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An Evolutionary Analysis of Turnout with Conformist Citizens

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An Evolutionary Analysis of Turnout With Conformist Citizens [1] [2]

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Abstract

We propose an evolutionary analysis of a voting game where citizens have a preference for conformism that adds to the instrumental preference for the electoral outcome. Multiple equilibria arise, and some generate high turnout. Simulations of best response dynamics show that high turnout is asymptotically stable if conformism matters but its likelihood depends on the reference group for conformism: high turnout is more likely when voters care about their own group's choice, as this better overrides the free rider problem of voting games. Comparative statics on the voting cost distribution, the population's size or the groups' composition are also done.

JEL classification: D72, C72, C73

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1 Introduction

The game theoretic literature of rational decision making predicts very low levels of turnout in large elections. This is because the probability of making a difference (i.e. being pivotal) is of smaller magnitude than the cost of voting for most of the citizens, which therefore prefer abstention to the costly action of going to the polls. However this prediction conflicts with the available evidence. People do vote and the observed turnout rates are well above the predicted ones. Pivot probability models can generate substantial turnout rates under specific circumstances: when there is no uncertainty and supporters of two alternatives are of equal size (Palfrey and Rosenthal, 1983). As soon as uncertainty is introduced, the high turnout equilibria disappear (Myerson, 1998; Palfrey and Rosenthal, 1985).

Recent attempts to rationalize higher levels of turnout within the pivot probability rely on the assumption of ethical preferences, so that voters act as rule utilitarian maximizers (see Coate and Conlin, 2004; Feddersen and Sandroni, 2006). In this context, caring about the welfare of the representative agent in one groups increases the expected benefit from turning out to vote. Similar effects can be obtained when voters are altruistic and therefore internalizes the benefits of their voting choice on others (see, e.g. Fowler, 2006; Jankowski, 2007) or group oriented (see, e.g. Morton, 1991; Uhlaner, 1986). [1]

However other questioned the relevance of the pivot probability in motivating individuals to vote. Meiroviz and Shots (2009) for example, show that when candidates are uncertain about policy preferences of the citizens', voting decisions can initially be dictated by signalling considerations so as to move future policies towards some more favorable to the voter, rather than by pivot probability considerations. In this setup turnout is substantial. Rotemberg (2009), instead, assumes that people are more altruistic towards individuals who agree with them and people's well being raises when they learn that other people share the same opinions. Those two elements act so as to reinforce expectations of voting towards one candidate and therefore induce higher turnout regardless of pivot probability considerations.

In our paper we rely on another possible explanation for high turnout rates, still within the pivot probability approach: social conformism. In particular, we analyze an evolutionary model of turnout where citizens have also preference for conformism; that is, citizens like to

[1] The literature on why people vote is quite vast. Recent surveys are given by Dhillon and Peralta (2002) and Geys (2006).

do what others in a reference group are doing and the decision of whether to vote is taken by weighting the benefit from being pivotal to the electoral outcome, the private cost of voting and the benefit of conforming with the majority. Contrary to the models that assume a preference (warm glow) for the act of voting (Riker and Ordeshook, 1968), in our model the act of voting generates a disutility if the majority of citizens decides to abstain. Therefore turnout is the result of the interplay between two types of mechanisms: one, which is standard and pushes turnout rates down, leads citizens who prefer the same alternative to free ride on others' efforts; the other, which is due to conformism, reinforces the low willingness to vote if there is an expectation that the others will stay home, but may also counteract it if there is an expectation that others will turn out.

The role of conformism in fostering turnout has been suggested by many in the social sciences (see, e.g. Coleman, 2004; De Matos and Barros, 2004; Klick and Parisi, 2008). However, to the best of our knowledge, nobody has looked at the coordination issues that are at the heart of the emergence of a norm.

The simple point we want to highlight is that when conformism matters, voting choices have a self-enforcing nature that generates multiplicity of Nash equilibria; besides the low turnout ones, there are also others where turnout can be significantly high. The exact magnitude of equilibrium turnout depends on the heterogeneity in the cost of voting and on the parameter governing the relative importance of conformism in citizens' utilities (see the companion paper Landi and Sodini, 2010).

Multiple equilibria generate prediction problems. In fact, one cannot consider conformism as a sufficient factor in causing high turnout rates, unless high turnout equilibria can be justified as being more plausible. In this paper, we contribute to the issue of equilibrium selection, by considering a simple best response dynamics, which is an adaptation of fictitious play (Brown, 1951) to voting games. We assume that time is discrete and that at every period each citizen knows the number of voters from the previous election and uses that information to generate expectations about the turnout in the incoming election. These expectations are needed to estimate the marginal benefit and the marginal cost of voting, which eventually determine individuals' decision to vote or not. These decisions translate into a new observed number of voters which will then be used to update further individuals' expectations for the next election.

Our analysis, which is done by considering the case where conformism depends on the entire population and the case where conformism depends on one's group, shows that different cases arise. If the weigh of conformism is low, we get a unique unstable interior equilibrium and the dynamics cycle around it. But if the weigh of conformism is sufficiently high, then multiple equilibria arise and the dynamical analysis allows us to compare the size of the basins of attraction of low and high turnout equilibria and to study the sensitivity of such basins to parameter variations. We find that: Interior high turnout equilibria are unstable when unique. When conformism depends on the behavior of the entire population, the basin of attraction of high turnout equilibria is smaller than the no turnout equilibrium. When conformism depends on the behavior of one's group the basin of attraction of the no turnout equilibrium shrinks significantly at the expenses of the high turnout equilibrium. In addition, new corner equilibria, that are asymptotically stable, arise. They are characterized by the fact that one group votes or abstains en masse. We then conclude that a norm of voting is more likely to emerge in the case of strong preference for conformism and smaller reference groups.

Dynamic models on turnout have been proposed in some recent papers. Substantial turnout rates are possible when: voters base their choice on not too noisy polls (Diermeier and Mieghem, 2008), since in this case the probability of being pivotal can increase substantially; voters care about their relative ranking in the population and switch to the action (vote or not) that will fare better than the average (Sieg and Schulz, 1995). However in this scenario many other steady state configurations are possible, so that the authors conclude that the evolution of turnout is ambiguous even though individual learning is unique; public spirited behavior has an evolutionary advantage over a selfish one (Conley et al., 2006), which requires cost of voting to be substantially nihil and low free riding opportunities from the selfish citizens.

Unlike our model, where convergence to steady state is pretty fast, more elaborated dynamics can be achieved in models of bounded rationality learning where agents update their actions when aspiration levels are not met (Bendor et al., 2003) or when individuals' decision to vote depends on their past action and whether their favorite party has won the election (Collins et al., 2008).

DeMichelis and Dhillon (2009) show that the only stable steady state in Palfrey and Rosenthal (1983) and Palfrey and Rosenthal (1985) involves low turnout. They analyze a selection process based on estimations of pivotal probability, endogenously updated taking into account information from polls or past elections. Our model is very close to DeMichelis and Dhillon (2009), in that we share the assumption that agents form expectations about pivotal probability via an adaptive updating rule. However, in our model, the selection process is also conditioned by the preference for conformity. The parameter indicating the relative importance of conformism will dictate the chances of high turnout equilibria to emerge.

The remainder of the paper is organized as follows. Section 2 describes the base ingredients of the model and provides the main results. Section 3 reports the simulation results' of our dynamics. Section 4 concludes the paper. All the proofs are relegated to the Appendix.

2 The static turnout model

Consider a large population of citizens who need to choose, via majority voting, between two exogenously given alternatives, R and L. The population is partitioned into two groups, one that prefers R to L (call it group R) and one that prefers L to R (call it group L). The utility received by citizens is set to 1 if their most preferred alternative is selected and 0 if their least preferred alternative is selected. Voting is a costly activity. Let the cost of voting, c, be an i.i.d. draw over the interval $[0, 1]$ according to a distribution $F(c) = 1-(1-c)^k$ with $k \geq 2$. [2] This distribution is chosen to reflect the intuition that the cost of voting is mainly small. Higher values of k imply higher probability mass around 0. Let $c_m = 1-2^{-1/k}$ denote the median of the cost distribution.

