

ISSN 1440-771X  
ISBN 0 7326 1083 4

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Australian Stock and Future Indices**

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**Working Paper 3/2001**

**May 2001**

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AND BUSINESS STATISTICS**

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# Market Architecture and Nonlinear Dynamics of Australian Stock and Futures Indices\*

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May 2001

## Abstract

This paper studies the All Ordinaries Index in Australia, and its futures contract known as the Share Price Index. We use a new form of smooth transition model to account for a variety of nonlinearities caused by transaction costs and other market/data imperfections, and given the recent interest in the effects of market automation on price discovery, we focus on how the nonlinear properties of the basis and returns have changed, now that floor trading in the futures contract has been replaced by electronic trading.

**Keywords:** Arbitrage, Electronic trading, Mean reversion, Nonlinear error correction, Smooth transition models, Thresholds, Transaction Costs.

*JEL classification:* C22, C23, E17, E37.

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\***Acknowledgments:** We wish to thank Professor Barry Goss and other participants of the conference on the “Growth, Performance and Concentration of International Financial Markets” held at the Monash University Centre in Prato, Italy, for their useful comments. We also thank an anonymous referee for many helpful suggestions. Heather Anderson acknowledges financial support from the conference organizers. Farshid Vahid acknowledges financial support from the Department of Econometrics and Business Statistics ARC Bounty Scheme.

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## I. INTRODUCTION

Researchers have studied the dynamic relationship between the price of stock futures contracts and the underlying cash market ever since 1982, when the Chicago Mercantile Exchange first introduced futures contracts based on Standard and Poor's (S&P) 500 Stock Index. Compelling empirical evidence that future indices lead stock indices and weaker evidence of feedback has attracted considerable attention, because standard "no arbitrage" arguments predict that in perfectly functioning markets there should only be contemporaneous correlations between the two return series, and zero cross-correlations at non-zero lags and leads. Several reasons for the observed "lead-lag" relationship between stock market and future indices have been put forward, and these include infrequent trading in components of the stock index, the use of transactions data rather than bid-ask quote data in index calculations, time delays in the computation and reporting of the stock index, and transaction costs associated with buying portfolios of stocks and futures contracts. Stoll and Whaley (1990) provide a useful discussion on possible causes of the "lead-lag" relationship, and Abhyankar (1998) surveys the empirical evidence on this issue.

The factors that give rise to the "lead-lag" relationship between stock indices and futures also imply nonlinearities in the behaviour of index returns. The most obvious of these is the nonlinear effect of transaction costs on portfolio adjustment, but infrequent trading in stocks can also introduce nonlinearities, as can the delays and other problems associated with the reporting of the stock index. The literature that studies nonlinearities in index returns is currently small, and it mostly focuses on thresholds caused by transaction costs. Authors such as Yadav et al (1994), Dwyer et al (1996) and Martens et al (1998) have studied the effects of transaction costs on arbitrage, by estimating threshold models of the basis and/or returns in the United States. These models assume identical transaction costs for all investors, and behavioural regimes that depend on whether arbitrage generates net profits after transaction costs. Statistically significant evidence of thresholds supports these models, but reconciliation of the estimated thresholds with independent estimates of transaction costs has been difficult.

One problem with threshold vs transaction cost comparisons is that both vary over different investors and different types of stocks. Individual investor and market specific thresholds then become blurred in an aggregate setting, and it becomes hard to relate any estimate of an "aggregate threshold" back

to a simple measure of transaction costs. Anderson (1997) studies this problem in a paper on arbitrage between bills of different maturity within the U.S. Treasury Bill Market. She models the yield adjustment process using a smooth transition error correction model in which transaction costs vary across market participants. The smooth transition allows for a continuum of regimes, which in the context of modelling stock returns can account for the nonlinear effects of infrequent trading and data reporting problems as well as heterogeneous transaction costs. Taylor et al (2000) use smooth transition error correction models to study the heterogeneity in transaction costs associated with trading FTSE100 stocks and futures.

This paper studies the All Ordinaries Index in Australia, and its futures contract known as the Share Price Index. Several papers have analysed the lead-lag relationship in the Australian context (see, for example, West (1997), Lin and Stevenson (1999) and Frino et al (2000)), but little direct work has been done on examining the nonlinearities in Australian markets. We introduce a new type of smooth transition model to account for nonlinearities caused by transaction costs and other market/data imperfections, and given the recent interest in the effects of market automation on price discovery, our study focuses on how the nonlinear properties of the basis and the returns have changed, now that floor-trading in the futures contract has been replaced by electronic trading.

The effects of screen trading have been studied by Grünbichler et al. (1994) who studied the German DAX index, and more recently, by Taylor et al. (2000) who studied the U.K. FTSE100 index. The Australian case differs from these other cases in that the recent automation involved the futures market, and created a situation in which both the spot and futures markets were then automated. In the German case, only the futures market was automated and stocks continued to be floor-traded, while in the UK case the stock market was automated while futures continued to be floor-traded. While one might expect screen trading to reduce the nonlinear effects of transaction costs in all three of these cases, one would also expect that the automation in the Australian case would remove asymmetries in dynamic behaviour that had been present because of the operational differences between trading in the spot market and trading in the futures market.

We find strong evidence of nonlinearity before the futures trading went on-line, and weaker evidence of nonlinearity after on-line trading. Our analysis

suggests that the automation of the futures market has removed the nonlinear properties of the basis, and made the nonlinear properties of the two returns series more similar. A particularly interesting finding is that prior to the automation of the futures market, the nonlinearities that characterise each market are different, whereas after the introduction of on-line futures trading, the returns in each market have a common nonlinear factor. Futures returns lead stock returns (with feedback) both before and after the introduction of screen-trading, and the futures lead increases only slightly after automation. The speed of mean-reversion in the basis is slow, and appears to be unchanged.

The remainder of this paper is organized as follows. Section 2 of this paper discusses the theoretical basis for our work, together with various econometric specifications that account for lead-lag relationships and nonlinearities. This section introduces our new smooth transition error correction model, which accounts for the possible effects of transaction costs, infrequent trading and asymmetries between trading in spots vs futures contracts. Section 3 discusses the institutional detail that underlies the Australian markets for equities and futures, and then provides details on the samples that are studied in this paper. Section 4 contains our empirical results, which compare the properties of the data before and after the cessation of floor trading in the futures market. Section 5 concludes.

## II. THEORETICAL BASIS

The relationship between the futures price of shares underlying a futures contract and the spot price on the cash market for the same shares is often described by the cost-of-carry model, which postulates that

$$F_t = S_t e^{(r-y)(T-t)}, \quad (1)$$

where  $F_t$  is the futures price of the index at time  $t$ ,  $S_t$  is the spot price of the index at time  $t$ ,  $r$  is the interest rate foregone while carrying the underlying stocks,  $y$  is the dividend yield on the stocks and  $T - t$  is the remaining life of the futures contract. Equation (1) is justified by a “no-arbitrage” assumption, since  $F_t > S_t e^{(r-y)(T-t)}$  would enable investors to profit by selling futures and buying stocks, while  $S_t e^{(r-y)(T-t)} > F_t$  would allow profits by buying futures and short selling stocks. The assumptions that underlie these arguments are that markets are perfectly efficient, and that transaction costs are zero. This

simple version of the model also assumes that the interest rate and dividend yield are constant over the life of the futures contract, although in practice they will vary, as will  $r - y$ , the net cost of carry of the underlying stocks.

Market efficiency implies that  $\ln S_t$  is a random walk, and that the returns denoted by  $s_t = \Delta \ln S_t$  are serially uncorrelated. The cost of carry model then implies that  $\ln F_t$  will be the sum of the random walk process in  $\ln S_t$  and the series  $(r - y)(T - t)$ . The dynamic properties of  $\ln F_t$  then depend on the assumptions about  $(r - y)(T - t)$ . When working with tick by tick data it is reasonable to assume that  $T$  is sufficiently distant to justify the treatment of  $(T - t)$  as a constant, and together with the assumptions that  $r$  and  $y$  are constant, the basis given by  $b_t = \ln F_t - \ln S_t$  is simply a constant<sup>1</sup>. Constant  $b_t$  are not observed in practice, but in this over-simplified case,  $\ln F_t$  follows the same random walk process as  $\ln S_t$ , and the returns denoted by  $f_t = \Delta \ln F_t$  are perfectly correlated with  $s_t$  and serially uncorrelated. The two return series  $f_t$  and  $s_t$  will have zero cross-correlations at all non-zero leads and lags. Given that  $b_t$  is not constant, it is common to allow for this by writing

$$b_t = \ln F_t - \ln S_t = \mu + v_t \quad (2)$$

in which  $\mu$  is interpreted as the expected cost of carry, and  $v_t = b_t - \mu$  (with  $E(v_t) = 0$ ) is known as the mis-pricing error. See Brenner and Kroner (1995) for further discussion on the stochastic implications of equation (1).

Empirical work has shown that  $\ln S_t$  and  $\ln F_t$  are not pure random walks, and that both include significant mean reverting components. There is also considerable evidence that  $\ln S_t$  and  $\ln F_t$  are cointegrated (i.e. they share the same random walk component). The literature has typically dealt with the observed correlations in returns data by attempting to “correct” for them (some examples include Stoll and Whaley (1990), and Shyy et al (1996)), or by explicitly modeling these dynamics (see, e.g., Wahab and Lashgari (1993), Brenner and Kroner (1995)). The latter approach, which also accounts for the cointegration is based on an error correction formulation given by

$$\begin{aligned} f_t &= c_f + \alpha_f(L)f_{t-1} + \beta_f(L)s_{t-1} + \gamma_f b_{t-d} + \varepsilon_t^f \\ s_t &= c_s + \alpha_s(L)f_{t-1} + \beta_s(L)s_{t-1} + \gamma_s b_{t-d} + \varepsilon_t^s \end{aligned} \quad (3)$$

in which  $\alpha_f(L)$ ,  $\beta_f(L)$ ,  $\alpha_s(L)$ , and  $\beta_s(L)$  are polynomials in the lag operator, and  $\varepsilon_t^f$  and  $\varepsilon_t^s$  are zero mean, serially uncorrelated errors that can be contemporaneously correlated. Equation (3) implies that returns will respond to

movements in the basis, consistent with arbitrage activities and corresponding mean reversion in the basis. As noted by Miller et al (1994), some of the mean reversion is due to corrections for infrequent trading, as stock indices “catch up” with futures. The parameter  $d$  takes no special meaning in a linear setting (since one can always reparameterise the lagged polynomials to obtain equivalent models regardless of  $d$ ), but it becomes important in the nonlinear models below. The assumption that the  $\gamma$  are not zero corresponds to a “no frictions” assumption, because it implies that returns respond to *all* movements in the basis. The presence of the lagged polynomials allows for short run dynamics, which might arise because of problems associated with the calculation and reporting of the indices.

