

Peer Group Effects, Sorting, and Fiscal Federalism

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Abstract

Suppose that, other things equal, an individual's utility increases with the fraction of residents in his community who are rich. Suppose further that the rich are more willing to pay for a local public than are the poor. Then the rich may over-provide a local public good, with the aim of dissuading the poor from moving into a community inhabited by the rich. We describe conditions under which the equilibrium will have mixed or homogeneous communities, and conditions under which the rich or the poor benefit from central government rules which constrain local decision making.

1 Introduction

An individual may want to live in a community where many of the residents are rich. The motive can arise from enjoying the high taxes paid by the rich, from peer-group effects in education, or from a desire for status that appears when living in a rich community. Such preferences can lead a community to adopt policies which appeal more to the rich than to the poor. In particular, if services provided by a local government are normal goods, then a community which provides high level of the service, and consequently imposes high taxes, can attract the rich but not the poor.

If people are mobile and residents cannot be excluded directly based on their income, the possibility arises of the poor chasing the rich. Policy makers

may use indirect instruments to reduce the attractiveness to the poor of a rich community: requiring minimum consumption of some private good; subsidizing the provision of some private good; distorting the provision of a public good.

As often noted, the possibility of the poor chasing the rich makes the sorting models in the theory of asymmetric information important for the economics of the local public sector. The outcomes of people's interjurisdictional mobility often share many of the characteristics of the separating or pooling equilibria in models of competitive insurance markets, and of other adverse selection problems. Voting choices complicate these outcomes, and may preclude the existence of well-behaved equilibria. To avoid these complications, and to highlight the implications of mobility, we assume instead that local policies are not chosen directly by residents.

The equilibria under mobility resemble those found in the literature on sorting. If the proportion of poor people is sufficiently high, the unique equilibrium will be a separating one: poor people reside in a jurisdiction which provides their preferred level of the local public good, and rich people live in a jurisdiction which over-provides the local public good in such a quantity so as to leave a poor person indifferent between jurisdictions. If, instead, the proportion of rich people in the general population is high, a pooling but not a separating equilibrium will exist. In this pooling equilibrium, all people, rich and poor, live in identical jurisdictions which provide a rich person's preferred level of the local public good.¹

Figures 1 and 2 illustrate these two types of equilibria. In the figures, the horizontal axis shows the level of public output (per capita) provided by a jurisdiction. The variable λ graphed along the vertical axis is the proportion of the population of a jurisdiction which is rich. The figures also show the indifference curves for the two groups of people. We suppose that the poor have steeper indifference curves than the rich.² And we suppose that both groups prefer the company of more rich people (higher values of λ).

One of our main concerns is the effect of nationally-imposed restrictions on the resulting equilibrium, and on the well-being of the two groups of residents. We consider several different types of restriction here: ceilings or floors on the level of local public good provision by city managers; grants or

¹So the equilibria here are exact analogies of those in Wilson's (1977) model of competitive insurance markets in which firms anticipate rivals' exit decisions.

²Actually, the assumption is that the slope of the indifference curves of the poor is higher, which actually means "less steep" if the indifference curves slope down.

subsidies to some jurisdictions (which must be financed by taxes on other jurisdictions, or on the general public); abolition of the local level of government, and its replacement by uniform national provision of the public output.

2 Literature

The consequences of the poor (potentially) chasing the rich have been explored in the literature. Many authors consider, for example, the distortions mobility might impose on policy choices in jurisdictions controlled by the rich. In models with a land market, minimum lot-size zoning has been explained as an exclusionary device. Examples are Hamilton (1975, 1976), Wheaton (1993), and Fernandez and Rogerson (1984). Wilson (1998) analyzes the incentives of rich jurisdictions to over-provide local public goods as an exclusionary device. Hoyt and Lee (2003) extend this analysis by showing that a rich jurisdiction may also wish to subsidize private goods in equilibrium, as an exclusionary device.

