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Abstract

The robustness of the LM tests for spatial error dependence of Burridge (1980) for the linear regression model and Anselin (1988) for the panel regression model are examined. While both tests are asymptotically robust against distributional misspecification, their finite sample behavior can be sensitive to the spatial layout. To overcome this shortcoming, standardized LM tests are suggested. Monte Carlo results show that the new tests possess good finite sample properties. An important observation made throughout this study is that the LM tests for spatial dependence need to be both mean- and variance-adjusted for good finite sample performance to be achieved. The former is, however, often neglected in the literature.

Key Words: Distributional misspecification; Group interaction; LM test; Moran's I Test; Robustness; Spatial panel models.

JEL Classification: C23, C5

1 Introduction.

The LM tests for spatial error correlation of Burridge (1980) for the linear regression model and Anselin (1988) for the panel regression model are both developed under the assumption that the model errors are normally distributed. This leads to a natural question on how robust these tests are against misspecification of the error distribution. While these tests are robust asymptotically against distributional misspecification, as can be inferred

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from the results of Kelejian and Prucha (2001) for the Moran's I test in the linear regression model, and proved in this article for the panel regression model, their finite sample behavior can be sensitive to the spatial layout. The main reason, as shown in this paper, is the lack of standardization of these tests, i.e., subtracting the mean and dividing by the standard deviation.[2] In particular, when each spatial unit has many neighbors (the number of neighbors grows with the number of spatial units), the mean of these tests can be far below zero even when the sample size is fairly large (for e.g., 1000), causing severe size distortion of the test.

Standardized LM (SLM) tests are recommended, which correct both the mean and variance of the existing LM tests under more relaxed assumptions on the error distributions. It is shown that these LM tests are not only robust against distributional misspecification, but are also quite robust against changes in the spatial layout. Monte Carlo simulations show that the SLM tests have excellent finite sample properties and significantly outperform their non-standardized counterparts. The Monte Carlo simulations also show that once size-adjusted, all the tests considered have similar power.

It is well known in the statistics and econometrics literature that standardizing an LM test improves its performance especially if asymptotic critical values are used. Moulton and Randolph (1989) emphasized this for the panel data regression model with random individual effects. See also Honda (1991) and Baltagi, Chang and Li (1992). Koenker (1981) showed that the standardization (or studentization in his terminology) leads to a robustified LM test for heteroscedasticity. This point, however, is not emphasized in the spatial econometrics literature, except for Anselin (2001), Kelejian and Prucha (2001), and Florax and de Graaff (2004), where the authors mainly stressed the variance correction but not the mean correction. Recently, Robinson (2008) proposed a general chi-square test for non-spherical disturbances, including spatial error dependence, in a linear regression model. He pointed out the test has an LM interpretation and may not provide a satisfactory approximation in smallish samples as well. He then introduced a couple of modifications directly on the chi-square statistic. Our approach of standardization is more in line with that of Koenker (1981). It works on the 'standard normal' version of an LM test, and thus is simpler. More importantly, our approach allows the errors to be nonnormal and is not restricted to linear regression models of non-spherical disturbances.

Our Monte Carlo simulation shows that the mean-correction as well as variance correction are *both* important to attain good size and power. Section 2 deals with the tests

[2] Honda (1985) shows that the LM test for random individual effects in the panel data regression model is uniformly most powerful and is robust against non-normality. Moulton and Randolph (1989) show that this test can perform poorly when the number of regressors is large or the interclass correlation of some of the regressors is high. They suggest a standardized LM test by centering and scaling Honda's LM test. They show that the standardized LM test performs better in small samples when asymptotic critical values from the normal distribution are used.

for spatial error dependence in a linear regression model, Section 3 deals with the tests for spatial error dependence in a panel data regression model, while Section 4 presents Monte Carlos results. Section 5 concludes the paper.

2 Tests for Spatial Error Dependence in a Linear Regression Model

2.1 Moran's I and Burridge's LM tests

The original form of Moran's I test (Moran, 1950) is based a sample of observations $Y = \{Y_1, Y_2, \cdots, Y_n\}'$ on a variable of interest Y, which takes the form

$$I = \frac{\sum_i \sum_j w_{ij}(Y_i - \bar{Y})(Y_j - \bar{Y})}{\sum_i (Y_i - \bar{Y})^2}, \tag{1}$$

where w_{ij}'s are the elements of an $N \times N$ spatial weight matrix W with $w_{ii} = 0$ and $\sum_{j=1}^{N} w_{ij} = 1, i = 1, \cdots, N$, and $\bar{Y}$ is the average of the Y_i's. If the observations are normal, then the null distribution of Moran's I test statistic is shown to be asymptotic normal. Cliff and Ord (1972) extended Moran's I test to the case of a spatial linear regression model:

$$Y = X\beta + u \tag{2}$$

where Y is an $N \times 1$ vector of observations on the response variable, X is an $N \times k$ matrix containing the values of explanatory (exogenous) variables, and u is an $n \times 1$ vector of disturbances of mean zero and variance σ_u^2. The extended Moran's I test takes the form

$$I = \frac{\tilde{u}' W \tilde{u}}{\tilde{u}' \tilde{u}}, \tag{3}$$

where $\tilde{u}$ is a vector of OLS residuals when regressing Y on X. If u is normal, then the distribution of I under the null hypothesis of no spatial error dependence is asymptotically normal distributed with mean and variance given by:

$$\begin{aligned} \mathrm{E}(I) &= \frac{1}{N-k}\mathrm{tr}(MW), \\ \mathrm{Var}(I) &= \frac{\mathrm{tr}(MWMW') + \mathrm{tr}((MW)^2) - \frac{2}{N-k}[\mathrm{tr}(MW)]^2}{(N-k)(N-k+2)}. \end{aligned}$$

Here $M = I_N - X(X'X)^{-1}X'$ and I_N is an N-dimensional identity matrix. In real applications, the test should be carried out based on $I^* = (I - \mathrm{E}I)/\mathrm{Var}^{\frac{1}{2}}(I)$, and referred to the standard normal distribution (Anselin and Bera, 1998). However, most of the literature suggested or hinted at the use of $I^o = I/\mathrm{Var}^{\frac{1}{2}}(I)$; see, e.g., Anselin (2001), Kelejian and Prucha (2001) and Florax and de Graaff (2004). The reason may be that the mean correction is asymptotically negligible or may be that $I^o = I/\mathrm{Var}^{\frac{1}{2}}(I)$ corresponds directly to the Burridge (1980) LM test described below.

Let us consider the case where u follows either a spatial autoregressive (SAR) process $u = \lambda W u + \varepsilon$ or a spatial moving average (SMA) process $u = \lambda W \varepsilon + \varepsilon$, where W is defined above, λ is the spatial parameter, and ε is a vector of independent and identically distributed (iid) normal innovations with mean zero and variance σ_ε^2. The hypothesis of no spatial error correlation can be expressed explicitly as $H_0 : \lambda = 0$ vs $H_a : \lambda \neq 0$. For this model specification, Burridge (1980) derived an LM test for H_0:

$$\mathrm{LM}_B = \frac{N}{\sqrt{S_0}} \frac{\tilde{u}' W \tilde{u}}{\tilde{u}'\tilde{u}}, \tag{4}$$

where $S_0 = \mathrm{tr}(W'W + W^2)$. Under the null hypothesis of no spatial error correlation, $\mathrm{LM}_B \xrightarrow{D} N(0,1)$. LM_B resembles I^o except for a scale factor. Our Monte Carlo simulations show that it is important to standardize it if one is using asymptotic critical values, especially for certain spatial layouts. Some discussion on this is given after Theorem 1.

2.2 The standardized LM test

The three test statistics (I^*, I^o and LM_B) are derived under the assumption that the errors are normally distributed. Theorem 1, given below, shows that all three tests behave well asymptotically under non-normality. But how do they behave under finite samples? We first present a modified version of these tests allowing the error distributions to be *non-normal*, and then give some discussion answering why the finite sample performance of I^o and LM_B can be poor. The following basic regularity conditions are necessary for studying the asymptotic behavior of these test statistics.

Assumption A1: *The innovations $\{\varepsilon_i\}$ are iid with mean zero, variance σ_ε^2, and excess kurtosis κ_ε. Also, the moment $E|\varepsilon_i|^{4+\eta}$ exists for some $\eta > 0$.*

Assumption A2: *The elements $\{w_{ij}\}$ of W are at most of order h_N^{-1} uniformly for all i, j, with the rate sequence $\{h_N\}$, bounded or divergent, satisfying $h_N/N \to 0$ as N goes to infinity. The $N \times N$ matrices $\{W\}$ are uniformly bounded in both row and column sums with $w_{ii} = 0$ and $\sum_j w_{ij} = 1$ for all i.*

Assumption A3: *The elements of the $N \times k$ matrix X are uniformly bounded for all N, and $\lim_{N\to\infty} \frac{1}{N} X'X$ exists and is nonsingular.*

Assumption A1 is taken from Kelejian and Prucha (2001) and is required for their central limit theorem of linear-quadratic forms. Assumption A2 is taken from Lee (2004a) and it identifies the different types of spatial dependence considered. Typically, one type of spatial dependence corresponds to the case where each unit has a fixed number of neighbors, which in turn means that h_N is bounded. The other type of spatial dependence corresponds to the case where the number of neighbors of each spatial unit grows as N goes to infinity, and in this case h_N is divergent. To limit the spatial dependence to a manageable degree, it is thus required that $h_N/N \to 0$ as $N \to \infty$.

Theorem 1: *Under Assumptions A1-A3, the standardized LM test for testing $H_0 : \lambda = 0$ vs $H_a : \lambda \neq 0$ (or $\lambda < 0$, or $\lambda > 0$) takes the form*

$$\mathrm{LM}_B^* = \frac{\tilde{u}'W\tilde{u}/\tilde{u}'\tilde{u} - S_1}{N^{-1}(\tilde{\kappa}_\varepsilon S_2 + S_3)^{\frac{1}{2}}}, \tag{5}$$

where $S_1 = \frac{1}{N-k}\mathrm{tr}(WM), S_2 = \sum_{i=1}^N a_{ii}^2$, and $S_3 = \mathrm{tr}(AA' + A^2)$, $A = MWM - S_1 M$, a_{ii} are the diagonal elements of A, and $\tilde{\kappa}_\varepsilon$ is the excess sample kurtosis of $\tilde{u}$. Under H_0, we have (i) $LM_B^* \xrightarrow{D} N(0,1)$; and (ii) the four test statistics, I^*, I^0, LM_B and LM_B^* are asymptotically equivalent.

