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Using a Constant Elasticity of Substitution Index to Estimate a Cost of Living Index: from Theory to Practice

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Using a Constant Elasticity of Substitution Index to estimate a Cost of Living Index: from Theory to Practice

by

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Abstract

Indexes often incorporate various biases due to their methods of construction. The Constant Elasticity of Substitution (CES) index can potentially eliminate substitution bias without needing current period expenditure data. The CES index requires an elasticity parameter. We derive a system of equations from which this parameter is estimated. We find that consumers are highly responsive to price changes at the elementary aggregation level. The results support the use of a geometric rather than arithmetic mean index at the elementary aggregate level. However, we find that even the use of a geometric mean index at the elementary aggregate level may not sufficiently account for the observed level of consumer substitution.

JEL Classifications: C43, E31
Key words: Price indexes, elasticity of substitution, scanner data.

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1. Introduction

The aim of the Consumer Price Index (CPI) is to provide a timely estimate of the amount of price change that has occurred between two periods. In the past, the measurement of price change for most statistical agencies has been based on obtaining a measure of 'pure' price change. This approach is concerned with the calculation of price change over time with reference to a fixed basket of goods and services. It does not allow for consumers to substitute between items and stores as relative prices change over the period.

In recent years there has been a shift by a number of statistical agencies towards a different basis for measurement of the CPI, that is, towards estimating changes in the cost of living. This type of approach is consistent with taking an *economic approach* to measuring price change. The economic approach is based on a consumer solving an optimization problem which typically involves either maximizing utility, or minimizing expenditure to achieve a certain level of utility, subject to a budget constraint; see e.g. Diewert (1976). From this type of framework, changes in the cost of living can be calculated by what is known as a Cost of Living Index (COLI). More specifically, a COLI aims to measure the change in the cost of purchasing a basket of goods and services in different time periods while holding utility constant. Fundamental to the COLI approach is that consumers are allowed to substitute between items and outlets in response to relative price changes. In other words, the basket of goods is *not* fixed in the COLI approach. To estimate a COLI some measure of utility is needed. However, in practice measuring utility has proved to be a difficult task, which in turn, has made the estimation of a COLI difficult as well.

Diewert (1976, 1978) overcame this problem by identifying a class of index numbers—which he termed superlative index numbers—that he showed could provide a second order approximation to a COLI. Importantly, superlative index numbers do not require some measure of utility in order to be calculated. A feature of superlative indexes is that they require price and expenditure share information on items bought in both a base period and *current* period. As current period information on expenditure shares is typically only available to statistical agencies with a time lag, this makes it difficult (if not impossible) for statistical agencies to produce an approximation to a COLI in a timely manner. As a result, most statistical agencies have kept using a fixed-basket type index.

However, there is an index which can provide an approximation to a COLI without the need for current period quantities or expenditure shares. This index is known as the Constant Elasticity of Substitution (CES) index. The only additional information that statistical agencies would require to estimate a CES index is an elasticity of substitution parameter. The estimation of this parameter is the primary focus of this paper.

Despite the potential for the CES index to be estimated in real time and therefore to be of practical use to statistical agencies, there has been very little research undertaken using this index, particularly in estimating the elasticity parameter. Shapiro and Wilcox (1997) were perhaps the first to attempt to provide an estimate of the elasticity of substitution using the CES index number formula, basically through a system of trial and error. The authors proposed a number of possible values for the elasticity parameter (i.e. $\sigma = 0.6$, $\sigma = 0.7$ and $\sigma = 0.8$) and the 'best fit' estimate was determined by identifying which parameter value minimized the difference between the superlative and CES indexes. They concluded that 'it is possible to produce an approximation to the Törnqvist [superlative] index that is both feasible in real time and quite accurate'. A similar approach is used in this paper, except that rather than using a system of trial and error, an algebraic solution is obtained for the elasticity parameter. This method will be referred to in this paper as the 'algebraic' method.[1]

Econometric methods can also be used to estimate the elasticity of substitution parameter. Shapiro and Wilcox (1997) noted that the estimation of a system of demand equations may be able to 'refine' estimation of the elasticity parameter. Econometric estimation also enables us to statistically test the value of the estimated parameters. In another original contribution, we derive such a system of demand equations for CES preferences.

[1] Opperdoes (2001), de Haan (2005) and Melser (2005) have used a similar approach, proposed by Balk (1999), to estimate the CES elasticity parameter. Melser (2005) combined Balk's (1999) approach with a grid search approach to find the most 'appropriate' elasticity of substitution.

We focus on the estimation of the elasticity parameter at the lowest level of aggregation in the CPI, the elementary aggregate level. This is in contrast to Shapiro and Wilcox (1997) who focused on the estimation of the elasticity parameter at the highest level of aggregation. The estimation and application of the elasticity parameter at the aggregate level is appropriate if we are only interested in obtaining an aggregate measure of price change which approximates a COLI. However, if we are interested in estimates of price change which account for consumer substitution at any of the subaggregate levels in the CPI then this type of approach is inadequate. The aggregate elasticity parameter cannot simply be applied to lower level aggregation units in the CPI as the elasticity of substitution is quite different at different levels of aggregation. This is because as we move to higher levels of aggregation in the CPI the items or item categories which comprise the aggregates become increasingly less homogenous and hence, less substitutable. The most substitution is thought to occur at the elementary aggregate level, where items which are relatively homogenous are grouped together. Therefore, it is at this level that price change estimates are most likely to be affected by the inability to appropriately account for consumer substitution. Importantly, this study will provide some insight into the most appropriate index number formula to be used in practice at the elementary aggregate level.

