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Juergen von Hagen and Haiping Zhang

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Jürgen von Hagen† and Haiping Zhang‡

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Abstract

We develop a tractable multi-country overlapping-generations model and show that cross-country differences in financial development explain three recent empirical patterns of international capital flows. Domestic financial frictions in our model distort interest rates and aggregate output in the less financially developed countries. International capital flows help ameliorate the two distortions.

International flows of financial capital and foreign direct investment affect aggregate output in each country directly through affecting the size of aggregate investment. In addition, they affect aggregate output indirectly through affecting the composition of aggregate investment and the size of aggregate savings. Under certain conditions, the indirect effects may dominate the direct effects so that, despite "uphill" net capital flows, full capital mobility may raise the steady-state aggregate output in the poor country as well as raise world output. However, if foreign direct investment is restricted, "uphill" financial capital flows strictly reduce the steady-state aggregate output in the poor countries and it is more likely that the steady-state world output is lower than under international financial autarky.

JEL Classification: E44, F41

Keywords: Capital account liberalization, financial frictions, financial development, foreign direct investment, world output gains

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†University of Bonn, Indiana University and CEPR. Lennestrasse. 37, D-53113 Bonn, Germany. E-mail: vonhagen@uni-bonn.de

‡Corresponding author. School of Economics, Singapore Management University. 90 Stamford Road, Singapore 178903. E-mail: hpzhang@smu.edu.sg

1 Introduction

This paper analyzes how the recent empirical patterns of international capital flows may affect the steady-state aggregate output at the country level as well as at the world level. According to the conventional neoclassical theory, capital should flow "downhill" from the rich country where the marginal return on capital is low to the poor country where the marginal return on capital is high. As a result, world output should be higher than under international financial autarky (IFA, hereafter). Meanwhile, there would be no difference between gross and net capital flows because capital flows would be unidirectional.

The recent empirical patterns of international capital flows are in stark contrast to these predictions (Lane and Milesi-Ferretti, 2001, 2007a,b). First, capital in the net term flows "uphill" from poor to rich countries (Prasad, Rajan, and Subramanian, 2006, 2007). Second, financial capital flows from poor to rich countries, while foreign direct investment (FDI, hereafter) flows in the opposite direction (Ju and Wei, 2010). Third, despite its negative net positions of international investment since 1986, the U.S. has been receiving a positive net investment income until 2005 (Gourinchas and Rey, 2007; Hausmann and Sturzenegger, 2007; Higgins, Klitgaard, and Tille, 2007). According to conventional neoclassical models (Matsuyama, 2004), "uphill" net capital flows reduce aggregate output in the poor country as well as world output. It implies that the recent wave of financial globalization is a negative-sum game where the welfare losses in the poor country exceeds the welfare gains in the rich country. However, before evaluating the welfare implications of international capital flows, we should first develop a model which has the theoretical predictions in line with the empirical patterns.

Recent research offers two main explanations to these empirical facts. Devereux and Sutherland (2009) and Tille and van Wincoop (2008, 2010) focus on the risk-sharing that investors can achieve by diversifying investment globally. International portfolio investment is determined by the cross-correlation patterns of aggregate shocks hitting individual economies. These models do not distinguish between FDI and portfolio investment. The second strand of literature emphasizes the implications of domestic financial market imperfections on the patterns of international capital flows (Antras and Caballero, 2009; Antras, Desai, and Foley, 2009; Aoki, Benigno, and Kiyotaki, 2009; Caballero, Farhi, and Gourinchas, 2008; Mendoza, Quadrini, and Rios-Rull, 2009; Smith and Valderrama, 2008). Ju and Wei (2008, 2010) show that cross-country differences in various aspects jointly generate the two-way flows of financial capital and FDI. The distinction between FDI and portfolio investment plays a key role in their models. However, most models address the current account determination without analyzing the aggregate output implications at the country or at the world level.

Following the second strand of the literature, we develop a tractable multi-country overlapping-generations model and show that cross-country differences in financial development explain these three empirical facts. Furthermore, under certain conditions, despite "uphill" net capital flows, full capital mobility may raise aggregate output in the poor country as well as raise world output in the steady state. Thus, poor countries may possibly benefit from the recent wave of financial globalization and it is feasible to achieve Pareto improvement at the world level through international transfer policy. These results are in contrast to the prediction of conventional neoclassical models.

Credit markets channel resources from the less to the more productive individuals. If credit markets were perfect, production would be conducted efficiently in the sense that the marginal rate of return is equalized across productive projects. In particular, the rates of return on loans and equity capital would be equal to the social rate of return. However, due to domestic financial frictions, the more productive individuals are subject to borrowing constraints. Under IFA, due to the constraint on the aggregate credit demand, the rate of return on loan, i.e., the loan rate, is inefficiently lower, while the rate of return to the equity capital of the more productive individuals, i.e., the equity rate, is higher than social rate of return. Thus, financial frictions distort the two interest rates. Furthermore, since the borrowing constraints keep the investment of the more (less) productive individuals lower (higher) than the socially efficiently level, aggregate output is also lower than in the frictionless case; due to the distortions on interest rates, aggregate savings are inefficiently lower than in the frictionless case and so is aggregate output. This way, financial frictions in our model also distort aggregate output through the ***investment composition*** channel and through the ***elastic savings*** channel.

We illustrate our major results intuitively in a two-country version of the multi-country model. The world economy consists of two countries, country N (North) and country S (South), which are fundamentally identical except that country N is more financially developed. Suppose that the two countries are in the steady state under IFA before capital mobility is allowed. Initially, the loan rate is higher, the equity rate is lower, and aggregate output is higher in country N than in country S. Under full capital mobility, the initial interest rate differentials drive financial capital flowing from from country S to country N and FDI flowing in the opposite direction. Since country N has a larger credit market, net capital flows are "uphill" from country S to country N. By receiving a higher return on its foreign assets than what it pays for its foreign liabilities, country N obtains a positive net investment income, despite its negative net international investment position. Intuitively, country N "exports" its financial services through two-way capital flows and receives a positive net return. This way, our model generates the theoretical predictions in line with the three empirical facts.

Full capital mobility affects aggregate output in each country directly through the size of aggregate investment and indirectly through the investment composition channel as well as through the elastic savings channel. Take country S as an example. First, net capital outflows directly reduce total resources available for domestic investment, which tends to reduce aggregate output. Second, financial capital outflows reduce domestic credit supply and FDI inflows raise domestic credit demand, which jointly raises the loan rate. The increase in the loan rate triggers the resource reallocation from the less to the more productive investment as well as raises aggregate savings, which tends to raise aggregate output. The direct effect depends on net capital flows, while the indirect effects depend on gross capital flows. Under full capital mobility, two-way capital flows imply that gross flows are much larger than net flows. Thus, if the initial cross-country differences in output distortions are large under IFA, the indirect effects may dominate and full capital mobility may raise aggregate output in country S, despite net capital outflows. However, if FDI flows are restricted, capital flows become one-way and net flows coincide with gross flows. Thus, the direct effect always dominates so that partial capital mobility strictly reduces aggregate output in country S.

Full capital mobility has the similar direct and indirect effects on world output. Take financial capital flows as an example. By ***cross-country resource reallocation***, "uphill" financial capital flows reduce (raise) the size of aggregate investment in country S (N), which tends to widen the cross-country aggregate output gap. Given the concave aggregate production with respect to aggregate investment in each country, "uphill" financial capital flows tend to reduce the steady-state world output, according to the Jensen's inequality. For simplicity, we call it the net investment size effect. Meanwhile, financial capital flows raise (reduce) the loan rate in country S (N), which indirectly triggers ***within-country resource reallocation*** among investment projects as well as between consumption and savings, as mentioned above. In particular, the indirect effects on aggregate output are positive (negative) for country S (N). Since the initial output distortions is more severe in country S than in country N, the output gains in country S dominate the output losses in country N. Thus, financial capital flows tend to raise the steady-state world output. For simplicity, we call it the net indirect effect. The same mechanism applies to FDI flows. Similar as the argument for aggregate output in country S, if the initial cross-country differences in output distortions are large under IFA, the net indirect effects may dominate and full capital mobility may raise world output, despite "uphill" net capital flows. However, if FDI flows are restricted, it is more likely that the direct effect dominates and partial capital mobility reduces world output.

As widely documented in the literature (Barlevy, 2003; Hsieh and Klenow, 2009; Jeong and Townsend, 2007; Levine, 1997; Midrigan and Xu, 2009), financial frictions distort production efficiency in the sense that some resources are inefficiently allocated into the less productive projects. If such distortions were not considered, the efficiency analysis of international capital flows would be incomplete and misleading. Our model is also related to the recent trade literature (Melitz, 2003). In particular, international trade leads to the reallocation of market shares from less to more productive firms, which generates a new channel for productivity and welfare gains. International capital flows in our model generate output gains by triggering similar resource reallocation among firms.

To summarize, financial frictions distort aggregate output through affecting resource allocation among investment projects with different productivity as well as through affecting resource allocation between consumption and savings. According to conventional neoclassical models (Matsuyama, 2004; von Hagen and Zhang, 2010), "uphill" net capital flows affect the size of aggregate investment in each country, which reduces aggregate output in the poor country as well as at the world level. As our contribution to the literature, we show that international capital flows indirectly trigger resource reallocation in these two dimensions, which generates output gains in the poor country as well as at the world level. Thus, our results complement the predictions of conventional neoclassical models.

The paper is organized as follows. Section 2 sets up the model under IFA and shows how domestic financial frictions distort interest rates and aggregate output. Section 3 shows the patterns of international capital flows and analyzes the implications on aggregate output. Section 4 addresses the output implications of partial capital mobility where FDI is restricted. Section 5 concludes and the appendix collects relevant proofs.

2 The Model under International Financial Autarky

2.1 The Model Setting

The world economy consists of $N \geq 2$ countries, which are fundamentally identical except the level of financial development as specified later. There are a final good, which is internationally tradable and serves as numeraire, and two types of intermediate goods, A and B, which are not traded internationally. The prices of intermediate goods in country $i \in \{1, 2, ..., N\}$ and period t are denoted by $v_t^{i,A}$ and $v_t^{i,B}$. In the following, variables in country i are denoted with the superscript i.

Individuals live for two periods and there is no population growth. In each country, the population size of each generation is normalized to one and each generation consists of two types of agents which we call *entrepreneurs* and *households*, each of mass η and $1-\eta$, respectively.[1] Individuals are endowed with one unit of labor when young and $\epsilon \geq 0$ units of labor when old, which they supply inelastically to aggregate production. Thus, the aggregate labor supply is $L = 1 + \epsilon$ in each period.

Final goods are produced contemporaneously using intermediate goods and labor in the Cobb-Douglas fashion. The input of labor, L, and intermediate goods, $M_t^{i,A}$ and $M_t^{i,B}$, are rewarded at their respective marginal products. To summarize,

$$Y_t^i = \left[\frac{M_t^{i,A}}{(1-\gamma)\alpha}\right]^{(1-\gamma)\alpha} \left(\frac{M_t^{i,B}}{\gamma\alpha}\right)^{\gamma\alpha} \left(\frac{L}{1-\alpha}\right)^{1-\alpha}, \text{where } \alpha \in (0,1),\ \gamma \in (0,1], \quad (1)$$

$$\omega_t^i L = (1-\alpha) Y_t^i \quad v_t^{i,A} M_t^{i,A} = (1-\gamma)\alpha Y_t^i, \qquad v_t^{i,B} M_t^{i,B} = \gamma\alpha Y_t^i. \quad (2)$$

Y_t^i and ω_t^i denote aggregate output of final goods and the wage rate, respectively. $(1-\alpha)$, $(1-\gamma)\alpha$ and $\gamma\alpha$ measure the respective factor shares of labor, intermediate goods A and B. There is no uncertainty in the economy. In this section, we assume that international capital flows are not allowed.

When young, individuals can produce intermediate goods from final goods and the production takes one period to complete. Entrepreneurs and households only differ in their endowment of production opportunities. In particular, young entrepreneurs can produce both types of intermediate goods while young households can only produce intermediate good A.[2] The rate of transformation from final goods to intermediate goods is normalized at unity in both sectors. Thus, the price of intermediate good $k \in \{A, B\}$ in period $t+1$, $v_{t+1}^{i,k}$, is also the rate of return to the sector-k investment in period t.

Assumption 1. $\eta \in (0, \gamma)$.

Assumption 1 ensures that aggregate entrepreneurial net worth is smaller than the socially efficient investment size in sector B. In equilibrium, entrepreneurs only produce intermediate good B and they finance part of their project investment using debt.

Individuals have an additive logarithm preference over consumption in two periods,

$$U_t^{i,j} = (1-\beta)\ln c_{y,t}^{i,j} + \beta \ln c_{o,t+1}^{i,j}, \quad (3)$$

[1]Matsuyama (2004) assumes that individuals are identical ex ante. Due to credit rationing, a fraction of individuals are randomly chosen to become entrepreneurs ex post and this fraction is endogenously determined. As shown in von Hagen and Zhang (2010), such an assumption is essential for the symmetry-breaking property of financial globalization, but FDI cannot be addressed. In order to analyze the joint determination of financial capital and FDI flows, we follow the assumption of Antras and Caballero (2009) by assuming that entrepreneurs account for a fixed fraction of population.

[2]We allow for the case of $\gamma = 1$ where intermediate good B and labor are used in the aggregate production. In equilibrium, intermediate good A vanishes.

where $c_{y,t}^{i,j}$ and $c_{o,t+1}^{i,j}$ denote its consumption when *y*oung and when *o*ld, respectively; $j \in \{e, h\}$ denotes the identity of *e*ntrepreneur and *h*ousehold, respectively; $\beta \in (0, 1]$ measures the relative weight of utility from consumption when old in the lifetime welfare.