Each citizen simultaneously choose whether to vote for R, or L or to stay home (abstain). The alternative that receives the majority of the votes is selected. A flip of a coin is used to break ties (so that each alternative is equally likely to be selected).

Since there are only two alternatives, to vote for the least preferred is weakly dominated. Therefore we can reduce the citizens' problem to either vote for the most preferred alternative or stay home. The payoff table can be simplified as follows:

[2] In our companion paper Landi and Sodini (2010) we characterize the equilibria of the static game for single peaked cost distributions.

		WIN	LOSE
$P1$	V	$1-c+\delta x$	$-c+\delta x$
	A	$1+\delta(1-x)$	$\delta(1-x)$

where $\delta > 0$ denotes the relative importance of conformism and x denotes the share of citizens who choose to vote. [3] This share will be determined within either the entire population (population wide conformism) or the group to which one voter belongs (group based conformism).

Following Myerson (1998, 2000), the population size is uncertain and distributed according to a Poisson process with mean n, where n is a finite but large integer. Each citizen belongs to group R with probability θ.

In this context, there is no loss in generality in focusing on strategies that take the form of a pair of cutoffs $(\bar{c}_R, \bar{c}_L)$, according to which any voter j belonging to group i votes if and only if $c_j \leq \bar{c}_{ji}$.

In pivot probability models each player chooses to vote whenever the marginal contribution to victory is no smaller than the marginal cost of voting. One's vote matters whenever it breaks a losing tie (so the two alternative have the same number of votes) or whenever it forces a winning tie (so an alternative is ahead by one vote).

By letting $\rho > 0$ ($\lambda > 0$) denote the probability that a group R (L) player votes, for large n the pivot probabilities can be approximated by the following equations (Myerson, 1998, 2000)

$$\begin{aligned} P_R(\rho,\lambda,\theta) &= \frac{e^{-n(\sqrt{\theta\rho}-\sqrt{(1-\theta)\lambda})^2}}{4\sqrt{n\pi\sqrt{\theta(1-\theta)\lambda\rho}}}\frac{\sqrt{\theta\rho}+\sqrt{(1-\theta)\lambda}}{\sqrt{\theta\rho}} \\ P_L(\rho,\lambda,\theta) &= \frac{e^{-n(\sqrt{\theta\rho}-\sqrt{(1-\theta)\lambda})^2}}{4\sqrt{n\pi\sqrt{\theta(1-\theta)\lambda\rho}}}\frac{\sqrt{\theta\rho}+\sqrt{(1-\theta)\lambda}}{\sqrt{(1-\theta)\lambda}} \end{aligned}$$

When conformism is population wide, the marginal cost of voting for a j citizen is given by

$$c_j + \delta\left[1 - 2(\theta\rho + (1-\theta)\lambda)\right]$$

where the first term is the private cost which is drawn from $F(c)$, whereas the second term is the cost arising from conformism: if the majority of the population votes, so that $\theta\rho +$

[3] Landi and Sodini (2010) analyze also the case of heterogenous δ.

$(1-\theta)\lambda \geq 1/2$, this term subtracts to the private cost of voting. On the other hand, if the majority of the population abstains, the cost of conforming adds to it.

A symmetric equilibrium is given by the pair of cutoffs $(\bar{c}_R, \bar{c}_L) \in [0,1]^2$, common to all players, such that no player gains from choosing a different one. The conditions to find these cutoffs are represented by the following set of equations

$$P_R(\rho,\lambda,\theta) = \bar{c}_R + \delta\left[1 - 2\left(\theta\rho + (1-\theta)\lambda\right)\right] \quad (1)$$

$$P_L(\rho,\lambda,\theta) = \bar{c}_L + \delta\left[1 - 2\left(\theta\rho + (1-\theta)\lambda\right)\right] \quad (2)$$

for interior equilibria, i.e. $(\bar{c}_R, \bar{c}_L) \in (0,1)^2$, and by

$$P_R(0,\lambda,\theta) \leq \delta\left[1-\lambda\right] \quad (3)$$

$$P_L(\rho,0,\theta) \leq \delta\left[1-\rho\right] \quad (4)$$

$$P_R(1,\lambda,\theta) \geq 1-\delta\lambda \quad (5)$$

$$P_L(\rho,1,\theta) \geq 1-\delta\rho \quad (6)$$

for corner equilibria, i.e. $\bar{c}_i \in \{0,1\}$ for some i. For example, an equilibrium of the type $(0,\lambda)$ requires all group R voters to abstain, which happens only when the marginal cost of voting exceeds its marginal benefit for all possible private costs. This means that inequality (3) has be satisfied. All the other inequalities can be interpreted in the same way. Note that ρ and λ are endogenously defined by

$$\rho = F(\bar{c}_R) = 1 - (1-\bar{c}_R)^k$$

and

$$\lambda = F(\bar{c}_L) = 1 - (1-\bar{c}_L)^k$$

We now proceed into the characterization of the equilibria for the polar case $\theta = 1/2$. We then show via simulations that in large populations the effect of θ on the equilibrium cutoffs is negligible. The first step consists in understanding that citizens in both groups pick the same cutoffs, which simplifies considerably the equations to be analyzed.

Proposition 1. *Suppose conformism is population wide and $\theta = 1/2$. The equilibrium* *cutoffs are the same, i.e.* $\bar{c}_R = \bar{c}_L = \bar{c}$ *and are characterized by the following equations*

$$\begin{aligned} [2n\pi F(\bar{c})]^{-1/2} = \bar{c} + \delta - 2\delta F(\bar{c}) &\quad \textit{for } \bar{c} \in (0,1) \\ \delta \geq \frac{1}{2} &\quad \textit{for } \bar{c} = 0 \\ \delta \geq 1 - \sqrt{\frac{1}{2n\pi}} &\quad \textit{for } \bar{c} = 1 \end{aligned}$$

Now let $(\hat{c}, \hat{\delta})$ be the unique solution to

$$\begin{aligned} [2n\pi F(c)]^{-1/2} &= c + \delta[1 - 2F(c)] \\ \frac{f(c)}{2F(c)\sqrt{2n\pi F(c)}} &= 2\delta f(c) - 1 \end{aligned}$$

In words, marginal cost and benefit of voting are tangent when evaluated at the pair $(\hat{c}, \hat{\delta})$. We are in the position to fully characterize the equilibria of the game via a simple graphical analysis (see figure 1).

[Figure 1 about here.]

Proposition 2. *Suppose conformism is population wide and* $\theta = 1/2$. *Let*

$$0 < \bar{c}^1 < c_m < \bar{c}^2 < \hat{c} < \bar{c}^3 < 1$$

and $\delta_1 = 1 - (2n\pi)^{-1/2}$. *The equilibria of the voting game are given by*

$$\begin{cases} (\bar{c}^1, \bar{c}^1) & \textit{if and only if } \delta < 1/2; \\ (0,0) & \textit{if and only if } 1/2 \leq \delta < \hat{\delta}; \\ (0,0) \ \ (\hat{c}, \hat{c}) & \textit{if and only if } \delta = \hat{\delta}; \\ (0,0) \ \ (\bar{c}^2, \bar{c}^2) \ \ (\bar{c}^3, \bar{c}^3) & \textit{if and only if } \hat{\delta} < \delta < \delta_1; \\ (0,0) \ \ (\bar{c}^2, \bar{c}^2) \ \ (1,1) & \textit{if and only if } \delta \geq= \delta_1; \end{cases}$$

As can be seen, when $\delta \geq \hat{\delta}$, we have multiple equilibria, and if δ is not too large, we have two interior equilibria. In this case voters whose private cost is below $\bar{c}^2$ vote in both equilibria, those whose private cost is above $\bar{c}^3$ abstain in both equilibria, and those whose private cost is between $\bar{c}^2$ and $\bar{c}^3$ do not vote if playing the equilibrium $(\bar{c}^2, \bar{c}^2)$ but vote when playing the equilibrium $(\bar{c}^3, \bar{c}^3)$. Therefore, the latter equilibrium generates a higher expected turnout. In general a higher cutoff means higher turnout, because turnout is given by $nF(\bar{c}^i)$.