The formal proof of Theorem 1 is given in the Appendix. To help understanding the theory, we outline the key steps leading to the standardization given in (5). First note that $\tilde{u}'W\tilde{u}$, the key quantity appeared in the numerators of (3)-(5), is not centered because $\mathrm{E}(\tilde{u}'W\tilde{u}) = \sigma_\varepsilon^2\mathrm{tr}(WM) \neq 0$. This motivates us to consider $\tilde{u}'W\tilde{u} - \sigma_\varepsilon^2\mathrm{tr}(WM)$, or its feasible version $\tilde{u}'W\tilde{u} - \frac{1}{n-k}(\tilde{u}'\tilde{u})\mathrm{tr}(WM) = u'Au$. Upon finding the variance of $u'Au$ and replacing σ_ε^2 in the variance expression by its MLE, we obtain (5). Some remarks follow.

Standardization of Moran's I given earlier works on $\tilde{u}'W\tilde{u}/\tilde{u}'\tilde{u}$, with its mean and variance derived under the assumption that $u \sim N(0, \sigma_\varepsilon^2 I_N)$. Robinson's (2010) approach works on LM_B^2 or $(\tilde{u}'W\tilde{u}/\tilde{u}'\tilde{u})^2$. Again, the derivations of the mean and variance depend on the normality assumption. Our approach works on the quadratic form $u'Au$ with its mean and variance readily available as long as the first four moments of the elements of u exist. Thus, our approach is simpler which does not depend on the normality assumption and is applicable to other models of more complicated structure.

Although both Moran's I and the LM_B test statistics are derived under the assumption that the innovations are normally distributed, Theorem 1 shows that they are asymptotically equivalent to the SLM test derived under relaxed conditions on the error distribution.[3] This means that all the four tests are robust against distributional misspecification when the sample size is large. But will the four tests behave similarly under finite sample? The following discussion points out that their finite sample performance may be different.

The major difference between LM_B and LM_B^* lies in the mean correction of the statistic $\tilde{u}'W\tilde{u}/\tilde{u}'\tilde{u}$. This correction may quickly become negligible as the sample size increases under certain spatial layouts, but not necessarily under other spatial layouts. From (A-1) in the

[3]That the LM test is asymptotically robust against the distributional misspecification is due to the special spatial structure built in the model and the fact that W has zero diagonal elements. However, if the spatial structure is changed, e.g., there are two error terms in the model and one is possibly spatially correlated, the regular LM test is no long robust, see Yang (2010).

appendix, we see that this mean correction factor is of the magnitude

$$\frac{NS_1}{(\tilde{\kappa}_\varepsilon S_2 + S_3)^{\frac{1}{2}}} = O_p((h_N/N)^{\frac{1}{2}}),$$

which shows that the magnitude of mean correction depends on the ratio $(h_N/N)^{\frac{1}{2}}$. For example, when $h_N = N^{0.8}$, $(h_N/N)^{\frac{1}{2}} = N^{-0.1}$. Thus, if $N = 20, 100$, and 1000, $N^{-0.1} = 0.74, 0.63$, and 0.50. This shows that the means of LM_B and I^o can differ from the means of LM_B^* and I^* by 0.74 when $N = 20$, 0.63 when $N = 200$ and 0.50 when $N = 1000$. Note that situations leading to $h_N = N^{0.8}$ may be the spatial layouts constructed under large group interactions, where the group sizes are large and the number of groups is small.[4] Our results show that in this situation, the non-standardized LM test or Moran's I test without the mean correction may be misleading. Monte Carlo simulations presented in Section 5 confirm these findings.

3 Tests for Spatial Error Dependence in a Panel Linear Regression Model

When repeated observations are made on the same set of N spatial units over time, Model (2) becomes

$$Y_t = X_t\beta + u_t, \;\; t = 1, \cdots, T, \tag{6}$$

resulting in a panel data regression model, where $\{Y_t, X_t\}$ denote the data collected at the tth time period. A defining feature of a panel data model is that the error vector u_t is allowed to possess a general structure of the form

$$u_{it} = \mu_i + \varepsilon_{it}, \;\; i = 1, \cdots, N, \;\; t = 1, \cdots, T, \tag{7}$$

where μ_i denotes the unobservable space-specific effect, due to aspects of regional structure, firm's specific feature, etc. Spatial units may be dependent. To allow for such a possibility, Anselin (1988) introduced a SAR process into the disturbance vector $\varepsilon_t = \{\varepsilon_{1t}, \cdots, \varepsilon_{Nt}\}'$,

$$\varepsilon_t = \lambda W \varepsilon_t + v_t, \;\; t = 1, \cdots, T, \tag{8}$$

where the spatial weight matrix W is defined similarly to that in Model (2), and v_t is an $N \times 1$ vector of iid remainder disturbances with mean zero and variance σ_v^2.

We are interested in testing the hypothesis $H_0 : \lambda = 0$. We consider the scenario where the time dimension T is small and the 'space' dimension N is large. This is the typical

[4] See Lee (2007) for a detailed discussion on spatial models with group interactions.

feature for many micro-level panel data sets. Let $B = I_N - \lambda W$. Stacking the vectors (Y_t, u_t, v_t) and the matrix X_t, the model can be written in matrix form:

$$Y = X\beta + u, \;\; u = (\iota_T \otimes I_N)\mu + (I_T \otimes B^{-1})v, \tag{9}$$

where ι_m represents an $m \times 1$ vector of ones, I_m represents an $m \times m$ identity matrix.

Assuming (i) the elements of μ are iid with mean zero and variance σ_μ^2, (ii) the elements of v are iid with mean zero and variance σ_v^2, and (iii) μ and v are independent. The log-likelihood function, assuming μ and v are both normally distributed, is given by:

$$\ell(\beta, \sigma_v^2, \sigma_\mu^2, \lambda) = -\frac{NT}{2}\log(2\pi\sigma_v^2) - \frac{1}{2}\log|\Sigma| - \frac{1}{2\sigma_v^2}u'\Sigma^{-1}u, \tag{10}$$

where $\Sigma = \frac{1}{\sigma_v^2}\mathrm{E}(uu') = \phi(J_T \otimes I_N) + I_T \otimes (B'B)^{-1}$, $\Sigma^{-1} = \bar{J}_T \otimes (T\phi I_N + (B'B)^{-1})^{-1} + E_T \otimes (B'B)$, $\phi = \sigma_\mu^2/\sigma_v^2$, $J_T = \iota_T\iota_T'$, $\bar{J}_T = \frac{1}{T}J_T$, and $E_T = I_T - \bar{J}_T$. See Anselin (1988) for details. Maximizing (10) gives the maximum likelihood estimator (MLE) of the model parameters if the error components are normally distributed, otherwise it gives a quasi-maximum likelihood estimator (QMLE).

Anselin (1988, p. 155) presents an LM test of $H_0 : \lambda = 0$ for Model (9), which can be written in the form

$$\mathrm{LM_A} = \frac{\tilde{u}'[\tilde{\rho}^2(\bar{J}_T \otimes W) + E_T \otimes W]\tilde{u}}{\tilde{\sigma}_v^2[(T - 1 + \tilde{\rho}^2)S_0]^{\frac{1}{2}}}, \tag{11}$$

where $S_0 = \mathrm{tr}(W'W) + W^2)$, $\tilde{\rho}$ is the constrained QMLE under H_0 of $\rho = \sigma_v^2/(T\sigma_\mu^2 + \sigma_v^2)$, and $\tilde{\sigma}_v^2$ the constrained QMLE of σ_v^2, and $\tilde{u}$ is the vector of constrained QMLE residuals.[5]

A nice feature of the LM test is that it requires only the estimates of the model under H_0. However, even under H_0, the constrained QMLE of ρ (or ϕ) does not posses an explicit expression, meaning that $\tilde{\rho}$ has to be obtained via numerical optimization. In fact, under H_0, the partially maximized log-likelihood (with respect to β and σ_v^2) is given by:

$$\ell_{\max}(\rho) = \text{constant} - \frac{NT}{2}\log\tilde{\sigma}_v^2(\rho) + \frac{N}{2}\log\rho, \tag{12}$$

where $\tilde{\sigma}_v^2(\rho) = \frac{1}{NT}\tilde{u}'(\rho)\Sigma^{-1}\tilde{u}(\rho)$, $\tilde{u}(\rho) = Y - X\tilde{\beta}(\rho)$, $\tilde{\beta}(\rho) = (X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1}Y$, and $\Sigma^{-1} = \rho\bar{J}_T \otimes I_N + E_T \otimes I_N$. Maximizing (12) gives the constrained QMLE (under H_0) $\tilde{\rho}$ of ρ, which in turn gives the constrained QMLEs $\tilde{\beta} = \tilde{\beta}(\tilde{\rho})$, $\tilde{\sigma}_v^2 = \tilde{\sigma}_v^2(\tilde{\rho})$, $\tilde{\Sigma}^{-1} = \tilde{\rho}\bar{J}_T \otimes I_N + E_T \otimes I_N$, and $\tilde{u} = \tilde{u}(\tilde{\rho})$, for β, σ_v^2, Σ^{-1} and $u(\rho)$, respectively.

[5]Baltagi, et al.(2003) considered the joint, marginal and conditional LM tests for λ and/or σ_μ^2, which includes (11) as a special case, and presented Monte Carlo results under spatial layouts with a fixed number of neighbors. Apparently, the LM test given in (11) does not fit into the framework of Robinson (2008), but it does if the test concerns $H_0 : \lambda = 0, \rho = 0$. We note that our approach is applicable to all scenarios similar to (11), i.e., testing spatial effect allowing other type of effects (such as random effects, heteroscedasticity, etc.) to exist in the model.

