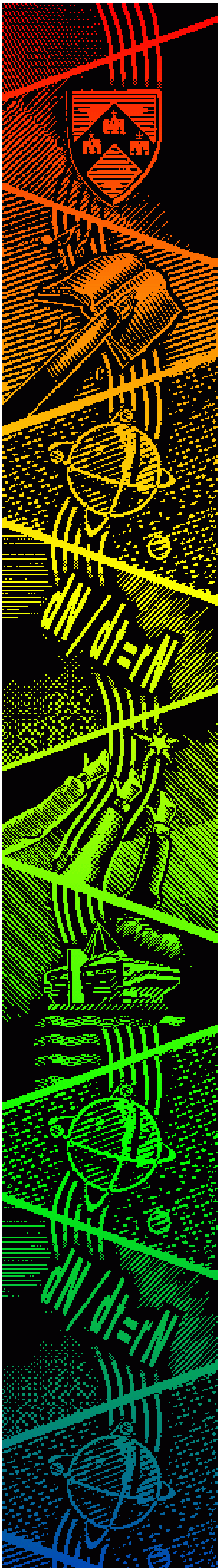

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Abstract

We study incentives for quality provision in markets where providers are motivated (semi-altruistic); prices are regulated and firms are funded by a combination of block grants and unit prices; competition is based on quality, and demand adjusts sluggishly. Health or education are sectors in which the mentioned features are the rule. We show that the presence of motivated providers makes dynamic competition tougher, resulting in higher steady-state levels of quality in the closed-loop solutions than in the benchmark open-loop solution, if the price is sufficiently high. However, this result is reversed if the price is sufficiently low (and below unit costs). Sufficiently low prices also imply that a reduction in demand sluggishness will lead to lower steady-state quality. Prices below unit costs will nevertheless be welfare optimal if the providers are sufficiently motivated.

Keywords: Quality competition; Differential games; Motivated agents.

JEL Classification: C73; H42; I18; I21; L13.

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1 Introduction

In markets for health care or education, prices are often regulated and consumer choices are mainly based on other criteria, such as travelling distance and quality. In both types of markets, competition between publicly funded providers has become an increasingly topical policy issue in recent years, as an increasing number of countries have introduced market-based reforms which give providers (hospitals or schools) incentives to compete for consumers (patients or students).[1] This has, in turn, spurred a considerable body of theoretical literature studying the nature of quality competition in regulated markets.[2] However, with very few exceptions, this literature has ignored two arguably important features of such markets, namely *motivated providers* and *sluggish demand*.

In the literature on health care supply, it has long been recognised that providers may exhibit semi-altruistic preferences.[3] For example, physicians are typically portrayed as 'imperfect agents' for their patients, trading off patient benefits against lower profits (see, e.g., McGuire, 2000). This notion has in recent years been complemented by an emerging literature on motivated agents in the broader public sector, where the assumption of 'mission oriented' workers (doctors, nurses, teachers) implies that the agents (e.g., hospitals or schools) to some extent share the objectives of the principal (government, in our examples).[4] Despite the emphasis given in the literature to the importance of motivated providers in the public sector in general, and in sectors like health care and education in particular, this aspect is largely absent in the existing literature on quality competition between publicly funded providers. A notable recent exception is Brekke, Siciliani and Straume (forthcoming), who analyse hospital competition with regulated prices and show that the presence of provider motivation can potentially reverse a previously established positive relationship between competition and quality.[5]

[1] These reforms typically include the combination of free choice of provider and activity-based payments. In health care, the original model for an activity-based payment system is the US Medicare and Medicaid programmes, where every hospital is paid a Diagnosis Related Group (DRG) tariff for every patient treated. Different variants of DRG pricing have now been introduced in a number of Western countries.

[2] See, e.g., Ma and Burgess (1993), Wolinsky (1997), Gravelle (1999), Lyon (1999), Del Rey (2001), Beitia (2003), Brekke, Nuscheler and Straume (2006, 2007), Karlsson (2007) and Matsumura and Matsushima (2007).

[3] See, e.g., Ellis and McGuire (1986), Chalkley and Malcolmson (1998), Eggleston (2005), Heyes (2005), Jack (2005) and Kaarbøe and Siciliani (2011).

[4] See, for example, Francois (2000), Murdock (2004), Glazer (2004), Besley and Ghatak (2005, 2006), Delfgaauw and Dur (2007, 2008) and Prendergast (2007). See also Francois and Vlassopoulos (2008) for an extensive review of the motivated agents literature.

[5] See Brekke, Siciliani and Straume (forthcoming) for a more extensive discussion of the assumption of motivated providers, with further references to relevant literature (including experimental evidence).

For both health care and education, quality is a key market variable. In health care, since consumers are insured against medical expenditures, the quality of care is usually a much more relevant variable than price for the patient's choice of provider. Similarly, in education markets tuition fees play a relatively minor role in most European countries (though they are on the rise in several countries like England or Italy), and the quality of the institution is typically much more important for the student's choice of school or university. However, since quality is much less readily observable than prices, it is also reasonable to assume that demand adjusts much more sluggishly to quality changes than to price changes. This effect may be particularly strong in the context of health care or education, due to consumer habits or trust in specific providers. If consumers have sluggish beliefs about quality, demand will adjust sluggishly to quality changes, implying that it takes some time before the potential demand increase due to an increase in quality is fully realised. The implications of sluggish demand for quality competition in regulated markets are analysed by Brekke et al. (forthcoming), using a differential-game framework where providers choose qualities in each period and demand adjusts sluggishly over time.[6] However, that paper follows the standard assumption in the literature on quality competition in regulated markets by assuming that providers are pure profit-maximisers.

In the present paper we combine the two above-mentioned features – *motivated providers* and *sluggish demand* – in a differential-game framework where providers are funded by a combination of block grants and unit prices, and compete on quality. We consider three different solution concepts: the open-loop, the memoryless closed-loop and the feedback closed-loop solutions. The purpose of our analysis is threefold. First, we compare the steady-state levels of quality in the different solution concepts to see whether more intense competition (closed-loop rules) actually yields higher quality levels in the steady-state solution of the game. Second, we investigate the effect on steady-state quality of increasing the degree of competition, either through lower travelling costs (increased substitutability) or less demand sluggishness. Third, we perform a welfare analysis where we derive the first-best optimal quality, both in and off steady state, and show how the optimal solution can be achieved by optimal price regulation, depending on the dynamic decision rules used by the providers. Throughout the analysis, a main concern for us is to show how the degree of provider motivation qualitatively affects our results.

[6] A more extensive discussion of the sluggish demand assumption is given in Brekke et al. (forthcoming).

When comparing the open-loop and closed-loop solutions of the game, we show that the presence of motivated providers changes the dynamic nature of quality competition and therefore makes a substantial difference. Furthermore, we show that the design of the provider payment system, i.e. the combination of block grants and unit prices, plays a crucial role in determining the outcome of dynamic quality competition. This is of policy relevance since payment systems change across countries and have changed over time. For example, hospitals in England used to be paid according to a block grant (with effectively a zero unit price) but are now paid according to an activity-based funding rule where the price varies for each type of procedure performed (similarly to the DRG – Diagnosis Related Groups – payment system within Medicare in the US). Several European countries like Italy and Spain have experimented with pricing rules where the unit price drops to 20-30% after a certain volume of activity has been reached: given high demand levels many hospitals are effectively operating at these lower unit prices. An interesting case is Norway where for several years prices have been set at a level which ranges between 40% and 60% of the average cost: the price is set every year by the government and has been set at either 40%, 55% or 60%.

In our differential game setting, the solution rules adopted by agents (i.e., the providers) capture the intensity of competition. The open-loop solution concept, used as a benchmark, implies that providers set their optimal plans at the beginning of the time considered, and then stick to them forever: in such a framework, competition is less intense as compared to behaviour rules in which the providers consider the interaction with their opponents at each point of time, like in the feedback-rule solution concept or in the memoryless closed-loop solution. We find that steady-state quality is higher when competition is more intense, if the price is sufficiently high. On the other hand, if prices are sufficiently low, and below unit costs, this result is reversed, with steady-state quality being higher in the open-loop benchmark case. Sufficiently low prices (below unit costs) also imply that lower travelling costs or less sluggish demand will reduce steady-state quality. The scope for such a negative relationship between competition intensity and steady-state quality is larger if the providers use closed-loop decision rules.

