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# Modelling the Risk and Return Relation Conditional on Market Volatility and Market Conditions

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## Abstract

This paper investigates whether the risk-return relation varies, depending on changing market volatility and up/down market conditions. Three market regimes based on the level of conditional volatility of market returns are specified – 'low', 'neutral' and 'high'. The market model is extended to allow for these three market regimes and a three-beta asset-pricing model is developed. For a set of US industry sector indices using a cross-sectional regression, we find that the beta risk premium in the three market volatility regimes is priced. These significant results are uncovered only in the pricing model that accommodates up/down market conditions.



KEY WORDS: CAPM, conditional market volatility, modelling conditional betas
JEL CODE: G12, G13

# I. Introduction

When testing the validity of asset pricing models, especially the CAPM, many studies (Kim and Zumwalt, 1979; Bhardwaj and Brooks, 1993; Pettengill, Sundaram and Mathur, 1995; Howton and Peterson, 1998; Crombez and Vander Vennet, 2000; and Faff, 2001) account for market movements, defined as up and down markets. To classify up and down markets various definitions have been used. For example, Kim and Zumwalt (1979) used three threshold levels, namely, average monthly market return, average risk-free rate and zero. When the realized market return is above (below) the threshold level the market is said to be in the up (down) market state.

Several studies have investigated the risk-return relationship in the tails of the market return distribution. For example, Crombez and Vander Vennet (2000) conducted an extensive investigation into this relationship. First, they defined up and down markets with two thresholds: (i) zero and (ii) the risk-free rate. Further, to define three regimes for market movements, namely, substantially upward moving, neutral, and substantial bear, the following threshold points were used: (iii) the average positive (negative) market return, (iv) the average positive (negative) market return plus (less) half the standard deviation of positive (negative) market returns, and (v) the average positive (negative) market return plus (less) three-quarters of the standard deviation of positive (negative) market returns. Crombez and Vander Vennet examined the beta risk-return relation in the aforementioned three market regimes and assessed the robustness of the regime classification by varying the width of the neutral interval. They found the conditional beta risk-return relation to be stronger as the classification of up and down markets was more pronounced.

An alternative approach to capture market movements is through various market volatility regimes. It has been argued in the finance literature and media that high volatility leads to high returns. Two interesting questions arise from this debate: (i) does the beta-return relationship depend on the various market volatility regimes? and (ii) are the betas corresponding to these volatility regimes priced?

The main objective of this paper is to investigate whether securities' responses to the market vary, depending on changing market volatility as defined by ARCH-type models. In particular, we aim to investigate whether market risks as measured by betas estimated across three different market conditions are useful in explaining asset/portfolio returns. Postulating three distinct betas across the three market volatility regimes, a three-state regime switching threshold model, with percentiles as threshold parameters, is employed.

There is empirical evidence raising concern about the ability of a single beta to explain cross-sectional variation of security/portfolio returns. See for example, Pettengill, Sundaram and Mathur (1995) and Jagannathan and Wang (1996). Security or portfolio systematic risk is known to vary considerably over time (Bos and Newbold, 1984; Episcopos, 1996; Brooks, Faff and Ho, 1997). Further, it is well known that the volatility of financial time series, particularly in high frequency data, changes over time. In this paper, we consider another possibility of incorporating market movements into asset pricing models by including the changes in the conditional market volatility. We achieve this by partitioning the market returns into three regimes corresponding to the size of the conditional market volatility modelled via an ARCH/GARCH-type process.

The paper is organized as follows: In the following section, a brief introduction to volatility models is given. In Section III, we derive a three-beta asset pricing model. The hypotheses of interest are given in Section IV followed by a description of the methodology in Section V. The data series used in this study are described in Section VI. Section VII is devoted to the empirical results and their analysis. Section VIII concludes the paper.

## II. Modelling Market Volatility

It is well-known that the security or portfolio return-generating processes in general are unstable and highly volatile. The model that has been used successfully to capture volatility in financial time series is the ARCH model due to the seminal paper by Engle (1982). The ARCH model allows the current conditional variance to be a function of the past squared error terms. This is consistent with volatility clustering. Bollerslev (1986) later generalized the ARCH (GARCH) model such that the current conditional variance is allowed to be a function of the past conditional variance and past squared error terms.

The return-generating process can be written as:

(1) ARMA($m$,$n$) mean: $$R_t = \mu + \sum_{i=1}^{m} \alpha_i R_{t-i} + \sum_{j=1}^{n} \beta_j \varepsilon_{t-j} + \varepsilon_t$$

where $R_t$ is the return in period $t$, $\varepsilon_t = \sigma_t v_t$, $v_t \sim N(0,1)$ and the conditional variance, $\sigma_t^2$ is defined as:

(2) GARCH($p$,$q$): $$\sigma_t^2 = \delta_0 + \sum_{i=1}^{p} \delta_1 \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \delta_2 \sigma_{t-j}^2 + \varepsilon_t$$

Engle, Lilien and Robins (1987) reveal that risk as measured by variance and expected returns, tends to be positively correlated. The fact that an increase in risk tends to result in higher expected returns is captured by the following GARCH model, by including a conditional variance or conditional standard deviation term in the mean equation given in (1) so that:

(3) $$R_t = \mu + \lambda \sigma_t^2 + \sum_{i=1}^{m} \alpha_i R_{t-i} + \sum_{j=1}^{n} \beta_j \varepsilon_{t-j}$$

The pair of equations (2) and (3) together are referred to as the GARCH-in-mean (GARCH-M($p$,$q$)) model. The parameter $\lambda$ is the contemporaneous returns response to the change of conditional variance.[1]

[1] Further extensions of the ARCH model are available in the vast literature on volatility modelling. For example, Engle and Ng (1993) argued that there is a negative relationship between security returns volatility and the sign of stock returns. The asymmetric volatility specification, referred to as the threshold ARCH model, can model this phenomenon. EGARCH($p$,$q$) model (see Nelson, 1991) and TGARCH($p$,q) model (see Glosten, Jagannathan and Runkle, 1989) are two other important conditional volatility models proposed in the literature. See Bollerslev, Engle and Nelson (1994) for a survey.

# III. Development of the Asset-Pricing Model

We define three market regimes and develop a conditional three-beta security return generating process. We then apply the security return generating process to a portfolio and obtain a three-beta asset-pricing model.

## A. Market Regimes

First, we fit a volatility model for daily market returns and obtain the estimates for conditional variance $\sigma_t^2$. Then, based on the magnitude of these estimates, we classify daily volatilities belonging to one of three market volatility regimes, using appropriately defined indicator functions.

