ISSN 1440-771X
ISBN 0 7326 1066 4

M O N A S H U N I V E R S I T Y

AUSTRALIA

A Test for the Difference Parameter of the ARIFMA Model Using the Moving Blocks Bootstrap

Elizabeth Ann Maharaj

Working Paper 11/99
September 1999

DEPARTMENT OF ECONOMETRICS AND BUSINESS STATISTICS

A TEST FOR THE DIFFERENCE PARAMETER OF THE ARFIMA MODEL USING THE MOVING BLOCKS BOOTSTRAP

Elizabeth Ann Maharaj

Department of Econometrics and Business Statistics

Monash University

Australia

Abstract

In this paper we construct a test for the difference parameter d in the fractionally integrated autoregressive moving-average (ARFIMA) model. Obtaining estimates by smoothed spectral regression estimation method, we use the moving blocks bootstrap method to construct the test for d. The results of Monte Carlo studies show that this test is generally valid for certain block sizes, and for these block sizes, the test has reasonably good power.

Keywords: Long memory, Periodogram regression, Smoothed periodogram regression, Block size.

1 *Introduction*

During the last two decades there has been considerable interest in the application of long memory time series processes in many fields, such as economics, hydrology and geology. Hurst (1951), Mandelbort (1971) and McLeod and Hipel (1978) among others, observed the long memory property in time series while working with different data sets. They observed the persistence of the observed autocorrelation function, that is, where the autocorrelations take far longer to decay than that of the same associated ARMA class. Granger (1978) was one of the first authors who considered fractional differencing in time series analysis.

Based on studies by McLeod and Hipel (1978) which are related to the original studies of Hurst (1951), a simple method for estimating d based on re-scaled adjusted range known as the Hurst coefficient method, was considered by various authors. However, Hosking (1984) concluded that this estimator of d cannot be recommended because the estimator of the equivalent Hurst coefficent is biased for some values of the coefficient and has large sampling variability.

Granger and Joyeux (1980) approximated the ARFIMA model by a high-order autoregressive process and estimated the difference parameter d by comparing variances for each different choice of d. Geweke and Porter-Hudak (1983) and Kashyap and Eom (1988) used a regression procedure for the logarithm of the periodogram to estimate d. Hassler (1993) suggested an estimator of d based on the regression procedure for the logarithm of the smoothed periodogram. Chen *et al.* (1993) and Reisen (1994) also considered estimators of d using the smoothed periodogram. Simulated results by these authors show that the smoothed periodogram regression estimator of d performs much better than the corresponding periodogram estimator of d. This is because, while the periodogram is asymptotically unbiased, it is not a consistent estimator of the spectral density function whereas the smoothed peroidogram is.

The maximum likelihood estimation method of d is another popular method where all other parameters present in the model are estimated together with d (see Cheung and Deibold (1994), Sowell (1992) and Hosking (1984)).

Hypothesis tests for d have been considered by various authors. Davies and Harte (1987) constructed tests for the Hurst coefficient (which is a function of difference parameter d) based on the beta-optimal principle, local optimality and the re-scaled range test. However these tests were restricted to testing for fractional white noise and fractional first-order autoregressive processes. Kwiatkowski *et al.* (1993) also considered tests for fractional white noise but these tests were based on the unit root approach.

Agiakloglou *et al.* (1993) have shown that if either the autoregressive or moving average operator in the ARFIMA model contains large roots, the periodiogram regression estimator of d will be biased and the asymptotic test based on it will not produce good results. Agiakloglou and Newbold (1994) developed Lagrange multiplier tests for the general ARFIMA model. However before testing for d, the order of ARMA model fitted to the series must be known. This can poise a problem since the order of the fitted model is influenced by the value of the difference parameter.

Hassler (1993) considered tests based on the asymptotic results of Kashyap and Eom (1988), of the periodogram regression estimator of d. However he showed that this test is only valid if the series are generated from a fractional white noise process. He also considered tests based on the empirical distribution of the smoothed periodogram regression estimator of d and concluded that this test is superior to the test based on the periodogram regression estimator of d, for discriminating between ARFIMA(p, 0, 0) and ARFIMA(p, d, 0) processes.

Reinsen (1994) considered tests based on the asymptotic results of Geweke and Porter-Hudak (1983), of the periodogram regression estimators of d, as well as asymptotic results based smoothed periodogram regression estimators of d. He concluded from simulated results that smoothed periodogram regression may be superior to periodogram regression for discriminating between ARFIMA(p, 0, q) and ARFIMA(p, d, q) processes.