Consider any particular household born in period t. In period t, it receives the labor income ω_t^i, consumes $c_{y,t}^{i,h}$, save $s_t^i = \omega_t^i - c_{y,t}^{i,h}$ in the form of the investment in its own production project $i_t^{i,h}$ and the loans to entrepreneurs d_t^i at the gross loan rate R_t^i. In period $t+1$, it receives the project revenue $v_{t+1}^{i,A} i_t^{i,h}$ and the gross deposit return $R_t^i d_t^i$. The household no-arbitrage condition is

$$R_t^i = v_{t+1}^{i,A}. \tag{4}$$

In addition, it receives the labor income $\epsilon\omega_{t+1}^i$. It consumes the total wealth $c_{o,t+1}^{i,e} = v_{t+1}^{i,A} i_t^{i,h} + R_t^i d_t^i + \epsilon\omega_{t+1}^i = R_t^i s_t^i + \epsilon\omega_{t+1}^i$ before exiting from the economy. Its consolidated lifetime budget constraint is $c_{y,t}^{i,h} + \frac{c_{o,t+1}^{i,h}}{R_t^i} = W_t^{i,h}$, where $W_t^{i,h} \equiv \omega_t^i + \frac{\epsilon\omega_{t+1}^i}{R_t^i}$ denotes the present value of its lifetime wealth. Given the logarithm utility function (3), the household optimal consumption-savings choices are,

$$c_{y,t}^{i,h} = (1-\beta) W_t^{i,h} \text{ and } c_{o,t+1}^{i,h} = R_t^i \beta W_t^{i,h}, \tag{5}$$

$$s_t^i = \omega_t^i - c_{y,t}^{i,h} = \beta\omega_t^i - (1-\beta)\frac{\epsilon\omega_{t+1}^i}{R_t^i}, \tag{6}$$

If we assume as in the literature that individuals only have the labor endowment when young, $\epsilon = 0$, or they only consume when old, $\beta = 1$, the second term on the right hand side of equation (6) vanishes and the savings are inelastic to the loan rate. Here, we allow for the case of elastic savings by assuming that individuals have the labor endowment when old, $\epsilon > 0$, and they care about consumption when young, $\beta \in (0, 1)$. Ceteris paribus, a rise in the loan rate induces the household to save more, $\frac{\partial s_t^i}{\partial R_t^i} > 0$.

Consider any particular entrepreneur born in period t. In period t, he receives the labor income ω_t^i, consumes $c_{y,t}^{i,e}$, and finances his investment $i_t^{i,e}$ using own funds $n_t^i = \omega_t^i - c_{y,t}^{i,e}$ together with debts $z_t^i = i_t^{i,e} - n_t^i$. In period $t+1$, he receives the project revenue $v_{t+1}^{i,B} i_t^{i,e}$ and the labor income $\epsilon\omega_{t+1}^i$. After repaying the debts, he consumes the rest, $c_{o,t+1}^{i,e} = v_{t+1}^{i,B} i_t^{i,e} - R_t^i z_t^i + \epsilon\omega_{t+1}^i$, before exiting from the economy. Due to limited commitment, the entrepreneur can borrow only up to a fraction of the future project revenue,

$$R_t^i z_t^i = R_t^i (i_t^{i,e} - n_t^i) \le \theta^i v_{t+1}^{i,B} i_t^{i,e}. \tag{7}$$

Following Matsuyama (2004, 2007), we use $\theta^i \in [0, 1]$ to measure the level of financial development in country i. It captures a wide range of institutional factors and is higher in countries with more sophisticated financial and legal systems, better creditor protection, and more liquid asset market, etc. We assume that countries only differ in the level of financial development, $0 \leq \theta^1 < \theta^2 < ... < \theta^N \leq 1$.

Define the rate of return on entrepreneurial equity capital as the equity rate,[3]

$$\Gamma_t^i \equiv \frac{v_{t+1}^{i,B} i_t^{i,e} - R_t^i z_t^i}{n_t^i} = v_{t+1}^{i,B} + (v_{t+1}^{i,B} - R_t^i)(\lambda_t^i - 1) \geq R_t^i, \tag{8}$$

where $\lambda_t^i \equiv \frac{i_t^{i,e}}{n_t^i}$ denotes the investment-equity ratio. For a unit of equity capital invested, the entrepreneur gets $v_{t+1}^{i,B}$ as the marginal return. In addition, he can borrow $(\lambda_t^i - 1)$ units of debt which provides him the extra return $(v_{t+1}^{i,B} - R_t^i)$. The term $(v_{t+1}^{i,B} - R_t^i)(\lambda_t^i - 1)$ captures the leverage effect, depending positively on the debt-equity ratio, $(\lambda_t^i - 1)$ and the spread, $(v_{t+1}^{i,B} - R_t^i)$. In equilibrium, the equity rate should be no less than the loan rate; otherwise, the entrepreneur would rather lend than borrow. The inequality (8) is equivalent to $R_t^i \leq v_{t+1}^{i,B}$ and we call it the participation constraint for the entrepreneur.

If $R_t^i < v_{t+1}^{i,B}$, the entrepreneur borrows to the limit, i.e., he finances the investment, $i_t^{i,e}$, using loans, $\frac{\theta^i v_{t+1}^{i,B} i_t^{i,e}}{R_t^i}$, and net worth, n_t^i, in period t. After repaying the debt in period $t+1$, he gets the return to his net worth, $(1-\theta^i)v_{t+1}^{i,B} i_t^{i,e}$. If $R_t^i = v_{t+1}^{i,B}$, the entrepreneur does not borrow to the limit and the leverage effect vanishes so that $\Gamma_t^i = v_{t+1}^{i,B}$. To summarize,

$$\Gamma_t^i = \begin{cases} \frac{(1-\theta^i)v_{t+1}^{i,B} i_t^{i,e}}{n_t^i} = \frac{(1-\theta^i)v_{t+1}^{i,B}}{1-\frac{\theta^i v_{t+1}^{i,B}}{R_t^i}}, & \text{if } R_t^i < v_{t+1}^{i,B}, \\ v_{t+1}^{i,B} & \text{if } R_t^i = v_{t+1}^{i,B}. \end{cases} \tag{9}$$

The entrepreneur's consolidated lifetime budget constraint is $c_{y,t}^{i,e} + \frac{c_{o,t+1}^{i,e}}{R_t^i} = W_t^{i,e}$, where $W_t^{i,e} \equiv \omega_t^i + \frac{\epsilon \omega_{t+1}^i}{\Gamma_t^i}$ denotes the present value of his lifetime wealth. Given the logarithm utility function (3), the entrepreneur's optimal consumption-savings choices are,

$$c_{y,t}^{i,e} = (1-\beta)W_t^{i,e} \text{ and } c_{o,t+1}^{i,e} = \Gamma_t^i \beta W_t^{i,e}, \tag{10}$$

$$n_t^i = \omega_t^i - c_{y,t}^{i,e} = \beta\omega_t^i - (1-\beta)\frac{\epsilon\omega_{t+1}^i}{\Gamma_t^i}, \tag{11}$$

where the entrepreneurial net worth n_t^i is elastic to the equity rate if $\epsilon > 0$ and $\beta \in (0,1)$.

Aggregate output of intermediate good A and B in period $t+1$ are respectively

$$M_{t+1}^{i,A} = (1-\eta)i_t^{i,h} \text{ and } M_{t+1}^{i,B} = \eta i_t^{i,e}. \tag{12}$$

The credit market and the final good market clear in each country,

$$(1-\eta)d_t^i = \eta z_t^i, \;\; \Rightarrow \;\; (1-\eta)(s_t^i - i_t^{i,h}) = \eta(i_t^{i,h} - n_t^i), \tag{13}$$

$$C_t^i + I_t^i = Y_t^i, \tag{14}$$

[3]Since the total external values of their projects are restricted by θ^i, entrepreneurs cannot issue equity after raising loans z_t^i. Thus, equity capital is restricted by the entrepreneurial savings.

where $C_t^i \equiv \eta(c_{y,t}^{i,e} + c_{o,t}^{i,e}) + (1-\eta)(c_{y,t}^{i,h} + c_{o,t}^{i,h})$ and $I_t^i \equiv \eta i_t^{i,e} + (1-\eta) i_t^{i,h}$ denote aggregate consumption and aggregate investment in country i and period t.

Definition 1. *Given the level of financial development θ^i, a market equilibrium in country $i \in \{1, 2, ..., N\}$ under IFA is a set of allocations of households, $\{i_t^{i,h}, s_t^i, c_{y,t}^{i,h}, c_{o,t}^{i,h}\}$, entrepreneurs, $\{i_t^{i,e}, n_t^i, c_{y,t}^{i,e}, c_{o,t}^{i,e}\}$, and aggregate variables, $\{Y_t^i, M_t^{i,A}, M_t^{i,B}, \omega_t^i, v_t^{i,A}, v_t^{i,B}, R_t^i, \Gamma_t^i\}$, satisfying equations* (1)-(2), (4)-(7), (9)-(13),

2.2 Equilibrium Analysis

For the notational convenience, we define some auxiliary parameters, $m \equiv \frac{(1-\beta)\epsilon}{(1+\epsilon)\rho}$, $\mathcal{Q} \equiv \frac{(1+\epsilon)\rho}{\beta}(1+m)$, $\bar{\theta} \equiv 1 - \frac{\eta}{\gamma}$, $\mathcal{A}^i \equiv 1 - \gamma\frac{\bar{\theta}-\theta^i}{1-\eta}$, $\mathcal{B}^i \equiv 1 + \gamma\frac{\bar{\theta}-\theta^i}{\eta}$. According to equations (6) and (11), if $\epsilon > 0$ and $\beta \in (0,1)$, savings are elastic to interest rates, $m > 0$; otherwise, $m = 0$ and savings are inelastic. In this sense, m is an indicator of elastic savings. As shown below, $\mathcal{Q}$ is the steady-state social rate of return in the frictionless case; $\bar{\theta}$ is a threshold value such that, for $\theta^i \in (0, \bar{\theta})$, the borrowing constraints are binding under IFA and

$$0 < \mathcal{A}^i < 1 < \mathcal{B}^i \quad \text{and} \quad \frac{\partial \mathcal{A}^i}{\partial \theta^i} > 0 > \frac{\partial \mathcal{B}^i}{\partial \theta^i}. \tag{15}$$

2.2.1 Intratemporal versus Intertemporal Relative Prices

Let $\chi_{t+1}^i \equiv \frac{v_{t+1}^{i,A}}{v_{t+1}^{i,B}}$ denotes the relative price of two intermediate goods and we call it the ***intratemporal relative price***.

In period t, I_t^i units of final goods are invested to produce $M_{t+1}^{i,A}$ and $M_{t+1}^{i,B}$ units of intermediate good A and B in period $t+1$, where $M_{t+1}^{i,A} + M_{t+1}^{i,B} = I_t^i$. Let $\Psi_t^i \equiv \frac{v_{t+1}^{i,A} M_{t+1}^{i,A} + v_{t+1}^{i,B} M_{t+1}^{i,B}}{I_t^i}$ denote the social rate of return to aggregate investment. Given the Cobb-Douglas aggregate production function, it is trivial to prove that $\Psi_t^i = \frac{v_{t+1}^{i,A}}{1-\gamma(1-\chi_{t+1}^i)}$.

The loan rate and the social rate of return are essentially the intertemporal prices of household savings and aggregate savings, respectively. Let $\psi_t^i \equiv \frac{R_t^i}{\Psi_t^i}$ denote their ratio and we call it the ***intertemporal relative price***. Substitute away $v_{t+1}^{i,A}$ using the household no-arbitrage condition (4), the two relative prices are positively related,

$$\psi_t^i = \frac{R_t^i}{\Psi_t^i} = 1 - \gamma(1 - \chi_{t+1}^i). \tag{16}$$

As shown below, the intra- and inter-temporal relative prices reflect the distortions of financial frictions on investment composition and individual welfare, respectively.

2.2.2 The General Solution to Equilibrium Allocation

The total savings of households in period t, $(1-\eta)s_t^i$, have the rate of return R_t^i, while those of entrepreneurs, ηn_t^i, have the rate of return Γ_t^i. In period $t+1$, aggregate revenue of intermediate goods $v_{t+1}^{i,A}M_{t+1}^{i,A} + v_{t+1}^{i,B}M_{t+1}^{i,B} = \rho L\omega_{t+1}^i$ is distributed among households and entrepreneurs. Using equations (6) and (11) to substitute away s_t^i and n_t^i, we get

$$(1-\eta)s_t^iR_t^i + \eta n_t^i\Gamma_t^i = \rho L\omega_{t+1}^i \quad \Rightarrow \quad (1-\eta)R_t^i + \eta\Gamma_t^i = \frac{\omega_{t+1}^i}{\omega_t^i}\mathcal{Q}. \tag{17}$$

Let X_{IFA}^i denote the steady-state value of variable X_t^i under IFA. If the borrowing constraints are binding, the model solutions are as follows,

$$I_t^i = \frac{\beta\omega_t^i}{m+1}\left[1 - \frac{m(1-\mathcal{A}^i)(\mathcal{B}^i-1)}{(m+\mathcal{A}^i)(m+\mathcal{B}^i)}\right], \tag{18}$$

$$\Gamma_t^i = \frac{\omega_{t+1}^i}{\omega_t^i}\mathcal{Q}\left(1+\frac{\mathcal{B}^i-1}{m+1}\right) = \frac{\omega_{t+1}^i}{\omega_t^i}\mathcal{Q}\left(1+\frac{\gamma}{1+m}\frac{\bar{\theta}-\theta^i}{\eta}\right), \tag{19}$$

$$R_t^i = \frac{\omega_{t+1}^i}{\omega_t^i}\mathcal{Q}\left(1-\frac{1-\mathcal{A}^i}{m+1}\right) = \frac{\omega_{t+1}^i}{\omega_t^i}\mathcal{Q}\left(1-\frac{\gamma}{1+m}\frac{\bar{\theta}-\theta^i}{1-\eta}\right), \tag{20}$$

$$\Psi_t^i = \frac{\omega_{t+1}^i}{\omega_t^i}\mathcal{Q}\left[1+\frac{m(1-\mathcal{A}^i)(\mathcal{B}^i-1)}{(m+1)(m+\mathcal{A}^i\mathcal{B}^i)}\right], \tag{21}$$

$$\psi_t^i = \psi_{IFA}^i = 1 - \frac{(1-\mathcal{A}^i)\mathcal{B}^i}{m+\mathcal{B}^i} = 1 - \frac{\gamma}{1+\frac{m}{1+\gamma\frac{\bar{\theta}-\theta^i}{\eta}}}\frac{\bar{\theta}-\theta^i}{1-\eta}, \tag{22}$$

$$\chi_{t+1}^i = \chi_{IFA}^i = 1 - \frac{1}{\gamma}\frac{(1-\mathcal{A}^i)\mathcal{B}^i}{m+\mathcal{B}^i} = 1 - \frac{1}{1+\frac{m}{1+\gamma\frac{\bar{\theta}-\theta^i}{\eta}}}\frac{\bar{\theta}-\theta^i}{1-\eta}, \tag{23}$$

$$\omega_{t+1}^i = \left(\frac{\Lambda_t^i}{\mathcal{Q}}\omega_t^i\right)^{\alpha} \quad \text{where} \quad \Lambda_t^i = \Lambda_{IFA}^i = \frac{(\chi_{IFA}^i)^{\gamma}}{1-\frac{\gamma}{1+m}\frac{\bar{\theta}-\theta^i}{1-\eta}}, \tag{24}$$

$$\frac{\partial \ln \Lambda_{IFA}^i}{\partial \theta^i} = \frac{m(\mathcal{B}^i-1)+\mathcal{B}^i(1-\mathcal{A}^i)(\frac{1}{\gamma}-1)}{\chi_{IFA}^i(\mathcal{B}^i+m)(\mathcal{A}^i+m)}\frac{\partial A^i}{\partial \theta^i} - \frac{m(1-\mathcal{A}^i)}{\chi_{IFA}^i(\mathcal{B}^i+m)^2}\frac{\partial \mathcal{B}^i}{\partial \theta^i}. \tag{25}$$

The two relative prices, χ_{t+1}^i and ψ_t^i, are time-invariant. The domestic production indicator Λ_t^i measures the efficiency of domestic production and is time-invariant. Aggregate output is proportional to the wage rate, $Y_t^i = \frac{(1+\epsilon)\omega_t^i}{(1-\alpha)}$. Thus, the model dynamics are characterized by the dynamics of wages. According to equation (24), given $\alpha \in (0,1)$, there exists a unique and stable steady state with the wage at $w_{IFA}^i = \left(\frac{\Lambda_{IFA}^i}{\mathcal{Q}}\right)^{\rho}$.