Define three indicator functions $I_{Lt}, I_{Nt}$ and $I_{Ht}$ as follows:

(4)
$$I_{Lt} = \begin{cases} 1 & if \quad \sigma_t^2 < \sigma_L^2 \\ 0 & otherwise \end{cases},$$

(5)
$$I_{Nt} = \begin{cases} 1 & if \quad \sigma_L^2 \le \sigma_t^2 \le \sigma_H^2 \\ 0 & otherwise \end{cases}$$

and

(6)
$$I_{Ht} = \begin{cases} 1 & if \quad \sigma_t^2 > \sigma_H^2 \\ 0 & otherwise \end{cases}$$

where $\sigma_L^2$ and $\sigma_H^2$ are the x$^{th}$ and (1-x)$^{th}$ percentiles of the conditional variance series, which are used as threshold parameters.[2] The preceding indicator functions are used to partition the market volatility into three groups: days with low $(I_{Lt} = 1)$, neutral $(I_{Nt} = 1)$ and high $(I_{Ht} = 1)$ market volatilities. Hence, three market regimes: low volatility market (LVM), neutral volatility market (NVM) and high volatility market (HVM) are defined.

[2] The choice of 'x'is arbitrary. We study the sensitivity of the results to several different percentiles in this empirical analysis later in Section VII E.

### B. Development of a Three-Beta Security Return Generating Process

In the empirical investigation of the single-factor CAPM, the beta is estimated using the market model given as:

(7) $$\text{Model A: } R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it}$$

where $R_{it}$ is return of security *i* in period *t*, $R_{mt}$ is return on market portfolio in period *t* and $\varepsilon_{it} \sim N(0, \sigma^2)$. We refer to Model A as the unconditional single-beta security return generating process.

To estimate the betas in the low, neutral and high volatility markets, we extend the market model given in (7) as:

(8) $$\text{Model B: } R_{it} = \alpha_i + \beta_{iL}\left(I_{Lt} R_{mt}\right) + \beta_{iN}\left(I_{Nt} R_{mt}\right) + \beta_{iH}\left(I_{Ht} R_{mt}\right) + \varepsilon_{it}$$

where, $\varepsilon_{it} \sim N(0, \sigma^2)$. The $\beta_{iL}$, $\beta_{iN}$ and $\beta_{iH}$ are defined as the systematic risks corresponding to the LVM, NVM and HVM regimes respectively. The model in (8) is a richer specification. It is a three-state regime-switching model with percentiles as threshold parameters.

Letting $R_{mt}^L = \left(I_{Lt} R_{mt}\right)$, $R_{mt}^N = \left(I_{Nt} R_{mt}\right)$ and $R_{mt}^H = \left(I_{Ht} R_{mt}\right)$ we obtain,

(9) $$R_{it} = \alpha_i + \beta_{iL} R_{mt}^L + \beta_{iN} R_{mt}^N + \beta_{iH} R_{mt}^H + \varepsilon_{it}$$

which we refer to as the unconditional three-beta security return generating process.

### C. Portfolio Analysis of the Three-Beta Model

In this section, to establish testable hypotheses on the three betas, we analyse the mean and variance of a portfolio comprised of securities with the return generating process given in (9).

Let us consider a portfolio comprised of *n* securities with weights $w_i$ such that $\sum_{i=1}^{n} w_i = 1$.

Now, from (9) we obtain the portfolio return, $R_{pt}$ as:

(10)
$$R_{pt} = \sum_{i=1}^{n} w_i \alpha_i + \left( \sum_{i=1}^{n} w_i \beta_{iL} \right) R_{mt}^{L} + \left( \sum_{i=1}^{n} w_i \beta_{iN} \right) R_{mt}^{N} + \left( \sum_{i=1}^{n} w_i \beta_{iH} \right) R_{mt}^{H} + \sum_{i=1}^{n} w_i \varepsilon_{it}$$

Letting $\alpha_p = \sum_{i=1}^{n} w_i \alpha_i$ , $\beta_{pL} = \left( \sum_{i=1}^{n} w_i \beta_{iL} \right)$, $\beta_{pN} = \left( \sum_{i=1}^{n} w_i \beta_{iN} \right)$, $\beta_{pH} = \left( \sum_{i=1}^{n} w_i \beta_{iH} \right)$ and $\varepsilon_{pt} = \sum_{i=1}^{n} w_i \varepsilon_{it}$ we can write (10) as:

(11)
$$R_{pt} = \alpha_p + \beta_{pL} R_{mt}^{L} + \beta_{pN} R_{mt}^{N} + \beta_{pH} R_{mt}^{H} + \varepsilon_{pt}$$

The mean and variance of the portfolio return are given by:

(12)
$$E\left(R_p\right) = \alpha_p + \beta_{pL} E\left(R_{mt}^{L}\right) + \beta_{pN} E\left(R_{mt}^{N}\right) + \beta_{pH} E\left(R_{mt}^{H}\right)$$

and

(13)
$$\begin{aligned} Var\left(R_p\right) = {} & \left(\beta_{pL}\right)^2 Var\left(R_{mt}^{L}\right) + \left(\beta_{pN}\right)^2 Var\left(R_{mt}^{N}\right) + \left(\beta_{pH}\right)^2 Var\left(R_{mt}^{H}\right) \\ & + 2\beta_{pL}\beta_{pN} Cov\left(R_{mt}^{L}, R_{mt}^{N}\right) + 2\beta_{pL}\beta_{pH} Cov\left(R_{mt}^{L}, R_{mt}^{H}\right) \\ & + 2\beta_{pN}\beta_{pH} Cov\left(R_{mt}^{N}, R_{mt}^{H}\right). \end{aligned}$$

For a well-diversified portfolio, the unsystematic portion of the variance $Var\left(\varepsilon_p\right)$ approaches zero. Further, let us examine the covariance terms:

(14)
$$\begin{aligned} Cov\left(R_{mt}^{k}, R_{mt}^{l}\right) &= E\left(R_{mt}^{k} R_{mt}^{l}\right) - E\left(R_{mt}^{k}\right) E\left(R_{mt}^{l}\right) \\ &= -E\left(R_{mt}^{k}\right) E\left(R_{mt}^{l}\right), \end{aligned}$$

where $k \neq l$ and $k, l = L, N, H$ . When the time period captured in the analysis is long enough, $E\left(R_{mt}^{L}\right)$ and $E\left(R_{mt}^{H}\right)$ will always be very small due to the large number of zeros in $\left\{R_{mt}^{L}\right\}$ and $\left\{R_{mt}^{H}\right\}$ series. This means at least one of the terms in (14) will be very small and therefore the covariance will be small too. The impact of an extended time period captured in the analysis on the magnitude of the variance terms in (13) will be much smaller. It is therefore reasonable to assume that the covariance terms in (13) are negligible compared to the variance terms.

