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Abstract

Volatility smiles arise in currency option markets when empirical exchange rate returns distributions exhibit leptokurtosis. This feature of empirical distributions is symptomatic of turbulent periods when exchange rate movements are in excess of movements based on the assumption of normality. In contrast, during periods of tranquility, movements in exchange rates are relatively small, resulting in unconditional empirical returns distributions with thinner tails than the normal distribution. Pricing currency options during tranquil periods on the assumption of normal returns yields implied volatility frowns, with over-pricing at both deep-in and deep-out-of-the-money contracts and under-pricing for at-the-money contracts. This paper shows how a parametric class of thin-tailed distributions based on the generalised Student t family of distributions can price currency options during periods of tranquility.

Key words: Option pricing; volatility frowns; thin-tails; generalised Student t.

JEL classification: C13, G13

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1 Introduction

Volatility smiles can arise in currency option markets when empirical exchange rate returns distributions exhibit leptokurtosis; see for example Hull (2000).¹ This feature of empirical distributions is characteristic of periods of turbulence, whereby movements in exchange rates are relatively large and in excess of movements based on the assumption of normality. In contrast, during periods of tranquility, movements in the spot prices, and hence in exchange rate returns, are relatively small, resulting in empirical distributions which exhibit relatively thinner tails than the normal distribution.

The implication of thin-tailed returns distributions for pricing currency options, and options in general, is that options based on the Black Scholes (1973) model are over-priced for deep in-the-money and deep out-of-the-money contracts, and under-priced for at-the-money options contracts. The mispricing of options manifests itself in implied volatility estimates across strike prices which are relatively lower for the in-the-money and out-of-the-money contracts and relatively higher for the at-the-money contracts. This feature is referred to as a volatility frown. Empirically the frown is less common than both volatility smiles and skews, which occur when the underlying returns distribution exhibits leptokurtosis; see for example, Das and Sundaram (1999) and the references therein. Establishing the link between the form of the underlying returns distribution and the relationship between implied volatility and strike prices helps to explain why volatility smiles are more commonly observed than volatility frowns, as leptokurtic currency returns distributions are more common than thin-tailed distributions.

The aim of this paper is to present a general pricing framework for pricing options under various market conditions. Whilst the emphasis is on pricing options in tranquil markets, the framework is, nonetheless, flexible enough

¹For a review of the empirical evidence pertaining to exchange rate returns, see de Vries (1994).

to enable the pricing of options in more turbulent markets. The approach adopted involves relaxing the normality assumption underlying the Black-Scholes model by modelling currency returns using a generalised Student t distribution; see Lye and Martin (1993), and Lim, Lye, Martin and Martin (1998).² This approach to modelling the distributional features of exchange rate returns, coupled with a time-varying volatility specification, ensures that the effects which cause implied volatility frowns are corrected for in the pricing of currency options. In particular, the volatility frowns observed in the data are shown to be manifestations of the misspecification of the underlying returns distribution, with the correct specification yielding constant volatilities across contracts at a given point in time.

The rest of the paper proceeds as follows. To help motivate the form of the option pricing model, an example of a volatility frown is provided in Section 2, based on European call options written on the US/BP exchange rate in June, 1998. An option pricing model which applies the generalised Student t distribution to define a flexible parametric risk neutral probability distribution, is presented in Section 3. This framework also includes a time-varying volatility structure that extends the Rosenberg and Engle (1997) specification by including a variable that captures mean reversion in the conditional volatility over the sample period investigated. A maximum likelihood estimation procedure is presented in Section 4. The main empirical results are given in Section 5 where the performance of various option pricing models are compared. The data consist of a panel of European currency call options over the period October 1, 1997, to June 16th, 1998, with all options maturing in September, 1998. The statistical and forecasting tests show that the proposed pricing model is superior to the Black-Scholes model, a model based

²Other approaches which allow for nonnormal returns include: the expansion of the Black-Scholes model to allow for higher order moments (Corado and Su, 1997); Binomial trees (Jackwerth and Rubinstein, 1996; and Dennis, 2001); mixtures of lognormals (Melick and Thomas, 1997); and non-parametric kernel methods (Ait-Sahalia, 1996, Ait-Sahalia and Lo, 1998, and Ghysels, Patilea, Renault and Torres, 1998).

on a lognormal mixture distribution, as well as some other special cases of the generalised Student t pricing framework. Section 6 provides a summary of the key results and some concluding remarks.

2 Volatility Frowns in the US/UK Currency Option Market

To help motivate the form of the currency option price model developed in the paper, Table 1 reports the end of day European currency call option prices, C , on the 16th of June, 1998, maturing in September, 1998, written on the US/BP exchange rate. Thus the maturity of this set of option contracts is $\tau = 0.252$. The strike prices are given by $X = \{163, 164, \dots, 178\}$, a total of 10 unique contracts traded on this day. The spot exchange rate is $S = 165.26$, and the US and UK risk free interest rates are respectively $r = 0.05156$ and $i = 0.072$, which are the 3-month treasury bill rates.

The Black-Scholes model for pricing European currency call options is also known as the Garman Kohlhagen (1983) option model. This model is equivalent to the Black-Scholes price for European call option contracts written on equities paying a continuous dividend stream equal to the foreign interest rate, i . The key assumptions of this model are that currency returns are identically and independently distributed as normal. The Garman-Kohlhagen (1983) currency option prices are presented in the last column of Table 1 using the formula

$$F_j^{GK} = Se^{-i\tau}N(d_1) - X_j e^{-r\tau}N(d_2), \quad j = 1, 2, \dots, 10, \quad (1)$$

where the j signifies the j th contract in the set of 10 unique contracts corre-

sponding to the full range of strike prices, and

$$\begin{aligned}
 d_1 &= \frac{\ln(S/X_j) + \left(r - i + \frac{\sigma^2}{2}\right) \tau}{\sigma\sqrt{\tau}}, \\
 d_2 &= \frac{\ln(S/X_j) + \left(r - i - \frac{\sigma^2}{2}\right) \tau}{\sigma\sqrt{\tau}}.
 \end{aligned}
 \tag{2}$$

The volatility parameter is set at $\sigma = 0.071$, which equals the annualised historical volatility estimate based on $\sqrt{250}$ times the standard deviation of returns over the sample period. For the first three contracts, $X = \{163, 164, 165\}$ and the last two contracts $X = \{176, 178\}$, the Garman-Kohlhagen prices exceed the actual prices, whereas for the remaining contracts the opposite occurs. This empirical result conflicts with existing empirical evidence whereby the reverse tends to be true for currency options; namely, Black-Scholes tends to under-price out-of-the money contracts and over-price at-the-money contracts.

