' and c'', respectively, then

$$\begin{aligned}
& \rho(r'') - \rho(r') = 0.5\overbrace{\left(C''_1 - C'_1\right)}^{\geq 0}\overbrace{\left[\cos(2\pi/N) - \cos(4\pi/N)\right]}^{\geq 0} \\
(\text{A20}) \quad & +0.5\overbrace{\left(C''_2 - C'_2\right)}^{\geq 0}\overbrace{\left[\cos(4\pi/N) - \cos(6\pi/N)\right]}^{\geq 0} + 0.5\overbrace{\left(C''_3 - C'_3\right)}^{\geq 0}\overbrace{\left[\cos(6\pi/N) - \cos(8\pi/N)\right]}^{\geq 0} \\
& + \ \dots\ + 0.5\overbrace{\left(C''_{(N-1)/2} - C'_{(N-1)/2}\right)}^{=0}\overbrace{\cos(\pi(N-1)/N)}^{<0}
\end{aligned}$$

when N is odd and replace the last term by $0.5(C''_{N/2} - C'_{N/2})\cos(\pi) = -0.5(C''_{N/2} - C'_{N/2})$ when N is even. Clearly condition set (10) together with $\cos(x)$ decreasing on $x \in (0,\pi)$ ensure that $\rho(r'') \geq \rho(r')$.

Part C): The transfers in question ensure satisfaction of condition set (10). ■

Proof of Proposition 5. Part A): We seek the extremes on (13) to establish when a K exists such that the r_n are in the interior of S given variance value v. Differentiate (12) and set equal zero as the necessary condition for a maximum or minimum:

$$(\text{A21}) \qquad -K_1\sin(2\pi n/N) + K_2\cos(2\pi n/N) = 0;$$

with solution set $n^* \in [0, N)$ where the solutions need not be in Ω. Re-write as

(A22) $$n^{*}=\frac{N}{2\pi}\tan^{-1}\left(K_2/K_1\right).$$

Consideration of the shape of the inverse tangent function shows that there are two solutions on $n^{*}\in[0,N)$. Furthermore, trigonometric periodicity ensures that $-K_1\sin(2\pi n/N)+K_2\cos(2\pi n/N)=K_1\sin\left[(2n+N)\pi/N\right]-K_2\cos\left[(2n+N)\pi/N\right]$, implying that the two solutions are π radians apart. Since (12) is continuous in n, one of the solutions must be the unique maximum and the other must be the unique minimum. So any candidate optima satisfy

(A23) $$r_n-\overline{r}=K_1\cos\left[\tan^{-1}\left(K_2/K_1\right)\right]+K_2\sin\left[\tan^{-1}\left(K_2/K_1\right)\right].$$

Now

(A24) $$\cos\left[\tan^{-1}\left(K_2/K_1\right)\right]=\pm\frac{K_1}{\sqrt{K_1^2+K_2^2}};\qquad \sin\left[\tan^{-1}\left(K_2/K_1\right)\right]=\pm\frac{K_2}{\sqrt{K_1^2+K_2^2}};$$

where the signs must match. Consequently the knowledge $v(r)=0.5K_1^2+0.5K_2^2$ provides

(A25) $$r_n-\overline{r}=\pm\sqrt{K_1^2+K_2^2}=\pm\sqrt{2v(r)}$$

as maximum and minimum. Thus for interior solutions on S we need $0<\overline{r}-\sqrt{2v(r)}$ and $\overline{r}+\sqrt{2v(r)}<\hat{q}$, or

(A26) $$v(r)<0.5\min\left[\overline{r}^2,(\hat{q}-\overline{r})^2\right]\equiv v^{+};$$

Part B): Use Wirtinger's inequality and specification (12) to re-write $\sum_{n=0}^{N-1}(r_n-r_{n-1})^2=4\sin^2(\pi/N)\sum_{n=0}^{N-1}(r_n-\overline{r})^2$ as $2\sum_{n=0}^{N-1}r_n^2-2\sum_{n=0}^{N-1}r_nr_{[n-1]}=4\sin^2(\pi/N)Nv$ or $\left(\sum_{n=0}^{N-1}r_n^2-\overline{r}^2\right)-\left(\sum_{n=0}^{N-1}r_nr_{[n-1]}-\overline{r}^2\right)=2\sin^2(\pi/N)Nv$ or $Nv-N\rho=2\sin^2(\pi/N)Nv$ or

(A27) $$\rho=v-2\sin^2(\pi/N)v=\cos(2\pi/N)v,$$

as $\cos(2\pi/N)\equiv 1-2\sin^2(\pi/N)$. ■

Proof of Proposition 6. Part A): Clearly, setting $v=0$ maximizes the value of $H(\bar{r},v)$ on $v\in[0,v^{\max}]$.

