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Single Source of Error State Space Approach to the Beveridge Nelson Decomposition

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Abstract

A well known property of the Beveridge Nelson decomposition is that the innovations in the permanent and transitory components are perfectly correlated. We use a single source of error state space model to exploit this property and perform a Beveridge Nelson decomposition. The single source of error state space approach to the decomposition is computationally simple, and in contrast to other methods of performing the Beveridge-Nelson decomposition, it incorporates the direct estimation of the long-run multiplier.

Keywords: Beveridge Nelson decomposition; Long-run multiplier; Single source of error; State-space models.

JEL classification: C22, C51, E32

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1. Introduction

Two stylised facts associated with most macroeconomic time series are that they exhibit long run growth and recurrent fluctuations around the growth path. This has often led to exercises which decompose macroeconomic series into permanent and transitory components, where the permanent component represents long run growth or the trend in the economy, and the transitory component is taken to represent the business cycle. There are many different ways in which this decomposition is undertaken (see Canova (1998) for a recent survey), and there is considerable debate about which decomposition (if any) leads to "trends" and "cycles" that best capture the features that economists typically associate with economic growth and business cycles.

One decomposition that has attracted considerable attention in the applied macroeconomics literature is the one first proposed by Beveridge and Nelson (BN) (1981). They defined the permanent component of an ARIMA $(p, 1, q)$ series as the level of the long run forecast of a series (minus the deterministic trend, if any), and the transitory component as the difference between the present level and the permanent component. This decomposition is based on forecasting considerations, because not only does the BN permanent component embody the (time t) long run forecast of the series, but the BN transitory component also embodies the forecastable momentum of the series at each point in time. A by-product of the BN decomposition is that the innovations in the permanent and transitory components are perfectly (and often negatively) correlated, which allows for the possibility that the "BN trend" and "BN cycle" are driven by the same innovation.

The forecasting literature has a long tradition of decomposing time series into trends and cycles, and like the macroeconomic literature, there are various ways in which this decomposition is undertaken and debate about which way is best. One popular decomposition that is often used in forecasting is the unobserved components (UC) decomposition advocated by Harvey (1985), in which the innovations in the trend and cyclical components have zero correlation by assumption. Watson (1986), Stock and Watson (1988), and Harvey and Koopman (2000) explore some of the properties of this decomposition. An alternative forecasting approach advocated by Ord, Koehler and

Snyder (1997) is the class of state space models with a single source of disturbance. In these latter models, the innovations of the unobserved state components as well as the observations are all perfectly correlated, because they are driven by the same disturbance. It is this similarity with the BN property that motivates the use of a single source of error (SSOE) state space forecasting approach to estimate the permanent and transitory components of the BN decomposition.

Harvey and Koopman (2000) have observed that the BN permanent and transitory components for an ARIMA$(0, 1, 1)$ model correspond to those from the UC trend and cycle decomposition with perfectly correlated disturbances. Here, we generalise this observation to point out that the SSOE state space forecasting approach can be used to obtain the BN components for any series with a ARIMA$(p, 1, q)$ process. Previous literature, including Miller (1988), Newbold (1990) and Morley (2002) has focussed on overcoming difficulties involved with truncating and estimating the infinite sums in the permanent component defined by BN (1982). In contrast, our SSOE state space forecasting approach focusses on the correlation between the unobserved components, and it avoids any need for truncation by working with the (equivalent) BN representation outlined in Stock and Watson (1988).

Features of the SSOE approach are that it incorporates the direct estimation of the long-run multiplier and it allows a straightforward comparison of the variances of the innovations for each component. This latter property is potentially useful for macroeconomists, who frequently interpret the BN permanent and transitory components in (the logarithms) of output as indicators of growth and cyclical behaviour in the economy, and then use measures of the ratio of the standard deviations of shocks to trends and output as measures of "persistence in output" (see, eg, Campbell and Mankiw (1987) and Stock and Watson (1988)).