An alternative way to highlight the differences in the two sets of prices presented in Table 1 is to compute the implied volatility for each contract. This is achieved by solving the nonlinear equations

$$C_j = F_j^{GK}(\sigma_j), \quad j = 1, 2, \dots, 10, \tag{3}$$

for σ_j , for each of the $j = 1, 2, \dots, 10$, strike prices. The implied volatility estimates are presented in Figure 1. The key characteristic is that the implied volatility estimates are not equal across strike prices even though the contracts are all written in the same market. The estimates range between 6.2% and 7.2% and display an inverted U-shape. This shape is in stark contrast to the volatility smile that is usually observed when analysing currency option data; see for example, Hull (2000). For this reason, the pattern observed in Figure 1 is referred to as a volatility frown.³

³Similar volatility frowns arise for other days in the sample.

Implied volatility smiles arise in currency option markets when the Garman-Kohlhagen price is applied because exchange rate returns are assumed to be normally distributed, when in fact their empirical distributions show them to be leptokurtic with fatter tails and sharper peaks than the normal distribution. In these circumstances, the fatness in the upper tail raises the probability that a currency option matures in the money thereby yielding a higher price for the contract. The opposite is true when the underlying distribution is thin-tailed as the probability that the option matures in-the-money is now smaller, which results in a lower price for the option.

The occurrence of a volatility frown in Figure 1 suggests that the empirical distribution of exchange rate returns exhibits thin-tails over the sample period. This is indeed the case, as indicated in Figure 2, where the empirical distribution of US/BP exchange rate returns over the period the 1st of October, 1997, to the 16th of June, 1998, is presented. Exchange rate returns are computed as differences of the natural logarithms of spot exchange rates and expressed as a percentage. A normal kernel density is used to compute the nonparametric density with a bandwidth equal to $\hat{\sigma}T^{-1/5}$, where $\hat{\sigma}$ is the estimate of the standard deviation of currency returns and $T = 178$ is the sample size. For comparison, the standardised normal distribution is also presented in Figure 2. The thin-tailed behaviour of the empirical distribution is evident, whilst the relative sharp peak shows that the number of days where the exchange rate exhibits very little movement, is also inconsistent with the normal distribution. The kurtosis coefficient calculated from the kernel density estimate is 0.357, which is significantly less than that associated with the normal distribution, namely a kurtosis coefficient of 3.

3 The Option Pricing Model

The empirical results of the previous section showing that a volatility frown is inconsistent with currency returns being normal, suggest that a more general

empirical model of option prices based on a nonnormal returns generating process is needed to reduce pricing biases. To this end, a pricing model is developed, whereby the risk-neutral probability distribution exhibits sufficient parametric flexibility to be able to capture the empirical characteristics highlighted above. A special feature of this parametric model is that it nests a number of pricing models, including the Garman-Kohlhagen model which is based on the assumption of normally distributed returns.

Let S_t be the spot exchange rate at time t of one unit of the foreign currency measured in the domestic currency. Defining r_t and i_t as the respective domestic and foreign risk free annualised interest rates at time t for maturity at time $t+n$, under uncovered interest rate parity, the expected depreciation of the exchange rate is given by

$$E_t \left[\ln \left(\frac{S_{t+n}}{S_t} \right) \right] = \left(r_t - i_t - \frac{\sigma_{t+n|t}^2}{2} \right) \tau, \quad (4)$$

where E_t is the conditional expectation operator based on information at time t , $\tau = n/365$ is a scale factor expressed as a proportion of a year, and $\sigma_{t+n|t}^2$, represents an annualised, time-varying risk premium. The actual movements in the exchange rate are assumed to be governed by

$$\ln \left(\frac{S_{t+n}}{S_t} \right) = \left(r_t - i_t - \frac{\sigma_{t+n|t}^2}{2} \right) \tau + \sigma_{t+n|t} \sqrt{\tau} z, \quad (5)$$

where z is a zero mean, unit variance random variable which represents unanticipated movements in exchange rates and which is uncorrelated with $r_t - i_t - \sigma_{t+n|t}^2/2$. To capture the empirical features of the unconditional US/BP exchange rate distribution identified in the previous section, z is assumed to be distributed as a generalised Student t distribution following the formulation of Lye and Martin (1993)

$$p(z) = k^{-1} \sigma_w \exp \left[\theta_1 \tan^{-1} \left(\frac{\mu_w + \sigma_w z}{\sqrt{\nu}} \right) + \theta_2 \ln (\nu + (\mu_w + \sigma_w z)^2) + \sum_{j=1}^4 \theta_{j+2} (\mu_w + \sigma_w z)^j \right], \quad (6)$$

where μ_w and σ_w are chosen to ensure that z is standardised to have zero mean and unit variance, and k is the normalising constant, defined by

$$k = \int p(z) dz. \quad (7)$$

The properties of the alternative parameterisations of this distribution are discussed below in the context of pricing currency options.⁴

The specification of the volatility structure is based on the formulation of Rosenberg and Engle (1997),

$$\sigma_{t+n|t} = \exp(\beta_0 + \beta_1 \ln(S_{t+n}/S_t)), \quad (8)$$

whereby conditional volatility is assumed to be a function of the exchange rate return over the life of the contract. This specification has the effect of rendering volatility stochastic, with $\sigma_{t+n|t}$ approaching $\exp(\beta_0)$, as $t \rightarrow t+n$. Alternative specifications of the volatility structure in option price models are GARCH (Engle and Mustafa, 1992; Duan, 1995; Sabbatini and Linton, 1998; Heston and Nandi, 2000; and Hafner and Herwartz, 2001) and stochastic volatility (Hull and White, 1987; Heston, 1993; Bates, 1996; Ghysels, Harvey and Renault, 1996; Guo, 1998; and Chernov and Ghysels, 2000).⁵ These alternative formulations are less attractive however, as option prices in the present framework can be computed using a one-dimensional integral which can be computed numerically. This, in turn, overcomes the need for pricing

⁴A form of this distribution has already been used by Lim, Lye, Martin and Martin (1998) to study currency option prices, as well as by Lim, Martin and Martin (2002) in pricing equity options.

⁵For further discussion of the specification and estimation of volatility models, see Harvey, Ruiz and Shephard (1994); Kim, Shephard and Chib (1998); amongst others.

options by Monte Carlo methods which tend to be relatively less accurate and computationally much slower.

Consider writing a European call option on S_t with strike price X , that matures in n periods. The price of the currency option is; see Hull (2000)

$$\begin{aligned} F(S_t) &= E \left[e^{-r_t \tau} \max(S_{t+n} - X, 0) | S_t \right] \\ &= e^{-r_t \tau} \int_X^\infty (S_{t+n} - X) g(S_{t+n} | S_t) dS_{t+n}, \end{aligned} \quad (9)$$

where

$$g(S_{t+n} | S_t) = |J| p(z), \quad (10)$$

J is the Jacobian of the transformation from z to S_{t+n} , given by

$$\begin{aligned} J &= \frac{dz}{dS_{t+n}} \\ &= \frac{1 + \beta_1 \sigma_{t+n|t}^2 \tau - \beta_1 \left(\ln(S_{t+n}/S_t) - \left(r_t - i_t - \frac{\sigma_{t+n|t}^2}{2} \right) \tau \right)}{S_{t+n} \sigma_{t+n|t} \sqrt{\tau}}, \end{aligned} \quad (11)$$

and $\sigma_{t+n|t}^2$ is as defined in (8). The price of the currency option in (9) nests a number of special cases. Setting

$$\beta_1 = 0,$$

in (8) results in a constant volatility model. Imposing the restrictions

$$\theta_4 = -0.5, \text{ and } \theta_j = 0, \forall j \neq 4,$$

yields the Garman-Kohlhagen (1983) model, as $p(z)$ in (6) reduces to the standardised normal distribution and $g(S_{t+n} | S_t)$ in (10) becomes lognormal.