The paper is set out as follows. The characteristics of elementary aggregate indexes and the CES index are described in Section 2. In Section 3, the 'algebraic' methodology and the econometric methodology used to estimate the elasticity of substitution parameter are detailed. The data used for estimation is described in Section 4, along with the results. Implications for the practical implementation of a CES index are discussed in Section 5.

2. Elementary Aggregate Indexes and the Constant Elasticity of Substitution Index

Elementary aggregates (EAs) are the lowest level of aggregation used in the compilation of the CPI. The ILO (2004) defines an elementary aggregate as being made up of 'expenditures on a small and relatively homogenous set of products defined within the consumption classification used in the CPI.' In practice, at this level of aggregation a number of (hopefully) representative items are sampled to obtain price quotes. Expenditure weights are typically not available.

An understanding of what occurs at the EA level of the CPI is extremely important as 'the elementary aggregates form the building blocks of the CPI …, and the choice of an inappropriate formula at this level can have a tremendous impact on the overall index' (ILO, 2004, p.356). By using highly detailed electronic-point-of-sale, or scanner data, it is possible to obtain an estimate of the 'true' level of substitution that occurs at the EA level. These estimates can provide some insight into which index is most appropriate to use at the EA level and how close we are in practice to capturing the amount of substitution that occurs.

2.1 Elementary Aggregates used in Practice

The Carli (1804), Dutot (1738) and Jevons (1863) indexes are the main elementary aggregate indexes that have been used in practice in the construction of the CPI. These indexes take the following forms:

$$Carli_{t-1}^{t} = \frac{1}{N}\sum_{i=1}^{N}\frac{p_i^t}{p_i^{t-1}} \quad , \tag{1}$$

$$Dutot_{t-1}^{t} = \frac{\sum_{n=1}^{N}\frac{p_n^t}{N}}{\sum_{n=1}^{N}\frac{p_n^{t-1}}{N}} \quad , \tag{2}$$

$$Jevons_{t-1}^{t} = \prod_{n=1}^{N}\left(\frac{p_n^t}{p_n^{t-1}}\right)^{\frac{1}{N}} \quad , \tag{3}$$

where p_n^t is the price of item n in period t, for n = 1…N items in each chosen period.

The Carli index in (1) is equal to the arithmetic mean of the *N* price ratios. The Dutot index in (2) is equal to the arithmetic mean of the *N* period t prices divided by the arithmetic mean of the *N* period t-1 prices. If the economic approach to index numbers is taken then, under certain sampling schemes, these two indexes are consistent with the assumption that no substitution occurs between goods at the elementary aggregate level; see e.g. Balk (2005). That is, the cross item elasticities are all assumed to be zero.

An alternative method for aggregating the price relatives at the EA level is by using a geometric mean 'Jevons' index.[2] If the relative prices of items in the commodity class are sampled using weights that are proportional to their base period expenditure shares, then the Jevons index is equivalent to a Cobb-Douglas index, where 1/N in (5) is replaced by the period 0 expenditure share for each item n. So, the Jevons index can be justified as an approximation to an underlying Cobb-Douglas index under an appropriate sampling scheme. The Cobb-Douglas index is consistent with the assumption that the cross item elasticities of substitution are one, and hence with appropriate sampling the Jevons index has the same property.

In recent years many statistical agencies have moved from the use of an arithmetic mean elementary aggregate index such as the Dutot or Carli indexes, to the use of the geometric mean Jevons index.[3] This largely reflects a move by statistical agencies to try to account for consumer substitution at the elementary aggregate level in the CPI. One of the main goals of this paper is to determine whether the Jevons index does in fact provide a closer representation of consumer behaviour than a Carli or Dutot index, and how close these indexes are to capturing the 'true' level of substitution at the elementary aggregate level. This can be done by comparing the implied elasticities of the standard elementary aggregate indexes to the 'actual' elasticity of substitution. In the next section we explain how the CES and superlative index number formulae can be used to calculate the elasticity of substitution.

[2] See Silver and Heravi (2007) for a theoretical and empirical comparison of the Dutot and Jevons indexes using a scanner data set on television sets.

[3] For example, Australia began using the Jevons index formula in 1998 but still uses the Dutot index in some cases where it considers the use of a Jevons index inappropriate, such as when the price of an item becomes zero (e.g. the government fully subsidises an item) or when consumers cannot substitute (e.g. land taxes) (ABS, 2005). The United States began using a geometric mean formula for most of its elementary indexes in January 1999 (BLS, 2006).

2.2 The Constant Elasticity of Substitution Index

A Constant Elasticity of Substitution (CES) index formula is also widely known as a Lloyd-Moulton Index, acknowledging that both Lloyd (1975) and Moulton (1996) showed that the CES index is 'exact' for a COLI if the utility of an economic agent comes from the CES family and if expenditure shares are taken from the same period as the denominator of the price relatives. In this context 'exact' refers to a price index which is found to be equal to the ratio of minimum costs needed to obtain a fixed level of utility using price vectors from two different periods (Diewert, 1976). The CES index is defined as follows:

$$CES_{t-1}^{t} \equiv \left[\sum_{n=1}^{N} s_n^{t-1} \left(\frac{p_n^t}{p_n^{t-1}} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \qquad (4)$$

where p_n^t is the price of item n in period t, s_n^{t-1} is the expenditure share of item n in period t-1, and σ is the constant elasticity of substitution between any pair of items, for items n = 1,…, N and periods t = 1,…,T.