2. Beveridge Nelson Decomposition

Assume that y_t is a I(1) variable with a Wold representation given by

$$\Delta y_t = \mu + \gamma (L) \varepsilon_t, \tag{2.1}$$

where μ is the long run growth or drift, $\gamma(L)$ is a polynomial in the lag operator L with $\gamma(0) = 1$ and $\Sigma_{i=0}^{\infty} |\gamma_i| < \infty$, and ε_t is an iid $\left(0, \sigma^2\right)$ one-step-ahead forecast error of y_t. Using the well known identity that $\gamma(L) = \gamma(1) + (1 - L)\gamma^*(L)$, we can rewrite (2.1) as

$$\Delta y_t = \mu + \gamma\left(1\right)\varepsilon_t + (1 - L)\gamma^*(L)\varepsilon_t, \tag{2.2}$$

or equivalently as

$$y_t = \frac{\mu}{(1-L)} + \gamma\left(1\right)\frac{\varepsilon_t}{(1-L)} + \gamma^*(L)\varepsilon_t. \tag{2.3}$$

The Beveridge Nelson permanent component is given by $\tau_t = \frac{\mu}{(1-L)} + \gamma\left(1\right)\frac{\varepsilon_t}{(1-L)}$ while the temporary component is $c_t = \gamma^*(L)\varepsilon_t$, and it is immediately clear that these two components are driven by the same innovation so that innovations to τ_t and c_t are perfectly correlated. Economists are often interested in the longrun multiplier $\gamma\left(1\right)$, which measures the long-run effect of a shock ε_t on y_t.

It is common to rewrite the expression for τ_t as

$$\tau_t = \mu + \tau_{t-1} + \gamma\left(1\right)\varepsilon_t, \tag{2.4}$$

which shows that the permanent component is a random walk with drift μ and a non-autocorrelated innovation given by $\gamma(1)\varepsilon_t$. Further, when $\gamma(L)$ is an an ARMA(p, q) process with $\gamma(L) = \frac{\theta(L)}{\phi(L)}$, $\theta(L) = 1 + \theta_1 L + \theta_2 L^2....\theta_q L^q$ and $\phi(L) = 1 + \phi_1 L + \phi_2 L^2....\phi_p L^p$, one can show that

$$c_t = \left(\frac{\gamma\left(L\right) - \gamma(1)}{(1-L)}\right)\varepsilon_t = \left(\frac{\theta(L) - \gamma(1)\phi(L)}{\phi(L)(1-L)}\right)\varepsilon_t = \frac{\psi(L)}{\phi(L)}\varepsilon_t, \tag{2.5}$$

where $\psi_0 = 1 - \gamma(1)$, and the order of $\psi(L)$ is n with $n \leq \max(p-1,\ q-1)$. Letting $\phi_p^*(L) = -\phi_1 L - \phi_2 L^2.... - \phi_p L^p$ and $\psi_n^*(L) = \psi_1 L + \psi_2 L^2.... + \psi_n L^n$, the expression for the transitory component becomes

$$c_t = \phi_p^*(L)c_t + \psi_n^*(L)\varepsilon_t + (1 - \gamma(1))\varepsilon_t, \tag{2.6}$$

which is used in our SSOE approach below. We use the perfect correlation between the contemporaneous innovations in equations (2.4) and (2.6) to parameterise our SSOE state space model. In contrast, Morley's (2002) state space approach to perform a BN decomposition is based on a parameterisation of first differenced y_t.

3. Single Source of Error State Space Models

The linear single source of error state space model proposed by Snyder (1985) is

$$y_t = \beta' x_{t-1} + e_t \tag{3.1a}$$

with

$$x_t = F x_{t-1} + \alpha e_t, \tag{3.1b}$$

where (3.1a) is known as the measurement equation and (3.1b) is known as the system equation. The k vector x_t represents the unobserved state of the underlying process at the beginning of period t, α is a fixed k vector of parameters, e_t is an iid $(0, \sigma^2)$ innovation, β is a fixed k vector, and F is a fixed $k \times k$ transition matrix. Often both β and F depend on a set of time invariant parameters. The key feature of this specification is that both equations are driven by the same innovation.

Snyder (1985) shows that the likelihood function associated with (3.1a) and (3.1b) is very simple, so that it is convenient to obtain maximum likelihood estimates of the parameters using the prediction error decomposition of the likelihood in conjunction with a suitable version of the Kalman filter. Further details relating to estimation are described in Snyder (1985) or Harvey (1989).