Part B): Setting $v=v^{+}$ maximizes the value of $H(\bar{r},v)$ on $v\in[0,v^{+}]$. The only solutions that have not been ruled out involve at least partial intensification.

Part C): Quasiconcavity rules out a solution on $v\in(v^{+},v^{\max}]$ given an interior maximum on $v\in[0,v^{+}]$. By Corollary 4.2, (13) maximizes the value of $\rho(r)$ for any given values of v and $\bar{r}$. Since $B_{\rho}(\cdot)>0$, any vector r that did not satisfy (13) could be improved upon by replacing it with (13).

Part D) Quasiconvexity with $H(\bar{r},0)<H(\bar{r},v^{+})$ ensures that the solution is in $[v^{+},v^{\max}]$ while all solutions on $v\in[v^{+},v^{\max}]$ must involve at least partial intensification. ■

Proof of Proposition 7. Part A): This is an adaptation of Remark 2, p. 110, in Yamamoto and Ikebe [1979]. Note, from (22), that $\phi_h\phi_l=1$, so that (23) can be rewritten as:

(A28)
$$p_{i,j}=\begin{cases}\left(\phi_h^{i}-\phi_l^{i}\right)\left(\phi_h^{N-1-j}-\phi_l^{N-1-j}\right)\Psi, & i\le j;\\ \left(\phi_h^{j}-\phi_l^{j}\right)\left(\phi_h^{N-1-i}-\phi_l^{N-1-i}\right)\Psi, & i>j.\end{cases}$$

Part B): Since $\Delta>0$ and $\phi_h^{N-1}>\phi_l^{N-1}$, therefore $\Psi>0$. Since $\phi_h>\phi_l, i>0, j>0$, and $N-1-i>0$, $N-1-j>0$ for all $i,j\in\{1,\ldots,N-2\}$, it immediately follows that all terms in parentheses in (A28) are strictly positive, proving the assertion.

Part C): For symmetry, arbitrarily assume that $i\le j$. Using (A28), $p_{i,j}$ and $p_{j,i}$ are:

(A29)
$$\begin{aligned}p_{a,b}&=\left(\phi_h^{a}-\phi_l^{a}\right)\left(\phi_h^{N-1-b}-\phi_l^{N-1-b}\right)\Psi, && a\le b;\\ p_{b,a}&=\left(\phi_h^{a}-\phi_l^{a}\right)\left(\phi_h^{N-1-b}-\phi_l^{N-1-b}\right)\Psi, && b\ge a.\end{aligned}$$

Centro-symmetry follows from inspection of (A28); e.g., $i\le j$ implies $N-1-i\ge N-1-j$ so that $p_{i,j} = (\phi_h^i - \phi_l^i)(\phi_h^{N-1-j} - \phi_l^{N-1-j})$ and

$$\text{(A30)} \qquad \begin{aligned} p_{N-1-i,N-1-j} &= \left(\phi_h^{N-1-j} - \phi_l^{N-1-j}\right)\left(\phi_h^{N-1-(N-1-i)} - \phi_l^{N-1-(N-1-i)}\right) \\ &= \left(\phi_h^{N-1-j} - \phi_l^{N-1-j}\right)\left(\phi_h^i - \phi_l^i\right) = p_{i,j}. \end{aligned}$$

Part D): This is also immediate from inspection of (A28). For $i < j$, and $\phi_h > 1 > \phi_l = \phi_h^{-1}$ then the top line of (A28) is increasing in *i*, whereas the bottom line is decreasing in *i*, implying that $p_{i,j}$ is increasing in *i* for $i < j$ and $p_{i,j}$ is decreasing in *i* for $i > j$, thus proving the conjecture. The second part of the statement, how $p_{i,j}$ varies with *j*, follows from the symmetry $p_{i,j} = p_{j,i}$ in part C).

Part E): Note from centro-symmetry in part C) that $p_{(N-2)/2,j} = p_{(N-2)/2,N-1-j}$. Then set $j = -k + (N-1)/2$ so that $p_{(N-2)/2,-k+(N-1)/2} = p_{(N-2)/2,N-1-(N-1)/2+k} = p_{(N-2)/2,(N-1)/2+k}$.