Similar to the LM test in the linear regression model, the numerator of $\mathrm{LM_A}$ given in (11) is again a quadratic form in the disturbance vector u, but now u contains two independent components. The large sample mean of this quadratic form is zero, but its finite sample mean is not necessarily zero. This may distort the finite sample distribution of the test statistic, in particular the tail probability. We now present a standardized version of the $\mathrm{LM_A}$ test, which corrects both the mean and the variance and has a better finite sample performance in the situation where each spatial unit has 'many' neighbors. Lemma 3 given in the Appendix is essential in deriving the modified test statistics. Some basic regularity conditions are listed below.

Assumption B1: *The random effects $\{\mu_i\}$ are iid with mean zero, variance σ_μ^2, and excess kurtosis κ_μ. The idiosyncratic errors $\{v_{it}\}$ are iid with mean zero, variance σ_v^2, and excess kurtosis κ_v. Also, the moments $E|\mu_i|^{4+\eta_1}$ and $E|v_{it}|^{4+\eta_2}$ exist for some $\eta_1, \eta_2 > 0$.*

Assumption B2: *The elements $\{w_{ij}\}$ of W are at most of order h_N^{-1} uniformly for all i, j, with the rate sequence $\{h_N\}$, bounded or divergent, satisfying $h_N/N \to 0$ as N goes to infinity. The $N \times N$ matrices $\{W\}$ are uniformly bounded in both row and column sums with $w_{ii} = 0$ and $\sum_j w_{ij} = 1$ for all i.*

Assumption B3: *The elements of the $NT \times k$ matrix X are uniformly bounded for all N and $\lim_{N\to\infty} \frac{1}{N} X'X$ exists and is nonsingular.*

Now, define $A(\rho) = \rho^2(\bar{J}_T \otimes W) + E_T \otimes W$, $M(\tilde{\rho}) = I_{NT} - X(X'\tilde{\Sigma}^{-1}X)^{-1}X'\tilde{\Sigma}^{-1}$, and $C(\rho) = M'(\rho)A(\rho)M(\rho)$. Let $diagv(A)$ be a column vector formed by the diagonal elements of a square matrix A. We have the following theorem.

Theorem 2: *Assume that the constrained QMLE $\tilde{\rho}$ under H_0 is a consistent estimator of ρ.*[6] *Under Assumptions B1-B3, for testing $H_0; \lambda = 0$, the standardized LM test which corrects both the mean and variance takes the form:*

$$\mathrm{LM}_\mathrm{A}^* = \frac{\tilde{u}'\tilde{A}\tilde{u}/\tilde{\sigma}_v^2 - \mathrm{tr}(\tilde{\Sigma}\tilde{C})}{[\tilde{\phi}^2\tilde{\kappa}_\mu\tilde{a}_1'\tilde{a}_1 + \tilde{\kappa}_v\tilde{a}_2'\tilde{a}_2 + \mathrm{tr}(\tilde{\Sigma}(\tilde{C}' + \tilde{C})\tilde{\Sigma}\tilde{C})]^{\frac{1}{2}}}, \tag{13}$$

where $\tilde{A} = A(\tilde{\rho})$, $\tilde{C} = C(\tilde{\rho})$, $\tilde{\kappa}_\mu$ is the sample excess kurtosis of $\tilde{\mu} = (\bar{J}_T \otimes I_N)\tilde{u}$, $\tilde{\kappa}_v$ is the sample excess kurtosis of $\tilde{v} = \tilde{u} - (\iota \otimes I_N)\tilde{\mu}$, $\tilde{a}_1 = \mathrm{diagv}[(\iota_T' \otimes I_N)\tilde{C}(\iota_T \otimes I_N)]$, and $\tilde{a}_2 = \mathrm{diagv}(\tilde{C})$. Under H_0, we have (i) $\mathrm{LM}_\mathrm{A}^* \xrightarrow{D} N(0,1)$, and (ii) the two LM tests (11) and (13) are asymptotically equivalent.

The proof of the theorem is again given in the Appendix. Similar to the results of Theorem 1, the results of Theorem 2 show that the mean correction factor for the standardized LM test is also of the order $O_p((h_N/N)^{\frac{1}{2}})$. Thus, the $\mathrm{LM_A}$ test can have large mean bias when h_N is large.

[6] *This condition may be relaxed to allow $\tilde{\rho}$ to be an arbitrary consistent estimator of ρ.*

4 Monte Carlo Results

The finite sample performance of the test statistics introduced in this paper are evaluated based on a series of Monte Carlo experiments. These experiments involve a number of different error distributions and a number of different spatial layouts. Comparisons are made between the standardized tests and their non-standardized counterparts to see the effects of the error distributions and the spatial layouts.

4.1 Spatial layouts and error distributions

Three general spatial layouts are considered in the Monte Carlo experiments and they are applied to all the test statistics involved in the experiments. The first is based on the Rook contiguity, the second is based on Queen contiguity and the third is based on the notion of group or social interactions with the number of groups $G = N^{\delta}$ where $0 < \delta < 1$. In the first two cases, the number of neighbors for each spatial unit stays the same (2-4 for Rook and 3-8 for Queen) and does not change when sample size N increases, whereas in the last case, the number of neighbors for each spatial unit increases with the increase of sample size but at a slower rate, and changes from group to group.

The details for generating the W matrix under Rook contiguity is as follows: (i) index the N spatial units by $\{1, 2, \cdots, N\}$, randomly permute these indices and then allocate them into a lattice of $r \times m(\geq N)$ squares, (ii) let $W_{ij} = 1$ if the index j is in a square which is on the immediate left, or right, or above, or below the square which contains the index i, otherwise $W_{ij} = 0$, and (iii) divide each element of W by its row sum. The W matrix under Queen contiguity is generated in a similar way, but with additional neighbors which share a common vertex with the unit of interest.

To generate the W matrix according to the group interaction scheme, (i) calculate the number of groups according to $G = \text{Round}(N^{\delta})$, and the approximate average group size $m = N/G$, (ii) generate the group sizes $(n_1, n_2, \cdots, n_G)$ according to a discrete uniform distribution from $m/2$ to $3m/2$, (iii) adjust the group sizes so that $\sum_{i=1}^{G} n_i = N$, and (iv) define $W = \text{diag}\{W_i/(n_i - 1), i = 1, \cdots, G\}$, a matrix formed by placing the submatrices W_i along the diagonal direction, where W_i is an $n_i \times n_i$ matrix with ones on the off-diagonal positions and zeros on the diagonal positions. In our Monte Carlo experiments, we choose $\delta = 0.2, 0.5$, and 0.8, representing respectively the situations where (i) there are few groups and many spatial units in a group, (ii) the number of groups and the sizes of the groups are of the same magnitude, and (iii) there are many groups with few elements in each. Clearly, under Rook or Queen contiguity, h_N defined in the theorems is bounded, whereas under group interaction h_N is divergent with rate $N^{1-\delta}$.[7]

[7] Clearly, this spatial layout covers the scenario considered in Case (1991). Lee (2007) shows that the group size variation plays an important role in the identification and estimation of econometric models with

The reported Monte Carlo results correspond to the following three error distributions: (i) standard normal, (ii) mixture normal, standardized to have mean zero and variance 1, and (iii) log-normal, also standardized to have mean zero and variance one. The standardized normal-mixture variates are generated according to

$$u_i = ((1-\xi_i)Z_i + \xi_i \tau Z_i)/(1-p+p*\tau^2)^{0.5},$$

where ξ is a Bernoulli random variable with probability of success p and Z_i is standard normal independent of ξ. The parameter p in this case also represents the proportion of mixing the two normal populations. In our experiments, we choose $p = 0.05$, meaning that 95% of the random variates are from standard normal and the remaining 5% are from another normal population with standard deviation τ. We choose $\tau = 10$ to simulate the situation where there are gross errors in the data. The standardized lognormal random variates are generated according to

$$u_i = [\exp(Z_i) - \exp(0.5)]/[\exp(2) - \exp(1)]^{0.5}.$$

This gives an error distribution that is both skewed and leptokurtic. The normal mixture gives an error distribution that is still symmetric like normal but leptokurtic. Other non-normal distributions, such as normal-gamma mixture and chi-squared, are also considered and the results are available from the author upon request. All the Monte Carlo experiments are based on 10,000 replications.

4.2 Performance of the tests for the linear regression model

The performance of the standardized LM test statistic (LM_B^*) introduced in Section 2 is compared with the standardized Moran's I (I^*), the Moran's I with only variance correction (I^0) and the LM statistics of Burridge (1980) (LM_B). The Monte Carlo experiments are carried out based on the following data generating process:

$$Y_i = \beta_0 + X_{1i}\beta_1 + X_{2i}\beta_2 + u_i$$

where X_{1i}'s are drawn from $10U(0,1)$ and X_{2i}'s are drawn from $5N(0,1)+5$. Both are treated as fixed in the experiments. The parameters $\beta = \{5, 1, 0.5\}'$ and $\sigma = 0.1$. Five different sample sizes are considered, i.e., $N = 50, 100, 200, 500,$ and 1000.

Size of the tests. The results in Table 1 show that LM_B and I^0 are undersized even in the normal case and things get worse for the normal mixture and lognormal distributions. In contrast, their standardized versions LM_B^* and I^* have size close to 5% for all experiments considered. The table also reports the empirical mean and standard deviation (SD) of these

group interactions, contextual factors and fixed effects. Yang (2010) shows that it also plays an important role in the robustness of the LM test of spatial error components.

statistics. It is clear that LM_B and I^0 have a downward mean shift, which can be sizable when N is not large, but decreases towards zero as N increases. Besides the mean shift, LM_B also has a downward SD shift, which can be sizable as well when N is not large, but goes zero as N increases. In contrast, LM_B^* and I^* have mean close to zero and SD close to 1 which explain why they have better size in all experiments. Recalling that LM_B corrects neither mean nor SD and that I^0 corrects only for SD, it is clear now why I^0 is undersized, and why LM_B is more severely undersized than I^0. Thus, the LM tests of spatial dependence need to be both mean- and variance-adjusted for good finite sample performance.

