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Estimating Advertising Half-life and the Data Interval Bias*.

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Abstract: We compare three methods of estimating the duration, or half-life, of advertising using computer simulation experiments. In particular, we investigate how well each method works with the data aggregated over different time intervals. In contrast with the existing theory on the, so called, data interval bias, our experiments are based upon realistic advertising schedules. Our results appear to indicate that the indirect "t-ratio" estimation procedure favoured by practitioners works well in the presence of such temporal aggregation. Additionally, we suggest a transformation that can be used in combination with the indirect "t-ratio" estimation procedure to obtain estimates of the underlying micro-period half-life from a variety of common (macro) data frequencies.

JEL Classification: M37, C13, C51

Keywords: Adstock, half-life, data interval bias.

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1. Introduction.

Recently there has been a rapid explosion in the availability of weekly data for use in market modeling. However, this data produces estimates for the duration of the advertising effect which differ markedly from those obtained from data of a lower periodicity. In particular, the longer the data interval, the longer the estimated duration of the advertising effect. This problem is known in the literature as the data interval bias. These differences in estimated advertising duration can have an enormous impact both in advertising strategy and, potentially, in the optimal allocation of elements of the marketing mix. For example, if the duration of the advertising effect is very short then it may well be appropriate to maintain a continous, or drip, advertising allocation over time. On the other hand, if the duration of the advertising effect is "long" then a burst schedule over time may be appropriate. For the purposes of this paper we will focus on the use of advertising half-life to measure the duration of the advertising effect. The advertising half-life is defined as the period by which half of the impact of the advertising on the response variable (*e.g.* sales) is felt.

The problem of data interval bias in measuring the duration and impact of advertising effects has been known for some twenty years and dates from the paper by Clarke (1976). With the increasing availability of data of a short time interval, *e.g.* weekly data, it might be argued that data interval bias is no longer a problem for researchers. However, we should note two points in contradiction to this view. Firstly, not every researcher has access to this high frequency data. In particular, not all marketing repotage systems operate on the same interval. Thus, it is crucial to understand how competing or interconnecting systems' reliance on different data intervals affects the use of the resultant data. In other words, it can be important to know about the bias induced by using lower frequency (*e.g.* four weekly or monthly) data in measuring the duration and impact of advertising effects. Secondly, many earlier studies, both published and unpublished, estimated duration and impacts of advertising from low frequency data. How reliable is this accumulated empirical evidence? Thus, it is perhaps time to reconsider the question of data interval bias.

Much work has now been published on the topic of data interval bias both in the marketing literature (see *inter alia* Assmus *et al.* (1984), Bass and Leone (1983), (1986), Blattberg and Jeuland (1981), Hanssens *et al.* (1990, Chapter 7), Russell (1988), Srinivasan and Weir (1988), Vanhonacker (1983), (1984), (1988) and Weiss *et al.* (1983)) and in related fields (see *inter alia* Moriguchi (1970), Mundlak (1961), Sasieni (1982), Tiao and Wei (1976), Vanhonacker (1987) and Zellner and Giesel (1970)). In a recent paper Leone (1995) summarizes the results from this literature as supporting two key results. The first of these is that data interval bias is real and that researchers need to exercise caution when using aggregated data. The second result is that, in certain circumstances, the data interval bias can be adjusted for and that, if this is done, the duration of the advertising effect is short (in the context of this paper Leone's results indicate that half-lives are in the range of 7 to 12 weeks). Unfortunately, the theoretical work on data interval bias has assumed that the advertising input (schedule) is a white noise random process. In practice this is far from the case. Discernible patterns exist in advertising data, due to the use of burst or drip schedules. Additionally, advertising and other marketing activity is typically correlated. These facts are seldom considered. Therefore, in this paper we will consider the question of data interval bias using a realistic advertising schedule.

The most popular model used both in the literature and in practice to model the impact of advertising on a response variable, such as sales, is the geometric distributed lag model. This model is formally equivalent to the model known as the adstock model (Broadbent (1979)). When using models incorporating adstock there are three approaches to estimation. First, the Koyck (Koyck (1954)) transformation can be applied and estimation conducted using a least squares procedure. Second, the lag parameter in adstock (the geometric lag) can be estimated directly as described in Johnston (1984). Third, as is often the case in practice, the half-life (and hence the lag parameter) can be estimated indirectly using a "t-ratio" approach. This last approach has been referred to as "finding the highest point on a billiard table" (Corlett (1985, p494)). To our knowledge, this last estimation technique has not been considered in the literature on the data interval bias.