The adaptation of equation (3) to account for transaction costs is based on the intuition that arbitrage will only occur when it generates net profits to investors. Defining  $c$  to be investors’ transaction cost and  $d$  to be a delay associated with making the appropriate trades, the arbitrage condition is given by  $|b_{t-d} - \mu| > c$ , which implies a no-arbitrage band given by  $-c < |b_{t-d} - \mu| < c$ . The transaction costs are assumed to be the same for all investors, and the same regardless of whether one is going short or long in the underlying stocks. The corresponding error correction model becomes

$$\begin{aligned} f_t &= c_{fi} + \alpha_{fi}(L)f_{t-1} + \beta_{fi}(L)s_{t-1} + \gamma_{fi}b_{t-d} + \varepsilon_t^f \\ s_t &= c_{si} + \alpha_{si}(L)f_{t-1} + \beta_{si}(L)s_{t-1} + \gamma_{si}b_{t-d} + \varepsilon_t^s \end{aligned} \quad (4)$$

which is a threshold error correction model (see, Balke and Fombey (1997)). The basis  $b_{t-d}$  drives the error correction process, and the threshold  $c$  defines three behavioural regimes in which

$$\begin{aligned} i &= 1 && \text{if } b_{t-d} - \mu < -c \text{ (i.e. if } v_{t-d} < -c) \\ i &= 2 && \text{if } -c < b_{t-d} - \mu < c \text{ (i.e. if } -c \leq v_{t-d} \leq c) \\ i &= 3 && \text{if } b_{t-d} - \mu > c \text{ (i.e. if } v_{t-d} > c) \end{aligned} \quad (4a)$$

We expect  $\gamma_{f2}$  and  $\gamma_{s2}$  to be zero because arbitrage will not generate net profits when  $|b_{t-d} - \mu| < c$ , (or equivalently when  $|v_{t-d}| < c$ ). One can generalise this model to allow for non-symmetric thresholds, and more than three behavioural regimes. The papers by Yadev (1994), Dywer et al (1996) and Martens et al (1998) are all set within this framework. They find statistically significant evidence in favour of transaction cost thresholds, but have difficulty in relating the estimated thresholds to average transaction costs. Martens et al (1998) find

evidence of many thresholds in their data, and attribute some of these thresholds to transaction costs and other thresholds to the effects of infrequent trading. These thresholds are not symmetrically distributed around  $\mu$ , which suggests differences between the responses to negative and positive pricing errors.

The smooth transition error correction model modifies (4) to obtain

$$\begin{aligned} f_t &= c_f^1 + \alpha_f^1(L)f_{t-1} + \beta_f^1(L)s_{t-1} + \gamma_f^1 b_{t-d} \\ &\quad + \Psi_f(v_{t-d})[c_f^2 + \alpha_f^2(L)f_{t-1} + \beta_f^2(L)s_{t-1} + \gamma_f^2 b_{t-d}] + \varepsilon_t^f \\ s_t &= c_s^1 + \alpha_s^1(L)f_{t-1} + \beta_s^1(L)s_{t-1} + \gamma_s^1 b_{t-d} \\ &\quad + \Psi_s(v_{t-d})[c_s^2 + \alpha_s^2(L)f_{t-1} + \beta_s^2(L)s_{t-1} + \gamma_s^2 b_{t-d}] + \varepsilon_t^s, \end{aligned} \quad (5)$$

in which  $\Psi_f$  and  $\Psi_s$  are exponential (ESTAR) transition functions, defined by

$$\Psi_j = [1 - \exp(-\frac{\lambda_j}{\sigma_v^2}(v_{t-d})^2)] \quad \text{for } j = f, s. \quad (5a)$$

The  $\Psi_j$  take values between zero and one, and they are monotonically increasing with the absolute size of the pricing error. As the  $\Psi_j$  vary, the VECM parameters also vary, implying that the nature of the price adjustment process changes with the size of the pricing error. As discussed in Anderson (1997), if one views the transition function  $\Psi$  as a cumulative density for the distribution of (non-negative) transaction cost thresholds, then relative to a baseline case without frictions, the parameters change more, when  $|v_{t-d}|$  is larger and a greater proportion of investors find the prospect of arbitrage more profitable. Taylor et al (2000) interpret their models in this way, setting  $\gamma_f^1 = \gamma_s^1 = 0$ , and then letting the smooth transition inherent in the  $\Psi_j$  account for multiple regimes implied by heterogenous transaction costs. In their model, the response to pricing errors becomes more pronounced, the larger the absolute size of that error, and error correction is effective only when  $|v_{t-d}|$  is sufficiently large.

[Insert Figure 1 about here]

One shortcoming of using smooth transition functions defined by the  $\Psi_j$ , rather than thresholds is that the symmetry in  $\Psi_j$  does not allow for asymmetries between the responses to negative and positive pricing errors. The use of transition functions such as  $\Psi_j$  also restrict the effective width of the implied “no-arbitrage bands”, unless the  $\lambda_j$  are very small and  $\Psi_j$  is approximately zero over a large range of  $v_{t-d}$ . Figure 1 compares the responses to pricing errors using the asymmetric threshold and a symmetric (ESTAR) smooth-transition



approach. While it is possible to modify the ESTAR model to account for asymmetries (see Anderson 1997), this paper specifies and employs a new smooth transition function that allows for wide “no-arbitrage” bands as well as asymmetries. This function allows for different behaviour, depending on whether the pricing error is positive or negative, and it is defined by

$$\Phi_{Pj} = \left[ \frac{1}{1 + \exp(-\frac{\lambda_{Pj}}{\sigma_v}(v_{t-d} - c_P))} - \frac{1}{1 + \exp(\frac{\lambda_{Pj}}{\sigma_v}c_P)} \right] \cdot \left[ \frac{1 + \exp(\frac{\lambda_{Pj}}{\sigma_v}c_P)}{\exp(\frac{\lambda_{Pj}}{\sigma_v}c_P)} \right] \quad (6a)$$

for  $v_{t-d} > 0$ , and

$$\Phi_{Nj} = \left[ \frac{1}{1 + \exp(\frac{\lambda_{Nj}}{\sigma_v}(v_{t-d} + c_N))} - \frac{1}{1 + \exp(\frac{\lambda_{Nj}}{\sigma_v}c_N)} \right] \cdot \left[ \frac{1 + \exp(\frac{\lambda_{Nj}}{\sigma_v}c_N)}{\exp(\frac{\lambda_{Nj}}{\sigma_v}c_N)} \right] \quad (6b)$$

for  $v_{t-d} \leq 0$ . The subscripts  $P$  and  $N$  respectively indicate those portions of the transition function that relate to positive and negative pricing errors. We call  $\Phi$  a U-STAR transition function, because its main characteristic is that it is shaped like a drunken U. The constants in the right hand brackets of these equations scale  $\Phi$  so that  $0 < \Phi < 1$ , and the constants inside the first brackets ensure that  $\Phi = 0$  at  $v_{t-d} = 0$ , and that  $\Phi$  is continuous at  $v_{t-d} = 0$ . See Figure 2 for some illustrations.

[Insert Figure 2 about here]

The U-STAR model then uses (6a) and (6b) in a specification given by

$$\begin{aligned} f_t &= c_f^0 + \alpha_f^0(L)f_{t-1} + \beta_f^0(L)s_{t-1} + \gamma_f^0 b_{t-d} \\ + I(v_{t-d} > 0) \cdot \Phi_{Pf}(v_{t-d}) [c_f^P + \alpha_f^P(L)f_{t-1} + \beta_f^P(L)s_{t-1} + \gamma_f^P b_{t-d}] \\ + I(v_{t-d} \leq 0) \cdot \Phi_{Nf}(v_{t-d}) [c_f^N + \alpha_f^N(L)f_{t-1} + \beta_f^N(L)s_{t-1} + \gamma_f^N b_{t-d}] + \varepsilon_t^f \\ s_t &= c_s^0 + \alpha_s^0(L)f_{t-1} + \beta_s^0(L)s_{t-1} + \gamma_s^0 b_{t-d} \\ + I(v_{t-d} > 0) \cdot \Phi_{Ps}(v_{t-d}) [c_s^P + \alpha_s^P(L)f_{t-1} + \beta_s^P(L)s_{t-1} + \gamma_s^P b_{t-d}] \\ + I(v_{t-d} \leq 0) \cdot \Phi_{Ns}(v_{t-d}) [c_s^N + \alpha_s^N(L)f_{t-1} + \beta_s^N(L)s_{t-1} + \gamma_s^N b_{t-d}] + \varepsilon_t^s, \quad (6) \end{aligned}$$

which implies a total response to pricing errors of  $\gamma_j^0 + I(v_{t-d} > 0) \cdot \gamma_j^P \Phi_{jP}(v_{t-d}) + I(v_{t-d} \leq 0) \cdot \gamma_j^N \Phi_{jN}(v_{t-d})$ . Since the automation of the futures market should lower transaction costs and make the adjustment of portfolios easier, we expect smaller “no arbitrage bands” in the latter part of our sample. For  $\gamma_j^0 \simeq 0$  this

corresponds to a thinner U (  $|c_P|$ , and  $|c_N|$  will be smaller), with steeper sides ( $\lambda_P$  and  $\lambda_N$  will be bigger). Since the stock market in Australia was automated prior to that in the futures market, the automation of the futures market should also reduce the differences between trade based on each index, and therefore reduce the presence of asymmetries. This corresponds to a decrease in the *difference* between  $|c_P|$ , and  $|c_N|$ , and a decrease in the *difference* between  $\lambda_P$  and  $\lambda_N$ . Similar patterns in  $\Phi_j$  might also be expected even when  $\gamma_j^0 \neq 0$ , although some response to pricing errors (i.e  $\gamma_j^0$ ) is now present at all times.