To highlight the properties of this option pricing model consider the case where the Garman-Kohlhagen model in (1) is used to price options when

returns are actually distributed according to the generalised Student t distribution in (6). More specifically, the particular distributional form used in these experiments is

$$p(z) = k\sigma_w \exp \left[- \left(\frac{1 + \nu}{2} \right) \ln (\nu + (\mu_w + \sigma_w z)^2) + \theta_6 (\mu_w + \sigma_w z)^4 \right], \quad (12)$$

with $\nu = 16$ and

$$\theta_6 = \{0.0, -0.05, -0.25\}.$$

Setting $\theta_6 = 0.0$ in (12) produces the Student t distribution with $\nu = 16$ degrees of freedom. As θ_6 becomes more negative the returns distribution becomes more thin-tailed. The range of moneyness of the option contracts considered is 160/165 to 180/165, with $S = 165$ as the exchange rate, which is approximately the same moneyness range corresponding to the empirical frown presented in Table 1. The maturity of the contracts is set at $\tau = 0.25$, and the domestic and foreign interest rates set at $r = 0.05$ and $i = 0.07$, respectively. The true volatility parameter is $\sigma = 0.07$.

Figure 3 shows the resultant volatility smiles and frowns. When returns are distributed as Student t with $\theta_6 = 0.0$ in (12), there is a volatility smile arising from the relative fatness in the tails of the distribution. Decreasing θ_6 from $\theta_6 = -0.05$ to $\theta_6 = -0.25$ causes the returns distribution to become thin-tailed which, in turn, results in a volatility frown.

To show the effects of skewness on the volatility frown the returns distribution is now specified as

$$p(z) = k\sigma_w \exp \left[- \left(\frac{1 + \nu}{2} \right) \ln (\nu + (\mu_w + \sigma_w z)^2) + \theta_3 (\mu_w + \sigma_w z) - 0.25 (\mu_w + \sigma_w z)^4 \right], \quad (13)$$

with $\nu = 0.64$ and θ_3 , which controls the degree of skewness, set at

$$\theta_3 = \{-1.0, 0.0, 1.0\}.$$

The exchange rate, strike prices, maturity and interest rates are the same as in the previous experiment. The results are presented in Figure 4. For symmetrical returns, $\theta_3 = 0.0$, the frown is relatively flat for a fairly wide range of

moneyness, turning into a frown only for the deep out-of-the-money options. Increasing the skewness parameter to $\theta_3 = 1.0$, causes the frown to become more distorted, whilst for $\theta_3 = -1.0$, the frown is relatively symmetric over the moneyness range reported.

The examples presented in Figures 3 and 4 show that volatility frowns can arise from misspecifying the form of the returns distribution. By assuming that returns are normal, when in fact they are not, the misspecification in the returns distribution is translated into a volatility structure that varies across strike prices. These examples also suggest that by specifying the returns distribution correctly the volatility structure across strike prices can become constant.

4 Estimation Procedures

In this section a statistical model is developed whereby observed option prices are used to estimate the parameters of the model. More formally, the relationship between $C_{j,t}$, the market price of the j^{th} call option contract at time t , and $F_{j,t}$, the theoretical price of the same option contract written at time t , is given by

$$C_{j,t} = F_{j,t} + \omega e_{j,t}, \tag{14}$$

where $e_{j,t}$ represents the pricing error with standard deviation ω . Following the approach of Engle and Mustafa (1992) and Sabbatini and Linton (1998), amongst others, the pricing error $e_{j,t}$, is assumed to be an *iid* standardised normal random variable; see also the discussion in Renault (1997) and Clement, Gouriéroux and Monfort (2000).⁶ The theoretical option price

⁶More general specifications of the pricing error in (14) could be adopted. For example, ω could be allowed to vary across the moneyness spectrum of option contracts, while a more general distributional structure for $e_{j,t}$, could be entertained; see, for example, Bates (1996, 2000). An alternative approach is to define the statistical model in terms of hedging errors; see Bakshi, Cao and Chen (1997).

is written as

$$F_{j,t} = F(S_t, X_{j,t}, \tau_j, r_t, i_t; \Omega), \quad (15)$$

where Ω is the vector of parameters which characterise the returns distribution and the volatility specification. In the special case of the Black-Scholes option pricing model, $\Omega = \{\beta_0\}$. Equation (14) may thus be viewed as a nonlinear regression equation, with parameter vector, Ω .

The unknown parameters of the model can be estimated by maximum likelihood. The logarithm of the likelihood function is

$$\ln L = -\frac{N}{2} \ln(2\pi\omega^2) - \frac{1}{2} \sum_{j,t} \left(\frac{C_{j,t} - F_{j,t}}{\omega} \right)^2, \quad (16)$$

where N is the number of observations in the panel of option prices. This function is maximised with respect to ω and Ω , using the GAUSS procedure MAXLIK. In maximising the likelihood, ω is concentrated out of the likelihood. The numerical integration procedure for computing the theoretical option price $F_{j,t}$, for the various models is based on the GAUSS procedure INTQUAD1. The accuracy of the integration procedure is ensured by checking that numerically and analytically derived Black-Scholes prices yield parameter estimates that are equivalent to at least four decimal points.⁷

5 Performance of Alternative Models

5.1 Data

The data set used in the empirical application consists of end-of-day European currency call options for the UK pound written on the US dollar over the period October 1st, 1997 to June 16th, 1998, a time period of 178 days. The data set is restricted to contracts which mature in September, 1998, so as to focus on volatility structures across strike prices. The prices of options are

⁷The calculation of the theoretical option prices by numerical integration is extremely fast and more accurate than pricing based on Monte Carlo methods.

specified as the bid prices. The complete set of strike prices over the sample period range from $X = 158$ to 178. Thus the data represent a panel data set where the cross-sectional units correspond to the strike prices, constituting $N = 736$ observations in total.

The US/BP exchange rate is presented in Figure 5. The US and UK risk free interest rates, taken as the 3-month Treasury bill rates, are presented in 6. The interest rates are relatively stable over the sample periods, deviating only slightly from their respective sample means of 5% and 7%.

5.2 Empirical Results

The performances of various currency option pricing models are now investigated. Four alternative models are examined. The first three models are based on the generalised Student t distribution in (6), whilst the fourth model is based on a mixture of lognormals used by Melick and Thomas (1997).