The elasticity parameter σ refers to the cross item elasticity of substitution. This captures the percentage change in the quantity of one good demanded from a 1 percent change in the price of the other good. The CES index number formula implies that the elasticities of substitution between any and all pairs of items are constant.

A feature of the CES index is that if σ equals zero then the index becomes the standard Laspeyres index:

$$Laspeyres_{t-1}^{t} = \sum_{i=1}^{N} s_i^{t-1} \left(\frac{p_i^t}{p_i^{t-1}} \right) \ . \qquad (5)$$

A Laspeyres-type index is typically used by statistical agencies to aggregate the sub-components of the CPI above the elementary aggregate level. The Laspeyres index is a fixed basket index which uses base period expenditure share weights to calculate price change. This index implicitly assumes no substitution. Another feature of the CES index is that as σ approaches 1 the index approaches the Jevons index of equation (3).[4]

An important feature of the CES index is that estimation of this index does not rely on current period expenditure weights. Compared with calculating a Laspeyres index the only additional information that is needed by a statistical agency is an estimate of the elasticity of substitution. With an accurate estimate of the elasticity parameter, a CES index can provide a technically feasible method for statistical agencies to produce a close approximation to a COLI in a timely manner.

An additional feature of the CES index is that the elasticity parameter is specified to be independent of time. For a CES index to be used in practice a fairly stable estimate of the elasticity of substitution parameter over time is needed. If, in practice, the parameter value is unstable it would have to be constantly updated and statistical agencies would only know with a lag whether the estimate used was in fact appropriate. If the estimated elasticity parameter is larger than the actual elasticity of substitution, then use of this parameter would lead to an over-adjustment of price change in any particular period, i.e. the estimate of price change would be too low. The converse is true if the elasticity parameter is too low. The cumulative impact of over-adjustment or under-adjustment in the CPI may, in the long run, be considerable. Therefore, the stability of the elasticity parameter is crucial to the successful application of the CES index in practice.

3. CES Estimation Methods

In this section the two alternative methods that will be used to estimate the CES parameter are more fully described. The methods are referred to in this paper as the 'algebraic method' and the 'econometric method'.

3.1. The Algebraic Method of Estimation

[4] Note that the CES index is undefined when $\sigma = 1$.

The algebraic method refers to an approach where a set of equations are solved algebraically to obtain the value of the CES parameter. To solve for an estimate of the elasticity of substitution using *direct* indexes the following equation was used:

$$0 = \left[\sum_{n=1}^{N} s_n^0 \left(\frac{p_n^t}{p_n^0} \right)^{1-\sigma_0^t} \right]^{\frac{1}{1-\sigma_0^t}} - Fisher_0^t \quad , \tag{6}$$

where σ_0^t is the constant elasticity of substitution between the base period 0 and current period t, for any pair of items in n =1,…, N, and $Fisher_0^t$ is the Fisher (1922) Ideal price index between periods 0 and t:

$$Fisher_0^t = \left[\sum_{n=1}^{N} s_n^0 \left(\frac{p_n^t}{p_n^0} \right) \Big/ \sum_{n=1}^{N} s_n^t \left(\frac{p_n^0}{p_n^t} \right) \right]^{0.5(s_n^0 + s_n^t)} .$$

To solve for an estimate of the elasticity of substitution using *chained* indexes the following equation was used:

$$0 = \left[\sum_{n=1}^{N} s_n^{t-1} \left(\frac{p_n^t}{p_n^{t-1}} \right)^{1-\sigma_{t-1}^t} \right]^{\frac{1}{1-\sigma_{t-1}^t}} - Fisher_{t-1}^t \tag{7}$$

where σ_{t-1}^t is the constant elasticity of substitution between the base period t-1 and current period t, for any pair of items in n =1,…, N, and $Fisher_{t-1}^t$ is the Fisher (1922) Ideal price index between periods t-1 and t:

$$Fisher_{t-1}^t = \left[\sum_{n=1}^{N} s_n^{t-1} \left(\frac{p_n^t}{p_n^{t-1}} \right) \Big/ \sum_{n=1}^{N} s_n^t \left(\frac{p_n^{t-1}}{p_n^t} \right) \right]^{0.5(s_n^{t-1} + s_n^t)} .$$

In equations (6) and (7) the estimates of σ depend on the time period used, t. In the original specification of the CES function the elasticity parameter is assumed to be independent of time. One of the aims of this paper is to establish whether, in fact, the estimated elasticity parameter is constant over time.

The software package Matlab was used to solve for the elasticity of substitution parameter. As the Fisher index approximates a COLI to the second order at a point, then a CES index using σ solved from (6) or (7) will similarly approximate a COLI.

3.2 Aggregation Methods Used in the Algebraic Approach

In practice there are a number of different ways that the CES and the Fisher indexes can be estimated. In this study the indexes were constructed in the following ways:

1. average prices and total quantities were aggregated over quarterly intervals;
2. goods were then, in turn, treated as the same good no matter which store they were purchased in (referred to as item aggregation over stores) or treated as different goods if they were not located in the same store (referred to as no item aggregation over stores) ; and
3. direct and chained indexes were estimated.