Then, equation (13) reduces to:

(15)
$$Var\left(R_p\right)=\left(\beta_{pL}\right)^2 Var\left(R_{mt}^L\right)+\left(\beta_{pN}\right)^2 Var\left(R_{mt}^N\right)+\left(\beta_{pH}\right)^2 Var\left(R_{mt}^H\right)$$
$$=V_L^2+V_N^2+V_H^2$$

where

(16)
$$V_L^2=\left(\beta_{pL}\right)^2 Var\left(R_{mt}^L\right)=$$ component of total portfolio variation systematically related to the LVM,

(17)
$$V_N^2=\left(\beta_{pN}\right)^2 Var\left(R_{mt}^N\right)=$$ component of total portfolio variation systematically related to the NVM

and

(18)
$$V_H^2=\left(\beta_{pH}\right)^2 Var\left(R_{mt}^H\right)=$$ component of total portfolio variation systematically related to the HVM.

## D. Relationship Between the Portfolio Betas and Returns

Assuming all components of the total portfolio variation are priced, we may express the expected portfolio return as:

(19)
$$E\left(R_p\right)=R_f+K_1V_L+K_2V_N+K_3V_H$$

where $K_1$, $K_2$ and $K_3$ are constants. Now replacing $V_L, V_N$ and $V_H$ with their explicit expressions given in (16-18), we can write (19) as:

(20)
$$E\left(R_p\right)=R_f+\left\{K_1\sqrt{Var\left(R_{mt}^L\right)}\right\}\beta_{pL}+\left\{K_2\sqrt{Var\left(R_{mt}^N\right)}\right\}\beta_{pN}$$
$$+\left\{K_3\sqrt{Var\left(R_{mt}^H\right)}\right\}\beta_{pH}$$

Hence it follows:

(21)
$$E\left(R_p\right)=R_f+\lambda_L\beta_{pL}+\lambda_U\beta_{pN}+\lambda_H\beta_{pH}$$

where

$\lambda_L = \left\{K_1 \sqrt{Var\left(R_{mt}^{L}\right)}\right\}$, $\lambda_N = \left\{K_2 \quad \sqrt{Var\left(R_{mt}^{N}\right)}\right\}$ and $\lambda_H = \left\{K_3 \sqrt{Var\left(R_{mt}^{H}\right)}\right\}$. We refer to (21) as three-beta asset pricing model.[3]

## IV. Hypotheses of Interest

We postulate that beta is unstable across the various volatility regimes. To test this proposition, we conduct hypothesis testing in two stages. First, we test separately whether the regression coefficients of the three-beta return-generating process defined in (9) are significantly different from zero, or not. The hypotheses to be tested are:

(22) $$H_0 : \beta_{ik} = 0 \text{ and } H_1 : \beta_{ik} \neq 0,\ k = L, N, H .$$

Second, we conduct multi-parameter testing, in that we test whether the regression coefficients in (9) are equal, or not. The hypotheses tested are:

(23) $$H_0 : \beta_{iL} = \beta_{iN} = \beta_{iH} = \beta_p$$

against $$H_1 : at \quad least \quad one \quad of \quad \beta_{iL}, \beta_{iN}, \beta_{iH} \quad is \neq \beta_p \ .$$

In line with the conventional assumption that the higher the variance the lower the preference for risky assets (i.e., investors are risk averse), we postulate further, that investors expect a premium to accept variation in the LVM, NVM and HVM. This means that we expect $K_1$, $K_2$ and $K_3$, which are the second, third and fourth terms in (19), to be positive and their significance can be investigated by testing

(24) $$H_0 : \lambda_l = 0 \text{ against } H_1 : \lambda_l > 0$$

where $l = L, N, H$ .

---

[3] For any portfolio with constant beta, (21) reduces to: $E\left(R_p\right) = R_f + \left(\lambda_L + \lambda_N + \lambda_H\right)\beta_p$. Now assuming that (21) is applicable for the market portfolio, which implies $\beta_{mL} = \beta_{mN} = \beta_{mH} = 1$, we obtain $E\left(R_m\right) = R_f + \lambda_L + \lambda_N + \lambda_H$. Alternatively, $\lambda_L + \lambda_N + \lambda_H = E\left(R_m\right) - R_f$. Hence, by substitution in the constant beta model follows: $E\left(R_p\right) = R_f + \beta_p\left\{E\left(R_m\right) - R_f\right\}$. This is the security market line version of the single-beta CAPM. This shows that when the three-beta asset-pricing model is assumed to be applicable to the market portfolio, the single-beta CAPM is just a special case of the three-beta asset-pricing model.

# V. Methodology

The analysis of the risk-return relationship is based on a two-stage procedure. In the first stage of the analysis, the systematic risks, $\beta_{iL}$, $\beta_{iN}$ and $\beta_{iH}$, are estimated. In the second stage we test whether the systematic beta risks are priced or not. This is followed by estimation of a pricing model incorporating up and down market movement.

*Stage-I: Beta estimation using time series data*

We estimate the model given in (8) using the method of ordinary least squares for a large group of sample portfolios using time series data in the first 655-day (2.5 years) period. In this stage of the analysis, through the empirical results of the hypothesis tests given in (22-23), we will be able to ascertain whether or not the beta is unstable across the three regimes.

*Stage-II: Estimation of cross-sectional relationship between returns and betas*

In each group of 131 days (0.5 year) that follows the sample period used in the estimation of the time series model in *Stage-I*, the daily sector returns are regressed on the beta estimates obtained in *Stage-I*, according to the cross-sectional relationship:

(25) Model C: $R_{it} = \lambda_0 + \lambda_L \beta_{iL} + \lambda_U \beta_{iN} + \lambda_H \beta_{iH} + \varepsilon_{it}$

where $\varepsilon_{it} \sim N(0, \sigma^2)$. Here, it is assumed that the sector betas estimated in *Stage-I* proxy betas of *Stage-II*. To ascertain whether betas in the three regimes are priced, the hypotheses given in (24) are tested for the averages of the slope coefficients in (25).

The above procedure will uncover possible non-stationarities of the regression coefficients – risk premiums – within the 131-day period. The two-stage estimation procedure is repeated using a rolling window technique, rolling forward six months at a time. This method enables 19 repetitions of the two-stage procedure allowing estimation of beta risk premium in 2489 consecutive days.

*Stage-III: Accommodating market movement*

We believe using the realized return in equation (25) instead of the expected as derived in (21) can introduce bias estimates. Therefore, following Pettengill, Sundaram and Mathur (1995), to ascertain whether beta in the three regimes is priced or not, the cross-sectional regression model given by

(26)
$$R_{pt} = \lambda_0 + \lambda_L^U \delta\beta_{pL} + \lambda_L^D (1-\delta)\beta_{pL} + \lambda_N^U \delta\beta_{pN} + \lambda_N^D (1-\delta)\beta_{pN} + \lambda_H^U \delta\beta_{pH} + \lambda_H^D (1-\delta)\beta_{pH} + \varepsilon_{pt}$$

where $\delta = 1$ for up market, $\delta = 0$ for down market and $\varepsilon_{pt} \sim N(0, \sigma_p^2)$ is estimated for each day in the testing period. We refer to (26) as the conditional three-beta risk-return relationship. Like Pettengill, Sundaram and Mathur (1995), we postulate that in the time periods where the market return in excess of the risk-free rate is negative it is reasonable to infer an inverse relationship between realized return and beta. Accordingly, we expect the beta risk premium in the up market to be positive and the beta risk premium in the down market to be negative. A positive and statistically significant beta risk premium in the up market and a negative and statistically significant beta risk premium in the down market is sufficient to suggest a systematic relationship between the beta in the low, neutral and high volatility regimes and the industry sector returns.

