ISSN 1440-771X

MONASH UNIVERSITY
AUSTRALIA

DEPARTMENT OF ECONOMETRICS
AND BUSINESS STATISTICS

Nonlinear Autoregressive Leading Indicator Models of Output in G-7 Countries

Heather M. Anderson, George Athanasopoulos
and Farshid Vahid

Working Paper 20/2002

Nonlinear Autoregressive Leading Indicator Models of Output in G-7 Countries

Heather M. Anderson*, George Athanasopoulos and Farshid Vahid
Department of Econometrics and Business Statistics
Monash University, Clayton, Victoria 3800.
AUSTRALIA

Abstract

This paper studies linear and nonlinear autoregressive leading indicator models of business cycles in G7 countries. The models use the spread between short-term and long-term interest rates as leading indicators for GDP, and their success in capturing business cycles is gauged by non-parametric shape tests, and their ability to predict the probability of recession. We find that bivariate nonlinear models of output and the interest rate spread can successfully capture the shape of the business cycle in cases where linear models fail. Also, our nonlinear leading indicator models for USA, Canada and the UK outperform other models of GDP with respect to predicting the probability of recession.

Keywords: Business Cycles, Leading Indicators, Model Evaluation, Nonlinear Models, Yield Spreads.

JEL classification: C22, C23, E17, E37.

*Corresponding author. E-mail: Heather.Anderson@BusEco.monash.edu.au
The authors acknowledge financial support from the Australian Research Council (Large Grant No. A00103258).

1. Introduction

Modeling the cyclical behavior of the aggregate output has always been an important question for macroeconomists, who often want to classify past and present patterns into particular phases of the business cycle, and forecast future turning points. There are lively debates about how to define and measure cycles in output, how to model them, and how to predict features such as turning points and recessions. Detrending issues fuel many of these debates (see e.g. Canova (1998)), but other important issues include the possible nonlinearity in business cycles (see e.g. Hamilton (1989)), and which variables are most useful for predicting output (Stock and Watson (1989) and (2001)).

The forecasting literature has often emphasized the ability of financial variables to predict various features of business cycles. In particular, Zellner and Hong (1989), Zellner et al (1991) and Zellner and Min (1999) show that adding (lags of) monetary and financial variables to univariate autoregressive models of output growth improves forecasts of turning points in many countries. These authors call their models "autoregressive leading indicator" (ARLI) models, a term that we use from now on. Related to forecasts and ARLI models is a large set of macroeconomic papers that document and explain why specific financial variables have leading information for the business cycle[1]. In their comprehensive review of this work, Stock and Watson (2001) conclude that "there is evidence that the term spread is a serious candidate as a predictor of output growth and recessions. The stability of this proposition in the U.S. is questionable, however, and its universality is unresolved". We interpret the lack of stability in the output growth/term spread relationship as evidence of nonlinearity, and this motivates our nonlinear approach to modeling output and the spread.

Almost all bivariate analyses of output and the spread are based on linear specifications. However, the empirical finance literature presents statistically significant evidence that the drift in the term structure of interest rate is nonlinear (see e.g. Aït-Sahalia 1996), and this suggests that a satisfactory bivariate model of output and the spread is likely to be nonlinear. Also, the apparent decline in the variance of output growth in the United States since the mid eighties (see e.g. Kim and Nelson (1999) and McConnell and Perez-Quiros (2000)) has led to a belief that it is necessary to include an exogenous

[1] A representative sample of this literature includes Davis and Fagan (1997), Estrella and Mishkin (1998), Friedman and Kuttner (1998), Gertler and Lown (1999), Hamilton and Kim (2002) and Kwark (2002).

structural break in models of output growth. However, it is possible that a self exciting nonlinear propagation mechanism can adequately generate changes in the important features of cycle characteristics, without any need for a structural break in the variance of shocks.

In this paper we develop linear and nonlinear autoregressive leading indicator models of output growth in G-7 countries. Our models use the spread between short-term and long-term interest rates as leading indicators for growth in GDP, and their success in capturing business cycles is gauged firstly by the non-parametric procedures developed by Harding and Pagan (2002), and then by their ability to forecast the probability of a recession (as in Fair (1993)). This contrasts with Teräsvirta and Anderson (1992), Clements and Krolzig (1998) and Jansen and Oh (1999), who used mean squared errors of one step ahead forecasts to evaluate various univariate nonlinear models of output. Our primary aim is to develop time series models that can predict important pre-specified events such as "recessions" and other salient features of business cycles. We are particularly interested in assessing the predictive ability of nonlinear specifications relative to linear specifications, and where applicable, relative to linear models that incorporate a structural break.

We build on the work in Anderson and Vahid (2001), who extended the class of linear autoregressive leading indicator (ARLI) models to include nonlinear autoregressive specifications (called NARLI models). NARLI models allow for differences in behavior over different phases of the business cycle, and they also allow for asymmetries in how the indicator leads output. In line with results of Stock and Watson (1989), Davis and Fagan (1997), Kozicki (1997), Friedman and Kuttner (1998), Estrella and Mishkin (1998) and others, we use yield spreads as our leading indicators. The predictive power of the spread is well established, but most research on this issue has stayed within the confines of conventional linear models of output and the spread. Notable exceptions include Estrella and Mishkin (1998), Karunaratne (1999) and Birchenhall et al (2000) who use a logit/probit model to explain a binary recession indicator, Galbraith and Tkacz (2000) who test for and find asymmetries in the link between the yield spread and output in G7 countries, and De Long and Summers (1988), Cover (1992), Karras (1996), Choi (1999) and Weiss (1999), who model asymmetries in the relationship between monetary policy and output.

We find that bivariate nonlinear models of output and the interest rate spread can predict the characteristic features of the business cycle in almost all cases where linear models fail. They can capture the amplitude and duration of both peak to trough states, and they can also capture the curvature in transition from trough to peak states. Linear leading indicator models of GDP fail to reproduce these properties for the US, Canada and the UK, as do univariate nonlinear models. Thus, the nonlinearity in a bivariate framework appears to be important. Our forecasting statistics are broadly consistent with our model evaluations. Relative to the linear models, our bivariate nonlinear specifications for US, Canada and the UK can predict the probability of recessions more accurately. Interestingly, for the USA and the UK, where a decline in volatility is observed, fixed parameter bivariate nonlinear models perform no worse than linear models with structural break. This provides evidence that the apparent decline in volatility may just be an implication of a nonlinear propagation mechanism in the conditional mean of output growth. For the other countries, the nonlinearity in the bivariate framework offers some improvement compared to univariate models, but only a small improvement relative to bivariate VARs.

The next section of this paper provides a description of our modelling methodology and develops the linear and nonlinear models that we use in our analysis. Then, in Section 3, we discuss the model evaluation techniques that we use and apply these evaluation techniques to our models. The paper concludes in Section 4 with a summary of our findings and some directions for future research.

2. Modeling methodology

2.1. Data

Our data consists of quarterly time series of real output (gross domestic product), short term interest rates and long term interest rates for the United States, Canada, the United Kingdom, France, Italy, Germany and Japan. We provide detailed information on data sources, our samples, and precise descriptions of our raw series in Appendix 1, and we base our benchmark analyses of business cycle characteristics on the natural logarithms of real GDP. Our spread variables are calculated by taking the difference between the interest rates on the long-term bond and the short term bond, and the variables in our parametric models are output growth (calculated as 100$\times$the differenced logarithms of real GDP) and the spread. We use the notation y_t to denote output growth (which we will call output) and s_t to denote the interest rate spread. Graphs of all variables are also provided in Appendix 1.

2.2. Linear and Nonlinear model specification

We develop our models, one country at a time, to make sure that we account for country specific characteristics. In each case, we estimate a univariate autoregressive specification, and then a VAR in output and the spread to provide a baseline bivariate model. We use AIC to guide our lag-length choices, but eliminate lagged variables if they are statistically insignificant and their removal does not lead to serially correlated residuals. We estimate our restricted VARs both equation by equation (with OLS), and as a SUR, but there is never much difference between the two and we only report the latter. The output equation in the restricted VAR is an autoregressive leading indicator (ARLI) model. We also estimate random walk models for each country, so that later we can compare the simulated properties of the data and random walk models with other linear models, and thereby assess how the lag structure and the financial indicator in each ARLI model can account for ability to capture business cycle characteristics.

We develop our nonlinear models by conducting specification tests on each equation in the VAR model. The nonlinear alternatives that we consider are threshold autoregressive (TAR) and logistic smooth transition autoregressive (LSTAR) models. We find these models attractive because they incorporate regimes that can easily be interpreted as recessionary and expansionary states, and changes between regimes depend on an observed transition variable, rather than on an unobservable state. A univariate LSTAR model of order p is defined by

$$y_t = (\pi_{10}+\pi_1' w_t)+(\pi_{20}+\pi_2' w_t)F(y_{t-d})+u_t, \quad \text{with } F(y_{t-d}) = [1+\exp[-\gamma(y_{t-d}-c)]\ ]^{-1},$$

for $w_t = (y_{t-1}, \ldots, y_{t-p})'$, $y_{t-d} \in w_t$, $\pi_j = (\pi_{j1}, \ldots, \pi_{jp})'$ for $j = 1, 2$, and $u_t \sim nid$. A TAR model is the limiting case of an LSTAR model when $\gamma \to \infty$. It is straightforward to generalize these models to bivariate models.

We use the Tsay (1989) test against the TAR alternative, and three tests by Luukkonen et al (1988) and Teräsvirta (1994) for evidence of STAR behavior. All of these tests require specifying the transition variable in advance. The Tsay (1989) test orders the data matrix (dependent and lagged variables) according to the transition variable, recursively estimates the model, and then tests whether the recursive residuals are orthogonal to the regressors. The three tests proposed by Luukkonen et al (1988) are all tests for omitted nonlinear terms. The simplest of these, which we call the "first order test", takes the cross product of the transition variable and all regressors to be the omitted variables. The "augmented first order test" adds the third power of the transition variable to the list of cross products considered by the first order test, and the "third order test" uses the cross products of the first, second and third powers of the transition variable with all the regressors. Luukkonen at al (1988) discuss the relative merits of these tests and note that the augmented first order test often can account for shortcomings associated with the other two tests. As suggested by these authors, we use F-test versions of the tests to account for the relatively small sample size.