### III. INSTITUTIONAL DETAILS AND DATA

Our empirical analysis is based on the All Ordinaries Index (AOI), calculated by the Australian Stock Exchange (ASX). Based on market capitalisation, the ASX is the 12th largest share market in the world, and the second largest in the Asia Pacific Region. At the end of 1999, the AOI was based on 253 actively traded stocks and it accounted for 91% of listed Australian equities. The Australian Stock Exchange trades between 10.00am and 4.00pm (EST) from Monday to Friday (public holidays excluded). Opening times for individual stocks are staggered but all stocks are trading by 10.10, and at the end of the day additional trading at volume weighted prices may continue until 4.20pm. Stock trading has been fully automated since 1991, when the Stock Exchange Automated Trading System (SEATS) was introduced. SEATS continuously matches bids and offers during normal trading hours, and updates the price indices. It also disseminates this updated information to data vendors, at frequencies which are usually more than once every minute.

The Sydney Futures Exchange (SFE) has been trading Share Price Index (SPI<sup>®</sup>) futures contracts based on the AOI since 1983, when it became the first exchange outside the USA to list index futures. Almost 20,000 SPI<sup>®</sup> contracts are traded each day, with most trading occurring in contracts with the next expiry month. Contracts mature at the end of March, June, September and December each year. Unlike the ASX, the SFE has not been fully automated until very recently, with trading in SPI<sup>®</sup> futures becoming fully automatic on October 4, 1999. Trading on the floor occurred between 9.50am to 12.30pm, and then from 2.00 pm until 4.10 pm (EST) from Monday to Friday (public holidays excluded) until November 12, 1999. Since then, day-time trading hours have been extended, with trading starting at 9.30am and continuing until 4.30pm.

Standard & Poors took over the management of the ASX indices in April

2000. This change has led to an expansion of the All Ordinaries Index to cover 500 stocks, and the introduction of two new indices known as the S&P/ASX 200 and the S&P/ASX 300, which are respectively based on 200 and 300 stocks. SPI<sup>®</sup> futures contracts based on the old All Industries Index are still being issued, but are being phased out as new futures contracts (called SPI200 and based on the S&P/ASX200) are being phased in. The first SPI200 contracts were listed in May 2000 and expired in June 2000, while the last SPI<sup>®</sup> contracts will expire in September 2000. The old All Ordinaries Index (based on 253 stocks) is still calculated, but will be discontinued after September 2001.

The data used in this study is tick by tick AOI data obtained from IRESS (Integrated Real Time Equity System) and matching tick by tick SPI<sup>®</sup> data obtained from the SFE. The samples covered the last two weeks of August in 1999, and the first two weeks of November 1999. The AOI is updated on IRESS approximately twice a minute, (to the nearest 0.1 index point), and this was converted to one observation per minute by using the last observation for each minute. The SPI data for August listed the time (to the nearest second), volume (number of contracts) and index value (to the nearest integer) for each transaction, but the November data recorded trade times in minutes, rather than in seconds. Minute by minute futures index values were obtained by weighting the index for each trade by its volume. The last available observation was used when data was missing. Only those contracts for futures expiring in September 1999 were included in the August data set, and only those contracts for futures expiring in December 1999 were included in the November data set.

Both markets were open between 10.00 am to 12.30 pm and then from 2.00 pm to 4.00, which led to 271 matched observations each day. However, the first 15 minutes of each day were discarded to avoid anomalies related to the staggered opening of the ASX. The daily samples of the stock and futures indices were each demeaned using the remaining observations (from 10.16 to 12.30 and 2.00pm to 4.00pm); demeaning the futures index accounted for dividends and interest rates (which were assumed to be constant for each day), while demeaning the stock index allowed the scaling of the basis to be centered on zero<sup>2</sup>. The analysis was then based on samples covering 10.30 am to 12.30 pm and 2.15 pm to 4.00 pm (226 observations) for each day, which allowed for the inclusion of up to 15 contiguous lags in our autoregressions. In total, there were thirteen days of data for August 1999 (2938 observations) and ten days of data for November 1999 (2260 observations), making 5198 observations altogether.

#### IV. EMPIRICAL ANALYSIS

It is useful to examine the properties of the returns and the basis prior to modelling their dynamics, and some summary statistics relating to the de-meaned data are provided in Table 1. Returns for futures were more volatile than those for stocks, and the variability of futures increased slightly after the automation of that market. The basis was slightly skewed, and although its variability increased after the automation of the futures market, its range decreased. All reported first order autocorrelation coefficients (excepting the futures return for November) were statistically significant, and formal tests indicated stronger first order autocorrelation in returns prior to the cessation of floor-trading in futures, and stronger negative first order autocorrelation in basis changes after the shift to electronic trading. The reported statistics for unit root analysis are the *averages* of Dickey Fuller unit root test statistics for each of the daily samples. Tests relating to the full samples would have been misleading given the removal of overnight returns and the use of different de-meaning transformations for different days. The averages of the Dickey Fuller are indicative only, but compared to the usual critical values (-2.88 for  $\ln(\text{price})$  and -1.95 for the returns and the basis) they suggest that the log prices have a unit root, and that returns and the basis are stationary. All 46 of the underlying tests on daily data supported a unit root in  $\ln(\text{prices})$ , all but four<sup>3</sup> of the tests rejected a unit root in returns, and all but six<sup>3</sup> rejected a unit root in the basis.

[Insert Tables 1A and 1B about here]

Table 2 reports some summary statistics relating to three linear VECM(12) specifications that provide the point of departure for our nonlinear modelling exercise. These models are based on the full sample, the August sample and the November sample, and full details are provided in Appendix 1. The above ADF tests justified our error correction representation, and while AIC suggested that five lags would be sufficient to model the *linear* dynamics of returns, we worked with a longer lag structure to allow for the possibility that AIC might not choose the optimal lag structure for our *nonlinear* models. The longer lag structure also provides a broader picture of the “lead-lag” relationship.

For the full sample, the error correction term is negative in the futures equation and positive in the stocks equation (as expected), with both results

being statistically significant. Each of the first ten lags of futures returns are statistically significant in the stock returns equation, in line with previous findings that returns for futures lead returns for stocks. The effect of lagged stock returns on futures is statistically significant for two lags, but changes sign at lag three and becomes insignificant after lag four.

[Insert Table 2 about here]

The VECM(12)s for the August and November subsamples are similar to the full sample model, in that the error correction coefficients are statistically significant and negative in the future returns equations and statistically significant and positive in the stock returns equations. Comparing the August and November equations, the future returns equation changes very little, although more lags of stocks have predictive power for futures in August (5 lags), than in November (2 lags). A heteroscedasticity corrected test of no change has a p-value of 0.1013. The changes in the equation for stock returns are more pronounced, with the futures lead increasing from about eight minutes in August to ten minutes in November. For this equation, a heteroscedasticity corrected test of no change strongly rejects the null, with a p-value of 0.0001. For each return, the overall level of significance (as measured by the p-value for the overall F-test) is lower in November than August, suggesting that returns have become less predictable since the automation in the futures market.

[Insert Table 3 about here]

Table 3 reports the results of heteroscedasticity corrected tests of the linear VECM(12) against various ESTAR alternatives. Each of these alternatives uses the lagged basis as a transition variable, but the transition lag is allowed to vary from one up to twelve. The tests are performed on a model of the basis as well as on the returns equations, because ESTAR behaviour in the basis will imply ESTAR behaviour in the returns; this model of the basis had the same explanators as the VECM(12). Given the similarities between the ESTAR and USTAR specifications, one would expect the ESTAR tests to have power against USTAR alternatives. The tests are based on second order approximations to the nonlinear alternative, and they assess whether the explanatory power of the linear equations increase, when one adds additional regressors that interact  $v_{t-d}^2$  with all of the VECM explanators.

Given the differences between the August and the November models noted above, and the fact that we wanted to assess the effect of closing futures floor trading, the nonlinearity tests were performed on each subsample separately. For August, nearly all of the tests found strong evidence of nonlinearity associated with movements in the basis. The p-values of the tests on returns and the basis were all minimised when  $d = 6$ , which suggests a lag of six minutes between pricing errors and nonlinear adjustment in returns. For November, the results were less significant and not as clear, but they supported a specification using  $d = 6$  for each of the returns equation. The basis did not show evidence of nonlinearity for any value of  $d$ , which together with the contrasting August results suggests that the process for pricing errors has changed since automation. Linearity in the basis also casts doubt about the presence of a “no-arbitrage band”. The fact that  $d = 6$  is not a suitable transition variable for the basis despite its suitability for each return is interesting for another reason, because this is consistent with a common nonlinear factor in returns. See Anderson and Vahid (1998) for details on common nonlinear factors.