Consider now the case where conformism is group based. The marginal cost of voting for player j becomes

$$\begin{cases} c_j + \delta(1-2\rho) & \text{if } j \text{ belongs to group } R \\ c_j + \delta(1-2\lambda) & \text{if } j \text{ belongs to group } L \end{cases}$$

regardless of θ. All the equilibria that we found for the case of population wide conformism still hold true. In fact, in this case the cutoffs for an interior equilibrium costs are obtained from

$$P_R(\rho,\lambda) = \bar{c}_R + \delta\left[1 - 2F(\bar{c}_R)\right] \quad (7)$$

$$P_L(\rho,\lambda) = \bar{c}_L + \delta\left[1 - 2F(\bar{c}_L)\right] \quad (8)$$

For $\theta = 1/2$ the equilibria for population wide conformism extend to this case. However there are also other corner equilibria, where all members of one group vote or abstain, and members of the other vote with positive probability. Thus we have:

Proposition 3. *When $\theta = 1/2$, any equilibrium found when conformism is population wide is also an equilibrium when conformism is group based.*

In addition, when conformism is group based, there are other corner equilibria, of the form $(0, \bar{c}_L)$, with $\bar{c}_L \in (c_m, 1]$ and $(\bar{c}_R, 0)$, with $\bar{c}_R \in (c_m, 1]$, and $(1, \bar{c}_L)$, with $\bar{c}_L \in (c_m, 1)$ and $(\bar{c}_R, 1)$, with $\bar{c}_R \in (c_m, 1)$, for δ sufficiently large.

Now let $(\tilde{\delta}, \tilde{c})$ denote the unique solution to

$$\frac{e^{-nF(c)/2}}{2} = c + \delta\left[1 - 2F(c)\right]$$
$$\frac{nf(c)e^{-nF(c)/2}}{4} = 1 - 2\delta f(c)$$

The values of $\tilde{\delta}$ and $\tilde{c}$ are computed in order to have a tangency between the marginal benefit and cost of voting, when the two functions are computed under the assumption that one group abstains en masse and the other votes with positive probability. Remark that $\tilde{\delta} > \hat{\delta}$ since $(2e^{-nF(c)/2})^{-1} < (2n\pi F(c))^{-1/2}$ for all $c \in [0,1]$.

In addition, let $\bar{\bar{\delta}}$ denote the lower bound of the values of δ for which an equilibrium of the type $(1, \bar{c}_L)$ or $(\bar{c}_R, 1)$ exist. Given the symmetry of the pivot probability when $\theta = 1/2$, this lower bound is obtained as

$$\bar{\bar{\delta}} = 1 - P_R(1, \bar{c}_L, 1/2) = 1 - P_R(\bar{c}_R, 1, 1/2)$$

We have that $\bar{\bar{\delta}}$ is a decreasing function of $\bar{c}_L$ and its minimum is when $\bar{c}_L = 0$. Note also that at the minimum, the pivot probability is given by $e^{-n/2}/2$. This allows us to conclude that $\bar{\bar{\delta}} > 1 - e^{-n/2}/2$ is the minimum value for δ that generates the above mentioned equilibrium. We are now ready to characterize the equilibria of the game under group based conformism.

Proposition 4. *Suppose conformism is group based and $\theta = 1/2$. Let*

$$0 < c^1 < c_m < c^4 < \hat{c} < c^5 < 1,$$

$$c_m < c^2 < \tilde{c} < c^3 < 1$$

$c_m < c^6 < 1$ *and* $\delta_1 = 1 - (2n\pi)^{-1/2}$. *The equilibria of the game are given by*

$$\begin{cases} (c^1, c^1) & \text{if and only if } \delta < 1/2; \\ (0,0) & \text{if and only if } 1/2 \leq \delta < \tilde{\delta}; \\ (0,0)\ (\hat{c},\hat{c}) & \text{if and only if } \delta = \hat{\delta}; \\ (0,0)\ (c^4,c^4)\ (c^5,c^5) & \text{if and only if } \hat{\delta} < \delta < \tilde{\delta}; \\ (0,0)\ (c^4,c^4)\ (c^5,c^5)\ (0,\tilde{c})\ (\tilde{c},0) & \text{if and only if } \delta = \tilde{\delta}; \\ (0,0)\ (c^4,c^4)\ (c^5,c^5)\ (0,c^2)\ (0,c^3)\ (c^2,0)\ (c^3,0) & \text{if and only if } \tilde{\delta} < \delta < \delta_1; \\ (0,0)\ (c^4,c^4)\ (1,1)\ (0,c^2)\ (0,c^3)\ (c^2,0)\ (c^3,0) & \text{if and only if } \delta_1 \leq \delta < 1-(2e^{n/2})^{-1}; \\ (0,0)\ (c^4,c^4)\ (1,1)\ (0,c^2)\ (0,1)\ (c^2,0)\ (1,0)\ (1,c^6)\ (c^6,1) & \text{if and only if } \delta \geq 1-(2e^{n/2})^{-1}; \end{cases}$$

When conformism is group based, we have a plethora of corner equilibria, for sufficiently high values of δ. In other words, if conformism matters, and only the behavior of one's group is relevant, whatever one group decides to coordinate upon is an equilibrium, regardless of what the other group is playing. In fact, when conformism is relatively important in the individuals' preferences and depends on one's group behavior, the free riding problem due to pivot probability considerations is less stringent: the cost of not conforming outweighs the cost of voting.

The next section is devoted to the analysis of the dynamic version of this model, which will help highlighting features related to equilibrium selection.

3 Dynamics

Our evolutionary analysis is based on a simple best response dynamics. To reduce considerably the computation time we assume the size of the population of players is constant at the value n. The share of R voters is given by θ. Each voter has a private cost of voting, which is not time dependent, randomly drawn from the cost distribution $F(c) = 1 - (1-c)^k$, with $k \geq 1$. Time is discrete and at every period $t+1$ each citizen knows the number of voters for every group at period t, $n_i(t)$, with $i = l,\ r$.

The expected number of voters in group R (resp. L) is given by $n(1-\theta)\lambda(t+1)$ (resp. $n\theta\rho(t+1)$). Citizens have static expectations about others decisions to vote, and use the equation for expected voters to infer the values of $\lambda(t+1)$ and $\rho(t+1)$ from the level of turnout observed at time t. Namely, voters estimates these two probabilities as

$$\lambda(t+1) = \frac{n_l(t)}{(1-\theta)n}$$

and

$$\rho(t+1) = \frac{n_r(t)}{\theta n}$$

Those values are then used to compute the pivot probabilities

$$\tilde{P}_L(n_r(t+1), n_l(t+1)) = \frac{e^{-(\sqrt{n_r(t)}-\sqrt{n_l(t)})^2}}{4\sqrt{\pi\sqrt{n_r(t)n_l(t)}}}\left(\frac{\sqrt{n_r(t)}+\sqrt{n_l(t)}}{\sqrt{n_l(t)}}\right) \tag{9}$$

and

$$\tilde{P}_R(n_r(t+1), n_l(t+1)) = \frac{e^{-(\sqrt{n_r(t)}-\sqrt{n_l(t)})^2}}{4\sqrt{\pi\sqrt{n_r(t)n_l(t)}}}\left(\frac{\sqrt{n_r(t)}+\sqrt{n_l(t)}}{\sqrt{n_r(t)}}\right) \tag{10}$$

and the net cost of participating

$$c_j + \delta\left[1 - \frac{2}{n}(n_r(t) + n_l(t))\right]$$

with c_j being the private cost of citizen j. At time $t+1$ all citizens need to decide whether to vote or not, and they do it by comparing the estimated marginal benefits to their marginal cost. All citizens whose expected marginal benefit does not fall below their marginal cost choose to vote. The others abstain. Specifically, for citizen j with private cost c_j and belonging to group i the behavioral rule becomes

$$\begin{cases} \text{if } \tilde{P}_i(n_r(t), n_l(t)) \geq c_j + \delta\left[1 - \frac{2}{n}(n_r(t)+n_l(t))\right] & \text{then } j \text{ votes} \\ \text{if } \tilde{P}_i(n_r(t), n_l(t)) < c_j + \delta\left[1 - \frac{2}{n}(n_r(t)+n_l(t))\right] & \text{then } j \text{ abstains} \end{cases} \tag{11}$$

These decisions generate a new observed number of voters which will then be used to update further each voters expectations at $t+2$.