The small sample distribution of the estimators of d is unknown in any estimation method. In this paper we develop a non-parametric test based on the moving blocks bootstrap (MBB) method, using the smoothed spectral regression estimator of d. The test can be applied to both small and large samples and does not depend on the distribution of the estimator of d.

In Section 2, we briefly describe the estimation of d using the periodogram and smoothed periodogram regression methods. We describe the moving blocks bootstrap test in Section 3, while in Section 4, we outline the experimental design and discuss the results of the simulation studies.

2 Regression Estimators of d

Using the notation of Box and Jenkins (1970), the autoregressive integrated moving average process, ARIMA(p,d,q), is defined as

$$f(B)(1-B)^d X_t = q(B)Z_t \qquad (2.1)$$

where B is the back-shift operator, Z_t is a white noise process with mean zero and variance σ^2 and $f(B) = 1 - f_1(B) - \ldots - f_p B^p$ and $q(B) = 1 - q_1 B - \ldots - q_q B^q$ are stationary autoregressive and invertible moving average operators of order p and q respectively. Granger and Joyeux (1980) and Hosking (1981) extended the model (2.1) by allowing d to take fractional values in the range (-0.5, 0.5). They expanded $(1 - B)^d$ using the binomial expansion

$$(1-B)^d = \sum_{k=0}^{\infty} \binom{d}{k} (-B)^k . \qquad (2.2)$$

Geweke and Porter-Hudak (1983) obtained the periodogram estimator of d as follows. The spectral density function of the ARIMA(p,d,q) process is

$$f(w) = f_u(w)\{2\,sin(w/2)\}^{-2d} \qquad (2.3)$$

where $f_u(w)$ is the spectral density of the ARMA(p,q) process. Hence

$$f(w) = \frac{\sigma^2}{2\pi} \frac{\left|\theta\left(e^{-iw}\right)\right|^2}{\left|\phi\left(e^{-iw}\right)\right|^2} \left\{2\,sin\left(\frac{w}{2}\right)\right\}, \qquad w \in (-\pi, \pi)$$

As $w \rightarrow 0$, $lim\{w^{2d} f(w)\}$ exists and is finite. Given a sample of size t, and given that that $w_j = 2\pi j/T$, (j = 1, 2 , . . . , T/2) is a set of harmonic frequencies, taking the logarithm of (2.3) gives

$$ln\{f(w_j)\} = ln\{f_u(w_j)\} - d\, ln\left\{2\, sin\left(\frac{w_j}{2}\right)\right\}^2 .$$

This may be written as

$$ln\{f(w_j)\} = ln\{f_u(0)\} - d\, ln\left\{2\, sin\left(\frac{w_j}{2}\right)\right\}^2 + ln\left(\frac{f_u(w_j)}{f_u(0)}\right). \qquad (2.4)$$

For a given series $x_1, x_2, \ldots, x_T$ the periodogram is given by

$$I_x(w) = \frac{1}{2\pi}\left\{R(0) + 2\sum_{k=1}^{T-1} R(k)cos(kw)\right\}, \qquad w \in (-\pi, \pi). \qquad (2.5)$$

Adding $I_x(w_j)$ to both sides of equation (2.4) gives

$$ln\{I(w_j)\} = ln\{f_u(0)\} - d\, ln\left\{2\, sin\left(\frac{w_j}{2}\right)\right\}^2 + ln\left(\frac{f_u(w_j)}{f_u(0)}\right) + ln\left(\frac{I(w_j)}{f(w_j)}\right). \qquad (2.6)$$

If the upper limit of j, say g, is chosen so that $g/T \rightarrow 0$ and if w_j is close to zero, then the term $ln\left(f_u(w_j)/f_u(0)\right)$ becomes negligible. Equation (2.6) can then be written as a simple regression equation

$$y_j = a + bx_j + u_j, \quad j = 1, 2, \ldots, g \qquad (2.7)$$

where $y_j = ln\{I(w_j)\}$, $x_j = ln\{2sin(w_j/2)\}$, $u_j = ln\{I(w_j)/f(w_j)\}+c$, $b = -d$, $a = ln\{f_n(0)\} - c$ and $c = E[-ln\{I(w_j)/f(w_j)\}]$. The estimator of d is then $-b$, where b is the least squares estimate of the slope of the regression equation (2.7), that is

$$\hat{b} = \frac{\sum_{j=1}^{g}(x_j - \bar{x})y_j}{\sum_{j=1}^{g}(x_j - \bar{x})^2}.$$