The results in Table 1 show that one of the major factors affecting the null distribution of LM_B and I^0 is the spatial layout, or rather the degree of spatial dependence. In situations of a large group interaction, e.g., $G = \texttt{Round}(N^{0.2})$ as in the first part of Table 1, the number of groups ranges from 2 to 4 for N ranging from 50 to 1000. Thus, there are only a few groups, each containing many spatial units which are all neighbors of each other. This 'heavy' spatial dependence distorts severely the null distributions of LM_B and I^0. In contrast, in situations of small group interaction, e.g., $G = \texttt{Round}(N^{0.8})$ as in the third part of Table 1, the number of groups ranges from 23 to 251 for N ranging from 50 to 1000. In this case, there are many groups each having only 2 to 4 units, giving a spatial layout with very week spatial dependence. As a result, the null distributions of LM_B and I^0 are much closer to $N(0, 1)$ though still not as close as those of the null distributions of LM_B^* and I^*. These observations are consistent with the discussion following Theorem 1.

Power of the tests. Empirical frequencies of rejection of the four tests are plotted in Figure 1 against the values of λ (horizontal line). Simulated critical values for each test are used, which means that the reported powers of the tests are size-adjusted. In each plot of Figure 1, each line we see is in fact the overlap of four lines corresponding to the four tests. This means that once size-adjusted, the four tests have almost identical power. This is not surprising as all four tests share the same term $\tilde{u}'W\tilde{u}/\tilde{u}'\tilde{u}$. The four tests differ mainly in their locations and scales, and thus have different sizes or null behaviors in general when referred to the standard normal. If, however, the exact critical values are used, they become essentially the same test. However, in real applications, one does not know the exact critical values and the asymptotic critical values are often used. In this case, it is important as we show to do the mean and variance correction to the test statistics so that the asymptotic critical values give a better approximation.

Figure 1 further reveals that the spatial layout and the sample size are the two important factors affecting the power of these tests. With less neighbors (plots on the right) or with a larger sample, the tests become more powerful. It is interesting to note that when the spatial dependence is strong, it is harder to detect the spatial dependence when the spatial parameter is negative than when it is positive (see the plots on the left). The error distribution also affect the power of the tests, but to a lesser degree.

4.3 Performance of the tests for the panel data regression model

The LM and SLM tests ($\mathrm{LM_A}$ and $\mathrm{LM_A^*}$) introduced in Section 3 are compared by Monte Carlo simulation based the following DGP

$$Y_t = \beta_0 + X_{1t}\beta_1 + X_{2t}\beta_2 + u_t, \text{ with } u_t = \mu + \varepsilon_t, \ t = 1, \cdots, T,$$

where the error components μ and ε_t can be drawn from any of the three distributions used in the previous two subsections, or the combination of any two distributions. For example, μ and ε_t can both be drawn from the normal mixture, or μ from the normal mixture but ε_t from the normal or log-normal distribution. The beta parameters are set at the same values as before, $\sigma_\mu^2 = 1.0$ and $\sigma_v^2 = 5$. For sample sizes, $T = 3, 10$; and $N = 20, 50, 100, 200, 500$. The same spatial layouts are used as described above.

Size of the tests. The results presented in Table 2 correspond to cases where both μ and v_t are normal, both are normal mixture, and both are log-normal. Essentially, the same conclusions hold as in the case of the spatial linear regression model . The SLM test outperforms its LM counterpart in all the experiments considered. Another interesting phenomenon is that the null behavior of $\mathrm{LM_A}$ also depends on the relative magnitude of the variance components σ_μ^2 and σ_v^2. The larger the ratio σ_v^2/σ_μ^2, the worse is the performance of the $\mathrm{LM_A}$ test. In contrast, the performance of $\mathrm{LM_A^*}$ is very robust.

Power of the tests. Empirical frequencies of rejection, based on the simulated critical values, of the two tests are plotted in Figure 2 against the values of λ (horizontal line). Now each line we see from each plot of Figure 2 is in fact an overlap of two lines, one for $\mathrm{LM_A}$ and the other for $\mathrm{LM_A^*}$. Similar to the case of the linear regression model, the two tests have almost identical power once they are size-adjusted. The power of the tests depend heavily on the degree of spatial dependence and on the sample size. It also depends on the error distributions, though to a lesser degree.

Some interesting details are as follows. The two plots in the first row of Figure 2 show that the two tests possess very low power and that the power does not seem to increase as N increase from 20 to 50 (with T fixed at 3). This is because the underlying spatial layout generates very strong spatial dependence. When N is increased from 20 to 50, the number of groups stays at $G = \texttt{Round}(N^{0.2}) = 2$. This means that under this spatial layout, the degree of spatial dependence at $N = 50$ is bigger than that at $N = 20$. As a result, the power does not go up, and might even go down slightly.

5 Conclusions

This paper recommends standardized LM tests of spatial error dependence for the linear as well as the panel regression model. We showed that when standardizing the LM tests for spatial effects it is important to adjust for *both* the mean and variance of the LM statistics. The mean adjustment is, however, often neglected in the literature. One important reason for the mean adjustment of the LM tests for spatial effects is that the degree of spatial dependence may grow with the sample size. This slows down the convergence speed of the maximum likelihood estimators (Lee, 2004a), making the concentrated score function (the key element of the LM test) more biased.

There are other LM tests for other spatial models that are derived under normal assumptions such as Baltagi, et al. (2003), and the LM test for spatial lag effect in the spatial autoregressive models (Anselin, 1988), which can be studied in a similar manner. This paper recommends the standardized version of these LM tests because it offers improvements in their finite sample performance, in addition to preserving the simplicity of the original LM tests so that they can be easily adopted by applied researchers.

In modifying the LM tests for robustness or for better finite sample performance, one is tempted to think of the bootstrap method. Unfortunately, the bootstrap method does not offer an easy and ready-to-use solution to the testing problems. The main difficulty is the generation of bootstrap data reflecting the null hypothesis. This is in a great contrast to problems of point estimation and confidence interval construction where the bootstrap method offer solutions to many complicated problem. This indicates that developing bootstrap LM tests for spatial effects is a very interesting topic of future research.

Appendix: Proofs of the Theorems

To prove the theorems, we need the following lemmas.

Lemma 1 (Lee, 2004a): *Let v be an $N \times 1$ random vector of iid elements with mean zero, variance σ^2, and finite excess kurtosis κ. Let A be an N dimensional square matrix. Then* $\mathrm{E}(v'Av) = \sigma^2 \mathrm{tr}(A)$ *and* $\mathrm{Var}(v'Av) = \sigma^4 \kappa \sum_{i=1}^N a_{ii}^2 + \sigma^4 \mathrm{tr}(AA' + A^2)$.

Lemma 2 (Lemma A.9, Lee, 2004b): *Suppose that A represents a sequence of $N \times N$ matrices that are uniformly bounded in both row and column sums. Elements of the $N \times k$ matrix X are uniformly bounded; and $\lim_{n\to\infty} \frac{1}{N} X'X$ exists and is nonsingular. Let $M = I_N - X(X'X)^{-1}X'$. Then*

(i) $\mathrm{tr}(MA) = \mathrm{tr}(A) + O(1)$
(ii) $\mathrm{tr}(A'MA) = \mathrm{tr}(A'A) + O(1)$
(iii) $\mathrm{tr}[(MA)^2] = \mathrm{tr}(A^2) + O(1)$, *and*
(iv) $\mathrm{tr}[(A'MA)^2] = \mathrm{tr}[(MA'A)^2] = \mathrm{tr}[A'A)^2] + O(1)$

Furthermore, if $A_{ij} = O(h_N^{-1})$ for all i and j, then

(vi) $\mathrm{tr}^2(MA) = \mathrm{tr}^2(A) + O(\frac{N}{h_N})$, *and*
(vii) $\sum_{i=1}^N [(MA)_{ii}]^2 = \sum_{i=1}^N (a_{ii})^2 + O(h_N^{-1})$,

where $(MA)_{ii}$ are the diagonal elements of MA, and a_{ii} are the diagonal elements of A.

Lemma 3: *Let $u = G_1\mu + G_2 v$, where u and v are two independent random vectors not necessarily of the same length containing, respectively, iid elements of means zero, variances σ_μ^2 and σ_v^2, skewness α_μ and α_v, and excess kurtosis κ_μ and κ_v; and G_1 and G_2 are two conformable non-stochastic matrices. Let A be a confirmable square matrix. Then,*

(*i*) $\mathrm{E}(u'Au) = \sigma_v^2 \mathrm{tr}(\Sigma A)$,
(*ii*) $\mathrm{Var}(u'Au) = \sigma_\mu^4 \kappa_\mu a_1' a_1 + \sigma_v^4 \kappa_v a_2' a_2 + \sigma_v^4 \mathrm{tr}[\Sigma(A' + A)\Sigma A]$,

where $\Sigma = \sigma_v^{-2}\mathrm{E}(uu') = \frac{\sigma_\mu^2}{\sigma_v^2} G_1 G_1' + G_2 G_2'$, $a_1 = \mathrm{diagv}(G_1' A G_1)$, *and* $a_2 = \mathrm{diagv}(G_2' A G_2)$.