Most of this literature takes the number of jurisdictions as fixed. In contrast, we explicitly consider free entry of new jurisdictions. This lets us more clearly relate the equilibrium policy choices with those obtaining in models of sorting under perfect competition with asymmetric information.³ A large literature examines entrepreneurial behavior in the local public sector. Important examples are Berglas (1976), Berglas and Pines (1981), Scotchmer and Wooders (1987), Brueckner and Lee (1989), Scotchmer (1997) and Conley and Wooders (1998). Much of that literature assumes that entrepreneurs are “small,” taking the utility attained by different types of resident as given. In contrast, we let each entrepreneur explicitly consider the effect of his policy choices on people’s location patterns.

A more important contrast between this paper and the earlier literature is our consideration of a higher level of government. The main focus here is a comparative-static analysis of the sorting equilibria. We allow policies of the national government to affect the policies chosen in equilibrium by local governments.

Several authors consider how the provision of public services can attract different types of residents. Tiebout (1956), in a footnote, speculates that

³The classic paper is Rothschild and Stiglitz (1976). Dionne, Doherty and Fombaron (2000) survey this literature.

individuals may desire to live near nice neighbors, but does pursue the implications of this idea for his theory. Strahilevitz (2006) does. Becker and Murphy (2000) observe that local governments may use restrictive zoning, housing codes, and high spending on schools that raise property taxes; such policies would appeal more to the rich or other groups the municipalities want to attract.

Epple and Romer (1991) present a static model where voters who choose local policies consider how policies affect the inter-community migration equilibrium. Their model focuses on intra-community redistribution, not public-good provision. Epple and Romano (2003) consider a peer-group effect in schools, and allow residents to vote on how much to spend on education. But in their model the vote on spending is made after people choose where to live, and therefore voters cannot use spending decisions to affect the composition of the schools. We, instead, allow people to move in response to spending decisions, and so spending can be strategic.

An elected official may bias services with the aim of attracting people who would likely vote for the official, and encourage the out-migration of his political opponents. Glaeser and Shleifer (2005) discusses Mayor Curley of Boston, who used wasteful redistribution to his poor Irish constituents and incendiary rhetoric to encourage richer citizens to emigrate from the city, thereby shaping the electorate in his favor. The model by Brueckner and Glazer (2008) resembles Glaeser and Shleifer's in considering how current policy affects migration and thus future policy. Whereas Glaeser and Shleifer (2005) focus on the incentives of vote-maximizing incumbent officials, Brueckner and Glazer (2008) consider the preferences of residents, and allow for a broader range of policies than redistribution.

Some work considers clubs where members of each type gain direct utility from the presence of the other type (Brueckner and Lee 1989). In his classic work, Becker (1957) explores a model where some individuals in a group prefer to work with persons of the same group. Under factor price equalization this leads to segregation in different sectors. Borjas (1982) assumes that white constituents prefer to be served by white clerks in a government agency, and that blacks prefer to be served by blacks. Arrow (1972) supposes that some whites do not like to work with blacks. Berglas (1976) and McGuire (1991) study the characteristics of a competitive equilibrium when firms hire workers with different skills.

The interaction of voting and migration is an important phenomenon which has been well analyzed in an extensive literature, including Ellickson

(1971), Westhoff (1977), Rose–Ackerman (1979), Epple, Filimon and Romer (1984, 1993), and de Bartolome (1990).

3 Assumptions

3.1 City managers

Equilibrium is often difficult to characterize when people vote on policy as well as voting with their feet, especially when no land market rations attractive jurisdictions. If we require that decisions about provision of a public good result from voting by residents of the jurisdiction under pairwise majority rule, and that people be perfectly mobile among jurisdictions, then equilibrium may fail to exist.