Let the state of the system be denoted by the pair $(n_r(t), n_l(t))$ which, again, indicates the number of group R and group L voters at time t. Condition (11) generates a deterministic transition dynamics

$$(n_r(t+1), n_l(t+1)) = G(n_r(t), n_l(t)) \tag{12}$$

Definition 1. *A steady state of (12) is a pair* $(\bar{n}_r, \bar{n}_l)$ *that is time invariant, that is* $G(\bar{n}_r, \bar{n}_l) = (\bar{n}_r, \bar{n}_l)$*. The basin of attraction of a steady state is the set of initial conditions* (n_r, n_l) *for which the dynamics (12) converge to that steady state. The size of the basin of attraction is given by the number of its elements divided by the total number of states.*

We are interested in the size of the basin of attraction of a steady state because it can be interpreted as the probability that the dynamics converge to that steady state. Note also that steady states of this best response dynamics and equilibria of the static game are related to each other. Specifically:

Proposition 5. *Let* $(\bar{c}_R, \bar{c}_L)$ *denote an equilibrium of the base game. The pair* $(\bar{n}_r, \bar{n}_l)$*, with* $\bar{n}_r = n\theta F(\bar{c}_R)$ *and* $\bar{n}_l = n(1-\theta)F(\bar{c}_L)$ *is a steady state of (12).*

Since the number n of players is finite, the set of possible states under dynamics (12) is finite, that is $(n_r, n_l) \leq (n\theta, n(1-\theta))$. Consequently, the following proposition holds:

Proposition 6. *Every orbit under dynamics (12) approaches either a steady state or a periodic orbit.*

We analyze the asymptotic behavior of dynamics (12) by numerical simulations. The simulations have been implemented according to the following algorithm:

1. Fix the values of the parameters of the model, such as population size (n), intensity of conformism in players' utility (δ), share of group R citizens (θ) and value of the parameter governing the cost distribution (k).

2. Generate the stochastic costs, drawn from the distribution $F(c) = 1-(1-c)^k$. This generation is done once and for all, so that the cost of voting does not change within one complete iteration of the simulation. Redrawing the cost of voting at each step would not change the results, since $n = 100,000$ (at least), but would increase significantly the computational time.

3. Compute the equilibria of the static game, which correspond to the steady states of the myopic best response under investigation.

4. Generate the initial conditions (i.e. who votes and who abstains in the entire population) and iterate the best response behavior as highlighted in (11). Note that, at each round, all the players update their behavior. By restricting the number of citizens that, at each time, can update their choice we potentially increase the chances of convergence of the evolutionary dynamics.

5. Check whether the state of the system has reached one of the steady states of dynamics or whether has oscillated between states.

The base model consists of equal size populations of R and L voters and conformism depends on the behavior of the entire population. Our results for the static model show that (see proposition 2) there can be one or two interior equilibria, one or two corner equilibria, and these equilibria can coexist depending on the underlying parameters. The simulations' results for the best response dynamics with static expectations are reported in table 1 and in figure 2

[Table 1 about here.]

[Figure 2 about here.]

[Figure 3 about here.]

Specifically, we have that for low values of δ, when only the low turnout equilibrium exists, the dynamics cycle around it (see the column called oscillating in the table, and the trajectories of the dynamics (12) in figure 2). As δ becomes larger, initially the equilibrium shifts to nobody votes, which becomes also asymptotically stable (see again the trajectories in figure 2). This further increases of δ make the high turnout interior equilibria to appear. The largest of the two is asymptotically stable (see the black coloring in figure 3). But the equilibrium where nobody votes still remains asymptotically stable. However, its basin of attraction diminishes with δ. Therefore the set of initial conditions that converge to the high turnout equilibrium increases with δ.

The next step of the simulations involves comparative statics on the parameters of the model, such as θ, n, δ and k. Table 2 reports the comparative statics for the change in population size, with $\delta = 0.58$, $k = 4$, and $\theta = 0.5$.

[Table 2 about here.]

As we can see, as n increases, the basin of attraction of the interior steady state remains practically unaltered. Given the specified set of parameters, the system converges to either the interior or the no vote steady state, depending on the initial conditions. Note also that the share of citizens voting in the interior steady state is substantially constant, when $n_L \geq 50,000$, and so is the shape of the basin of attraction.
Next we study the roles played by θ and by δ. Table 3 reports the results of the simulations with $n = 100,000$ and $k = 4$. As the equilibria are symmetric with respect to θ, to economize on space table 3 (and the similar tables who follow) reports only the values of $\theta \geq 0.5$.

[Table 3 about here.]

We can observe some patterns that were already highlighted in the symmetric case. Namely:

1. As δ increases the high turnout equilibrium approaches to all vote.

2. The basin of attraction for the high turnout equilibrium increases with δ, at any level of θ: the stronger the preference for conformism, the more likely it is to select the high turnout equilibrium.

3. The shape and size of the basins of attraction of the asymptotically stable equilibria is affected by θ: as we move away from $\theta = 0.5$, the basins of attraction lose in symmetry (see figure 4 for a sample case) as the slope of the line separating them changes. In addition, the set of initial conditions that converge to the interior equilibrium expands.

[Figure 4 about here.]

Last, we study the role of k, the parameter governing the cost distribution. The larger k, the more right skewed the costs are, so that the higher the mass about low levels of costs. Table 4 reports the results of the simulations for groups of size $100,000$ each.

[Table 4 about here.]

We observe that as k grows, the turnout level evaluated at the interior steady state increases and eventually converges to all vote; furthermore, its basin of attraction grows, as can also be seen from figure 5, where the black area increases with k.

[Figure 5 about here.]

A similar analysis can be done for the case where conformism depends on the behavior of one's group. Our analysis of the static game has showed that in this context there are other corner equilibria, where one group votes and the other abstains. Our simulations show that these equilibria are also asymptotically stable steady states of the best response dynamics. First we report the simulation results for the case where groups are of equal size, and δ varies. They are reported in table 5 and in figure 7, for the parameter specification $n = 200,000$, $\theta = 0.5$ and $k = 4$.

[Table 5 about here.]

As for the base case with conformism that depends on the whole population's behavior, with low levels of δ there is only a low turnout equilibrium and the dynamics cycle around it. As δ grows, the no vote equilibrium appears and becomes asymptotically stable. For further increases of δ the interior high turnout equilibria and the corner equilibria where one group votes and the other abstains arise. Both are stable. Two new tones of grey are observable in figure 7, on the top left and bottom right corners. They indicate the set of all initial conditions for which the system converges to these new corner equilibria. Their basins of attraction increase with δ, at the expense of the no vote equilibrium, whose basin of attraction shrinks. When conformism depends on one's group behavior, the chances of observing equilibria with high turnout are much larger.

[Figure 6 about here.]

[Figure 7 about here.]

Like before, an increase in n is not changing significantly the results. The next step therefore involves analyzing the behavior of the dynamics as δ and θ change. Table 6 reports the simulation results for the parameter specification $n = 100,000$ and $k = 4$.

[Table 6 about here.]

As before, an increase in δ makes the interior equilibrium converge to the point where everybody votes. In addition, its basin of attraction increases with δ. A weak increase is also observed for the corner equilibria where one group votes and the other abstains. Overall the equilibrium in which nobody votes becomes less likely to emerge when δ increases. Note that in this case the value of θ does not matter: the symmetry of the basins of attraction is preserved regardless of θ (see figure 8). This depends on the fact that θ does not affect the conformism equations, while it affects only marginally the pivot probabilities.

[Figure 8 about here.]

Last, table 7 reports the comparative statics with respect to changes in k.