Proof: The result (i) is trivial. For ii), we have,

$$u'Au = \mu' G_1' A G_1 \mu + v' G_2' A G_2 v + \mu' G_1'(A + A')G_2 v.$$

It is easy to see that the three terms are uncorrelated. Thus,

$$\mathrm{Var}(u'Au) = \mathrm{Var}(\mu' G_1' A G_1 \mu) + \mathrm{Var}(v' G_2' A G_2 v) + \mathrm{Var}[\mu' G_1'(A' + A)G_2 v].$$

From Lemma 1, we obtain $\mathrm{Var}(\mu' G_1' A G_1 \mu) = \sigma_\mu^4 \kappa_\mu a_1' a_1 + \sigma_\mu^4 \mathrm{tr}[AG_1G_1'(A' + A)G_1G_1']$, and $\mathrm{Var}(v' G_2' A G_2 v) = \sigma_v^4 \kappa_v a_2' a_2 + \sigma_v^4 \mathrm{tr}[AG_2G_2'(A' + A)G_2G_2']$. It is easy to show that $\mathrm{Var}(\mu' G_1'(A' + A)G_2 v) = \sigma_\mu^2 \sigma_v^2 \mathrm{tr}[(A' + A)G_2G_2'(A' + A)G_1G_1']$. Putting these three expressions together leads to (ii). *Q.E.D.*

Proof of Theorem 1: First, we note that

$$\tilde{u}'W\tilde{u} - S_1\tilde{u}'\tilde{u} = \tilde{u}'(W - S_1I_N)\tilde{u} = u'M(W - S_1I_N)Mu = u'Au.$$

Under H_0 and Assumption A1, Lemma 1 is applicable to $u'Au$, which gives $\mathrm{E}u'Au = \sigma_\varepsilon^2 \mathrm{tr}A = 0$ and $\mathrm{Var}(u'Au) = \sigma_\varepsilon^4\kappa_\varepsilon \sum_{i=1}^n a_{ii}^2 + \sigma_\varepsilon^4[\mathrm{tr}(AA') + \mathrm{tr}(A^2)]$. Letting $W^* = W - S_1I_N$, we have $A = MW^*M$. By Lemma 2(i) and Assumption A2, $\mathrm{tr}(WM) = O(1)$ which gives $S_1 = O(N^{-1})$. Hence, the elements of W^* are of uniform order $O(h_N^{-1})$. Under Assumption A3, M is uniformly bounded in both row and column sums (Lee, 2004a, Appendix A). It follows that the elements of A are of uniform order $O(1/h_N)$, and that the row and column sums of the matrix A are uniformly bounded. Thus, the generalized central limit theorem for linear-quadratic forms of Lee (2004a, Appendix A) is applicable,[8] which shows that $u'Au$ is asymptotically normal, or equivalently,

$$\frac{u'Au}{\sigma_\varepsilon^2(\kappa_\varepsilon S_2 + S_3)^{\frac{1}{2}}} = \frac{\tilde{u}'W\tilde{u} - S_1\tilde{u}'\tilde{u}}{\sigma_\varepsilon^2(\kappa_\varepsilon S_2 + S_3)^{\frac{1}{2}}} \xrightarrow{D} N(0,1).$$

Now, it is easy to show that under H_0 $\tilde{\sigma}_\varepsilon^2 \equiv \tilde{u}'\tilde{u}/N \xrightarrow{p} \sigma_\varepsilon^2$ and $\tilde{\kappa}_\varepsilon \equiv \frac{1}{n\tilde{\sigma}_\varepsilon^4}\sum_{i=1}^n \tilde{u}_i^4 - 3 \xrightarrow{p} \kappa_\varepsilon$ (see Yang (2010) for the proof of a similar result). The result (i) thus follows from Slutsky's theorem by replacing σ_ε by $\tilde{\sigma}_\varepsilon$ and κ_ε by $\tilde{\kappa}_\varepsilon$.

To prove the asymptotic equivalence of LM_B and LM_B^*, we note that

$$\mathrm{LM}_B^* = \left(\frac{S_0}{\tilde{\kappa_\varepsilon} S_2 + S_3}\right)^{\frac{1}{2}} \mathrm{LM}_B - \frac{NS_1}{(\tilde{\kappa_\varepsilon} S_2 + S_3)^{\frac{1}{2}}}. \tag{A-1}$$

Thus, it is sufficient to show that the factor in front of LM_B is $O_p(1)$ and the second term is $o_p(1)$. As the elements $\{w_{ij}^*\}$ of W^* are uniformly $O(h_N^{-1})$, Lemma 2(vi) and Assumption A2 ($w_{ii} = 0$) lead to $S_2 = \sum_{i=1}^n a_{ii}^2 = \sum_{i=1}^N (w_{ii}^*)^2 + O(h_N^{-1}) = O(h_N^{-1})$. Lemma 2(ii) and (iii) lead to $S_3 = S_0 + O(1)$. Since the elements of W are uniformly $O(h_N^{-1})$ and the row sums of W are uniformly bounded, it follows that the elements of WW' and W^2 are uniformly $O(h_N^{-1})$. Hence, S_0 is $O(N/h_N)$, and so is S_3. Furthermore, $\tilde{\kappa}_\varepsilon = O_p(1)$. These lead to $(S_0/(\tilde{\kappa_\varepsilon} S_2 + S_3))^{\frac{1}{2}} = O_p(1)$ and $NS_1/(\tilde{\kappa_\varepsilon} S_2 + S_3)^{\frac{1}{2}} = O_p((h_N/N)^{\frac{1}{2}}) = o_p(1)$, showing $\mathrm{LM}_B \sim \mathrm{LM}_B^*$. Similarly, one can show that $\mathrm{Var}(I) \sim S_0$, and hence $\mathrm{LM}_B \sim I^*$. Finally, it is evident that $I^o \sim I^*$ *Q.E.D.*

Proof of Theorem 2: We have $\tilde{u} = Y - X\tilde{\beta} = Y - X(X'\tilde{\Sigma}^{-1}X)^{-1}X'\tilde{\Sigma}^{-1}Y \equiv M(\tilde{\rho})Y$. The numerator of $\mathrm{LM_A}$ becomes $\tilde{u}'A(\tilde{\rho})\tilde{u} = Y'M'(\tilde{\rho})A(\tilde{\rho})M(\tilde{\rho})Y = u'M'(\tilde{\rho})A(\tilde{\rho})M(\tilde{\rho})u = u'C(\tilde{\rho})u$. By the mean value theorem,

$$u'C(\tilde{\rho})u = u'C(\rho)u + u'\dot{C}(\bar{\rho})u\ (\tilde{\rho} - \rho),$$

[8]Lee (2004a) generalized the results of Kelejian and Prucha (2001) to cover the case where h_N is unbounded. Lee's results require the matrix A to be symmetric. If it is not, it can be replaced by $\frac{1}{2}(A + A')$.

where $\bar{\rho}$ lies between $\tilde{\rho}$ and ρ, $\dot{C}(\rho) = \frac{d}{d\rho}C(\rho) = M'(\rho)[2\rho(\bar{J}_T \otimes W) - 2(\bar{J}_T \otimes I_N)P(\rho)A(\rho)]M(\rho)$, and $P(\rho) = X(X'\Sigma^{-1}X)^{-1}X'$. It is easy to see the elements of $C(\rho)$ are of uniform order $O(1/h_N)$ uniformly in ρ, and so are the elements of $\dot{C}(\bar{\rho})$. As $\tilde{\rho}$ is a consistent estimator of ρ, it follows that $u'C(\tilde{\rho})u \sim u'C(\rho)u$. Now, $u'C(\rho)u$ can be decomposed into the following three terms,

$$\mu'(\iota_T' \otimes I_N)C(\rho)(\iota_T \otimes I_N)\mu + v'C(\rho)v + \mu'(\iota_T' \otimes I_N)C(\rho)v,$$

which are either independent or asymptotically independent. Thus, the asymptotic normality of the first two terms on the right hand side of the above equation follow from the generalized central limit theorem for linear-quadratic forms of Lee (2004a, Appendix A). This generalizes the results of Kelejian and Prucha (2001). The asymptotic normality of the last term follows from the fact that the two random vectors involved are independent. The mean and variance of $u'C(\rho)u$ can be easily obtained from Lemma 3 in the Appendix. In fact, $E(u'C(\rho)u) = \sigma_v^2 \mathrm{tr}(\Sigma C(\rho))$, and

$$\mathrm{Var}(u'C(\rho)u) = \sigma_v^4\{\phi^2\kappa_\mu a_1'a_1 + \kappa_v a_2'a_2 + \mathrm{tr}[\Sigma(C(\rho)' + C(\rho))\Sigma C(\rho)]\}.$$

Thus the result in (i) follows and $\mathrm{LM}_{\mathrm{A}}^* \xrightarrow{D} N(0,1)$.

To prove the result in (ii), let $X(\rho) = \Sigma^{-1/2}X$ and $M^*(\rho) = I_{NT} - X(\rho)[X'(\rho)X(\rho)]^{-1}X'(\rho)$. Assumption 3 and the structure of $\Sigma^{-1/2}$ guarantee that the elements of $X(\rho)$ are bounded uniformly in both N and ρ. Thus, Lemma 2 in the Appendix is applicable on $M^*(\rho)$ for each ρ. We have $C(\rho) = M'(\rho)A(\rho)M(\rho) = \Sigma^{1/2}M^*(\rho)A(\rho)M^*(\rho)\Sigma^{-1/2}$. Thus,

$$\begin{aligned}
\mathrm{tr}[\Sigma C(\rho)] &= \mathrm{tr}[M^*(\rho)A(\rho)M^*(\rho)\Sigma] \\
&= \mathrm{tr}[A(\rho)M^*(\rho)\Sigma] + O(1) \quad \text{(by Lemma 2, Appendix)} \\
&= \mathrm{tr}[M^*(\rho)\Sigma A(\rho)] + O(1) \\
&= \mathrm{tr}[\Sigma A(\rho)] + O(1) \quad \text{(by Lemma 2, Appendix)} \\
&= O(1).
\end{aligned}$$

Similarly, by successively applying Lemma 2, one shows that

$$\begin{aligned}
\mathrm{tr}[\Sigma(C(\rho)' + C(\rho))\Sigma C(\rho)] &= \mathrm{tr}[M^*(\rho)(A(\rho)' + A(\rho))M^*(\rho)\Sigma M^*(\rho)A(\rho)M^*(\rho)\Sigma] \\
&= \mathrm{tr}[(A(\rho)' + A(\rho))\Sigma A(\rho)\Sigma] + O(1) \\
&= (T - 1 + \rho^2)S_0 + O(1).
\end{aligned}$$

Under Assumption B2, the elements of W^2 and WW' are of uniform order $O(1/h_N)$. It follows that $S_0 = O(N/h_N)$. Hence,

$$\mathrm{tr}[\Sigma(C(\rho)' + C(\rho))\Sigma C(\rho)] \sim (T - 1 + \rho^2)S_0 = O(N/h_N).$$