Table 1: Evidence of Nonlinearity

	USA	Canada	UK	France	Germany	Italy	Japan
Univariate Models of Output	-	-	-	-	y^*_{t-1} FAST y_{t-2} FAST y_{t-3} FAST y_{t-4} S	y^*_{t-3} AST	y_{t-4} T y^*_{t-5} T
Bivariate Models of Output	s_{t-1} FAST s^*_{t-2} FAST s_{t-3} FAST	y^*_{t-1} FST s_{t-1} FT s_{t-2} F	y_{t-1} S y^*_{t-2} FAST y_{t-3} FS s_{t-1} S s_{t-2} S	-	y^*_{t-1} FAST y_{t-2} FAS y_{t-3} FAST	y^*_{t-3} AS s_{t-1} AT s_{t-2} FA	y^*_{t-4} S s_{t-1} S
Bivariate Models of Spread	y_{t-1} FAST y_{t-2} FAST y_{t-3} FAST s^*_{t-1} FAST s_{t-2} FAST s_{t-3} FAST	y^*_{t-1} ST s_{t-1} ST s_{t-2} S	y_{t-2} FA	y_{t-1} FAST y_{t-2} FAST s^*_{t-1} FAST s_{t-2} FAST	y_{t-2} S y^*_{t-3} FAST s_{t-1} T s_{t-2} FAST s_{t-3} FAS s_{t-4} FA	y_{t-3} S s^*_{t-1} S	y_{t-2} S s_{t-1} S s^*_{t-4} FAS s_{t-5} T

Entries in the table relate to rejections of the null hypothesis of linearity (at the 5% significance level). See the text for descriptions of the tests.

We consider each lag of output and the spread as a possible transition variable and perform nonlinearity tests for each country. The results are summarized in Table 1. For each country, this table shows if there is evidence of nonlinearity in the univariate autoregressive model of output, in the output equation of a bivariate model of output and the spread, and in the spread equation of the bivariate model. An entry like y_{t-2}

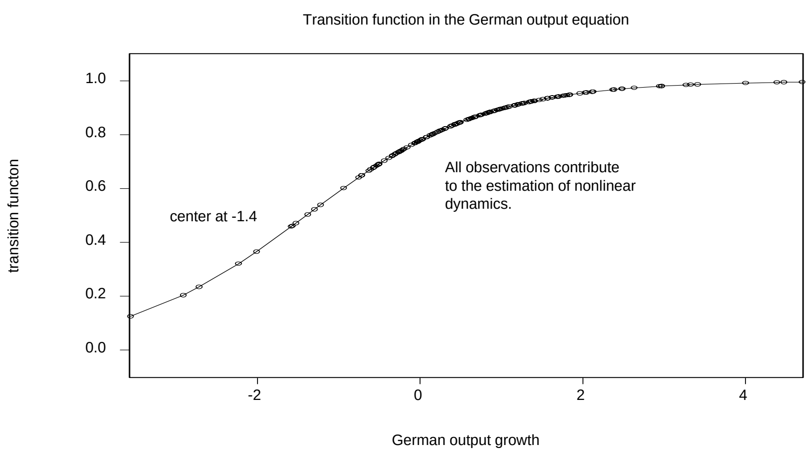
means that the null of linearity was rejected at the 5% level of significance when the transition variable was y_{t-2}. After the transition variable, we report which test or tests rejected linearity. The letters 'F', 'A', 'S' and 'T' stand for first order, augmented first order, third order and TAR tests respectively. Finally, the transition variable that we have selected for our final specification for each equation is marked by a star superscript.

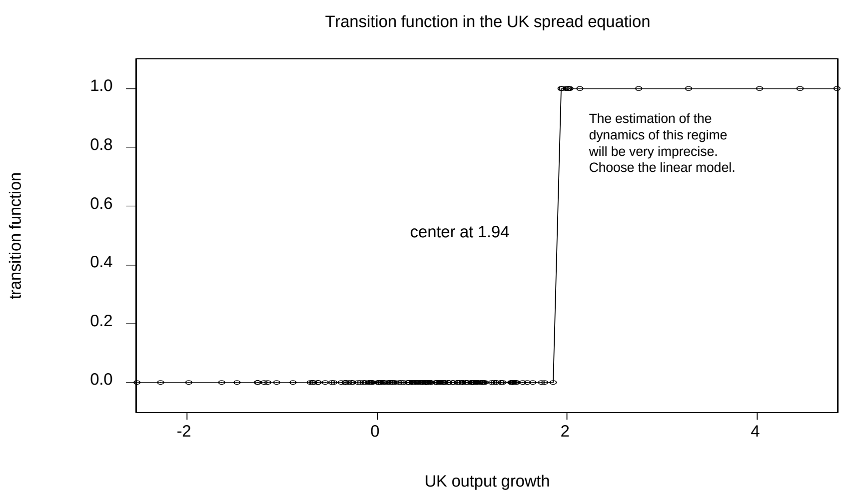
The null of linearity is not rejected in the univariate autoregressive models of output for the US, Canada, UK and France. This is consistent with work in Anderson and Vahid (2001), who show that the force-fitting of univariate nonlinear autoregressive models to US output growth does not improve model performance, relative to an AR model. However, in the bivariate setting, we find significant evidence of nonlinearity in both output and spread equations, in all cases except the French output equation.

We use the results of the nonlinearity tests to guide our specification for each equation, and as above, we remove statistically insignificant explanators from equations provided that their removal does not lead to serially correlated residuals. If evidence of nonlinearity is found for more than one transition variable, we fit separate nonlinear models for each transition variable, and then choose the model with best fit. Since TAR models are special cases of STAR models when $\gamma \to \infty$, we begin by fitting a STAR model in all cases where nonlinearity is found, but we monitor the likelihood function, and switch to a TAR specification if the global maximum seems to occur when the transition parameter is very large.

It is well-known that nonlinearity tests are sensitive to "outliers" (see van Dijk, et al (1997)), and that it is appropriate to disregard evidence of nonlinearity that arises because of recording errors or one time exogenous events. However, if outliers are not generated by errors or rare events, then they will be very important for the identification of the nonlinear propagation mechanism. We attend to the potential outlier problem by restricting the sorts of STAR models that we are willing to entertain (we require at least 10% of observations to lie on each side of the transition threshold c) and we monitor the maximization of the likelihood function. If the likelihood function is maximized when the parameter c is on our imposed boundary and γ implies threshold behavior, then we classify this as nonlinearity due to outliers, and we switch back to a linear specification. We choose the nonlinear specification in all other cases. In particular, when c is on a boundary but the transition function is smooth, we choose the nonlinear specification. In this case observations on both sides of c contribute to the estimation of parameters in both regimes. This decision rule is illustrated in the Figure 1, and we have found that it leads to stable nonlinear models that do not explode in repeated simulations.

Figure 1





The estimated models for each country are presented in Appendix 2. For each country, a random walk with drift model, a univariate linear autoregressive model (if different from the random walk), a univariate non-linear autoregressive model, a bivariate linear model of output and the spread and a bivariate nonlinear model of output and the spread are reported. Non-linear models are fitted only if tests of linearity reject the linear autoregressive models and if non-linear models allow for sufficient number of observations in each regime as discussed above. As discussed above, we also present univariate autoregressive models with a structural break for the US and UK. Residual tests indicate that none of the dynamic models have serially correlated residuals (according to Lagrange multiplier tests), but many of the linear specifications show strong evidence of heteroskedasticity (according to White tests and ARCH tests), structural change (according to Ramsey reset tests) and nonlinearity as indicated in Table 1.

3. Model Evaluation

We evaluate our models according to their ability to capture and predict business cycle characteristics. This places a direct focus on the likely requirements of model users, who will typically want to study and forecast business cycles. Given that relatively long samples are needed to reveal business cycle characteristics, we have chosen evaluation techniques that track model performance over the entire sample, rather than over a short post-sample evaluation period. This avoids the possibility that an out-of-sample evaluation period is uneventful, or does not contain sufficient information to allow evaluation to be meaningful. Our entire sample for each G7 country contains several recessionary and expansionary periods, so that the evaluation of performance with respect to a particular business cycle characteristic is based a sufficient number of relevant data points.

3.1. Predicting Business Cycle Characteristics (BCCs)

Harding and Pagan (2002) point out the gap between policy makers' focus on turning points in the levels of output and academic interest in modeling the moments of detrended data. They advocate using a cycle dating algorithm to identify the turning points in the levels, and measuring various business cycle characteristics (BCCs) based on these turning points. These BCCs include the duration and amplitude of a cycle from peak to trough and from trough to peak, as well as cumulative movements and asymmetries within these phases. We follow their suggested techniques for dating cycles and measuring eight BCCs, and then we evaluate our models by comparing the BCCs in our samples with the BCCs predicted by our models. This model evaluation technique can be seen as a test for model admissibility, in the sense that it tells us if a model cannot produce a feature that is actually observed in the data. A brief summary of this procedure is provided below.

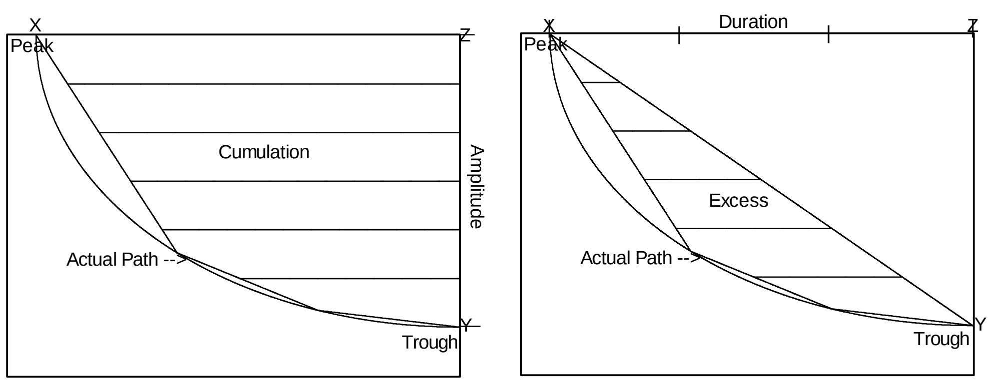
The cycle dating algorithm is an adaptation of the Bry-Boschan (1971) algorithm, and it identifies turning points when

$$\log GDP_t > (<) \log GDP_{t\pm k} \quad \text{for } k = 1, 2 \text{ quarters,}$$

provided that each phase of a cycle lasts at least two quarters and the whole cycle lasts at least five quarters. This algorithm is applied to both raw data, and data series that have been generated using DGPs implied by our estimated models.

Figure 2 illustrates the measurement of four BCCs over a peak to trough phase, while the economy moves along the curved path from point X to point Y. The length of the line XZ shows the duration of the phase, i.e. how long it takes (in quarters) for the phase to be completed, while the length of the line YZ shows the amplitude of the phase, i.e. the total change in output as the economy moves from X to Y. We convert the latter into a percentage change. The shaded area labeled "cumulation" measures the impact of the recession, by approximating the total accumulated loss in output as the economy moves from peak to trough. We convert this measure to a percentage. The final BCC (labeled "excess" in the right hand diagram in Figure 2) measures the difference between the cumulated output loss and a crude triangle approximation (given by triangle XYZ) to this loss. This measures the curvature of the phase of the cycle. We divide this measure by the duration and convert it to a percentage.

Figure 2: Calculation of Business Cycle Characteristics
(Duration, Amplitude, Cumulation and Excess)



The above measures relate to a single recession, but one can summarize the business cycle characteristics of a given series by calculating the means of each BCC for all peak-to-trough and all trough-to-peak phases. These eight summary statistics (calculated without any prior detrending of the series) provide a natural benchmark for evaluating a business cycle model, because a good model should imply the same BCCs as those that in the data. Parametric models are, of course, typically modelling detrended data.