In this paper we use computer simulation (Monte Carlo) techniques to investigate the impact of temporal data aggregation on the estimation of the half-life in adstock models. In particular, we investigate how the results from weekly and four weekly data compare when the true underlying process that generates the data is at the daily level. The experiments used are based upon a realistic advertising schedule and thus should give an indication of the comparative merits of different estimation procedures and the impact of temporal data aggregation in practice. The plan of the rest of this paper is as follows. Section 2 describes the adstock model, details the estimation techniques typically used by applied researchers to estimate the adstock model and discusses the impact of temporal data aggregation on the estimation techniques. Section 3 describes the simulation experiments and the summary measures that we use to evaluate the performance of the estimation techniques and section 4 discusses the results of our experiments. In section 5 we attempt to use our results to provide guidance to practitioners on the estimation of the true, underlying, half life when the model estimation is conducted using temporally aggregated data. Finally, section 6 contains some concluding remarks.

2. The Model.

The underlying model for the response variable R in period i is a distributed lag model in terms of the advertising data, T, of the form:

$$R_i = \alpha + \beta \sum_{j=0}^{\infty} \lambda^j T_{i-j} + u_i, \; i = 1, \ldots, n \tag{2.1}$$

$0 < \lambda < 1$. It is often the case that the response process is parameterized to ensure that the sum of the lagged impacts of the advertising is unity. This is achieved by using $\beta^* = \beta(1-\lambda)$ as the sum of the lagged impacts is $1/(1-\lambda)$.

As Clarke (1976) and Johnston (1984), *inter alia*, show, this model is consistent with a range of underlying hypotheses including adaptive expectations and partial adjustment. Furthermore, and pertinent to this current study, Vakratsas and Ambler (1995) show that this model is also formally equivalent to the adstock model of Broadbent (1979). Thus, this functional form embodies a number of the main hypotheses concerning how advertising affects a response variable. The questions of the existence of data interval bias and its impact in this model have been widely studied in the literature cited above. One estimation technique that has been proposed in that literature that has "good" properties in the presence of temporal data aggregation is a Direct grid search procedure (Srinivasan and Weir (1988)). This technique is discussed below.

Estimation of the model (2.1) can be carried out in a number of ways. In this paper we consider three estimation techniques: ordinary least squares, Direct (grid search) and an Indirect method based upon t-ratios. We now briefly describe each of these estimation techniques - without the complication of temporal aggregation. The first technique is ordinary least squares (OLS). To use OLS in this model we first need to transform (2.1) into an appropriate form. The transformation used is termed the Koyck transform (Koyck (1954)) and is used to form the difference $R_i - \lambda R_{i-1}$ which after re-arrangement yields the estimating equation:

$$R_i = \alpha(1-\lambda) + \lambda R_{i-1} + \beta(1-\lambda)T_i + v_i,\ i = 2, \ldots, n. \tag{2.2}$$

Although other suitable techniques such as Instrumental Variables exist, this equation can be and often is estimated by OLS.

The Direct estimation technique is described in detail in Johnston (1984, pp 358-360). Again the model (2.1) is transformed. This time the resultant estimating equation is:

$$R_i = \alpha + \beta T_i^* + \gamma\lambda^i + e_i,\ i = 1, \ldots, n \tag{2.3}$$

where $T_i^* = T_i + \lambda T_{i-1} + \ldots + \lambda^{i-1}T_1$. The model is estimated by grid searching (2.3) by OLS over values for λ in the interval $0 < \lambda < 1$ and choosing the results that minimize the residual sum of squares.

The final estimation procedure is an Indirect one based upon the adstock formulation of Broadbent (1979). In the adstock formulation in which "first period counts full" (2.1) is rewritten as:

$$R_i = \alpha + \beta A_i + u_i, \ i = 1, \ldots, n. \tag{2.4}$$

with the weights in the formula defining A_i adding to one to ensure that in the long run adstock does not exceed the gross rating point (GRP) input. Estimation of the adstock model is usually carrried out by the researcher *a priori* calculating a range of adstocks from the data on GRPs. These adstocks are typically calculated using the recursion:

$$A_i = T_i + \lambda A_{i-1}, \ i = 1, \ldots, n. \tag{2.5}$$

The adstocks chosen are defined by a list of half-lives (η) that the researcher believes may be appropriate for the response variable R. The half-life is defined as the period by which half of the impact of the advertising is felt and is related to the lag parameter λ since $\eta = \ln(0.5)/\ln(\lambda)$. Thus, the researcher selects a range of half-lives, η, and for each value finds the corresponding value of λ. Then using (2.5) the observations on that adstock are *calculated*. The results of Leone

(1995) suggest that when we are interested in the short term effects of advertising, $7 \leq \eta \leq 12$ weeks. Consistent with this finding, Broadbent and Fry (1995) suggest that half-lives in excess of 13 weeks are probably related to the medium/long term effects of advertising.