Hassler (1993), Chen *et al.* (1993) and Reisen (1994) all independently suggested an estimate of d based on the regression procedure for the logarithm of the smoothed periodogram. The smoothed periodogram estimate of the spectral density function is

$$\hat{f}_x(w) = \frac{1}{2p}\left\{R(0)l_0 + 2\sum_{k=1}^{m} l_k R(k)cos(kw)\right\}, \qquad w \in (-p, p), \tag{2.8}$$

where $\{\lambda_k\}$ are a set of weights called the lag window, and $m < T$ is called the truncation point. The smoothed periodogram estimates are consistent if m is chosen so that as $T \to \infty$ and $m \to \infty$, $m/T \to 0$. While the periodogram estimates are asymptotically unbiased they are not consistent. Various lag windows can be chosen to smooth the periodogram. The Parzen window is given by

$$\lambda_k = \begin{cases} 1 - 6\left(\frac{k}{m}\right)^2 + 6\left(\frac{k}{m}\right)^3, & 0 \le k \le m/2\,, \\ 2\left(1 - \frac{k}{m}\right)^3, & m/2 \le k \le m\,, \end{cases}$$

and has the desirable property that it always produces positive estimates of the spectral density function.

Equation (2.6) can then be written as

$$ln\left\{\hat{f}(w_j)\right\} = ln\{f_u(0)\} - d\, ln\left\{2\,sin\left(\frac{w_j}{2}\right)\right\}^2 + ln\left(\frac{f_u(w_j)}{f_u(0)}\right) + ln\left(\frac{\hat{f}(w_j)}{f(w_j)}\right). \tag{2.9}$$

Then equation (2.9) can be expressed as equation (2.7) with $y_j = ln\left\{\hat{f}(w_j)\right\}$ and

$u_j = ln\left\{\hat{f}(w_j)/\, f(w_j)\right\}$. The smoothed spectral regression estimator of d is then

$$\hat{d} = \hat{b} = \frac{\sum_{j=1}^{g}(x_j - \bar{x})y_j}{\sum_{j=1}^{g}(x_j - \bar{x})^2}\;. \tag{2.10}$$

Geweke and Porter-Hudak (1983) showed that the periodogram regression estimator of d is asymptotically normally distributed with mean d and variance

$$\frac{\pi^2}{6\sum_{j=1}^{g}\left(x_j - \overline{x}\right)^2},$$

while Kashyap and Eom (1988) showed that that periodogram regression estimator of d is asymptotically normally distributed with mean d and variance $T^{1/2}$. The smoothed periodogram regression estimator is also asymptotically normally distributed with mean d. In particular for a Parzen window, Reisen (1994) showed that the variance of this estimator is approximately

$$\frac{0.53928\, m}{T\sum_{j=1}^{g}\left(x_j - \overline{x}\right)^2}.$$

3 Moving Blocks Bootstrap Test

3.1 Methodology

Efron (1979) initiated the standard bootstrap and jackknife procedures based on independent and identically distributed observations. This set-up gives a better approximation to the distribution of statistics compared to the classical large sample approximations (Bickel and Freedman 1981, Singh 1981, Babu 1986). It was pointed out by Singh (1981) that when the observations are not independent, as is the case for time series data, the bootstrap approximation may not be good. In such a situation the ordinary bootstrap procedure fails to capture the dependency structure of the data.

A general bootstrap procedure for weakly stationary data, free of specific modelling, has been formulated by Kunsch (1989). A similar procedure has been suggested independently by Liu and Singh (1988). This procedure, referred to as the moving blocks bootstrap method, does not require one to first fit a parametric or semi-parametric model to the dependent data. The procedure works for arbitrary stationary processes with short range dependence. Moving blocks bootstrap samples are drawn from a stationary dependent data set as follow: Let X_1, X_2, . . . , X_T be a sequence of stationary dependent random variables with common distribution function for each X_i. Let $B_1, B_2, \ldots, B_{T-b+1}$ be the moving blocks where b is the size of each block. B_j stands for the jth block consisting of b consecutive observations, that is $B_j = \{x_j, x_{j+1}, \ldots, x_{j+b-1}\}$. k independent block samples are drawn with replacement from $B_1, B_2, \ldots, B_{T-b+1}$. All observations of the k-sampled blocks are then pasted together in succession to form a bootstrap sample. The number of blocks k is chosen so that $T \cong kb$. With the moving blocks bootstrap, the idea is to choose a block size b large enough so that the observations more than b units apart are nearly independent. We simply cannot resample from the individual observations because this would destroy the correlation that we are trying to capture. However by sampling the blocks of length b, the correlation present in observations less than b units apart is retained. The choice of block size can be quite important. Hall *et al*. (1995) addressed this issue. They pointed out that optimal block size depends on the context of the problem and suggested some data driven procedures.