The above state space model is stable if the matrix $(F - \alpha\beta')$, also known as the discount matrix, has eigenvalues with absolute value less than one (Ord, Koehler, and Snyder (1997)). Letting $(F - \alpha\beta') = D$, it can be shown that

$$x_t = \sum_{j=0}^{\infty} D^j \alpha y_{t-j} \tag{3.2}$$

and

$$y_t = \sum_{j=1}^{\infty} \beta' D^j \alpha y_{t-j} + e_t \tag{3.3}$$

Hence, when D is strictly stable, $D^j \longrightarrow 0$ when $j \longrightarrow \infty$, and past observations have a declining effect as one moves further back in time.

4. Single Source of Error State Space Approach to BN Decomposition

Consider a time series y_t with an ARIMA$(p,1,q)$ process represented by (2.1) with $\gamma(L) = \frac{\theta(L)}{\phi(L)}$. The I(1) term allows the series to be broken down into its permanent (τ_t) and transitory (c_t) components in accordance with the BN decomposition so that

$$y_t = \tau_t + c_t \tag{4.1}$$

$$\text{with } \tau_t = \mu + \tau_{t-1} + \alpha\varepsilon_t \text{ and } c_t = \phi_p^*(L)c_t + \psi_n^*(L)\varepsilon_t + (1-\alpha)\varepsilon_t, \tag{4.2}$$

where $\alpha = \gamma(1)$ from equations (2.4) and (2.6).

Substituting (4.2) into (4.1) gives a single source of error measurement equation

$$y_t = \mu + \tau_{t-1} + \phi_p^*(L)c_t + \psi_n^*(L)\varepsilon_t + \varepsilon_t, \tag{4.3}$$

with the state transition equations given by the two equations in (4.2).

The state transition equations in (4.2), are somewhat similar to the UC decompositions in Watson (1986), Stock and Watson (1988), and Harvey and Koopman (2000). However, the two equations are driven by the same innovation and are perfectly correlated, unlike the standard UC decomposition in which the trend and cycle disturbances have zero correlation. Morley et al (2003) have estimated the correlation between innovations to UC trend and cycle components for an ARIMA(2,1,2) model of (the logarithms of) US GDP and found it to be -0.91, but this sort of correlation can only be identified in an ARIMA framework when $p > q + 2$.

Following the convention of calling the permanent component of the BN decomposition "the trend" and the transitory component "the cycle", the parameter of interest in empirical studies of (the logarithms) of output is typically α, which measures the long run increase in GDP resulting from a 1% shock in GDP in one quarter. In practice, if $\alpha < 1$ then the trend and cycle will have perfect positive correlation and both components will share in the variation of the data. However, if $\alpha > 1$, then the innovations in the trend and cycle will have perfect negative correlation, and the trend τ_t will be more variable than y_t. Some researchers (see eg Proietti, 2002) have questioned whether one should call τ_t a "trend" when it is more volatile than output itself, but as pointed out by Morley et al (2003) (who observed that $\alpha > 1$ for real US GDP), a shock to output can shift the trend so that output is behind trend until it catches up. Thus it is quite reasonable for "trend innovations" to be negatively correlated with "cycle innovations" and for the former innovations to be more variable than output innovations.

5. Applications

We illustrate the use of the single source error state space approach to compute the Beveridge Nelson permanent/transitory decompositions for ARIMA(0,1,1), ARIMA(1,1,0) and ARIMA(2,1,2) models of the logarithms of real output for the United States, the United Kingdom and Australia. The US models coincide with those used by Stock and Watson (1988) in their study of the contribution of the trend component to real US GNP, and we broaden the scope to include decompositions for UK and Australia to demonstrate the relative contribution of trends in other countries. We use quarterly GNP data for the USA (from 1947:1 to 2003:1) , and quarterly GDP data for the UK and Australia (from 1979:3 to 2003:3). As noted above, our parameter of interest is α, which is Campbell and Mankiw's (1987) persistence measure that predicts the long run increase in output resulting from a 1% shock in output in one quarter. Since researchers are often interested in the fraction of the variance in the quarterly change in real output that can be attributed to changes in its stochastic trend, we use our computed BN trends to calculate Stock and Watson's (1988) R^2 measure of this ratio. The empirical results are presented in Table 1, and we outline details relating to the SSOE state space formulation below.