In the welfare analysis, we show that the optimal price which implements first-best quality in steady state is lower under open-loop behaviour than under closed-loop behaviour, unless providers are highly motivated. We also show that, if providers are sufficiently motivated, they are optimally funded by a combination of block grants and unit prices, where the price does not fully cover unit costs. Furthermore, the scope for the optimal price to be below unit costs is larger if the providers use closed-loop decision rules.

The remainder of the paper is organised as follows. The model is presented in Section 2. This model is then analysed for open-loop behaviour in Section 3 and closed-loop behaviour (feedback and memoryless) in Section 4. Section 5 compares the steady-state outcomes of the three different solution concepts, while the relationship between competition intensity and steady-state quality is explored in Section 6. A welfare analysis is presented in Section 7, while Section 8 concludes the paper.

2 Model

Consider a market with two providers which are located at either end of the unit line $S = [0, 1]$. Consumers are uniformly distributed on S with a total mass normalised to 1. We assume unit demand, where each consumer demands one unit of the good from her most preferred provider. The utility of a consumer who is located at $x \in S$ and chooses Provider i, located at z_i, is given by

$$u(x, z_i) = v + kq_i - \tau |x - z_i|, \tag{1}$$

where v is the gross valuation of consumption, $q_i \geq \underline{q}$ is the quality offered by Provider i, k is the marginal utility of quality and τ is the marginal disutility of travelling. We assume that $v > \tau$ in order to ensure that the market is always fully covered. The lower bound $\underline{q}$ is the minimum quality the providers are allowed to offer[7] and is, for simplicity, set equal to 0. We also normalise by setting $k = 1$, implying that τ measures the importance of travelling costs relative to quality.

The consumer who is indifferent between Provider i and Provider j is located at $\widehat{D}$, which is implicitly given by

$$v - \tau\widehat{D} + q_i = v - \tau\left(1 - \widehat{D}\right) + q_j.$$

The *potential demand* of Provider i is then given by

$$\widehat{D} = \frac{1}{2} + \frac{q_i - q_j}{2\tau}. \tag{2}$$

[7] We can interpret $\underline{q}$ as a minimum quality standard set by a regulator. If $q_i < \underline{q}$, Provider i might lose his license to operate in the market. In the context of health care, $q_i < \underline{q}$ can be interpreted as malpractice.

As in Brekke et al. (forthcoming), we assume that demand adjusts sluggishly to quality changes. Sluggish demand adjustments can be due to habitual behaviour or imperfect information about quality among consumers, implying that it takes some time before changes in provider quality is observed an acted upon in the market. Suppose that, at each point in time, only a fraction $\gamma \in (0,1)$ of consumers become aware of changes in relative quality offered by the providers. This means that, at each point in time, only a fraction γ of any changes in potential demand is realised. Defining $D(t)$ as the *actual demand* of Provider i at time t, the law of motion of actual demand is given by

$$\frac{dD(t)}{dt} := \dot{D}(t) = \gamma(\widehat{D}(t) - D(t)). \tag{3}$$

The lower is γ, the more sluggish is demand. The parameter γ is therefore an inverse measure of the degree of demand sluggishness in the market. Notice that, since total demand is inelastic, the dynamics of the demand for Provider i automatically determines the demand for Provider j, so that both providers face the same dynamic constraint, given by (3).[8]

Providers are partially motivated and maximise a weighted sum of consumers' utility and profits. The instantaneous objective function of Provider i is

$$\Omega_i\left(t\right) = T + pD(t) - C\left(D\left(t\right), q_i\left(t\right)\right) + \alpha B_i(q_i(t), D(t)), \tag{4}$$

where T is a lump-sum transfer from the regulator and p is a price per unit of output provided. The cost of provision is given by a cost function $C\left(D\left(t\right), q_i\left(t\right)\right)$, which for simplicity is assumed to take the following linear-quadratic form:

$$C\left(D, q_i\right) = cD_i + \frac{\theta}{2}q_i^2 + F, \tag{5}$$

where $c > 0$ is a constant unit cost of production, $\theta > 0$ measures the cost of quality provision

[8] The law of motion of actual demand for Provider j is

$$\begin{aligned} \frac{d\left(1 - D\left(t\right)\right)}{dt} &= \gamma\left(\left(1 - \widehat{D}\left(t\right)\right) - \left(1 - D\left(t\right)\right)\right) \\ &= \gamma\left(D\left(t\right) - \widehat{D}\left(t\right)\right), \end{aligned}$$

which is the same as

$$\frac{dD\left(t\right)}{dt} = \gamma\left(\widehat{D}\left(t\right) - D\left(t\right)\right).$$

and $F > 0$ is fixed costs. In order to ensure that the second-order conditions are satisfied in all optimisation problems considered throughout the analysis, we assume that the parameters θ and τ take values such that $\theta\tau > 1$. Provider motivation is captured by the last term in (4), where

$$B_i(q_i, D) \equiv \int_0^D (v + q_i - \tau x)\, dx \tag{6}$$

is the instantaneous aggregate gross surplus of consumers attending Provider i. The degree to which providers are motivated is measured by $\alpha \in (0,1)$.

We will compare different game-theoretic solution concepts: the *open-loop* solution on one side, and the *feedback* and *memoryless* closed-loop solutions on the other side. The open-loop solution concept assumes that each provider knows the initial state of the system but cannot observe quality (and thus potential demand) in subsequent periods; thus, each provider computes his optimal plan at the beginning of the game and then sticks to it forever. Under the closed-loop solution concepts, on the other hand, each provider can observe, and therefore react to, the value of the state variable(s) in each period of time.

Within the closed-loop concepts, we focus on two different rules: the feedback and the memoryless closed-loop rules. According to the feedback rule, the optimal choice is derived from the Bellman equation, and the choice variable of each provider at any time is related to the state variable at the current date. The memoryless closed-loop rule is based on the Hamiltonian solution technique, but – differently from the open-loop – takes explicitly into account the interaction between the rival's choice and the state variable(s) at any point in time. Both the feedback and the memoryless closed-loop solutions are strongly time consistent.[9]

In order to ensure that steady-state quality is non-negative under all solution concepts considered, we assume that the price is above a certain threshold level. More specifically, we assume that

$$p \geq \overline{p} := c - \alpha\left(v + \tau\left(\frac{1}{2} + \frac{\rho}{\gamma}\right)\right). \tag{7}$$

[9] See Mehlman (1988) or Basar and Olsder (1995; Ch. 6) for details. See also Brekke et al. (forthcoming) or Cellini and Lambertini (2004) for a comprehensive discussion of the mentioned solution concepts, with recent references to the literature on differential games.

3 Open-loop solution

Defining ρ as the preference discount rate, Provider i's maximisation problem is given by

$$\underset{q_i}{\text{Maximise}} \int_0^{+\infty} \Omega_i\left(t\right) e^{-\rho t} dt, \tag{8}$$

$$\begin{aligned} \text{subject to } \dot{D}(t) &= \gamma(\widehat{D}(t) - D(t)), && (9) \\ D(0) &= D_0 > 0, && (10) \end{aligned}$$

where q_i is the control variable. Denoting the current-value co-state variable associated with the state equation by $\mu_i(t)$, the current-value Hamiltonian is[10]

$$H_i = T + (p-c)\,D - \frac{\theta}{2}q_i^2 - F + \alpha B^i(q_i, D) + \mu_i \gamma\left(\frac{1}{2} + \frac{q_i - q_j}{2\tau} - D\right), \tag{11}$$

where μ_i is the current-value co-state variable associated with the state equation. The solution is given by

$$\begin{aligned} \frac{\partial H_i}{\partial q_i} &= \alpha D + \frac{\gamma}{2\tau}\mu_i - \theta q_i = 0, && (12) \\ \dot{\mu}_i &= \rho\mu_i - \frac{\partial H_i}{\partial D} = \mu_i\left(\rho + \gamma\right) - \left(p - c + \alpha\left(v + q_i - \tau D\right)\right), && (13) \\ \dot{D} &= \frac{\partial H_i}{\partial \mu_i} = \gamma(\frac{1}{2} + \frac{q_i - q_j}{2\tau} - D), && (14) \end{aligned}$$

and the transversality condition $\lim_{t \to +\infty} e^{-\rho t}\mu_i(t)D(t) = 0$. The second order conditions[11] are satisfied for $\theta\tau > 1$. By totally differentiating (12), and substituting (13) and (14), we obtain

$$\dot{q}_i = \frac{\alpha\gamma}{\theta}(\frac{1}{2} + \frac{q_i - q_j}{2\tau} - D) - \frac{\gamma}{2\tau\theta}\left(p - c + \alpha\left(v + q_i - \tau D\right)\right) + \left(\rho + \gamma\right)\left(q_i - \frac{\alpha}{\theta}D\right). \tag{15}$$

An analogous condition is obtained for $\dot{q}_j$.