An alternative approach is to assume that jurisdictions are controlled by profit-maximizing entrepreneurs, charging residents admission fees to the jurisdiction, which are not required to equal the per capita cost of the public sector.⁴

In keeping with the notion of competitive markets under asymmetric information, we assume that jurisdictions’ policies are under private control, with free entry of new jurisdictions. But we assume that each jurisdiction’s proprietor takes as given the policies offered by competing jurisdictions. In the canonical model of competitive insurance under asymmetric information (Rothschild and Stiglitz 1976), firms try to earn positive profits from the introduction of new insurance contracts, taking as given the contracts offered by other firms. An equilibrium is a set of contracts for which no such profitable entry by new firms is possible.

Our assumption that public output levels in the model are set by “city managers,” rather than set directly by residents can be relaxed without changing the results. The equilibrium menus will be the same, whether jurisdictions are run by city managers, or by profit-maximizing entrepreneurs who can select admission fees. The menus will also be the same if public output levels are chosen by voters — provided that an equilibrium in pure strategies exists in this latter case.

⁴We assume throughout that the cost per capita of a given level of local public good provision does not vary with the number of people. This assumption of constant returns to scale in population is consistent with most of the literature (starting with Tiebout 1956), and with some empirical evidence.

3.2 Population

The population consists of $\bar{P} > 0$ poor people, and of $\bar{R} > 0$ rich people. The fraction of rich people in the population is $\bar{\lambda}$:

$$\bar{\lambda} \equiv \frac{\bar{R}}{\bar{R} + \bar{P}} \quad (1)$$

The preference of a type- i ($i \in \{P, R\}$) person can be represented by a utility function $U_i(g, \lambda)$, where g is the level provided of the public good in the jurisdiction in which the person lives, and where λ is the proportion of the population — in that jurisdiction — which is rich.

As argued below, several phenomena can be represented by this reduced-form utility measure. In all the cases described below, exogenously set rules determine how much each resident pays in taxes to finance the provision of the public good.

For example, suppose the cost of the local public sector is shared equally by all residents in a jurisdiction. Suppose also that residents care directly about the income composition of their jurisdiction, perhaps arising from concerns about status, or from peer-group effects. Thus, their utility could be represented by some function $V_i(x, g, \lambda)$, where x is consumption of a numéraire private good. Then if the cost per person per unit of the local public good is c , $U_i(g, \lambda) \equiv V_i(y_i - cg, g, \lambda)$, where y_i is the exogenous income of a type- i person.

Alternatively, if the cost of the public good is financed by a proportional income tax, then the tax rate t in a jurisdiction satisfies

$$t(g, \lambda) = \frac{cg}{\lambda y_R + (1 - \lambda)y_P}, \quad (2)$$

so that $U_i(g, \lambda) = V_i((1 - t(g, \lambda))y_i, g, \lambda)$, which would depend on the population composition even if the direct utility measure $U_i(\cdot, \cdot, \cdot)$ were independent of the income composition λ . Since the cost of paying for the public sector is subsumed in the utility measure $U_i(g, \lambda)$, increases in g may increase or decrease a resident's utility. We make the following “standard” assumptions about this utility measure:

1. For each income class $i \in \{P, R\}$ and each possible population composition $\lambda \in [0, 1]$ there is a unique preferred level of public output $g_i^*(\lambda)$

such that

$$\frac{\partial U_i(g, \lambda)}{\partial g} > 0 \quad \text{if and only if} \quad g < g_i^*(\lambda).$$

2. $\frac{\partial U_i(g, \lambda)}{\partial \lambda} > 0$ for all $i \in \{P, R\}$, $g > 0$ and $\lambda \in [0, 1]$
3. For some finite $g_1^E > g_P^*(0)$,

$$U_P(g_1^E, 1) = U_P(g_P^*(0), 0).$$

4. If $g' > g$, and if $U_P(g, \lambda) = U_P(g', \lambda')$, then $U_R(g', \lambda') > U_R(g, \lambda)$

Assumption ?? makes sense if the poor must pay some share of the cost of the public sector, no matter how small. Then a sufficiently high level of public output would drive their private consumption to zero, and it seems reasonable to assume that no peer–group or status effect can compensate for starvation.