To solve the initialization problem (knowledge of A_0 and T_0) in (2.5) the adstock calculations can be made for not just the n periods required but also for a number of preceeding periods. If the GRP input is not available for this prior period then $A_0 = T_0 = \bar{T}$ where $\bar{T}$ is the average GRP level over an appropriate period. We then estimate (2.4) by OLS. That is, the response variable R is then regressed against each of these calculated adstocks in turn. The adstock chosen is the one that yields the highest t-ratio on the associated estimated coefficient $\hat{\beta}$ or the highest R^2 for the equation. Intuitively what this estimation procedure does is to grid search over values of η as opposed to the direct approach that grid searches over λ. However, in practice the indirect method is easier to implement.

3. Simulation Study.

To evaluate the performance of the estimation techniques above in the presence of temporal data aggregation we conduct a simulation, or Monte Carlo, study. Interested readers are referred to Davidson and MacKinnon (1993, Chap. 21) for further details of Monte Carlo methodology. Briefly, our simulation study consists of repeated execution of the following steps:

1. Generate data at the micro-period level (daily) and aggregate it to a macro-period level (either weekly or four-weekly).

2. Estimate the model (2.1) using the macro-period data. Record estimator performance.

If the process is repeated sufficiently many times (*i.e.* the number of replications, N, is sufficiently large) then the Monte Carlo method will produce accurate results for the properties of the estimators in finite samples. In our experiments we use 500 (=N) replications.

An integral part of a simulation study is the design of the experiment, as the results are conditional upon the design used. Thus it is important that the design is representative of the situation researchers might meet in practice. In our experiments the micro-period (daily) data on R_i is generated from:

$$R_i = \alpha + \beta A_i + \sigma u_i,\ i = 1, \ldots, 1456. \tag{3.1}$$

A_i is an adstock variable with half life η (= 2 days, 4 days, 1, 2, 3, 4, 6, 12 and 24 weeks) and u_i is a standard Normal ($N(0,1)$) variable. The values chosen for α and β in all experiments were *12* and *0.5*. The resultant model has an implied advertising elasticity of approximately *0.2* for the half-lives considered.

Adstock is calculated from a daily GRP schedule with "first period counts full" and weights normalized to sum to one. Three values were chosen for σ (*1, 2,* and *4*) corresponding to small, medium and large amounts of "noise" in the underlying data generating process.

The daily GRP schedule used as input to the adstock calculations was generated to be representative of the type of schedule that might be used in practice. The GRP data was generated as follows. We consider consecutive "blocks" of 10 days duration. These are either advertised or not. A block is advertised with probability 0.2. If a block is advertised then any particular day within the block may or may not carry advertising. A day carries advertising with a probability of 0.4 and if a day carries advertising then 50 GRPs appear. This produces a schedule with an expected 10 day burst size of 200 GRPs at an average of 20 GRPs a day. We generated a series of 1456 daily GRPs from this scheme and these were then assumed as fixed input into the adstock calculations for all experiments. The actual (or, realised) daily average GRPs for our entire input schedule is 6 GRPs and plots of this schedule at the weekly level (208 weeks) and four-weekly level (52 periods) can be found in Figures 1 and 2.

INSERT Figures 1 and 2 about here

Thus prior to step 1 in the experiments we calculated daily adstocks with 2 day, 4 day, 1, 2, 3, 4, 6, 12 and 24 week half lives. In step 1, for each half life in turn, consistent with (3.1) we computed $12 + 0.5 \times A_i$ and added to that a value of σ times a random drawing from the standard Normal ($N(0, 1)$) distribution. This was carried out for each of the 1456 days and yielded a sample of 1456 daily observations on R_i. These daily observations were then summed over either 7 or 28 day blocks to yield a sample of 208 weekly or 52 four-weekly observations on R for step 2.

A further question arises in considering the aggregation of the daily GRP schedule to the macro-period for estimation in step 2. For the OLS and Direct estimation procedures it is sufficient to aggregate the daily GRPs to the appropriate macro-period level (weekly or four-weekly). However, for the Indirect method there is a choice to be made concerning the calculation of adstocks for the macro-period. The Indirect method requires a range of macro-period adstocks and these could be calculated either using macro-period GRPs as input or by using micro-period GRPs as input and aggregating these micro-period adstocks to the macro-period level. The latter method will be more accurate as it takes into account the variation in the micro-period schedule.

From a practical viewpoint this is saying that although the researcher only has data on the response variable at the macro-period level s/he may have GRP data at either the micro or macro period level. To investigate this issue we consider variants of the Indirect method in our experiments. For the daily to weekly aggregation T-Stat(A) uses daily GRP input to the adstock calculations and T-Stat(B) uses weekly GRP data as input. For the daily to four-weekly aggregation T-Stat(A) and T-Stat(B) are the same as the days to weeks case but T-Stat(C) uses four weekly GRP data as input to the adstock calculations.