We next set  $d = 6$  and then estimated the implied USTAR models. Given the long lag structure and the complicated nature of the nonlinearity, we used a two stage estimation process, that involved a grid search for the transition parameters during the first stage. For the August sample we chose to work with estimates of the transition function for the basis, since the nonlinearity in the returns was associated with the nonlinear movement in the basis. The estimated transition parameters for the basis were then incorporated as fixed transition parameters in the second stage of estimation, which involved estimating the other parameters for the equations for stock returns and futures returns. For November we adopted a different approach, since the basis was linear. Here, we first used a grid search to estimate the transition function for the stock returns (which we chose in preference to the futures equation because stock returns are more predictable than futures returns), and we then estimated the other parameters for the stock return equation. We then used this, together with an estimated linear equation for the basis, to deduce the futures equation. This latter technique imposed the common factor restriction implied by the nonlinearity tests.

[Insert Tables 4A and 4B about here]

Summary statistics relating to each nonlinear VECM(12) are presented in

Table 4, and the estimated transition functions are illustrated in Figure 3. Full details are provided in Appendix 2. The lag structure in these nonlinear models was richer than that in the linear VECMS. In the nonlinear model, the futures lead over stocks was 10 minutes in August and increased to 11 minutes in November, while stocks could predict futures for up to 12 minutes ahead in August, but for only 8 minutes in November. An important property of these models is that the predictability of each type of return decreased after automation (the  $R^2$  dropped quite substantially), consistent with a decline in the *strength* of the lead-lag relationship. The error correction terms were statistically significant in most regimes for the August equations (including the middle regimes), which provides evidence against restricting  $\gamma_{f1} = 0$  and  $\gamma_{s1} = 0$  (as in Taylor et al (2000)). Thus, although the strength of error correction changes with the pricing error, the usual transactions cost interpretation of “no-arbitrage” bands does not provide the whole story in this case. There are other factors affecting the mean reversion process, which include infrequent trading and complications associated with trading portfolios of stocks. For November, the error correction terms were not statistically significant, although the lagged basis still played a crucial role in that it generated the transition between different behavioural regimes<sup>4</sup>.

[Insert Figure 3 about here]

Figure 3 shows that the boundaries of the behavioural regimes for August and November are different, with the inner band for pricing errors being more symmetric for November than for August, and thinner. The increased symmetry suggests that the automation of the futures market has reduced some of the practical differences between responding to positive and negative pricing errors, and the thinner band implies that smaller pricing errors will now induce regime shifts.

It is hard to interpret autoregressive parameters in time series models, and nonlinearity further complicates interpretation. We therefore study the dynamic properties of our models by analysing their generalised impulse response functions<sup>5</sup>. We trace the impacts of shocks to futures and stocks on movements in the basis, assuming that the basis is initially zero and the market is in equilibrium. The size of the shocks that we consider are approximately one standard deviation of the basis (about 0.07) and two standard deviations, and given that it is often believed that shocks to the stock index are firm specific and differ-

ent from shocks to the futures index which reflect macroeconomic shocks (see Frino et al, 2000), we consider two extreme cases. In the first case a positive (negative) shock to the basis is caused purely by a positive (negative) shock in the futures market, and we call this sort of shock a “macroeconomic” shock. In the second case a positive (negative) shock to the basis is caused purely by a negative (positive) shock to the stock index, and we call this sort of shock a “firm specific” shock.

The generalised impulse response functions are illustrated in Figures 4 and 5. There are minor differences between the effects of the two types of shocks, and minor asymmetries between the effects of positive and negative shocks. There are no significant differences between the response functions for August and November. The half lives of all shocks are short, but in each case it takes more than an hour for equilibrium to be restored.

[Insert Figure 4 and Figure 5 about here]

## V. CONCLUSION

This paper examines the impact of screen trading in futures on the nonlinear properties of the basis and returns. We use a new form of smooth transition model to account for nonlinearities caused by transaction costs and other market/data imperfections, and we study the properties of these models by inspecting their implied responses to various shocks. We find strong evidence of nonlinearity before the futures trading went on-line, and weaker evidence of nonlinearity after on-line trading. Our analysis suggests that the automation of the futures market has made the nonlinear properties of the stock market and the futures market more similar, and that after the introduction of on-line futures trading, the returns in each market have a common nonlinear factor. Futures returns lead stock returns (with feedback) both before and after the introduction of screen-trading, and the futures lead increases only slightly after automation. The speed of mean-reversion in the basis is slow, and appears to be unchanged. We find that even though the models are statistically different, their implications as shown by their impulse response functions are virtually the same.



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## Footnotes:

1. In practice, researchers working with tick by tick data often account for daily changes in  $(r - y)(T - t)$  by removing the daily averages of  $\ln S_t$  and  $\ln F_t$  from the data. See, eg, Dwyer et al (1996).
2. The assumptions that are made when demeaning the daily samples are that  $r$  and  $y$  are constant throughout each day, and that the futures contract expiry date  $T$  is far enough into the future to ensure that  $(T - t)$  is approximately constant throughout each day. The demeaning implies that  $b_t = v_t$  in our empirical work.
3. Two of these exceptions relate to Melbourne Cup Day, when Australians are much more interested in a horse race than they are in the stock market.
4. Further exploration based on unit root tests for the basis found weak evidence of “no-arbitrage bands” given by  $-0.009 < b_{t-6} < 0.009$  for August and  $-0.003 < b_{t-6} < 0.003$  for November, but we do not pursue this issue further here.
5. Unlike linear models, the expected response to shocks cannot be derived analytically, and are therefore derived by averaging over many simulated response paths (See Koop et al, 1996).

**TABLE 1A: SUMMARY STATISTIC RELATING TO THE  
INDICES AND RETURNS**

	August		November		Full Sample	
	Futures	Stocks	Futures	Stocks	Futures	Stocks
Max Price Index	3113.8	3083.4	3018.7	3009.0	3113.8	3083.4
Min Price Index	2905.5	2934.1	2880.2	2887.2	2880.2	2887.2
ADF for $\ln(\text{Price})$	-1.349	-1.327	-1.629	-1.855	-1.471	-1.557
Max Return	0.1360	0.1169	0.1296	0.1171	0.1360	0.1171
Min Return	-0.1240	0.1238	-0.1602	-0.1023	-0.1602	-0.1238
St Dev Return	0.0299	0.0209	0.0319	0.0197	0.0308	0.0204
$\rho_1$ for Return	0.1590	0.2294	0.0049	0.0837	0.0874	0.1705
ADF for Return	-4.330	-4.262	-4.229	-4.670	-4.389	-4.158
No of Observations	2938	2938	2260	2260	5198	5198

**TABLE 1B: SUMMARY STATISTICS RELATING TO THE  
BASIS**

	August	November	Full Sample
Mean	0.0000	0.0000	0.0000
Median	-0.0007	-0.0026	-0.0013
Max	0.3896	0.2377	0.3896
Min	-0.3113	-0.2122	-0.3113
St Dev	0.0688	0.0785	0.0732
Skewness	0.1511	0.1263	0.1388
Kurtosis	5.1954	2.6947	3.8450
$\rho_1$ for $\Delta b_t$	-0.1336	-0.1727	-0.1528
ADF	-2.1157	-2.1333	-2.1233
No of Observations	2938	2260	5198

Notes:  $\rho_1$  is the first order autocorrelation coefficient. The reported ADF statistics are arithmetic averages of the ADF test statistics for the daily samples. The ADF regressions for the  $\ln(\text{price})$  contained a constant and 12 lags, while the ADF regressions for the returns and the basis contained 12 lags.

**TABLE 2: LINEAR ERROR CORRECTION MODELS OF  
RETURNS FOR FUTURES AND STOCKS**

**FULL SAMPLE**

**Model of Futures Returns:**

Last statistically significant (at 5% level) lag of stock returns:  $s_{t-4}$   
Error correction coefficient ( $v_{t-1}$ ) with het. c.t-stat: -0.0287 (-3.9731)  
Summary statistics:  $R^2 = 0.049$ ,  $s.e. = 0.030$

**Model of Stock Returns:**

Last statistically significant (at 5% level) lag of futures returns:  $f_{t-10}$   
Error correction coefficient ( $v_{t-1}$ ) with het. c.t-stat: 0.0254 (5.369)  
Summary statistics:  $R^2 = 0.161$ ,  $s.e. = 0.019$

**AUGUST SAMPLE**

**Model of Futures Returns:**

Last statistically significant (at 5% level) lag of stock returns:  $s_{t-5}$   
Error correction coefficient ( $v_{t-1}$ ) with het. c.t-stat: -0.0276 (-2.5394)  
Summary statistics:  $R^2 = 0.069$ ,  $s.e. = 0.029$

**Model of Stock Returns:**

Last statistically significant (at 5% level) lag of futures returns:  $f_{t-8}$   
Error correction coefficient ( $v_{t-1}$ ) with het. c.t-stat: 0.0280 (3.9015)  
Summary statistics:  $R^2 = 0.212$ ,  $s.e. = 0.019$

**NOVEMBER SAMPLE**

**Model of Futures Returns:**

Last statistically significant (at 5% level) lag of stock returns:  $s_{t-2}$   
Error correction coefficient ( $v_{t-1}$ ) with het. c.t-stat: -0.0301 (-3.1313)  
Summary statistics:  $R^2 = 0.044$ ,  $s.e. = 0.031$

**Model of Stock Returns:**

Last statistically significant (at 5% level) lag of futures returns:  $f_{t-10}$   
Error correction coefficient ( $v_{t-1}$ ) with het. c.t-stat: 0.0215 (3.5346)  
Summary statistics:  $R^2 = 0.121$ ,  $s.e. = 0.019$

**TABLE 3: P-VALUES OF HETEROSCEDASTICITY  
CONSISTENT TESTS OF  
 $H_0$  : No Nonlinearity vs  $H_1$  : ESTAR type nonlinearity**

Transition Lag	August			November		
	Stocks	Futures	Basis	Stocks	Futures	Basis
1	.0004	.0000	.0041	.0466	.3900	.8914
2	.0009	.0000	.0016	.2432	<b>.0029</b>	.4106
3	.0000	.0011	.0026	.0258	.1892	.5832
4	.0004	.0001	.0009	.1360	.6489	.7423
5	.0001	.0000	.0000	.0789	.0789	<b>.1068</b>
6	<b>.0000</b>	<b>.0000</b>	<b>.0000</b>	.0211	.0202	.1611
7	.0000	.0001	.0000	.0057	.0797	.3576
8	.0014	.0032	.0313	.1149	.5574	.7632
9	.0615	.0142	.0586	.0100	.4731	.8928
10	.0034	.0067	.0200	.0380	.0750	.3305
11	.0373	.0330	.0407	.0436	.1842	.8670
12	.0004	.0184	.0231	<b>.0001</b>	.2348	.9009

Note: The minimum p-value found for each set of linearity tests is indicated in bold type.