Assumption ?? is the usual single–crossing property. It ensures that the the indifference curve of a rich person through any (p, λ) combination is flatter than the indifference curve through the same point of a poor person — when g is depicted on the horizontal axis and λ on the vertical. (Since indifference curves can slope down or up, “less than” here means greater in absolute value if both groups’ indifference curves sloped down.)

Each person chooses from a menu of jurisdictions. Each individual ignores her own influence on the income distribution in the different jurisdictions. So she chooses her most preferred jurisdiction from those available, taking as given the income distribution parameter λ in each jurisdiction.

Of course this parameter λ must be consistent with the location choices people make. We assume that a city manager cannot directly exclude people from a jurisdiction. Either a person’s income is private information when she chooses where to live, or a legal proscription prohibits any discrimination based on income.

Since people’s perception of the income distribution must be based on the actual income distribution, we define a distribution of population as **consistent** with the available menu of local public output levels, and with the perceived income distribution, if the perceived proportion of rich people

in a jurisdiction equals the actual proportion, after people make their optimal location choices.

Figure 1 illustrates indifference curves for preferences which are consistent with the above assumptions. Curve IP represents the indifference curve of a poor person through $(g_P^*(0), 0)$, here represented by the point Poorville. Curve IR is the indifference curve of a rich person through $(g_1^E, 1)$, here represented by the point Richville.

3.3 Definition of equilibrium

Our equilibrium concept is analogous to a competitive equilibrium in sorting models. Private entrepreneurs seek to make profits by charging admission fees to new jurisdictions for which they have guaranteed some level of provision for the local public good. An equilibrium is a set of local public good provision levels such that no new entrepreneur can make a profit by providing some distinct level of local public good provision.

We modify the above definition in two ways. First, we allow entrepreneurs to introduce (different) multiple jurisdictions. However, no cross-subsidization is allowed; an entrepreneur cannot lose money on any single jurisdiction. Second, we restrict the entrepreneurs to breaking even with their contracts, rather than making positive profit. This zero-profit condition lets us analyze the existence of equilibrium using two-dimensional graphs. But an outcome will be an equilibrium under this restriction if and only if it is an equilibrium when entrepreneurs are free to choose whatever admission fee they wish.⁵ As in competitive insurance models, competition drives profits down to zero even without this restriction.

The equilibrium concept used here differs in an important respect from that in Rothschild and Stiglitz (1976). Consider the effect of entry by a new firm in the model by Rothschild and Stiglitz. Cream-skimming by a new entrant may alter the mix of consumers served by existing firms, and this cream-skimming may cause some of the contracts offered by existing firms to become unprofitable. As is usual in models of competitive behavior, new entrants are assumed not to anticipate the effect of their entry on other firms' profits, and on the potential exit of no-longer-profitable firms.

In the model presented here, cream-skimming affects not the profits of existing firms, but the value of λ in other jurisdictions. This change will affect

⁵Provided that they charge the same admission fee to all.

the consistent allocations. In other words, in the Rothschild–Stiglitz (1976) model, firms play Nash with regard to their strategic variables, the price–quantity pairs they offer, and assume that altering their own behavior will not change the other firms’ contracts. In the current model, city managers play Nash in output levels g . The λ ’s are not strategic variables, but consequences of the city managers’ output choices. A new entrant assumes its own behavior will not change other city managers’ output choices, but will change the allocation of residents, since the residents are not strategic players but instead respond passively to city managers’ choices.

So the model here is formally closer to the modification of the Rothschild–Stiglitz model made by Wilson (1977), ensuring the existence of equilibrium. Not surprisingly, the equilibrium outcomes here are more similar to those in this modified insurance model.

The timing of the model is as follows. First a national legislature decides which restrictions, if any, to impose on prospective city managers who decide local policies. Then city managers make their choices of local public good levels, anticipating the migration responses of residents. Then residents sort themselves, so that they reside only in the jurisdictions which they most prefer (which choice depends, of course, on the location decisions of others).