According to equations (19)-(20), $\bar{\theta}$ is the threshold value in the sense that for $\theta^i \in (0,\bar{\theta})$, $R_t^i < \Gamma_t^i$ and the borrowing constraints are binding. For $\theta^i \in (\bar{\theta},1]$, the borrowing constraints are slack, $R_t^i = \Gamma_t^i$, and the frictionless allocation is obtained by plugging $\theta^i = \bar{\theta}$ and $\mathcal{A}^i = \mathcal{B}^i = 1$ into equations (18)-(24), which are summarized in lemma 1.

Lemma 1. *For $\theta^i \in [\bar{\theta},1]$, the borrowing constraints are slack and there exists a unique and stable non-zero steady state in country i with the wage at $\omega_{IFA} = \mathcal{Q}^{-\rho}$.*

The private and social rates of return coincide, $R_t^i = \Gamma_t^i = \Psi_t^i = v_{t+1}^{i,A} = v_{t+1}^{i,B} = \frac{\omega_{t+1}^i}{\omega_t^i}\mathcal{Q}$. *In the steady state,* $R_{IFA}^i = \Gamma_{IFA}^i = \Psi_{IFA}^i = \mathcal{Q}$.

Aggregate investment $\frac{\beta\omega_t^i}{1+m}$ *is allocated in the two sectors, proportional to their respective factor shares,* $M_{t+1}^{i,A} = (1-\gamma)\frac{\beta\omega_t^i}{1+m}$ *and* $M_{t+1}^{i,B} = \gamma\frac{\beta\omega_t^i}{1+m}$. *The relative prices and the domestic production indicator are constant at unity,* $\chi_{IFA}^i = \psi_{IFA}^i = \Lambda_{IFA}^i = 1$.

We take this frictionless allocation as the benchmark and analyze the case of $\theta^i \in (0, \bar{\theta})$ where the borrowing constraints are binding. In particular, we address the distortions of financial frictions on interest rates and aggregate output.

2.2.3 Financial Frictions and Interest Rates

In the frictionless case, the private and social rates of return coincide. In the case with financial frictions, the constraint on aggregate credit demand keeps the loan rate lower than the social rate of return in order to clear the credit market, and entrepreneurs benefit from the inefficiently low loan rate so that the equity rate is higher than the social rate of return, i.e., $R_t^i < \Psi_t^i < \Gamma_t^i$. Thus, ***financial frictions distort the interest rates***, which has the distributional effect on individual welfare.

2.2.4 Financial Frictions and Aggregate Output

Aggregate output in the steady state is positively related to the domestic production indicator, $Y_{IFA}^i = \frac{1+\epsilon}{1-\alpha}\omega_{IFA}^i = \frac{1+\epsilon}{1-\alpha}\mathcal{Q}^{-\rho}(\Lambda_{IFA}^i)^\rho$. In the frictionless case, $\Lambda_{IFA}^i = 1$. In the case with financial frictions, $\Lambda_{IFA}^i < 1$, if $m > 0$ and (or) $\gamma \in (0,1)$. In the following, we show that financial frictions distort aggregate output through two distinct channels, i.e., the investment composition channel and the elastic savings channel.

The Investment Composition Channel

In order to highlight the investment composition channel, we shut down the elastic savings channel by setting $m = 0$.[4] According to equations (6) and (11), the individual savings become inelastic to interest rates and aggregate investment is proportional to the aggregate labor income of young individuals, $I_t^i = \beta\omega_t^i$.

In the frictionless case, given $\gamma \in (0,1)$, the sectoral investments of intermediate good A and B are proportional to their relative factors share $(1-\gamma)$ and γ. In the case with financial frictions, the binding borrowing constraints lead to the over- (under-) proportional investment in sector A (B). Since two intermediate goods are imperfect substitutes

[4] It can be done by assuming that individuals do not have the labor endowment when old, $\epsilon = 0$, or they only consume when old, $\beta = 1$.

in aggregate production, the distortion on the cross-sector investment reduces aggregate production efficiency and the steady-state aggregate output.[5] This way, financial frictions distort aggregate output through the ***investment composition channel***. According to equation (25), a rise in θ^i improves cross-sector investment and the domestic production indicator is higher, $\frac{\partial \ln \Lambda^i_{IFA}}{\partial \theta^i} = \frac{(1-\mathcal{A}^i)(\frac{1}{\gamma}-1)}{\chi^i_{IFA}\mathcal{A}^i}\frac{\partial A^i}{\partial \theta^i} > 0$, given $m = 0$ and $\gamma \in (0,1)$.

In the frictionless case, $\chi^i_{IFA} = 1$; in the case with financial frictions, the distortion on the cross-sector investment keeps the price of intermediate good A (B) lower (higher) than the socially efficient level so that $\chi^i_{IFA} \in (0,1)$. According to equation (23), χ^i_{IFA} rises in θ^i and has the maximum value of one at $\theta^i = \bar{\theta}$. Thus, a higher intratemporal relative price reflects smaller output distortion through the investment composition channel.

The Elastic Savings Channel

In order to highlight the elastic savings channel, we shut down the investment composition channel by setting $\gamma = 1$. Final goods are produced using labor and intermediate good B. In equilibrium, intermediate good A vanishes and only entrepreneurs produce intermediate goods. The intratemporal relative price becomes meaningless and is substituted by the intertemporal relative price, $\psi^i_t = \chi^i_{t+1}$, according to equation (16).

In the frictionless case, the loan rate coincides with the social rate of return and $I^i_t = \frac{\beta\omega^i_t}{1+m}$. In the case with financial frictions, the loan rate is lower while the equity rate is higher than the social rate of return. The inefficiently low loan rate depresses the household savings, while the inefficiently high equity rate encourages the entrepreneurial savings. According to equation (18), $I^i_t < \frac{\beta\omega^i_t}{1+m}$ so that the steady-state aggregate output is lower than in the frictionless case. Thus, financial frictions distort aggregate output through the ***elastic savings channel***. According to equation (25), a rise in θ^i raises aggregate savings and investment so that the domestic production indicator is higher, $\frac{\partial \ln \Lambda^i_{IFA}}{\partial \theta^i} = \frac{m}{\psi^i_{IFA}(m+\mathcal{B}^i)}\left(\frac{\mathcal{B}^i-1}{\mathcal{A}^i+m}\frac{\partial A^i}{\partial \theta^i} - \frac{1-\mathcal{A}^i}{\mathcal{B}^i+m}\frac{\partial \mathcal{B}^i}{\partial \theta^i}\right) > 0$, given $m > 0$ and $\gamma = 1$.

In the frictionless case, $\psi^i_{IFA} = 1$; in the case with financial frictions, the constraint on aggregate credit demand keeps the loan rate lower than the social rate of return so that $\psi^i_{IFA} \in (0,1)$. According to equation (22), ψ^i_{IFA} rises in θ^i and has the maximum value of one at $\theta^i = \bar{\theta}$. Thus, given $m > 0$, a higher intertemporal relative price reflects smaller output distortion through the elastic savings channel.

[5] In the New Keynesian monetary models with sticky nominal prices, a non-zero inflation rate distorts production among monopolistic-competitive firms, which generates efficiency losses. By analogy, financial frictions distort the cross-sector investment and aggregate output is lower than in the frictionless case.

The Case with No Distortion on Aggregate Output

The two channels can be shut down together by setting $\gamma = 1$ and $m = 0$. Since only intermediate good B is produced and aggregate savings are inelastic to interest rates, financial frictions do not distort aggregate output through the two channels, $\Lambda^i_{IFA} = 1$, according to equation (25). In this case, financial frictions still distort interest rates.

Proposition 1 summarizes the property of market equilibrium with financial frictions.

Proposition 1. *For $\theta^i \in [0, \bar{\theta})$, the borrowing constraints are binding and there exists a unique and stable non-zero steady state in country i with the wage at $\omega^i_{IFA} = \left(\frac{\Lambda^i_{IFA}}{Q}\right)^\rho$.*

Financial frictions create a wedge between the private and social rates of return, $R^i_t < \Psi^i_t < \Gamma^i_t$. In the steady state, the loan rate is higher while the equity rate is lower in the country with a higher level of financial development.

Financial frictions may distort aggregate output through the investment composition channel and (or) the elastic savings channel, depending on the parameter values of m and γ. In the presence of output distortions, the intra- and inter-temporal relative prices reflect the distortions through the two channels, respectively, and the steady-state aggregate output increases in the level of financial development.

3 Full Capital Mobility

We consider *full capital mobility* where individuals are allowed to lend and make direct investments abroad. Without loss of generality, we assume $0 \leq \theta^1 < \theta^2 < ... < \theta^N \leq \bar{\theta}$ such that the borrowing constraints are binding in the steady state in all countries under IFA and under full capital mobility. We also assume that all countries are initially in the steady state under IFA before capital mobility is allowed from period $t = 0$ on.

Let Φ^i_t and Ω^i_t denote the aggregate outflows of financial capital and FDI from country i in period t, respectively, with negative values indicating capital inflows. Financial capital outflows reduce the domestic credit supply, $(1-\eta)(s^i_t - i^{i,A}_t) - \Phi^i_t$, while FDI outflows reduce the aggregate equity capital for domestic investment, $\eta n^i_t - \Omega^i_t$. Thus, FDI flows raise (reduce) the aggregate credit demand in the host (source) country. With these changes, the analysis in section 2 carries through, due to the linearity of preferences, productive projects, and borrowing constraints. In particular, financial capital flows equalize the loan rate across the border and the credit markets clear in each country as well as at the world level; FDI flows equalize the equity rate across the border and the world equity capital market clears; FDI flows directly affect aggregate output of intermediate good B in each country. To summarize,

$$\sum_{i=1}^{N}\Phi_t^i = \sum_{i=1}^{N}\Omega_t^i = 0, \qquad R_t^i = R_t^*, \qquad \Gamma_t^i = \Gamma_t^*,$$
$$(1-\eta)(s_t^i - i_t^{i,A}) = (\lambda_t^i - 1)(\eta n_t^i - \Omega_t^i) + \Phi_t^i, \quad M_{t+1}^{i,B} = \lambda_t^i(\eta n_t^i - \Omega_t^i).$$

Except them, the equations of market equilibrium in each country are same as under IFA.

Aggregate savings of households in all countries in period t, $(1-\eta)\sum_{i=1}^{N} s_t^i$, have the same rate of return at R_t^*, while those of entrepreneurs, $\eta\sum_{i=1}^{N} n_t^i$, have the same rate of return Γ_t^*. In period $t+1$, aggregate revenue of intermediate goods $\sum_{i=1}^{N}(v_{t+1}^{i,A}M_{t+1}^{i,A} + v_{t+1}^{i,B}M_{t+1}^{i,B}) = \rho L\sum_{i=1}^{N}\omega_{t+1}^i$ is distributed among households and entrepreneurs in all countries. Let $\omega_t^w \equiv \frac{\sum_{i=1}^{N}\omega_t^i}{N}$ denote the world average wage in period t. Using equations (6) and (11) to substitute away s_t^i and n_t^i, we get

$$(1-\eta)R_t^* + \eta\Gamma_t^* = \frac{\omega_{t+1}^w}{\omega_t^w}\mathcal{Q}. \tag{26}$$

Define a country-specific auxiliary parameter, $p_{IFA}^i \equiv \frac{\Gamma_{IFA}^i}{\mathcal{Q}} = 1 + \frac{\gamma}{1+m}\frac{\bar{\theta}-\theta^i}{\eta}$. Let X_{FCM} denotes the steady-state value of variable X under full capital mobility. The model solutions under full capital mobility are,

$$\Gamma_t^i = \frac{\omega_{t+1}^w}{\omega_t^w}\Gamma_{IFA}^i - \frac{\omega_{t+1}^w}{\omega_t^w}\mathcal{Z}_{FCM}^i, \quad \text{where} \quad \mathcal{Z}_{FCM}^i \equiv \frac{(\chi_{FCM}^i - \chi_{IFA}^i)\Gamma_{IFA}^i}{(\chi_{FCM}^i - \chi_{IFA}^i) + \frac{1-\theta^i}{(1-\eta)p_{IFA}^i}} \tag{27}$$

$$R_t^i = \frac{\omega_{t+1}^w}{\omega_t^w}R_{IFA}^i + \frac{\eta}{1-\eta}\frac{\omega_{t+1}^w}{\omega_t^w}\mathcal{Z}_{FCM}^i. \tag{28}$$

$$\chi_{FCM}^i = \frac{(1-\theta^i)R_t^i}{\Gamma_t^i} + \theta^i, \tag{29}$$

$$\psi_{FCM}^i = 1 - \gamma(1-\chi_{FCM}^i), \tag{30}$$

$$\Phi_t^i = (1-\eta)\beta\omega_t^i\left[1 - \frac{\omega_{t+1}^i}{\omega_t^i}\frac{R_{IFA}^i}{R_t^*}\right] \tag{31}$$

$$\Omega_t^i = \eta\beta\omega_t^i\left[1 - \frac{\omega_{t+1}^i}{\omega_t^i}\frac{\Gamma_{IFA}^i}{\Gamma_t^*}\right] \tag{32}$$

$$\Omega_t^i + \Phi_t^i = \beta\omega_t^i\left\{1 - \frac{\omega_{t+1}^i}{\omega_t^i}\left[\eta\frac{\Gamma_{IFA}^i}{\Gamma_t^*} + (1-\eta)\frac{R_{IFA}^i}{R_t^*}\right]\right\} \tag{33}$$

$$\omega_{t+1}^i = \left[\frac{(1-\theta^i)R_t^*}{\Gamma_t^*} + \theta^i\right]^{\gamma\rho}(\frac{1}{R_t^*})^{\rho}. \tag{34}$$