3.3 Econometric Method of Estimation

We derive a system of equations from which the CES parameter can be estimated. An important advantage of using econometric methods over the algebraic method is that the parameter estimates will have standard errors. The standard errors allow us to statistically test whether the elasticity parameter is equal to zero, one or any other value of interest.

3.4 Derivation of the Econometric Model

Assume that the consumer's utility function f(q), where q = quantity of items 1…N, is linearly homogenous and the corresponding unit cost function is c(p) is defined as follows:

$$c(p) \equiv \alpha_0 \left[\sum_{n=1}^{N} \alpha_n p_n^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \qquad \text{if } \sigma \neq 1, \text{ or} \qquad (8)$$

$$\ln c(p) \equiv \alpha_0 + \sum_{n=1}^{N} \alpha_n \ln p_n \qquad \text{if } \sigma = 1, \qquad (9)$$

where α_n and σ are non-negative parameters and $\sum_{n=1}^{N} \alpha_n = 1$.

The unit cost function defined in equation (8) corresponds to a Constant Elasticity of Substitution (CES) aggregator function, where σ equals the elasticity of substitution.

Assume cost minimizing behaviour by the consumer in period 0 and that the aggregate (unobservable) level of period 0 utility is $u_0 = f(q_0)$. Then, applying Shephard's Lemma to equation (8) we obtain:

$$q_n^0 = u^0 \alpha^0 \left[\sum_{k=1}^{N} \alpha_k (p_k^0)^r \right]^{\frac{1}{r}-1} \alpha_n (p_n^0)^{r-1}, \qquad r \equiv 1 - \sigma \neq 0, \quad (10)$$

$$= \frac{u^0 c(p^0) \alpha_n (p_n^0)^{r-1}}{\sum_{k=1}^{N} \alpha_k (p_k^0)^r}, \qquad n=1,\ldots,N. \quad (11)$$

We can rewrite equation (11) as:

$$\frac{p_n^0 q_n^0}{u^0 c(p^0)} = \frac{\alpha_n (p_n^0)^r}{\sum_{k=1}^{N} \alpha_k (p_k^0)^r} \quad . \quad (12)$$

To eliminate the utility parameter, set period 0 expenditure equal to observed expenditure (Y_0) on the N commodities during period 0, so that $u_0 c\left(p^0\right) = Y^0 \equiv \sum_{n-1}^{N} p_n^0 q_n^0$ and substitute this into equation (12). This yields an equation for the consumer's n^th^ expenditure share in period 0:

$$s_n^0 \equiv \frac{p_n^0 q_n^0}{Y^0} = \frac{\alpha_n (p_n^0)^{(r)}}{\sum_{k=1}^{N} \alpha_k (p_k^0)^{(r)}} \quad . \quad (13)$$

Similarly, the expenditure share in period t, s_n^t, can be written as:

$$s_n^t \equiv \frac{p_n^t q_n^t}{Y^t} = \frac{\alpha_n (p_n^t)^{(r)}}{\sum_{k=1}^{N} \alpha_k (p_k^t)^{(r)}} \quad . \tag{14}$$

To identify the $\alpha_1, \alpha_2, \ldots, \alpha_n$ parameters, each equation in (14) can be divided by, for example, the first equation in period t, to obtain the following system of equations:

$$\frac{s_n^t}{s_1^t} = \frac{\alpha_n \left(p_n^t\right)^r}{\alpha_1 \left(p_1^t\right)^r} \,, \qquad n = 2,\ldots, N. \tag{15}$$

Taking logs of this system and adding error terms gives the following:

$$\ln\left[\frac{s_n^t}{s_1^t}\right] = \beta_n + r\ln\left[\frac{p_n^t}{p_1^t}\right] + e_n^t \quad , \tag{16}$$

where $\beta_n = \ln\alpha_n - \ln\alpha_1$ and e_n^t is a stochastic error term, for items n = 2,…,n and periods t = 0, 1,…, T.

In the above system of equations the first commodity plays an asymmetric role in estimation. To circumvent this, we can divide the price of an item by the geometric mean of prices of all items in that period. [5]

The geometric mean of α_n is defined as:

$$\alpha_\bullet \equiv \left[\prod_{n=1}^{N} \alpha_n\right]^{\frac{1}{N}} . \tag{17}$$

The geometric mean of the period t expenditure shares, s_n^t , is defined as :

$$s_\bullet^t \equiv \left[\prod_{n=1}^{N} s_n^t\right]^{\frac{1}{N}} . \tag{18}$$

The geometric mean of the period t prices, p_n^t , is defined as :

[5] Alternatively, the geometric mean of the expenditure shares in equation (14) can be taken to circumvent the asymmetry problem, leading to equation (21).

$$p_{\bullet}^{t} \equiv \left[\prod_{n=1}^{N} p_{n}^{t}\right]^{\frac{1}{N}}. \tag{19}$$

Using equation (14) and the definitions from equations (17), (18) and (19) we can obtain:

$$s_{\bullet}^{t} = \frac{\alpha_{\bullet}(p_{\bullet}^{t})^{r}}{\sum_{k=1}^{N}\alpha_{k}(p_{k}^{t})^{r}}. \tag{20}$$