Finally, Lemma 2(vii) in the Appendix leads to $a_1'a_1 = O(1/h_N)$ and $a_2'a_2 = O(1/h_N)$. The result in (ii) thus follows and the two LM tests given in (11) and (13) are asymptotically equivalent. *Q.E.D.*

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Table 1. Empirical Means, SDs and Tail Probabilities at 5% Level: Linear Regression

		Normal			Normal Mixture			Log-normal		
N	Test	Mean	SD	Prob	Mean	SD	Prob	Mean	SD	Prob
		`Spatial Layout: Large Group Interaction with` $G = N^{0.2}$								
50	LM_B	-0.4925	0.7296	0.0161	-0.5003	0.5967	0.0051	-0.4977	0.6449	0.0099
	I^0	-0.6894	1.0213	0.0310	-0.7003	0.8352	0.0151	-0.6967	0.9027	0.0206
	I^*	0.0099	1.0213	0.0542	-0.0011	0.8352	0.0348	0.0025	0.9027	0.0443
	LM_B^*	0.0103	1.0642	0.0582	-0.0011	0.8639	0.0380	0.0026	0.9360	0.0479
100	LM_B	-0.3736	0.8496	0.0250	-0.3983	0.7184	0.0100	-0.3884	0.7759	0.0163
	I^0	-0.4479	1.0188	0.0370	-0.4776	0.8614	0.0181	-0.4658	0.9303	0.0258
	I^*	0.0224	1.0188	0.0557	-0.0073	0.8614	0.0344	0.0045	0.9303	0.0435
	LM_B^*	0.0228	1.0396	0.0580	-0.0074	0.8783	0.0356	0.0046	0.9489	0.0453
200	LM_B	-0.4177	0.8048	0.0196	-0.4134	0.7433	0.0108	-0.4109	0.7566	0.0150
	I^0	-0.5121	0.9868	0.0291	-0.5068	0.9113	0.0188	-0.5037	0.9276	0.0257
	I^*	-0.0093	0.9868	0.0503	-0.0040	0.9113	0.0403	-0.0009	0.9276	0.0428
	LM_B^*	-0.0094	0.9968	0.0514	-0.0040	0.9189	0.0407	-0.0009	0.9358	0.0436
500	LM_B	-0.4129	0.8112	0.0181	-0.3968	0.7862	0.0169	-0.4105	0.7984	0.0178
	I^0	-0.5049	0.9920	0.0313	-0.4852	0.9614	0.0272	-0.5020	0.9764	0.0285
	I^*	-0.0067	0.9920	0.0521	0.0130	0.9614	0.0481	-0.0039	0.9764	0.0474
	LM_B^*	-0.0067	0.9960	0.0524	0.0130	0.9651	0.0483	-0.0039	0.9801	0.0477
1000	LM_B	-0.3480	0.8710	0.0243	-0.3573	0.8474	0.0212	-0.3505	0.8620	0.0209
	I^0	-0.4022	1.0065	0.0329	-0.4129	0.9793	0.0279	-0.4050	0.9962	0.0307
	I^*	0.0076	1.0065	0.0507	-0.0031	0.9793	0.0462	0.0048	0.9962	0.0496
	LM_B^*	0.0076	1.0085	0.0509	-0.0031	0.9812	0.0463	0.0048	0.9981	0.0498
		`Spatial Layout: Group Interaction with` $G = N^{0.5}$								
50	LM_B	-0.2831	0.9286	0.0257	-0.2566	0.7907	0.0177	-0.2599	0.8425	0.0182
	I^0	-0.3031	0.9939	0.0384	-0.2747	0.8463	0.0252	-0.2781	0.9017	0.0248
	I^*	-0.0247	0.9939	0.0450	0.0036	0.8463	0.0287	0.0002	0.9017	0.0310
	LM_B^*	-0.0258	1.0356	0.0510	0.0037	0.8752	0.0323	0.0002	0.9348	0.0359
100	LM_B	-0.2261	0.9534	0.0303	-0.2305	0.8353	0.0222	-0.2240	0.8845	0.0239
	I^0	-0.2382	1.0045	0.0388	-0.2429	0.8801	0.0286	-0.2360	0.9319	0.0295
	I^*	0.0007	1.0045	0.0464	-0.0040	0.8801	0.0303	0.0029	0.9319	0.0359
	LM_B^*	0.0007	1.0251	0.0500	-0.0040	0.8940	0.0319	0.0030	0.9484	0.0384
200	LM_B	-0.1616	0.9694	0.0328	-0.1810	0.8798	0.0282	-0.1798	0.9158	0.0264
	I^0	-0.1670	1.0015	0.0388	-0.1870	0.9089	0.0329	-0.1858	0.9461	0.0301
	I^*	0.0164	1.0015	0.0434	-0.0036	0.9089	0.0344	-0.0024	0.9461	0.0342
	LM_B^*	0.0166	1.0116	0.0451	-0.0037	0.9170	0.0354	-0.0024	0.9549	0.0362
500	LM_B	-0.1522	0.9765	0.0402	-0.1290	0.9439	0.0364	-0.1287	0.9460	0.0336
	I^0	-0.1554	0.9971	0.0444	-0.1317	0.9637	0.0406	-0.1314	0.9659	0.0367
	I^*	-0.0096	0.9971	0.0442	0.0140	0.9637	0.0420	0.0144	0.9659	0.0410
	LM_B^*	-0.0096	1.0011	0.0451	0.0141	0.9674	0.0425	0.0144	0.9696	0.0417
1000	LM_B	-0.1094	0.9865	0.0435	-0.1270	0.9662	0.0409	-0.1040	0.9645	0.0402
	I^0	-0.1111	1.0018	0.0467	-0.1290	0.9811	0.0445	-0.1056	0.9795	0.0432
	I^*	0.0140	1.0018	0.0466	-0.0039	0.9811	0.0439	0.0195	0.9795	0.0430
	LM_B^*	0.0140	1.0038	0.0470	-0.0039	0.9830	0.0445	0.0196	0.9814	0.0433

Table 1. Cont'd

N	Test	Normal			Normal Mixture			Log-normal		
		Mean	SD	Prob	Mean	SD	Prob	Mean	SD	Prob
		`Spatial Layout: Small Group Interaction with` $G = N^{0.8}$								
50	LM_B	-0.1136	0.9836	0.0441	-0.1171	0.8330	0.0295	-0.1269	0.8776	0.0294
	I^0	-0.1154	0.9999	0.0471	-0.1191	0.8468	0.0311	-0.1290	0.8922	0.0321
	I^*	0.0027	0.9999	0.0462	-0.0009	0.8468	0.0307	-0.0109	0.8922	0.0341
	LM_B^*	0.0028	1.0419	0.0575	-0.0010	0.8752	0.0345	-0.0113	0.9247	0.0395
100	LM_B	-0.1064	0.9851	0.0469	-0.0956	0.8594	0.0328	-0.1029	0.9124	0.0343
	I^0	-0.1075	0.9957	0.0494	-0.0967	0.8687	0.0335	-0.1040	0.9223	0.0350
	I^*	-0.0085	0.9957	0.0477	0.0023	0.8687	0.0327	-0.0050	0.9223	0.0386
	LM_B^*	-0.0087	1.0161	0.0540	0.0024	0.8827	0.0346	-0.0051	0.9387	0.0408
200	LM_B	-0.0890	0.9999	0.0503	-0.0708	0.9044	0.0390	-0.0800	0.9333	0.0372
	I^0	-0.0894	1.0042	0.0514	-0.0711	0.9083	0.0395	-0.0803	0.9374	0.0384
	I^*	-0.0146	1.0042	0.0531	0.0037	0.9083	0.0391	-0.0055	0.9374	0.0404
	LM_B^*	-0.0148	1.0144	0.0554	0.0037	0.9164	0.0401	-0.0056	0.9461	0.0416
500	LM_B	-0.0313	0.9938	0.0499	-0.0528	0.9696	0.0484	-0.0426	0.9516	0.0382
	I^0	-0.0314	0.9960	0.0502	-0.0529	0.9717	0.0489	-0.0427	0.9537	0.0385
	I^*	0.0149	0.9960	0.0496	-0.0066	0.9717	0.0477	0.0036	0.9537	0.0395
	LM_B^*	0.0150	1.0000	0.0506	-0.0066	0.9754	0.0487	0.0036	0.9573	0.0400
1000	LM_B	-0.0491	0.9912	0.0468	-0.0366	0.9699	0.0505	-0.0254	0.9655	0.0411
	I^0	-0.0491	0.9927	0.0472	-0.0367	0.9713	0.0505	-0.0254	0.9669	0.0412
	I^*	-0.0112	0.9927	0.0476	0.0013	0.9713	0.0510	0.0126	0.9669	0.0416
	LM_B^*	-0.0112	0.9946	0.0480	0.0013	0.9732	0.0514	0.0126	0.9687	0.0418
		`Spatial Layout: Queen's Contiguity`								
50	LM_B	-0.2412	0.9375	0.0374	-0.2267	0.7980	0.0189	-0.2379	0.8462	0.0231
	I^0	-0.2570	0.9991	0.0528	-0.2416	0.8504	0.0286	-0.2535	0.9018	0.0314
	I^*	-0.0052	0.9991	0.0508	0.0103	0.8504	0.0258	-0.0017	0.9018	0.0332
	LM_B^*	-0.0054	1.0410	0.0596	0.0106	0.8792	0.0311	-0.0018	0.9348	0.0376
100	LM_B	-0.1661	0.9735	0.0447	-0.1538	0.8363	0.0267	-0.1462	0.9071	0.0307
	I^0	-0.1709	1.0015	0.0508	-0.1582	0.8604	0.0321	-0.1504	0.9333	0.0351
	I^*	-0.0083	1.0015	0.0497	0.0043	0.8604	0.0300	0.0122	0.9333	0.0380
	LM_B^*	-0.0085	1.0220	0.0546	0.0044	0.8754	0.0325	0.0124	0.9506	0.0409
200	LM_B	-0.1259	0.9769	0.0457	-0.1077	0.9145	0.0395	-0.1238	0.9281	0.0344
	I^0	-0.1281	0.9938	0.0486	-0.1096	0.9303	0.0427	-0.1260	0.9441	0.0373
	I^*	-0.0028	0.9938	0.0489	0.0157	0.9303	0.0413	-0.0007	0.9441	0.0385
	LM_B^*	-0.0028	1.0038	0.0510	0.0158	0.9380	0.0429	-0.0007	0.9525	0.0397
500	LM_B	-0.0727	0.9986	0.0510	-0.0813	0.9665	0.0484	-0.0752	0.9707	0.0416
	I^0	-0.0732	1.0052	0.0525	-0.0819	0.9729	0.0498	-0.0757	0.9771	0.0428
	I^*	-0.0076	1.0052	0.0527	-0.0010	0.9729	0.0504	0.0052	0.9771	0.0429
	LM_B^*	0.0077	1.0093	0.0532	-0.0010	0.9765	0.0508	0.0052	0.9807	0.0434
1000	LM_B	-0.0426	0.9972	0.0486	-0.0546	0.9901	0.0498	-0.0484	0.9801	0.0444
	I^0	-0.0427	1.0007	0.0492	-0.0548	0.9936	0.0508	-0.0485	0.9836	0.0452
	I^*	0.0152	1.0007	0.0502	0.0031	0.9936	0.0503	0.0093	0.9836	0.0459
	LM_B^*	0.0152	1.0027	0.0506	0.0031	0.9955	0.0509	0.0093	0.9855	0.0462