However, this doesn't prevent the simulation of detrended data and then the integration of the simulated series to obtain an analogue of the original data together with its BCC measures. For each parametric model, we undertake 10000 simulations in order to estimate the empirical density functions for each of the eight characteristics of interest, and then we compare these densities with the relevant characteristics in the original data. If an observed BCC lies in the upper 5% or lower 5% tails of the simulated density, then this provides evidence against the parametric model.

3.2. Forecasting Recessions

In line with previous work done by Neftçi (1982), Diebold and Rudebusch (1989), Zellner at al. (1991) and Fair (1993), we evaluate models according to their ability in predict business cycle events. Prediction is based on simulation, since some of our models are nonlinear and taking expectations is not straightforward. The technique for predicting the probability of an event involves defining the event of interest as a property of a sequence of multi-step ahead predictions, classifying each predicted sequence from the Monte Carlo as either having or not having that property, and then setting the estimated probability equal to the proportion of Monte Carlo sequences that have the property. See Fair (1993) for further details.

We use Fair's (1993) two definitions of a recession, which are:

A: At least two consecutive quarters of negative growth in real GDP over the next five quarters; and

B: At least two quarters of negative growth in real GDP over the next five quarters. As noted in Fair, the first of these definitions is in common use. The other definition is broader, and allows us to assess how predictions might change, as we change the event that we are trying to predict. Neither definition coincides with the peak to trough phases identified via the cycle dating algorithm, but one could usefully adapt the Monte Carlo simulations to focus on an event such as reaching a peak, or completing a peak to trough phase, if one wanted to. We take lagged observations and our estimated parameters as given for each observation in our sample, and then use the simulation process to estimate the probabilities of events A and B. This leads to series of probabilities (P_t), which can be compared against indicator variables (D_t) for events A and B, where each of P_t and D_t relate to an event over the five quarter period between (t) and $(t+4)$, and P_t is predicted from the information set at time $(t-1)$.

It is useful to note the similarities and differences between our probability predictions, and others. Firstly, we define a recession as an observable event. Therefore, we do not need to make inference about an unobservable state, as is done in Markov Switching models. Also, since our definition of recession is directly related to one to five period ahead forecasts of output, an appropriate model for forecasting the probability of recession is one that delivers the one to five period ahead predictive density of output. In this context, binary dependent variable models are problem specific, and if there is interest in estimating the probabilities associated with other events, then the dependent variable needs to be redefined in each case, and the model needs to be re-estimated.

We evaluate our probability forecasts using Brier's (1950) quadratic probability scores (QPS) and log probability scores (LPS), which are respectively defined by

$$\begin{aligned} QPS &= \frac{1}{T}\sum_{t=1} 2(P_t - D_t)^2 \qquad (0 < QPS < 2), \quad \text{and} \\ LPS &= -\frac{1}{T}\sum_{t=1} [(1-D_t)\ln(1-P_t) + D_t \ln P_t] \qquad (0 < LPS < \infty). \end{aligned}$$

for a sample of T forecasts. QPS provides a probability analogue to the usual mean squared error criterion, while LPS penalizes large mistakes more than QPS. Like the mean squared error measure, low QPS and low LPS imply accurate forecasts. See Diebold and Rudebusch (1989) for further discussion on these evaluation criteria.

Our forecasts about recessions are not genuine out-of-sample forecasts, but the QPS and LPS criteria differ from the loss functions that are minimized when the parameters are estimated. These criteria are not independent of the in-sample sum of squared errors, but there is little reason to believe that they necessarily improve with the fit of the model. Indeed, our results show that larger models do not necessarily outperform more parsimonious models.

3.3. Results

Summary statistics of the business cycle characteristics for each of our log(GDP) series are provided in Table 2. Here, it is clear that each characteristic varies from country to country, and that the characteristics of the peak-to-trough phase are quite different from those of the trough-to-peak phase. Of course, given that the log(GDP) series have a positive trend and that the table shows the characteristics of the actual series (as opposed to the detrended series), it is not surprising that the trough-to-peak duration and amplitude of the cycles are much larger than the peak-to-trough ones. One striking observation is the small trough-to-peak characteristics, in particular the cumulative gain, of Japanese business cycles relative to other countries. We attribute this to the clear break in the trend in the Japanese GDP since 1990. As we will discuss below, we cannot reproduce this break endogenously with a nonlinear self-exciting model.

Summary tables of performance of different models in reproducing the business cycle characteristics for each of the G-7 countries are reported in Panel 1. Similarly, Panel 2 contains summary tables reporting the performance of different models in predicting the probability of recessions for each country. We highlight the main findings for each country below.

Table 2: Benchmark Business Cycle Characteristics

	USA	Canada	UK*	France	Germany	Italy	Japan
Time Span	61:1-00:4	61:4-00:3	60:1-00:2	70:4-98:4	61:2-99:4	71:3-99:4	71:2-99:4
Duration							
PT	3.8	4.0	4.4	3.0	4.5	2.8	3.6
TP	20.4	16.0	25.5	32.5	19.2	14.8	8.0
Amplitude							
PT	-2.1	-3.2	-3.2	-1.6	-2.3	-1.5	-1.9
TP	22.9	17.2	21.5	21.3	20.1	11.5	4.9
Cumulation							
PT	-4.2	-6.6	-9.6	-2.0	-5.3	-3.0	-6.3
TP	342	257	381	358	253	130	19
Excess							
PT	-0.1	0.3	-0.1	0.0	0.1	-0.1	-0.1
TP	1.4	1.4	-0.5	-0.3	0.8	0.3	-0.1

Note: The UK figures relate to a cycle with a minimum length of 4 quarters rather than 5. This makes our analysis comparable to Harding and Pagan (2002)

The United States: As previously documented by Harding and Pagan (2002), linear autoregressive models fail to reproduce the curvature of the cycles in the US. The linear ARLI model fails this test as well, whereas the bivariate NARLI model passes this data admissibility test. Allowing for a break in the variance and the autoregressive structure in 1984:2 also produces a data admissible model. However, the probability forecast scores in the US table in Panel 2 show that all univariate models, including the model with a break at 1984:2, do worse than the bivariate models of output and the spread. This is particularly interesting because in the forecasting exercise with the break model we assume that the break is recognized as soon as it happens, an assumption that gives an informational advantage to the break model. These results lead us to conclude that the bivariate nonlinear leading indicator model for the US fits the characteristics of the US business cycles without any need for an exogenous structural break.

Given the recent recession in the US, we also used our estimated models to produce genuine out-of-sample predictions of the probability of recession. As for the estimation period, bivariate models of output and the spread performed much better than the univariate models in predicting this recession. Both bivariate models began to predict a higher than average probability of a recession two periods before 2001:1, which is the first quarter of negative growth in the out of sample period. The bivariate nonlinear model predicted a higher probability of Event A than the bivariate linear model only one period before 2001:1, but it predicted a higher probability for Event B from two periods before 2001:1. This is particularly interesting, because the ability of the spread to predict recessions was questioned after many models of output and the spread "missed" the 1990 recession.

Canada: The Canadian results are qualitatively similar to the US case. In the bivariate models of output and the spread, significant nonlinearity is found in each equation. Unlike the US case where the transition variables are lags of spread, in the Canadian models the first lag of output is the transition variable in each equation. This opens up the possibility of common nonlinearity in these equations, but a test for common nonlinearity suggested by Anderson and Vahid (1998) rejects this hypothesis. As in the US model, the transition function in the spread equation is much smoother than the transition function in the output equation. All models but the bivariate NARLI model fail to produce the shape of Canadian business cycles. In predicting recessions, those models that include the spread perform much better than the univariate models, and the bivariate NARLI model has the best scores. As in the US case, we conclude that the bivariate NARLI model fits the characteristics of the Canadian business cycles, and that it is well suited for predicting recessions.

United Kingdom: As in the previous two cases, no nonlinearity is found in the univariate model of UK output, but significant nonlinearity is found in the bivariate models of output and spread. However, using the procedure described in Section 2, we conclude that the evidence of nonlinearity in the spread equation is due to just a few outlying observations. Therefore, we use a linear model for the spread. We also estimate a univariate autoregressive model with an exogenous break estimated at 1990:4. As in the previous cases, univariate autoregressive models and the linear ARLI model fail to capture the shape of the business cycle. Interestingly, the autoregressive model with a single structural break also fails this task. The bivariate NARLI model is the only model that can reproduce the shape of UK business cycles. This model also does best in predicting the probability of recessions, although the improvement in the scores is not as dramatic as in the US and Canadian cases.

France: The French case is the only case where no nonlinearity is found in either the univariate output equation or in the ARLI model of output. Unlike the previous cases, the univariate autoregressive model of output can capture the shape of the business cycles in France. However, the univariate autoregressive model scores worse than the random walk model in predicting the probability of recessions. The bivariate ARLI model of output and the spread also passes the data admissibility tests, and scores much better than the univariate models in predicting the probability of recessions. The bivariate NARLI model, in which only the spread equation is nonlinear, does not perform better than the bivariate linear model.

Germany: This is the first case where the univariate autoregressive model of output shows significant signs of nonlinearity, and hence we have also estimated a univariate LSTAR model for output. However, the univariate nonlinear model does not produce better probability forecasts than the univariate linear autoregressive model. All models (except the random walk model) pass the data admissibility test. Bivariate models score better in predicting the probability of recessions, with bivariate NARLI model improving the scores only slightly over the bivariate linear model.

Italy: As in the German case, linearity is rejected even in the linear autoregressive model of output, and all estimated models capture the shape of the business cycle. Unlike the German case however, the univariate nonlinear model of output scores considerably better than the univariate linear model in predicting recessions. Unlike all previous cases, the contribution of the spread to the output equation is quite weak. Even though the coefficient of the lag of the spread in the output equation is significantly different from zero, the estimated standard errors of the univariate autoregressive model of output and that of the output equation in the linear bivariate model are equal. Indeed, if we had allowed for five lags in the bivariate model, we would have ended up with no spread variable in the output equation. The forecast accuracy measures confirm that the addition of the spread does not help predict recessions in Italy and the univariate nonlinear model does best.

Japan: The Japanese case is unique in the sense that without allowing for an exogenous structural break, all models fail to capture the shape of the business cycles in Japan. We attribute this to the fact that in Japan there has been a significant decline in the output trend, unlike the US or the UK cases where the evidence of a break is in the variance and the persistence of output growth. We estimate the break date to be 1991:2. Linear models with the break can capture the shape of the business cycles in Japan. The ARLI model with break produces the best forecasts for the probability of recessions. We emphasize again that in the forecasting exercise, we assume that the structural break is recognized immediately after it happens. We conclude from our analysis that a bivariate time series model of output and the spread is not rich enough to explain the important features of the business cycles in Japan.