**TABLE 4A: NONLINEAR ERROR CORRECTION MODEL OF  
RETURNS FOR FUTURES AND STOCKS**

**AUGUST SAMPLE**

**Model of Futures Returns:**

Last statistically significant (at 5% level) lag of stock returns:  $s_{t-12}$

Coefficients and heteroscedasticity corrected t-stats for error correction terms:

$$\gamma^N: \quad 0.0375 \quad (0.692)$$

$$\gamma^0: \quad -0.0442 \quad (-2.528)$$

$$\gamma^P: \quad -0.1321 \quad (-1.256)$$

Summary statistics:  $R^2 = 0.102$ ,  $s.e. = 0.028$

**Model of Stock Returns:**

Last statistically significant (at 5% level) lag of futures returns:  $f_{t-10}$

Coefficients and heteroscedasticity corrected t-stats for error correction terms:

$$\gamma^N: \quad -0.1076 \quad (-2.648)$$

$$\gamma^0: \quad 0.0518 \quad (4.788)$$

$$\gamma^P: \quad 0.1664 \quad (2.492)$$

Summary statistics:  $R^2 = 0.248$ ,  $s.e. = 0.018$

Correlation between errors for the two equations is 0.365

Implied s.e. of basis = 0.028.

Transition Function:

$$\Phi_{Pt} = \left[ \frac{1}{1 + \exp(-\frac{3.42}{.0688}(v_{t-6} - 0.13))} - \frac{1}{1 + \exp(\frac{3.42}{.0688} \times 0.13)} \right] \cdot \left[ \frac{1 + \exp(\frac{3.42}{.0688} \times 0.13)}{\exp(\frac{3.42}{.0688} \times 0.13)} \right]$$

for  $v_{t-6} > 0$

$$\Phi_{Nt} = \left[ \frac{1}{1 + \exp(\frac{10}{0.0688}(v_{t-6} + 0.09))} - \frac{1}{1 + \exp(\frac{10}{0.0688} \times 0.09)} \right] \cdot \left[ \frac{1 + \exp(\frac{10}{0.0688} \times 0.09)}{\exp(\frac{10}{0.0688} \times 0.09)} \right]$$

for  $v_{t-6} < 0$

The transition variable  $v_{t-6}$  is  $< -0.09$  for 216 observations, between  $-0.09$  and  $0.13$  for 2616 observations, and  $> 0.13$  for 106 observations.

**TABLE 4B: NONLINEAR ERROR CORRECTION MODEL OF  
RETURNS FOR FUTURES AND STOCKS**

**NOVEMBER SAMPLE**

**Model of Futures Returns:**

Last statistically significant (at 5% level) lag of stock returns:  $s_{t-8}$

Coefficients and heteroscedasticity corrected t-stats for error correction terms:

$$\gamma^N: \quad -0.0929 \quad (-1.294)$$

$$\gamma^0: \quad -0.0091 \quad (-0.282)$$

$$\gamma^P: \quad -0.0171 \quad (-0.395)$$

Summary statistics:  $R^2 = 0.043$ ,  $s.e. = 0.031$

**Model of Stock Returns:**

Last statistically significant (at 5% level) lag of future returns:  $f_{t-11}$

Coefficients and heteroscedasticity corrected t-stats for error correction terms:

$$\gamma^N: \quad 0.0560 \quad (1.379)$$

$$\gamma^0: \quad 0.0208 \quad (1.180)$$

$$\gamma^P: \quad 0.0146 \quad (0.567)$$

Summary statistics:  $R^2 = 0.163$ ,  $s.e. = 0.018$

Correlation between errors for the two equations is 0.274.

Implied s.e. of basis = 0.031.

Transition Function:

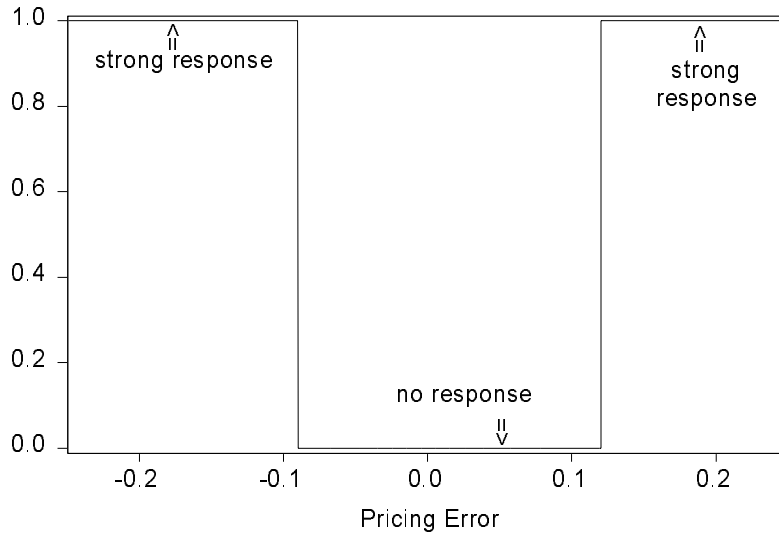
$$\begin{aligned} \Phi_{Pt} &= \left[ \frac{1}{1 + \exp(-\frac{10}{.0785}(v_{t-6} - 0.04))} - \frac{1}{1 + \exp(\frac{10}{.0785} \times 0.04)} \right] \cdot \left[ \frac{1 + \exp(\frac{10}{.0785} \times 0.04)}{\exp(\frac{10}{.0785} \times 0.04)} \right] \\ \text{for } v_{t-6} &> 0 \\ \Phi_{Nt} &= \left[ \frac{1}{1 + \exp(\frac{10}{.0785}(v_{t-6} + 0.09))} - \frac{1}{1 + \exp(\frac{10}{.0785} \times 0.09)} \right] \cdot \left[ \frac{1 + \exp(\frac{10}{.0785} \times 0.09)}{\exp(\frac{10}{.0785} \times 0.09)} \right] \\ \text{for } v_{t-6} &< 0 \end{aligned}$$

The transition variable  $v_{t-6}$  is  $< -0.09$  for 216 observations, between  $-0.09$  and  $0.04$  for 2009 observations, and  $> 0.04$  for 713 observations.



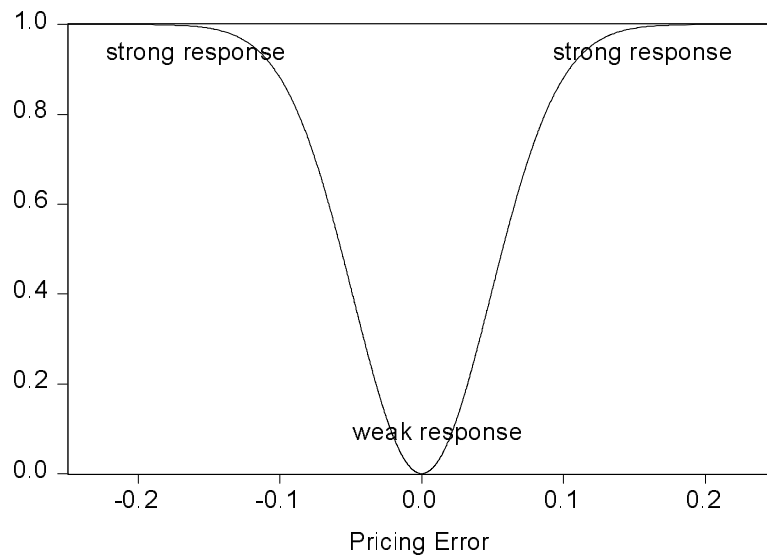
**Figure 1: Response to Pricing Errors**

Threshold Transition Function ( $\gamma$  in eqn (4))



In this illustration,  $\gamma_1 = \gamma_3 = 1$ , and  $\gamma_2 = 0$ . The symmetric thresholds (i.e.  $-c$  and  $c$  in eqn (4a)) have been replaced by asymmetric thresholds ( $-0.09$  and  $.12$ ).

ESTAR Transition Function ( $\Psi$  in eqn (5))



In this illustration,  $\frac{\lambda_i}{\sigma_v^2} = 1$  in eqn (5a).