## VI. Data

The dataset comprises the daily price series of 127 US industry sectors (portfolios) obtained from Datastream. The return series on the US market and the US 3-month Treasury-Bill rate are used as proxies for the market portfolio return and the risk-free rate, respectively. The daily returns are calculated as the change in the logarithm of the closing prices of successive days. The time period we investigate is from January 1, 1990 to January 17, 2002.

Due to the large number of sectors involved we do not report their summary statistics. A few brief comments follow. The mean returns vary widely across the sectors, with the highest being 0.109 per cent for Biotechnology and the lowest being –0.025 per cent for the Mining sector. The mean market return is 0.049, with the lowest and the highest returns being –7.020 per cent and 5.335 per cent, respectively. The standard deviation of the market return distribution, 0.978 per cent, is closer to the lower end to that of the sectors, of which the lowest is 0.835 per cent for Utilities and the highest is 3.028 per cent for Funeral and Cemeter. The market and seventy sectors are negatively skewed. As expected with daily data, there is considerable evidence of excess kurtosis across the sample portfolios. Indeed, the excess kurtosis of the market return distribution is 4.869.

## VII. Empirical Results and Analysis

### A. GARCH Specification

We consider that the market return has the mean equation:

(27) $$R_{mt} = \mu + \varepsilon_t$$

and the conditional variance equation:

(28) $$\sigma_t^2 = \delta_0 + \delta_1 \varepsilon_{t-1}^2 + \delta_2 \sigma_{t-1}^2$$

This GARCH(1,1) model was estimated using the maximum likelihood approach for the US market returns assuming conditional normality in the standardised residuals:[4]

(29) $$\sigma_t^2 = \underset{(0.0000)}{0.0000} + \underset{(0.0045)}{0.0583}\varepsilon_{t-1}^2 + \underset{(0.0048)}{0.9368}\sigma_{t-1}^2 .$$

The figures in parentheses are standard errors. All parameters in (29) are significantly different from zero at the 1% level. This is the model we use to characterise the low, neutral and high volatility regimes.

### B. Testing for the Validity of the Assumptions on the Covariance Terms

[4] Most studies that implement GARCH($p$,$q$) models adopt low orders for $p$ ad $q$. They seem to be sufficient to model the variance dynamics over long periods (Bollerslev, Chou and Kroner, 1992).

In the development of the pricing model given in (21), we made two assumptions: (i) the model is valid for well-diversified portfolios and (ii) compared to the variance terms, the covariance terms in (13) are negligible. The variance and covariance terms of the market portfolio returns corresponding to the three volatility regimes with threshold percentiles (15%, 85%), given in Table 1 suggest that the assumption (ii) above appears to hold in this case.

## C. Beta vs Realized Returns

In this section, we investigate the beta risk premium estimated in the low, neutral and high market volatility regimes. We consider five sets of percentiles: (1%, 99%), (5%, 95%), (10%, 90%), (15%, 85%) and (20%, 80%) as threshold parameters that define the three volatility regimes. The parameter estimates of Model C, given in (27) with the betas corresponding to the low, neutral and high volatility markets as explanatory variables, are reported in Table 2. When the threshold parameters are taken as (5%, 95%), (10%, 90%), (15%, 85%) and (20%, 80%) the beta risk premium is significantly different from zero only in the low volatility regime. As the low and high volatility regimes become more pronounced with (1%, 99%) as the threshold parameters, the beta risk premium is significantly different from zero in the neutral volatility regime only. As evidenced in Panels A-E in Table 2, the beta risk premium in the high volatility regime is not significant and has the sign opposite to what was expected. On the other hand, the beta risk premium in the neutral volatility regime though not significant with any set of threshold parameters, always has the correct sign.

Clearly, the above results are inconsistent across the market volatility regimes. We believe that these inconsistencies might be due to the bias that creeps in as a result of using the realized return in equation (25) instead of the expected as derived in (21). Therefore, we estimated equation (26), the conditional three-beta return generating process, with the five sets of threshold parameters defined earlier. Of the 2489 days in the testing period 1296 (52.1%)[5]

[5] Pettengill, Sundaram and Mathur (1995) using monthly US data from Jan 1926 through Dec 1990, reported 57.6% of the months correspond to 'up market' days. Faff (2001) study that used monthly Australian data over the period 1974 to 1995 reported that 54.2% of the months provide positive excess market returns.

are ‘up market’ days and 1193 are ‘down market’ days. An analysis of the results reported in Table 3 indicates that the risk premium in most instances is significantly different from zero and always has the correct sign. Therefore, in the dataset that we have considered, there is evidence to suggest that the beta risk premium in the ‘up market’ is positive and the beta risk premium in the ‘down market’ is negative and this is true with the beta in the low, neutral and high market volatility regimes. This significant result is strongly evident when the threshold parameters are taken as (15%, 85%) and (20%, 80%). The unconditional model failed to uncover a systematic relation between the beta in the low, neutral and high volatility regimes and the industry sector returns but the conditional model does.

## D. Time Variation in Beta

As outlined in Section V, we repeated the two-stage estimation process 19 times by rolling forward 131 days at a time. In each application of the estimation process we constructed three indicator variables defined in (4-6) in a 655-day period in *Stage-I* of the two-stage process. These indicator variables establish the low, neutral and high market volatility regimes according to a chosen set of threshold parameters. As reported earlier, we obtained very strong favourable results in the conditional three-beta risk-return relationship model with (15%, 85%) as the threshold parameters. Therefore, in this section we analyse the results further with the same set of threshold parameters.

First, we evaluated the number of low, neutral and high volatility days within each of the nineteen 655-day *Stage-I* estimation periods. These results are reported in Table 4.[6] The Table 4 entries suggest that the nineteen estimation periods can be sensibly divided into four (overlapping) sub-sample periods: (1) Jan 90 - Dec 92, (2) Jan 91 - Jun 97, (3) Jul 95 - Dec 98 and (4) Jan 97 - Jun 01. In the second sub-sample period that accounts for 1179 days (4.5 years), none of the days belong to the high volatility regime. Similarly, in the fourth sub-sample period of 655 days (2.5 years) none of the days belong in the low market volatility

[6] It should be noted here that due to the rolling window technique, two consecutive 655-day *Stage-I* estimation periods overlap through 524 days (2 years).

regime. In the sub-sample periods 1 and 3, all three categories of low, neutral and high market volatility days are present. These observations clearly indicate the volatility-clustering phenomenon in the US market return series.