Define as $Q := q_i - q_j$ the difference in quality between the two providers. The dynamics of the

[10] For the remainder of the analysis, the indication of time (t) is omitted to ease notation.

[11] These are given by $H_{q_i q_i} = -\theta < 0$, $H_{DD} = -\alpha\tau < 0$ and $H_{DD}H_{q_i q_i} - \left(H_{Dq_i}\right)^2 = \alpha\left(\tau\theta - \alpha\right) > 0$.

equilibrium is described by

$$\dot{Q} = \frac{1}{\theta}\left[\alpha\left(3\gamma+2\rho\right)\left(\frac{1}{2}-D\right)+\left(\theta\left(\gamma+\rho\right)+\frac{\alpha}{2\tau}\gamma\right)Q\right] \tag{16}$$

and

$$\dot{D} = \gamma\left(\frac{1}{2}+\frac{1}{2\tau}Q-D\right), \tag{17}$$

which can be represented in a phase diagram in D-Q-space (Figure 1). If the initial demand for Provider i is above one half ($D > \frac{1}{2}$), then the quality difference Q is strictly positive and converges towards zero as D converges towards the steady-state level ($\frac{1}{2}$).[12] Intuitively, if the initial demand is above one half, the marginal benefit from quality is higher for Provider i as quality affects a larger number of consumers.[13] Thus, for $D_0 > \frac{1}{2}$, Provider i has a stronger incentive than Provider j to provide quality in the initial period of the game, implying a positive initial quality difference: $Q(0) > 0$. However, on the equilibrium dynamic path, the quality difference is sufficiently small such that $\widehat{D}(Q) < D_0$, implying that Provider i's potential demand is lower than its actual demand. As demand for Provider i reduces over time, this provider's incentive to invest in quality reduces correspondingly, while the opposite is true for the rival provider. This process continues until the steady state where quality and demand differences vanish.

[Figure 1 about here]

The steady-state level of quality (where $D = \frac{1}{2}$ and $\dot{q}_i = 0$) is given by

$$q^{OL} = \frac{2\left(p-c\right)\gamma+\alpha\left(\gamma\left(2v+\tau\right)+2\tau\rho\right)}{4\theta\tau\left(\gamma+\rho\right)-2\alpha\gamma}. \tag{18}$$

It is straightforward to see that steady-state quality will be higher with a higher price or with more motivated providers. The relationship between steady-state quality and the intensity of competition (measured by τ^{-1} or γ), in the open- and closed-loop solutions, will be explored later in Section 6.

[12] The condition $\alpha < \overline{\alpha}$ also ensures stability in the saddle-path sense, i.e., there is only one admissible path which leads to the steady state. Details are available upon request.

[13] From (6), for a given level of demand, the marginal benefit of quality is

$$\frac{\partial B_i}{\partial q_i} = D$$

and thus increasing with demand.

4 Closed-loop solutions

4.1 Memoryless closed-loop rule

According to this rule, the problem (8), (9) and (10) of Provider i, with the corresponding Hamiltonian (11), must be solved by taking into account the interaction between the rival's control variable q_j and the state D at any point in time. More specifically, the first-order condition (12), the constraint (14), and transversality condition remain unchanged. However, the adjoint equation (13) has to be replaced by the following:

$$\dot{\mu}_i = \rho\mu_i - \frac{\partial H_i}{\partial D} - \frac{\partial H_i}{\partial \widehat{q}_j}\frac{\partial \widehat{q}_j}{\partial D}, \tag{19}$$

where the term $\frac{\partial H_i}{\partial \widehat{q}_j}\frac{\partial \widehat{q}_j}{\partial D}$ captures the interaction between the rival's choice and the state. Provider i explicitly considers the fact that, at any point in time, the state variable affects the rival's optimal quality, which in turns affects his own choice. Since $\frac{\partial \widehat{q}_j}{\partial D} = -\frac{\alpha}{\theta}$ (i.e., higher demand from Provider i reduces the quality of Provider j); and $\frac{\partial H_i}{\partial \widehat{q}_j} = -\frac{\gamma}{2\tau}\mu_i$ (i.e., a higher quality of Provider j reduces the utility of Provider i),[14] condition (19) can be written as

$$\dot{\mu}_i = \mu_i\left(\rho+\gamma\right) - \left(p - c + \alpha\left(v + q_i - \tau D\right)\right) - \frac{\alpha\gamma\mu_i}{2\theta\tau}. \tag{20}$$

The standard solution procedure[15] leads to

$$\dot{q}_i = \frac{\alpha\gamma}{\theta}\left(\frac{1}{2} + \frac{q_i - q_j}{2\tau} - D\right) - \frac{\gamma}{2\tau\theta}\left(p - c + \alpha(v + q_i - \tau D)\right) + \left[(\rho+\gamma) - \frac{\alpha\gamma}{2\tau\theta}\right](q_i - \frac{\alpha}{\theta}D). \tag{21}$$

In the symmetric steady state, where $D = 1/2$ and $q_i = q_j$, we obtain the steady-state level of quality in the memoryless closed-loop solution by solving for $\dot{q}_i = 0$, yielding

$$q^{ML} = \frac{2\theta\gamma\left(p-c\right) + \alpha\left(2\theta\left(\rho\tau + \gamma v\right) + \gamma(\tau\theta - \alpha)\right)}{4\theta\left(\tau\theta(\gamma+\rho) - \gamma\alpha\right)}, \tag{22}$$

which is also increasing in α and p.

[14] These properties are also due to the fact that providers' motivation is such that each of them derives utility from the satisfation of his own clients. Notice also that the property $\partial\widehat{q}_j/\partial D < 0$ is consistent with the intertemporal strategic substitutability of choice variables mentioned below.

[15] I.e., obtain q_i from (12) and differentiate it with respect to time; substitute the dymanics of D and μ_i and substitute μ_i by its expression from (12).

4.2 Feedback closed-loop rule

When solving for the feedback closed-loop solution, we restrict attention to stationary Markovian strategies, obtaining a stationary Markovian Nash equilibrium in linear strategies. The full derivation of the feedback solution is given in the Appendix. The equilibrium dynamic decision rules are found to be

$$q_i = \phi_i(D) = \frac{\alpha}{\theta} D + (\sigma_1 + \sigma_2 D)\, \frac{\gamma}{2\tau\theta} \tag{23}$$

and

$$q_j = \phi_j(D) = \frac{\alpha}{\theta}(1-D) + (\sigma_1 + \sigma_2(1-D))\, \frac{\gamma}{2\tau\theta}, \tag{24}$$

where

$$\sigma_1 = \frac{4\theta\tau^2 (p - c + \alpha v) + 2\tau\gamma\sigma_2 (\theta\tau - \alpha) - \gamma^2\sigma_2^2}{4\tau (\theta\tau (\gamma + \rho) - \alpha\gamma) - \gamma^2\sigma_2} \tag{25}$$

and

$$\sigma_2 = \frac{\tau}{3\gamma^2} \left((2\theta\tau (2\gamma + \rho) - 4\alpha\gamma) - \sqrt{(2\theta\tau (2\gamma + \rho) - 4\alpha\gamma)^2 + 12\alpha\gamma^2 (\theta\tau - \alpha)} \right) < 0. \tag{26}$$

From (23)-(24), notice that $\partial q_i / \partial (1-D) < 0$ and $\partial q_j / \partial D < 0$. Thus, according to the definition given by Jun and Vives (2004), qualities are *intertemporal strategic substitutes*. That is, the control (quality) of each player responds negatively to a positive change in the state (demand) of the other player.[16]

Applying the steady-state condition $D = 1/2$ to (23)-(24), steady-state quality in the feedback solution is

$$q^{FB} = \frac{12\theta\gamma (p - c + \alpha v) + 10\theta\alpha\tau\rho + 2\alpha\gamma (\theta\tau - \alpha) + \alpha\sqrt{(2\theta\tau (2\gamma + \rho) - 4\alpha\gamma)^2 + 12\alpha\gamma^2 (\theta\tau - \alpha)}}{2\theta \left(8\gamma (\theta\tau - \alpha) + 10\theta\tau\rho + \sqrt{(2\theta\tau (2\gamma + \rho) - 4\alpha\gamma)^2 + 12\alpha\gamma^2 (\theta\tau - \alpha)} \right)}. \tag{27}$$

As in the open-loop and memoryless closed-loop solutions, steady-state quality is increasing in p and α.