Liu and Singh (1988) have shown that for a general statistic that is a smooth functional, the moving blocks bootstrap procedure is consistent, assuming that $b \to \infty$ at a rate such that $b/T \to 0$ as $T \to \infty$.

Variations of the moving blocks bootstrap, in particular subsampling of blocks have been discussed by Politis *et al.* (1997). However this procedure is far too computationally intensive and has not been considered here.

3.2 Hypothesis Testing

It has been suggested by Hinkley (1988) and Romano (1988) that when bootstrap methods are used in hypothesis testing, the bootstrap test statistic should be in the form of a metric. Romano (1988) has shown that test statistics in the form of metrics yield valid tests against all alternatives.

To test $H_0 : d = 0$ against $H_1 : d \neq 0$, we use the test statistic $W = \left|\hat{d} - d\right|$, where $\hat{d}$ is the smoothed periodogram regression estimate based on the Parzen window (Equation 2.10). Even though the consistency results of Liu and Singh (1988) will not strictly apply here, since the estimator of d is not a smooth functional, we will nevertheless obtain a moving blocks bootstrap estimator of d and use it in the hypothesis testing procedure that follows.

The bootstrap provides a first-order asymptotic approximation to the distribution of W under the null hypothesis. Hence, the null hypothesis can be tested by comparing W to a bootstrap-based critical value. This is equivalent to comparing a bootstrap-based *p-value* to a nominal significance level a. Consider the sample $x_1, x_2, \ldots, x_T$. The bootstrap-based *p-value* is obtained by the following steps: (1) Sample with replacement k times from the set $\{B_1, B_2, \ldots, B_{T-b+1}\}$. This produces a set of blocks $\{B_1^*, B_2^* \ldots, B_k^*\}$, which when laid end-to-end forms a new time series, that is the bootstrap sample of length T, $x_1^*, x_2^*, \ldots, x_T^*$. (2) Using this sample, the test statistic $W^* = \left|\hat{d}^* - \hat{d}\right|$ is calculated. $\hat{d}^*$ is the bootstrap estimate of d and is obtained from Equation (2.10) with y_j obtained from $x_1^*, x_2^*, \ldots, x_T^*$. Steps (1) and (2) are then repeated J times. The empirical distribution of the J values of W^* is the bootstrap estimate of the distribution of W. The bootstrap-based *p-value* p^* is an estimate of the *p-value* that would be associated with test statistics W, and it is obtained as follows: $p^* = \#\,(W^* > W)/J$. For a nominal significance level a, H_0 is rejected if $p^* < a$.

Note that choosing $\left|\hat{d}^* - \hat{d}\right|$ and not $\left|\hat{d}^* - d\right|$ as the bootstrap test statistic has the effect of increasing the power of the test (see Hall and Wilson (1991)). Beran (1988) and Fisher and Hall (1990) suggest that a bootstrap test should be based on pivotal test statistics because it improves the accuracy of the test. However since the standard deviation of $\hat{d}^*$ cannot be obtained from the sample, a pivotal test is not possible in this case.

4 Simulation Studies

4.1 Experimental Design

Time series of lengths T =100 and 300 were generated from the ARFIMA(p,d,q) processes, using the method suggested by Hosking (1981). The white noise process is assumed to be normally distributed with mean 0 and standard deviation 1. With the parameter $d \in$ [-0.45, 0.45] in increments of 0.15, the series were generated from ARFIMA(1,*d*,0), with f = 0, 0.1, 0.3, 0.5, 0.7, ARIFMA(0,*d*,1) with q = 0, 0.1, 0.3, 0.5, 0.7, and ARIFMA(1,d,1) with f = -0.6, θ=0.3. These models were chosen, so that both first and second order processes as well as a range of parameter values would be considered. Block sizes of the order $b = T^k$, $0.3 < k < 0.8$ in increments of 0.05 were considered. This range of k values was selected to ensure that the blocks did not contain too few observations or too many observations.