4. Conclusion and Directions for Further Research

In this paper we ask if bivariate nonlinear autoregressive models of output growth and the term spread can explain and forecast important features of business cycles in G-7 countries. We evaluate our models by assessing whether or not they imply the cyclical features that are present in the observed data, and how well they can forecast the probability of well defined events such as "two consecutive quarters with negative growth in the next five quarters". For the US, Canada and the UK, we find that the bivariate nonlinear leading indicator model of output and the spread is the only model that can capture the shape of the business cycles without any need for exogenous time variation (i.e. structural breaks). In these cases, the bivariate NARLI models are also clearly the best models for predicting recessions. For France, Germany and Italy we found that all autoregressive models of output could capture the shape of the business cycles. Addition of the term spread improves the probability forecasts for France and Germany, but not for Italy. We found no model of output and term spread that could explain the important characteristics of the Japanese business cycles without recourse to a structural break in the mean.

Relative to other research that has looked at the link between yield spreads and output, the distinctive feature of our work is that we explicitly model nonlinearities in output and the spread. We follow a stepwise procedure for developing our models which starts from finding the best linear model and then moves to nonlinear models only if nonlinear models are warranted by the data. The procedures that we follow to decide between threshold or smooth transition models, and between "genuine" nonlinearity or "outlier" behavior are likely to be of interest in other applications.

The apparent reduction in the variance of output growth in the US has attracted a lot of attention lately, and similar declines in the variance of output growth in many other industrial countries will doubtless motivate further parallel research on this topic. Our results show that for the US and the UK, one cannot rule out the possibility that the post-war data have been generated by a fixed parameter self-exciting nonlinear model of output and the spread. That is, there is no need to look beyond the information set containing output and the spread, and no need to allow for exogenous structural breaks to explain the salient features of the business cycles in these countries.

Some researchers have advocated the use of error correction terms from models of financial markets in models of real variables, and we think that this insight is important. The yield spread is, of course a valid error correction term in modeling the bond market, and as such, it summarizes many features of the bond sector. Recent related work by Sensier et al (2002) provides evidence of the usefulness of short-term interest rates in Germany for predicting recession in Italy and France. We believe that further research that uses carefully chosen error correction terms from international financial markets as predictors for output within a multivariate nonlinear framework, may lead to superior models for capturing the shape and the turning points of business cycles.

References

Aït-Sahalia, Y. (1996), "Testing Continuous Time Models of the Spot Interest Rate", The Review of Financial Studies, 9, 385 - 426.

Anderson, H.M. and F. Vahid (2001), "Predicting the Probability of a Recession with Nonlinear Autoregressive Leading Indicator Models", Macroeconomic Dynamics, 3, 1 - 50.

Anderson, H. M. and F. Vahid (1998), "Testing Multiple Equation Systems for Common Nonlinear Factors", Journal of Econometrics, 84, 1 - 37.

Birchenhall, C.R., H. Jessen, D.R. Osborn and P. Simpson (1999), " Predicting U.S. Business Cycle Regimes", Journal of Business and Economic Statistics, 17, 313-323.

Brier, G.W. (1950) "Verification of Forecasts Expressed in Terms of Probability", Monthly Weather Review, 75, 1-3.

Bry, G. and C. Boschan (1971), Cyclical Analysis of Time Series: Selected Procedures and Computer Programs, New York, NBER.

Canova, F. (1998), "Detrending and Business Cycle Facts", Journal of Monetary Economics, 41, 475-512.

Choi, W.G. (1999), "Asymmetric Monetary Effects on Interest Rates across Monetary Policy Stances", Journal of Money, Credit and Banking, 31, 386-416.

Cover, J. P. (1992) "Asymmetric Effects of Positive and Negative Money-Supply Shocks", Quarterly Journal of Economics, Vol CVII, pp. 1261-82,

Clements, M.P. and H. Krolzig (1998), "A Comparison of the Forecast Performance of Markov Switching and Threshold Autoregressive Models of US GNP", Econometrics Journal, 1, c47-c75.

Davis, E. P. and G. Fagan (1997), "Are Financial Spreads Useful Indicators of Future Inflation and Output Growth in EU Countries?", Journal of Applied Econometrics, 12, 701 - 714.

De Long, B. J. and Summers, L. (1988). "How Does Macroeconomic Policy Affect Output?", Brookings Papers on Economic Activity, 2, p 433-80.

Diebold, F.X. and G.D. Rudebusch (1989), "Scoring the Leading Indicators", Journal of Business, 62, 369-391.

Estrella A. and F. S. Mishkin (1998), "Predicting U.S. Recessions: Financial Variables as Leading Indicators", Review of Economics and Statistics, 80, 45 - 61.

Fair, R.C. (1993), "Estimating Event Probabilities from Macroeconometric Models Using Stochastic Simulation", Chapter 3 in J.H. Stock and M.W. Watson (eds) **Business Cycles, Indicators and Forecasting**, NBER Studies in Business Cycles, 28, NBER: Chicago.

Friedman, B.M. and K.N. Kuttner (1998), " Indicator Properties of the Paper-Bill Spread: Lessons from Recent Experience", The Review of Economics and Statistics, LXXX, 34-44.

Galbraith, J. W. and G. Tkacz (2000), "Testing for Asymmetry in the Link between the Yield Spread and Output in the G-7 Countries", Journal of International Money and Finance, 19, 657 - 672.

Gertler, M. and C.S. Lown (1999), "The Information in the High-Yield Bond Spread for the Business Cycle: Evidence and Some Implications", Oxford Review of Economic Policy, 15, 132-150.

Hamilton, J.D. and D. H. Kim (2000), "A Re-examination of the Predictability of Economic Activity Using the Yield Spread", Discussion Paper # 2000-23, University of California at San Diego.

Hamilton, J. D. (1989), "A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle", Econometrica, 57, 357-384.

Harding, A. and A.R. Pagan (2002), "Dissecting the Cycle: A Methodological Investigation", Journal of Monetary Economics, 49, 365 - 381.

Jansen, D.W. and W. Oh (1999), "Modeling Nonlinearity of Business Cycles: Choosing Between the CDR and STAR Models", Review of Economics and Statistics, 81, 2, 344-349.

Karras, G., (1996), ‘Are the Output Effects of Monetary Policy Asymmetric? Evidence from a Sample of European Countries”, Oxford Bulletin of Economics and Statistics, 58, 2, 267 - 278.

Karunaratne, N. D. (1999), “The Yield Curve as a Predictor of Growth and Recessions in Australia”, Mimeo, University of Queensland, Australia.

Kim, C. and C.R. Nelson (1999), “Has the U.S. Economy Become More Stable? A Bayesian Approach Based on a Markov-Switching Model of the Business Cycle,” Review of Economics and Statistics, 81, 608-616.

Kozicki, S. (1997), “Predicting Real Growth and Inflation with the Yield Spread”, Federal Reserve Economic Review.

Kwark, N-S. (2002), “Default Risk, Interest Rate Spreads, and Business Cycles: Explaining the Interest Rate Spread as a Leading Indicator”, Journal of Economic Dynamics and Control, 26, 271-302.

Luukkonen, R., P. Saikkonen and T. Teräsvirta (1988), “Testing Linearity Against Smooth Transition Autoregressive Models”, Biometrika, 75, 491 - 499.

McConnell, M.M. and G. Perez-Quiros (2000), “Output Fluctuations in the United States: What Has Changed Since the Early 1980s?” American Economic Review, 90, 1464-1476.

Neftçi, S.N. (1982), “Optimal Prediction of Cyclical Downturns”, Journal of Economic Dynamics and Control, 4, 225-241.

Sensier, M., M. Artis, D.R. Osborn and C.R. Birchenhall (2002), “Domestic and International Influences on Business Cycle Regimes in Europe”, Discussion paper number 011, University of Manchester.

Stock, J. H., and M. W. Watson, (1989) “New Indexes of Coincident and Leading Economic Indicators”, **N.B.E.R. Macroeconomics Annual**, N.B.E.R.

Stock, J.H. and M.W. Watson (2001), “Forecasting Output and Inflation: The Role of Asset Prices”, Working Paper on Mark Watson’s Web page, Princeton University.

Teräsvirta, T. and H.M. Anderson (1992), "Characterizing Nonlinearities in Business Cycles Using Smooth Transition Autoregressive Models", Journal of Applied Econometrics, 7, S119-S136.

Teräsvirta, T. (1994), "Specification, Estimation and Evaluation of Smooth Transition Autoregressive Models", Journal of the American Statistical Association, 89, 208-218.

Tsay, R. (1989) "Testing and Modeling Threshold Autoregressive Processes", Journal of the American Statistical Association", 84, 231-240.

van Dijk, D., P.H. Frances and A. Lucas, (1999), "Testing for Smooth Transition Nonlinearity in the presence of Outliers", Journal of Business and Economic Statistics, 17, 217 - 235.

Weise, C. L. (1999), "The Asymmetric Effects of Monetary Policy: A Nonlinear Vector Autoregressive Approach", Journal of Money, Credit and Banking, 31, 85-108.

Zellner, A. and C. Hong (1989), "Forecasting International Growth Rates Using Bayesian Shrinkage and other Procedures", Journal of Econometrics, 40, 183-202.

Zellner, A., C. Hong and C. Min (1991), "Forecasting Turning Points in International Growth Rates Using Bayesian Exponentially Weighted Autoregression, Time-Varying Parameter and Pooling Techniques", Journal of Econometrics, 49, 275-304.

Zellner, A. and C. Min (1999), "Forecasting Turning Points in Countries' Output Growth Rates", Journal of Econometrics, 88, 203-306.

APPENDIX 1: DATA

Precise descriptions of the raw series that we use in this analysis are given below. Unless otherwise stated, we have drawn all data for the first four countries from the OECD portion of the DX database (Australia), and we have extracted all data for the last three countries from the data files available on Mark Watson's web page. We use the logarithms of real GDP when we undertake our benchmark analysis, and our models are functions of output growth ($y_t = 100 \times \Delta \ln(GDP)$) and the interest rate spread ($s_t =$ Long-term interest rate - Short-term interest rate). The effective samples used for analysis are shorter than the raw series because of lagged variables in the models.

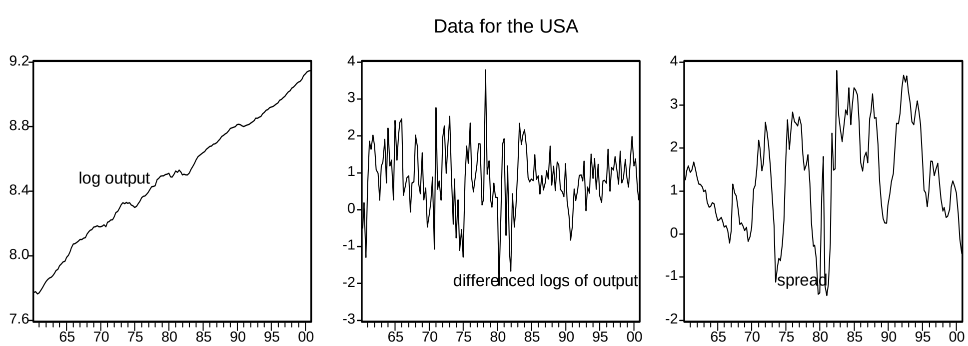
USA (1960:1 to 2000:4)

Output: Real Gross Domestic Product: (Billions of Chained 1996 Dollars, seasonally adjusted at annual rates, from the U.S. Department of Commerce, Bureau of Economic Analysis).