[16] From (23)-(24), it can easily be shown that $\frac{\partial q_i}{\partial (1-D)} < 0$ and $\frac{\partial q_j}{\partial D} < 0$ if $\alpha < \frac{\theta\tau(2\rho + 5\gamma)}{2\gamma}$. Our assumption of $\alpha < \overline{\alpha}$ ensures that this condition is always met.

5 Comparison of steady-state quality levels

One of our main objectives is to investigate which solution concept yields the most competitive outcome in terms of steady-state quality levels. A comparison of the previously derived steady-state quality levels yields the following result:

Proposition 1 *(i) If $\alpha = 0$ or $p = \widetilde{p}$, then $q^{ML} = q^{FB} = q^{OL}$;*

(ii) If $\alpha > 0$ and $p > \widetilde{p}$, then $q^{ML} > q^{FB} > q^{OL}$;

(iii) If $\alpha > 0$ and $p < \widetilde{p}$, then $q^{OL} > q^{FB} > q^{ML}$; where

$$\widetilde{p} := c - \alpha \left(\frac{\alpha}{2\theta} + v - \frac{\tau}{2} \right) > \overline{p}. \tag{28}$$

Proof. From (18) and (27): $q^{FB} > (<) \, q^{OL}$ if $2\gamma\Phi \left[2\theta (p - c) + \alpha \left(\alpha + 2\theta \left(v - \frac{\tau}{2} \right) \right) \right] > (<) \, 0$ where

$$\Phi := 2 \left(2\theta\tau\gamma + \theta\tau\rho + \alpha\gamma \right) - \sqrt{\left(2 \left(2\theta\tau\gamma + \theta\tau\rho + \alpha\gamma \right) \right)^2 - 12\theta\alpha\tau\gamma \left(3\gamma + 2\rho \right)} > 0.$$

From (18) and (22): $q^{ML} > (<) \, q^{OL}$ if $2\gamma^2\alpha \left[2\theta (p - c) + \alpha \left(\alpha + 2\theta \left(v - \frac{\tau}{2} \right) \right) \right] > (<) \, 0$. Notice that $\Phi = 0$ if $\alpha = 0$, implying that $q^{FB} = q^{OL} = q^{ML}$ if $\alpha = 0$. For $\alpha > 0$, the comparison between q^{FB} and q^{OL}, and the comparison between q^{ML} and q^{OL}, both depend on the sign of $2\theta (p - c) + \alpha \left(\alpha + 2\theta \left(v - \frac{\tau}{2} \right) \right)$. From (27) and (22): $q^{ML} > (<) \, q^{FB}$ if

$$\frac{\gamma \left(\sqrt{\Psi} - \left(2\gamma \left(\theta\tau - \alpha \right) + \theta\tau\rho \right) \right) \left[2\theta (p - c) + \alpha \left(\alpha + 2\theta \left(v - \frac{\tau}{2} \right) \right) \right]}{4\theta \left(\gamma \left(\theta\tau - \alpha \right) + \theta\tau\rho \right) \left(4\gamma \left(\theta\tau - \alpha \right) + 5\theta\tau\rho + \sqrt{\Psi} \right)} > (<) \, 0,$$

where $\Psi := \left(\theta\tau \left(2\gamma + \rho \right) \right)^2 - \alpha\gamma \left(\theta\tau \left(5\gamma + 4\rho \right) - \alpha\gamma \right) > 0$. Notice that $\sqrt{\Psi} - \left(2\gamma \left(\theta\tau - \alpha \right) + \theta\tau\rho \right) > 0$ since $\Psi - \left(2\gamma \left(\theta\tau - \alpha \right) + \theta\tau\rho \right)^2 = 3\alpha\gamma^2 \left(\theta\tau - \alpha \right) > 0$. Thus, the comparison between q^{ML} and q^{FB} also depend on the sign of $2\theta (p - c) + \alpha \left(\alpha + 2\theta \left(v - \frac{\tau}{2} \right) \right)$. The sign of this expression is determined by the critical value of the price given by $\widetilde{p}$, which then determines the quality ranking stated in the Proposition. Finally, notice that $\widetilde{p} - \overline{p} = \frac{\alpha}{2\theta\gamma} \left(\gamma \left(2\theta\tau - \alpha \right) + 2\theta\tau\rho \right) > 0$. ■

The first part of the proposition confirms a result already provided by Brekke et al. (forthcoming). Under pure profit-maximising behaviour and with production costs that are linear in output, there is an absence of strategic interaction that yields the stated coincidence results.[17] The second and third parts of the proposition present results that, to the best of our knowledge, are new to the literature. In regulated markets where motivated providers compete dynamically on quality, steady-state quality is highest in the memoryless closed-loop solution and lowest in the open-loop solution, if the price is above a certain threshold level. Otherwise, if the price is sufficiently low, steady-state quality is highest in the open-loop solution and lowest in the memoryless closed-loop solution. Notice that the threshold value of p is such that the price mark-up on marginal cost is negative, i.e., $\widetilde{p} < c$.

The intuition for these new results is found by noticing how the presence of motivated providers affects the strategic nature of quality competition. Suppose that Provider i increases its quality. This reduces the number of consumers patronising Provider j and therefore also reduces the marginal benefit of quality investments for altruistic reasons (i.e., $\partial B_j / \partial q_j$ is reduced). Consequently, Provider j responds by reducing its quality. In other words, qualities are *strategic substitutes* at each point in time.

If the price is sufficiently high, $p > \widetilde{p}$, this strategic substitutability makes dynamic competition tougher in the closed-loop solutions (under memoryless or feedback rules), where players can set their quality choices according to the evolution of demand and taking into account the strategic interaction at each instance of time. By increasing its quality today, Provider i can provoke a quality reduction from its competitor tomorrow (and vice versa). Due to this strategic nature of the dynamic competition, and due to the lack of any form of commitment over time, steady-state quality turns out to be higher in the closed-loop solutions as compared to the open-loop case.

However, this conclusion holds only if each provider has a sufficiently strong incentive to increase demand. The lower the price is, the lower is the incentive to attract more consumers for profit-oriented reasons. If the price is sufficiently low, such that the providers face a negative price-cost margin ($p < c$), the incentive to attract more consumers for altruistic reasons is counteracted by incentives to dampen demand for financial reasons. Indeed, if the price is sufficiently below marginal costs ($p < \widetilde{p} < c$), implying that the incentives to compete for consumers are relatively weak, a more collusive outcome with lower steady-state quality levels, is achieved under closed-loop rules. Notice that provider motivation ($\alpha > 0$) still ensures that quality levels are positive even if providers face

[17] Brekke et al. (forthcoming) only consider the open-loop and the feedback closed-loop solution. Here we confirm that the coincidence result also applies to the memoryless closed-loop solution.

negative price-cost margins. With purely profit-oriented providers, interior solutions would not exist if $p < c$: for example, if hospitals maximise profits and are paid through block grants they would have no incentives to provide quality above the minimum level, which seems an obvious theoretical conclusion, but an implausible scenario for the real world.