If the N period equations from equation (14) are divided by equation t in (20) then the following system of equations is obtained:

$$\frac{s_{n}^{t}}{s_{\bullet}^{t}} = \frac{\alpha_{n}(p_{n}^{t})^{(r)}}{\alpha_{\bullet}(p_{\bullet}^{t})^{(r)}}, \qquad n = 1,\ldots, N;\ t = 0, 1,\ldots, T. \tag{21}$$

Taking logarithms of both sides of equation (21) and adding an error term e_{n}^{t} generates the following system of equations:

$$\ln\frac{s_{n}^{t}}{s_{\bullet}^{t}} = \gamma_{n} + r\ln\left[\frac{p_{n}^{t}}{p_{\bullet}^{t}}\right] + e_{n}^{t}, \quad n = 1,\ldots, N;\ t = 0, 1,\ldots, T, \tag{22}$$

where $\gamma_n = \ln\alpha_n - \ln\alpha_{\bullet}$. This system of equations is linear in its parameters, γ_n and $r = (1-\sigma)$. From equation (17) and using the definition of γ_n, it can be shown that:

$$\sum_{n=1}^{N}\gamma_{n} = 0. \tag{23}$$

From equations (18), (19), (22), and (23) we obtain:

$$0 = \sum_{n=1}^{N}\ln\left[\frac{s_{n}^{t}}{s_{\bullet}^{t}}\right] \tag{24}$$

$$= \sum_{n=1}^{N}\gamma_{n} + r\sum_{n=1}^{N}\ln\left[\frac{p_{n}^{t}}{p_{\bullet}^{t}}\right] + \sum_{n=1}^{N}e_{n}^{t}$$

$$= 0 + r[0] + \sum_{n=1}^{N} e_n^t$$

$$= \sum_{n=1}^{N} e_n^t \tag{25}$$

For any period t, the errors sum to zero and hence are not independently distributed. To estimate the system of equations as specified in equation (22) one of the equations from the time period t must be dropped, leading to (N-1)(T+1) degrees of freedom. If Seemingly Unrelated Regression (SUR) is used to estimate the system then the parameter estimates will be invariant to the equation that is dropped (Zellner, 1962, 1963).

The system of equations can be further simplified so that econometric modelling is not required to estimate the CES parameter. Using results from equations (23) and (25), we get the following simplified equation:

$$\frac{\sum_{n=1}^{N} \ln\left[\frac{s_n^t}{s_\bullet^t}\right]}{\sum_{n=1}^{N} \ln\left[\frac{p_n^t}{p_\bullet^t}\right]} = r\,. \tag{26}$$

Thus, equations (22) and (26) provide two ways the CES parameter could potentially be estimated in this framework. Initially equation (26) was used. The resulting CES estimates were unsatisfactory as they were found to be highly volatile across aggregation methods, and in some cases, quite unrealistic. Due to the unsatisfactory nature of these results we turned to econometric estimation.

A SUR approach was used to estimate the system of equations as specified in equation (22). In this system there were T+1 observations for each equation and N 'normalised expenditure share' and 'normalised price' variables. Each equation regresses the price of item n on the expenditure share of item n for all periods t = 0, 1,…, T.

An issue that should be addressed before proceeding is that of potential endogeneity between prices and quantities (or expenditure). In the economic approach to index numbers a consumer (or household) solves a utility maximization (or cost minimization) problem. The consumer chooses a bundle of goods to maximize utility (or minimize cost) subject to some budget constraint. In this framework, prices are assumed to be exogenous. In essence, households regard the observed price data as given, while the quantity data are regarded as solutions to the various optimization problems. This assumption appears to be fairly reasonable for our analysis as we use data on items which are bought in a supermarket, where prices are generally taken as given by consumers.

3.5 Aggregation Methods used in the Econometric Approach

Aggregation methods used in the econometric approach vary somewhat to those used in the algebraic approach. This was mainly due to data considerations. When using the econometric approach, as the number of items in each item category rises data management and program coding becomes extremely cumbersome and increasingly difficult. Therefore, econometric estimation was not undertaken for the aggregation method which gave the most disaggregated data set – that is, no item aggregation over stores. Aggregation methods used in the econometric method were:

1) average prices and total quantities were aggregated over weekly, monthly and quarterly intervals respectively; and
2) goods were treated as the same good no matter which store they were in (ie. identical items were aggregated over stores in forming total quantities and unit value prices).

4. Data and Results

This study uses a scanner data set collected by A.C. Nielsen. The data set contains information on four supermarket chains located in one of the major capital cites in Australia. In total, over 100 stores are included in this data set with these stores accounting for approximately 80% of grocery sales in this city (Jain and Abello, 2001). The data set contains 65 weeks of data, collected between February 1997 and April 1998. Information on 19 different supermarket item categories, such as bread and biscuits, are included. Within each product category, information is available on the range of brands found within the item category in each of the stores.

The data set includes a large number of observations for nine item categories over a 65 week period. An observation here refers to the average weekly price (weekly unit value) and total weekly quantity of each item sold in each store in each week. The smallest number of observations for a particular item category is approximately 246,000 for tin tomatoes, while the largest number of observations is just under two and a half million for the item category soft drinks. The highly detailed nature of the data enables us to estimate a superlative index and obtain an estimate of the elasticity of substitution.