5.1. ARIMA(0,1,1) model

The BN permanent and transitory components for an ARIMA$(0,1,1)$ model are

$$\tau_t = \mu + \tau_{t-1} + \alpha\varepsilon_t \quad \text{and}$$

$$c_t = (1-\alpha)\varepsilon_t,$$

where, in terms of the ARMA coefficients for Δy_t, $\alpha = \gamma(1) = 1 + \theta_1$. These equations can be cast into single source of error state space form with

$$y_t = \mu + \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \tau_{t-1} \\ c_{t-1} \end{bmatrix} + \varepsilon_t$$

as the measurement equation and

$$\begin{bmatrix} \tau_t \\ c_t \end{bmatrix} = \begin{bmatrix} \mu \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \tau_{t-1} \\ c_{t-1} \end{bmatrix} + \begin{bmatrix} \alpha \\ 1-\alpha \end{bmatrix} \varepsilon_t.$$

Forecasts for these state space equations can be computed by using a suitable version of the Kalman filter and the maximum likelihood estimates of the parameters (α and μ) are obtained using the prediction error decomposition of the likelihood function. Note that it is α, rather than the MA(1) parameter that is directly estimated. The eigenvalues of the discount matrix $(F - \alpha\beta')$ (from equation 3.1) need to be within the unit circle to ensure stability, and this condition is satisfied for each of the three decompositions undertaken here.

The estimated αs and implied variance ratios for USA, UK and Australian output are shown in Table 1. Here it is interesting to note that while $\alpha > 1$ for the USA and Australia, implying that innovations to the "trend" and "cycle" are negatively correlated, the same is not true for the UK. Turning to the R^2 measures of the fraction of the variance in the quarterly change in real output that can be attributed to changes in its stochastic trend, we see that trend makes a relatively lower contribution in the USA and Australia, than it does in the UK.

The implied transitory components are illustrated in the left hand side graphs in Figure 1, together with reference recessions published by the NBER and the ECRI. While there are often pronounced declines in the transitory components around the NBER/ECRI peak to trough episodes, there are also clear differences between BN-cycles based on ARIMA(0,1,1) models of output and conventional business cycles. This is hardly surprising, given that each type of cycle has been constructed to serve different purposes, and has been based on quite different information sets.

5.2. ARIMA(1,1,0) model

For an ARIMA$(1,1,0)$ model the permanent trend component is the same as above, although in this case $\alpha = \frac{1}{1+\phi_1}$ in terms of the ARMA coefficients for Δy_t. The cycle component is given by

$$c_t = -\phi_1 c_{t-1} + (1-\alpha)\varepsilon_t.$$

Arranging the model into state space form, the measurement equation is

$$y_t = \mu + \begin{bmatrix} 1 & -\phi_1 \end{bmatrix} \begin{bmatrix} \tau_{t-1} \\ c_{t-1} \end{bmatrix} + \varepsilon_t$$

and the transition equation is

$$\begin{bmatrix} \tau_t \\ c_t \end{bmatrix} = \begin{bmatrix} \mu \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & -\phi_1 \end{bmatrix} \begin{bmatrix} \tau_{t-1} \\ c_{t-1} \end{bmatrix} + \begin{bmatrix} \alpha \\ 1-\alpha \end{bmatrix} \varepsilon_t.$$

Estimation of the state space model imposes the identity that $\phi_1 = \frac{1-\alpha}{\alpha}$ (which arises from the observation that $\alpha = \frac{1}{1+\phi_1}$), and provides a direct estimate of α. As above, appropriate stability conditions (in terms of the eigenvalues for the discount matrix) are satisfied for each country. Results are provided in Table 1 and the implied transitory components are illustrated in the center graphs of Figure 1. As for the ARIMA(0,1,1) model, $\alpha > 1$ for the USA and Australia, while $\alpha < 1$ for the UK. Also, the implied R^2 for the USA and Australia are much smaller than that for the UK, reflecting a comparatively less noisy transitory component in the latter country.