[Table 7 about here.]

As before, an increase in k makes the basins of attraction of the interior and the corner equilibria larger.

An interesting feature of the model comes from the role of the preference for conformism: when conformism is based on one's group's behavior, the high turnout interior equilibrium is less like to emerge at the expenses of corner solutions where one of the two groups votes and the other abstains. When the preference for conformism is strong enough, in other words, people prefer to behave with the majority in their own group regardless of the pivot probabilities. Paradoxically, there can be results where the majority stays home and the minority turns out and wins the election! In comparison to De Matos and Barros (2004), who analyzed the role of contagion in a structured network where initially only few vote, setting aside any electoral competition effect. What they find, therefore, is consistent with our within group behavior in this context.

4 Conclusion

This paper presents an evolutionary analysis of a model of turnout where citizens have preference for conformism. If conformism plays a non marginal role in citizens' utilities, the voting game has multiple equilibria, and high turnout rates are observable. However low turnout rates are always observable.

Both high and low turnout equilibria are asymptotically under our simple best response selection dynamics. The basin of attraction of high turnout equilibria increases in size as conformism becomes more important in citizens' utility. However the low turnout equilibrium has always a larger basins of attraction when conformism depends on the aggregate choice in the entire population, whereas it has a smaller basins of attraction when conformism depends on the choice made by one's political group. Therefore the type of reference group matters in stimulating high turnout rates.

Finally, in this model, turnout is interpreted as the solution of two types of coordination problems: one, which is standard to pivot probability models, is due by the tendency of citizens who are on the same side of the political spectrum to free ride on others' efforts. This (lack of) coordination pushes turnout rates down. The other, with the population at large (but not necessarily so) whereby others' actions directly influence one's utility. Such a coordination effect may reinforce the low willingness to vote if there is an expectation that the many will stay home, but may also counteract it if there is an expectation that many will turn out.

We have seen how important is the role of the reference group for conformism in directing coordination towards high or low turnout equilibria. The next step of this research agenda aims at pointing the magnifying glass to the network structures that make coordination on high turnout rates more likely to emerge.

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A Proof of main results

Proof of Proposition 1. An interior equilibrium needs to solve equations (1) and (2). By taking the ratio between these two equations, one condition can be written as

$$\sqrt{\frac{F(\bar{c}_L)}{F(\bar{c}_R)}} = \frac{\bar{c}_R + \delta\,(1 - F(\bar{c}_L) - F(\bar{c}_R))}{\bar{c}_L + \delta\,(1 - F(\bar{c}_L) - F(\bar{c}_R))} \tag{13}$$

Note that $\bar{c}_R = \bar{c}_L = \bar{c}$ is a solution to this equation. Moreover it is unique. Suppose in fact that $\bar{c}_L > \bar{c}_L$ is another solution. Then the left hand side of equation (13) is greater than 1 while the right hand side is smaller than 1. The opposite holds if $\bar{c}_L < \bar{c}_L$.

Next observe that $(0,0)$ is an equilibrium if and only if conditions (3) and (4) are met. Since the pivot probability at the origin are $1/2$, those conditions require $\delta \geq 1/2$.

Moreover, $(1,1)$ is an equilibrium if and only if conditions (5) and (6) are met. The pivot probabilities when everybody votes can be approximated by $(2n\pi)^{-1/2}$. Hence we need $\delta \geq 1 - (2n\pi)^{-1/2}$. Landi and Sodini (2010) show there are no other corner equilibria. □

Proof of Proposition 3. By repeating the steps undertaken in the proof of Proposition 1 one immediately sees that the interior equilibria and the corner equilibria of the type $(0,0)$ and $(1,1)$ are the same.

In addition, since $\theta = 1/2$, the conditions for corner equilibria of the type $(0, \bar{c}_L)$ and $(\bar{c}_R, 0)$, or $(1, \bar{c}_L)$ and $(\bar{c}_R, 1)$ are the same. Hence it is enough to consider one case each.

$(0, \bar{c}_L)$ is an equilibrium if and only if

$$\begin{aligned} P_R(0, F(\bar{c}_L), 1/2) &\leq \delta \\ P_L(0, F(\bar{c}_L), 1/2) &= \bar{c}_L + \delta(1 - 2F(\bar{c}_L)) \end{aligned}$$

By plugging the corresponding values for the pivot probabilities and rearranging, one gets

$$\delta \geq \frac{e^{-nF(\bar{c}_L)/2}}{2}\left[1 + \frac{nF(\bar{c}_L)}{2}\right] \tag{14}$$

$$\frac{e^{-nF(\bar{c}_L)/2}}{2} = \bar{c}_L + \delta\left[1 - 2F(\bar{c}_L)\right] \tag{15}$$

First observe that the right hand side in inequality (14) is a decreasing function of $\bar{c}_L$ at has a peak at zero which is given by $1/2$. Moreover, the right and the left hand side of equation (15) behave similarly to those in figure 1. This tells us that a necessary condition for both equality and inequality to happen is that $\delta \geq 1/2$. In fact, for lower values of δ, the solution to equation (15) would happen at a low value of $\bar{c}_L$ which would require a higher value of δ for inequality (14) to hold.

Following the logic from figure 1, if we let $(\tilde{\delta}, \tilde{c})$ denote the unique solution to

$$\begin{aligned} \frac{e^{-nF(c)/2}}{2} &= c + \delta\left[1 - 2F(c)\right] \\ \frac{nf(c)e^{-nF(c)/2}}{4} &= 1 - 2\delta f(c) \end{aligned}$$

we have that equation (15) has two interior solutions for any $\delta > \tilde{\delta}$. One of these solutions converges to 1 as δ approaches $1 - (2e^{n/2})^{-1}$. Both solutions are when δ is large enough to satisfy inequality (14).

Similarly, $(1, \bar{c}_L)$ is an equilibrium if and only if

$$P_R(1, F(\bar{c}_L), 1/2) \geq 1 - \delta \tag{16}$$

$$P_L(1, F(\bar{c}_L), 1/2) = \bar{c}_L + \delta(1 - 2F(\bar{c}_L)) \tag{17}$$

Observe that both probabilities are increasing in $\bar{c}_L$ and therefore reach a peak when $\bar{c}_L = 1$. This means that equation (17) has no solutions for δ sufficiently small, whereas it has two and then one solutions are δ grows. If we let $\bar{\bar{\delta}}$ denote the lower bound of the values of δ for which an equilibrium of the type $(1, \bar{c}_L)$ exist, this lower bound is obtained as

$$\bar{\bar{\delta}} = 1 - P_R(1, \bar{c}_L, 1/2) = 1 - P_R(\bar{c}_R, 1, 1/2)$$

The values of δ for which solutions to equation (17) exist, and for which inequality (16) is satisfied are going to be very close to 1. □

Proof of Proposition 5. Suppose we have an equilibrium $(\bar{c}_R, \bar{c}_R)$. Therefore this pair satisfies the set of conditions (1) to (6). By the law of large numbers the number of voters for each group is

$$\begin{aligned} n_r(t) &= \theta n F(\bar{c}_R) \\ n_l(t) &= (1-\theta) n F(\bar{c}_L)) \end{aligned}$$

and therefore the estimated probability of vote for each type is given by

$$\begin{aligned} \rho_{t+1} &= F(\bar{c}_R) \\ \lambda_{t+1} &= F(\bar{c}_R) \end{aligned}$$

This implies that the estimated pivot probabilities corresponds to the equilibrium pivot probabilities. Hence group i voter with private cost c_j votes if and only if $c_j \leq \bar{c}_i$. $\square$

Figure 1: Equilibria as a function of δ, with $n = 1,00$ and $F(c) = c(2-c)$. Interior equilibria are obtained as the intersection between two curves, the marginal benefit of voting (in blue), which decreases exponentially with c, and the marginal cost of voting, which changes with respect to δ, but passes through (c_m, c_m) for all values of δ. Corner equilibria are achieved by comparing marginal costs and benefits at the extremes, $c = 0$ and $c = 1$.