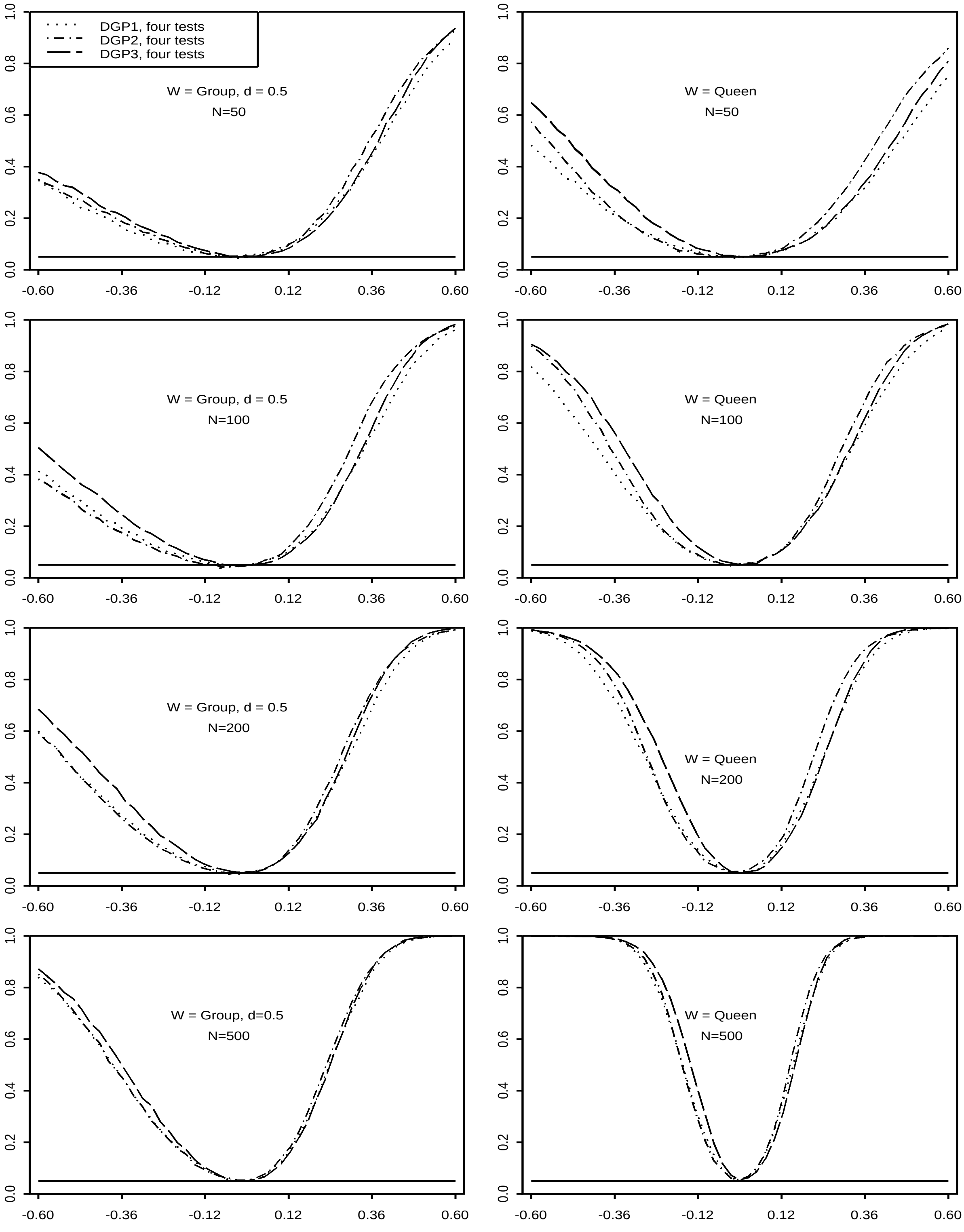

Figure 1. Size-Adjusted Empirical Powers of the Four Tests

Table 2. Empirical Means, SDs and Tail Probabilities at 5% Level: Panel Regression

$T = 3$	Normal			Normal Mixture			Log-normal		
N	Mean	SD	Prob	Mean	SD	Prob	Mean	SD	Prob
	`Spatial Layout: Large Group Interaction,` $G = N^{0.2}$								
20	-0.2527	0.9195	0.0240	-0.2573	0.7854	0.0180	-0.2645	0.8462	0.0203
	0.0056	1.0295	0.0514	-0.0038	0.8704	0.0334	-0.0097	0.9419	0.0374
50	-0.2601	0.9168	0.0251	-0.2525	0.8416	0.0159	-0.2625	0.8490	0.0201
	0.0108	1.0090	0.0476	0.0171	0.9182	0.0336	0.0068	0.9291	0.0381
100	-0.2391	0.9411	0.0251	-0.2038	0.9009	0.0293	-0.2069	0.9168	0.0249
	-0.0275	0.9944	0.0401	0.0079	0.9487	0.0397	0.0051	0.9663	0.0390
200	-0.2066	0.9565	0.0332	-0.2129	0.9337	0.0304	-0.1953	0.9305	0.0268
	0.0069	1.0067	0.0468	-0.0005	0.9811	0.0440	0.0180	0.9774	0.0416
500	-0.2055	0.9511	0.0307	-0.2048	0.9359	0.0271	-0.2011	0.9433	0.0273
	0.0132	1.0014	0.0454	0.0132	0.9845	0.0419	0.0174	0.9926	0.0433
	`Spatial Layout: Group Interaction,` $G = N^{0.5}$								
20	-0.2071	0.9515	0.0321	-0.2161	0.8339	0.0235	-0.2016	0.8747	0.0251
	-0.0079	1.0253	0.0490	-0.0204	0.8929	0.0324	-0.0035	0.9383	0.0368
50	-0.0984	0.9813	0.0420	-0.1134	0.8948	0.0332	-0.1112	0.9227	0.0325
	0.0156	1.0085	0.0474	-0.0011	0.9175	0.0379	0.0014	0.9466	0.0406
100	-0.1024	0.9846	0.0448	-0.1057	0.9256	0.0336	-0.1280	0.9386	0.0352
	0.0093	1.0039	0.0484	0.0053	0.9430	0.0389	-0.0172	0.9563	0.0401
200	-0.0943	0.9830	0.0461	-0.0813	0.9535	0.0388	-0.0929	0.9555	0.0382
	-0.0008	0.9951	0.0479	0.0121	0.9649	0.0413	0.0002	0.9668	0.0417
500	-0.0737	0.9878	0.0466	-0.0635	0.9837	0.0451	-0.0661	0.9808	0.0429
	0.0045	0.9954	0.0482	0.0147	0.9910	0.0465	0.0121	0.9880	0.0449
	`Spatial Layout: Small Group Interaction,` $G = N^{0.8}$								
20	-0.0648	0.9981	0.0493	-0.0647	0.8497	0.0294	-0.0541	0.9146	0.0372
	0.0140	1.0402	0.0596	0.0127	0.8838	0.0329	0.0246	0.9520	0.0469
50	-0.0235	0.9918	0.0473	-0.0463	0.8969	0.0383	-0.0214	0.9259	0.0352
	0.0182	1.0099	0.0526	-0.0056	0.9121	0.0394	0.0200	0.9421	0.0381
100	-0.0621	0.9885	0.0490	-0.0543	0.9385	0.0425	-0.0595	0.9506	0.0389
	-0.0074	0.9977	0.0518	0.0000	0.9470	0.0430	-0.0051	0.9592	0.0418
200	-0.0486	0.9969	0.0486	-0.0418	0.9721	0.0493	-0.0547	0.9600	0.0382
	-0.0108	1.0016	0.0496	-0.0042	0.9766	0.0496	-0.0170	0.9644	0.0396
500	-0.0117	0.9957	0.0489	-0.0162	0.9916	0.0542	-0.0238	0.9738	0.0408
	0.0137	0.9977	0.0502	0.0091	0.9936	0.0542	0.0016	0.9757	0.0421
	`Spatial Layout: Queen's Contiguity`								
20	-0.1518	0.9654	0.0416	-0.1556	0.8194	0.0252	-0.1595	0.8858	0.0290
	0.0159	1.0602	0.0634	0.0081	0.8946	0.0352	0.0058	0.9696	0.0454
50	-0.1122	0.9824	0.0479	-0.1311	0.8859	0.0342	-0.1252	0.9200	0.0333
	0.0097	1.0334	0.0580	-0.0110	0.9298	0.0434	-0.0047	0.9664	0.0407
100	-0.0862	0.9837	0.0462	-0.0926	0.9385	0.0438	-0.0943	0.9456	0.0389
	0.0068	1.0148	0.0529	-0.0007	0.9674	0.0491	-0.0022	0.9748	0.0455
200	-0.0580	0.9948	0.0483	-0.0760	0.9593	0.0443	-0.0555	0.9730	0.0433
	0.0131	1.0167	0.0541	-0.0057	0.9801	0.0470	0.0153	0.9941	0.0485
500	-0.0321	0.9880	0.0422	-0.0469	0.9860	0.0495	-0.0540	0.9801	0.0447
	0.0116	1.0033	0.0465	-0.0035	1.0012	0.0526	-0.0108	0.9953	0.0490

Note: under each N, the first row corresponds to $\mathrm{LM_A}$ and the second corresponds to $\mathrm{LM_A^*}$.