5.2.1 Generalised Student t Models

The specifications of the models based on the generalised Student t distribution are

$$\text{Normal: } \theta_4 = -0.5, \text{ and } \theta_j = 0, \forall j \neq 4,$$

$$\text{Student: } \theta_1 = 0, \theta_2 = -(1 + \gamma^2)/2, \theta_j = 0, \forall j > 2,$$

$$\text{Thin-tailed: } \theta_1 = 0, \theta_2 = -(1 + \gamma^2)/2, \theta_3 \neq 0, \theta_4 = \theta_5 = 0, \theta_6 = -0.25.$$

The Normal model corresponds to the Garman-Kohlhagen option price model. The Student model is based on the Student t distribution which allows for fatness in the tails of the distribution but not skewness. As this distribution does not exhibit thinned tailed behaviour, it is conjectured that this model should misprice options during tranquil periods. In contrast, the Thin-tailed model allows for thinness in the tails of the distribution as $\theta_6 < 0$, and thus

should yield smaller mispricing errors in tranquil markets.⁸ This model also allows for skewness as $\theta_3 \neq 0$.

The parameter estimates of the Thin-tailed, Student and Normal option price models are contained in Table 2, with standard errors based on the inverse of the Hessian given in parentheses. For all models the estimate of β_1 is statistically significant providing evidence that volatility is not constant over the sample period. There is also strong evidence of skewness in returns as the estimate of θ_3 in the Thin-tailed model is statistically significant. Both of these results represent strong statistical evidence against the Garman-Kohlhagen, Normal, pricing model.

Figure 7 gives the estimated residuals of the Thin-tailed model across all contracts at each point in time in the sample, with the residuals ordered in contract blocks at each point in time. This plot shows that for the last part of the sample, the Thin-tailed model is consistently overpricing options; that is, the pricing errors are negative. A similar result occurs for the other estimated models. To understand this result, Figure 8 gives the implicit volatilities computed for all options in the sample with moneyness of $|S/X| < 0.01$, based on equation (3). The striking feature of the implicit volatility estimates is that for most of the period they are falling from values around 10% early in the sample period, to around 7% near the end of the sample period. This suggests that the implied volatility is mean reverting to its long-run value.⁹ It further suggests that the implied volatility estimates based on (8) yield predictions that are too high near the end of the sample, which, in turn,

⁸The choice of $\theta_6 = -0.25$, is a convenient normalisation, however other choice could be adopted.

⁹To establish the value of the long-run value of volatility, a GARCH(1,1) model is estimated over the sample period using daily returns data. The estimated model is

$$\begin{aligned} 100 (\ln S_t - \ln S_{t-1}) &= 0.0241 + e_t \\ \hat{\sigma}_t^2 &= 0.0183 + 0.0409e_{t-1}^2 + 0.8745\hat{\sigma}_{t-1}^2. \end{aligned}$$

This yields a long-run value of the squared volatility of $0.0183 / (1 - 0.0409 - 0.8745) = 0.2163$. The long-run annualised volatility estimate is then $\sqrt{(250)(0.2163)} = 7.354\%$, which is consistent with the implied volatility estimates reported in Figure 8.

yield prices that are too high relative to the observed market prices. To capture this feature of the data the volatility specification in (8) is extended to include maturity

$$\sigma_{t+n|t} = \exp(\beta_0 + \beta_1 \ln(S_{t+n}/S_t) + \beta_2 \tau_t + \beta_3 \tau_t^2), \quad (17)$$

where τ_t represents maturity at time t . As the option contracts in the sample period all mature in September 1998, the value of τ_t is continually decreasing over time.¹⁰

The parameter estimates of the Thin-tailed, Student and Normal models based on the extended volatility specification in (17), are given in Table 3. The estimates of β_2 and β_3 for all three models in Table 3 are statistically significant at conventional significance levels showing that volatility over the sample period is a function of maturity.

The reductions in pricing errors yielded by adopting the extended volatility specification in (17) are highlighted in Table 4. This table presents estimates of the residual variance ω^2 , given in (14), for each model, as well as the AIC and SIC statistics. The residual variance is defined as

$$\omega^2 = \frac{\sum_{j,t} (C_{j,t} - F_{j,t})^2}{N}, \quad (18)$$

where $C_{j,t}$ and $F_{j,t}$ are respectively the actual and expected call option prices. The results show that there is a large reduction in mispricing errors from the adoption of the extended volatility specification. The results also show that the Thin-tailed model yields the smallest average mispricing errors compared to the Student t and Normal models, although the relative difference between the models falls with the adoption of the extended volatility specification in (17).¹¹

¹⁰An alternative extension of the volatility specification in (8) is to include higher order powers of returns. This specification was tried but yielded inferior results to the volatility specification in (17), and for certain models, did not even converge.

¹¹Plots of the residuals using the extended volatility specification in (17) show that the models no longer continually misprice options, especially in the latter part of the sample.

5.2.2 Mixture of Lognormals

The fourth model investigated in the empirical analysis is the lognormal mixture distribution suggested by Melick and Thomas (1997). The option pricing model in this case is

$$F(S_t) = \phi F^{GK}(\sigma_{1,t+n|t}) + (1 - \phi) F^{GK}(\sigma_{2,t+n|t}), \quad (19)$$

where $F^{GK}(\sigma_{i,t+n|t})$, $i = 1, 2$, is the Garman-Kohlhagen price as defined in (1), $0 \leq \phi \leq 1$, is the mixing parameter which weights the two subordinate lognormal distributions, and the subordinate volatilities are specified as¹²

$$\begin{aligned} \sigma_{1,t+n|t} &= (\exp \beta_{1,0} + \beta_{1,1} \ln(S_{t+n}/S_t) + \beta_{1,2}\tau_t + \beta_{1,3}\tau_t^2) \\ \sigma_{2,t+n|t} &= (\exp \beta_{2,0} + \beta_{2,1} \ln(S_{t+n}/S_t)). \end{aligned} \quad (20)$$

The lognormal mixture distribution can generate both skewed and thin-tailed risk neutral densities and thus represents a competitor to the generalised Student t distribution discussed above.

The results from estimating the mixture of lognormal model by maximum likelihood are presented in Table 5. For completeness, the results based on volatilities being independent of maturity, $\beta_{1,2} = \beta_{1,3} = 0$ in (20), are also presented. Corresponding estimates of mispricing errors based on the estimates of the residual variance in (18) and the AIC and SIC statistics, are presented in Table 6. Comparing Tables 4 and 6 shows that when the volatility is not a function of maturity, the lognormal mixture model is the second best performer behind the Thin-tailed option price model. Extending the volatility specification to include maturity, results in the lognormal mixture model being ranked last. This suggests that this model is not able to capture the skewness and thin-tailed behaviour of the currency returns distribution

¹²A more general volatility model was tried initially whereby both subordinate volatility specifications were functions of maturity. This more general model did not converge suggesting that the additional parameterisation was redundant, and hence was excluded from the final set of empirical results.

as well as the Thin-tailed model which is based on the generalised Student t distribution.