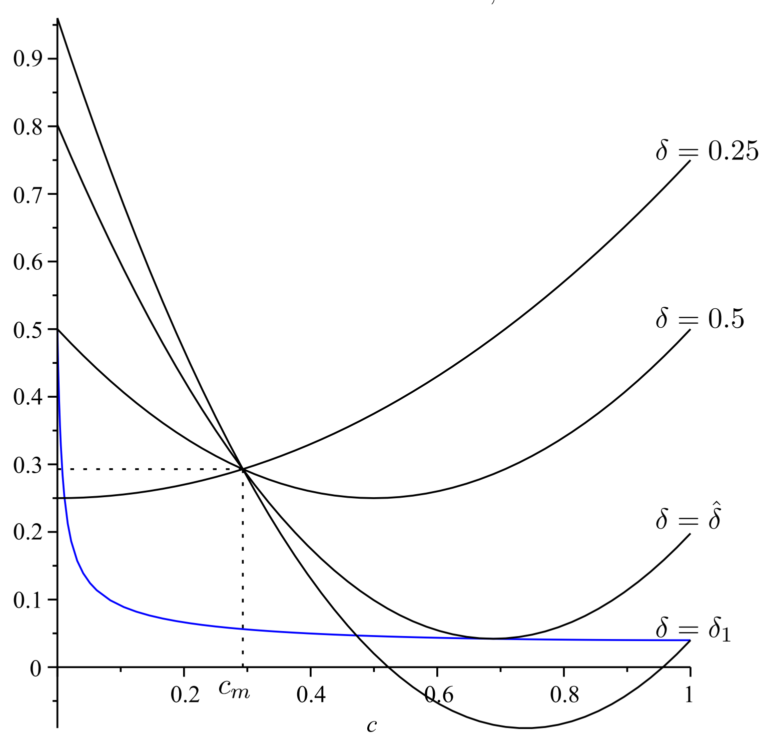
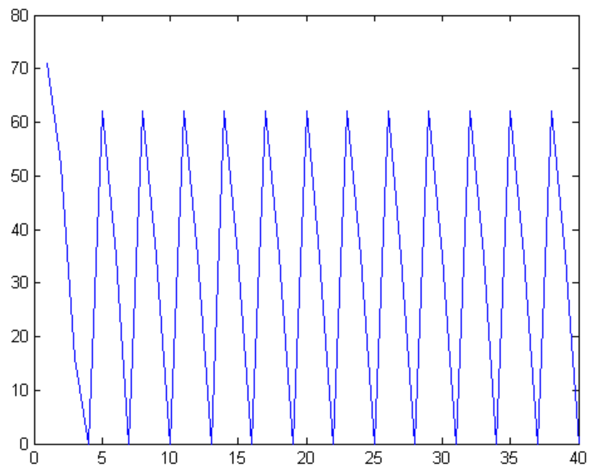
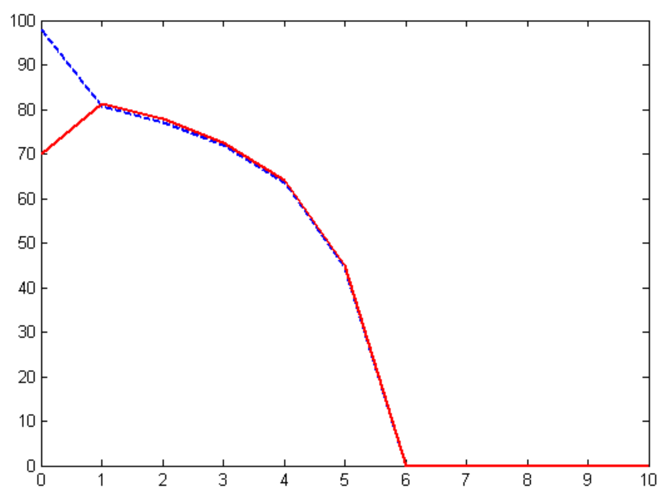


Figure 2: Trajectories of the dynamics when δ is small.

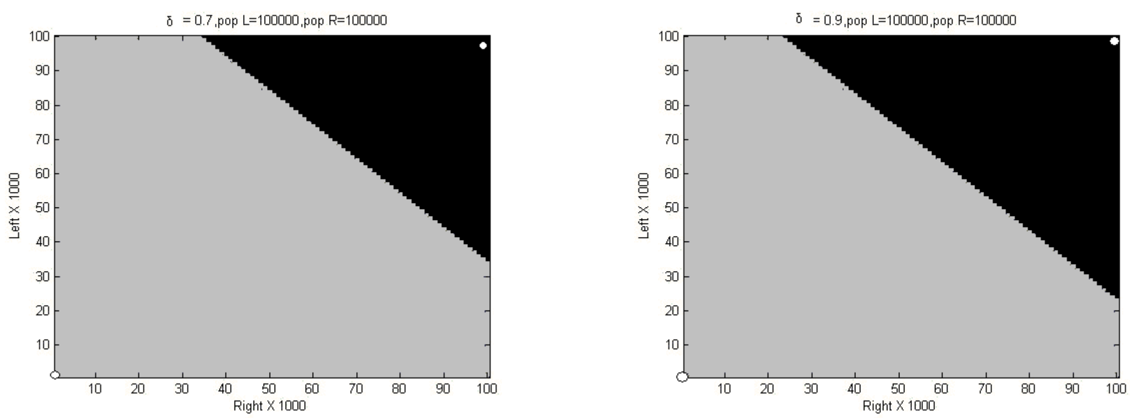


(a) Trajectories when $\delta = 0.1$. The system cycles about the low turnout equilibrium.



(b) Trajectories when $\delta = 0.5$. The system converges to the no vote equilibrium.

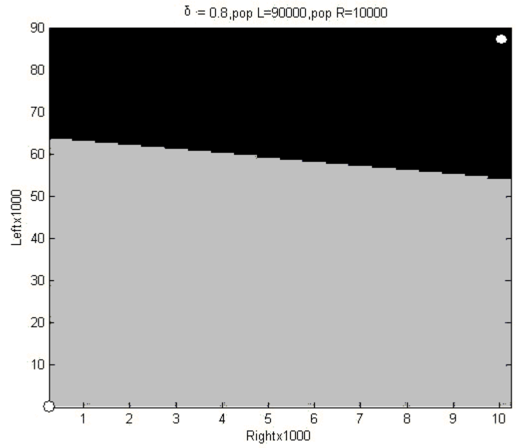
Figure 3: Basins of attraction for the asymptotically stable steady states as δ changes. The black area represents the basin of attraction for the high turnout equilibrium, the light grey area is for the no vote equilibrium. Steady states are represented by white circles.



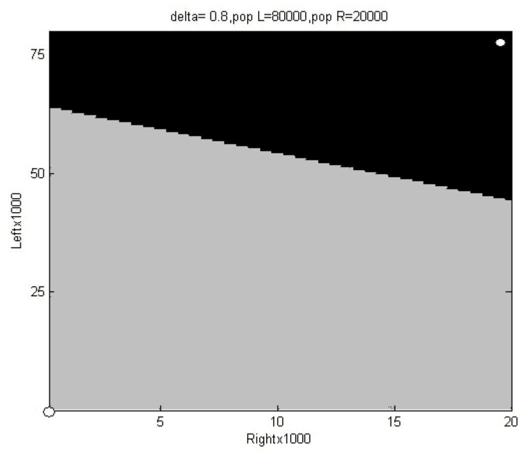
(a) Basins of attraction when $\delta = 0.7$.

(b) Basins of attraction when $\delta = 0.9$.

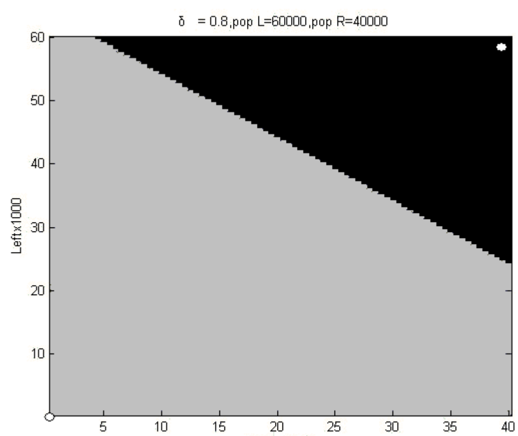
Figure 4: Basins of attractions for the asymptotically stable steady states when groups have different sizes. The black area represents the basin of attraction for the high turnout equilibrium, the light grey area is for the no vote equilibrium. Steady states are represented by white circles.