To briefly recap, the elasticity of substitution is estimated for a number of different aggregation methods. For the algebraic method, estimates are based on quarterly aggregation over time, with item aggregation over stores and also, no item aggregation over stores. For the econometric method, estimates were based on quarterly, monthly and weekly time aggregation, and item aggregation over stores. These different aggregation methods are taken into account when comparing estimation results across methods.

The results calculated using the algebraic method for chained indexes with item aggregation over stores and no item aggregation over stores are presented in table 2. The corresponding results for direct indexes are presented in table 3. To provide a check on the values of the calculated elasticity parameters the values for σ were substituted into the CES index and the CES index was then calculated. The results for the CES index were then compared with the Fisher index. This comparison shows that, in general, the estimated CES index approximates the Fisher index quite closely. Absolute differences between the Fisher and CES chained index links (i.e. indexes estimated between periods 1-2, 2-3, 3-4 and 4-5) range from 0.00% to 0.177%, with a mean difference of 0.023%. Absolute differences between Fisher and CES direct indexes (i.e. indexes between periods 1-2, 1-3, 1-4 and 1-5) range from 0.00% to 0.130%, with a mean difference of 0.014% .

Elasticity estimates using the econometric method were estimated for weekly, monthly and quarterly time aggregation with item aggregation over stores. An interesting point to note is that a small number of the estimates of the elasticity of substitution were found to be negative. This result is contrary to what standard economic theory predicts. If two goods, items A and B, are close substitutes then an increase in the price of item A should lead to an increase in the quantity purchased of item B. A negative elasticity parameter is a result that was also found by Melser (2005) and provides evidence of some possible 'non-standard' behaviour by consumers. A possible behavioural explanation for this finding may be consumer loyalty to particular brands or stores. Time constraints faced by consumers may also play a role. For example, if a consumer is pressed for time they may purchase the most convenient item rather than the cheapest or best value item. More information about consumer purchasing behaviour could be important in helping understand whether, and in what contexts, a negative elasticity parameter is in fact reasonable.

In general, the elasticity parameter estimates indicate that quite a high degree of consumer substitution occurs for most of the item categories included in this study. In the overwhelming majority of cases the value of the elasticity parameter was found to be much closer to one than to zero. The size of the elasticity parameter for many of the item categories was surprisingly high. When the algebraic method was used just over 90% of the estimates were found to have a value higher than one. For example, elasticity estimates using the algebraic method for the item category toilet paper ranged from 1.68 to 4.98 over all different types of aggregation. These results indicate that for many item categories there is a high level of consumer substitution.

Table 4 reports results from applying the econometric approach of equation (22). Using this approach we are able to formally test the value of the estimated r parameter. As the parameter estimate $r = 1 - \sigma$, a test for $r = 0$ is equivalent to a test for unit elasticity, $\sigma = 1$. Hence, the hypothesis tested was H_o: $r = 0$, H_1: $r \neq 0$, using a standard t test. For the case of weekly aggregation, based on the t-values in table 4, with a critical value of 1.96, the hypothesis that $r = 0$ and hence $\sigma = 1$ was rejected for all but one of the nine item categories at the 5% significance level. With monthly aggregation, the hypothesis was rejected for six of the nine item categories at the same significance level. With quarterly aggregation, the hypothesis was rejected for seven of the nine item categories. In section 2.2 we saw that when σ =0, the CES index becomes the standard Laspeyres index. It was of interest to test whether the assumption of no item substitution was appropriate for any of our item categories. The hypothesis H_o: $r = 1$ (or σ =0) , H_1: $r \neq 1$ (or σ =0) was tested. Again, using the standard t-test and a critical value of 1.96 the hypothesis that r = 1 (or σ =0) was rejected in the overwhelming majority of cases. In only two cases, one with monthly aggregation and one with quarterly aggregation, the hypothesis could not be rejected.

Overall, the results show that in the majority of cases the value of σ was found to be either not statistically significantly different from one, or significantly greater than one. These results provide strong support for the use of a Jevons index rather than a Carli or Dutot index at the elementary aggregate level. The results also indicate that for some item categories even the use of Jevons index, which assumes a cross-item elasticity of one, may not be adequate to capture the 'true' level of consumer substitution.

An important finding is that the method of aggregation used appears to have a considerable impact on the estimate of the CES parameter. Diewert (1974) showed that, in theory, increasingly higher levels of aggregation should lead to increasingly smaller elasticities of substitution, and conversely, increasingly lower levels of aggregation should lead to increasingly higher elasticities of substitution. In general, this seems to occur with our estimates. When using the algebraic method, the elasticity parameter is typically found to be lower when we compare estimates where item prices and quantities have been aggregated over stores to when there is no aggregation of prices and quantities of items over stores; see table 3. Similarly, when using the econometric method, as aggregation over time increases—from weekly to monthly to quarterly—we generally observe a corresponding fall in the elasticity parameter. For example, if we look at the estimates from the econometric method for the item category biscuits, the elasticity parameter goes from 2.38 to 1.05 to -0.65 where the only change that occurs between these estimates is that of the time period aggregation. These types of changes in the elasticity parameter are not negligible and make it difficult to come up with one estimate which is thought to reflect the 'true' value of the elasticity parameter. The results show that the t statistics for weekly time aggregation are an order of magnitude higher than for monthly or quarterly time aggregation (see table 4). This indicates that weekly time aggregation may be the appropriate unit of aggregation to use when estimating the elasticity parameter. It does not seem implausible to suggest that many shoppers may do their grocery shopping on a weekly basis. In this case, weekly time aggregation would be appropriate.