5.3. ARIMA(2,1,2) model

The ARIMA(2,1,2) model of output has been used by Morley et al (2003) for US GDP, and if one restricts attention to just ARIMA(0,1,1), ARIMA(1,1,0) and ARIMA(2,1,2) models, it is the model chosen by AIC for both the USA and the UK. (AIC chooses the ARIMA(1,1,0) for Australia). As usual, the permanent component is given by the first equation in (4.2), while the transitory component is given by

$$c_t = -\phi_1 c_{t-1} - \phi_2 c_{t-2} + \theta_1 \varepsilon_{t-1} + (1-\alpha)\varepsilon_t.$$

In this case $\alpha = \frac{1+\theta_1+\theta_2}{1+\phi_1+\phi_2}$ in terms of the ARMA coefficients for Δy_t, although this relationship does not affect the following estimation.

The model can be cast into a single source of error state space form with

$$y_t = \mu + \begin{bmatrix} 1 & -\phi_1 & -\phi_2 & \theta_1 \end{bmatrix} \begin{bmatrix} \tau_{t-1} \\ c_{t-1} \\ c_{t-2} \\ \varepsilon_{t-1} \end{bmatrix} + \varepsilon_t$$

being the measurement equation, and

$$\begin{bmatrix} \tau_t \\ c_t \\ c_{t-1} \\ \varepsilon_t \end{bmatrix} = \begin{bmatrix} \mu \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\phi_1 & -\phi_2 & \theta_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tau_{t-1} \\ c_{t-1} \\ c_{t-2} \\ \varepsilon_{t-1} \end{bmatrix} + \begin{bmatrix} \alpha \\ 1-\alpha \\ 0 \\ 1 \end{bmatrix} \varepsilon_t$$

being the transition equation.

Table 1 reports the estimation results and Figure 1 illustrates the implied transitory components. As above, appropriate stability conditions (in terms of the eigenvalues for the discount matrix) are satisfied for all countries. In the UK case, $\widehat{\theta}_1$ is statistically insignificant and is set to zero. The reported results are similar to those in Sections 5.1 and 5.2, excepting that the estimated α for the UK is now greater than one. Once again, the results suggest that the permanent component in the US and Australian decompositions are relatively less volatile than the corresponding component in the UK decompositions.

6. Conclusion

In this paper a single source of error state space approach has been proposed to exactly compute the permanent and transitory components of the BN decomposition, in accordance with the original BN property that the two components are perfectly correlated. This approach offers a simple and straight forward formulation of both components in state space form to fit a given ARIMA model, and it allows direct inference on the long-run multiplier α as opposed to indirect inference based on the ARIMA coefficients.

7. References:

Beveridge, S., and C. R. Nelson (1981). "A New Approach to Decomposition of Economic Time Series into Permanent and Transitory Components with Particular Attention to Measurement of the Business Cycle". Journal of Monetary Economics, 7, 151-174.

Campbell, J. and N.G.Mankiw (1987a). "Permanent and Transitory Components in Macroeconomic Fluctuations", The American Economic Review, 77, 111-117.

Canova, F. (1998). "Detrending and Business Cycle Facts," Journal of Monetary Economics, 41, 475 - 512.

Economic Cycle Research Institute, http://www.businesscycle.com

Harvey, A. C. (1989). "Forecasting, Structural Time Series Models and the Kalman Filter". Cambridge University Press.

Harvey, A.C., and S. J. Koopman (2000). "Signal Extraction and the Formulation of Unobserved Components Models". The Econometrics Journal, Vol 3, 84-107.

Miller, M. (1988). "The Beveridge-Nelson Decomposition of Economic Time Series - Another Economical Computational Method", Journal of Monetary Economics, 21, 141-142.

Morley, J. C. (2002). "A State-Space Approach to Calculating the Beveridge-Nelson Decomposition", Economics Letters, 75, 123-127.

Morley, J. C., C. R. Nelson, and E. Zivot (2003). "Why are the Beveridge-Nelson and Unobserved Components Decompositions of GDP so Different?", The Review of Economics and Statistics, Vol 85(2), 235-243.

National Bureau of Economic Research, http://www.nber.org

Newbold, P. (1990). "Precise and Efficient Computation of Beveridge-Nelson Decomposition of Economic Time Series", Journal of Monetary Economics, 26, 453-457

Ord, J .K., A. B. Koehler, and R. D. Snyder (1997). "Estimation and Prediction for a Class of Dynamic Nonlinear Statistical Models", Journal of the American Statistical Association, Vol .92, No.440, 1621-1629.