Figure 2: U-STAR Transition Functions

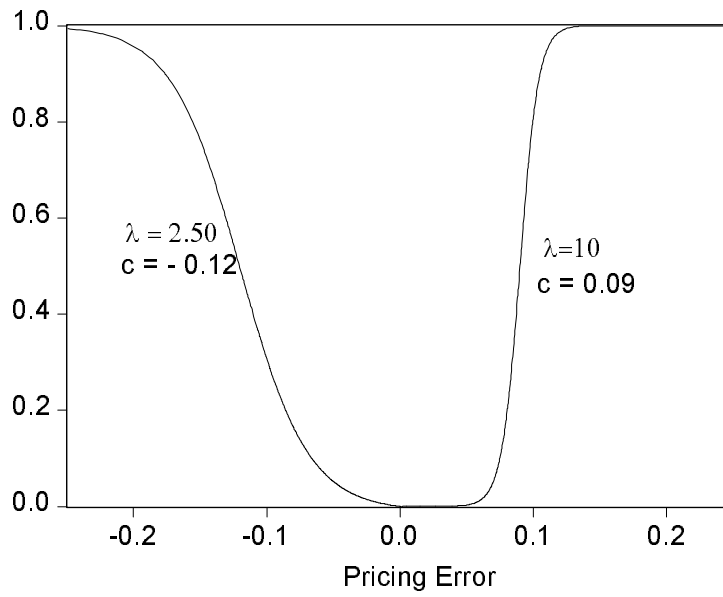
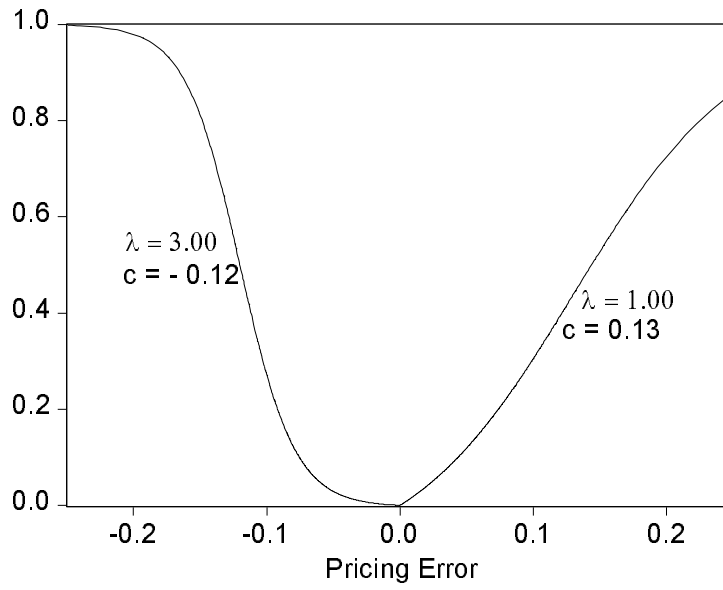


Figure 3: Estimated Transition Functions

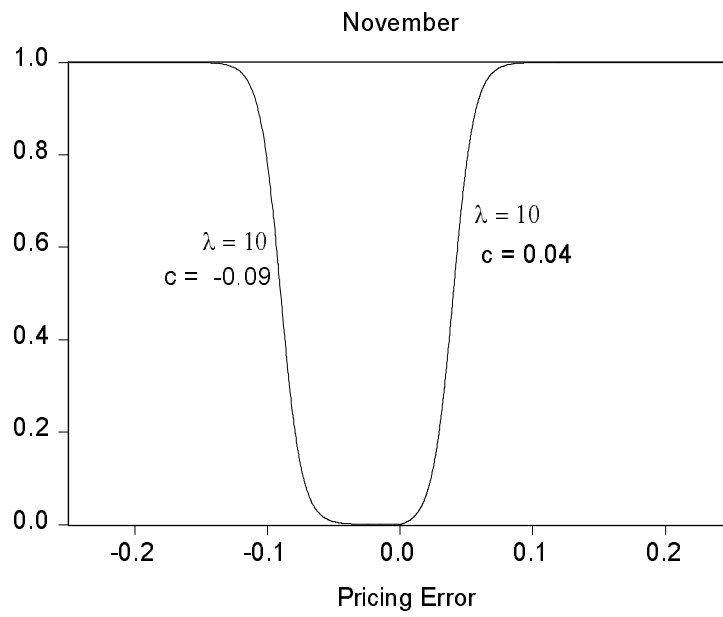
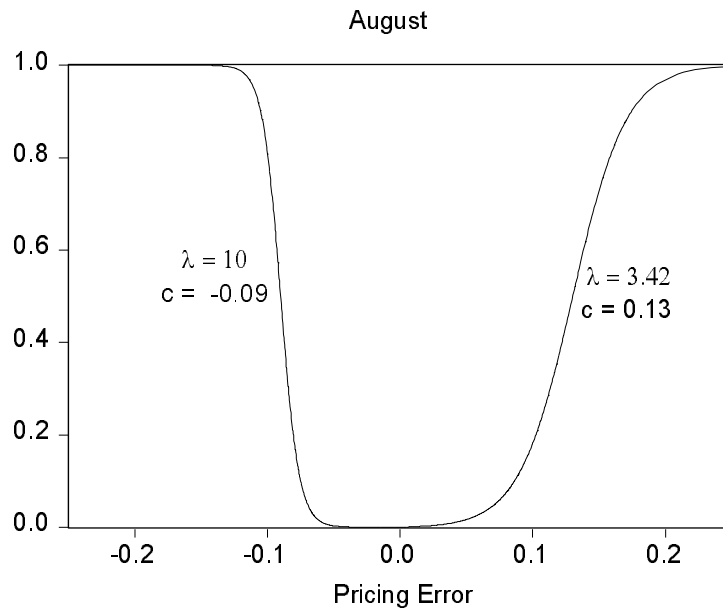


Figure 4: Mean Reversion in the Basis

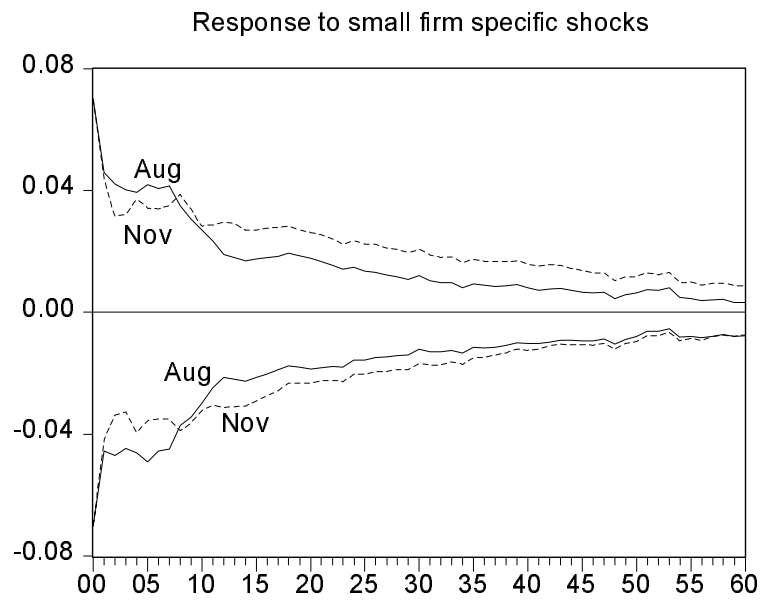
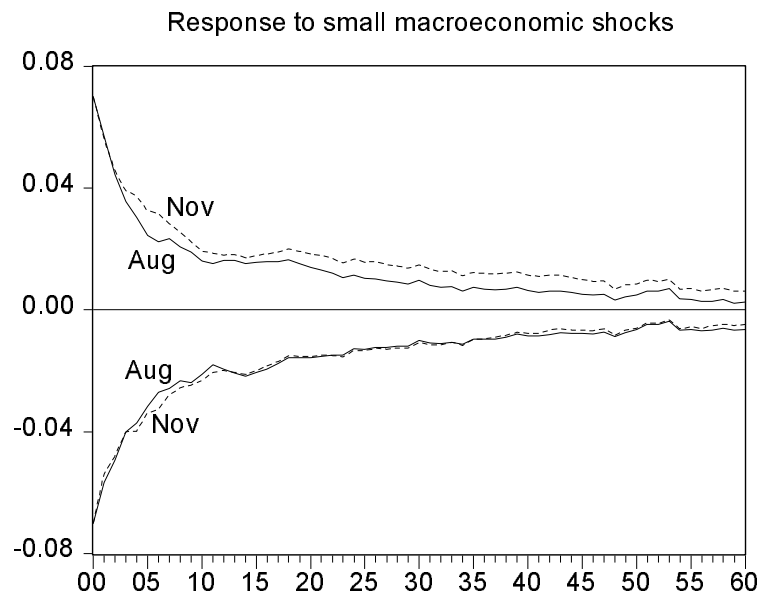
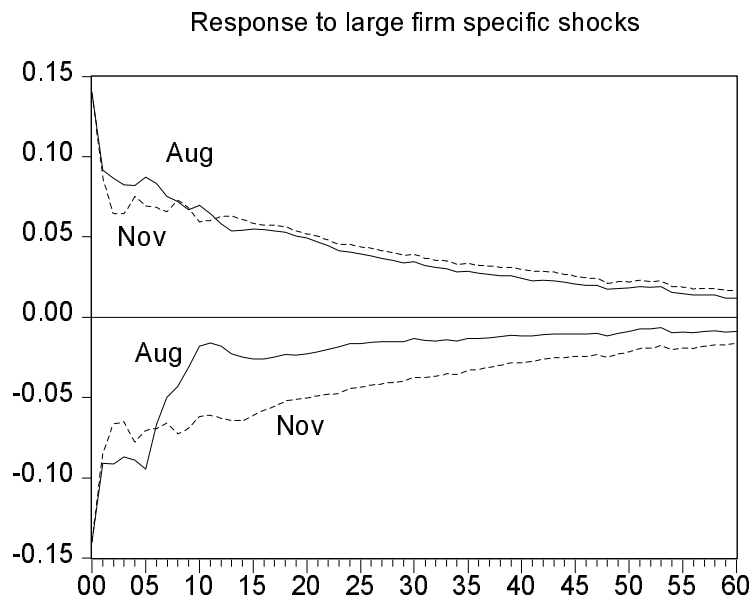
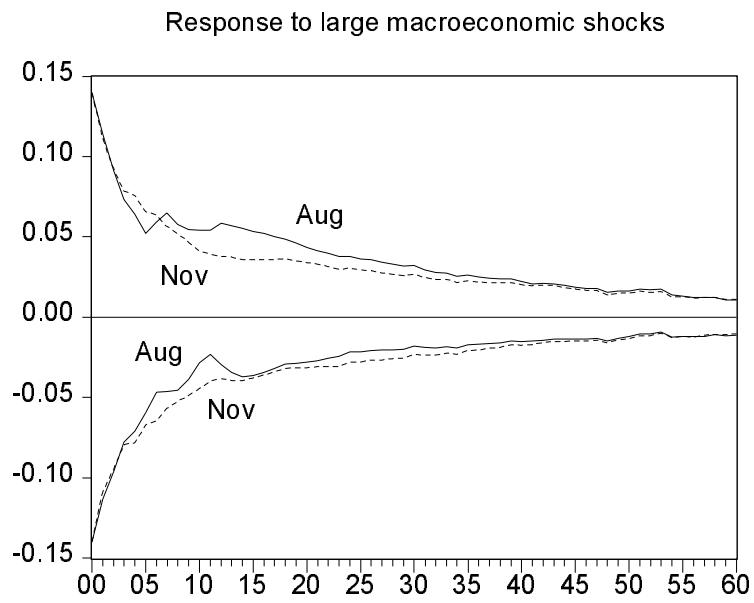