(a) Basins of attraction when $\theta = 0.9$.

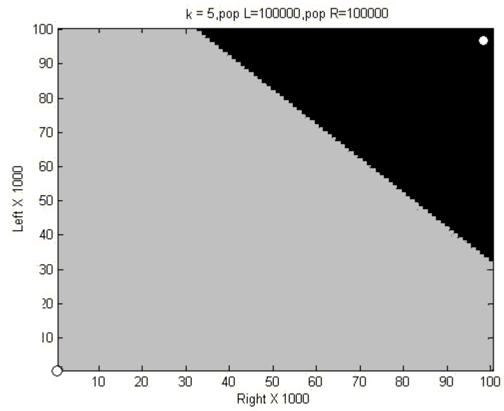


(b) Basins of attraction when $\theta = 0.8$.

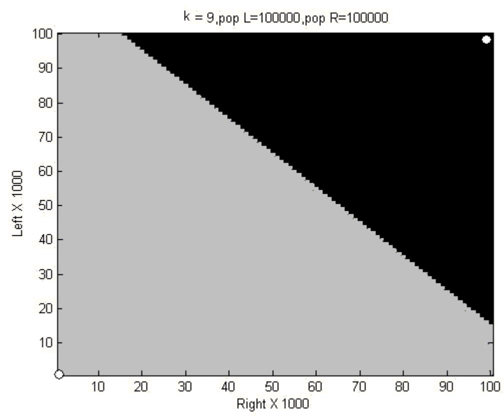


(c) Basins of attraction when $\theta = 0.4$.

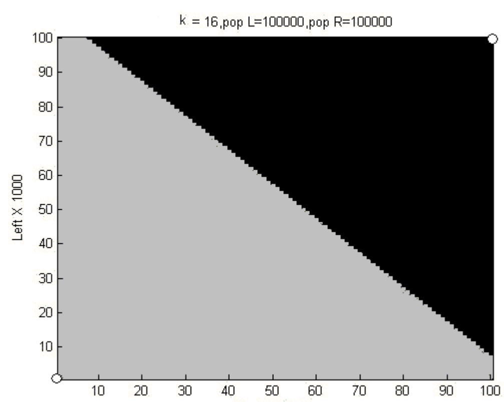
Figure 5: Basins of attractions for the asymptotically stable steady states as k changes. The black area represents the basin of attraction for the high turnout equilibrium, the light grey area is for the no vote equilibrium. Steady states are represented by white circles.



(a) Basins of attraction when $k = 5$.

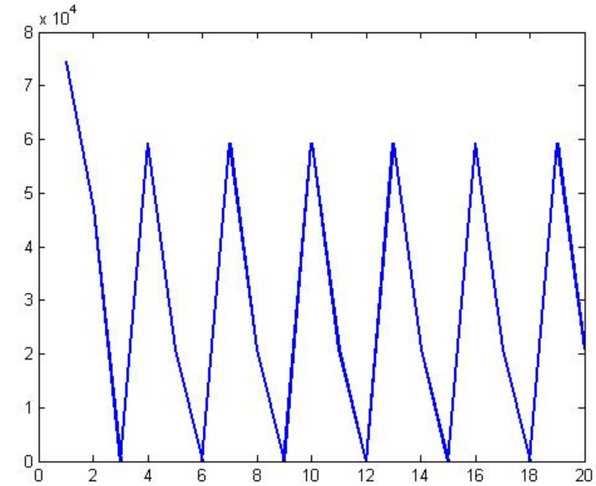


(b) Basins of attraction when $k = 9$.

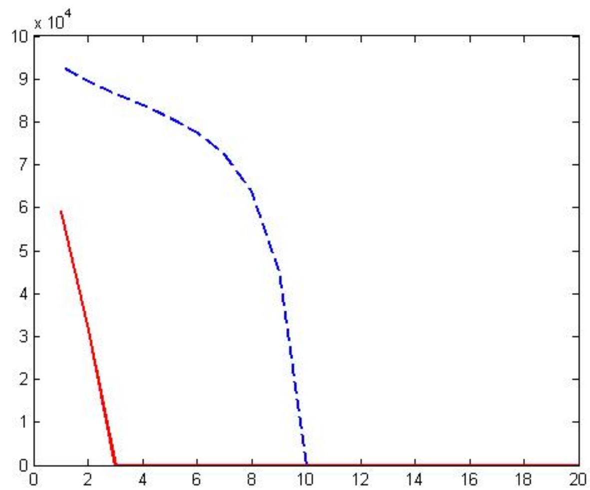


(c) Basins of attraction when $k = 16$.

Figure 6: Trajectories for low values of δ.

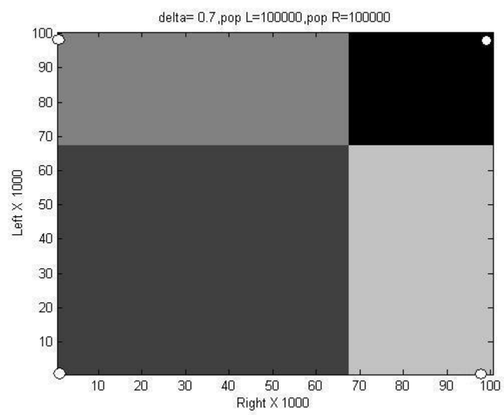


(a) Trajectories when $\delta = 0.3$. The system cycles about the low turnout equilibrium.

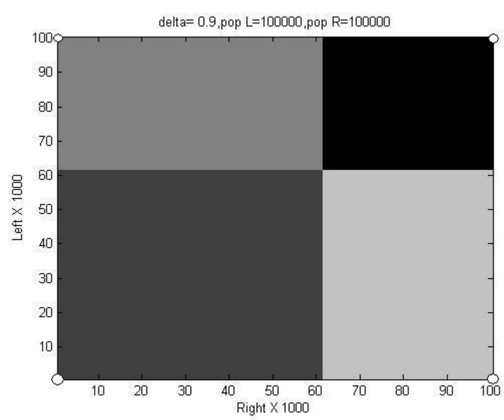


(b) Trajectories when $\delta = 0.5$. The system converges to the no vote equilibrium.

Figure 7: Basins of attraction for the asymptotically stable steady states as δ increases, and conformism depends on one's group behavior. The black area represents the basin of attraction of the high turnout equilibrium, the grey for the no vote equilibrium. The other shades of light grey represent the basin of attraction of the corner equilibria where one group abstains and the other votes with high probability.

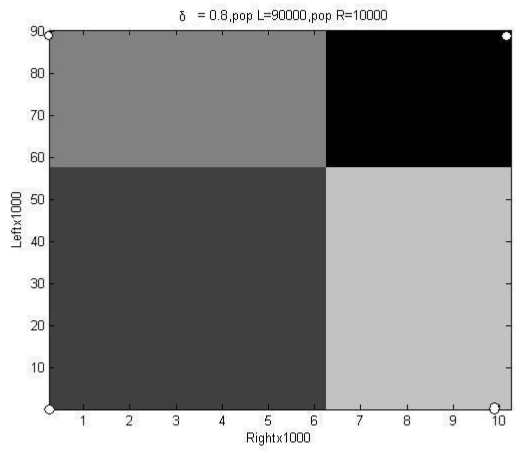


(a) Basins of attraction when $\delta = 0.7$.