Short-Term Interest Rates: 3-Month Treasury (Secondary) Bill Market Rates (Averages over business days expressed as a percentage, H15 Release from the Federal Reserve Board of Governors).

Long-Term Interest Rates: 10-Year Treasury Bond Constant Maturity Rates (Averages over business days expressed as a percentage, H15 Release from the Federal Reserve Board of Governors).

The effective sample consisted of 160 observations, dating from 1961:1 to 2000:4.



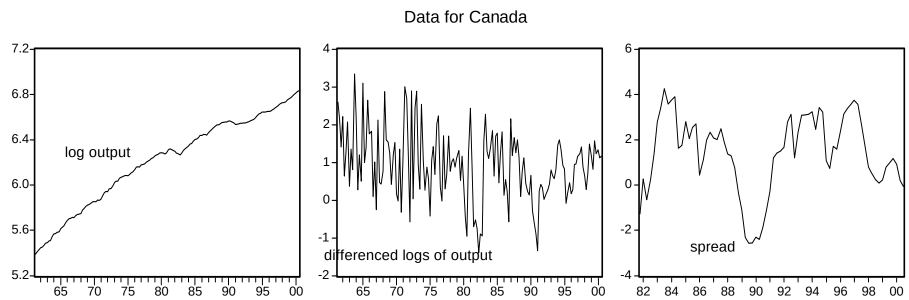
CANADA (1961:1 to 2000:3)

Output: Real Gross Domestic Product (seasonally adjusted in constant 1992 prices, series CAN.NAGVTT01.NCALSA).

Short-Term Interest Rates: Interest rates on 90 day deposit receipts. (expressed as a percentage pa, series CAN.IRT3DR01.ST).

Long-Term Interest Rates: Yields on long term government bonds (>10 Years). (expressed as a percentage pa, series CAN.IRLGV06.ST).

The effective sample consisted of 156 observations, dating from 1961:4 to 2000:3.



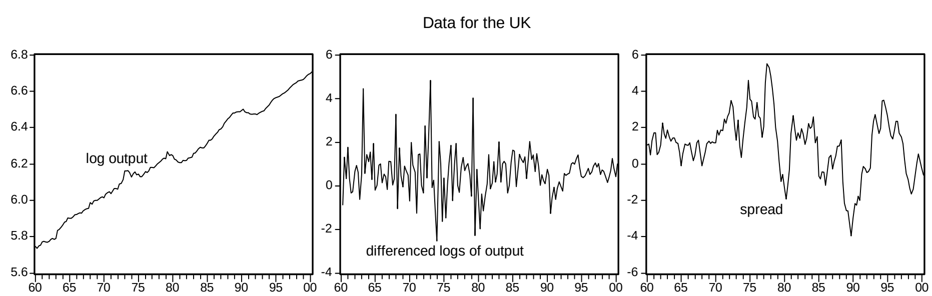
UNITED KINGDOM (1960:1 to 2000:2)

Output: Real Gross Domestic Product (seasonally adjusted in constant 1995 prices, series GBR.NAGVTT01.NCALSA).

Short-Term Interest Rates: 3 Month Treasury Bill Rates. (expressed as a percentage pa, series 11260C..ZF... from the IFS portion of the DX database).

Long-Term Interest Rates: Yields on 10 Year Government Bonds (expressed as a percentage pa, series GBR.IRLTGV02.ST).

The effective sample consisted of 158 observations, dating from 1961:1 to 2000:2



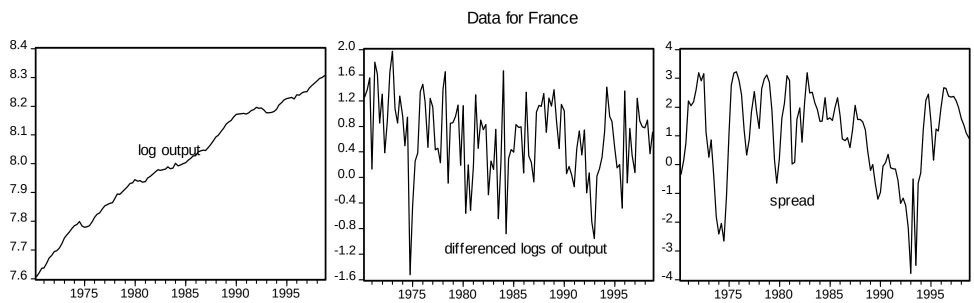
FRANCE (1970:1 to 1998:4)

Output: Real Gross Domestic Product (seasonally adjusted in constant 1980 prices, series FRA.NAGVTT01h.NCALSA).

Short-Term Interest Rates: Interest Rate on 3 Month PIBOR (expressed as a percentage pa, series FRA.IRT31B01.ST).

Long-Term Interest Rates: Interest Rates on 10 year Bonds when issued (expressed as a percentage pa, series FRA.IRLTOT02.ST).

The effective sample consisted of 113 observations, dating from 1970:4 to 1998:4.



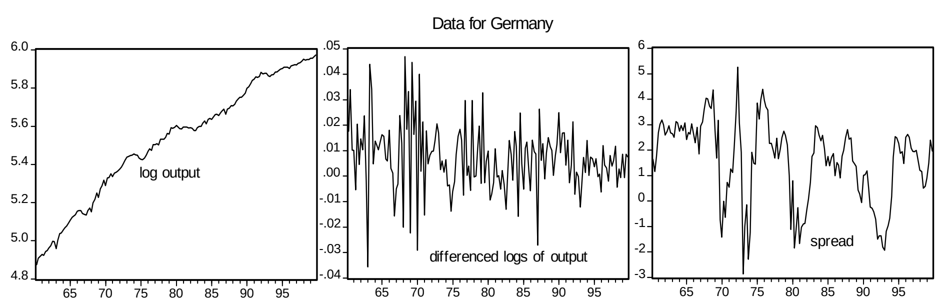
GERMANY (1960:1 to 1999:4)

Output: Real Gross Domestic Product, (seasonally adjusted, series I 199bv&r@c134, originally from the IFS data base).

Short-Term Interest Rates: Overnight Interest Rate (expressed as a percentage pa, series I 160c@c134, from the IFS data base).

Long-Term Interest Rates: Interest rate on a long term Government bond (expressed as a percentage pa, series I 161@c134, from the IFS data base).

The effective sample consisted of 155 observations, dating from 1961:2 to 1999:4.



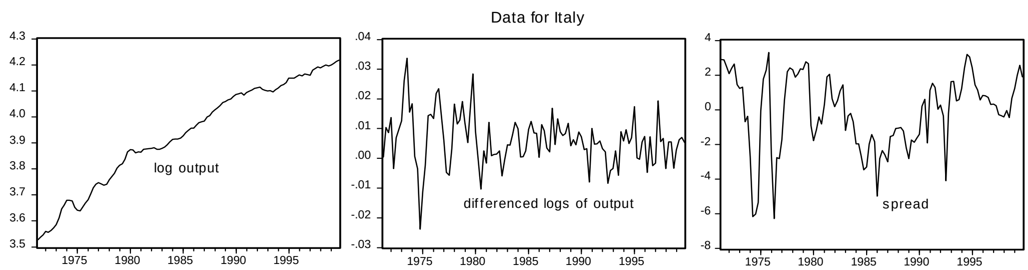
ITALY (1971:4 to 1998:4)

Output: Real Gross Domestic Product, (seasonally adjusted, series I 199bv&r@c136, originally from the IFS data base).

Short-Term Interest Rates: Overnight Interest Rate (expressed as a percentage pa, series I 160c@c136, from the IFS data base).

Long-Term Interest Rates: Interest Rate on a Long Term Government Bond (expressed as a percentage pa, series I 161@c136, from the IFS data base).

The effective sample consisted of 110 observations, dating from 1972:3 to 1998:4.



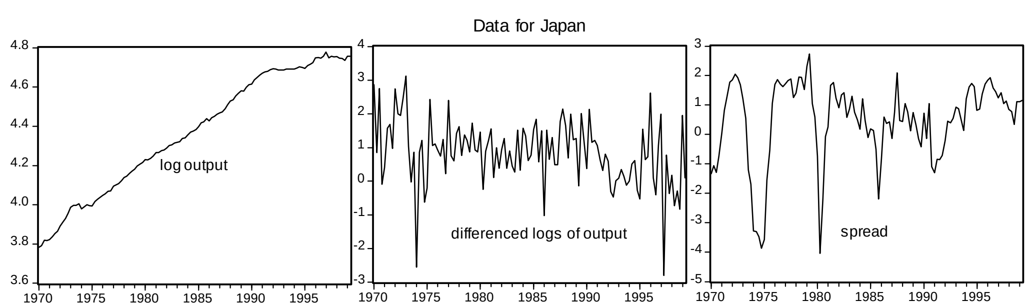
JAPAN (1969:4 to 1999:4)

Output: Real Gross Domestic Product, (seasonally adjusted in 1990 prices, series I 199bv&r@c158, originally from the IFS data base).

Short-Term Interest Rates: Overnight Interest Rate (expressed as a percentage pa, series I 160b@c158, from the IFS data base).

Long-Term Interest Rates: Interest Rate on a Long Term Government Bond (expressed as a percentage pa, series I 161@c158, from the IFS data base).

The effective sample consisted of 113 observations, dating from 1971:2 to 1999:4.