Figure 5: Mean Reversion in the Basis



**APPENDIX 1A: LINEAR ERROR CORRECTION MODEL FOR  
THE FULL SAMPLE**

Variable	Returns for Futures		Returns for Stocks	
	Coef	het.c t-stat	Coef	het.c. t-stat
const	-0.0002	-0.4523	-0.0000	-0.1316
$v_{t-1}$	-0.0287	-3.9731	0.0254	5.3686
$f_{t-1}$	0.0400	2.0193	0.1771	15.6057
$f_{t-2}$	-0.0340	-1.8107	0.1117	10.3028
$f_{t-3}$	-0.0296	-1.5672	0.0933	8.2279
$f_{t-4}$	-0.0006	-0.0320	0.0545	4.9544
$f_{t-5}$	-0.0095	-0.5568	0.0565	5.0410
$f_{t-6}$	-0.0034	-0.1933	0.0246	2.2409
$f_{t-7}$	0.0103	0.6051	0.0264	2.4878
$f_{t-8}$	-0.0027	-0.1648	0.0233	2.0659
$f_{t-9}$	0.0015	0.0853	0.0213	1.9916
$f_{t-10}$	-0.0149	-0.8755	0.0206	1.9126
$f_{t-11}$	-0.0104	-0.6304	0.0190	1.777
$f_{t-12}$	0.0059	0.3594	0.0087	0.8354
$s_{t-1}$	0.2903	10.92224	-0.0084	-0.4713
$s_{t-2}$	0.0627	2.3194	-0.0413	-2.2830
$s_{t-3}$	-0.0186	-0.7046	-0.0548	-3.1965
$s_{t-4}$	-0.06422	-2.4054	-0.0349	-2.0721
$s_{t-5}$	-0.0276	-1.0767	-0.0338	-1.9466
$s_{t-6}$	-0.0059	-0.2295	-0.0191	-1.1448
$s_{t-7}$	-0.0444	-1.8100	-0.0437	-2.6382
$s_{t-8}$	-0.0149	-0.6180	-0.0169	-1.0355
$s_{t-9}$	0.0337	1.4646	0.0032	0.2001
$s_{t-10}$	0.0147	0.6366	-0.0242	-1.6130
$s_{t-11}$	0.0233	1.1217	-0.0140	-0.9581
$s_{t-12}$	-0.0033	-0.1593	-0.0265	-1.7854
$R^2$	0.0488		0.1612	
$s.e$	0.0301		0.0187	

Note: The pricing error  $v_{t-1} = b_{t-1} - \mu$  is used as the error correction term.

**APPENDIX 1B: LINEAR ERROR CORRECTION MODEL FOR  
THE AUGUST SAMPLE**

Variable	Returns for Futures		Returns for Stocks	
	Coef	het.c t-stat	Coef	het.c. t-stat
const	-0.0003	-0.5047	0.0000	0.0220
$v_{t-1}$	-0.0276	-2.5394	0.0280	3.9015
$f_{t-1}$	0.1030	4.1596	0.2168	13.3864
$f_{t-2}$	-0.0370	-1.5210	0.1243	8.2632
$f_{t-3}$	-0.0035	-0.1317	0.0985	6.1992
$f_{t-4}$	-0.0173	-0.7257	0.0466	2.8822
$f_{t-5}$	0.0067	0.2765	0.0567	3.4756
$f_{t-6}$	0.0064	0.2541	0.0332	2.1668
$f_{t-7}$	0.04487	1.8601	0.0242	1.6036
$f_{t-8}$	0.0140	0.5904	0.0321	2.0885
$f_{t-9}$	-0.0069	-0.2797	0.0154	0.9983
$f_{t-10}$	-0.0266	-1.1316	0.0093	0.6117
$f_{t-11}$	-0.0030	-0.1288	0.0214	1.4354
$f_{t-12}$	0.0138	0.5723	0.0015	0.1025
$s_{t-1}$	0.2831	7.9086	0.0000	0.0040
$s_{t-2}$	0.0205	0.5771	-0.0336	-1.4428
$s_{t-3}$	-0.0291	-0.8341	-0.0643	-2.7648
$s_{t-4}$	-0.0673	-1.8982	-0.0516	-2.2869
$s_{t-5}$	-0.0773	-2.2647	-0.0536	-2.3922
$s_{t-6}$	-0.0188	-0.5383	-0.0211	-0.9796
$s_{t-7}$	-0.0553	-1.6394	-0.0533	-2.4407
$s_{t-8}$	-0.0056	-0.1738	-0.0483	-2.2176
$s_{t-9}$	0.0325	1.0124	-0.0031	-0.1376
$s_{t-10}$	0.0184	0.6242	-0.0036	-0.1712
$s_{t-11}$	0.0384	1.3737	-0.0090	-0.4696
$s_{t-12}$	0.0054	0.1945	-0.0362	-1.7486
$R^2$	0.0694		0.2121	
$s.e$	0.0290		0.0187	

Note: The pricing error  $v_{t-1} = b_{t-1} - \mu$  is used as the error correction term

**APPENDIX 1C: LINEAR ERROR CORRECTION MODEL FOR  
THE NOVEMBER SAMPLE**

Variable	Returns for Futures		Returns for Stocks	
	Coef	het.c t-stat	Coef	het.c. t-stat
const	0.0000	0.0116	-0.0001	-0.2701
$v_{t-1}$	-0.0301	-3.1313	0.0215	3.5346
$f_{t-1}$	-0.0294	-0.9750	0.1334	8.5887
$f_{t-2}$	-0.0402	-1.4318	0.0930	6.0773
$f_{t-3}$	-0.0554	-2.1399	0.0861	5.4026
$f_{t-4}$	0.0063	0.2420	0.0640	4.289
$f_{t-5}$	-0.0243	-1.0098	0.0617	4.0238
$f_{t-6}$	-0.0186	-0.7495	0.0194	1.2551
$f_{t-7}$	-0.0278	-1.1550	0.0311	2.0984
$f_{t-8}$	-0.0307	-1.3406	0.0141	0.8690
$f_{t-9}$	0.0033	0.1357	0.0268	1.8369
$f_{t-10}$	-0.0080	-0.3314	0.0318	2.1267
$f_{t-11}$	-0.0177	-0.7459	0.0171	1.1424
$f_{t-12}$	-0.0085	-0.3784	0.0145	1.0294
$s_{t-1}$	0.2850	7.2584	-0.0483	-1.8785
$s_{t-2}$	0.1211	3.0051	-0.0645	-2.2729
$s_{t-3}$	0.0202	0.5010	-0.0433	-1.6646
$s_{t-4}$	-0.0392	-0.9514	-0.0099	-0.3764
$s_{t-5}$	0.0431	1.0931	-0.0015	-0.0567
$s_{t-6}$	0.0178	-0.4589	-0.0009	-0.0365
$s_{t-7}$	-0.0269	-0.7348	-0.0161	-0.6480
$s_{t-8}$	-0.0213	-0.5792	0.0328	1.3321
$s_{t-9}$	0.0347	1.0309	0.0175	0.7482
$s_{t-10}$	0.0026	0.0711	-0.0482	-2.2835
$s_{t-11}$	-0.0118	-0.3730	-0.0273	-1.2324
$s_{t-12}$	-0.0306	-0.9448	-0.0293	-1.3929
$R^2$	0.0436		0.1205	
$s.e$	0.0313		0.0186	

Note: The pricing error  $v_{t-1} = b_{t-1} - \mu$  is used as the error correction term



**APPENDIX 2A: NONLINEAR ERROR CORRECTION MODEL  
OF FUTURES (AUGUST SAMPLE)**

Variable	Superscript 0 Regime		Superscript P Regime		Superscript N Regime	
	Coef	t-stat	Coef	t-stat	Coef	t-stat
const	-0.001	-1.525	0.035	1.987	0.0003	0.0325
$v_{t-6}$	-0.044	-2.528	-0.132	-1.256	0.037	0.692
$f_{t-1}$	0.093	3.151	0.299	2.320	0.052	0.510
$f_{t-2}$	-0.027	-0.941	0.034	0.250	0.065	0.642
$f_{t-3}$	-0.026	-0.839	0.463	2.773	0.210	1.812
$f_{t-4}$	-0.015	-0.536	0.154	1.208	0.003	0.024
$f_{t-5}$	-0.021	-0.736	0.530	3.158	0.044	0.400
$f_{t-6}$	0.005	0.174	-0.084	-0.650	0.005	0.054
$f_{t-7}$	0.042	1.579	0.163	1.363	-0.082	-0.846
$f_{t-8}$	0.044	1.661	-0.318	-2.727	-0.197	-2.261
$f_{t-9}$	0.025	0.926	-0.230	-2.230	-0.168	-1.549
$f_{t-10}$	-0.012	-0.449	0.179	1.276	-0.282	-3.322
$f_{t-11}$	-0.006	-0.224	0.238	1.423	-0.141	-1.420
$f_{t-12}$	0.020	0.746	-0.009	-0.068	-0.052	-0.567
$s_{t-1}$	0.266	6.564	-0.165	-0.769	0.031	0.219
$s_{t-2}$	0.014	0.341	-0.277	-1.255	0.007	0.053
$s_{t-3}$	-0.022	-0.546	-0.249	-1.324	-0.094	-0.630
$s_{t-4}$	-0.042	-1.016	-0.430	-1.751	-0.136	-0.847
$s_{t-5}$	-0.089	-2.221	-0.229	-1.121	0.129	0.850
$s_{t-6}$	-0.027	-0.710	0.073	0.373	0.174	1.312
$s_{t-7}$	-0.088	-2.361	0.270	1.278	0.240	2.034
$s_{t-8}$	-0.022	-0.635	-0.161	-0.888	0.243	1.986
$s_{t-9}$	0.006	0.180	0.170	0.946	0.221	2.010
$s_{t-10}$	0.011	0.331	-0.238	-1.472	0.155	1.327
$s_{t-11}$	0.016	0.514	0.007	0.044	0.220	1.916
$s_{t-12}$	0.030	0.967	0.032	0.221	-0.213	-2.097
Transition $\lambda$	not applicable		3.42		10	
Transition $c$	not applicable		0.13		-0.09	

See equations (6), (6a) and 6(b) for the model specification. All t statistics are corrected for heteroscedasticity. The  $R^2$  is 0.102 and the *s.e.* is 0.028.