Estimates of d were obtained by the method of smoothed periodogram regression using the Parzen window. The number of regression terms i in Equation (2.10) was chosen to be T^g, where $g = 0.5$ while the truncation point in the Parzen lag window was chosen to be T^m, where $m = 0.7$. (Chen *et al.* (1993) and Reisen (1994) showed that these values of g and m produced good estimates of d in their simulation studies.) Other values of g and m in the range (0,1) where also trialed, but were found to produce poor estimates of d.

A total of 500 moving blocks bootstrap replications were generated for each Monte Carlo trial which was repeated 1000 times for each case. All programming was done in *Gauss.*

4.2 Discussion

For both T = 100 and 300 with block sizes $b = T^k$, $0.3 < k < 0.6$, the size of the test was mostly underestimated. However for T = 100 with block size $b = T^{0.6} = 16$, for series generated from the processes AR(1) , f = 0.5 and MA(1), q = 0.5, the size estimates were fairly close to the nominal levels of significance. For block sizes $b = T^{0.65} = 20$ and $b = T^{0.7} = 25$, the size estimates were fairly close to the nominal levels of significance for series generated from all other selected processes, except for AR(1) , f = 0.5, 0.7 and MA(1), q = 0.5, 0.7 for which it was overestimated. For T = 300, with block sizes $b = T^{0.65} = 41$ and $b = T^{0.7} = 54$, the size estimates were fairly close to the nominal levels of significance for series generated from all other selected processes except for AR(1) , f = 0.7 and MA(1), q = 0.7. For these processes, size was overestimated. For both T = 100 and 300 with block size

$b = T^k$, $0.7 < k < 0.8$, the size of the test was considerably overestimated in all cases. Some of the size estimates are given in Tables 1 and 2.

<Tables 1 and 2 >

Power estimates for both T = 100 and 300, were obtained for block sizes b = $T^{0.65}$ and $T^{0.7}$, which had produced reasonably good size estimates. These power estimates for $b = T^{0.65}$ are given in Tables 3 and 4. For T = 100, it can be seen that the test has fairly good power as d approaches 0.45. However as d approaches -0.45, the increase in power is not as good, especially for series generated from the AR(1), f = 0.5 and the ARMA processes. Similar observations were made for T = 100 with $b = T^{0.7}$.

For T = 300 with $b = T^{0.65}$, it can be seen that the test has fairly good power as d approaches both 0.45 and -0.45 except when the series are generated from the ARMA process with d approaching –0.45. Similar observations were made for T = 300 with $b = T^{0.7}$.

<Table 3 and 4 >

Overall then, it appears that our test performs reasonably well for block sizes $b = T^{0.65}$ and $T^{0.7}$. However as the parameter values of the autoregressive and moving average processes from which the series are generated, tend to the boundary value of 1 (that is, the series approach non-stationarity), the test tends to become invalid. However it takes longer for this to happen for T =300 than for T = 100. As expected the power of the test improves with increasing series length. It would appear then from this simulation study that the optimal block size for the purposes of testing for d, is in the interval [$T^{0.65}$, $T^{0.7}$].

To compare this test for d, to the asymptotic test for d which is based on the smoothed spectral regression estimates, as considered by Reinsen (1993), series where generated from some of the processes mentioned above for g = 0.5 and for m = 0.7 and 0.9. Reinsen (1993) showed that for m = 0.9 and g = 0.5, this test has fairly good power. However our results in Table 5, clearly show that this test is not valid since its size is considerably overestimated. Since our test is generally valid for certain block sizes and since it has reasonably good power for these block sizes, it would appear to be more reliable than this asymptotic test.

<Table 5>

5 Concluding Remarks

Overall this method of testing for *d* using the moving blocks bootstrap appears to produce fairly good results for certain block sizes. That is, in most cases, for these block sizes, the test is generally valid with reasonably good power. We believe that this test has an advantage over the other tests in the literature because it is free of any of the restrictions that are imposed on these other tests, and it can adequately differentiate between ARFIMA (*p*,*d*,*q*) and ARFIMA(*p*,0,*q*) models.

Acknowledgments

This work was supported by a grant from the *Monash University Research Fund*. Thanks to Mizanur Laskar for the programming assistance and to John Nankervis for the useful discussions on ARFIMA models and the bootstrap.