APPENDIX 2: MODELS OF OUTPUT AND SPREAD

A. USA: 1961:1 - 2000:4

(Standard errors are in brackets)

Random walk model of output:

$$\hat{y}_t = \underset{(0.07)}{0.87} \qquad \hat{\sigma}_{\mathsf{MLE}} = .87$$

AR(2) model of output without a break:

$$\hat{y}_t = \underset{(0.10)}{0.53} + \underset{(0.08)}{0.26}\, y_{t-1} + \underset{(0.08)}{0.13}\, y_{t-2} \qquad \hat{\sigma}_{\mathsf{MLE}} = .82$$

AR model of output with a break at 1984:2:

$$\begin{aligned} y_t &= \underset{(0.14)}{0.64} + \underset{(0.10)}{0.29}\, y_{t-1} + \hat{\varepsilon}_t & \hat{\sigma}_{\mathsf{MLE}} = 1.02 \quad \text{for 1961:1 to 1984:1} \\ y_t &= \underset{(0.13)}{0.44} + \underset{(0.12)}{0.23}\, y_{t-1} + \underset{(0.12)}{0.22}\, y_{t-2} + \hat{\varepsilon}_t & \hat{\sigma}_{MLE} = 0.48 \quad \text{for 1984:2 to 2000:4} \end{aligned}$$

ARLI model of output and spread:

$$\begin{aligned} \hat{y}_t &= \underset{(0.11)}{0.33} + \underset{(0.08)}{0.18}\, y_{t-1} + \underset{(0.07)}{0.12}\, y_{t-2} + \underset{(0.05)}{0.20}\, s_{t-2} & \hat{\sigma}_{\mathsf{MLE}} = 0.79 \\ \hat{s}_t &= \underset{(0.08)}{0.26} - \underset{(0.05)}{0.15}\, y_{t-2} + \underset{(0.08)}{1.06}\, s_{t-1} - \underset{(0.11)}{0.35}\, s_{t-2} + \underset{(0.08)}{0.18}\, s_{t-3} & \hat{\sigma}_{\mathsf{MLE}} = 0.55 \end{aligned}$$

Bi-NARLI model of output and spread:

$$\begin{aligned} \hat{y}_t &= -\underset{(0.23)}{0.52}\, y_{t-1} + \underset{(0.24)}{0.49}\, y_{t-2} + \underset{(0.29)}{0.50}\, y_{t-3} - \underset{(0.26)}{0.66}\, s_{t-1} + \underset{(0.36)}{1.37}\, s_{t-2} + \\ &\quad f_{yt} \times \left(\underset{(0.16)}{0.81} + \underset{(0.24)}{0.71}\, y_{t-1} - \underset{(0.25)}{0.43}\, y_{t-2} - \underset{(0.29)}{0.57}\, y_{t-3} + \underset{(0.29)}{0.73}\, s_{t-1} - \underset{(0.37)}{1.40}\, s_{t-2}\right) \\ f_{yt} &= \left(1 + \exp\{-14\,(s_{t-2} - 0.024)\}\right)^{-1} \qquad \hat{\sigma}_{\mathsf{MLE}} = 0.71 \end{aligned}$$

$$\begin{aligned} \hat{s}_t &= \underset{(0.09)}{0.45} - \underset{(0.08)}{0.24}\, y_{t-2} + \underset{(0.13)}{1.19}\, s_{t-1} - \underset{(0.14)}{0.56}\, s_{t-2} + \\ &\quad f_{st} \times \left(\underset{(0.25)}{0.21}\, y_{t-2} - \underset{(0.07)}{0.11}\, y_{t-3} - \underset{(0.13)}{0.47}\, s_{t-1} + \underset{(0.20)}{0.35}\, s_{t-2} + \underset{(0.09)}{0.34}\, s_{t-3}\right) \\ f_{st} &= \left(1 + \exp\{-6.79\,(s_{t-1} - 1.24)\}\right)^{-1} \qquad \hat{\sigma}_{\mathsf{MLE}} = 0.49 \end{aligned}$$

B. Canada: 1961:4 - 2000:3

(Standard errors are in brackets)

Random walk model of output:

$$\hat{y}_t = \underset{(0.07)}{0.91} \qquad \hat{\sigma}_{\mathsf{MLE}} = .91$$

AR(1) model of output:

$$\hat{y}_t = \underset{(0.10)}{0.63} + \underset{(0.08)}{0.31}\, y_{t-1} \qquad \hat{\sigma}_{\mathsf{MLE}} = .87$$

ARLI model of output and spread:

$$\begin{aligned}
\hat{y}_t &= \underset{(0.10)}{0.55} + \underset{(0.08)}{0.19}\, y_{t-1} + \underset{(0.06)}{0.20}\, s_{t-2} & \hat{\sigma}_{\mathsf{MLE}} = 0.82 \\
\hat{s}_t &= \underset{(0.09)}{0.23} - \underset{(0.07)}{0.12}\, y_{t-1} + \underset{(0.08)}{1.02}\, s_{t-1} - \underset{(0.08)}{0.16}\, s_{t-2} & \hat{\sigma}_{\mathsf{MLE}} = 0.76
\end{aligned}$$

Bi-NARLI model of output and spread:

$$\begin{aligned}
\hat{y}_t &= \underset{(0.16)}{0.37} + \underset{(0.20)}{0.64}\, y_{t-2} + \underset{(0.11)}{0.42}\, s_{t-1} + \underset{(0.04)}{0.13}\, s_{t-2} + \\
&\quad f_{yt} \times (\underset{(0.21)}{0.21} + \underset{(0.08)}{0.22}\, y_{t-1} - \underset{(0.20)}{0.64}\, y_{t-2} - \underset{(0.11)}{0.42}\, s_{t-1}) \\
f_{yt} &= (1 + \exp\{-41.69\,(y_{t-1} + 0.042)\})^{-1} & \hat{\sigma}_{\mathsf{MLE}} = 0.76
\end{aligned}$$

$$\begin{aligned}
\hat{s}_t &= \underset{(0.73)}{2.37} + \underset{(0.89)}{1.68}\, y_{t-1} - \underset{(0.38)}{0.61}\, y_{t-2} + \underset{(0.15)}{0.56}\, s_{t-2} + \\
&\quad f_{st} \times (\underset{(0.73)}{-2.37} - \underset{(0.89)}{1.68}\, y_{t-1} + \underset{(0.38)}{0.61}\, y_{t-2} + \underset{(0.09)}{1.14}\, s_{t-1} - \underset{(0.17)}{0.79}\, s_{t-2}) \\
f_{st} &= (1 + \exp\{-4.72\,(y_{t-1} + 0.32)\})^{-1} & \hat{\sigma}_{\mathsf{MLE}} = 0.70
\end{aligned}$$

C. UK: 1960:1 - 2000:2

(Standard errors are in brackets)

Random walk model of output:

$$\hat{y}_t = \underset{(0.08)}{0.60} \qquad \hat{\sigma}_{\mathsf{MLE}} = 1.03$$

AR(1) model of output:

$$\begin{aligned} y_t &= \underset{(0.11)}{0.61} + \hat{\varepsilon}_t && \hat{\sigma}_{\mathsf{MLE}} = 1.16 \quad \text{for 1960:1 to 1990:4} \\ y_t &= \underset{(0.08)}{0.22} + \underset{(0.11)}{0.67}\, y_{t-1} + \hat{\varepsilon}_t && \hat{\sigma}_{MLE} = 0.31 \quad \text{for 1991:1 to 2000:2} \end{aligned}$$

ARLI model of output and spread:

$$\begin{aligned} \hat{y}_t &= \underset{(0.10)}{0.38} + \underset{(0.08)}{0.16}\, y_{t-3} + \underset{(0.05)}{0.12}\, s_{t-2} && \hat{\sigma}_{MLE} = 1.01 \\ \hat{s}_t &= \underset{(0.07)}{0.20} - \underset{(0.05)}{0.16}\, y_{t-3} + \underset{(0.07)}{1.15}\, s_{t-1} - \underset{(0.08)}{0.27}\, s_{t-2} && \hat{\sigma}_{MLE} = 0.67 \end{aligned}$$

Bi-NARLI model of output and spread:

$$\begin{aligned} \hat{y}_t &= \underset{(0.11)}{0.26} + \underset{(0.11)}{0.22}\, y_{t-2} + \underset{(0.47)}{1.10}\, s_{t-1} - \underset{(0.61)}{1.30}\, s_{t-2} + \underset{(0.36)}{0.75}\, s_{t-3} + \\ &\quad f_{yt} \times \left(\underset{(0.26)}{0.77}\, y_{t-1} - \underset{(0.15)}{0.39}\, y_{t-3} - \underset{(0.78)}{1.70}\, s_{t-1} + \underset{(1.03)}{2.11}\, s_{t-2} - \underset{(0.65)}{1.21}\, s_{t-3}\right) \\ f_{yt} &= \left(1 + \exp\left\{-1.44\left(y_{t-2} - 0.38\right)\right\}\right)^{-1} && \hat{\sigma}_{MLE} = 0.94 \end{aligned}$$

$$\hat{s}_t = \underset{(0.07)}{0.20} - \underset{(0.05)}{0.16}\, y_{t-3} + \underset{(0.07)}{1.15}\, s_{t-1} - \underset{(0.08)}{0.27}\, s_{t-2} \qquad \hat{\sigma}_{MLE} = 0.67$$

D. France: 1970:4 - 1998:4

(Standard errors are in brackets)

Random walk model of output:

$$y_t = \underset{(0.06)}{0.61} + \hat{\varepsilon}_t \qquad \hat{\sigma}_{MLE} = 0.64$$

AR(2) model of output:

$$y_t = \underset{(0.09)}{0.36} + \underset{(0.09)}{0.21} y_{t-1} + \underset{(0.09)}{0.18} y_{t-2} + \hat{\varepsilon}_t \qquad \hat{\sigma}_{MLE} = 0.61$$

ARLI model of output and spread:

$$\begin{aligned} \hat{y}_t &= \underset{(0.08)}{0.35} + \underset{(0.09)}{0.19} y_{t-2} + \underset{(0.04)}{0.13} s_{t-2} && \hat{\sigma}_{MLE} = 0.58 \\ \hat{s}_t &= \underset{(0.10)}{0.22} + \underset{(0.06)}{0.80} s_{t-1} && \hat{\sigma}_{MLE} = 0.91 \end{aligned}$$

Bi-NARLI model of output and spread:

$$\begin{aligned} \hat{y}_t &= \underset{(0.08)}{0.34} + \underset{(0.09)}{0.19} y_{t-2} + \underset{(0.04)}{0.14} s_{t-2} && \hat{\sigma}_{MLE} = 0.58 \\ \hat{s}_t &= -\underset{(0.38)}{1.64} + \underset{(0.49)}{0.58} y_{t-2} + \\ (s_{t-1} &> -0.33) \times \left(\underset{(0.41)}{2.18} - \underset{(0.49)}{0.58} y_{t-2} + \underset{(0.10)}{1.01} s_{t-1} - \underset{(0.09)}{0.38} s_{t-2} \right) && \hat{\sigma}_{MLE} = 0.78 \end{aligned}$$

E. Germany: 1961:2 - 1999:4

(Standard errors are in brackets)

Random walk model of output:

$$y_t = \underset{(0.10)}{0.67} + \hat{\varepsilon}_t \qquad \hat{\sigma}_{MLE} = 1.32$$

AR(4) model of output:

$$y_t = \underset{(0.12)}{0.55} - \underset{(0.07)}{0.17}\, y_{t-1} + \underset{(0.07)}{0.33}\, y_{t-4} + \hat{\varepsilon}_t \qquad \hat{\sigma}_{MLE} = 1.23.$$

STAR(4) model of output:

$$\hat{y}_t = \underset{(0.38)}{2.32}\, y_{t-3} + (1 + \exp\{-1.52\,(y_{t-1} + 1.26)\})^{-1} \times \left(\underset{(0.13)}{0.51} - \underset{(0.43)}{2.57}\, y_{t-3} + \underset{(0.08)}{0.28}\, y_{t-4} \right) \qquad \hat{\sigma}_{MLE} = 1.07.$$

ARLI model of output and spread:

$$\hat{y}_t = \underset{(0.14)}{0.26} - \underset{(0.07)}{0.23}\, y_{t-1} + \underset{(0.07)}{0.30}\, y_{t-4} + \underset{(0.06)}{0.23}\, s_{t-3} \qquad \hat{\sigma}_{MLE} = 1.17$$

$$\hat{s}_t = \underset{(0.10)}{0.36} + \underset{(0.06)}{0.82}\, s_{t-1} + \underset{(0.09)}{0.24}\, s_{t-3} - \underset{(0.08)}{0.29}\, s_{t-4} \qquad \hat{\sigma}_{MLE} = 0.87$$