By drawing on the analysis of Brekke et al. (forthcoming), we can also say something about the robustness of Proposition 1 with respect to alternative cost assumptions. In our analysis we have, for simplicity, assumed constant marginal production costs. In a similar modelling framework, Brekke et al. (forthcoming) show that steady-state quality is higher in the open-loop solution than in the feedback closed-loop solution with increasing marginal production costs and profit-maximising providers. Thus, allowing for cost convexity is likely to increase the threshold level of provider motivation above which steady-state quality is higher under closed-loop rules. More specifically, our conjecture is that, with convex provision costs, quality will be higher under closed-loop decision rules if the degree of provider motivation is sufficiently high relative to the degree of cost convexity.

6 Competition intensity and steady-state quality

Let us now see how steady-state quality depends on the intensity of competition under the different solution concepts. In our model there are two reasonable measures of competition intensity; the degree of competition increases if travelling costs become lower (a decrease in τ) and/or if demand becomes less sluggish (an increase in γ). Notice that both of these measures can be influenced by policy. For example, by making publicly available quality indicators measuring the performance of, e.g., hospitals or schools, the government can increase consumer awareness about quality and thereby reducing demand sluggishness.[18]

For tractability reasons, we restrict attention to the open-loop and the memoryless closed-loop solutions. From Proposition 1 we know that these two rules always yield the highest and lowest steady-state quality levels. From (18) and (22), we derive the following results:

Proposition 2 *Consider the steady-state quality in the open-loop and memoryless closed-loop solu-*

[18] An example of such policy measures is the publication of hospital and school 'League Tables' in the UK.

tions, respectively. The effects of lower travelling costs are given by

$$-\frac{\partial q^{OL}}{\partial \tau}=\gamma\left(\frac{\alpha\left(\alpha\left(\gamma+2\rho\right)+4v\theta\left(\gamma+\rho\right)\right)+4\theta\left(\gamma+\rho\right)\left(p-c\right)}{2\left(\gamma\left(2\theta\tau-\alpha\right)+2\theta\tau\rho\right)^{2}}\right)>(<)\,0 \quad if \quad p>(<)\,p_{\tau}^{OL}$$

and

$$-\frac{\partial q^{ML}}{\partial \tau}=\gamma\left(\frac{2\theta\left(\gamma+\rho\right)\left(p-c+v\alpha\right)+\alpha^{2}\rho}{4\left(\gamma\left(\theta\tau-\alpha\right)+\theta\tau\rho\right)^{2}}\right)>(<)\,0 \quad if \quad p>(<)\,p_{\gamma}^{ML},$$

where

$$p_{\tau}^{ML}:=c-\alpha\left(v+\frac{\alpha\rho}{2\theta\left(\gamma+\rho\right)}\right)>p_{\tau}^{OL}:=c-\alpha\left(v+\frac{\alpha\left(\gamma+2\rho\right)}{4\theta\left(\gamma+\rho\right)}\right)>\overline{p},$$

while the effects of less sluggish demand are given by

$$\frac{\partial q^{OL}}{\partial \gamma}=\tau\rho\left(\frac{2\theta\left(p-c\right)+\alpha\left(\alpha+\theta\left(2v-\tau\right)\right)}{\left(\gamma\left(2\theta\tau-\alpha\right)+2\theta\tau\rho\right)^{2}}\right)>(<)\,0 \quad if \quad p>(<)\,p_{\gamma}$$

and

$$\frac{\partial q^{ML}}{\partial \gamma}=\tau\rho\left(\frac{2\theta\left(p+v\alpha\right)-\alpha\left(\theta\tau-\alpha\right)}{4\left(\gamma\left(\theta\tau-\alpha\right)+\theta\tau\rho\right)^{2}}\right)>(<)\,0 \quad if \quad p>(<)\,p_{\gamma},$$

where

$$p_{\gamma}:=c-\alpha\left(v-\frac{1}{2}\tau+\frac{\alpha}{2\theta}\right)>p_{\tau}^{ML}.$$

If providers face a positive price-cost margin ($p>c$), there is an unambiguously positive relationship between competition intensity and steady-state quality. Lower travelling costs or less sluggish demand leads to a higher steady-state quality level both under open-loop and closed-loop rules. This is what we would expect, as lower travelling costs and less sluggish demand make actual demand more quality-elastic and therefore stimulate each provider's incentives to increase quality.

However, the relationship between competition intensity and steady-state quality changes if the providers' face a sufficiently low price (that is lower than marginal production costs). If the price-cost margin is negative, the providers' optimal quality choices result from two counteracting incentives, as previously discussed. The providers have an incentive to increase quality for altruistic reasons ($\alpha>0$) but they also have an incentive to reduce quality for profit-oriented reasons (since $p<c$). More quality-elastic demand, due to lower travelling costs or less sluggish demand, will strengthen both these incentives, but the profit incentive will increase more if the price is sufficiently low,

implying that steady-state quality will decrease.[19]

This result has potentially important policy implications. If policy makers try to stimulate quality competition by publishing quality indicators in order to make demand less sluggish, this will have the intended effect only if the providers receive a sufficiently high unit price. Otherwise, if the price is too low, policy measures to stimulate quality competition by reducing demand sluggishness may be counterproductive.

The scope for a negative relationship between competition intensity and steady-state quality can be further explored by comparing the different threshold values of p reported in Proposition 2:

Corollary 1 *Since $p_\gamma > p_\tau^{ML} > p_\tau^{OL}$, a negative relationship between competition intensity and steady-state quality is more likely (i.e., applies for a larger set of parameter values) if we consider a reduction in demand sluggishness rather than travelling costs, and it is more likely if the providers use closed-loop rather than open-loop rules.*

7 Welfare

In this section we derive first-best quality, in and off steady state, and analyse how first-best quality can be achieved by optimal price regulation. When analysing optimal price regulation, we restrict attention to the open-loop and memoryless closed-loop solutions.

We define social welfare as the sum of producers' and consumers' surplus net of third-party payments. Under the assumption that the providers face a limited liability constraint, the transfer T will be set such that each provider breaks even. Social welfare at time t is then given by

$$\begin{aligned} W(t) &= (1+\beta\alpha)\left(\int_0^{D(t)} (v + q_i(t) - \tau x)\, dx + \int_{D(t)}^1 (v + q_j(t) - \tau(1-x))\, dx\right) \\ &\quad - (1+\lambda)\left(c + \frac{\theta}{2}\left(q_i(t)^2 + q_j(t)^2\right) + 2F\right), \end{aligned} \tag{29}$$

where $\lambda > 0$ is the opportunity cost of public funds. A non-trivial issue when defining social welfare in the presence of motivated providers is whether the altruistic part of provider preferences should be included (implying that consumer utility is 'double-counted') in the welfare function or not. By

[19] These results resemble some of those reported in Brekke, Siciliani and Straume (forthcoming) who find, in a static setting, that the relationship between competition and quality is ambiguous in the presence of motivated providers.

including a binary parameter $\beta = \{0, 1\}$, we make the welfare expression sufficiently flexible to incorporate both alternatives, double-counting ($\beta = 1$) and no double-counting ($\beta = 0$).

7.1 Steady-state analysis

We start out by deriving the first-best optimal quality in steady state, where each provider serve half of the market. Maximising (29) with respect to qualities yields the first-best level of quality in steady state:

$$q_i = q_j = q^* = \frac{1 + \alpha\beta}{2\theta(1 + \lambda)}. \tag{30}$$

Intuitively, higher costs of quality provision (θ) or higher opportunity costs of public funds (λ) will reduce the first-best level of quality. On the other hand, allowing for double-counting of consumer utility ($\beta = 1$) increases the first-best quality, with the effect of double-counting being stronger the more motived providers are.