The stability of the CES parameters over time, and preferably over estimation methods, is extremely important if the CES index is to be adopted in practice. For the chained estimates from the algebraic method, stability of the estimates across time does not appear to hold for most item categories, with estimates showing considerable variation from quarter to quarter. For example, if we take the case of soft drinks where there is item aggregation over stores (see table 3) the estimates of elasticity for each of the four periods are 1.93, 2.22, 3.39 and 1.49. This high period-to-period variation in the estimated elasticity parameter was also found by Opperdoes (2001) and de Haan (2005). When direct indexes are used to estimate the elasticity parameter a high degree of variation in the estimates is still observed. It may be that the quarterly time period over which the elasticity parameter is estimated is not long enough to average out the influence of large short-term shifts in consumer purchases due to short-term sales. With the econometric method, we estimated an elasticity parameter over five quarters but cannot compare this across time as no more data were available. It seems possible that the estimation of perhaps a yearly CES parameter would provide more stable CES estimates over time.

Using the parameter standard errors from the econometric method, it is possible to formally test whether the elasticity estimates from the algebraic and econometric methods are equivalent. Statistical tests were performed on the set of estimates where quarterly time aggregation and item aggregation over stores was considered; see table 5. The t-tests are for the equality of the econometric elasticity parameter estimate and its algebraic counterpart, which include the algebraic estimates for periods 0-1, 1-2, 2-3 and 3-4 respectively. The results show that in the majority of cases (28 of the 36) the parameter estimates for the algebraic method were found to be significantly different to the econometric estimates at the 5% significance level.

It was of interest to see whether averaging the CES estimates over the 4 periods and testing this average estimate against the econometric method would provide more consistent results; see table 6. Significant disparities between the two estimation methods still remained. Six out of the nine tests for equality across the average algebraic CES estimates and the econometric CES estimates were rejected at the 5% significance level. Based on these results we find that the econometric and algebraic methods for estimating the CES parameter seem to generate significantly different results in the majority of cases.

The variability of the elasticity parameter as we move across different estimation methods provides some cause for concern. Clearly, the choice of estimation method has non trivial implications.

5. Discussion

Substitution bias has been identified as an important source of bias in the CPI (Boskin Commission, 1996). The CES index is an index which can (approximately) account for consumer substitution and which has the potential to be implemented in real time. The only additional information statistical agencies need to do this is an estimate of the elasticity parameter. A key contribution of this paper is the derivation of a system of equations from which the CES parameter can be estimated. Importantly, this approach allows us to statistically test the value of the elasticity parameter.

One of the key findings of this work is that consumers appear to be very responsive to price changes at the elementary aggregation level and that item substitution is an important characteristic of consumer shopping behaviour. Our results indicate that the move by many statistical agencies in recent years from the use of an arithmetic to a geometric mean index at the elementary aggregate should better capture actual consumer behaviour. However, our results also suggest that even the use of a geometric mean index at the elementary aggregate level may not sufficiently account for the 'true' level of consumer substitution.

The results also highlight some important issues in the practical application of the CES index. This paper has shown that the elasticity parameter is sensitive to the method of aggregation used, both over items and over time. Currently there is very little evidence to guide price statisticians on the appropriate method of aggregation to be used when detailed price and quantity data is available; for a recent contribution, see Ivancic, Diewert and Fox (2009). The identification of appropriate aggregation methods which lead to a more stable elasticity of substitution parameter would mean that the CES index could move from being a theoretical possibility to an index that could be reliably applied in practice.

Table 1. Overview of the Data

Items	Weekly Observations	Number of items
Biscuits	2,452,797	1,322
Coffee	514,945	149
Oil	483,146	314
Pasta	1,065,204	706
Soft drinks	2,140,587	964
Spreads	283,676	102
Sugar	254,453	114
Tin tomatoes	246,187	164
Toilet paper	438,525	128

2. Algebraic Method: Chained Index Elasticity Estimates

Item Aggregation Over Stores				
	σ_0^1	σ_1^2	σ_2^3	σ_3^4
Biscuits	2.42552	2.67365	3.39576	-1.43016
Coffee	2.38707	2.63075	0.00001	3.49111
Oil	2.95129	2.35201	3.68231	3.89971
Pasta	0.00001	0.00001	0.72025	2.31540
Soft drinks	1.93019	2.22218	3.38553	1.49117
Sugar	0.00001	0.00001	0.00001	-0.50455
Spreads	0.00001	0.00001	0.00001	0.00001
Tin tomatoes	3.09080	-0.50464	3.38012	2.35791
Toilet paper	3.47736	3.64392	3.40452	1.67633

No Item Aggregation Over Stores				
	σ_0^1	σ_1^2	σ_2^3	σ_3^4
Biscuits	2.97642	3.0222	3.01771	0.99669
Coffee	3.09551	2.61364	4.31422	4.07921
Oil	3.47684	2.73481	3.23877	3.35277
Pasta	1.46939	1.57228	1.32993	1.57560
Soft drinks	2.93036	3.71392	3.79730	2.81774
Sugar	3.52211	2.67001	2.75629	0.58465
Spreads	3.60853	1.86547	2.75098	2.79398
Tin tomatoes	2.42661	2.73356	2.78944	2.66333
Toilet paper	4.18811	4.97844	4.07722	3.10417

Note: σ_{t-1}^{t} = constant elasticity of substitution between the base period t-1, and current period t, t = 1,…, T, for any pair of items in n =1,…, N.