Lemma 2. *Under full capital mobility, the two relative prices are time-invariant and there exists a unique and stable steady state.*

Proof. See appendix B. □

3.1 The Steady-State Patterns of International Capital Flows

In the steady state, $\frac{\omega^i_{t+1}}{\omega^i_t} = 1$. Substituting it into equations (27)-(33), the steady-state patterns of interest rates and capital flows are,

$$\Gamma^i_{FCM} = \Gamma^i_{IFA} - \mathcal{Z}^i_{FCM}, \tag{35}$$

$$R^i_{FCM} = R^i_{IFA} + \frac{\eta}{1-\eta}\mathcal{Z}^i_{FCM}, \tag{36}$$

$$\Phi^i_{FCM} = (1-\eta)\beta\omega^i_{FCM}\left(1 - \frac{R^i_{IFA}}{R^*_{FCM}}\right) = \eta\beta\omega^i_{FCM}\frac{\mathcal{Z}^i_{FCM}}{R^*_{FCM}}, \tag{37}$$

$$\Omega^i_{FCM} = \eta\beta\omega^i_{FCM}\left(1 - \frac{\Gamma^i_{IFA}}{\Gamma^*_{FCM}}\right) = -\eta\beta\omega^i_{FCM}\frac{\mathcal{Z}^i_{FCM}}{\Gamma^*_{FCM}}, \tag{38}$$

$$\Phi^i_{FCM} + \Omega^i_{FCM} = \eta\beta\omega^i_{FCM}\mathcal{Z}^i_{FCM}\frac{(\Gamma^*_{FCM} - R^*_{FCM})}{\Gamma^*_{FCM}R^*_{FCM}}. \tag{39}$$

Proposition 2. *In the steady state, there exists a threshold value of the country index $\hat{N}$ such that the world interest rates are $R^*_{FCM} \in (R^{\hat{N}}_{IFA}, R^{\hat{N}+1}_{IFA}]$ and $\Gamma^*_{FCM} \in [\Gamma^{\hat{N}+1}_{IFA}, \Gamma^{\hat{N}}_{IFA})$. In country $i \in \{1, 2, ..., \hat{N}\}$, full capital mobility raises the relative prices, $\chi^i_{FCM} > \chi^i_{IFA}$ and $\psi^i_{FCM} > \psi^i_{IFA}$, the gross and net capital flows are $\Phi^i_{FCM} > 0 > \Omega^i_{FCM}$ and $\Phi^i_{FCM} + \Omega^i_{FCM} > 0$; the opposite applies for country $i \in \{\hat{N}+1, \hat{N}+2, ..., N\}$. Similar as under IFA, the relative prices increase in θ^i, $\chi^{i+1}_{FCM} > \chi^i_{FCM}$ and $\psi^{i+1}_{FCM} > \psi^i_{FCM}$. Gross international investment return sums up to zero in each country, $\Phi^i_{FCM}R^*_{FCM} + \Omega^i_{FCM}\Gamma^*_{FCM} = 0$.*

Proof. See appendix B. □

In the presence of output distortion, the steady-state aggregate output is higher in country $i \in \{\hat{N}+1, \hat{N}+2, ..., N\}$ than in country $i \in \{1, 2, ..., \hat{N}\}$ under IFA. Under full capital mobility, country $i \in \{\hat{N}+1, \hat{N}+2, ..., N\}$, which has the level of financial development above the world average, imports financial capital and exports FDI. Since the rate of return to its foreign assets (FDI outflows) is higher than the interest rate it pays for its foreign liabilities (financial capital inflows), $\Gamma^*_{FCM} > R^*_{FCM}$, country $i \in \{\hat{N}+1, \hat{N}+2, ..., N\}$ receives a positive net international investment income, $\Phi^i_{FCM}(R^*_{FCM}-1) + \Omega^i_{FCM}(\Gamma^*_{FCM}-1) = 0 - (\Phi^i_{FCM} + \Omega^i_{FCM}) > 0$, despite its negative international investment position, $\Phi^i_{FCM} + \Omega^i_{FCM} < 0$. This way, our model shows analytically that cross-country differences in financial development explain the three recent empirical evidences.

In the following, we address the implication of full capital mobility on aggregate output. For simplicity, we focus on the two-country version, i.e., the world economy consists of country S (South) and country N (North) with $0 \leq \theta^S < \theta^N < \bar{\theta}$. In other words, country S represents the group of country $i \in \{1, 2, ..., \hat{N}\}$, while country N represents the group of country $i \in \{\hat{N}+1, \hat{N}+2, ..., N\}$.

3.2 International Capital Flows and Aggregate Output

In the absence of output distortion, i.e., $m = 0$ and $\gamma = 1$, aggregate output is identical in both countries at $\omega^i_{IFA} = \mathcal{Q}^{-\rho}$ under IFA. In the steady state under full capital mobility, "uphill" net capital flows reduce (raise) the size of aggregate investment in country S (N) so that aggregate output in country S (N) is strictly lower (higher) than under IFA, which we call the investment size effect; given the concave aggregate production with respect to intermediate goods, world output is lower than under IFA, which we call the net investment size effect. This way, net capital flows affect aggregate output at the country and at the world level directly through cross-country resource reallocation.

In the presence of output distortion, international capital flows affect aggregate output directly through cross-country resource reallocation as mentioned above and indirectly through the investment composition channel and the elastic savings channel.

3.2.1 The Investment Composition Channel

We focus on the investment composition channel by setting $m = 0$ and $\gamma \in (0, 1)$. Under full capital mobility, capital flows affect directly the size and indirectly the composition of aggregate investment in each country. Take country S as an example. Net capital outflows directly reduce the size of aggregate investment. Meanwhile, financial capital outflows reduce the credit supply and FDI inflows raise the credit demand, which jointly raises the loan rate. The rise in the loan rate induces households to reduce their investment in sector A. Since sector A is initially over-invested under IFA, full capital mobility indirectly improves the cross-sector investment, reflected by the rise in the intratemporal relative price. The direct effect is negative for aggregate output and depends on net capital flows, while the indirect effect is positive and depends on gross capital flows. Under full capital mobility, two-way capital flows imply that gross flows are much larger than net flows. Under certain conditions, the indirect effect may dominate the direct effect so that full capital mobility may raise aggregate output in country S, ***despite net capital outflows***.

For the illustration purpose, we show the steady-state patterns in a numerical example. The benchmark values of parameters are chosen as follows: $\eta = 0.1$ implies that entrepreneurs account for 10% of population, $\alpha = 0.36$ implies that the labor income accounts for 64% of aggregate output, $\gamma = 0.5$ implies that intermediate goods A and B have the equal factor share in the aggregate production function, $\beta = 0.4$ implies that individuals consume 60% of their lifetime income when young and save 40% for future, $\epsilon = 0$ implies that individuals do not have labor endowment when old and thus, $m = 0$. The threshold value is $\bar{\theta} = 1 - \frac{\eta}{\gamma} = 0.8$.

Figure 1 compare the steady-state patterns of the model economy under full capital mobility versus under IFA, given $\theta^N = \bar{\theta}$. The horizontal axes denote $\theta^S \in [0, \bar{\theta}]$, the vertical axis of the bottom-right panel denotes the percentage difference of world output under the two scenarios, while the vertical axes of other panels denote the levels.

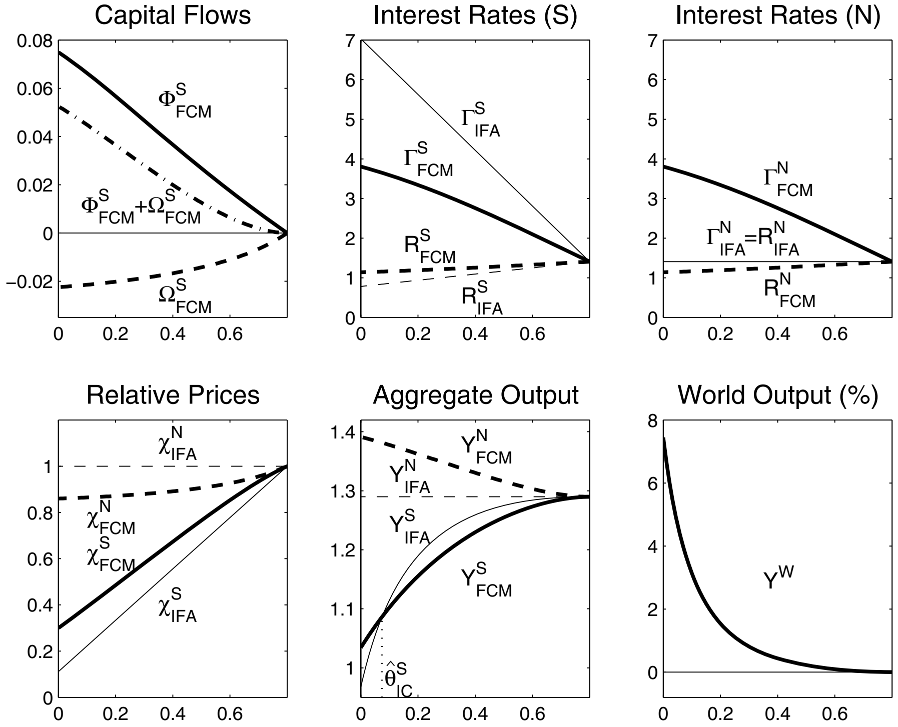

Figure 1: Steady-State Patterns under Full Capital Mobility versus under IFA

The steady-state patterns of capital flows, interest rates, and the intratemporal relative prices confirm the results in propositions 2. In the following, we focus on aggregate output.

Proposition 3. *The positive investment size effect strictly dominates the negative composition effect in country N so that* $Y^N_{FCM} > Y^N_{IFA}$. *Given* θ^N, *there exists a threshold value* $\hat{\theta}^S_{IC}$. *For* $\theta^S \in [0, \hat{\theta}^S_{IC})$, *the positive investment composition effect dominates the negative investment size effect in country S so that* $Y^S_{FCM} > Y^S_{IFA}$; *for* $\theta^S \in (\hat{\theta}^S_{IC}, \theta^N)$, *the opposite applies. Furthermore,* $Y^S_{FCM} < Y^N_{FCM}$.

Proof. See appendix B. □

Figure 2 shows the threshold value $\hat{\theta}^S_{IC}$ in the parameter space of (θ^N, θ^S), given $\gamma \in \{0.2, 0.5, 0.8\}$, respectively. Take $\gamma = 0.5$ as an example and $\bar{\theta}_2 \equiv 1 - \frac{\eta}{0.5}$ denotes the corresponding threshold value for θ^N. Given the assumption of $0 \leq \theta^S < \theta^N \leq \bar{\theta}_2$, the feasible parameter space of (θ^N, θ^S) is denoted by the triangular region below the 45 degree line and to the left of the middle dashed line. The downward-sloping solid line denoted by $\gamma = 0.5$ specifies $\hat{\theta}^S_{IC}$ as a function of $\theta^N \in (0, \bar{\theta}_2)$. As long as the parameter values are in the region above this solid line, full capital mobility reduces the steady-state aggregate output in country S; otherwise, the opposite applies.

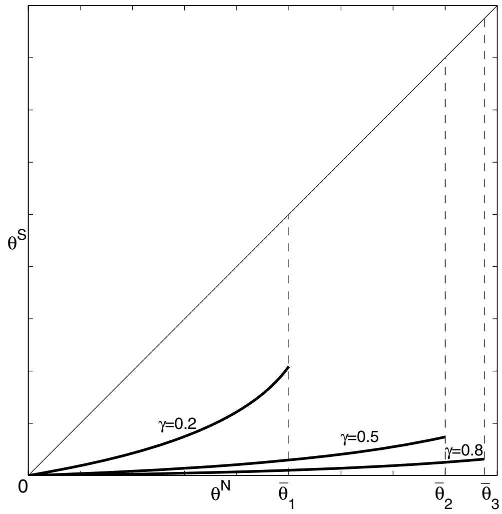

Figure 2: $\hat{\theta}^S_{IC}$ as a Function of θ^N: $\gamma \in \{0.2, 0.5, 0.8\}$

The direct and indirect effects on aggregate output in country S depend ultimately on the cross-country differences in financial development. Given θ^N, the smaller θ^S is, the larger the cross-country interest rate differentials and the output distortions in country S are under IFA. It is more likely that the indirect effect dominates the direct effect. By the same logic, the threshold value $\hat{\theta}^S_{IC}$ declines in θ^N, given γ.

Similarly, besides the negative net investment size effect, full capital mobility affect world output indirectly through the investment composition channel. In particular, both financial capital and FDI flows improve (worsen) production efficiency in country S (N) by affecting the cross-sector investment. Given $0 \leq \theta^S < \theta^N \leq \bar{\theta}$, aggregate output is distorted more severely in country S than in country N under IFA. Thus, the efficiency gains in country S dominate the efficiency losses in country N so that gross capital flows tend to affect world output positively. We call it the net investment composition effect. The net investment size effect depends on net capital flows, while the net investment composition effect depends on gross capital flows. According to figure 1, given the benchmark values of parameters, the net composition effect dominates the net size effect so that, ***despite "uphill" net capital flows***, full capital mobility raises world output, which is in contrast to the predictions of the conventional neoclassical models. The logic is same as mentioned above for aggregate output in country S.

In the numerical example with $\gamma = 0.5$ where financial frictions distort investment composition under IFA, full capital mobility raises world output for $\theta^N = \bar{\theta}$ and $\theta^S \in [0, \bar{\theta})$, while in the case of $\gamma = 1$ where financial frictions do not distort investment composition under IFA, full capital mobility reduces world output. In the following, we analyze how γ may reshape the world output implications of full capital mobility.