Bi-NARLI model of output and spread:

$$\begin{aligned} \hat{y}_t &= \underset{(0.42)}{0.99}\, y_{t-2} + \underset{(0.35)}{1.80}\, y_{t-3} + \underset{(0.06)}{0.18}\, s_{t-1} + (1 + \exp\{-0.90\,(y_{t-1} + 1.4)\})^{-1} \times \\ &\quad \left(\underset{(0.16)}{0.50} - \underset{(0.54)}{1.21}\, y_{t-2} - \underset{(0.42)}{2.24}\, y_{t-3} \right) \qquad \hat{\sigma}_{MLE} = 1.06 \end{aligned}$$

$$\begin{aligned} \hat{s}_t &= \underset{(0.07)}{0.81}\, s_{t-1} + \underset{(0.68)}{1.43}\, s_{t-2} - \underset{(0.58)}{1.17}\, s_{t-4} + (1 + \exp\{-1.41\,(y_{t-3} + 0.90)\})^{-1} \times \\ &\quad \left(\underset{(0.12)}{0.33} - \underset{(0.72)}{1.59}\, s_{t-2} + \underset{(0.10)}{0.21}\, s_{t-3} + \underset{(0.62)}{1.06}\, s_{t-4} \right) \qquad \hat{\sigma}_{MLE} = 0.78 \end{aligned}$$

F. Italy: 1971:3 - 1999:4

(Standard errors are in brackets)

Random walk model of output:

$$y_t = \underset{(0.11)}{0.59} + \hat{\varepsilon}_t \qquad \hat{\sigma}_{MLE} = 0.86$$

AR(5) model of output:

$$y_t = \underset{(0.10)}{0.45} + \underset{(0.08)}{0.46} y_{t-1} - \underset{(0.08)}{0.22} y_{t-5} + \hat{\varepsilon}_t \qquad \hat{\sigma}_{MLE} = 0.73$$

STAR(5) model of output:

$$\begin{aligned} y_t &= \underset{(017)}{-0.77} y_{t-3} + \frac{1}{1+\exp(-2.04(y_{t-3}-0.61))} \times \\ &\left(\underset{(0.43)}{1.87} + \underset{(0.16)}{0.85} y_{t-1} - \underset{(0.15)}{0.41} y_{t-5} \right) + \hat{\varepsilon}_t \qquad \hat{\sigma}_{MLE} = 0.65 \end{aligned}$$

ARLI model of output and spread:

$$\begin{aligned} \hat{y}_t &= \underset{(0.08)}{0.36} + \underset{(0.08)}{0.43} y_{t-1} + \underset{(0.03)}{0.09} s_{t-2}, \qquad \hat{\sigma}_{MLE} = 0.73 \\ \hat{s}_t &= \underset{(0.16)}{0.48} + \underset{(0.09)}{0.87} s_{t-1} - \underset{(0.12)}{0.30} s_{t-2} + \underset{(0.09)}{0.17} s_{t-3} - \underset{(0.15)}{0.49} y_{t-1} - \underset{(0.15)}{0.40} y_{t-3}, \qquad \hat{\sigma}_{MLE} = 1.25 \end{aligned}$$

Bi-NARLI model of output and spread:

$$\begin{aligned} \hat{y}_t &= \underset{(0.08)}{0.43} + \underset{(0.10)}{0.35} y_{t-1} + (1+\exp\{-6.56\,(y_{t-3}-1.40)\})^{-1} \times \\ &\left(\underset{(0.29)}{0.81} y_{t-1} - \underset{(0.15)}{0.37} y_{t-3} + \underset{(0.09)}{0.30} s_{t-2} \right) \qquad \hat{\sigma}_{MLE} = 0.65 \end{aligned}$$

$$\begin{aligned} \hat{s}_t &= \underset{(0.17)}{0.39} + \underset{(0.09)}{0.78} s_{t-1} - \underset{(0.11)}{0.32} s_{t-2} + \underset{(0.08)}{0.18} s_{t-3} - \underset{(0.14)}{0.48} y_{t-1} - \underset{(0.16)}{0.57} y_{t-3} + \\ &(1+\exp\{-7.47\,(s_{t-1}-2.0)\})^{-1} \times \left(\underset{(2.13)}{7.63} - \underset{(0.80)}{2.85} s_{t-1} + \underset{(0.41)}{0.82} y_{t-3} \right) \qquad \hat{\sigma}_{MLE} = 1.13 \end{aligned}$$

G. Japan: 1971:2 - 1999:4

(Standard errors are in brackets)

Random walk model of output:

$$y_t = \underset{(0.10)}{1.06} - \underset{(0.18)}{0.80} D_t + \hat{\varepsilon}_t \qquad \hat{\sigma}_{MLE} = 0.87$$

AR(5) model of output:

$$y_t = \underset{(0.12)}{1.07} - \underset{(0.18)}{0.79} D_t + \underset{(0.10)}{0.19} y_{t-3} - \underset{(0.09)}{0.20} y_{t-5} + \hat{\varepsilon}_t \qquad \hat{\sigma}_{MLE} = 0.84$$

TAR model of output:

$$y_t = \underset{(0.15)}{0.41} + \underset{(0.11)}{0.51} y_{t-3} + \underset{(0.14)}{0.24} y_{t-4} - \underset{(0.12)}{0.16} y_{t-5} + (y_{t-4} \geq 1.061) \left(\underset{(0.15)}{0.26} y_{t-2} - \underset{(0.11)}{0.51} y_{t-3} \right) + \hat{\varepsilon}_t \qquad \hat{\sigma}_{\varepsilon} = 0.84$$

ARLI model of output and spread:

$$\begin{aligned} \hat{y}_t &= \underset{(0.15)}{1.04} - \underset{(0.19)}{0.90} D_t + \underset{(0.09)}{0.15} y_{t-3} - \underset{(0.08)}{0.18} y_{t-5} + \underset{(0.06)}{0.15} s_{t-1} && \hat{\sigma}_{MLE} = 0.82 \\ \hat{s}_t &= \underset{(0.08)}{0.14} + \underset{(0.06)}{0.89} s_{t-1} - \underset{(0.05)}{0.17} s_{t-4} && \hat{\sigma}_{MLE} = 0.75 \end{aligned}$$

Bi-NARLI model of output and spread:

$$\begin{aligned} \hat{y}_t &= \underset{(0.11)}{0.47} + \underset{(0.08)}{0.29} y_{t-3} + \underset{(0.08)}{0.21} y_{t-4} + \underset{(0.11)}{0.22} s_{t-1} - \underset{(0.09)}{0.19} s_{t-2} + \\ & \quad f_{yt} \times \left(-\underset{(0.53)}{1.10} + \underset{(0.41)}{0.85} y_{t-2} - \underset{(0.28)}{0.93} s_{t-4} \right) \\ \hat{f}_{yt} &= \left(1 + \exp\left\{-12.69\left(y_{t-5} - 2.00\right)\right\}\right)^{-1} && \hat{\sigma}_{MLE} = 0.77 \\ \hat{s}_t &= \underset{(0.08)}{0.14} + \underset{(0.06)}{0.89} s_{t-1} - \underset{(0.05)}{0.17} s_{t-4} && \hat{\sigma}_{MLE} = 0.75 \end{aligned}$$

PANEL 1

Performance of Different Models in Capturing the Shape of Business Cycles

The values in parentheses are bounds of 90% confidence intervals derived from the simulated distributions. The asterisks highlight those sample statistics whose 90% bounds do not contain the observed cycle characteristic.

A. USA

	Raw Data	RW + Drift	AR(2)	AR(2)+ Break	ARLI	Bi NARLI
Duration						
PT	3.8	2.4* (2.0,3.4)	3.1 (2.0,4.5)	3.0 (2.0,4.5)	3.3 (2.2,5.0)	2.9 (2.0,4.0)
TP	20.4	36.5 (13.0,80.0)	31.0 (15.0,64.5)	24.2 (10.6,47.7)	27.6 (14.3,49.3)	37.0 (17.3,73.0)
Amplitude						
PT	−2.1	−1.0* (−1.7,−0.5)	−1.4 (−2.3,−0.7)	−1.7 (−2.8,−0.8)	−1.5 (−2.5,−0.8)	−1.7 (−3.1,−0.7)
TP	22.9	34.2 (12.2,75.6)	30.9 (15.0,64.5)	25.7 (11.1,50.9)	27.0 (13.3,50.3)	35.5 (16.6,69.9)
Cumulation						
PT	−4.2	−1.4* (−2.7,−0.5)	−2.6 (−6.0,−0.7)	−3.0 (−7.2,−0.8)	−3.2 (−7.4,−0.9)	−2.8 (−6.2,−0.7)
TP	342	1025 (97,3451)	865 (156,2649)	562 (82,1678)	655 (136,1841)	1107 (195,3254)
Excess						
PT	−0.10	0.00 (−0.15,0.15)	0.00 (−0.16,0.16)	−0.00 (−0.20,0.19)	0.00 (−0.16,0.17)	0.02 (−0.21,0.27)
TP	1.36	0.05 (−1.43,1.58)	−0.00* (−1.40,1.34)	0.21 (−1.07,1.64)	0.04* (−1.12,1.28)	0.08 (−1.20,1.38)

B. Canada

	Raw Data	RW + Drift	AR(1)	ARLI	Bi NARLI
Duration					
PT	4.0	2.4* (2.0,3.4)	2.8 (2.0,4.0)	2.9 (2.0,4.0)	3.7 (2.0,6.0)
TP	16.0	36.4 (12.6,82.0)	27.6 (12.5,56.0)	27.6 (12.1,58.0)	36.9 (10.0,89.0)
Amplitude					
PT	−3.2	−1.1* (−1.8,−0.5)	−1.4* (−2.2,−0.7)	−1.4* (−2.3,−0.7)	−2.5 (−5.8,−0.7)
TP	17.2	35.9 (12.3,79.8)	28.8 (12.4,60.0)	28.5 (11.8,60.0)	39.0 (9.5,95.6)
Cumulation					
PT	−6.6	−1.4* (−2.9,−0.5)	−2.2* (−4.8,−0.8)	−2.5* (−5.5,−0.8)	−6.7 (−21.5,−0.7)
TP	257	1061 (91,3604)	694 (106,2128)	691 (96,2177)	1192 (55,4378)
Excess					
PT	0.27	−0.00* (−0.16,0.16)	0.00* (−0.15,0.14)	0.00* (−0.16,0.16)	0.06 (−0.19,0.35)
TP	1.35	0.04 (−1.54,1.68)	0.03 (−1.31,1.40)	0.02 (−1.44,1.44)	0.14 (−1.81,2.24)