3. Algebraic Method: Direct Index Elasticity Estimates

Item Aggregation Over Stores				
	σ_0^1	σ_0^2	σ_0^3	σ_0^4
Biscuits	2.42551	1.13374	-1.3552	0.76635
Coffee	2.38714	2.30405	1.76114	2.21414
Oil	2.95129	1.45592	0.96131	1.22036
Pasta	0.00001	-0.48086	1.77028	2.04616
Soft drinks	1.93029	1.65979	1.92837	1.68923
Sugar	0.00100	0.00100	1.33781	0.64028
Spreads	0.00100	1.07248	2.19402	3.09059
Tin tomatoes	3.09092	-0.35002	1.06939	0.00001
Toilet paper	3.47710	2.41553	2.90105	1.90301

No Item Aggregation Over Stores				
	σ_0^1	σ_0^2	σ_0^3	σ_0^4
Biscuits	2.97609	2.56594	2.25314	2.4264
Coffee	3.09596	2.51715	2.48718	3.08167
Oil	3.47700	1.69209	1.25222	1.25128
Pasta	1.46924	1.20454	1.37977	1.16107
Soft drinks	2.93036	2.9030	2.92257	2.94198
Sugar	3.52154	2.57774	2.19588	1.54602
Spreads	3.61055	3.24788	3.82685	2.48245
Tin tomatoes	2.42661	2.46470	2.56142	2.30435
Toilet paper	4.18815	4.19266	3.37434	3.13625

Note: σ_0^t = constant elasticity of substitution between the base period 0, and period t=1…5, for any pair of items in n =1,…, N.

Table 4. Econometric Method: Quarterly, Monthly and Weekly Elasticity Estimates

Quarterly Time Aggregation

	Parameter Estimate (r)	Standard Error	T-value	CES Estimate of σ=(1-r)
Biscuits	1.652	0.169	9.76	-0.652
Coffee	1.762	0.291	6.05	-0.762
Oil	-0.490	0.190	-2.58	1.490
Pasta	-1.544	0.231	-6.68	2.544
Soft drinks	1.458	0.200	7.30	-0.458
Sugar	0.009	0.367	0.02	0.991
Spreads	0.928	0.208	4.45	0.072
Tin tomatoes	-0.336	0.626	-0.54	1.336
Toilet paper	-2.485	0.500	-4.97	3.485

Monthly Time Aggregation

	Parameter Estimate (r)	Standard Error	T-value	CES Estimate of σ=(1-r)
Biscuits	-0.045	0.082	-0.55	1.045
Coffee	-0.782	0.161	-4.85	1.782
Oil	-1.002	0.081	-12.44	2.002
Pasta	-0.247	0.108	-2.29	1.247
Soft drinks	-1.090	0.086	-12.71	2.090
Sugar	0.248	0.157	1.58	0.752
Spreads	-1.450	0.346	-4.20	2.450
Tin tomatoes	0.599	0.454	1.32	0.401
Toilet paper	-3.239	0.270	-12.01	4.239

Weekly Time Aggregation

	Parameter Estimate (r)	Standard Error	T-value	CES Estimate of σ=(1-r)
Biscuits	-1.382	0.045	-30.46	2.382
Coffee	-1.951	0.065	-29.81	2.951
Oil	-2.029	0.050	-40.59	3.029
Pasta	-1.989	0.031	-63.26	2.989
Soft drinks	-2.658	0.024	-111.94	3.658
Sugar	-0.153	0.079	-1.94	1.153
Spreads	-1.751	0.145	-12.08	2.751
Tin tomatoes	-2.896	0.137	-21.17	3.896
Toilet paper	-4.727	0.075	-63.38	5.727

Table 5. t-values for Hypothesis Test: σ (Econometric) = σ_{t-1}^{t} (Algebraic)

	t-values			
	Period 0-1	**Period 1-2**	**Period 2-3**	**Period 3-4**
Biscuits	-18.21	-19.68	-23.95	4.60
Coffee	-10.82	-11.66	-2.62	-14.62
Oil	-7.69	-4.54	-11.54	-12.68
Pasta	-6.68	-6.68	-9.80	-16.71
Soft drinks	-11.94	-13.40	-19.22	-9.75
Spreads	2.70	2.70	2.70	4.08
Sugar	0.35	0.35	0.35	0.35
Tin tomatoes	-2.80	2.94	-3.27	-1.63
Toilet paper	0.02	-0.32	0.16	3.62

Note: σ_{t-1}^{t} (Algebraic) refers to the elasticity parameters from the algebraic method with chaining, for periods t = 1,…, 4; see equation (9). σ (Econometric) refers to the elasticity estimate from the econometric method; see equation (24).

Table 6. t-values for Hypothesis Test: σ (Econometric) = σ (Algebraic)

	T-values
Biscuits	-14.31
Coffee	-9.93
Oil	-9.11
Pasta	7.73
Soft drinks	-13.58
Spreads	3.04
Sugar	0.35
Tin tomatoes	-1.19
Toilet paper	0.87

Note: σ (Algebraic) refers to the average quarterly elasticity estimate.

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