In step 2 we take our aggregated data and estimate the response model (2.1) using each of the techniques. We then record how well the techniques have performed. For each replication we recorded the estimated half-life ($\hat{\eta}$). Averaged over all 500 replications this yields the average estimated half-life. We also recorded deviations of estimated from true parameter values to estimate bias ($E(\hat{\lambda} - \lambda)$) and mean square error ($E(\hat{\lambda} - \lambda)^2$) for the lag parameter. The final measure that we considered was a measure of "nearness". For each replication, we recorded which estimator was closest in absolute value to the true value of λ. Averaged over all 500 replications this yields the proportion of times that each estimator was nearest to the true value. For space reasons we only report the results for estimated half-lives and nearness. These results give a clear picture of estimator performance and are confirmed by the results on bias and mean square error. However, the other results are available on request.

4. Results.

Tables 1 and 2 contain the average half-life estimates for the two sets of experiments of daily to weekly and daily to four-weekly aggregation. The entries in the body of these tables give the average estimated half-life in weeks from the 500 replications of that data generating and estimation process. Tables 3 and 4 contain the measures of nearness. The entries in the body of these tables give the proportion of times in the 500 replications of the data generating and estimation process that a given estimator was closest in absolute value to the true parameter value.

Considering first the case of daily to weekly aggregation. In table 1 we see that for the Direct and Indirect methods data interval bias is present. Additionally, for the Indirect method the magnitude of this bias increases the more noise there is in the underlying micro-period data generating process. Comparing the two methods we see that T-Stat(A) has negligible data interval bias and that T-Stat(B) and Direct have comparable levels of bias. These findings make sense. T-Stat(A) uses daily GRP input into adstock calculations and thus the weekly adstocks created by the temporal aggregation will be a more accurate representation of the underlying advertising weight than those in T-Stat(B) that have weekly GRP data as input. Furthermore, T-Stat(B) and Direct are both “grid search” techniques using the macro-period data. T-Stat(B) searches over η using adstock variables and Direct searches over λ using GRP data. Since $\eta = \ln(0.5)/\ln(\lambda)$ it is not surprising that these methods are similar in the results. Table 3 confirms that T-Stat(A) is systematically better than the other estimation techniques in the daily to weekly case. However, there is some evidence in the nearness results in table 3 that the performance of T-Stat(A) deteriorates as the underlying true half-life increases. Of the two techniques using weekly advertising data in the estimation process T-Stat(B) appears better.

Tables 2 and 4 relate to the daily to four-weekly aggregation experiments. A very similar pattern of results emerges from these tables. T-Stat(A) does best, followed by T-Stat(B) and then there is little to choose between T-Stat(C) and Direct. Once again these results stem primarily from the level of aggregation used in the input to the advertising variable used in the estimation. If we have less aggregated data then it should be used. The nearness results in table 4 suggest that there is again some evidence of a deterioration of the performance of T-Stat(A) as the underlying true half-life increases and that of the two techniques using four-weekly advertising data in the estimation process T-Stat(C) appears better.

The exception to this pattern for both sets of experiments is the OLS estimation method. For OLS data interval bias exists for the short half-lives but as the underlying half-life increases the OLS method actually underestimates the underlying true half-life. This contrary result is caused by the simple fact that there is nothing in the OLS procedure to enforce the constraint that $0 < \hat{\lambda} < 1$. Indeed as the underlying half-life increases the OLS procedure produces values of $\hat{\lambda}$ that are negative. Such values average out with $\hat{\lambda}$ values that lie in the admissible range and yield downward bias in the estimated half-life. The two sets of nearness results in tables 3 and 4 do, however, suggest that there are times when OLS can get "near" to the true value. These seem to occur in the region of half-life where the OLS procedure crosses over from overestimating to underestimating the true half-life. In practice, we are unlikely to know whether we are in this region. In our opinion, the results seem to strongly support the view that researchers should not just apply the Koyck transform and use OLS to estimate the response model.

In summary our results clearly show that the Indirect method often used by practitioners performs well. Data interval bias does exist but is of a relatively small magnitude. Thus the use of these methods appears to be supported.

5. Application.

The question that we now pose is the following. If we estimate the half life with an Indirect method using temporally aggregated data as $\hat{\eta}$ what do our simulation results suggest the underlying true half-life is? In an attempt to answer this question and hence provide further guidelines for the correction of the data interval bias in the Indirect methods we carried out some regression modeling based on the data in tables 1 and 2. We regressed the true half life upon a constant and the estimated half-life. For T-Stat(A) in both aggregations the constant proved insignificant. Note that we did not include the value of σ as a regressor as, in practice, this would not be known.

Our results were as follows:

Daily to Weekly - T-Stat(A):

$$\widehat{True} = \underset{(420.9)}{0.9881} \times Estimated$$

27 observations, $\bar{R}^2 = 0.961$. T statistics in parentheses.