4 Equilibria

The concept of equilibrium is the usual subgame perfect Nash equilibrium. City managers anticipate the locational choices people will make. They choose to enter if and only if each of the jurisdictions they operate can attract a positive population, given the choices made by other city managers, and given the subsequent location decisions of individuals. Since the population distributions consistent with a given menu of local public output levels need not be unique, we must place further restrictions on the population distributions which result from city managers’ decisions. But we defer these formalities to section ?? (and subsequent sections).

4.1 Separating equilibrium

We shall present the results graphically here. The Appendix gives formal analysis and proofs. In Figure 1, curve IP is an indifference curve of a poor person. It is constructed to be tangent to the horizontal axis; that is, it repre-

sents the highest utility a poor person can obtain if no rich people live in the community in which he lives. A poor person who lives in a community with only the poor maximizes his utility by choosing the level of g represented by point Poorville (this is the tangency point with the horizontal axis). Anywhere above this curve represents a higher level of utility for a poor person than points on curve IP .

The point Richville is constructed to satisfy the selection constraint, that no poor person prefers to move into rich jurisdictions. It lies on the right-hand side of indifference curve IP , where the fraction of rich people in the community is 1. Curve IR is the indifference curve of a rich person through point Richville. The horizontal dashed line in the middle of the graph represents the proportion of the population (aggregated over all communities) that is rich, an analogue to the “pooling line” in models of insurance.

In this figure, the indifference curve IR of the rich lies above this pooling line. Under these conditions, a separating equilibrium exists. All rich people live in communities consisting only of rich people. Each enjoys consumption at point Richville, on indifference curve IR . All poor people live in communities consisting only of poor people. Each enjoys consumption at point Poorville, on indifference curve IP . Note that if these points represent the allocations, then no rich person would want to move to any newly established community which gives a consumption point below curve IR , so no community which lies between curve IR and IP is feasible. And no poor person would want to live in a community which lies below curve IP .

The poor and the rich would both prefer to live in a community which gives a consumption point in the area above curve IR . But no such community could be established—if it were, all the poor and all the rich would want to live there, so that the proportion of the rich in such a community would necessarily be the same as in the population as a whole, or at the level indicated by the horizontal dashed line. Since this horizontal line lies below curve IR , entry of such a new community is impossible.

The high level of g at point Richville can be interpreted to mean that the rich want to provide high g (with concomitant high taxes) to deter the poor from moving into their communities; in the absence of such a consideration, the rich would prefer to consume somewhere to the left of point Richville, where less g is provided.

4.2 Pooling equilibrium

A different equilibrium, with pooling, appears if the rich are a sufficiently large fraction of the population. Indifference curve IP in Figure 2 is the same as in Figure 1; indifference curve $IR1$ in Figure 2 is the same as indifference curve IR in Figure 1. Curve $IR2$ is tangent at point Pooling to the dashed horizontal pooling line. This horizontal line represents the fraction of the population that is rich, and is higher than in Figure 1.

Point Pooling is an equilibrium. Any point above $IR2$ is infeasible: it would attract all the rich and the poor in the population, implying that the proportion of rich in each community is greater than their proportion of the population — an impossibility. Any community with an allocation below indifference curve $IR2$ would be less attractive to the rich than is the community at point Pooling. Therefore, any such community would attract no rich people, leading the poor to live in a community at point Poorville, which each finds inferior to living in a community at point Pooling.

Notice that the rich are better off at point Pooling than at point Richville. As long as the pooling line is above the indifference curve $IR1$ through the point Richville, the rich will benefit from an increase in the proportion of the rich in the population.⁶ And since the poor are better off at point Pooling than at point Poorville, they too benefit from an increase in the proportion of the rich in the population.