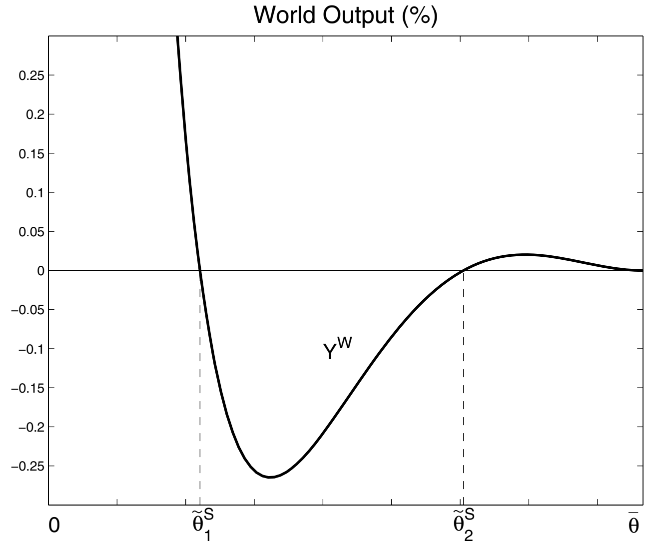

Figure 3: Full Capital Mobility and World Output: $\gamma = 0.75$

For the illustration purpose, we set $\gamma = 0.75$ while keeping other parameter values same as in the benchmark case. Given $\theta^N = \bar{\theta}$, figure 3 shows the percentage differences of world output under full capital mobility and under IFA, with $\theta^S \in [0, \theta^N)$ on the horizontal axis. There exists two threshold values: $0 < \tilde{\theta}^S_1 < \tilde{\theta}^S_2 < \theta^N$. For $\theta^S \in (\tilde{\theta}^S_1, \tilde{\theta}^S_2)$, full capital mobility reduces world output; for $\theta^S \in (0, \tilde{\theta}^S_1) \cup (\tilde{\theta}^S_2, \theta^N)$, the opposite applies.

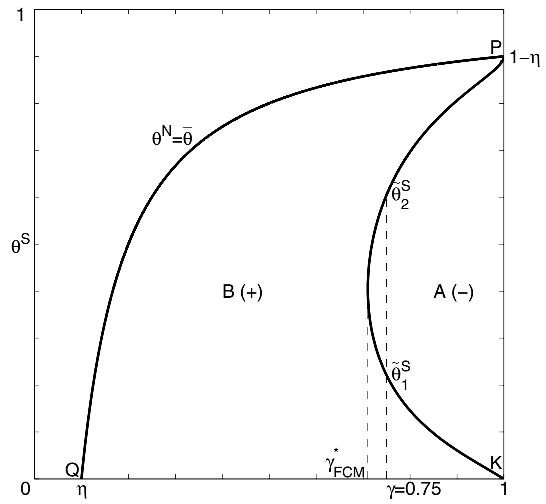

Figure 4: Full Capital Mobility and World Output: Threshold Values

More generally, figure 4 shows the threshold values $\bar{\theta}$, $\tilde{\theta}^S_1$, and $\tilde{\theta}^S_2$ in the space of (γ, θ^S), given $\theta^N = \bar{\theta}$. The horizontal axis denotes $\gamma \in (0, 1)$ and the vertical axis denotes $\theta^S \in (0, 1)$. Curve PQ represents the threshold value $\bar{\theta} \equiv 1 - \frac{\eta}{\gamma}$. We focus on the case of $0 \le \theta^S < \theta^N = \bar{\theta}$, i.e., the region below curve PQ. There exists a threshold value γ^*_{FCM} such that if $\gamma \in (\eta, \gamma^*_{FCM})$, full capital mobility strictly raises world output; if $\gamma \in (\gamma^*_{FCM}, 1)$, full capital mobility may or may not raise world output for $\theta^S \in [0, \theta^N)$. Curve PK in figure 3 represents the two threshold values $\tilde{\theta}^S_1$ and $\tilde{\theta}^S_2$ as functions of γ.

To sum up, region A denotes the region to the right of curve PK and region B denotes the region between curve PK and PQ. For (γ, θ^S) in region B, full capital mobility raises world output; for (γ, θ^S) in region A, the opposite applies.

Whether full capital mobility raises the steady-state world output depends on the relative magnitude of the net investment composition effect and the net investment size effect, which is determined ultimately by the interest rate differentials and the cross-country differences in output distortion under IFA.

3.2.2 The Elastic Savings Channel

We focus on the elastic savings channel by setting $m > 0$ and $\gamma = 1$. Under full capital mobility, capital flows affect directly the size of domestic investment and indirectly the size of domestic savings. Take country S as an example. Domestic investment is financed by the difference between domestic savings and net capital outflows, $I^i_t = (1-\eta)s^i_t + \eta n^i_t - (\Phi^i_t + \Omega^i_t)$. Net capital outflows directly reduce the size of domestic investment. Meanwhile, financial capital outflows and FDI inflows jointly raise the loan rate and reduce the equity rate, which is reflected by the rise in the intertemporal relative price. The rise in the loan rate induces households to raise their savings while the decline in the equity rate induces entrepreneurs to reduce their savings. In the net term, gross capital flows indirectly raise domestic savings. The direct effect is negative for aggregate output and depends on net capital flows, while the indirect effect is positive and depends on gross capital flows. By the same logic as mentioned in subsection 3.2.1, under certain conditions, the indirect effect may dominate the direct effect so that full capital mobility may raise aggregate output in country S, ***despite net capital outflows***.

We calculate the steady-state patterns by setting $\gamma = 1$ and $\epsilon = 1$ while keeping the values for others parameters same as in the benchmark case of subsection 3.2.1. In other words, we assume that only intermediate good B is used in the aggregate production and that individuals are endowed with one unit of labor when old and thus, $m > 0$. The steady-state patterns of capital flows, interest rates, relative prices, aggregate output at the country and at the world level are qualitatively identical as in figure 1.

Proposition 4. *For $\theta^N = \bar{\theta}$, it always holds that $Y^N_{FCM} > Y^N_{IFA}$. There always exits a threshold value $\hat{\theta}^S_{ES} \in (0, \theta^N]$ as a function of θ^N such that for $\theta^S \in (0, \hat{\theta}^S_{ES})$, the positive* *savings effect dominates the negative investment size effect in country S, $Y^S_{FCM} > Y^S_{IFA}$. Furthermore, $Y^S_{FCM} < Y^N_{FCM}$.*

In general, if $\eta \in (0, 0.5)$, there are three scenarios as follows.

1. *if $m \in (0,1)$, it holds that $Y^S_{FCM} > Y^S_{IFA}$, for $\theta^S \in (0, \frac{1-\sqrt{1-4m^2(1-\eta)\eta}}{2})$, and that $Y^N_{FCM} > Y^N_{IFA}$, for $\theta^N \in (\frac{1-\sqrt{1-4m^2(1-\eta)\eta}}{2}, \bar{\theta})$;*

2. *If $m \in (1, \frac{[\eta(1-\eta)]^{-0.5}}{2})$, it holds that $Y^S_{FCM} > Y^S_{IFA}$, for $\theta^S \in (0, \frac{1-\sqrt{1-4m^2(1-\eta)\eta}}{2}) \cup (\frac{1+\sqrt{1-4m^2(1-\eta)\eta}}{2}, \bar{\theta})$, and that $Y^N_{FCM} > Y^N_{IFA}$, for $\theta^N \in (\frac{1-\sqrt{1-4m^2(1-\eta)\eta}}{2}, \frac{1+\sqrt{1-4m^2(1-\eta)\eta}}{2})$.*

3. *If $m > \frac{[\eta(1-\eta)]^{-0.5}}{2}$, it holds that $Y^S_{FCM} > Y^S_{IFA}$, for $\theta^S \in (0, \bar{\theta})$.*

If $\eta \in (0.5, 1)$, there are two scenarios as follows.

1. *if $m \in (0,1)$, it holds that $Y^S_{FCM} > Y^S_{IFA}$, for $\theta^S \in (0, \frac{1-\sqrt{1-4m^2(1-\eta)\eta}}{2})$, and that $Y^N_{FCM} > Y^N_{IFA}$, for $\theta^N \in (\frac{1-\sqrt{1-4m^2(1-\eta)\eta}}{2}, \bar{\theta})$;*

2. *If $m > 1$, it holds that $Y^S_{FCM} > Y^S_{IFA}$, for $\theta^S \in (0, \bar{\theta})$.*

Proof. See the proof of proposition 3 in appendix B. □

For θ^N not in the ranges specified in the three cases of proposition 4, the negative savings effect may dominate the positive investment size effect in country N so that full capital mobility may reduce aggregate output in country N, despite net capital inflows.

Our benchmark parameter values of ϵ, β, and α imply that $m < 1$. Thus, the first scenario of proposition 4 applies. Figure 5 shows the threshold value $\hat{\theta}^S_{ES}$ in the parameter space of (θ^N, θ^S), given $\epsilon \in \{0.2, 0.6, 1\}$, respectively. Take $\epsilon = 1$ as an example. Given the assumption of $0 \leq \theta^S < \theta^N \leq \bar{\theta}$, the feasible parameter space of (θ^N, θ^S) is denoted by the triangular region below the 45 degree line. The downward-sloping solid line denoted by $\epsilon = 1$ specifies $\hat{\theta}^S_{ES}$ as a function of $\theta^N \in (0, \bar{\theta})$. As long as the parameter values are in the region below this solid line, full capital mobility raises the steady-state aggregate output in country S, ***despite net capital outflows***; otherwise, the opposite applies. The intuition is same as mentioned in subsection 3.2.1 for aggregate output in country S.

Besides the negative net investment size effect, full capital mobility affects world output indirectly through the elastic savings channel. In particular, financial capital and FDI flows raise (reduce) aggregate savings in country S (N) by affecting interest rates. Given $0 \leq \theta^S < \theta^N \leq \bar{\theta}$, aggregate savings is depressed more severely in country S than in country N under IFA. Thus, under full capital mobility, the rise in the aggregate savings in country S dominate the decline in country N so that gross capital flows affect world output positively. We call it the net savings effect. The net investment size effect depends

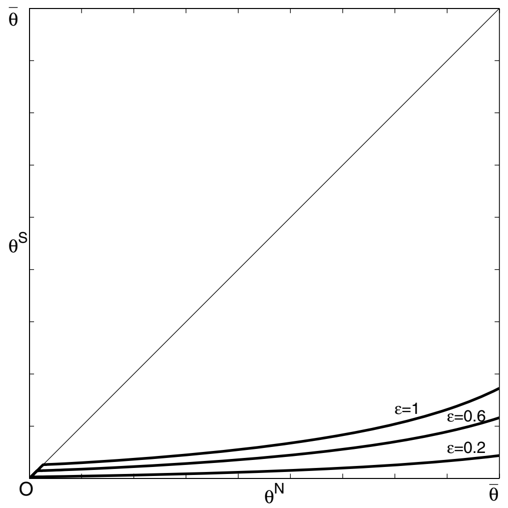

Figure 5: $\hat{\theta}_{ES}^S$ as a Function of θ^N: $\epsilon \in \{0.2, 0.6, 1\}$

on net capital flows, while the net savings effect depends on gross capital flows. Given our parameter values, in particular, $\epsilon = 1$ and $\gamma = 1$, the net savings effect dominates the net investment size effect so that, ***despite "uphill" net capital flows***, full capital mobility raises world output. The logic is same as mentioned in subsection 3.2.1.

By setting $\epsilon = 0.2$ and $\gamma = 1$ while keeping other parameter values same as in the benchmark case of subsection 3.2.1, we calculate the percentage differences of the steady-state world output under full capital mobility and under IFA for $\theta^S \in [0, \theta^N)$, given $\theta^N = \bar{\theta}$. The patterns are qualitatively identical as in figure 3. In particular, there exists two threshold values: $0 < \tilde{\theta}_1^S < \tilde{\theta}_2^S < \theta^N$, such that for $\theta^S \in (\tilde{\theta}_1^S, \tilde{\theta}_2^S)$, full capital mobility reduces world output; for $\theta^S \in (0, \tilde{\theta}_1^S) \cup (\tilde{\theta}_2^S, \theta^N)$, the opposite applies.

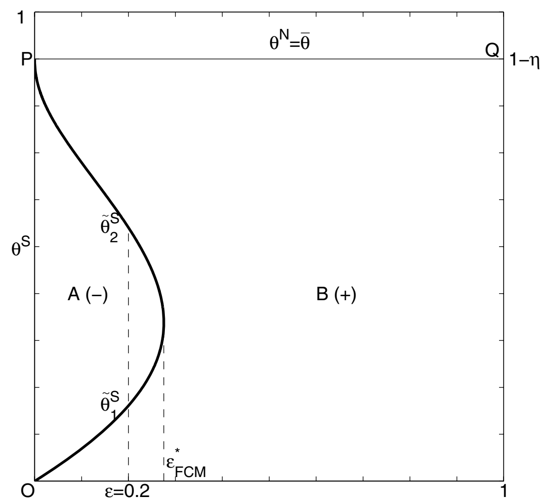

Figure 6: Full Capital Mobility and World Output: Threshold Values

More generally, figure 6 shows the threshold values $\bar{\theta}$, $\tilde{\theta}_1^S$, and $\tilde{\theta}_2^S$ in the space of (ϵ, θ^S), given $\theta^N = \bar{\theta}$. The horizontal axis denotes $\epsilon \in (0, 1)$. The threshold value $\bar{\theta} \equiv 1 - \eta$ is independent of ϵ and represented by line PQ. We focus on the case of $0 \leq \theta^S < \theta^N = \bar{\theta}$, i.e., the region below line PQ. There exists a threshold value ϵ^*_{FCM} such that for $\epsilon > \epsilon^*_{FCM}$, full capital mobility raises world output; otherwise, full capital mobility may or may not raise world output. The two threshold values $\tilde{\theta}^S_1$ and $\tilde{\theta}^S_2$ mentioned above are functions of ϵ and represented by curve PO.

To sum up, region A (B) denotes the region to the left (right) of curve PO. For (ϵ, θ^S) in region B, full capital mobility raises world output; otherwise, the opposite applies. The logic is same as mentioned in subsection 3.2.1.

In the presence of domestic financial frictions, capital does not flow to the place where the marginal product of capital is higher but to the place where the private rate of return is higher. Thus, international capital flows may not necessarily raise world output in our model. Such a result has a significant welfare and policy implications. As shown in von Hagen and Zhang (2010), capital flows have opposite welfare implications to different individuals within and across countries. As long as capital mobility can raise world output, financial globalization is still a positive-sum game and it is possible to achieve Pareto improvement at the world level through domestic and international transfers; otherwise, financial globalization is a negative-sum game and strictly reduces world welfare, which cannot be offset through international transfers.