What is the optimal price that ensures that steady-state quality will be at the first-best level? This depends on the decision rules used by the providers. Under *open-loop* rules, the optimal price, p^{OL}, is implicitly given by $q^{OL}\left(p^{OL}\right) = q^*$, yielding

$$p^{OL} = c + \frac{(1 + \alpha\beta)(2\theta\tau(\gamma + \rho) - \alpha\gamma) - \theta\alpha(1 + \lambda)(2(\upsilon\gamma + \tau\rho) + \tau\gamma)}{2\gamma\theta(1 + \lambda)}. \tag{31}$$

Under *memoryless closed-loop* rules, the optimal price, p^{ML}, is implicitly given by $q^{ML}\left(p^{ML}\right) = q^*$, yielding

$$p^{ML} = c + \frac{2(1 + \alpha\beta)(\theta\tau(\gamma + \rho) - \alpha\gamma) - \alpha(1 + \lambda)(2\theta(\upsilon\gamma + \tau\rho) + \gamma(\theta\tau - \alpha))}{2\gamma\theta(1 + \lambda)}. \tag{32}$$

A comparison of these two optimal prices yields the following result

Proposition 3 *The optimal price is higher (lower) under the open-loop than under the memoryless closed-loop solution if* $\alpha < (>)\, \widehat{\alpha} := \frac{1}{1-\beta+\lambda}$.

Notice that the critical value $\widehat{\alpha}$ is such that $\min\left\{p^{OL}, p^{ML}\right\} > \widetilde{p}$ for $\alpha < \widehat{\alpha}$ and $\max\left\{p^{OL}, p^{ML}\right\} < \widetilde{p}$ for $\alpha > \widehat{\alpha}$. Thus, if the degree of provider motivation is sufficiently low ($\alpha < \widehat{\alpha}$) and the providers use memoryless closed-loop rules, first-best quality is achieved by setting a price $p^{ML} > \widetilde{p}$. At this price, steady-state quality is lower if the players instead use open-loop rules, i.e., $q^{OL}\left(p^{ML}\right) < q^{ML}\left(p^{ML}\right) = q^*$. Therefore, a higher price ($p^{OL} > p^{ML}$) is needed to induce first-best quality in the open-loop solution. However, this result is reversed if provider motivation is sufficiently strong ($\alpha > \widehat{\alpha}$). In this case, first-best quality is induced in the open-loop solution by setting a price $p^{OL} < \widetilde{p}$. At this price, steady-state quality is now lower when the providers use memoryless closed-loop rules and the optimal price under the closed-loop solution is therefore higher. The latter case, where $p^{ML} > p^{OL}$, is more likely to occur in the absence of double-counting ($\beta = 0$) and when the opportunity cost of public funds is high, which imply that the first-best quality level is relatively low. Notice that a positive opportunity cost of public funds ($\lambda > 0$) is a necessary condition for $p^{ML} > p^{OL}$, since $\widehat{\alpha} \geq 1$ for $\lambda = 0$.

A further characterisation of the optimal price under the different solution concepts is given by the following result:

Proposition 4 *Suppose that social welfare is defined such that $\beta = 0$. In this case, there exist two threshold values $\widetilde{\alpha}_0$ and $\widetilde{\alpha}_1$, where $\widetilde{\alpha}_0 < \widetilde{\alpha}_1 < 1$, such that the optimal price that implements first-best quality in steady state implies:*

(i) a positive price-cost margin under both the open-loop and the memoryless closed-loop solution if $\alpha < \widetilde{\alpha}_0$;

(ii) a negative price-cost margin under the memoryless closed loop solution and a positive price-cost margin under the open-loop solution if $\widetilde{\alpha}_0 < \alpha < \widetilde{\alpha}_1$;

(iii) a negative price-cost margin under both the open-loop and the memoryless closed-loop solutions if $\alpha > \widetilde{\alpha}_1$.

Proof. From (31) we have that $p^{OL}\,(>) < c$ if

$$\alpha\,(<) > \widetilde{\alpha}_1 := \frac{2\theta\tau\left(\gamma+\rho\right)}{\gamma+\theta\left(1+\lambda\right)\left(\gamma\left(2v+\tau\right)+2\tau\rho\right)} < 1.$$

It is straightforward to confirm that $\widetilde{\alpha}_1 < \widehat{\alpha}$. Thus, for $\alpha = \widetilde{\alpha}_1$, the optimal price under the memoryless closed-loop solution is even lower than under the open-loop solution; i.e., $p^{ML}\left(\widetilde{\alpha}_1\right) < p^{OL}\left(\widetilde{\alpha}_1\right) = c$. Since $p^{ML} > c$ for $\alpha = 0$ and p^{ML} is monotonically decreasing in α, there exists a threshold value $\widetilde{\alpha}_0 \in (0, \widetilde{\alpha}_1)$ such that $p^{ML} < c$ if $\alpha > \widetilde{\alpha}_0$. ■

This result has interesting policy implications. If providers are sufficiently motivated, they should be optimally financed by a combination of lump-sum transfers (T) and unit prices (p), where the price does not fully cover the unit costs. Furthermore, the scope for an optimal price below unit costs is larger if the providers use memoryless closed-loop rules. These conclusions hold if social welfare is defined such that consumer utility is not double-counted (i.e., if $\beta = 0$). If we allow for double-counting ($\beta = 1$), the optimal price would be higher under both solution concepts (in order to induce the higher first-best quality level). This would reduce the scope for optimal prices below unit costs.

It is also worth noticing that $\widetilde{\alpha}_1$ is decreasing in γ. Thus, less sluggish demand increases the scope for an optimal price below unit costs under both solution concepts.

7.2 Dynamic welfare analysis

We now extend the welfare analysis to consider optimal price regulation off the steady state. In order to derive the first-best quality benchmark, suppose that the regulator can directly set the providers' quality levels at each point in time. The first-best dynamic quality paths are then given by the solution to the following problem:

$$\underset{q_i, q_j}{\text{Maximise}} \int_0^{+\infty} W(t)\, e^{-\rho t} dt, \tag{33}$$

subject to

$$\frac{dD(t)}{dt} \equiv \dot{D}(t) = \gamma(\widehat{D}(t) - D(t)), \tag{34}$$

$$D(0) = D_0 > 0, \tag{35}$$

where $W(t)$ is given by (29). Let $\mu(t)$ be the current value co-state variable associated with the state equation. The current-value Hamiltonian is[20]

$$\begin{aligned} H = {} & (1+\beta\alpha)\left[\int_0^D (v + q_i - \tau x)\, dx + \int_D^1 (v + q_j - \tau(1-x))\, dx\right] \\ & - (1+\lambda)\left[c + \frac{\theta}{2}\left(q_i^2 + q_j^2\right) + 2F\right] + \mu\gamma\left(\frac{1}{2} + \frac{q_i - q_j}{2\tau} - D\right). \end{aligned} \tag{36}$$

[20] To ease notation, we drop the time index t.

The solution is given by (a) $\partial H/\partial q_i = 0$, $\partial H/\partial q_j = 0$, (b) $\dot{\mu} = \rho\mu - \partial H/\partial D$, (c) $\dot{D} = \partial H/\partial \mu$, or more extensively:

$$\begin{aligned}
(1+\beta\alpha)\,D + \frac{\gamma}{2\tau}\mu &= (1+\lambda)\,\theta q_i, & (37)\\
(1+\beta\alpha)\,(1-D) - \frac{\gamma}{2\tau}\mu &= (1+\lambda)\,\theta q_j, & (38)\\
\dot{\mu} &= \mu\,(\rho+\gamma) - (1+\beta\alpha)\left[q_i - q_j + \tau\,(1-2D)\right], & (39)\\
\dot{D} &= \gamma(\frac{1}{2} + \frac{q_i - q_j}{2\tau} - D), & (40)
\end{aligned}$$

to be considered along with the transversality condition $\lim_{t\to+\infty} e^{-\rho t}\mu(t)D(t) = 0$. The above conditions are also sufficient if H is concave in (q_i, q_j, D), which is the case for $\theta\tau > 1$.[21] By totally differentiating (37), and using (39)-(40), the optimal solution is provided by:

$$\begin{aligned}
\dot{q_i}^* &= (\rho+\gamma)\left(q_i^* - \frac{1+\beta\alpha}{(1+\lambda)\,\theta}D^*\right), & (41)\\
\dot{q_j}^* &= (\rho+\gamma)\left(q_j^* - \frac{1+\beta\alpha}{(1+\lambda)\,\theta}(1-D^*)\right), & (42)\\
\dot{D}^* &= \gamma(\frac{1}{2} + \frac{q_i^* - q_j^*}{2\tau} - D^*). & (43)
\end{aligned}$$