Part F): From (A28),

$$\text{(A31)} \qquad p_{i,i} = \left(\phi_h^i - \phi_l^i\right)\left(\phi_h^{N-1-i} - \phi_l^{N-1-i}\right)\Psi = \left(\phi_h^i - \phi_h^{-i}\right)\left(\phi_h^{N-1-i} - \phi_h^{-N+1+i}\right)\Psi.$$

Specifying $\Delta_i \equiv p_{i+1,i+1} - p_{i,i}$, it follows that

$$\text{(A32)} \qquad \begin{aligned} \Delta_i &= \left\{-\left(\phi_h^i - \phi_h^{-i}\right)\left(\phi_h^{N-1-i} - \phi_h^{-N+1+i}\right) + \left(\phi_h^{i+1} - \phi_h^{-i-1}\right)\left(\phi_h^{N-2-i} - \phi_h^{-N+2+i}\right)\right\}\Psi \\ &= \left\{-\phi_h^{N-1} + \phi_h^{-N+1+2i} + \phi_h^{N-1-2i} - \phi_h^{-N+1} + \phi_h^{N-1} - \phi_h^{-N+3+2i} - \phi_h^{N-3-2i} + \phi_h^{-N+1}\right\}\Psi \\ &= \left\{-\phi_h^{-N+1+2i}\left(\phi_h^2 - 1\right) + \phi_h^{N-3-2i}\left(\phi_h^2 - 1\right)\right\}\Psi = \left\{\phi_h^{N-3-2i}\left(\phi_h^2 - 1\right)\left(1 - \phi_h^{4i+4-2N}\right)\right\}\Psi. \end{aligned}$$

Since $\phi_h > 1$, Δ_i has the sign of $2N - 4i - 4$, i.e., $\Delta_i \geq 0$ if and only if $i \leq (N-2)/2$. For *i* even and $i' \equiv (N/2) - 1$ then $p_{i,i}$ is increasing in *i* when $i < i'$, $p_{i',i'} = p_{i'+1,i'+1}$, and $p_{i,i}$ is decreasing in *i* when $i > i'$. For *i* odd and $i'' \equiv (N-1)/2$, $p_{i,i}$ increases when $i < i''$, reaches a maximum at $i = i''$ and decreases when $i \geq i''$.

Part G): This requires writing out the relevant terms,

(A33)
$$p_{i,1} = \Psi(\phi_h - \phi_l)\left(\phi_h^{N-1-i} - \phi_l^{N-1-i}\right);$$
$$p_{i,N-2} = \Psi\left(\phi_h^i - \phi_l^i\right)\left(\phi_h^{N-1-(N-2)} - \phi_l^{N-1-(N-2)}\right) = \Psi\left(\phi_h - \phi_l\right)\left(\phi_h^i - \phi_l^i\right).$$

Thus we may define

(A34)
$$A \equiv p_{i,1} + p_{i,N-2} = \Psi\left(\phi_h - \phi_l\right)\left[\left(\phi_h^{N-1-i} - \phi_l^{N-1-i}\right) + \left(\phi_h^i - \phi_l^i\right)\right].$$

Similarly, with

(A35)
$$p_{i-1,1} = \Psi\left(\phi_h - \phi_l\right)\left(\phi_h^{N-i} - \phi_l^{N-i}\right); \qquad p_{i-1,N-2} = \Psi\left(\phi_h^{i-1} - \phi_l^{i-1}\right)\left(\phi_h - \phi_l\right);$$
$$B \equiv p_{i-1,1} + p_{i-1,N-2} = \Psi\left(\phi_h - \phi_l\right)\left[\left(\phi_h^{N-i} - \phi_l^{N-i}\right) + \left(\phi_h^{i-1} - \phi_l^{i-1}\right)\right];$$

then

(A36)
$$\begin{aligned}\Delta_A(i) \equiv A - B &= \left\{\left[p_{i,1} + p_{i,N-2}\right] - \left[p_{i-1,1} + p_{i-1,N-2}\right]\right\} \\ &= \Psi\left(\phi_h - \phi_l\right)\left[\left(\phi_h^{N-1-i} - \phi_l^{N-1-i}\right) + \left(\phi_h^i - \phi_l^i\right) - \left(\phi_h^{N-i} - \phi_l^{N-i}\right) - \left(\phi_h^{i-1} - \phi_l^{i-1}\right)\right] \\ &= \Psi\left(\phi_h - \phi_l\right)\left[\left(\phi_h^{N-1-i}\left(1 - \phi_h\right) - \phi_l^{N-1-i}\left(1 - \phi_l\right)\right) + \phi_h^{i-1}\left(\phi_h - 1\right) + \phi_l^{i-1}\left(1 - \phi_l\right)\right] \\ &= \Psi\left(\phi_h - \phi_l\right)\left(\phi_h - 1\right)\left[\left(-\phi_h^{N-1-i} - \phi_l^{N-1-i}\phi_h^{-1}\right) + \phi_h^{i-1} + \phi_l^{i-1}\phi_h^{-1}\right],\end{aligned}$$

where $(1 - \phi_l) = \phi_h^{-1}(\phi_h - 1)$ has been used. Factorize further to obtain