Secondly, we tested whether or not the three betas[7] estimated in equation (8) are equal for each of the 127 sectors in each of the nineteen *Stage-I* estimation periods. In Table 5, we report the results of the fourteen sectors where the null hypothesis of equal beta is rejected (in favour of at least one beta being different) in at least five out of the nineteen estimation periods. The bottom row in Table 5 gives the totals across all 127 industry sectors. It is clearly evident in Table 5 that the largest numbers of rejections occur in the beta estimation periods 1-2 and 12-19. These periods fall in the first, third and the fourth sub-sample periods which include high market volatility days. Therefore, it appears that the beta estimated in different market volatility regimes is likely to be different in the time periods that include high market volatility regimes compared to the time periods where the market volatility is neutral or low[8].

## E. Sensitivity of Three-Beta Risk-Return Relation

*Time Period Used for Testing Beta Risk Premiums*

Here we separately examine the three-beta risk-return relationship in the time periods that include the days (i) where the market volatility has not been very high, (ii) where the market volatility has not been very low and (iii) where the market volatility has been mixed. We noted such occasions when we investigated the nineteen beta estimation periods in the previous section. We estimated the conditional three-beta risk-return relationship with (15%, 85%) as threshold parameters in the sub-sample periods 1 and 3, 2, and 4 separately. The results are given in Table 6.

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[7] The explanatory variables in (8) are not correlated and therefore will not pose multicollinearity problems in estimating the regression parameters. In all sectors, the betas in the low, neutral and high market volatility regimes are positive and significantly different from zero at the 1% level.
[8] There are sophisticated methods of investigating time variation in beta. One such method is bi-variate GARCH model. We do not pursue this in this paper.

The results shown in Panels B and C in Table 6 provide very strong evidence in support of the conditional relationship with positive and statistically significant premiums in the 'up market' and negative and significant premiums in the 'down market'. The results in Panel A, Table 6 where the beta risk premium estimates in all three volatility regimes are available, provide statistical evidence only in support of a systematic relation between the beta in the low and neutral volatility regimes and the returns. Though the beta risk premium in the high volatility regime is not statistically significant, it is positive in the 'up market' and negative in the 'down market' as expected.

*Alternative Definitions of Up and Down Markets*

We repeated the analysis with up and down markets defined as the positive and negative market returns respectively, instead of excess market returns. Then, of the 2489 days available for the testing, 1373 (55.16%) were up market days. The estimates of the conditional asset-pricing model are reported in Table 7. The results obtained here are consistent with those reported in Table 3 where a positive market return in excess of the risk-free rate is defined as an 'up market'.[9] A reason for this similarity in results appears to be that the number of 'up market' days observed over the sample period is not significantly different under the two definitions.

Following Crombez and Vander Vennet (2000), we investigated the conditional relationship in three market regimes defined with the following threshold points: (i) the average positive market return and average negative market return, (ii) the average positive market return plus half the standard deviation of positive market returns and average negative market return less half the standard deviation of negative market returns, and (iii) the average positive market return plus three-quarters of the standard deviation of positive market returns

[9] Kim and Zumwalt (1979) in their analysis of security returns based on a two-beta model divided the returns into up or down markets using three alternative cut-off levels: average monthly market return, average risk-free rate and zero. They reported that the different cut-off levels produced virtually identical results.

('substantial bull market') and average negative market return less three-quarters of the standard deviation of negative market returns ('substantial bear market'). In all three cases the results are qualitatively the same and, for the sake of brevity, we report in Table 8 the outcome for case (iii) only. From the table we see that there is very strong evidence of negative (positive) and statistically significant beta risk premium in the 'substantial bear market' ('substantial bull market') regimes. When the market is neutral, the beta risk premium is significant only in the low volatility market. In this case the premium is positive.

## VIII. Conclusions

In this paper, we examined the empirical validity of a conditional three-beta CAPM. Specifically, having modelled the market return volatility as a GARCH(1,1) process, we defined three volatility regimes based on the size of the conditional volatilities. Using a three-state volatility-switching model, with various percentile cut-offs of the estimated conditional volatility as threshold parameters (e.g., $15^{th}$ and $85^{th}$ percentiles), a three-beta asset-pricing model is specified and tested. The three betas correspond to the low, neutral and high market volatility regimes specified by the threshold parameters.

An analysis of the results overwhelmingly suggests that the betas in the low, neutral and high volatility regimes are positive and significant. In most of the industry sectors the betas were not found to be significantly different in the three regimes. The betas in the three regimes however, are more likely to be different when the estimation period includes high market volatility days than otherwise.

We also investigated whether or not the betas are priced in a cross-sectional regression framework. We find that the beta risk premium in the three market volatility regimes is priced. Notably, these significant pricing results are uncovered only in the pricing model conditioned on the sign of the realized market return, while the unconditional model does not reveal such a relation. In the conditional three-beta asset-pricing model, the beta risk premiums are positive and significantly different from zero in the 'up market' and are negative and significantly different from zero in the 'down market'. That is, we have strong evidence to suggest that the components of the total portfolio return variations systematically related to the low, neutral and high market volatility regimes are priced. As such, our evidence provides further insights into the conditional risk-return relation established by Pettengill, Sundaram and Mathur (1995).

An extension of this study to emerging markets would give further insights into how the three-beta pricing model would work in different economies. As discussed in the introduction, this paper assumes that volatility is known and that the beta's response to various volatility regimes is abrupt. Application of smooth transition and Markov-switching processes to model the CAPM-beta might provide some fruitful results, which would be worthy topics for future research.

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Table 1. Variance-covariance of US market returns corresponding to volatility regimes

| | US Market |
|---|---|
| $Var\left(R_{mt}^{L}\right)$ | 0.0000038 |
| $Var\left(R_{mt}^{N}\right)$ | 0.0000554 |
| $Var\left(R_{mt}^{H}\right)$ | 0.0003650 |
| $Cov\left(R_{mt}^{L}, R_{mt}^{N}\right)$ | -2.28 x $10^{-8}$ |
| $Cov\left(R_{mt}^{L}, R_{mt}^{H}\right)$ | -1.37 x $10^{-8}$ |
| $Cov\left(R_{mt}^{H}, R_{mt}^{N}\right)$ | -3.68 x $10^{-8}$ |

Notes: Sample period is January 1 1990 through January 17, 2002. The estimates are based on 3144 observations. The threshold parameters used are (15%, 85%) percentiles.