Setting $\dot{q_i}^* = 0$ and differentiating yields

$$\frac{\partial D^*}{\partial q_i^*}|_{\dot{q}_i^*=0} = \theta > 0. \quad (44)$$

From (37) and (38), it also follows that $q_j^* = \frac{1+\beta\alpha}{(1+\lambda)\theta} - q_i^*$, which means that we can re-write (43) as

$$\dot{D}^* = \gamma\left[\frac{1}{2} - D^* + \frac{1}{\tau}\left(q_i^* - \frac{1+\beta\alpha}{2\,(1+\lambda)\,\theta}\right)\right]. \quad (45)$$

Setting $\dot{D}^* = 0$ and differentiating yields $\frac{\partial D^*}{\partial q_i^*}|_{\dot{D}=0} = \frac{1}{\tau} > 0$. The first-best solution is described in Figure 2. Similarly to the equilibrium paths under the open-loop or memory-less closed-loop

[21] H is concave in (q_i, q_j, D) if the Hessian matrix

$$\begin{bmatrix} H_{q_iq_i} & H_{q_iq_j} & H_{q_iD} \\ H_{q_jq_i} & H_{q_jq_j} & H_{q_jD} \\ H_{Dq_i} & H_{Dq_j} & H_{DD} \end{bmatrix} = \begin{bmatrix} -\theta & 0 & 1 \\ 0 & -\theta & -1 \\ 1 & -1 & -2\tau \end{bmatrix}$$

is negative semidefinite. This is true if $\theta\tau > 1$.

solutions, quality and demand move together on the socially optimal path. For $D_0 > \frac{1}{2}$, the quality for Provider i is higher than Provider j. Intuitively, if initially Provider i has more than half of the market, the marginal benefit from quality is higher for Provider i, as quality affects a larger number of consumers. As demand for Provider i reduces over time, the optimal quality for Provider i (j) reduces (increases). This process continues until the steady state where quality and demand differences vanish.

[Figure 2 about here]

Let us now see how the regulator can implement the first-best quality paths off steady state by choosing provider-specific and time-varying prices. We want to investigate how the optimal prices depend on whether quality and demand are off the steady state or not, and how these prices differ according to whether the providers use open-loop or closed-loop rules.

Comparing (41) with (15), the optimal price for Provider i under the *open-loop* scenario is

$$p_i^{OL} = c + \frac{2\tau}{\gamma}(\rho+\gamma)\left(\frac{1+\beta\alpha}{1+\lambda} - \alpha\right) D^* - \alpha\left(v + q_i^* - \tau D^*\right) + 2\tau\alpha\left(\frac{1}{2} + \frac{q_i^* - q_j^*}{2\tau} - D^*\right). \qquad (46)$$

Recall that p^{OL}, given by (31), is the optimal steady-state price when the providers use open-loop rules. Whether the optimal off-steady-state price for Provider i is higher or lower than the steady-state one depends on the sign of the following expression:

$$\begin{aligned} p_i^{OL} - p^{OL} \quad = \quad & \frac{2\tau}{\gamma}(\rho+\gamma)\left(\frac{1+\beta\alpha}{1+\lambda} - \alpha\right)\left(D^* - \frac{1}{2}\right) - \alpha\left(\left(q_i^* - \frac{1}{2\theta}\right) - \tau\left(D^* - \frac{1}{2}\right)\right) \\ & + 2\alpha\tau\left(\frac{1}{2} + \frac{q_i^* - q_j^*}{2\tau} - D^*\right). \end{aligned} \qquad (47)$$

Suppose that $D_0 > 1/2$ and $\frac{1+\beta\alpha}{1+\lambda} - \alpha > 0$, so that first-best quality and initial demand are above the steady-state levels for Provider i. There are three terms to account for. The first term takes into account the fact that the provider does provide higher quality when the demand is high, but not as much as the regulator would like. The regulator needs therefore to increase the price. The second term takes into account the extra utility for the provider from treating the marginal consumer. There are two counteracting effects: on one hand quality is higher than in the first best but transportation costs are also higher. Depending on the net effect (i.e., whether the utility of the marginal consumer is higher or lower off steady state or in the steady state), the price tends to be higher or lower compared to the steady state. The third term is negative as demand is decreasing over time ($\dot{D}^* < 0$) and therefore the provider has a lower incentive to provide quality, which needs to be compensated with a higher price.

Comparing (41) with (21), the optimal price for Provider i under the *memoryless closed-loop* scenario is given by

$$p_i^{ML} = c + \frac{2\tau}{\gamma}(\rho+\gamma)\left(\frac{1+\beta\alpha}{1+\lambda} - \alpha\right)D^* - \alpha\left(v + q_i^* - \tau D^*\right) + 2\tau\alpha\left(\frac{1}{2} + \frac{q_i^* - q_j^*}{2\tau} - D^*\right) - \alpha\left(q_i^* - \frac{\alpha}{\theta}D^*\right). \tag{48}$$

Using (46) and (48), the optimal dynamic prices under the open-loop and memoryless closed-loop scenarios compare as follows:

$$p_i^{OL}(t) - p_i^{ML}(t) = \alpha\left(q_i^*(t) - \frac{\alpha}{\theta}D^*(t)\right). \tag{49}$$

Using the optimal steady-state prices under the two solution concepts, this expression can be rewritten as

$$p_i^{OL}(t) - p_i^{ML}(t) = \left(p^{OL} - p^{ML}\right) + \alpha\left[\frac{1}{\theta}\left(\frac{1+\beta\alpha}{1+\lambda} - \alpha\right)\left(D^* - \frac{1}{2}\right) + \left(q_i^* - \frac{1+\beta\alpha}{(1+\lambda)\,\theta}D^*\right)\right]. \tag{50}$$

Once more, suppose that $\frac{1+\beta\alpha}{1+\lambda} - \alpha > 0$ and $D_0 > 1/2$, implying that first-best quality and initial demand are above the steady-state levels for Provider i. The first term in (50) gives the difference in optimal steady-state prices between the open-loop and memoryless closed-loop case. This difference is positive for $\frac{1+\beta\alpha}{1+\lambda} - \alpha > 0$ (cf. Proposition 4). The second term is also positive by the assumption $D_0 > 1/2$. The third term is always negative since $\dot{q_i}^* = (\rho+\gamma)\left(q_i^* - \frac{1+\beta\alpha}{(1+\lambda)\theta}D^*\right) < 0$, as we can see from the phase diagram in Figure 2. Therefore, the difference in optimal prices between the open-loop and memoryless closed-loop cases can be larger or smaller off the steady state.[22] An interesting special case is when $\beta = 0$ (no 'double-counting' of consumer utility), so that q_i^* and D^* do not depend on altruism. In this case, when α is sufficiently high the second term is small, which suggests that the price difference will be smaller off the steady-state (as the third term is negative) and then increase as the system converges to the steady state. This is most clearly seen by considering the limit

[22] Notice that in the steady state the second and third terms are equal to zero.

case $\alpha = \frac{1+\beta\alpha}{1+\lambda}$, where $p^{OL} = p^{ML}$ while $p_i^{OL}(t) < p_i^{ML}(t)$ off steady state. If α is marginally larger than $\frac{1+\beta\alpha}{1+\lambda}$, the optimal price for Provider i is higher under open-loop than under closed-loop rules in steady state ($p^{OL} > p^{ML}$), while the opposite is true off steady state ($p_i^{OL}(t=0) < p_i^{ML}(t=0)$).

8 Concluding remarks

In this paper we have analysed quality competition between publicly funded providers in markets with sluggish demand, the prime applications of our analysis being health care and education (hospital or school competition). We have shown that, in such markets, the presence of provider motivation makes a crucial difference for the dynamic nature of quality competition. In contrast to previous results in the literature, we have shown that steady-state quality is higher under closed-loop rules (when competition is more intense) than under open-loop rules, if the providers face sufficiently high unit prices. Any price in excess of unit costs is sufficient to produce this result. However, the result is reversed if the price is sufficiently below unit costs). Interestingly, we have shown that the ranking of steady-state qualities in the three different solution concepts considered depends on a single threshold value of the price. An implication of this is that both the highest and the lowest steady-state quality levels are always to be found either in the open-loop or in the memoryless closed-loop solutions. In contrast, the feedback closed-loop solution always provides intermediate levels of steady-state quality, regardless of the price.