Daily to Weekly - T-Stat(B):

$$\widehat{True} = \underset{(-12.24)}{-0.5261} + \underset{(225.3)}{0.9754} \times Estimated$$

27 observations, $\bar{R}^2 = 0.999$. T statistics in parentheses.

Daily to Four-Weekly - T-Stat(A):

$$\widehat{True} = \underset{(425.8)}{0.9895} \times Estimated$$

27 observations, $\bar{R}^2 = 0.999$. T statistics in parentheses.

Daily to Four-Weekly - T-Stat(B):

$$\widehat{True} = \underset{(-10.29)}{-0.3758} + \underset{(262.6)}{0.9789} \times Estimated$$

27 observations, $\bar{R}^2 = 0.999$. T statistics in parentheses.

Daily to Four-Weekly - T-Stat(C):

$$\widehat{True} = \underset{(-9.34)}{-1.7251} + \underset{(56.82)}{0.9150 \times Estimated}$$

27 observations, $\bar{R}^2 = 0.992$. T statistics in parentheses.

These regressions can be used to answer the question posed above. For example, suppose that a researcher using four-weekly data on R and as input to adstocks estimates using the Indirect method (in this case T-Stat(C)) that the half-life of advertising is 6 weeks. If we believe that the true generating process is at the daily level then we can use the regression above to produce an estimate for the "true" half-life. In this example the estimate would be *3.7649 (=-1.7251+0.915×6)* weeks.

A range of such calculations have been tabulated in table 5 to assist in these regression based translations. We will consider two examples. If we estimated the half-life to be 4 weeks using weekly data then the "true" half-life is 3.376 weeks and the equivalent half-life estimate for this "true" half-life of 3.376 weeks from 4-weekly data would be 5.574 weeks. On the other hand, if we estimated the half-life to be 4 weeks using 4-weekly data then the "true" half-life is 1.935 weeks and the equivalent half-life estimate for this "true" half-life of 1.935 weeks from weekly data would be 2.523 weeks.

Leone (1995) in his paper derives a formula to link the decay parameters in the micro and macro period models. This formula is based upon the results and assumptions used in Bass and Leone (1983). Re-writing the Leone formula in terms of half-lives we have the micro half-life, η, given in terms of the (measured) macro half-life, h:

$$\eta = \frac{\ln(0.5)}{\ln\left(\frac{m \times 0.5^{1/h}}{1+(m-1)\times 0.5^{1/h}}\right)}, \tag{5.1}$$

where m is the number of micro-periods contained in a macro-period (e.g. $m = 7$ for days to weeks). We note that the Leone formula (5.1) does not take into account how the macro period model is *estimated* to measure the half-life and thus will give different equivalent half-lives to our regression based formulae for each of the indirect estimation methods. For comparative purposes, however, we include the Leone equivalent half-life in table 5. We see that (5.1) does indeed reduce the estimated (measured) half-life, but not as far as our regression based formulae. The biggest difference between the "True" and the Leone half-life values occurs in the case of short half-lives estimated from 4-weekly data. In most other cases the difference is not large.

In practice researchers are still faced with estimating models using data that is (predominately) either monthly or bi-monthly in periodicity. If the true data generating process is assumed to be daily then half-life estimates from either of these temporal aggregated data periods would be subject to the data interval bias. If we could produce an equivalent half-life estimate ($\tilde{\eta}$) to correspond to the estimate ($\hat{\eta}$) obtained from this monthly (or bi-monthly) data we could then use table 5 to produce our value for the "true" half-life.

Table 1A in Leone (1995) provides some evidence that the average half-life estimated from bi-monthly data is approximately twice that estimated from monthly date. Thus, as a rule of thumb, we suggest converting bi-monthly based estimates of half-life to monthly equivalents by multiplying them by 0.5. To convert monthly based estimates of half-life to 4-weekly equivalents we suggest multipying them by 0.9231 (=12/13). To illustrate this approach assume that a researcher has an estimate of half-life of 6 weeks using bi-monthly data. The monthly equivalent estimate is 3 weeks and the 4-weekly equivalent is 2.769 weeks. Using the regression based methods underlying table 5 (in this case for T-Stat(C)) the "true" half-life is 0.809 weeks. This value of 0.809 weeks is dramatically different in its practical implications (*e.g.* for budget allocation over time a drip schedule may be suggested) than the original estimate of 6 weeks (*e.g.* for budget allocation over time a burst schedule may be suggested)!

6. Conclusions.

Using computer simulation (Monte Carlo) techniques we have investigated the impact of temporal data aggregation on the estimation of the half-life in adstock models. In particular, we were interested in the performance of three commonly used estimation techniques based upon weekly or four-weekly data when the true underlying process that generates the data is at the daily level. The estimation techniques considered were ordinary least squares on a suitably transformed model, Direct (grid searched) estimation of a transformed version of the model and Indirect estimation using t-ratios. The latter technique having some popularity amongst practitioners in advertising agencies.