Table 2. Risk premium estimates in the unconditional three-beta CAPM

| | $\lambda_0$ | $\lambda_L$ | $\lambda_N$ | $\lambda_H$ |
|---|---|---|---|---|
| Panel A: Low and high conditional volatility cuts off at 1st and 99th percentiles | | | | |
| Mean adjusted R-square = 0.0877 | | | | |
| Estimate | -0.0000 | -0.0000 | 0.0006 | -0.0001 |
| Standard deviation | 0.0101 | 0.0022 | 0.0154 | 0.0065 |
| t-value | -0.1619 | 0.9400 | 1.8977*** | -0.8977 |
| Panel B: Low and high conditional volatility cuts off at 5th and 95th percentiles | | | | |
| Mean adjusted R-square = 0.0913 | | | | |
| Estimate | 0.0000 | 0.0002 | 0.0004 | -0.0001 |
| Standard deviation | 0.0101 | 0.0046 | 0.0163 | 0.0098 |
| t-value | 0.0561 | 1.7604*** | 1.2375 | -0.4769 |
| Panel C: Low and high conditional volatility cuts off at 10th and 90th percentiles | | | | |
| Mean adjusted R-square = 0.0938 | | | | |
| Estimate | 0.0000 | 0.0002 | 0.0004 | -0.0002 |
| Standard deviation | 0.0100 | 0.0058 | 0.0162 | 0.0116 |
| t-value | 0.1635 | 1.6612*** | 1.2926 | -0.7177 |
| Panel D: Low and high conditional volatility cuts off at 15th and 85th percentiles | | | | |
| Mean adjusted R-square = 0.0943 | | | | |
| Estimate | 0.0000 | 0.0003 | 0.0004 | -0.0002 |
| Standard deviation | 0.0098 | 0.0073 | 0.0153 | 0.0129 |
| t-value | 0.0885 | 1.8823*** | 1.2055 | -0.7185 |
| Panel E: Low and high conditional volatility cuts off at 20th and 80th percentiles | | | | |
| Mean adjusted R-square = 0.0969 | | | | |
| Estimate | -0.0000 | 0.0003 | 0.0002 | -0.0001 |
| Standard deviation | 0.0097 | 0.0073 | 0.0160 | 0.0142 |
| t-value | -0.0159 | 1.9862** | 0.7172 | -0.2052 |

Notes: Sample period is January 1 1990 through January 17, 2002. The estimates are based on 2489 observations. Mean adjusted R-square is the average of the R-square values in the 2489 cross-sectional regressions. The model estimated is:

$$R_{pt} = \lambda_0 + \lambda_L \beta_{pL} + \lambda_N \beta_{pN} + \lambda_H \beta_{pH} + \varepsilon_{pt}$$

* significant at the 1% level
** significant at the 5% level and
*** significant at the 10% level.

Table 3. Risk premium estimates in the conditional three-beta CAPM (conditioned on excess market return)

| | Up market [ $(R_{mt} - R_{ft}) > 0$ ] | | | | Down market [ $(R_{mt} - R_{ft}) < 0$ ] | | | |
|---|---|---|---|---|---|---|---|---|
| | $\lambda_0^U$ | $\lambda_L^U$ | $\lambda_N^U$ | $\lambda_H^U$ | $\lambda_0^D$ | $\lambda_L^D$ | $\lambda_N^D$ | $\lambda_H^D$ |
| Panel A: Low and high conditional volatility cuts off at 1st and 99th percentiles | | | | | | | | |
| Mean adjusted R-square (up market) = 0.0810 | | | | | Mean adjusted R-square (down market) = 0.0950 | | | |
| Estimate | -0.0007 | 0.0000 | 0.0075 | 0.0001 | 0.0007 | -0.0001 | -0.0069 | -0.0003 |
| SD | 0.0099 | 0.0024 | 0.0134 | 0.0060 | 0.0103 | 0.0021 | 0.0138 | 0.0071 |
| t-value | -2.7296* | 0.0504 | 20.205* | 0.5246 | 2.5037* | 1.3876 | -17.275* | -1.6499*** |
| Panel B: Low and high conditional volatility cuts off at 5th and 95th percentiles | | | | | | | | |
| Mean adjusted R-square (up market) = 0.0837 | | | | | Mean adjusted R-square (down market) = 0.0996 | | | |
| Estimate | -0.0006 | 0.0003 | 0.0063 | 0.0009 | 0.0007 | -0.0000 | -0.0060 | -0.0012 |
| SD | 0.0099 | 0.0047 | 0.0148 | 0.0093 | 0.0104 | 0.0044 | 0.0155 | 0.0102 |
| t-value | -2.3697** | 2.5994* | 15.244* | 3.5124* | 2.4304** | -0.2444 | -13.336* | -3.9825* |
| Panel C: Low and high conditional volatility cuts off at 10th and 90th percentiles | | | | | | | | |
| Mean adjusted R-square (up market) = 0.0868 | | | | | Mean adjusted R-square (down market) = 0.1014 | | | |
| Estimate | -0.0006 | 0.0006 | 0.0053 | 0.0015 | 0.0007 | -0.0002 | -0.0049 | -0.0020 |
| SD | 0.0098 | 0.0059 | 0.0154 | 0.0111 | 0.0102 | 0.0056 | 0.0154 | 0.0118 |
| t-value | -2.0721** | 3.5497* | 12.367* | 4.9818* | 2.3265** | -1.4557 | -10.932* | -5.8769* |
| Panel D: Low and high conditional volatility cuts off at 15th and 85th percentiles | | | | | | | | |
| Mean adjusted R-square (up market) = 0.0873 | | | | | Mean adjusted R-square (down market) = 0.1018 | | | |
| Estimate | -0.0005 | 0.0011 | 0.0040 | 0.0022 | 0.0006 | -0.0007 | -0.0036 | -0.0027 |
| SD | 0.0096 | 0.0072 | 0.0146 | 0.0126 | 0.0100 | 0.0073 | 0.0150 | 0.0129 |
| t-value | -1.7960*** | 5.7449* | 9.8325* | 6.2172* | 1.9133*** | -3.1516* | -8.2111* | -7.3823* |
| Panel E: Low and high conditional volatility cuts off at 20th and 80th percentiles | | | | | | | | |
| Mean adjusted R-square (up market) = 0.0899 | | | | | Mean adjusted R-square (down market) = 0.1046 | | | |
| Estimate | -0.0004 | 0.0015 | 0.0032 | 0.0026 | 0.0005 | -0.0010 | -0.0030 | -0.0030 |
| SD | 0.0095 | 0.0071 | 0.0150 | 0.0135 | 0.0100 | 0.0073 | 0.0164 | 0.0144 |
| t-value | -1.6952*** | 7.4172* | 7.6443* | 6.9546* | 1.6497*** | -4.6492* | -6.2778* | -7.1003* |

Notes: Sample period is January 1 1990 through January 17, 2002. The estimates in the 'up market' are based on 1296 observations and the estimates in the 'down market' are based on 1193 observations. SD = standard deviation.
The model estimated is:

$$R_{pt} = \lambda_0 + \lambda_L^U \delta\beta_{pL} + \lambda_L^D(1-\delta)\beta_{pL} + \lambda_N^U \delta\beta_{pN} + \lambda_N^D(1-\delta)\beta_{pN} + \lambda_H^U \delta\beta_{pH} + \lambda_H^D(1-\delta)\beta_{pH} + \varepsilon_{pt}$$ where

$\delta = 1$ when excess market return in day t is positive and $\delta = 0$ when excess market return in day t is negative. Excess market return is the market return in excess of the risk-free return.