In markets with sluggish demand, policy makers can try to reduce demand sluggishness by collecting and publishing quality indicators on a regular basis, in order to make consumers more aware of quality differences between providers. As a policy measure to stimulate quality competition, we have shown that this may be counterproductive if the providers face a price that is below unit costs. Therefore policies with high unit prices and policies which increase information are complements. If the price is sufficiently below unit costs, more quality-responsive demand will reduce quality in steady state, and this is more likely to happen if providers use closed-loop decision rules. Nevertheless, in our welfare analysis we have shown that the optimal design of the provider payment system implies prices below unit costs if the degree of provider motivation is sufficiently high.

Appendix: Solving for the feedback closed-loop solution

Using (6), Provider i's instantaneous objective function is

$$T + (p-c)\,D - \frac{\theta}{2}q_i^2 - F + \alpha\left[(v+q_i)\,D - \frac{\tau D^2}{2}\right], \tag{A1}$$

which, together with (3), defines a linear-quadratic problem. The value function of Provider i is therefore defined as

$$V^i(D) = \sigma_0 + \sigma_1 D + (\sigma_2/2)D^2. \tag{A2}$$

Focusing on stationary Markovian linear strategies, defined by $q_i = \phi_i(D)$ and $q_j = \phi_j(D)$, the value function must satisfy the Hamilton-Jacobi-Bellman (HJB) equation, which, for Provider i, is given by

$$\rho V^i(D) = \max\left\{T + (p-c)\,D - \frac{\theta}{2}q_i^2 - F + \alpha((v+q_i)\,D - \tau\frac{D^2}{2}) + V_D^i\gamma\left(\frac{1}{2} + \frac{q_i - q_j}{2\tau} - D\right)\right\} \tag{A3}$$

Maximisation of the right-hand-side of (A3) yields $\alpha D - \theta q_i + V_D^i\frac{\gamma}{2\tau} = 0$, which, after substitution of $V_D^i = \sigma_1 + \sigma_2 D$, yields

$$q_i = \phi_i(D) = \frac{\alpha}{\theta}D + (\sigma_1 + \sigma_2 D)\,\frac{\gamma}{2\tau\theta}, \tag{A4}$$

and, by symmetry,

$$q_j = \phi_j(D) = \frac{\alpha}{\theta}(1-D) + (\sigma_1 + \sigma_2(1-D))\,\frac{\gamma}{2\tau\theta}. \tag{A5}$$

(A3) can therefore be expressed as

$$\rho V^i(D) = \left\{\begin{array}{c} T + (p-c)\,D - \frac{\theta}{2}\left(\frac{\alpha}{\theta}D + (\sigma_1+\sigma_2 D)\,\frac{\gamma}{2\tau\theta}\right)^2 - F \\ +\alpha\left(\left(v + \left(\frac{\alpha}{\theta}D + (\sigma_1+\sigma_2 D)\,\frac{\gamma}{2\tau\theta}\right)\right)D - \tau\frac{D^2}{2}\right) \\ +(\sigma_1+\sigma_2 D)\,\gamma\left(\frac{1}{2} + \frac{\left(\frac{\alpha}{\theta}D + (\sigma_1+\sigma_2 D)\frac{\gamma}{2\tau\theta}\right) - \left(\frac{\alpha}{\theta}(1-D) + (\sigma_1+\sigma_2(1-D))\frac{\gamma}{2\tau\theta}\right)}{2\tau} - D\right) \end{array}\right\}. \tag{A6}$$

For the above equality to hold, the parameters must satisfy the following equations:

$$\begin{aligned}
\rho\sigma_0 - \frac{1}{2}\gamma\sigma_1 - T + F + \frac{1}{8\theta\tau^2}\gamma^2\sigma_1^2 + \frac{1}{4\theta\tau^2}\gamma^2\sigma_1\sigma_2 + \frac{1}{2\theta}\frac{\alpha}{\tau}\gamma\sigma_1 &= 0 \quad \text{(A7)} \\
\left(\gamma\sigma_1 + \rho\sigma_1 - \frac{\alpha\gamma}{\theta\tau}\sigma_1 - (p-c) - v\alpha - \frac{\gamma}{2}\sigma_2 + \frac{\gamma^2}{4\theta\tau^2}\sigma_2^2 + \frac{\alpha}{2\theta\tau}\gamma\sigma_2 - \frac{\gamma^2}{4\theta\tau^2}\sigma_1\sigma_2\right) D &= 0 \quad \text{(A8)} \\
\left(\frac{\alpha\tau}{2} - \frac{1}{2\theta}\alpha^2 + \sigma_2\left(\gamma + \frac{\rho}{2} - \frac{1}{\theta}\frac{\alpha}{\tau}\gamma\right) - \frac{3\gamma^2}{8\theta\tau^2}\sigma_2^2\right) D^2 &= 0 \quad \text{(A9)}
\end{aligned}$$

Solving (A8) for σ_1 yields (25), while solving (A9) for σ_2 yields two candidate solutions:

$$\sigma_2 = \frac{\tau}{3\gamma^2}\left(\left(2\theta\tau\left(2\gamma + \rho\right) - 4\alpha\gamma\right) \pm \sqrt{\left(2\theta\tau\left(2\gamma+\rho\right) - 4\alpha\gamma\right)^2 + 12\alpha\gamma^2\left(\theta\tau - \alpha\right)}\right).$$

Since the value function must be concave in order to ensure stable strategies, we select the negative root. The steady-state level of quality is found by substituting the derived expressions for σ_1 (from (A8)) and σ_2 into (A4) or (A5) and setting $D = \frac{1}{2}$.

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Figure 1. Phase diagram for the open-loop solution

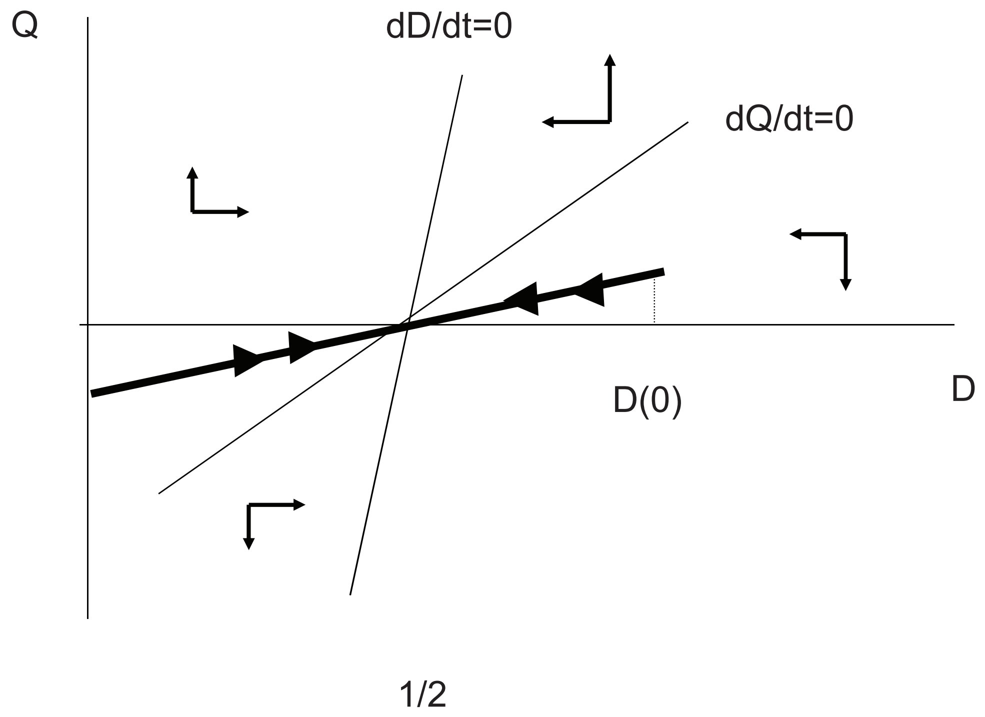

Figure 2. First best quality