References

Agiakloglou, C., Newbold, P. and Wohar, M. (1993) Bias in an estimator of the fractional difference parameter. *Journal of Time Series Analysis* 14, 3, 235-246.

Agiakloglou, C. and Newbold, P. (1994) Langrange multiplier tests for fractional difference. *Journal of Time Series Analysis* 15, 3, 251–262.

Babu, G. J. (1986) Bootstraping statistics with linear combinations of chi-square as a weak limit. *Sankhya* A, 56, 85-93.

Beran, R. (1988) Prepivoting test statistics: A bootstrap view and asymptotic refinements. *Journal of the American Statistical Association* 83, 687-697.

Bickel, P. J. and Freedman, D. A. (1981) Some asymmetric theory for the bootstrap. *Annals of Statistics,* 9, 1196-1217.

Box G. E. and Jenkins, G. M. (1970) *Time series analysis, forecasting and control*. San Francisco, CA: Holden-Day.

Chen, G., Abraham, B. and Peiris, S. (1993) Lag window estimation of the degree of differencing in fractionally integrated time series model. *Journal of Time Series Analysis* 15, 473-487.

Cheung, Y. and Diebold, F. X. (1994) On maximum likelihood estimation of the differencing parameter of fractionally-integrated noise with unknown mean. *Journal of Econometrics* 62, 301-316.

Davies, R. B. and Harte, D. S. (1987) Tests for Hurst effect. *Biometrika* 1, 95-101.

Efron, B. (1979) Bootstrap methods: Another look at the jackknife. *Annals of Statistics* 7, 1-26.

Fisher, N. I. and Hall, P.(1990) On bootstrap hypothesis testing. *Australian Journal of Statistics*, 1977-1990.

Geweke, J. and Porter-Hudak, S. (1983) The estimation and application of long memory time series models. *Journal of Time Series Analysis* 4, 221-238.

Granger, C. W. J. (1978) New classes of time series models. *Statistician* 27, 237-253.

Granger, C. W. J. and Joyeux, R. (1980) An introduction to long memory time series models and fractional differencing. *Journal of Time Series Analysis* 1, 15-39.

Hall, P., Horowitz, J. L. and Jing, B. (1995) On blocking rules for the bootstrap with dependent data. *Biometrika* 82, 561-574.

Hall, P. and Wilson, S. R. (1991) Two guidelines for bootstrap hypothesis testing. *Biometrics* 47, 757-762.

Hassler, U. (1993) Regression of spectral estimators with fractionally integrated time series. *Journal of Time Series Analysis* 14, 369-379.

Hinkley, D. V. (1988) Bootstrap methods. *Journal of the Royal Statistical Society*. 50, 321-337.

Hosking, J. R. M. (1981) Fractional differencing. *Biometrika* 68, 165-76.

Hosking, J. R. M. (1984) Modelling persistance in hydrological time series using fractional differencing. *Water Resources Research* 20, 1898-1908.

Hurst, H. E. (1951) Long-term storage capacity of reservoirs. *Transactions of the American Society of Civil Engineers* 116, 770-799.

Kashyap, R. L. and Eom, K. B. (1988) Estimation in long-memory time series models. *Journal of Time Series Analysis* 9, 35-41.

Kunsch, H. R. (1989) The jacknife and bootstrap for general stationary observations. *Annals of Statistics* 17, 1217-1241.

Kwiatkowski, D., Phillips, P. C. B., Schmidt, P. and Shin, Y. (1992), Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root? *Journal of Econometrics* 54, 159-178.

Liu, R. Y. and K. Singh (1988) *Exploring the Limits of the Bootstrap*. (Editor: R. Lepage and L. Billard) John Wiley and Sons.

Mandelbort, B. B. (1971) A first fractional Gaussian noise generator. *Water Resource Research* 7, 543-553.

McLeod, A. I. and Hipel, K. W. (1978) Preservation of the rescaled adjusted range 1: A reassessment of the Hurst phenomenon. *Water Resource Research* 14, 491-508.

Politis, D. N., Romano, J. P. and Wolf, M. (1997) Subsampling from heteroskedastic time series. *Journal of Econometrics* 81, 281-317.

Reisen, V. A. (1994) Estimation of the fractional differencing parameter in the ARIMA(p,d,q) model using the smoothed periodogram. *Journal of Time Series Analysis* 15, 335-350.

Romano, J. (1988) A bootstrap revival and some nonparametric distance tests. *Journal of the American Statistical Association* 83, 698-708.