* significant at the 1% level

** significant at the 5% level and

*** significant at the 10% level.

Table 4. Distribution of conditional volatility across beta estimation period

| Sub-sample period | Beta estimation period | Beta estimation time interval | Number of days | | |
|---|---|---|---|---|---|
| | | | Low volatility | Neutral volatility | High volatility |
| 1 | 1 | Jan 1990 - Jun 1992 | 2 | 613 | 40 |
| | 2 | Jul 1990 - Dec 1992 | 50 | 565 | 40 |
| 2 | 3 | Jan 1991 - Jun 1993 | 81 | 574 | 0 |
| | 4 | Jul 1991 - Dec 1993 | 185 | 470 | 0 |
| | 5 | Jan 1992 - Jun 1994 | 223 | 432 | 0 |
| | 6 | Jul 1992 - Dec 1994 | 265 | 390 | 0 |
| | 7 | Jan 1993 - Jun 1995 | 315 | 340 | 0 |
| | 8 | Jul 1993 - Dec 1995 | 368 | 287 | 0 |
| | 9 | Jan 1994 - Jun 1996 | 269 | 386 | 0 |
| | 10 | Jul 1994 - Dec 1996 | 250 | 405 | 0 |
| | 11 | Jan 1995 - Jun 1997 | 206 | 449 | 0 |
| 3 | 12 | Jul 1995 - Dec 1997 | 108 | 517 | 30 |
| | 13 | Jan 1996 - Jun 1998 | 24 | 601 | 30 |
| | 14 | Jul 1996 - Dec 1998 | 19 | 539 | 97 |
| 4 | 15 | Jan 1997 - Jun 1999 | 0 | 542 | 113 |
| | 16 | Jul 1997 - Dec 1999 | 0 | 523 | 132 |
| | 17 | Jan 1998 - Jun 2000 | 0 | 436 | 219 |
| | 18 | Jul 1998 - Dec 2000 | 0 | 367 | 288 |
| | 19 | Jan 1999 - Jun 2001 | 0 | 349 | 306 |

Notes: Low and high conditional volatility cuts off at 15$^{th}$ and 85$^{th}$ percentiles, respectively. In each interval shown in column 3, the beta in the low, neutral and high volatility time periods are estimated using time series data over 655 days. As indicated in the shaded cells, none of the days in sub-period 2 belong in the high market volatility regime and none of the days in sub-period 4 belong in the low market volatility regime.

Table 5. Sectors with significantly different beta estimates in the low, neutral and high volatility regimes in at least five of the nineteen rolling periods

| | Beta estimation period | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | |
| Beverages | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 6 |
| Discount & Spr. Stores | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 5 |
| Electronic Equipment | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 5 |
| Gold Mining | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 6 |
| Mining | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 5 |
| Non Cyc Cons Gds | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 7 |
| Pharm. & Biotech | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 6 |
| Pharmaceuticals | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 7 |
| Soft Drinks | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 5 |
| Support Services | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 6 |
| Telecom Fxd. Line | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 6 |
| Total | 3 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 5 | 6 | 0 | 8 | 7 | 2 | 10 | 10 | 8 | |
| Total (all 127 sectors) | 6 | 12 | 1 | 3 | 0 | 0 | 0 | 0 | 0 | 2 | 6 | 24 | 11 | 41 | 32 | 17 | 16 | 26 | 26 | |

Notes: Hypotheses tested are $H_0 : \beta_{pL} = \beta_{pN} = \beta_{pH} = \beta_p$ against $H_A : at\ least\ one\ of\ \beta_{pL}, \beta_{pN}\ and\ \beta_{pH}\ is \neq \beta_p$. Significance is tested at the 5% level. Rolling periods 1-2, 3-11, 12–14 and 15-19 corresponds to sub-sample periods 1,2,3 and 4 respectively. Sub-sample periods are defined in Table 3.

Table 6. Risk premium estimates in the conditional three-beta CAPM in sub-sample periods

| | Up market [ $(R_{mt} - R_{ft}) > 0$ ] | | | | Down market [ $(R_{mt} - R_{ft}) < 0$ ] | | | |
|---|---|---|---|---|---|---|---|---|
| | $\lambda_0^U$ | $\lambda_L^U$ | $\lambda_N^U$ | $\lambda_H^U$ | $\lambda_0^D$ | $\lambda_L^D$ | $\lambda_N^D$ | $\lambda_H^D$ |
| Panel A: Sub-sample periods 1 and 3 | | | | | | | | |
| Up market days = 357 and down market days = 298 | | | | | | | | |
| Estimate | -0.0016 | 0.0015 | 0.0061 | 0.0009 | 0.0012 | -0.0011 | -0.0057 | -0.0007 |
| SD | 0.0101 | 0.0064 | 0.0154 | 0.0112 | 0.0118 | 0.0075 | 0.0162 | 0.0115 |
| t-value | -3.0021*** | 4.3512*** | 7.4582*** | 1.5144 | 1.6835* | -2.6118*** | -6.1042*** | -1.0693 |
| Panel B: Sub-sample period 2 | | | | | | | | |
| Up market days = 641 and down market days = 538 | | | | | | | | |
| Estimate | 0.0006 | 0.0015 | 0.0029 | | 0.0000 | -0.0009 | -0.0037 | |
| SD | 0.0063 | 0.0089 | 0.0116 | | 0.0075 | 0.0093 | 0.0114 | |
| t-value | 2.6033*** | 4.2299*** | 6.3967*** | | 0.0155 | -2.1285** | -7.5290*** | |
| Panel C: Sub-sample period 4 | | | | | | | | |
| Up market days = 298 and down market days = 357 | | | | | | | | |
| Estimate | -0.0015 | | 0.0038 | 0.0084 | 0.0009 | | -0.0016 | -0.0086 |
| SD | 0.0138 | | 0.0186 | 0.0221 | 0.0117 | | 0.0182 | 0.0199 |
| t-value | -1.9340* | | 3.4978*** | 6.5209*** | 1.4301 | | -1.6590* | -8.1493*** |

Notes: Low and high conditional volatility cuts off at 15th and 85th percentiles, respectively. The shaded cells indicate that the corresponding beta risk premium is not estimated. The model estimated is:

$$R_{pt} = \lambda_0 + \lambda_L^U \delta\beta_{pL} + \lambda_L^D (1-\delta)\beta_{pL} + \lambda_N^U \delta\beta_{pN} + \lambda_N^D (1-\delta)\beta_{pN} + \lambda_H^U \delta\beta_{pH} + \lambda_H^D (1-\delta)\beta_{pH} + \varepsilon_{pt}$$ where

$\delta = 1$ when excess market return in day t is positive and $\delta = 0$ when excess market return in day t is negative. Excess market return is the market return in excess of the risk-free return.