Table 2. Cont'd

$T=10$	Normal			Normal Mixture			Log-normal		
N	Mean	SD	Prob	Mean	SD	Prob	Mean	SD	Prob
	Spatial Layout: Large Group Interaction, $G=N^{0.2}$								
20	-0.1280	0.9754	0.0389	-0.1096	0.9059	0.0340	-0.1280	0.9130	0.0308
	-0.0111	0.9997	0.0427	0.0079	0.9262	0.0378	-0.0110	0.9345	0.0367
50	-0.1080	0.9877	0.0430	-0.1222	0.9438	0.0336	-0.1118	0.9528	0.0340
	0.0084	1.0051	0.0490	-0.0056	0.9599	0.0358	0.0052	0.9697	0.0405
100	-0.1043	1.0048	0.0465	-0.0876	0.9775	0.0416	-0.0895	0.9619	0.0382
	-0.0099	1.0160	0.0491	0.0078	0.9883	0.0448	0.0067	0.9727	0.0421
200	-0.1044	0.9838	0.0458	-0.0977	0.9712	0.0370	-0.0966	0.9735	0.0398
	-0.0089	0.9942	0.0482	-0.0013	0.9814	0.0403	-0.0001	0.9835	0.0444
500	-0.0978	0.9915	0.0412	-0.1199	0.9738	0.0385	-0.0682	0.9867	0.0455
	-0.0035	1.0009	0.0440	-0.0256	0.9831	0.0406	0.0271	0.9961	0.0496
	Spatial Layout: Group Interaction, $G=N^{0.5}$								
20	-0.0806	0.9945	0.0484	-0.0723	0.9163	0.0392	-0.0928	0.9287	0.0356
	0.0001	1.0105	0.0493	0.0092	0.9298	0.0413	-0.0114	0.9434	0.0383
50	-0.0472	0.9886	0.0460	-0.0467	0.9451	0.0451	-0.0617	0.9679	0.0407
	0.0108	0.9958	0.0477	0.0118	0.9519	0.0462	-0.0027	0.9750	0.0418
100	-0.0422	1.0053	0.0481	-0.0511	0.9679	0.0447	-0.0434	0.9743	0.0442
	0.0062	1.0097	0.0491	-0.0024	0.9721	0.0458	0.0055	0.9783	0.0455
200	-0.0412	0.9846	0.0448	-0.0318	0.9814	0.0436	-0.0413	0.9863	0.0455
	0.0023	0.9876	0.0458	0.0121	0.9843	0.0443	0.0027	0.9893	0.0472
500	-0.0202	1.0025	0.0497	-0.0467	1.0034	0.0506	-0.0460	0.9914	0.0471
	0.0143	1.0041	0.0504	-0.0122	1.0050	0.0505	-0.0113	0.9929	0.0484
	Spatial Layout: Small Group Interaction, $G=N^{0.8}$								
20	-0.0252	0.9851	0.0456	-0.0402	0.9183	0.0401	-0.0320	0.9316	0.0359
	0.0131	0.9964	0.0481	-0.0017	0.9279	0.0410	0.0065	0.9417	0.0383
50	-0.0171	0.9969	0.0497	-0.0251	0.9639	0.0494	-0.0298	0.9714	0.0421
	0.0092	1.0021	0.0507	0.0014	0.9687	0.0497	-0.0033	0.9764	0.0438
100	-0.0162	1.0136	0.0524	-0.0257	0.9775	0.0490	0.0014	0.9863	0.0445
	0.0053	1.0162	0.0530	-0.0040	0.9799	0.0489	0.0232	0.9888	0.0454
200	-0.0218	0.9911	0.0492	-0.0075	0.9977	0.0529	-0.0034	1.0046	0.0474
	-0.0065	0.9923	0.0499	0.0079	0.9990	0.0533	0.0121	1.0058	0.0475
500	0.0034	1.0119	0.0534	-0.0137	0.9946	0.0531	-0.0148	0.9957	0.0485
	0.0147	1.0125	0.0540	-0.0024	0.9952	0.0531	-0.0034	0.9962	0.0490
	Spatial Layout: Queen's Contiguity								
20	-0.0660	0.9845	0.0472	-0.0797	0.8990	0.0406	-0.0682	0.9333	0.0366
	0.0142	1.0317	0.0567	0.0010	0.9410	0.0467	0.0131	0.9774	0.0452
50	-0.0550	0.9933	0.0465	-0.0673	0.9547	0.0447	-0.0499	0.9585	0.0406
	0.0000	1.0204	0.0539	-0.0121	0.9805	0.0493	0.0061	0.9846	0.0452
100	-0.0458	0.9936	0.0480	-0.0258	0.9812	0.0470	-0.0433	0.9741	0.0444
	-0.0049	1.0129	0.0523	0.0158	1.0002	0.0509	-0.0017	0.9928	0.0496
200	-0.0268	1.0011	0.0532	-0.0232	0.9901	0.0514	-0.0185	0.9760	0.0442
	0.0026	1.0167	0.0562	0.0065	1.0054	0.0552	0.0114	0.9911	0.0471
500	-0.0304	0.9954	0.0484	-0.0077	1.0024	0.0508	-0.0109	1.0002	0.0510
	-0.0113	1.0083	0.0511	0.0118	1.0155	0.0542	0.0085	1.0132	0.0538

Note: under each N, the first row corresponds to $\mathrm{LM_A}$ and the second corresponds to $\mathrm{LM_A^*}$.

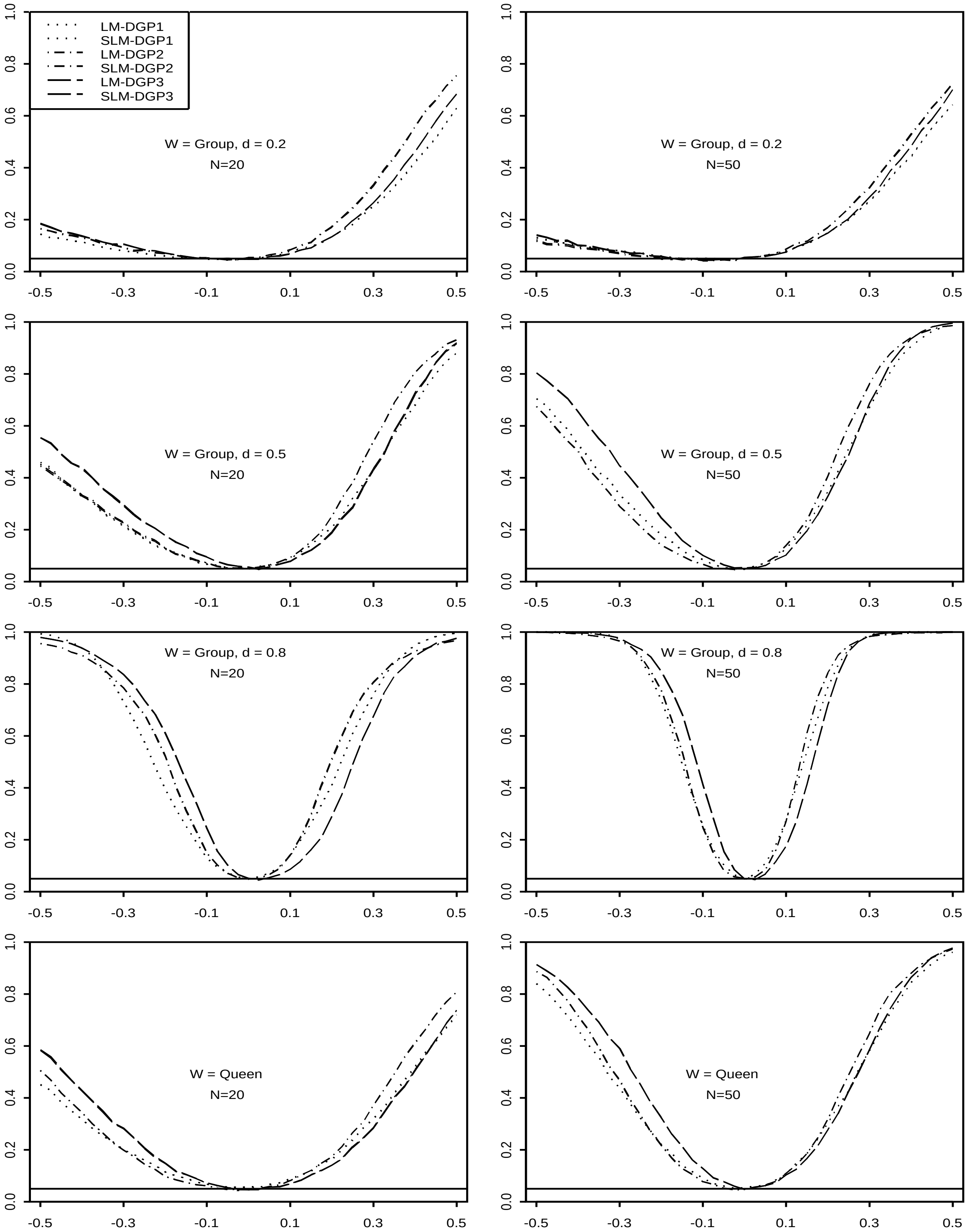

Figure 2. Size-Adjusted Empirical Powers of Panel LM and SLM Tests