Singh, K. (1981) On the asymptotic accuracy of Efron's bootstrap. *Annals of Statistics* 9, 1187-1195.

Sowell, F. (1992) Maximum likelihood estimation of stationary univariate fractionally integrated time series models. *Journal of Econometrics* 53, 165-18.

Table 1 **Size Estimates for H_0: $d = 0$, H_1 $d \neq 0$, for $T = 100$**

			AR(1)				MA(1)			ARMA(1,1)
k	**Block size T^k**	**Significance Level**	**0**	**0.1**	**0.3**	**0.5**	**0.1**	**0.3**	**0.5**	**-0.6 0.3**
0.60	**16**	**10%**	0.073	0.075	0.083	0.138	0.073	0.073	0.130	0.049
		5%	0.025	0.025	0.032	0.069	0.025	0.031	0.053	0.013
		1%	0.003	0.003	0.004	0.011	0.002	0.001	0.007	0.000
0.65	**20**	**10%**	0.122	0.124	0.132	0.190	0.120	0.127	0.191	0.095
		5%	0.060	0.062	0.067	0.114	0.062	0.068	0.109	0.042
		1%	0.012	0.011	0.014	0.030	0.013	0.150	0.026	0.005
0.70	**25**	**10%**	0.136	0.141	0.144	0.201	0.139	0.143	0.226	0.118
		5%	0.079	0.080	0.080	0.136	0.084	0.088	0.132	0.052
		1%	0.019	0.022	0.027	0.049	0.020	0.020	0.039	0.006

Table 2 **Size Estimates for H_0: $d = 0$, H_1 $d \neq 0$, for $T = 300$**

			AR(1)				MA(1)			ARMA(1,1)
k	**Block size T^k**	**Significance Level**	**0**	**0.1**	**0.3**	**0.5**	**0.1**	**0.3**	**0.5**	**-0.6 0.3**
0.60	**31**	**10%**	0.063	0.067	0.071	0.084	0.063	0.060	0.067	0.035
		5%	0.022	0.022	0.022	0.038	0.020	0.019	0.025	0.007
		1%	0.003	0.003	0.003	0.004	0.002	0.002	0.003	0.000
0.65	**41**	**10%**	0.098	0.095	0.096	0.111	0.097	0.095	0.101	0.065
		5%	0.045	0.043	0.042	0.053	0.043	0.045	0.050	0.024
		1%	0.004	0.003	0.005	0.004	0.004	0.005	0.003	0.000
0.70	**54**	**10%**	0.119	0.118	0.117	0.132	0.123	0.119	0.132	0.099
		5%	0.062	0.065	0.072	0.070	0.064	0.062	0.078	0.046
		1%	0.020	0.021	0.020	0.022	0.019	0.019	0.024	0.013

Table 3 **Power Estimates for H_0: d = 0, H_1 $d \neq 0$, for T = 100, Block size $T^{0.65} = 20$**

		AR(1)				MA(1)			ARMA(1,1)
d	**Significance Level**	**0**	**0.1**	**0.3**	**0.5**	**0.1**	**0.3**	**0.5**	**-0.6 0.3**
-0.45	**10%**	0.706	0.692	0.609	0.434	0.720	0.773	0.861	0.539
-0.30		0.432	0.422	0.354	0.233	0.454	0.519	0.689	0.396
-0.15		0.224	0.210	0.175	0.124	0.237	0.277	0.412	0.228
0.00		0.122	0.124	0.132	0.190	0.120	0.127	0.191	0.095
0.15		0.255	0.268	0.320	0.458	0.244	0.195	0.145	0.147
0.30		0.603	0.621	0.672	0.783	0.589	0.520	0.392	0.462
0.45		0.884	0.892	0.915	0.947	0.877	0.849	0.791	0.820
-0.45	**5%**	0.515	0.497	0.438	0.306	0.531	0.579	0.682	0.268
-0.30		0.302	0.289	0.239	0.146	0.311	0.363	0.480	0.188
-0.15		0.134	0.130	0.099	0.058	0.142	0.173	0.273	0.100
0.00		0.060	0.062	0.067	0.114	0.062	0.068	0.109	0.042
0.15		0.175	0.184	0.222	0.342	0.162	0.131	0.073	0.076
0.30		0.478	0.497	0.557	0.672	0.464	0.414	0.290	0.352
0.45		0.823	0.831	0.862	0.908	0.815	0.785	0.683	0.751
-0.45	**1%**	0.227	0.223	0.188	0.110	0.224	0.218	0.243	0.020
-0.30		0.094	0.088	0.078	0.041	0.098	0.105	0.157	0.013
-0.15		0.038	0.036	0.027	0.011	0.038	0.047	0.068	0.009
0.00		0.012	0.011	0.014	0.030	0.013	0.150	0.026	0.005
0.15		0.066	0.073	0.100	0.160	0.059	0.039	0.013	0.015
0.30		0.298	0.316	0.355	0.435	0.280	0.243	0.161	0.195
0.45		0.662	0.678	0.707	0.772	0.652	0.603	0.522	0.567