* significant at the 1% level

** significant at the 5% level and

*** significant at the 10% level.

Table 7. Risk premium estimates in the conditional three-beta CAPM (conditioned on raw market return)

| | Up market [ $R_{mt} > 0$ ] | | | | Down market [ $R_{mt} < 0$ ] | | | |
|---|---|---|---|---|---|---|---|---|
| | $\lambda_0^U$ | $\lambda_L^U$ | $\lambda_N^U$ | $\lambda_H^U$ | $\lambda_0^D$ | $\lambda_L^D$ | $\lambda_N^D$ | $\lambda_H^D$ |
| Panel A: Low and high conditional volatility cuts off at 1st and 99th percentiles | | | | | | | | |
| Estimate | -0.0007 | 0.0000 | 0.0071 | 0.0001 | 0.0008 | 0.0001 | -0.0074 | -0.0003 |
| SD | 0.0096 | 0.0023 | 0.0232 | 0.0059 | 0.0106 | 0.0022 | 0.0141 | 0.0073 |
| t-value | -2.7328*** | 0.1257 | 20.120*** | 0.3054 | 2.5296** | 1.3022 | -17.607*** | -1.4775 |
| Panel B: Low and high conditional volatility cuts off at 5th and 95th percentiles | | | | | | | | |
| Estimate | -0.0006 | 0.0003 | 0.0059 | 0.0008 | 0.0008 | -0.0000 | -0.0064 | -0.0012 |
| SD | 0.0096 | 0.0046 | 0.0145 | 0.0090 | 0.0107 | 0.0046 | 0.0159 | 0.0106 |
| t-value | -2.3723** | 2.5764*** | 15.220*** | 3.4580*** | 2.4543** | -0.2225 | -13.495*** | -3.9388*** |
| Panel C Low and high conditional volatility cuts off at 10th and 90th percentiles | | | | | | | | |
| Estimate | -0.0005 | 0.0006 | 0.0050 | 0.0014 | 0.0007 | -0.0003 | -0.0052 | -0.0021 |
| SD | 0.0096 | 0.0058 | 0.0150 | 0.0108 | 0.0104 | 0.0057 | 0.0158 | 0.0122 |
| t-value | -2.0954** | 3.6001*** | 12.373*** | 4.8908*** | 2.3740** | -1.5363 | -11.073*** | -5.8164*** |
| Panel D: Low and high conditional volatility cuts off at 15th and 85th percentiles | | | | | | | | |
| Estimate | -0.0005 | 0.0011 | 0.0038 | 0.0020 | 0.0006 | -0.0007 | -0.0038 | -0.0029 |
| SD | 0.0094 | 0.0070 | 0.0143 | 0.0123 | 0.0103 | 0.0075 | 0.0154 | 0.0132 |
| t-value | -1.8208* | 5.8293*** | 9.8597*** | 6.0884*** | 1.9537* | -3.2694*** | -8.3284*** | -7.3123*** |
| Panel E: Low and high conditional volatility cuts off at 20th and 80th percentiles | | | | | | | | |
| Estimate | -0.0004 | 0.0014 | 0.0030 | 0.0024 | 0.0005 | -0.0011 | -0.0032 | -0.0031 |
| SD | 0.0092 | 0.0070 | 0.0147 | 0.0132 | 0.0103 | 0.0075 | 0.0168 | 0.0147 |
| t-value | -1.7204* | 7.5385*** | 7.6768*** | 6.7448*** | 1.6865* | -4.8474*** | -6.4078*** | -7.0138*** |

Notes: Sample period is January 1 1990 through January 17, 2002. The estimates in the up market are based on 1373 observations and the estimates in the down market are based on 1116 observations. SD = standard deviation. The model estimated is:
$R_{pt} = \lambda_0 + \lambda_L^U \delta\beta_{pL} + \lambda_L^D (1-\delta)\beta_{pL} + \lambda_N^U \delta\beta_{pN} + \lambda_N^D (1-\delta)\beta_{pN} + \lambda_H^U \delta\beta_{pH} + \lambda_H^D (1-\delta)\beta_{pH} + \varepsilon_{pt}$ where $\delta = 1$ when market return in day t is positive and $\delta = 0$ when market return in day t is negative. Excess market return is the market return in excess of the risk-free return.

* significant at the 1% level

** significant at the 5% level and

*** significant at the 10% level.

Table 8. Risk premium estimates in the conditional three-beta CAPM (substantial bear and bull market regimes)

| | $\lambda_0$ | $\lambda_L$ | $\lambda_N$ | $\lambda_H$ |
|---|---|---|---|---|
| Panel A: Regime 1 – Substantial bear market: market return ≤ (mean negative market return - 0.75 SD of negative market return) Number of days = 205 | | | | |
| Estimate | -0.0006 | -0.0024 | -0.0056 | -0.0110 |
| SD | 0.0144 | 0.0091 | 0.0212 | 0.0195 |
| t-value | -0.6313 | -3.7916*** | -3.7921*** | -8.0755*** |
| Panel B: Regime 2 – Neutral market: (mean negative market return - 0.75 SD of negative market return) < market return < (mean positive market return + 0.75 SD of positive market return) Number of days = 2051 | | | | |
| Estimate | 0.0004 | 0.0004 | -0.0001 | -0.0003 |
| SD | 0.0087 | 0.0072 | 0.0138 | 0.0098 |
| t-value | 2.0464** | 2.7333*** | -0.1679 | -1.3126 |
| Panel C: Regime 3 – Substantial bull market: market return ≥ (mean positive market return + 0.75 SD of positive market return) Number of days = 234 | | | | |
| Estimate | -0.0027 | 0.0012 | 0.0093 | 0.0101 |
| SD | 0.0134 | 0.0061 | 0.0178 | 0.0203 |
| t-value | -3.0817*** | 3.1219*** | 7.9606*** | 7.6289*** |

Notes: Sample period is January 1 1990 through January 17, 2002. Low and high conditional volatility cuts off at 15th and 85th percentiles, respectively. The model estimated is:

$$R_{pt} = \lambda_0 + \lambda_L \beta_{pL} + \lambda_N \beta_{pN} + \lambda_H \beta_{pH} + \varepsilon_{pt}$$

* significant at the 1% level
** significant at the 5% level and
*** significant at the 10% level.