Proietti, T. (2002), "Forecasting with Structural Time Series Models", in Clements and Hendry (eds), A Companion to Economic Forecasting", (Prentice-Hall).

Snyder, R. D. (1985). "Recursive Estimation of Dynamic Linear Models", Journal of the Royal Statistical Society, series B, 47, 272-276.

Stock, J. H., and M. W. Watson (1988). "Variable Trends in Economic Time Series", Journal of Economic Perspectives, Vol 2(3), 147-174.

Watson, M. W. (1986). "Univariate Detrending Methods with Stochastic Trends", Journal of Monetary Economics, 18, 49-75.

Table 1

Measures of the importance of trend in real log GNP/GDP

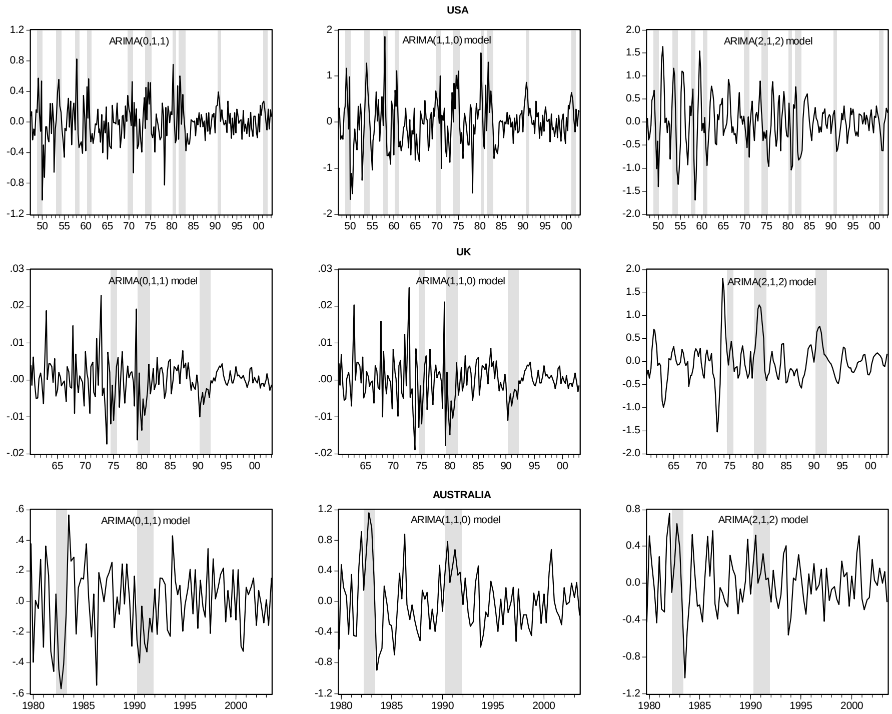
Univariate Statistical Model	Long-run change in GNP predicted from a 1% shock change in GNP in one quarter (α^*)	Variance ratios R^2
US GNP	Data from 1947:I to 2003:I	
ARIMA(0,1,1)	1.2701 (0.0552)[1]	0.9339
ARIMA(1,1,0)	1.5226 (0.0.1464)	0.8817
ARIMA(2,1,2)	1.2653 (0.1459)	0.8458
UK GDP	Data from 1960:I to 2003:I	
ARIMA(0,1,1)	0.9945 (0.0724)	0.9999
ARIMA(1,1,0)	0.9940 (0.0759)	0.9999
ARIMA(2,1,2)	1.2267 (0.1587)	0.9686
Australia GDP	Data from 1979:1 to 2003:3	
ARIMA(0,1,1)	1.3000 (0.0878)	0.9175
ARIMA(1,1,0)	1.4942 (0.0110)	0.8882
ARIMA(2,1,2)	1.3733 (0.0460)	0.8822

[1] std. error in parenthesis

*estimates of α. Estimates of other coefficients can be requested from the authors

The R^2 statistic is obtained by regressing the quarterly change in GNP against the change in the BN trend

Figure 1: Transitory components for ARIMA models for the USA, UK and AUSTRALIA



Note: The shaded areas on the graphs indicate peak to trough episode (recessions) recorded by the NBER (for the USA) and the ECRI (for the UK and Australia)