C. UK

	Raw Data	RW + Drift	AR(1)+ Break	ARLI	Bi NARLI
Duration					
PT	4.4	3.0* (2.3,4.0)	3.2* (2.3,4.3)	3.4 (2.4,4.7)	3.9 (2.5,5.8)
TP	25.5	14.7* (8.6,24.5)	13.6* (7.6,23.0)	15.3 (8.9,25.5)	16.6 (9.1,29.0)
Amplitude					
PT	−3.2	−1.7* (−2.3,−1.1)	−1.9* (−2.8,−1.3)	−1.9* (−2.7,−1.2)	−2.3 (−3.8,−1.3)
TP	21.5	12.2* (7.3,20.2)	11.9* (6.9,19.9)	12.9 (7.3,21.6)	14.8 (7.4,27.0)
Cumulation					
PT	−9.6	−3.0* (−5.4,−1.4)	−3.7* (−7.0,−1.6)	−3.9* (−7.8,−1.6)	−6.4 (−15.9,−1.8)
TP	381	158 (45,394)	141* (37,356)	173 (47,435)	233 (51,626)
Excess					
PT	−0.14	−0.00* (−0.13,0.13)	0.00 (−0.17,0.15)	0.00 (−0.15,0.15)	0.01 (−0.17,0.20)
TP	−0.50	0.00 (−0.53,0.53)	0.03 (−0.55,0.63)	−0.00 (−0.61,0.61)	0.04 (−0.73,0.79)

D. France

	Raw Data	RW + Drift	AR(2)	ARLI	Bi NARLI
Duration					
PT	3.0	2.5 (2.0,3.5)	3.2 (2.0,5.0)	3.1 (2.0,5.0)	3.8 (2.0,7.0)
TP	32.5	28.7 (8.7,68.0)	25.2 (9.0,56.0)	28.0 (8.0,66.0)	22.6 (6.0,54.0)
Amplitude					
PT	−1.6	−0.8* (−1.3,−0.4)	−1.1 (−1.9,−0.5)	−1.0 (−1.8,−0.4)	−1.2 (−2.4,−0.5)
TP	21.3	19.2 (5.8,45.1)	17.5 (5.1,40.9)	19.0 (4.6,46.6)	14.6 (2.8,38.3)
Cumulation					
PT	−2.0	−1.1 (−2.2,−0.3)	−2.1 (−5.4,−0.5)	−1.9 (−4.9,−0.4)	−3.4 (−10.5,−0.5)
TP	358	434 (27,1545)	367 (28,1257)	438 (22,1572)	314 (10,1188)
Excess					
PT	0.03	0.00 (−0.12,0.12)	0.00 (−0.13,0.14)	0.00 (−0.14,0.14)	0.00 (−0.14,0.15)
TP	−0.37	0.03 (−1.04,1.15)	0.02 (−1.17,1.23)	0.01 (−1.32,1.33)	0.03 (−1.05,1.14)

E. Germany

	Raw Data	RW + Drift	AR(4)	LSTAR(4)	ARLI	Bi NARLI
Duration						
PT	4.5	3.3* (2.4,4.4)	3.4 (2.3,5.1)	3.2 (2.0,5.0)	3.8 (2.3,5.8)	3.4 (2.2,5.2)
TP	19.2	13.9 (8.5,22.8)	18.5 (10.1,32.7)	18.2 (9.9,31.7)	19.1 (10.1,35.0)	20.2 (10.7,37.7)
Amplitude						
PT	-2.3	−2.4 (−3.3,−1.6)	−2.1 (−3.1,−1.3)	−2.1 (−3.4,−1.1)	−2.3 (−3.5,−1.3)	−1.9 (−3.3,−1.1)
TP	20.1	14.0 (8.6,22.4)	16.5 (8.6,30.0)	16.5 (8.6,29.2)	17.3 (8.6,32.5)	17.4 (9.3,31.7)
Cumulation						
PT	-5.3	−4.6 (−8.5,−2.1)	−4.5 (−9.8,−1.6)	−4.3 (−11.1,−1.2)	−5.7 (−12.9,−1.7)	−4.5 (−11.8,−1.2)
TP	253	161 (49,401)	270 (60,731)	264 (61,686)	294 (61,831)	311 (71,858)
Excess						
PT	.11	.00 (−.18,.18)	.00 (−.19,.19)	.05 (−.14,.26)	.00 (−.21,.21)	.00 (−.15,.21)
TP	.79	.01 (−.63,.64)	.02 (−.83,.85)	.35 (−.47,1.26)	.04 (−.87,.97)	.22 (−.56,1.04)

F. Italy

	Raw Data	RW + Drift	AR(5)	STAR(5)	ARLI	Bi NARLI
Duration						
PT	2.8	2.8 (2.0,4.0)	3.5 (2.4,4.7)	3.0 (2.3,3.9)	3.6 (2.4,5.0)	3.2 (2.3,4.5)
TP	14.8	20.6 (9.3,43.0)	15.6 (8.8,27.7)	16.0 (9.0,28.8)	15.5 (8.7,27.0)	15.2 (8.4,26.7)
Amplitude						
PT	-1.5	−1.2 (−1.9,−0.7)	−1.7 (−2.6,−0.9)	−1.5 (−2.4,−0.8)	−1.8 (−2.8,−1.0)	−2.1 (−3.7,−1.0)
TP	11.5	15.2 (6.9,31.2)	13.3 (7.5,22.7)	13.2 (7.4,22.7)	13.3 (7.3,22.8)	12.2 (6.6,20.6)
Cumulation						
PT	-3.0	−2.0 (−4.1,−0.7)	−3.5 (−6.9,−1.3)	−2.3 (−4.1,−1.0)	−3.9 (−8.1,−1.3)	−4.3 (−10.4,−1.3)
TP	130	264 (41,837)	158 (38,424)	157 (40,425)	159 (40,420)	135 (34,355)
Excess						
PT	-0.06	0.00 (−0.14,0.15)	0.00 (−0.14,0.15)	0.00 (−0.10,0.14)	0.00 (−0.15,0.15)	−0.04 (−0.29,0.14)
TP	0.27	0.02 (−0.81,0.88)	0.01 (−0.68,0.73)	−0.18 (−0.99,0.56)	0.02 (−0.74,0.78)	−0.37 (−1.26,0.33)

G. Japan

	Raw Data	RW + Drift+Break	AR(5)+ Break	TAR(5)	ARLI +Break	Bi NARLI
Duration						
PT	3.6	3.5 (2.0,6.0)	3.6 (2.0,6.0)	3.6 (2.0,6.7)	4.2 (2.3,7.0)	4.1 (2.0,6.5)
TP	8.0	17.5 (4.0,42.0)	18.6 (4.5,43.5)	27.1 (8.0,66.0)	17.8 (5.0,38.5)	22.5* (9.7,47.0)
Amplitude						
PT	−1.9	−1.7 (−2.0,−0.6)	−1.7 (−3.0,−0.6)	−1.6 (−3.2,−0.6)	−2.2 (−3.0,−0.6)	−5.3 (−12.2,−1.2)
TP	4.9	15.4* (2.4,40.5)	16.5 (2.6,42.7)	24.5* (5.2,63.1)	17.4 (3.1,39.8)	25.8* (8.7,55.8)
Cumulation						
PT	−6.3	−3.8 (−10.4,−0.8)	−4.0 (−10.6,−0.9)	−4.2 (−12.6,−0.6)	−6.4 (−16.4,−1.3)	−17.2 (−51.4,−1.6)
TP	19	353 (5,1348)	383 (6,1473)	554* (23,2119)	355 (9,1244)	440* (49,1448)
Excess						
PT	−0.13	0.00 (−0.23,0.22)	0.00 (−0.23,0.23)	0.00 (−0.22,0.21)	0.00 (−0.26,0.26)	−0.25 (−1.18,0.36)
TP	−0.13	0.34 (−0.63,1.84)	0.36 (−0.68,1.93)	0.04 (−1.80,1.89)	0.28 (−0.75,1.72)	−0.39 (−2.41,1.91)

PANEL 2

Summary of Probability Forecasts

A. USA

Pr(A) = 0.1410, Pr(B) = 0.2000

	Event A		Event B	
Model	QPS	LPS	QPS	LPS
Constant	0.242	0.407	0.303	0.480
RW	0.246	0.416	0.303	0.481
AR(2)	0.235	0.388	0.284	0.453
AR(2)+Break	0.234	0.392	0.276	0.433
ARLI	0.169	0.267	0.197	0.320
Bi-NARLI	0.133	0.251	0.176	0.301

B. Canada

Pr(A)=0.1192, Pr(B)=0.1457

	Event A		Event B	
Model	QPS	LPS	QPS	LPS
Constant	0.209	0.364	0.248	0.413
RW	0.210	0.367	0.250	0.417
AR(1)	0.202	0.351	0.242	0.405
ARLI	0.111	0.210	0.136	0.258
Bi-NARLI	0.088	0.149	0.121	0.215

C. UK

Pr(A)=0.1558, Pr(B)=0.2662

	Event A		Event B	
Model	QPS	LPS	QPS	LPS
Constant	0.263	0.433	0.391	0.579
RW	0.278	0.455	0.447	0.640
AR(1)+Break	0.267	0.428	0.411	0.581
VAR	0.238	0.404	0.412	0.602
Bi-NARLI	0.206	0.367	0.359	0.545

D. France

Pr(A) = 0.0734, Pr(B) = 0.1651

	Event A		Event B	
Model	QPS	LPS	QPS	LPS
Constant	0.136	0.262	0.276	0.448
RW	0.138	0.268	0.280	0.456
AR(2)	0.151	0.290	0.296	0.470
ARLI	0.113	0.181	0.255	0.402
Bi-NARLI	0.111	0.176	0.258	0.402

E. Germany

Pr(A) = 0.1987, Pr(B) = 0.4503

Model	Event A QPS	Event A LPS	Event B QPS	Event B LPS
Constant	0.318	0.499	0.495	0.688
RW	0.333	0.518	0.498	0.691
AR(4)	0.327	0.508	0.509	0.703
STAR(4)	0.330	0.508	0.495	0.688
ARLI	0.285	0.457	0.487	0.683
Bi-NARLI	0.283	0.448	0.462	0.653

F. Italy

Pr(A) = 0.2432, Pr(B) = 0.3153

Model	Event A QPS	Event A LPS	Event B QPS	Event B LPS
Constant	0.380	0.568	0.443	0.635
RW	0.384	0.574	0.442	0.634
AR(5)	0.340	0.517	0.395	0.579
STAR(5)	0.297	0.463	0.353	0.528
ARLI	0.367	0.560	0.450	0.649
Bi-NARLI	0.289	0.469	0.358	0.546

G. Japan

Pr(A)=0.1468, Pr(B)=0.2661

Model	Event A QPS	Event A LPS	Event B QPS	Event B LPS
Constant	0.250	0.417	0.391	0.579
RW	0.200	0.330	0.244	0.403
AR(5)	0.196	0.312	0.229	0.373
TAR(5)	0.194	0.314	0.291	0.462
ARLI	0.180	0.269	0.219	0.343
BiNARLI	0.192	0.319	0.262	0.401