5.3 Forecasting

The comparisons of the models presented above are all based on within sample statistical properties. In this section, following Bakshi, Cao and Chen (1997) and Sarwar and Krehbiel (2000), the relative out-of-sample forecasting performance of the four models is investigated. Each model is re-estimated over a restricted sample period which excludes those contracts written on the last day in the data set; namely, June 16th, 1998. These options are then priced using information available prior to June 16th, with the predicted prices compared with the actual prices.

The results of the forecasting performance of the competing models are presented in Table 7. For completeness, the results based on the initial and extended volatility specifications, equations (8) and (17) respectively, are presented. The statistical measure adopted is the RMSE which is computed as

$$RMSE = \sqrt{\frac{\sum_{j=1}^{10} (C_{j,t} - F_{j, \text{June 16th}|I_{t-1}})^2}{10}}, \quad (21)$$

where $F_{j, \text{June 16th}|I_{t-1}}$ represents option prices quoted on June 16th using each of the four models, based on previous information, denoted as I_{t-1} . Focussing on the extended volatility specification results, the RMSE statistics are smallest for the Thin-tailed model, showing that this model prices option contracts written on the next day more accurately than do the other three models. The Student t and normal models yield the same RMSEs, while the lognormal mixture model yields the largest RMSE.

6 Conclusions

The aim of the paper was to specify a model to price currency options during tranquil periods characterised by small changes in spot prices and thin-tailed returns distributions. During these periods, option models based on the assumption of normality were found to over-price deep-in and deep-out of the money contracts and under-price at-the-money contracts. This yielded a volatility frown, which was in contrast with the more usual phenomenon of volatility smiles in currency markets. This establishment of a link between volatility frowns and thin-tailed returns distributions provided an explanation as to why volatility smiles were more commonly observed than volatility frowns.

The option price model was based on a parametric specification of the risk neutral probability distribution which was designed to capture thin-tails in exchange returns distributions during tranquil currency markets. A general volatility specification was also adopted which included the currency return over the remaining life of the option as well a maturity term which captured mean reversion in exchange rate volatility over the sample period. The model was applied to pricing European currency call options on the UK pound written on the US dollar over the period October 1st, 1997 to June 16th, 1998. The analysis was performed on a panel of call options with prices computed jointly on contracts within days as well as across days.

The key empirical results showed that the proposed option price model resulted in large reductions in pricing errors and improvements in forecasting, compared to a range of existing option models. The proposed model was also shown to correct for volatility frowns, thereby demonstrating that volatility frowns are a manifestation of misspecifying the risk neutral probability distribution.

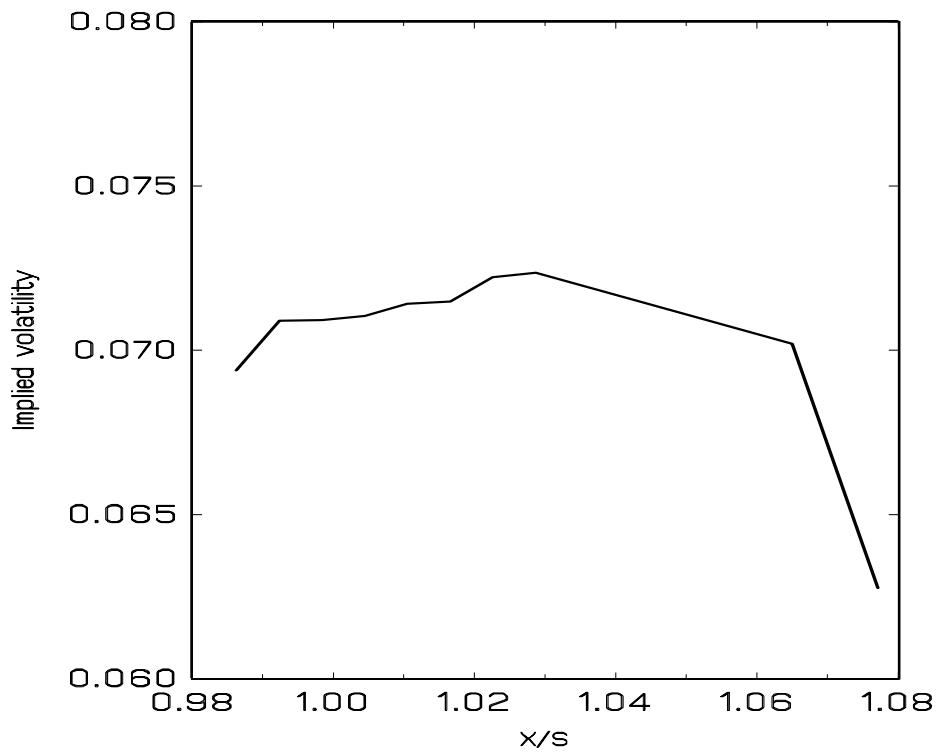


Figure 1: Implied volatility frown for US/BP European currency call options written on the 16th of June, 1998, and maturing in September, 1998.

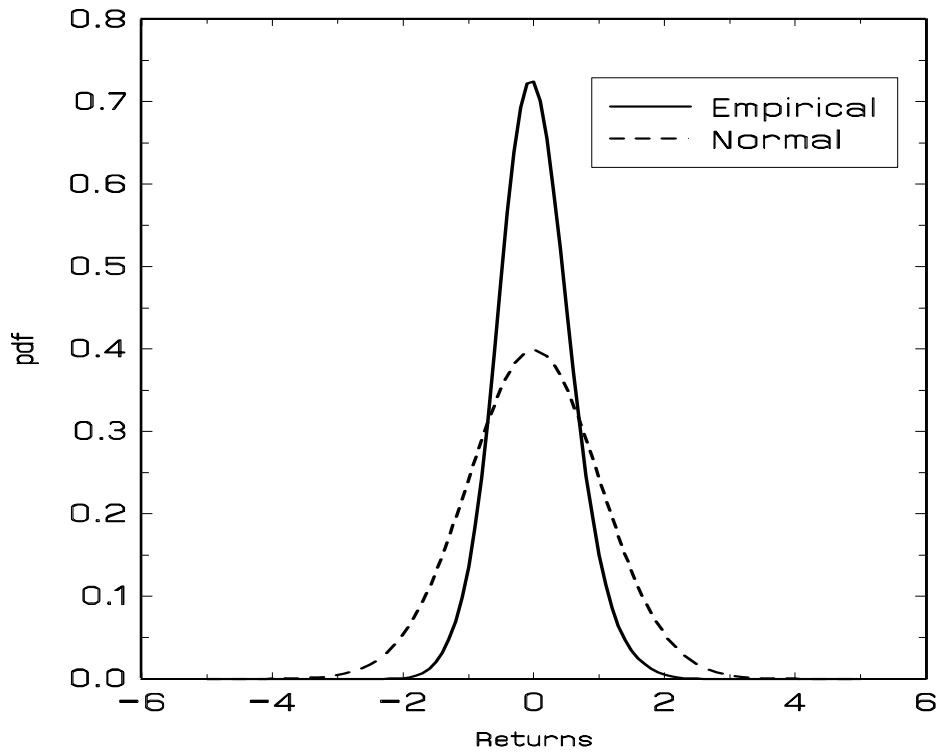


Figure 2: Empirical distribution of standardised US/BP foreign exchange returns, 11th of September, 1997, to the 16th of June, 1998.