The simulation experiments used were based upon a realistic advertising schedule and thus should give an indication of the comparative merits of different estimation procedures and the impact of temporal data aggregation in practice. We found that OLS is not a suitable estimation procedure for the model discussed. The Direct and Indirect methods suffered from data interval bias but the magnitude of this bias was relatively small. These two techniques were of similar accuracy when the advertising data input was of the macro-period frequency. However, the indirect method was more accurate if micro or intermediate period frequency data was used as input.

Additionally we used our simulation results to estimate a simple correction equation to move from the half-life estimated from temporally aggregated data to an estimate of the "true" underlying half-life. Given the simplicity of the Indirect method and these regression results linking estimated to "true" half-lives we conclude that this method has much to recommend it to practitioners.

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Tables.

Table 1: Half-life Estimates ($\widehat{\eta}$) - Daily to Weekly Aggregation.

Half-life	OLS			DIRECT		
Weeks	$\sigma = 1$	$\sigma = 2$	$\sigma = 4$	$\sigma = 1$	$\sigma = 2$	$\sigma = 4$
0.2857	0.5871	0.5734	0.5275	0.6002	0.5999	0.5992
0.5714	1.0222	0.9663	0.8056	1.0060	1.0055	1.0045
1	1.6267	1.4348	1.0137	1.5407	1.5403	1.5394
2	2.7937	1.9993	1.0388	2.6848	2.6848	2.6825
3	3.5625	2.0891	0.9256	3.7864	3.7863	3.7777
4	3.9883	2.0047	0.8257	4.8706	4.8698	4.8530
6	4.2185	1.7537	0.6915	6.9765	6.9722	6.9368
12	3.8406	1.3472	0.5403	12.8067	12.7479	12.5155
24	3.7598	1.2427	0.5028	24.4364	24.3023	23.8061

Half-life	T-Stat(A)			T-Stat(B)		
Weeks	$\sigma = 1$	$\sigma = 2$	$\sigma = 4$	$\sigma = 1$	$\sigma = 2$	$\sigma = 4$
0.2857	0.2857	0.2857	0.2857	0.5714	0.5726	0.5849
0.5714	0.5714	0.5714	0.5714	1.0000	1.0017	1.0060
1	1.0000	1.0020	1.0037	1.5580	1.5389	1.5400
2	2.0046	2.0054	2.0149	2.6580	2.6626	2.6749
3	3.0040	3.0100	3.0309	3.7283	3.7369	3.7589
4	4.0097	4.0149	4.0469	4.7743	4.7823	4.8131
6	6.0129	6.0263	6.0797	6.8006	6.8177	6.8657
12	12.0106	12.0414	12.1826	12.7600	12.7909	12.9403
24	24.0337	24.1894	24.7769	24.7809	24.9423	25.4780

Table 2: Half-life Estimates ($\hat{\eta}$) - Daily to Four-Weekly Aggregation.

Half-life		OLS			DIRECT		
4-Weeks	Weeks	$\sigma = 1$	$\sigma = 2$	$\sigma = 4$	$\sigma = 1$	$\sigma = 2$	$\sigma = 4$
0.0714	0.2857	1.0714	1.0611	1.0273	1.1746	1.1737	1.1737
0.1429	0.5714	1.5050	1.4858	1.4206	1.7536	1.7530	1.7529
0.2500	1	2.0896	2.0484	1.9089	2.6207	2.6201	2.6193
0.5000	2	3.4301	3.2800	2.8235	4.5364	4.5332	4.5202
0.7500	3	4.8091	4.4406	3.4757	6.1984	6.1909	6.1614
1.0000	4	6.1858	5.4734	3.8850	7.6331	7.6205	7.5788
1.5000	6	8.7472	7.0275	4.2011	9.9790	9.9544	9.8505
3.0000	12	14.2730	9.0249	4.1367	15.3444	15.2708	14.9275
6.0000	24	21.6206	11.0257	4.3273	24.0157	25.8923	25.3793

Half-life		T-Stat(A)			T-Stat(B)		
4-Weeks	Weeks	$\sigma = 1$	$\sigma = 2$	$\sigma = 4$	$\sigma = 1$	$\sigma = 2$	$\sigma = 4$
0.0714	0.2857	0.2857	0.2857	0.2886	0.5363	0.5171	0.5094
0.1429	0.5714	0.5714	0.5723	0.5746	0.8574	0.8574	0.8583
0.2500	1	1.0006	1.0017	1.0046	1.3543	1.3540	1.3574
0.5000	2	2.0054	2.0031	2.0114	2.4614	2.4626	2.4734
0.7500	3	2.9997	3.0057	3.0223	3.5346	3.5403	3.5574
1.0000	4	3.9983	4.0057	4.0246	4.5840	4.5917	4.6111
1.5000	6	6.0037	6.0011	6.0500	6.6240	6.6294	6.6663
3.0000	12	11.9951	12.0177	12.1503	12.5843	12.5980	12.7317
6.0000	24	24.0157	24.1649	24.7406	24.5549	24.7017	25.2503