(A37)
$$\begin{aligned}\Delta_A(i) &= \Psi\left(\phi_h - \phi_l\right)(\phi_h - 1)\left[\left(-\phi_h^{N-1-i} - \phi_h^{-N+i}\right) + \phi_h^{i-1} + \phi_h^{-i}\right] \\ &= \Psi\left(\phi_h - \phi_l\right)(\phi_h - 1)\left[\phi_h^{i-1}\left(1 - \phi_h^{-N+1}\right) - \phi_h^{-i}\left(\phi_h^{N-1} - 1\right)\right] \\ &= \Psi\left(\phi_h - \phi_l\right)(\phi_h - 1)\left(\phi_h^{N-1} - 1\right)\phi_h^{-i}\left(\phi_h^{2i-N} - 1\right).\end{aligned}$$

For *N* even, let $i' \equiv N/2$. Then $\Delta_A(i)$ is negative on $i < i'$ and positive on $i > i'$, implying $p_{i,1} + p_{i,N-2} < p_{i-1,1} + p_{i-1,N-2}$ on $i \in \{2, 3, \ldots, (N/2) - 1\}$, $p_{i,1} + p_{i,N-2} = p_{i-1,1} + p_{i-1,N-2}$ when $i = N/2$ and $p_{i,1} + p_{i,N-2} > p_{i-1,1} + p_{i-1,N-2}$ on $i \in \{(N/2) + 1, \ldots, N-2\}$. For *N* odd, the same reasoning implies $p_{i,1} + p_{i,N-2} < p_{i-1,1} + p_{i-1,N-2}$ on $i \in \{2, 3, \ldots, (N-1)/2\}$, and $p_{i,1} + p_{i,N-2} > p_{i-1,1} + p_{i-1,N-2}$ on $i \in \{(N+1)/2, \ldots, N-2\}$.

Part H): From part C), $p_{i,j} = p_{N-2-i,N-2-j}$. Sum each over *j* to conclude that $P_i = P_{N-2-i}$.

Rewrite (21) in the text as:

(A38)
$$\mathbf{\Gamma r}=\mathbf{b}; \qquad \mathbf{\Gamma}\equiv\begin{pmatrix} 1+2\xi & -\xi & 0 & \cdots & 0 & 0 \\ -\xi & 1+2\xi & -\xi & \cdots & 0 & 0 \\ 0 & -\xi & 1+2\xi & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1+2\xi & -\xi \\ 0 & 0 & 0 & \cdots & -\xi & 1+2\xi \end{pmatrix};$$
$$\mathbf{r}\equiv\begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ \vdots \\ r_{N-3} \\ r_{N-2} \end{pmatrix}; \qquad \mathbf{b}\equiv\frac{1}{\kappa}\begin{pmatrix} \alpha_1+\tau\hat{r}_0 \\ \alpha_2 \\ \alpha_3 \\ \vdots \\ \alpha_{N-3} \\ \alpha_{N-2}+\tau\hat{r}_{N-1} \end{pmatrix}.$$

By definition, $p_{i,j}$ is the element in the i^{th} row and j^{th} column of $\mathbf{\Gamma}^{\mathbf{-1}}$. By definition:

(A39)
$$\begin{pmatrix} P_1 \\ P_2 \\ \vdots \\ P_{N-3} \\ P_{N-2} \end{pmatrix}=\mathbf{\Gamma}^{\mathbf{-1}}\mathbf{U} \qquad \text{where} \qquad \mathbf{U}=\begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ 1 \end{pmatrix}.$$

Also,

(A40)
$$\mathbf{\Gamma U}=\begin{pmatrix} 1+\xi \\ 1 \\ \vdots \\ 1 \\ 1+\xi \end{pmatrix}=\mathbf{U}+\mathbf{E} \qquad \text{where} \qquad \mathbf{E}=\begin{pmatrix} \xi \\ 0 \\ \vdots \\ 0 \\ \xi \end{pmatrix}.$$

Thus:

(A41)
$$\mathbf{U}=\mathbf{\Gamma}^{\mathbf{-1}}\mathbf{\Gamma U}=\mathbf{\Gamma}^{\mathbf{-1}}\left(\mathbf{U}+\mathbf{E}\right)=\mathbf{P}+\mathbf{\Gamma}^{\mathbf{-1}}\mathbf{E}.$$