**APPENDIX 2B: NONLINEAR ERROR CORRECTION MODEL  
OF STOCKS (AUGUST SAMPLE)**

Variable	Superscript 0 Regime		Superscript P Regime		Superscript N Regime	
	Coef	t-stat	Coef	t-stat	Coef	t-stat
const	0.0001	0.277	-0.030	-2.700	-0.009	-1.552
$v_{t-6}$	0.052	4.788	0.166	2.492	-0.108	-2.648
$f_{t-1}$	0.185	9.761	-0.019	-0.231	0.116	1.544
$f_{t-2}$	0.094	5.437	-0.103	-1.175	0.188	2.660
$f_{t-3}$	0.068	3.852	-0.090	-0.964	0.180	2.090
$f_{t-4}$	0.015	0.830	-0.266	-2.612	0.250	2.989
$f_{t-5}$	0.021	1.040	-0.147	-1.755	0.191	2.286
$f_{t-6}$	0.017	0.991	-0.168	-2.309	0.186	3.466
$f_{t-7}$	0.017	1.029	0.037	0.440	0.073	1.224
$f_{t-8}$	0.042	2.443	-0.096	-1.472	-0.059	-1.059
$f_{t-9}$	0.007	0.384	-0.074	-0.920	0.119	1.735
$f_{t-10}$	0.001	0.052	-0.050	-0.572	0.019	0.356
$f_{t-11}$	0.011	0.697	-0.048	-0.553	0.057	0.740
$f_{t-12}$	0.009	0.558	-0.242	-2.578	-0.012	-0.202
$s_{t-1}$	0.014	0.521	0.097	0.692	-0.041	-0.362
$s_{t-2}$	0.023	0.906	0.027	0.203	-0.336	-3.266
$s_{t-3}$	-0.022	-0.899	0.434	3.408	-0.412	-4.426
$s_{t-4}$	-0.020	-0.819	0.224	1.722	-0.236	-2.368
$s_{t-5}$	-0.011	-0.432	0.030	0.268	-0.244	-2.449
$s_{t-6}$	-0.018	-0.779	0.048	0.345	-0.091	-1.115
$s_{t-7}$	-0.024	-0.986	-0.196	-1.839	-0.176	-2.446
$s_{t-8}$	-0.067	-2.868	0.063	0.461	0.140	1.856
$s_{t-9}$	-0.006	-0.253	0.118	0.989	-0.018	-0.206
$s_{t-10}$	0.010	0.417	0.070	0.601	-0.151	-2.049
$s_{t-11}$	-0.026	-1.280	0.176	1.481	0.124	1.751
$s_{t-12}$	-0.038	-1.778	0.266	2.799	-0.115	-1.654
Transition $\lambda$	not applicable		3.42		10	
Transition $c$	not applicable		0.13		-0.09	

See equations (6), (6a) and 6(b) for the model specification. All t statistics are corrected for heteroscedasticity. The  $R^2$  is 0.248 and the *s.e.* is 0.018.

**APPENDIX 2C: NONLINEAR ERROR CORRECTION MODEL  
OF FUTURES (NOVEMBER SAMPLE)**

Variable	Superscript 0 Regime		Superscript P Regime		Superscript N Regime	
	Coef	t-stat	Coef	t-stat	Coef	t-stat
const	0.0004	0.372	-0.001	-0.294	-0.008	-0.875
$v_{t-6}$	-0.009	-0.282	-0.017	-0.395	-0.093	-1.294
$f_{t-1}$	-0.137	-2.367	0.194	2.421	0.316	3.018
$f_{t-2}$	-0.061	-1.197	-0.005	-0.070	0.098	1.012
$f_{t-3}$	-0.099	-1.961	0.092	1.327	0.069	0.653
$f_{t-4}$	0.020	0.380	-0.065	-0.873	0.007	0.070
$f_{t-5}$	-0.003	0.056	-0.090	-1.335	0.028	0.258
$f_{t-6}$	0.022	0.551	-0.077	-1.269	-0.067	-0.802
$f_{t-7}$	-0.062	-1.706	0.121	2.143	-0.016	-1.192
$f_{t-8}$	-0.042	-1.275	0.034	0.624	0.060	0.750
$f_{t-9}$	-0.007	-0.224	0.019	0.309	0.032	0.359
$f_{t-10}$	-0.023	-0.650	0.046	0.800	-0.049	-0.601
$f_{t-11}$	-0.003	-0.082	-0.040	-0.706	-0.024	-0.285
$f_{t-12}$	0.006	0.197	-0.019	-0.363	-0.043	-0.518
$s_{t-1}$	0.355	5.128	-0.104	-1.073	-0.203	-1.443
$s_{t-2}$	0.171	2.454	-0.027	-0.262	-0.249	-1.884
$s_{t-3}$	0.056	0.764	-0.086	-0.848	0.007	0.049
$s_{t-4}$	-0.053	-0.728	0.034	0.317	-0.010	-0.075
$s_{t-5}$	0.055	0.720	-0.029	-0.280	0.095	0.727
$s_{t-6}$	-0.084	-1.307	0.197	2.144	0.144	1.183
$s_{t-7}$	0.066	-1.076	-0.167	-1.903	-0.256	-2.205
$s_{t-8}$	-0.066	-1.141	0.058	0.657	0.229	1.917
$s_{t-9}$	0.034	0.670	-0.001	-0.008	-0.106	-0.838
$s_{t-10}$	0.063	1.049	-0.099	-1.186	-0.095	-0.769
$s_{t-11}$	-0.010	-0.217	-0.098	-1.313	0.172	1.667
$s_{t-12}$	-0.069	-1.348	0.083	1.130	0.067	0.650
Transition $\lambda$	not applicable		10		10	
Transition $c$	not applicable		0.04		-0.09	

See equations (6), (6a) and 6(b) for the model specification. All t statistics are corrected for heteroscedasticity. The  $R^2$  is 0.043 and the *s.e.* is 0.031.

**APPENDIX 2D: NONLINEAR ERROR CORRECTION MODEL  
OF STOCKS (NOVEMBER SAMPLE)**

Variable	Superscript 0 Regime		Superscript P Regime		Superscript N Regime	
	Coef	t-stat	Coef	t-stat	Coef	t-stat
const	-0.001	-1.322	0.0002	0.097	0.011	1.990
$v_{t-6}$	0.021	1.180	0.015	0.567	0.056	1.379
$f_{t-1}$	0.105	3.657	0.049	1.191	0.028	0.436
$f_{t-2}$	0.082	3.108	-0.011	-0.255	0.045	0.807
$f_{t-3}$	0.096	3.505	-0.018	-0.412	-0.127	-2.458
$f_{t-4}$	0.075	2.683	-0.037	-0.874	-0.089	-1.585
$f_{t-5}$	0.076	2.974	-0.027	-0.595	-0.065	-1.097
$f_{t-6}$	0.020	0.987	-0.008	-0.211	0.036	0.729
$f_{t-7}$	0.038	1.881	-0.002	-0.042	-0.012	-0.255
$f_{t-8}$	0.034	1.595	-0.053	-1.357	-0.025	-0.469
$f_{t-9}$	0.018	0.934	0.006	0.152	0.069	1.240
$f_{t-10}$	0.018	0.902	0.044	1.225	-0.023	-0.430
$f_{t-11}$	0.005	0.296	0.029	0.813	0.005	0.092
$f_{t-12}$	0.010	0.535	0.003	0.093	0.029	0.521
$s_{t-1}$	-0.030	-0.786	-0.069	-1.074	0.085	0.946
$s_{t-2}$	-0.089	-2.377	0.072	1.095	0.050	0.572
$s_{t-3}$	-0.075	-1.791	0.060	0.927	0.127	1.380
$s_{t-4}$	-0.034	-0.833	0.029	0.436	0.201	2.216
$s_{t-5}$	-0.064	-1.637	0.144	2.189	0.106	1.308
$s_{t-6}$	-0.035	-1.030	0.068	1.100	0.020	0.229
$s_{t-7}$	0.028	0.786	-0.095	-1.587	-0.136	-1.752
$s_{t-8}$	0.055	1.786	-0.063	-1.147	-0.008	-0.092
$s_{t-9}$	0.035	1.126	-0.039	-0.712	-0.036	-0.393
$s_{t-10}$	0.011	0.399	-0.123	-2.587	-0.115	-1.473
$s_{t-11}$	0.022	0.831	-0.093	-1.936	-0.080	-0.825
$s_{t-12}$	-0.010	-0.330	0.005	0.103	-0.103	-1.234
Transition $\lambda$	not applicable		10		10	
Transition $c$	not applicable		0.04		-0.09	

See equations (6), (6a) and 6(b) for the model specification. All t statistics are corrected for heteroscedasticity. The  $R^2$  is 0.163 and the  $s.e.$  is 0.018.