Half-life		T-Stat(C)		
4-Weeks	Weeks	$\sigma = 1$	$\sigma = 2$	$\sigma = 4$
0.0714	0.2857	1.1566	1.1611	1.1434
0.1429	0.5714	1.7374	1.7426	1.7383
0.2500	1	2.5966	2.5943	2.6037
0.5000	2	4.3760	4.3800	4.3937
0.7500	3	5.8231	5.8274	5.8414
1.0000	4	7.0606	7.0657	7.0871
1.5000	6	9.2349	9.2397	9.2751
3.0000	12	15.2686	15.2834	15.4120
6.0000	24	27.1706	27.3074	27.6834

Table 3: Nearness of $\widehat{\lambda}$ - Daily to Weekly Aggregation.

Half-life	OLS			DIRECT		
Weeks	$\sigma = 1$	$\sigma = 2$	$\sigma = 4$	$\sigma = 1$	$\sigma = 2$	$\sigma = 4$
0.2857	0	0	0	0	0	0
0.5714	0	0	0.012	0	0	0
1	0	0	0.330	0	0	0
2	0	0.324	0	0	0	0.044
3	0.012	0.002	0	0	0.014	0.066
4	0.282	0	0	0	0.026	0.124
6	0	0	0	0.008	0.094	0.202
12	0	0	0	0.130	0.206	0.270
24	0	0	0	0.240	0.302	0.320

Half-life	T-Stat(A)			T-Stat(B)		
Weeks	$\sigma = 1$	$\sigma = 2$	$\sigma = 4$	$\sigma = 1$	$\sigma = 2$	$\sigma = 4$
0.2857	1	1	1	0	0	0
0.5714	1	1	0.988	0	0	0
1	1	1	0.670	0	0	0
2	1	0.674	0.902	0	0.002	0.054
3	0.988	0.978	0.804	0	0.006	0.130
4	0.718	0.908	0.714	0	0.066	0.162
6	0.950	0.774	0.592	0.042	0.132	0.206
12	0.700	0.524	0.412	0.170	0.270	0.318
24	0.446	0.382	0.296	0.314	0.316	0.384

Table 4: Nearness of $\widehat{\lambda}$ - Daily to Four-Weekly Aggregation.

Half-life		OLS			DIRECT		
4-Weeks	Weeks	$\sigma = 1$	$\sigma = 2$	$\sigma = 4$	$\sigma = 1$	$\sigma = 2$	$\sigma = 4$
0.0714	0.2857	0	0	0	0	0	0
0.1429	0.5714	0	0	0	0	0	0
0.2500	1	0	0	0	0	0	0
0.5000	2	0	0	0.032	0	0	0
0.7500	3	0	0	0.196	0	0	0
1.0000	4	0	0.002	0.254	0	0	0
1.5000	6	0	0.134	0.110	0	0	0.018
3.0000	12	0.048	0.084	0	0	0.032	0.156
6.0000	24	0.110	0.002	0	0.152	0.172	0.242

Half-life		T-Stat(A)			T-Stat(B)		
4-Weeks	Weeks	$\sigma = 1$	$\sigma = 2$	$\sigma = 4$	$\sigma = 1$	$\sigma = 2$	$\sigma = 4$
0.0714	0.2857	1	1	0.976	0	0	0.024
0.1429	0.5714	1	1	0.990	0	0	0.010
0.2500	1	1	1	0.968	0	0	0.032
0.5000	2	1	0.962	0.784	0	0.038	0.184
0.7500	3	0.982	0.862	0.562	0.018	0.138	0.242
1.0000	4	0.944	0.770	0.460	0.056	0.228	0.286
1.5000	6	0.872	0.616	0.500	0.128	0.250	0.344
3.0000	12	0.654	0.510	0.472	0.298	0.342	0.242
6.0000	24	0.430	0.424	0.342	0.276	0.206	0.134

Half-life		T-Stat(C)		
4-Weeks	Weeks	$\sigma = 1$	$\sigma = 2$	$\sigma = 4$
0.0714	0.2857	0	0	0
0.1429	0.5714	0	0	0
0.2500	1	0	0	0
0.5000	2	0	0	0
0.7500	3	0	0	0
1.0000	4	0	0	0
1.5000	6	0	0	0.028
3.0000	12	0	0.032	0.130
6.0000	24	0.032	0.196	0.282

Table 5: Half-life equivalents measured in weeks.