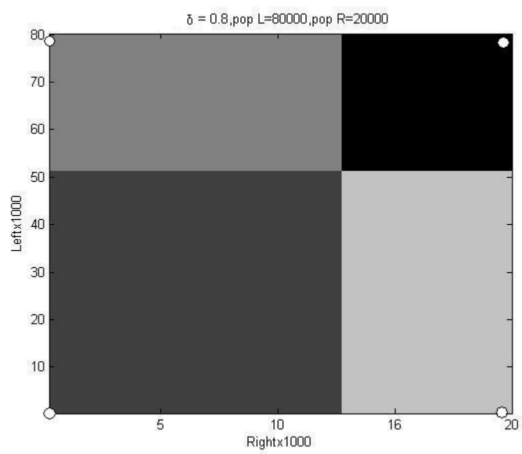


(b) Basins of attraction when $\delta = 0.9$.

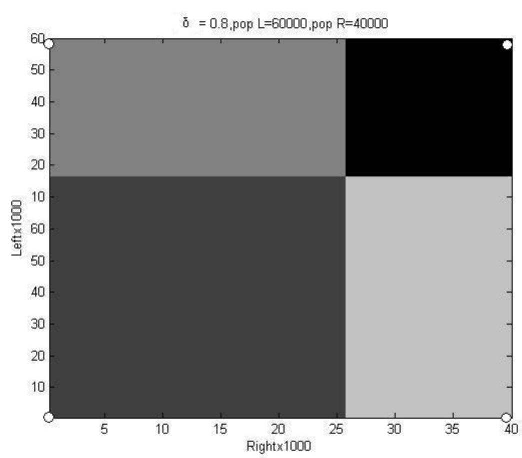
Figure 8: Basins of attraction for the asymptotically stable steady states when groups have different size. The black are represents the basin of attraction of the high turnout equilibrium, the grey for the no vote equilibrium. The other shades of light grey represent the basin of attraction of the corner equilibria where one group abstains and the other votes with high probability.



(a) Basins of attraction when $\theta = 0.9$.



(b) Basins of attraction when $\theta = 0.8$.



(c) Basins of attraction when $\theta = 0.6$.

Table 1: Size of basin of attraction and value of steady states as a function of δ, with $k = 4$, $\theta = 0.5$ and $n = 200,000$. pr(i) denotes the fraction of initial conditions that converge to an interior high turnout steady state; pr(nobody) denotes the share of the initial conditions that converge to the steady stated where nobody votes; pr(oscillating) denotes the share of initial conditions that cycle; s.s.L (s.s.R.) indicates the steady state number of voters for group L (R).

δ	pr(i)	pr(nobody)	pr(oscillating)	s.s.L=s.s.R.
0.1	0.00	0.00	1.00	-
0.2	0.00	0.00	1.00	-
0.3	0.00	0.00	1.00	-
0.4	0.00	0.00	1.00	-
0.5	0.00	1.00	0.00	-
0.6	0.15	0.85	0.00	96,008
0.7	0.22	0.78	0.00	99,047
0.8	0.26	0.74	0.00	99,834
0.9	0.30	0.70	0.00	99,990

Table 2: Size of basin of attraction and value of interior steady state as a function of n, with $\delta = 0.58$, $k = 4$, and $\theta = 0.5$.

$n_L = n_R$	pr(i)	pr(nobody)	s.s.L=s.s.R
100	0.10	0.88	97
50,000	0.12	0.88	47,357
60,000	0.12	0.88	56,820
70,000	0.12	0.88	66,282
80,000	0.13	0.87	75,744
90,000	0.12	0.88	85,205
100,000	0.13	0.87	94,666
120,000	0.12	0.88	113,587

Table 3: Size of basin of attraction and value of interior steady state as a function of δ and θ, with $n = 100,000$ and $k = 4$.

δ	θ	pr(i)	pr(nobody)	s.s.L	s.s.R
0.5	0.5	0.00	1.00	-	-
0.5	0.7	0.00	1.00	-	-
0.5	0.9	0.00	1.00	-	-
0.7	0.5	0.22	0.78	49,526	49,526
0.7	0.7	0.25	0.75	29,716	69,337
0.7	0.9	0.31	0.69	9,905	89,147
0.9	0.5	0.30	0.70	49,995	49,995
0.9	0.7	0.34	0.66	29,997	69,993
0.9	0.9	0.38	0.63	9,999	89,991

Table 4: Size of basin of attraction and value of interior steady state as a function of k, with $n = 200,000$, $\theta = 0.5$, and $\delta = 0.6$.

k	pr(i)	pr(nobody)	s.s.L = s.s.R.
4	0.15	0.85	96,008
5	0.23	0.77	98,789
6	0.28	0.71	99,563
7	0.32	0.68	99,833
8	0.35	0.65	99,935
9	0.37	0.63	99,974
10	0.38	0.62	99,990
11	0.39	0.61	99,996
12	0.41	0.59	99,998
13	0.42	0.58	99,999
14	0.42	0.58	100,000
15	0.43	0.57	100,000
16	0.44	0.56	100,000
17	0.44	0.56	100,000
18	0.45	0.55	100,000
19	0.45	0.55	100,000
20	0.45	0.55	100,000

Table 5: Size of basin of attraction and value of steady states as a function of δ, with $k = 4$, $\theta = 0.5$ and $n = 200,000$. Conformism depends on one's group's behavior. pr(cor R = 0) [pr(cor L = 0)] denotes the share of initial conditions that converge to a steady state where all group R [L] citizens abstain.

δ	pr(i)	pr(nobody)	pr(cor R=0)	pr(cor L=0)	pr(oscillating)	s.s.L. = s.s.R.
0.1	0.00	0.00	0.00	0.00	1.00	-
0.2	0.00	0.00	0.00	0.00	1.00	-
0.3	0.00	0.00	0.00	0.00	1.00	-
0.4	0.00	0.00	0.00	0.00	1.00	-
0.5	0.00	1.00	0.00	0.00	0.00	-
0.6	0.07	0.53	0.20	0.20	0.00	96,008
0.7	0.11	0.45	0.22	0.22	0.00	99,047
0.8	0.13	0.41	0.23	0.23	0.00	99,834
0.9	0.15	0.37	0.24	0.24	0.00	99,990

Table 6: Size of basin of attraction and value of interior steady state as a function of δ and θ, with $n = 100,000$ and $k = 4$.

δ	θ	pr(i)	pr(nobody)	pr(corner L=0)	pr(corner R=0)	s.s.L.	s.s.R.
0.5	0.50	0.00	1.00	0.00	0.00	-	-
0.5	0.70	0.00	1.00	0.00	0.00	-	-
0.5	0.90	0.00	1.00	0.00	0.00	-	-
0.7	0.50	0.11	0.45	0.22	0.22	49,526	49,526
0.7	0.70	0.11	0.45	0.22	0.22	29,710	69,324
0.7	0.90	0.11	0.44	0.21	0.24	9,903.37	89,130.34
0.9	0.50	0.15	0.37	0.24	0.24	49,995	49,995
0.9	0.70	0.15	0.37	0.23	0.25	29,997	69,993
0.9	0.90	0.15	0.37	0.23	0.25	9,998.99	89,990.93

Table 7: Size of basin of attraction and value of interior steady state as a function of k, with $n = 200,000$, $\theta = 0.5$, and $\delta = 0.6$.

k	pr(i)	pr(nobody)	pr(corner L=0)	pr(corner R=0)	s.s.L. = s.s.R.
4	0.07	0.53	0.20	0.20	96,008
5	0.12	0.44	0.22	0.22	98,789
6	0.14	0.38	0.24	0.24	99,563
7	0.16	0.36	0.24	0.24	99,833
8	0.18	0.34	0.24	0.24	99,935
9	0.18	0.32	0.25	0.25	99,974
10	0.19	0.31	0.25	0.25	99,990
11	0.20	0.30	0.25	0.25	99,996
12	0.20	0.30	0.25	0.25	99,998
13	0.21	0.29	0.25	0.25	99,999
14	0.21	0.29	0.25	0.25	100,000
15	0.21	0.29	0.25	0.25	100,000
16	0.22	0.28	0.25	0.25	100,000
17	0.22	0.28	0.25	0.25	100,000
18	0.22	0.28	0.25	0.25	100,000
19	0.22	0.28	0.25	0.25	100,000
20	0.22	0.28	0.25	0.25	100,000