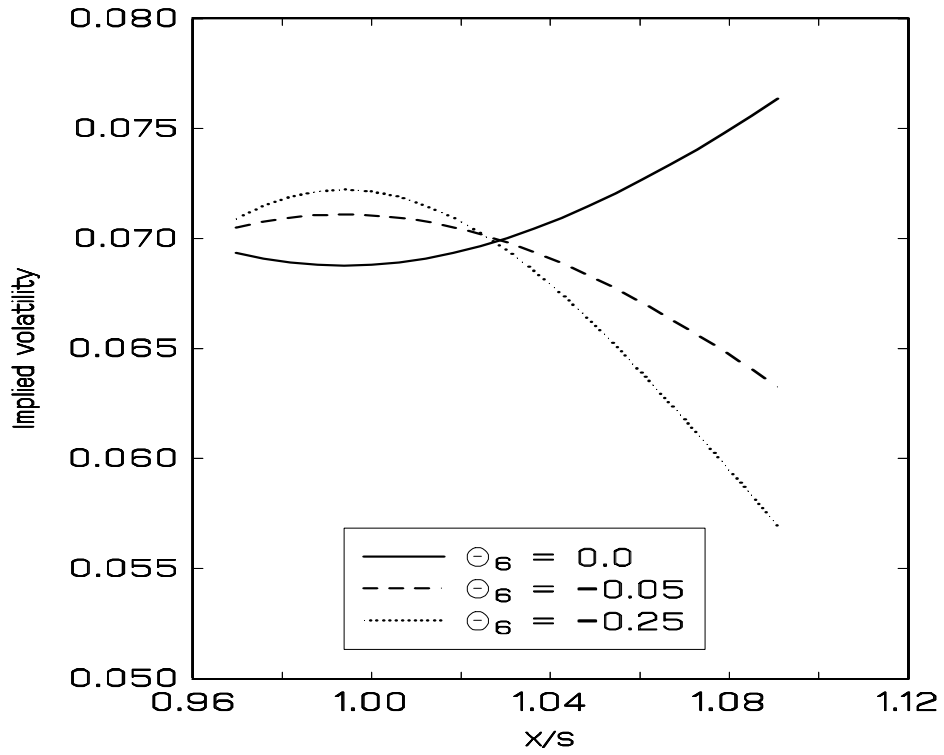


Figure 3: Volatility smiles and frowns generated when returns are distributed as generalised Student t with $\nu = 16$, and varying “thinness parameter” θ_6 : true volatility is $\sigma = 0.07$.

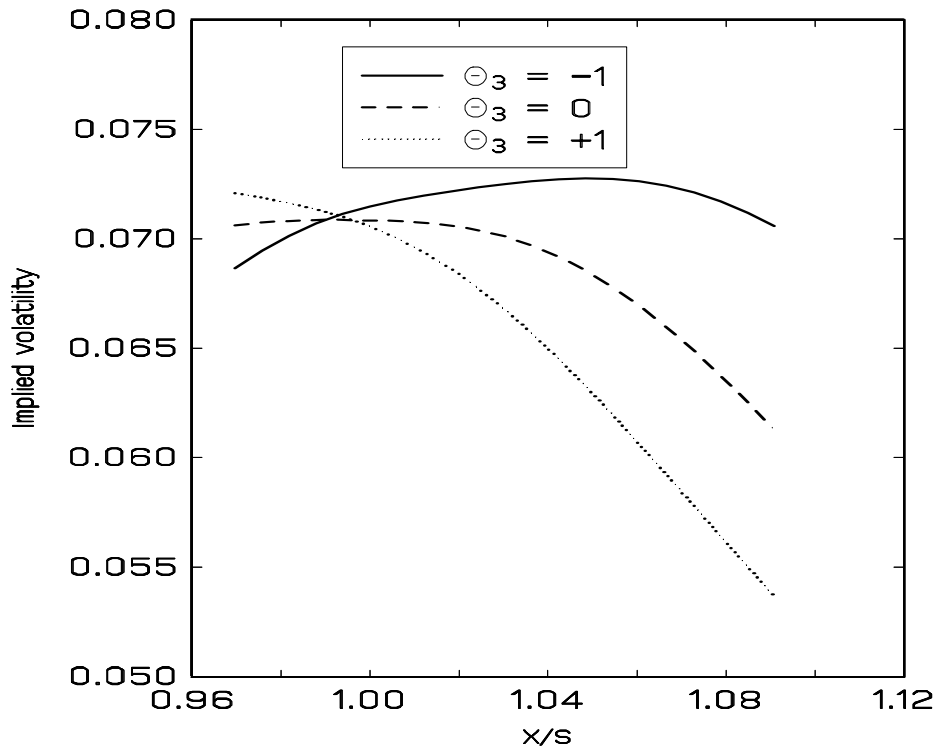


Figure 4: Volatility smiles and frowns generated when returns are distributed as generalised Student t with $\nu = 0.64$, and varying “skewness parameter” θ_3 : true volatility is $\sigma = 0.07$.