Table 4 Power Estimates for H_0: $d = 0$, H_1 $d \neq 0$, for $T = 300$, Block size $T^{0.65} = 41$

d	Significance Level	AR(1)				MA(1)			ARMA(1,1)
		0	0.1	0.3	0.5	0.1	0.3	0.5	-0.6 0.3
-0.45	**10%**	0.914	0.914	0.908	0.871	0.914	0.911	0.903	0.557
-0.30		0.684	0.681	0.667	0.560	0.691	0.697	0.735	0.456
-0.15		0.306	0.300	0.279	0.206	0.312	0.336	0.390	0.235
0.00		0.098	0.095	0.096	0.111	0.097	0.095	0.101	0.065
0.15		0.354	0.360	0.394	0.461	0.343	0.312	0.230	0.262
0.30		0.818	0.824	0.846	0.884	0.814	0.793	0.744	0.777
0.45		0.966	0.967	0.969	0.980	0.965	0.965	0.952	0.959
-0.45	**5%**	0.805	0.810	0.808	0.749	0.795	0.776	0.697	0.209
-0.30		0.527	0.522	0.492	0.408	0.520	0.511	0.494	0.194
-0.15		0.193	0.190	0.169	0.120	0.193	0.201	0.227	0.104
0.00		0.045	0.043	0.042	0.053	0.043	0.045	0.050	0.024
0.15		0.233	0.241	0.278	0.355	0.225	0.197	0.135	0.162
0.30		0.731	0.737	0.758	0.809	0.725	0.694	0.635	0.678
0.45		0.947	0.949	0.951	0.963	0.944	0.939	0.921	0.931
-0.45	**1%**	0.422	0.454	0.455	0.403	0.399	0.294	0.168	0.008
-0.30		0.194	0.206	0.196	0.148	0.185	0.158	0.112	0.010
-0.15		0.048	0.045	0.040	0.023	0.047	0.046	0.040	0.002
0.00		0.004	0.003	0.005	0.004	0.004	0.005	0.003	0.000
0.15		0.075	0.076	0.091	0.137	0.071	0.057	0.029	0.041
0.30		0.490	0.499	0.529	0.585	0.478	0.439	0.367	0.407
0.45		0.863	0.863	0.879	0.900	0.861	0.846	0.814	0.836

Table 5 **Size estimates for H_0: $d = 0$, H_1 $d \neq 0$, for the asymptotic test**

			AR(1)				MA(1)			ARMA(1,1)
T	***m***	**Significance Level**	**0**	**0.1**	**0.3**	**0.5**	**0.1**	**0.3**	**0.5**	**-0.6 0.3**
100	**7**	**10%**	0.363	0.365	0.396	0.471	0.359	0.357	0.463	0.368
		5%	0.278	0.276	0.290	0.374	0.281	0.287	0.384	0.292
		1%	0.163	0.163	0.169	0.240	0.163	0.177	0.238	0.186
	9	**10%**	0.279	0.272	0.275	0.347	0.270	0.270	0.320	0.266
		5%	0.188	0.186	0.209	0.258	0.190	0.200	0.241	0.195
		1%	0.087	0.087	0.092	0.142	0.088	0.094	0.127	0.097
300	**7**	**10%**	0.487	0.490	0.485	0.517	0.491	0.481	0.503	0.487
		5%	0.400	0.404	0.411	0.431	0.404	0.414	0.422	0.416
		1%	0.272	0.267	0.267	0.297	0.276	0.271	0.300	0.273
	9	**10%**	0.316	0.322	0.306	0.338	0.330	0.316	0.333	0.316
		5%	0.219	0.249	0.241	0.244	0.236	0.234	0.242	0.232
		1%	0.121	0.121	0.112	0.124	0.123	0.109	0.125	0.114