4 Partial Capital Mobility

In order to compare with the scenario of full capital mobility, we consider here the scenario of *partial capital mobility* under which individuals are allowed to lend abroad but not to make direct investment abroad.[6] The steady-state patterns of capital flows and relative prices are similar as under full capital mobility. We put the detailed analysis in appendix A and focus here on the output implications of partial capital mobility.

The initial cross-country loan rate differentials under IFA drive financial capital flows from country S to country N under partial capital mobility. Besides the direct investment size effect, financial capital flows have an indirect effect through the investment composition channel and (or) the elastic savings channel. Under partial capital mobility, capital flows are one-way and net flows coincide with gross flows. Thus, the direct effect

[6] There exists another scenario where individuals are allowed to make direct investment but not to lend abroad. In that case, households in country i can lend domestically, make direct investment domestically or abroad. The non-arbitrage condition leads to the cross-country loan rate equalization, despite the restriction on financial capital flows. The allocation is identical as in the scenario of full capital mobility.

always dominates the indirect effect so that the steady-state aggregate output strictly rises (declines) in country N (S) under partial capital mobility.

Besides the negative net investment size effect through widening the cross-country output gap, financial capital flows have the indirect positive effect on the steady-state world output through the investment composition channel and (or) the elastic savings channel. In the case with the investment composition channel only, there exists a threshold value γ^*_{PCM} such that for $\gamma \in (\gamma^*_{PCM}, 1)$, there exists $\tilde{\theta}^S_{PCM}$. Given $\theta^N = \bar{\theta}$, for $\theta^S \in (0, \tilde{\theta}^S)$, partial capital mobility raises the steady-state world output; for $\theta^S \in (\tilde{\theta}^S, \theta^N)$, the opposite applies. In the case with the elastic savings channel only, given $\epsilon \in (0, 1)$, there exists a threshold value $\tilde{\theta}^S$ such that for $\theta^S \in (\tilde{\theta}^S, \theta^N)$, partial capital mobility reduces the steady-state world output; otherwise, the opposite applies.

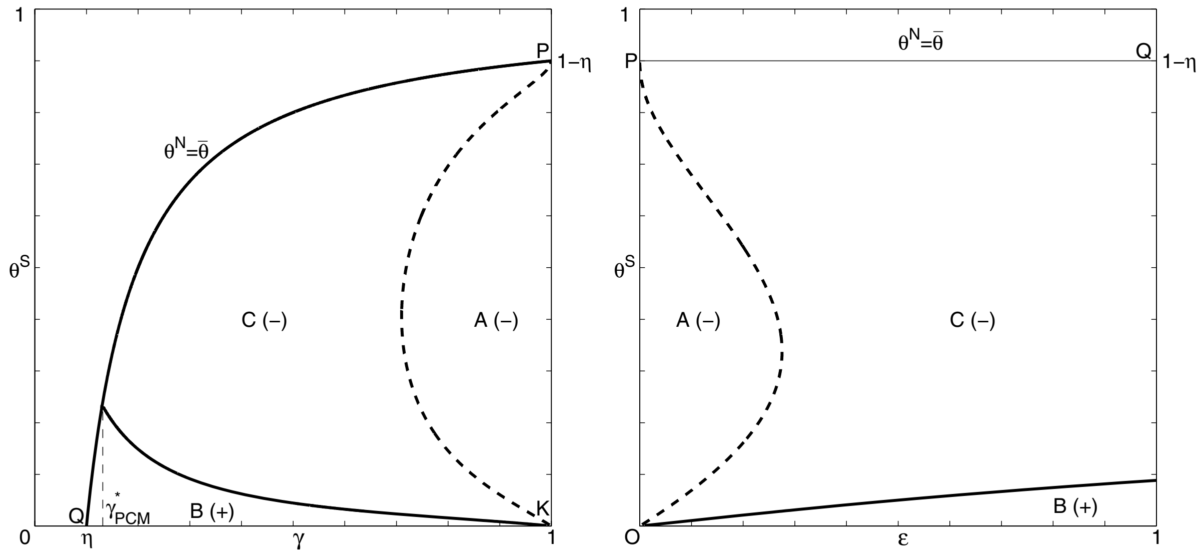

Figure 7: Partial Capital Mobility and World Output: Threshold Values

We calculate the threshold values in the two cases and compare them with those under full capital mobility. The left and the right panels of figure 7 correspond to figure 4 and 6, respectively. Take the left panel as an example. The dashed line refers to the threshold values under full capital mobility, while the downward sloping curve between region B and C refers to the threshold value under partial capital mobility. If the parameter values of (γ, θ^S) is in region C, full (partial) capital mobility raises (reduces) the steady-state world output. Similar results prevail in the case with the elastic savings channel only. Intuitively, as mentioned in subsection 3.2, the net investment size effect depends on net flows while the indirect positive effect depends on gross flows. Under partial capital mobility, net and gross capital flows coincide. Thus, it is more likely that the negative net investment size effect dominates so that the steady-state world output declines.

5 Conclusion

We develop a tractable multi-country model where domestic financial frictions distort interest rates. Given the cross-country differences in financial development, the interest rate differentials drive international capital flows and the theoretical predictions are consistent with the empirical patterns in the recent past.

We then use this model to address the aggregate output implications of international capital flows. Besides the direct effects on aggregate output through cross-country resource reallocation, both financial capital flows and FDI have the indirect effects through within-country resource reallocations in the investment composition channel and in the elastic savings channel. Under certain conditions, the indirect effects may dominate so that, despite "uphill" net capital outflows, full capital mobility may raise the steady-state aggregate output in the poor country as well as world output. Our results complement conventional neoclassical models by identifying the two distinct channels.

Our model differs from conventional neoclassical models only in the presence of financial frictions. International capital flows ameliorate the distortions of financial frictions on interest rates and aggregate output. The theoretical results of our model may serve as the benchmark for further investigations on the implications of international capital flows in the presence of increasing returns, endogenous growth, technology-promoting FDI, etc.

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A Partial Capital Mobility

Financial capital flows equalize the loan rate across the border and the credit markets clear in each country as well as at the world level,

$$R_t^i = R_t^*, \quad (1-\eta)(s_t^i - i_t^{i,A}) = (\lambda_t^i - 1)\eta n_t^i + \Phi_t^i, \quad \text{and} \quad \sum_{i=1}^{N} \Phi_t^i = 0.$$

Except them, the equations of market equilibrium in each country are same as under IFA. The model solutions are

$$\Gamma_t^i = \frac{\omega_{t+1}^i}{\omega_t^i}\Gamma_{IFA}^i \tag{40}$$

$$R_t^i = \frac{\omega_{t+1}^i}{\omega_t^i}R_{IFA}^i + \frac{\omega_{t+1}^i}{\omega_t^i}\mathcal{Z}_{t+1}^i, \quad \text{where} \quad \mathcal{Z}_{t+1}^i \equiv \frac{(\chi_{t+1}^i - \chi_{IFA}^i)\Gamma_{IFA}^i}{1-\theta^i}, \tag{41}$$

$$\chi_{t+1}^i = \frac{(1-\theta^i)R_t^i}{\Gamma_t^i} + \theta^i, \tag{42}$$

$$\psi_t^i = 1 - \gamma(1 - \chi_{t+1}^i), \tag{43}$$

$$\Phi_t^i = (1-\eta)\beta\omega_t^i\left(1 - \frac{\omega_{t+1}^i}{\omega_t^i}\frac{R_{IFA}^i}{R_t^i}\right) \tag{44}$$

$$\omega_{t+1}^i = \left(\frac{\Lambda_t^i}{\mathcal{Q}}\omega_t^i\right)^{\alpha}, \quad \text{where} \quad \Lambda_t^i \equiv \frac{(\chi_{t+1}^i)^{\gamma}(1-\theta^i)}{(\chi_{t+1}^i - \theta^i)p_{IFA}^i}, \tag{45}$$

$$\frac{\partial \ln \Lambda_t^i}{\partial \chi_{t+1}^i} = -\frac{\chi_{t+1}^i(1-\gamma) + \gamma\theta^i}{\chi_{t+1}^i(\chi_{t+1}^i - \theta^i)} < 0 \tag{46}$$

Let X_{PCM} denote the steady-state value of variable X under partial capital mobility.

Lemma 3. *There exists a unique and stable steady state under partial capital mobility.*

Proof. See appendix B. □

In the steady state, the interest rates and financial capital flows are

$$\Gamma^i_{FCM} = \Gamma^i_{IFA}, \tag{47}$$

$$R^i_{PCM} = R^i_{IFA} + \mathcal{Z}^i_{PCM}, \quad \text{where} \quad \mathcal{Z}^i_{PCM} \equiv \frac{(\chi^i_{PCM} - \chi^i_{IFA})\Gamma^i_{IFA}}{1-\theta^i}, \tag{48}$$

$$\Phi^i_{FCM} = (1-\eta)\beta\omega^i_{FCM}\frac{\mathcal{Z}^i_{PCM}}{R^*_{FCM}}, \tag{49}$$

Proposition 5. *In the steady state, there exists a threshold value of the country index* $\tilde{N}$ *such that* $R^{\tilde{N}}_{IFA} < R^*_{FCM} \leq R^{\tilde{N}+1}_{IFA}$. *The world loan rate is* $R^*_{FCM} \in (R^{\tilde{N}}_{IFA}, R^{\tilde{N}+1}_{IFA}]$ *and the equity rate in each country is same as under IFA,* $\Gamma^i_{PCM} = \Gamma^i_{IFA}$. *In country* $i \in \{1, 2, ..., \tilde{N}\}$, *partial capital mobility leads to financial capital outflows,* $\Phi^i_{PCM} > 0$, *which raises the relative prices,* $\chi^i_{PCM} > \chi^i_{IFA}$ *and* $\psi^i_{PCM} > \psi^i_{IFA}$, *and reduces aggregate output,* $Y^i_{PCM} < Y^i_{IFA}$; *the opposite applies for country* $i \in \{\tilde{N}+1, \tilde{N}+2, ..., N\}$.

The steady-state relative prices increase in θ^i, $\chi^{i+1}_{PCM} > \chi^i_{PCM}$ *and* $\psi^{i+1}_{PCM} > \psi^i_{PCM}$.

Proof. See appendix B. □

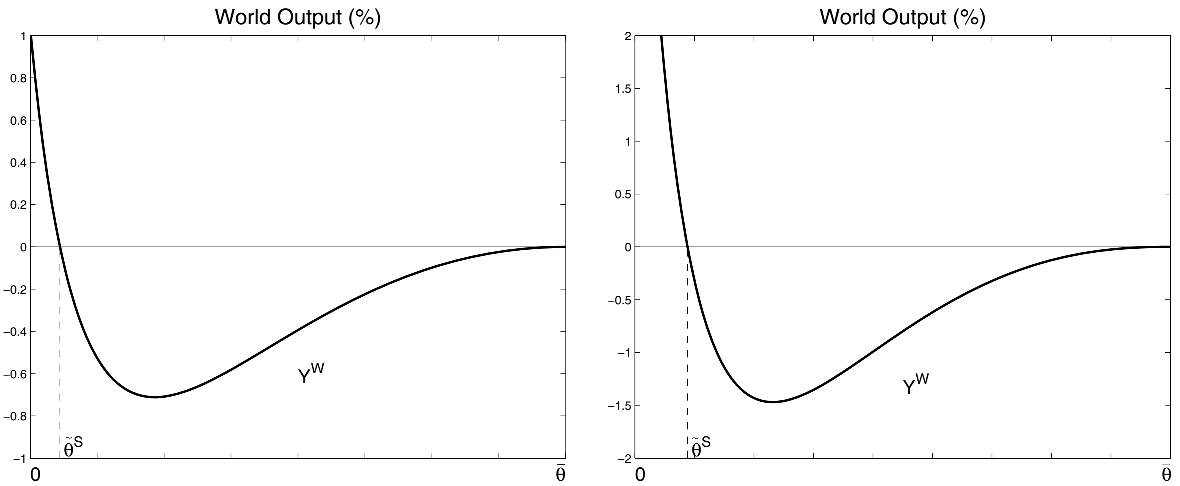

Figure 8: Partial Capital Mobility and World Output

For the illustration purpose, we show the percentage differences of the steady-state world output under partial capital mobility versus under IFA in two cases, given $\theta^N = \bar{\theta}$ and $\theta^S \in [0, \bar{\theta}]$. In the first case, we set $\gamma = 0.5$ and $\epsilon = 0$ while keeping the values of other parameters same as in the benchmark case so that we feature the investment composition channel only, while in the second case, we set $\gamma = 1$ and $\epsilon = 1$ while keeping the values of other parameters same as in the benchmark case so that we feature the elastic savings channel only. The left and the right panels of figure 8 show the two cases, respectively and the axis scalings are same as in figure 3. In each case, there exists a threshold value $\tilde{\theta}^S$ such that for $\theta^S \in (0, \tilde{\theta}^S)$, partial capital mobility raises the steady-state world output; otherwise, the opposite applies. See section 4 for further analysis.

B Proofs

Proof of Lemma 2

Proof. The proof consists of three steps. First, we prove that equation (27) is the solution to the equity rate under full capital mobility. If the borrowing constraints are binding, it holds under IFA and under full capital mobility,

$$\chi^i_{t+1} = \frac{R^i_t(1-\theta^i)}{\Gamma^i_t} + \theta^i, \quad \Rightarrow \quad \frac{\Delta\chi^i_{t+1}}{1-\theta^i} = \frac{R^i_t}{\Gamma^i_t} - \frac{R^i_{IFA}}{\Gamma^i_{IFA}}, \quad \text{where } \Delta^i_{t+1} \equiv \chi^i_{t+1} - \chi^i_{IFA}. \tag{50}$$

According to equation (17), $(1-\eta)R^i_{IFA} + \eta\Gamma^i_{IFA} = \mathcal{Q}$. Substituting R^i_t and R^i_{IFA} with Γ^i_t and Γ^i_{IFA} using equation (26) and $R^i_{IFA} = \frac{1}{(1-\eta)}(\mathcal{Q} - \eta\Gamma^i_{IFA})$, we solve the equity rate from equation (50). Plug in the solution to the equity rate in equation (26) to solve R^i_t.