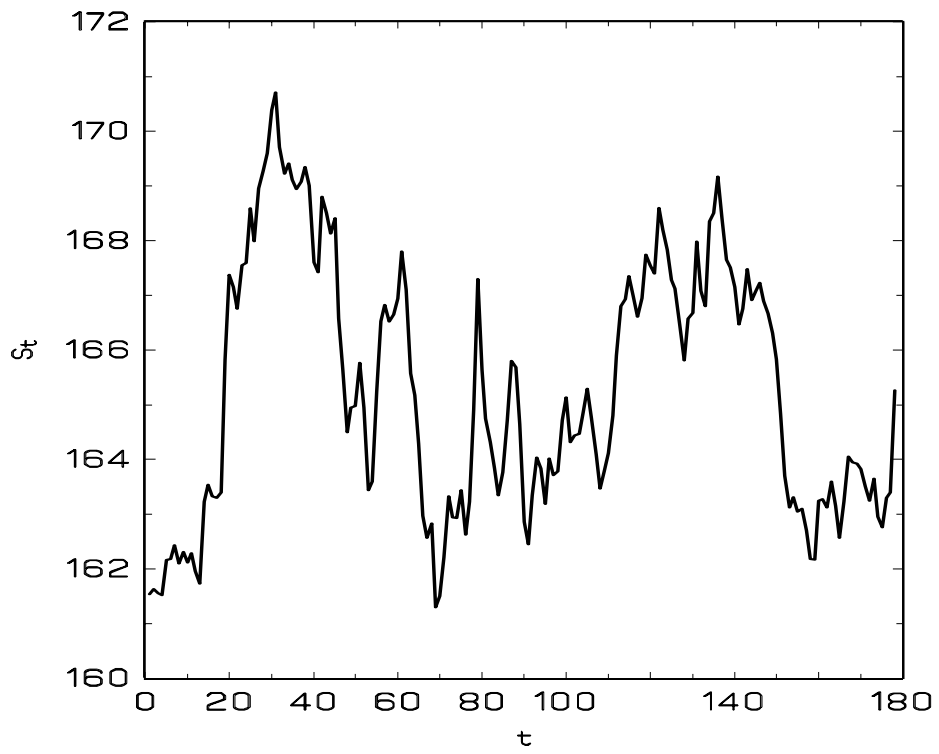


Figure 5: US/BP exchange rate, October 1st, 1997 to June 16th, 1998.

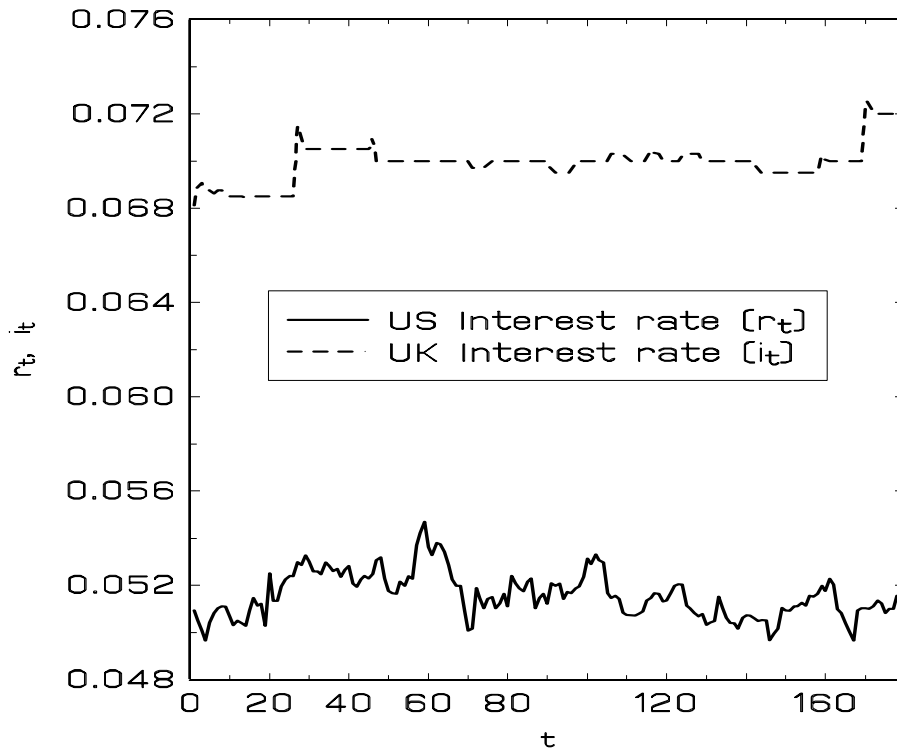


Figure 6: US and UK 3-month bill rates, October 1st, 1997 to June 16th, 1998.

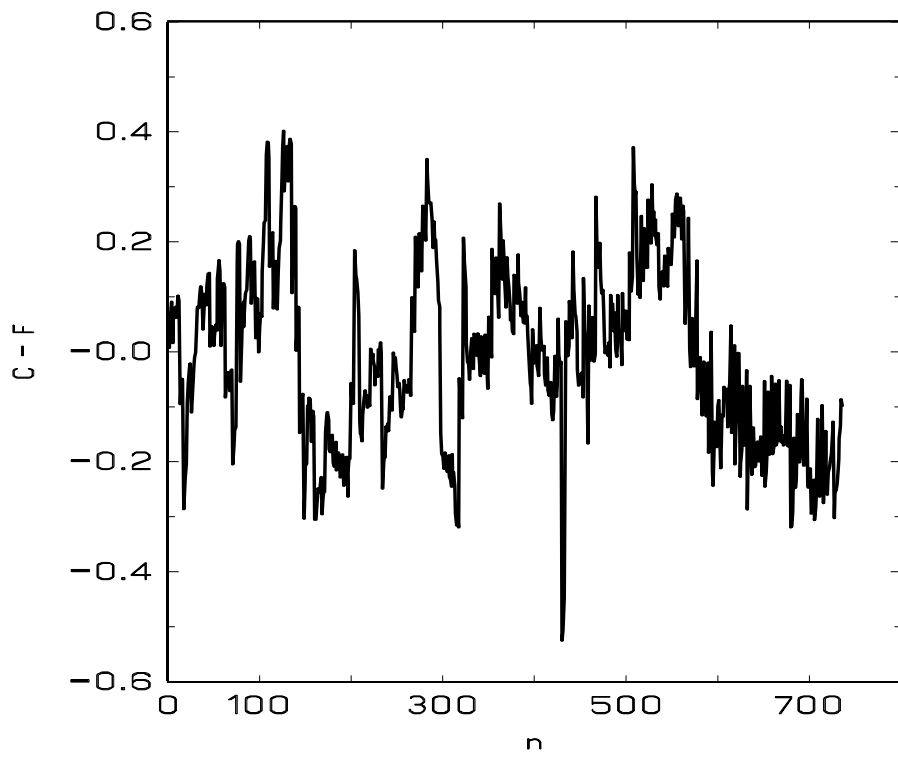


Figure 7: Residuals of the Thin-tailed model across contracts and time, based on the volatility specification given in equation (8).