True:	*Leone:*	*Estimated from:* Weekly	4-Weekly
0.449	0.742	1	2.376
1.425	1.722	2	3.442
2.400	2.716	3	4.508
3.376	3.713	4	5.574
4.351	4.711	5	6.640
5.326	5.710	6	7.706
6.302	6.709	7	8.772
7.277	7.708	8	9.838
8.253	8.707	9	10.904
9.228	9.707	10	11.970
10.203	10.707	11	13.037
11.179	11.706	12	14.103
12.154	12.706	13	15.169
*	0.231	*	1
0.105	0.973	0.647	2
1.020	1.873	1.585	3
1.935	2.822	2.523	4
2.850	3.791	3.461	5
3.765	4.769	4.399	6
4.680	5.754	5.337	7
5.595	6.743	6.275	8
6.510	7.734	7.213	9
7.425	8.727	8.152	10
8.340	9.721	9.090	11
9.255	10.717	10.028	12
10.170	11.712	10.966	13

Figure 1: Weekly Advertising Schedule

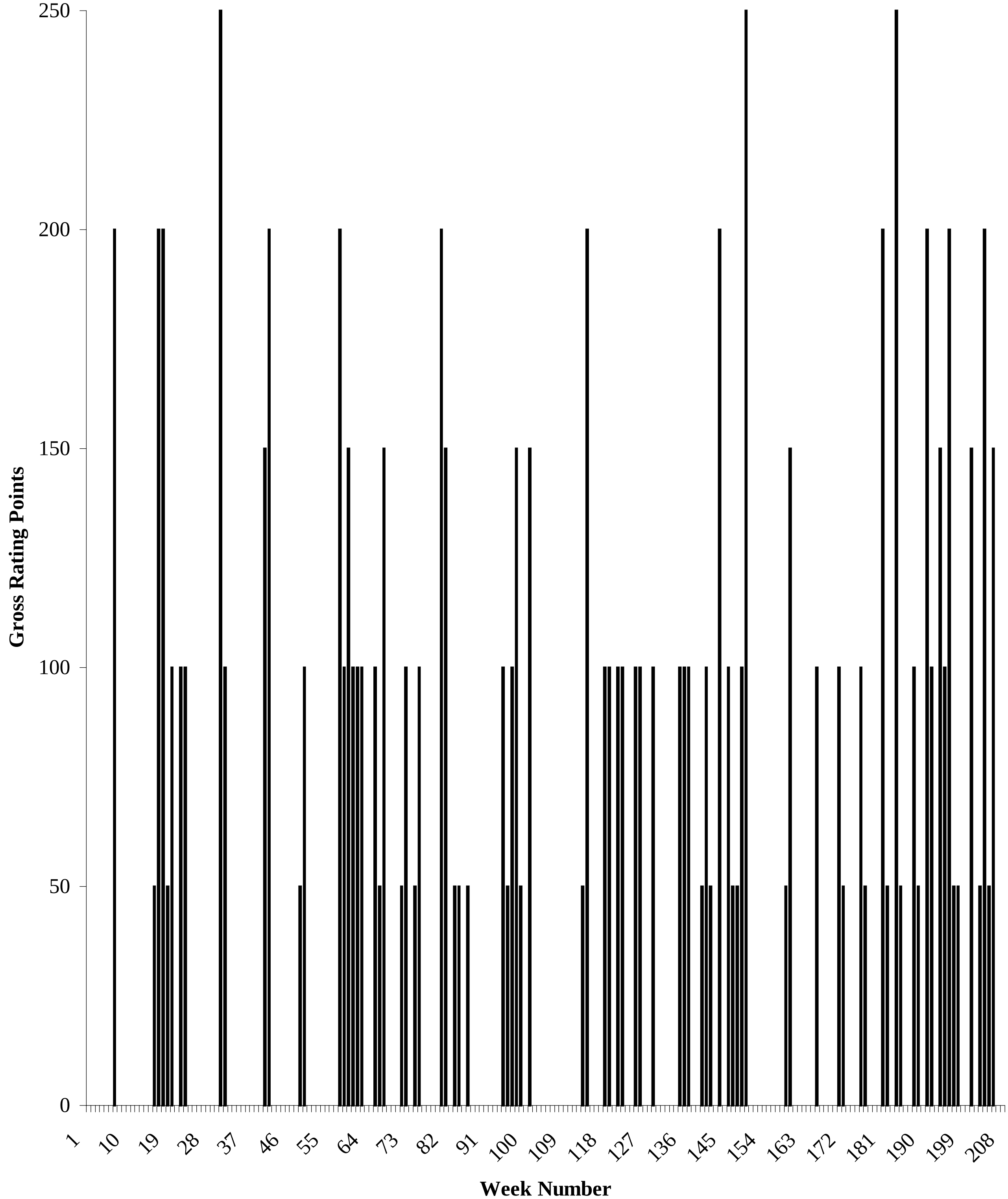

Figure 2: Four Weekly Advertising Schedule