Second, we prove that χ^i_{t+1} is constant under full capital mobility. Let us assume that χ^i_{t+1} is time variant and so is the auxiliary variable Z^i_{t+1} defined in equation (27). According to equation (27), the equity rate equalization in country i and N implies that

$$\Gamma^i_{IFA} - \mathcal{Z}^i_{t+1} = \Gamma^N_{IFA} - \mathcal{Z}^N_{t+1}, \quad \Rightarrow \quad (1-\eta)\left(\frac{\Delta\chi^i_{t+1}}{1-\theta^i} - \frac{\Delta\chi^N_{t+1}}{1-\theta^N}\right) = \frac{1}{p^N_{IFA}} - \frac{1}{p^i_{IFA}}, \tag{51}$$

$$\Rightarrow \quad \frac{\partial\Delta\chi^i_{t+1}}{\partial\Delta\chi^N_{t+1}} = \frac{1-\theta^i}{1-\theta^N} > 0. \tag{52}$$

Using equations (27) and (32), we rewrite the condition, $\sum_{i=1}^N \Omega^i_t = 0$, into

$$\sum_{i=1}^N \mathcal{Z}^i_{t+1}\omega^i_{t+1} = 0 \tag{53}$$

Given the Cobb-Douglas production function, $\omega^i_{t+1} = (\chi^i_{t+1})^{\gamma\rho}(R^i_t)^{-\rho}$. Combining it with the loan rate equalization, $R^i_t = R^*_t$, we rewrite equation (53) into

$$\sum_{i=1}^N \mathcal{K}^i_{t+1} = 0, \quad \text{where} \quad \mathcal{K}^i_{t+1} \equiv \frac{p^i_{IFA}\Delta\chi^i_{t+1}(\Delta\chi^i_{t+1} + \chi^i_{IFA})^{\gamma\rho}}{\Delta\chi^i_{t+1} + \frac{1-\theta^i}{(1-\eta)p^i_{IFA}}}, \tag{54}$$

$$\frac{\partial\mathcal{K}^i_{t+1}}{\partial\Delta\chi^i_{t+1}} = \mathcal{K}^i_{t+1}\left\{\frac{1}{\Delta\chi^i_{t+1}\left[\frac{(1-\eta)p^i_{IFA}}{1-\theta^i}\Delta\chi^i_{t+1} + 1\right]} + \frac{\gamma\rho}{\Delta\chi^i_{t+1} + \chi^i_{IFA}}\right\} > 0. \tag{55}$$

Substituting $\Delta\chi^i_{t+1}$ with $\Delta\chi^N_{t+1}$ using equations (51), the left-hand side of equation (54) becomes a monotonically increasing function of $\Delta\chi^N_{t+1}$,

$$\frac{\partial\sum_{i=1}^N \mathcal{K}^i_{t+1}}{\partial\Delta\chi^N_{t+1}} = \sum_{i=1}^N \frac{\partial\mathcal{K}^i_{t+1}}{\partial\Delta\chi^i_{t+1}}\frac{\partial\Delta\chi^i_{t+1}}{\partial\Delta\chi^N_{t+1}} > 0. \tag{56}$$

For $\Delta\chi^N_{t+1} = 0$, equation (51) implies that $\Delta\chi^i_{t+1} > 0$. Given $\Delta\chi^i_{t+1} > 0$, $\mathcal{K}^i_{t+1} > 0$. Thus, the left-hand side of equation (54) is larger zero for $\Delta\chi^N_{t+1} = 0$. There exits a unique solution to $\Delta\chi^N_{t+1}$ which is smaller than zero and time-invariant. Using equations (51), we can then solve $\Delta\chi^i_{t+1}$ for $i \in \{1, 2, ..., N-1\}$, accordingly.

Finally, we prove the existence of a unique and stable steady state under full capital mobility. χ^i_{t+1} is time-invariant and so is $\mathcal{Z}^i_{t+1}$. Let $R^i_{FCM} \equiv R^i_{IFA} + \frac{\eta}{1-\eta}\mathcal{Z}^i_{FCM}$ which is same across countries, $R^i_{FCM} = R^*_{FCM}$. Thus, the loan rate depends on the dynamics of the world-average wages and so is the wage in country i,

$$\omega^i_{t+1} = \left(\frac{\omega^w_{t+1}}{\omega^w_t}R^*_{FCM}\right)^{-\rho}(\chi^i_{FCM})^{\rho\gamma}.$$

The dynamics of the world-average wages are

$$\omega^w_{t+1} = \frac{\sum_{i=1}^N \omega^i_{t+1}}{N} = \left(\frac{\omega^w_{t+1}}{\omega^w_t}R^*_{FCM}\right)^{-\rho}\frac{\sum_{i=1}^N(\chi^i_{FCM})^{\rho\gamma}}{N},$$

$$\omega^w_{t+1} = \left(\frac{\omega^w_t}{R^*_{FCM}}\right)^{\alpha}\left[\frac{\sum_{i=1}^N(\chi^i_{FCM})^{\rho\gamma}}{N}\right]^{1-\alpha}$$

Given $\alpha \in (0, 1)$, the phase diagram of the world-average wage is concave. Thus, there exists a unique and stable steady state. Proportional to the wage, aggregate output in country i is determined by the world output dynamics. □

Proof of Proposition 2

Proof. According to equation (20), the steady-state loan rate in country i monotonically increases in θ^i under IFA, which together with equation (37) and the world credit market clearing condition, $\sum_{t=1}^N \Phi^i_{FCM} = 0$, implies that there exists a threshold value of the country index $\hat{N}$ such that $Z^{\hat{N}}_{FCM} > 0 \geq Z^{\hat{N}+1}_{FCM}$. Thus, the world loan rate is $R^*_{FCM} \in (R^{\hat{N}}_{IFA}, R^{\hat{N}+1}_{IFA}]$. According to equations (17) and (26), it holds in the steady state that $(1-\eta)R^i_j + \eta\Gamma^i_j = \mathcal{Q}$, where $j \in \{IFA, FCM\}$ denotes the scenario of IFA and full capital mobility. Thus, $\Gamma^*_{FCM} \in [\Gamma^{\hat{N}+1}_{IFA}, \Gamma^{\hat{N}}_{IFA})$.

Given that $\mathcal{Z}^i_{FCM}$ monotonically increases in $\Delta\chi^i_{FCM}$ and $R^*_{FCM} \in (R^{\hat{N}}_{IFA}, R^{\hat{N}+1}_{IFA})$, it is obvious that full capital mobility raises the relative prices in country $i \in \{1, 2, ..., \hat{N}\}$, $\chi^i_{FCM} > \chi^i_{IFA}$ and $\psi^i_{FCM} > \psi^i_{IFA}$. The gross equity premium is by definition, $\frac{\Gamma^i_{FCM}}{R^i_{FCM}} =$

$\frac{1-\theta^i}{\chi^i_{FCM}-\theta^i}$, and the cross-country equalization implies that $\frac{1-\chi^i_{FCM}}{1-\theta^i} = \frac{1-\chi^{i+1}_{FCM}}{1-\theta^{i+1}} = \frac{\chi^{i+1}_{FCM}-\chi^i_{FCM}}{\theta^{i+1}-\theta^i} > 0$. Given $\theta^{i+1} > \theta^i$, it holds that $\chi^{i+1}_{FCM} > \chi^i_{FCM}$. According to equation (30), we get $\psi^{i+1}_{FCM} > \psi^i_{FCM}$. Similar as under IFA, the relative prices under full capital mobility monotonically increase in θ^i. According to equations (37) and (38), the changes in the interest rates imply that in country $i \in \{1, 2, ..., \hat{N}\}$, $\Phi^i_{FCM} > 0 > \Omega^i_{FCM}$. Since $\Gamma^*_{FCM} > R^*_{FCM}$, the steady-state net capital flows have the same sign as $\mathcal{Z}^i_{FCM}$, according to equation (39). Thus, $\Phi^i_{FCM} + \Omega^i_{FCM} > 0$ in country $i \in \{1, 2, ..., \hat{N}\}$. The opposite applies to country $i \in \{\hat{N}+1, \hat{N}+2, ..., N\}$.

According to equations (37) and (38), the gross international investment returns are

$$R^*_{FCM}\Phi^i_{FCM} + \Gamma^*_{FCM}\Omega^i_{FCM} = \rho\omega^i_{FCM}(1-\eta)(\mathcal{Z}^i_{FCM} - \mathcal{Z}^i_{FCM}) = 0.$$

□

Proof of Proposition 3

Proof. According to equations (17), (26), (34), the steady-state relative prices, interest rates, and wages have the same relationship under full capital mobility and under IFA,

$$\omega^i_j = (\chi^i_j)^{\gamma\rho}(R^i_j)^{-\rho}, \quad \chi^i_j = \frac{R^i_j}{\Gamma^i_j}(1-\theta^i) + \theta^i, \quad \eta\Gamma^i_j + (1-\eta)R^i_j = \mathcal{Q}. \tag{57}$$

where $j \in \{IFA, FCM\}$ refers to the scenarios of IFA and full capital mobility, respectively. Under full capital mobility, $\omega^i_{FCM} = (\chi^i_{FCM})^{\gamma\rho}(R^i_{FCM})^{-\rho}$, $R^S_{FCM} = R^N_{FCM}$ and $\chi^S_{FCM} < \chi^N_{FCM}$ jointly imply that $\omega^S_{FCM} < \omega^N_{FCM}$, or equivalently, $Y^S_{FCM} < Y^N_{FCM}$.

For the notational convenience, we normalize interest rates by $\mathcal{Q}$ and define two auxiliary variables, $r^i_j \equiv \frac{R^i_j}{\mathcal{Q}}$ and $p^i_j \equiv \frac{\Gamma^i_j}{\mathcal{Q}}$. Equation (57) implies that $r^i_j = \frac{1-\eta p^i_j}{1-\eta}$. Given θ^i, w^i_j is a function of the normalized equity rate p^i_j,

$$\omega^i_j = \frac{1}{\mathcal{Q}^\rho}\left[\frac{(1-\theta^i)r^i_j}{p^i_j} + \theta^i\right]^{\gamma\rho}(r^i_j)^{-\rho}. \tag{58}$$

The allocation under IFA is a special case of that under full capital mobility where $\theta^S = \theta^N$. In the steady state, given θ^S, the world equity rate changes with θ^N and the wage rate in country S changes accordingly. The first derivative of ω^i_j with respect to p^i_j is

$$\frac{\partial\omega^i_j}{\partial p^i_j} = \omega^i_j\rho\frac{\theta^i\left[\frac{(1-r^i_j)^2}{\eta} + \frac{1}{1-\eta}\right] - r^i_j\left[r^i_j - (1-\theta^i)\frac{(1-\gamma)}{1-\eta}\right]}{[(1-\theta^i)r^i_j + \theta^i p^i_j]p^i_j r^i_j}. \tag{59}$$

Since $\frac{\partial\mathcal{A}^i}{\theta^i} = \frac{\gamma}{1-\eta} > 0$, we get $\mathcal{A}^i_{min} = \frac{1-\gamma}{1-\eta}$ for $\theta^i = 0$.

$$r^i_{min} = \frac{m + A^i_{min}}{m+1} > A^i_{min} = \frac{1-\gamma}{1-\eta} > (1-\theta^i)\frac{(1-\gamma)}{1-\eta} \quad \Rightarrow \quad r^i_j > (1-\theta^i)\frac{(1-\gamma)}{1-\eta}.$$

The second component in the numerator of equation (59) is strictly positive. $\frac{\partial \omega_j^i}{\partial p_j^i}$ has the same sign as the numerator $\mathcal{N}_j^i \equiv \theta^i \left[\frac{(1-r_j^i)^2}{\eta} + \frac{1}{1-\eta}\right] - r_j^i \left[r_j^i - (1-\theta^i)\frac{(1-\gamma)}{1-\eta}\right]$. If $\mathcal{N}_j^i > 0$, a marginal decline in r_j^i keeps $\frac{\partial \omega_j^i}{\partial p_j^i} > 0$; if $\mathcal{N}_j^i < 0$, a marginal rise in r_j^i keeps $\frac{\partial \omega_j^i}{\partial p_j^i} < 0$.

According to equations (19)-(20), $p_{IFA}^i = \frac{m+B^i}{m+1}$ and $r_{IFA}^i = \frac{m+A^i}{m+1}$. Evaluate $\frac{\partial \omega_j^i}{\partial p_j^i}$ in the steady state under IFA by substituting p_{IFA}^i and r_{IFA}^i into equation (59), we get

$$\frac{\partial \omega_{IFA}^i}{\partial p_{IFA}^i} = \rho \omega_{IFA}^i \frac{[A^i B^i - m^2 - (B^i + m) B^i \frac{(1-\gamma)\eta}{(1-\eta)\gamma}](1-A^i)(m+1)}{(m+A^i)(m+B^i)(m+A^i B^i)} \tag{60}$$

We first consider the case with the investment composition channel only, i.e., $m = 0$ and $\gamma \in (0,1)$. Equation (60) is simplified into

$$\frac{\partial \omega_{IFA}^i}{\partial p_{IFA}^i} = \rho \omega_{IFA}^i \gamma \frac{\theta^i(\bar{\theta} - \theta^i)}{(1-\eta)^2 (A^i)^2 B^i} \tag{61}$$

For $\theta^i = 0$ or $\theta^i = \bar{\theta}$, $\frac{\partial \omega_{IFA}^i}{\partial p_{IFA}^i} = 0$; for $\theta^i \in (0, \bar{\theta})$, $\frac{\partial \omega_{IFA}^i}{\partial p_{IFA}^i} > 0$.

Consider country N. Full capital mobility reduces the steady-state loan rate, which raises the numerator $\mathcal{N}_j^N$ of equation (59). For $\theta^N = \bar{\theta}$, $\mathcal{N}_{IFA}^N = 0$. Thus, it always hold that $\mathcal{N}_{FCM}^N > \mathcal{N}_{IFA}^N = 0$ or equivalently, $\frac{\partial \omega_{FCM}^N}{\partial p_{FCM}^N} > 0$. Thus, by raising the steady-state equity rate, full capital mobility raises the steady-state aggregate output, $Y_{FCM}^N > Y_{IFA}^N$. By analogy, we can prove that $Y_{FCM}^N > Y_{IFA}^N$ for $\theta^N \in (0, \bar{\theta})$.