Now pick off the i^{th} row in $\mathbf{U}$ and write it as U_i. From (A41) it is $U_i=1=P_i+$ $(p_{i,1}+p_{i,N-2})\xi$. Since part B) of the proposition has $p_{i,j}>0$, therefore

(A42) $$P_i = 1-\left(p_{i,1}+p_{i,N-2}\right)\xi < 1$$

Hence:

(A43) $$P_i - P_{i-1} = \xi\left\{\left(p_{i-1,1}+p_{i-1,N-2}\right)-\left(p_{i,1}+p_{i,N-2}\right)\right\} = \frac{\tau}{\kappa}\left\{p_{i-1,1}+p_{i-1,N-2}-p_{i,1}-p_{i,N-2}\right\}.$$

From part G) of the proposition, if N is odd, the expression on the right-hand side of (A43) is positive for $i \in \{2,3,\ldots,(N-1)/2\}$ and negative for $i \in \{(N+1)/2,\ldots,N-2\}$, implying P_i increases for $i \le (N-1)/2$, attains its maximum at $i=(N-1)/2$ and decreases thereafter. Similarly, for N even, the expression on the right-hand side of (A43) is positive for $i \in \{2,3,\ldots,(N/2)-1\}$, zero when $i = N/2$ and negative for $i \in \{(N/2)+1,\ldots,N-2\}$, implying P_i increases for $i \le (N-2)/2$, reaches a maximum at $P_{(N-2)/2} = P_{N/2}$, and decreases thereafter. ■

Proof of Corollary 7.1. From (21) and (23), $dr_n / d\hat{r}_0 = p_{n,1}\tau/\kappa$. Parts D) and C) of the proposition have that $p_{n,1} \equiv p_{1,n}$ is decreasing in the value of n. Similarly, $dr_n / d\hat{r}_{N-1} = p_{n,N-2}\tau/\kappa$ while parts D) and C) have that $p_{n,N-2} \equiv p_{N-2,n}$ is increasing in the value of n. ■

Proof of Corollary 7.2. From inverting (21) in the text we can write

(A44) $$r_i = P_i\alpha + \tau\left(p_{i,1}\hat{r}_0 + p_{i,N-2}\hat{r}_{N-1}\right) = \alpha + \tau\left[p_{i,1}\left(\hat{r}_0 - \frac{\alpha}{\kappa}\right) + p_{i,N-2}\left(\hat{r}_{N-1} - \frac{\alpha}{\kappa}\right)\right],$$

where, in (A44), we use the result from the proof of part H), Proposition 7, that:

(A45) $$P_i = 1-\left(p_{i,1}+p_{i,N-2}\right)\xi; \qquad \xi \equiv \tau/\kappa.$$

Part A): For $\hat{r}_0 = \hat{r}_{N-1} > \alpha/\kappa$, (A44) becomes:

(A46) $$r_i = \alpha + \tau\left(\hat{r}_0 - \frac{\alpha}{\kappa}\right)\left(p_{i,1}+p_{i,N-2}\right).$$

Since part G) of Proposition 7 establishes that the sequence $p_{i,1}+p_{i,N-2}$ is U-shaped, with minimum in the middle of the interval $\{1, \ldots, N-2\}$, and since $\hat{r}_0 = \hat{r}_{N-1} > \alpha / \kappa$, it immediately follows from (A46) that the sequence r_i is also U-shaped with $r_{\min} > \alpha$.

Part B): The logic is the same as part A), except that since $\hat{r}_0 - \alpha / \kappa < 0$, the sequence r_i forms an inverted U, with maximum $r_{\max} < \alpha$ in the middle of the interval, and relative minima at the end points.

Part C): Note that, from Proposition 7, part D), $p_{i,1}$ is monotonically decreasing in *i* and that $p_{i,N-2}$ is monotonically increasing in *i*. Thus, for $\hat{r}_0 - \alpha / \kappa < 0 < \hat{r}_{N-1} - \alpha / \kappa$, it follows that $p_{i,1}\left(\hat{r}_0 - \alpha / \kappa\right) + p_{i,N-2}\left(\hat{r}_{N-1} - \alpha / \kappa\right)$ is monotonically increasing in *i*, proving the assertion.

Part D): Proof of D) is identical to C), except that since $\hat{r}_0 - \alpha / \kappa > 0 > \hat{r}_{N-1} - \alpha / \kappa$, $p_{i,1}\left(\hat{r}_0 - \alpha / \kappa\right) + p_{i,N-2}\left(\hat{r}_{N-1} - \alpha / \kappa\right)$ - and hence r_i - is monotone decreasing in *i*. ■

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