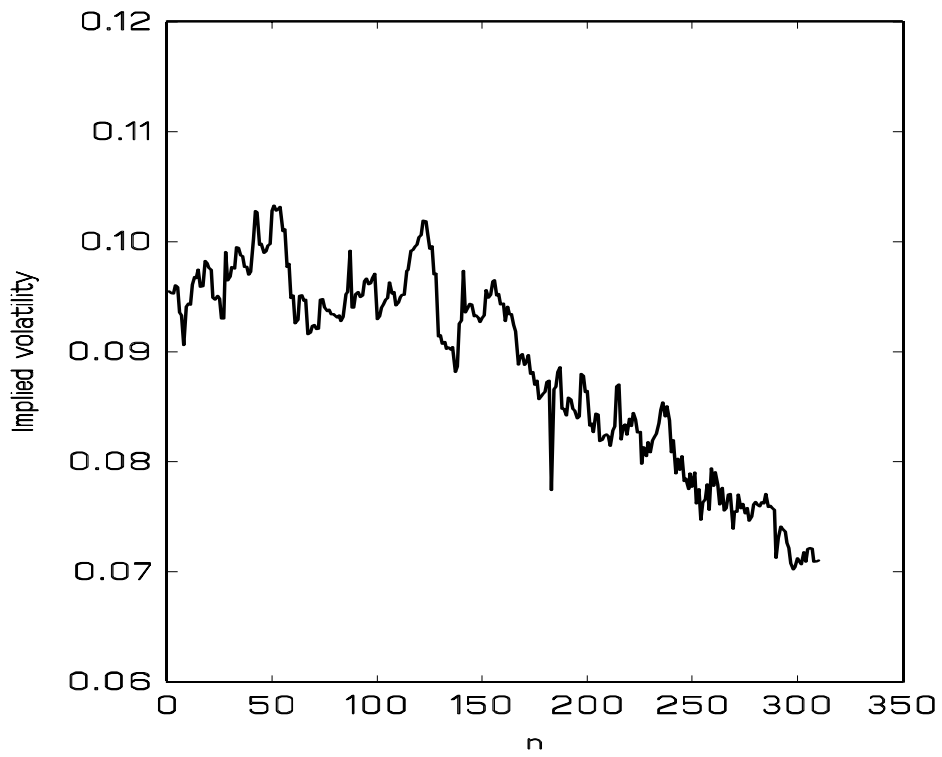


Figure 8: Implicit volatility estimates across all contracts at each point in time for contracts moneyness $|S/X| < 0.01$, using equation (3).

Table 1:
 US/BP end of day European currency call option prices,
 16th of June, 1998, maturing in September, 1998.

Strike Price X	Observed Call Price C	Garman-Kohlhagen Price $F^{GK} (\sigma = 0.071)$
163	3.01	3.061
164	2.51	2.513
165	2.03	2.032
166	1.62	1.618
167	1.28	1.268
168	0.99	0.977
169	0.77	0.740
170	0.58	0.551
176	0.06	0.064
178	0.01	0.027

Table 2:

Maximum likelihood estimates of generalised Student t option price models using volatility specification (8): standard errors in brackets.

Parameter	Thin-tailed	Student	Normal
β_0	-2.608 (0.004)	-2.487 (0.006)	-2.459 (0.003)
β_1	0.759 (0.001)	0.244 (0.005)	0.431 (0.001)
$\gamma = \sqrt{\nu}$	0.727 (0.155)	2.088 (0.040)	n.a.
θ_1	0.0	0.0	0.0
θ_2	$-0.5(1+\gamma^2)$	$-0.5(1+\gamma^2)$	0.0
θ_3	0.700 (0.179)	0.0	0.0
θ_4	0.0	0.0	-0.5
θ_5	0.0	0.0	0.0
θ_6	-0.25	0.0	0.0
$\ln L/N$	0.401	0.084	-0.210

(a) n.a. = not applicable.

Table 3:

Maximum likelihood estimates of generalised Student t option price models using volatility specification (17): standard errors in brackets.

Parameter	Thin-tailed	Student	Normal
β_0	-2.902 (0.019)	-2.992 (0.015)	-3.002 (0.034)
β_1	0.577 (0.002)	0.280 (0.095)	0.373 (0.032)
β_2	1.089 (0.068)	1.556 (0.048)	1.528 (0.043)
β_3	-0.615 (0.051)	-0.939 (0.034)	-0.924 (0.033)
$\gamma = \sqrt{\nu}$	0.612 (0.058)	4.122 (1.159)	n.a.
θ_1	0.0	0.0	0.0
θ_2	$-0.5(1+\gamma^2)$	$-0.5(1+\gamma^2)$	0.0
θ_3	0.452 (0.093)	0.0	0.0
θ_4	0.0	0.0	-0.5
θ_5	0.0	0.0	0.0
θ_6	-0.25	0.0	0.0
$\ln L/N$	0.634	0.609	0.607

(a) n.a. = not applicable.

Table 4:
Mispricing estimates of the generalised Student t models.

Statistic	Model		
	Thin-tailed	Student	Normal
<i>Volatility: Equation (8)</i>			
Residual variance ^(a)	0.026	0.049	0.089
AIC ^(b)	-581.626	-117.247	313.292
SIC ^(c)	-563.221	-103.443	322.494
<i>Volatility: Equation (17)</i>			
Residual variance ^(a)	0.016	0.017	0.017
AIC ^(b)	-922.042	-886.306	-885.630
SIC ^(c)	-894.435	-863.300	-867.225

(a) Based on equation (18).

(b) $AIC = -2\ln L + 2k$, where L is the likelihood and k is the number of estimated parameters.

(c) $SIC = -2\ln L + \ln(N)k$, where L is the likelihood, N is the sample size and k is the number of estimated parameters.

Table 5:

Maximum likelihood estimates of the mixture of lognormal option price model for various volatility specifications: standard errors in brackets.

Parameter	Volatility Specification, (20)	
	$\beta_{1,2} = \beta_{1,3} = 0$	$\beta_{1,2}, \beta_{1,3} \neq 0$
$\beta_{1,0}$	-2.327 (0.019)	-3.963 (0.610)
$\beta_{1,1}$	0.368 (0.016)	0.192 (0.689)
$\beta_{1,2}$	0.000	6.130 (1.978)
$\beta_{1,3}$	0.000	-3.941 (1.145)
$\beta_{2,0}$	-3.512 (0.065)	-2.746 (0.057)
$\beta_{2,1}$	0.522 (0.047)	0.416 (0.077)
ϕ	0.374 (0.010)	0.125 (0.019)
$\ln L/N$	0.224	0.598

(a) n.a. = not applicable.

Table 6:

Mispricing estimates of the mixture of lognormal models.

Statistic	Volatility Specification, (20)	
	$\beta_{1,2} = \beta_{1,3} = 0$	$\beta_{1,2}, \beta_{1,3} \neq 0$
Residual variance ^(a)	0.037	0.018
AIC ^(b)	-319.130	-866.294
SIC ^(c)	-296.124	-834.085

(a) Based on equation (18).

(b) AIC = $-2\ln L + 2k$, where L is the likelihood and k is the number of estimated parameters.

(c) SIC = $-2\ln L + \ln(N)k$, where L is the likelihood, N is the sample size and k is the number of estimated parameters.

Table 7:
Forecasting performance of alternative option
price models on June 16th, 1998: RMSE.

Volatility specification	Thin-tailed	Student	Normal	Mixture
Without maturity ^(a)	0.209	0.277	0.440	0.222
With maturity ^(b)	0.026	0.036	0.036	0.058

- (a) Based on equation (8) for the Thin-tailed, Student and Normal models, and equation (20) with $\beta_{1,2} = \beta_{1,3} = 0$, for the mixture of lognormal model.
- (b) Based on equation (17) for the Thin-tailed, Student and Normal models, and equation (20) with $\beta_{1,2}, \beta_{1,3} \neq 0$, for the mixture of lognormal model.

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