Consider country S. Full capital mobility raises the steady-state loan rate, which reduces the numerator $\mathcal{N}_j^S$ of equation (59). For $\theta^S = 0$, $\mathcal{N}_{IFA}^S = 0$. Thus, it always hold that $\mathcal{N}_{FCM}^S < \mathcal{N}_{IFA}^S = 0$ or equivalently, $\frac{\partial \omega_{FCM}^S}{\partial p_{FCM}^S} < 0$. Thus, by reducing the steady-state equity rate, full capital mobility raises the steady-state aggregate output. For $\theta^S \in (0, \theta^N)$, $\frac{\partial \omega_{IFA}^S}{\partial p_{IFA}^S} > 0$ and $\mathcal{N}_{FCM}^S < \mathcal{N}_{IFA}^S$. If θ^S is slightly lower than θ^N, it is likely that $\mathcal{N}_{FCM}^S$ is still positive, or equivalently, $\frac{\partial \omega_{FCM}^S}{\partial p_{FCM}^S} > 0$. Thus, by reducing the steady-state equity rate, full capital mobility reduces the steady-state aggregate output, $Y_{FCM}^S < Y_{IFA}^S$. In contrast, for θ^S much lower than θ^N, it is likely that $\mathcal{N}_{FCM}^S < 0$, or equivalently, $\frac{\partial \omega_{FCM}^S}{\partial p_{FCM}^S} < 0$. Thus, by reducing the steady-state equity rate, full capital mobility raises the steady-state aggregate output, $Y_{FCM}^S > Y_{IFA}^S$. Thus, there exists a threshold value $\hat{\theta}_{IC}^S$ such that for $\theta^S \in [0, \hat{\theta}_{IC}^S)$, $Y_{FCM}^S > Y_{IFA}^S$, and for $\theta^S \in (\hat{\theta}_{IC}^S, \theta^N)$, the opposite applies.

Let us then consider the case with the elastic savings channel only, i.e., $m > 0$ and $\gamma = 1$. Equation (60) is simplified into

$$\frac{\partial \omega_{IFA}^i}{\partial p_{IFA}^i} = \rho \omega_{IFA}^i \frac{[A^i B^i - m^2](1-A^i)(m+1)}{(m+A^i)(m+B^i)(m+A^i B^i)} \tag{62}$$

The sign of $\frac{\partial \omega_{IFA}^i}{\partial p_{IFA}^i}$ depends on the numerator $[A^i B^i - m^2](1-A^i) = \left[\frac{\theta^i(1-\theta^i)}{\eta(1-\eta)} - m^2\right] \frac{(\bar{\theta}-\theta^i)}{1-\eta}$.

For $\theta^N = \bar{\theta}$, the numerator is equal to zero and $\frac{\partial \omega^i_{IFA}}{\partial p^i_{IFA}} = 0$. As mentioned above for the case with the investment composition channel, full capital mobility strictly raises the steady-state aggregate output in country N. For $\theta^S = 0$, the numerator is smaller than zero and $\frac{\partial \omega^i_{IFA}}{\partial p^i_{IFA}} < 0$. By analogy, full capital mobility strictly raises the steady-state aggregate output in country S.

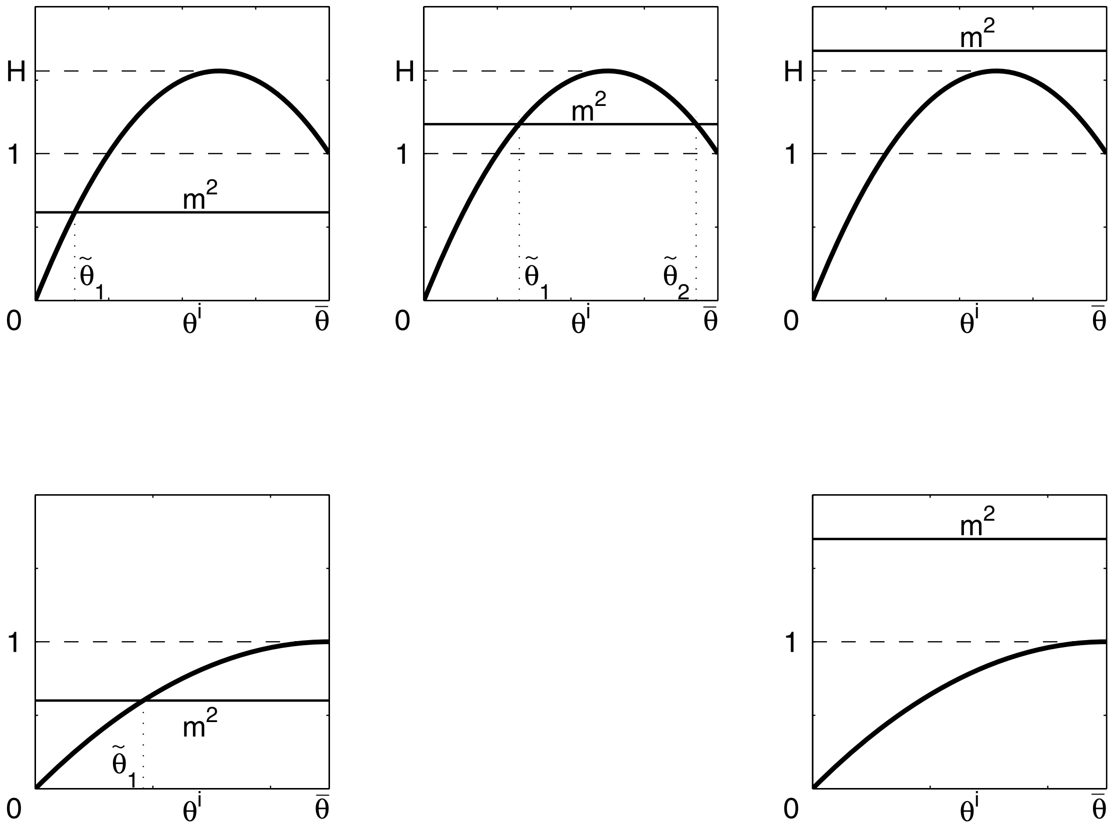

Figure 9: Threshold Values under Various Scenarios

Figure 9 shows all possible cases on the relative size of $\frac{\theta^i(1-\theta^i)}{\eta(1-\eta)}$ and m^2 where the three panels in the first row show the cases with $\eta \in (0, 0.5)$, the two panels in the second row show the cases with $\eta \in (0, 5, 1)$, and the horizontal axis shows $\theta^i \in (0, \bar{\theta})$.

Given $\eta \in (0, 0.5)$, $\frac{\theta^i(1-\theta^i)}{\eta(1-\eta)}$ is a hump-shaped function of $\theta^i \in (0, 1-\eta)$. In particular, $\frac{\theta^i(1-\theta^i)}{\eta(1-\eta)} \in (0, \frac{1}{4\eta(1-\eta)})$ and point H denotes its highest value $\frac{1}{4\eta(1-\eta)} > 1$.

- If $m \in (0, 1)$, there exists a threshold value $\tilde{\theta}_1 = \frac{1-\sqrt{1-4m^2(1-\eta)\eta}}{2}$ such that, for $\theta^i \in (0, \tilde{\theta}_1)$, $\left[\frac{\theta^i(1-\theta^i)}{\eta(1-\eta)} - m^2\right] \frac{(\bar{\theta}-\theta^i)}{1-\eta} < 0$ and, for $\theta^i \in (\tilde{\theta}_1, \bar{\theta})$, the opposite applies.

- If $m \in (1, \frac{[\eta(1-\eta)]^{-0.5}}{2})$, there exists two threshold values $\tilde{\theta}_1 = \frac{1-\sqrt{1-4m^2(1-\eta)\eta}}{2}$ and $\tilde{\theta}_2 = \frac{1+\sqrt{1-4m^2(1-\eta)\eta}}{2}$ such that for $\theta^i \in (\tilde{\theta}_1, \tilde{\theta}_2)$, $\left[\frac{\theta^i(1-\theta^i)}{\eta(1-\eta)} - m^2\right] \frac{(\bar{\theta}-\theta^i)}{1-\eta} > 0$ and, for $\theta^i \in (0, \tilde{\theta}_1) \cup (\tilde{\theta}_2, \bar{\theta})$, the opposite applies.

- If $m > \frac{[\eta(1-\eta)]^{-0.5}}{2}$, for $\theta^i \in (0, \bar{\theta})$, it holds that $\left[\frac{\theta^i(1-\theta^i)}{\eta(1-\eta)} - m^2\right] \frac{(\bar{\theta}-\theta^i)}{1-\eta} < 0$.

Given $\eta \in (0, 0.5)$, $\frac{\theta^i(1-\theta^i)}{\eta(1-\eta)}$ is a monotonically increasing function of $\theta^i \in (0, 1-\eta)$. In particular, $\frac{\theta^i(1-\theta^i)}{\eta(1-\eta)} \in (0, 1)$.

- If $m \in (0, 1)$, there exists a threshold value $\tilde{\theta}_1 = \frac{1-\sqrt{1-4m^2(1-\eta)\eta}}{2}$ such that, for $\theta^i \in (0, \tilde{\theta}_1)$, $\left[\frac{\theta^i(1-\theta^i)}{\eta(1-\eta)} - m^2\right] \frac{(\bar{\theta}-\theta^i)}{1-\eta} > 0$ and, for $\theta^i \in (\tilde{\theta}_1, \bar{\theta})$, the opposite applies.
- If $m > 1$, for $\theta^i \in (0, \bar{\theta})$, $\left[\frac{\theta^i(1-\theta^i)}{\eta(1-\eta)} - m^2\right] \frac{(\bar{\theta}-\theta^i)}{1-\eta} < 0$.

By the same logic as in the case of the investment composition channel, we can prove the results in the five scenarios of Proposition 4. □

Proof of Lemma 3

Proof. Combining equations (41) and (45), we rewrite the dynamic equation of wages,

$$\ln \omega_{t+1}^i = -\rho \ln R_t^* + \gamma\rho \ln \left(\frac{\omega_t^i}{\omega_{t+1}^i} R_t^* \frac{(1-\theta^i)}{\Gamma_{IFA}^i} + \theta^i \right). \tag{63}$$

The first and the second derivatives of ω_{t+1}^i with respect to ω_t^i are

$$\frac{\partial \omega_{t+1}^i}{\partial \omega_t^i} = \frac{\omega_{t+1}^i}{\omega_t^i} \frac{\rho\gamma}{\rho\gamma + 1 + \frac{\theta^i}{\chi_{t+1}^i - \theta^i}} \in \left(0, \frac{\omega_{t+1}^i}{\omega_t^i}\right)$$

$$\frac{\partial \omega_{t+1}^i}{\partial^2 \omega_t^i} = -\frac{\partial \omega_{t+1}^i}{\partial \omega_t^i} \frac{\omega_t^i}{(\omega_{t+1}^i)^2} \left(\frac{\omega_{t+1}^i}{\omega_t^i} - \frac{\partial \omega_{t+1}^i}{\partial \omega_t^i} \right) \left[\frac{\omega_t^i}{\omega_{t+1}^i}(1+\rho\gamma) + \frac{\theta^i}{1-\theta^i} \frac{\Gamma_{IFA}^i}{R_t^*} \right]^{-1}$$

Since $\frac{\partial \omega_{t+1}^i}{\partial \omega_t^i} \in \left(0, \frac{\omega_{t+1}^i}{\omega_t^i}\right)$, we get $\frac{\partial \omega_{t+1}^i}{\partial^2 \omega_t^i} < 0$. Thus, the phase diagram of wages is a concave function under partial capital mobility if the borrowing constraints are binding.

According to equation (63), for $\omega_t^i = 0$, the phase diagram has a positive intercept on the vertical axis at $\omega_{t+1}^i = (R_t^*)^{-\rho}(\theta^i)^{(\gamma\rho)}$. Define a threshold value $\bar{\omega}_t^i = \Gamma_{IFA}^i (R_t^*)^{-\frac{1}{1-\alpha}}$. For $\omega_t^i \in (0, \bar{\omega}_t^i)$, the phase diagram of wages is monotonically increasing and concave. For $\omega_t^i > \bar{\omega}_t^i$, aggregate saving and investment in sector B is so high that the intratemporal relative price is equal to one, or equivalently, $R_t^i = v_{t+1}^{i,B}$. Thus, the borrowing constraints are slack and the phase diagram is flat with $\omega_{t+1}^i = \bar{\omega}_{t+1}^i = (R_t^*)^{-\rho}$. Given $R_t^* < \mathcal{Q} < \Gamma_{IFA}^i$, we get $\bar{\omega}_{t+1}^i < \bar{\omega}_t^i$. In other words, the kink point is below the 45 degree line.

Thus, the phase diagram of wages crosses the 45 degree line once and only once from the left, and the intersection is in its concave part. Thus, the model economy has a unique and stable steady state under partial capital mobility. □

Proof of Proposition 5

Proof. Following the proof of proposition 2, there exists a threshold value of the country index $\tilde{N}$ such that $R^*_{PCM} \in (R^{\tilde{N}}_{PCM}, R^{\tilde{N}+1}_{PCM}]$. In the steady state, partial capital mobility raises the relative prices and equation (46) implies that partial capital mobility reduces aggregate output in country $i \in \{1, 2, ..., \tilde{N}\}$, i.e., $\chi^i_{PCM} > \chi^i_{IFA}$, $\psi^i_{PCM} > \psi^i_{IFA}$, and $Y^i_{PCM} < Y^i_{IFA}$. The opposite applies to country $i \in \{\tilde{N}+1, \tilde{N}+2, ..., N\}$.

In the steady state, $\Gamma^i_{PCM} = \Gamma^i_{IFA}$ and equation (19) imply that $\Gamma^i_{PCM} > \Gamma^{i+1}_{PCM}$. The loan rate equalization implies that

$$\begin{aligned} \Gamma^i_{PCM}\left(1 - \frac{1-\chi^i_{PCM}}{1-\theta^i}\right) &= \Gamma^{i+1}_{PCM}\left(1 - \frac{1-\chi^{i+1}_{PCM}}{1-\theta^{i+1}}\right), &\Rightarrow \quad & \frac{1-\chi^i_{PCM}}{1-\theta^i} > \frac{1-\chi^{i+1}_{PCM}}{1-\theta^{i+1}} \\ 1-\theta^i &> 1-\theta^{i+1}, &\Rightarrow \quad & 1-\chi^i_{PCM} > 1-\chi^{i+1}_{PCM} \end{aligned}$$

Thus, the steady-state relative prices rise in θ^i, i.e., $\chi^{i+1}_{PCM} > \chi^i_{PCM}$. Given $\omega^i_{PCM} = \left[\frac{(\chi^i_{PCM})^\gamma}{R^*_{PCM}}\right]^\rho$, the steady-state wage under partial capital mobility also rises in θ^i. □