This upsetting of the separating equilibrium with a pooling contract operates exactly as in adverse selection models. However, unlike the result in Rothschild and Stiglitz, an equilibrium can exist in this situation. Pooling outcomes cannot be upset so easily by cream-skimming by new entrants. A new entrant who attempts to attract only the rich residents of a mixed jurisdiction would lower the value of λ in the existing mixed jurisdiction. In insurance markets, this reduces the profits of an existing firm below zero, but it does not change the attractiveness of the existing contract to high-risk (or low-risk) customers. Here the fall in λ harms directly the poor (and rich) residents of an existing jurisdiction. In doing so, it may induce the poor, as well as the rich, to leave the existing jurisdiction. This of course would undo the new entrant's plan, of attracting only the rich.

⁶Once the pooling line is below this indifference curve, as in Figure 1, further decreases in the proportion of rich people have no effect on either group's equilibrium utility.

5 Spending limits

The separating equilibrium has the poor potentially worse off than they would be in an equilibrium where communities are heterogeneous. The feasible outcome the poor would most like is an equilibrium in which each community has a proportion of the rich and poor identical to that in the population as a whole.

Suppose then that the poor were a majority in the population, controlling a central government which could restrict local governments. The poor could gain by limiting local spending on the locally provided good. Such a restriction could eliminate the separating equilibrium, to the benefit of the poor. This motive could explain restrictions on local spending, such as adopted by California's Proposition 13, or the Serrano decision, which required equalization of spending across school districts.

In a separating equilibrium the high level of g and associated taxes dissuade the poor from living among the rich. The poor may therefore benefit from a spending limit on g . Figure 4 shows how. In the absence of a spending constraint, Figure 4, like Figure 1, shows a separating equilibrium with the rich consuming at Richville and the poor consuming at Poorville. In Figure 4, indifference curve IP is the same as in Figure 1—it shows the highest utility the poor can get in a community consisting only of the poor. Indifference curve $IR2$ is tangent to the horizontal line, at point T .

Now suppose a constraint is imposed (say by the central government) that the maximum level of g not exceed the level associated with point T . Then the equilibrium is a pooling one, with all communities represented by point T . At point T , the poor are better off than at the separating equilibrium where the poor consume at Poorville. The rich would prefer a community above indifference curve $IR2$, but the constraint on g means that any such community would also attract the poor, and so is unattainable.

Though a spending limit can benefit the poor, as just seen, a small spending limit may hurt the rich without helping the poor. This is shown in Figure 3. It differs from Figure 4 in having a less stringent spending limit. If the original equilibrium (with no spending limit) were a separating equilibrium, then the outcome of a spending limit which is just binding must be the one depicted in this figure. The equilibrium in this situation would have all the rich and some of the poor live in a community with allocation at point Mixed, and some of the poor consuming at Poorville. The limit on spending on g

hurts the rich, but does not help the poor.⁷

We have so far discussed caps on spending by local governments. But we note that an effective cap can be imposed by setting some (nonlinear) spending requirements. Suppose the central government says that if a city provides some service, then it must provide at least $G + H + \Delta$. An example would be building codes for theaters in schools. The constraint would induce the rich to go below g_H , thereby effectively imposing a cap.

6 Uniform service

The second generation of fiscal federalism literature emphasizes that uniformity of public good provision differs from centralization. But uniform provision is feasible if policy of the local public sector is dictated by the national government. Consider then the choice between uniform provision by the central government, and the equilibrium prevailing under competitive behavior by city managers (with none of the central government regulations described in the previous section).

Consider a first, “constitutional” stage, at which residents chose whether to have (i) free-entry, competitive provision by many city managers, with no central government regulation, or (ii) uniform provision, with the level chosen by majority rule at the national level. If the poor are in the majority nationally (that is, if $\bar{\lambda} < 1/2$), then the poor must do better under uniformity: they get to set the level of g that maximizes their utility, and also enjoy the benefits of living in a community with rich people. This must make them better off than under decentralization: in a separating equilibrium they do not enjoy the benefits of living in a community with rich people, and in a pooling equilibrium they do not get to set their preferred level of g . While uniform central provision is better for the poor in this situation, it must be strictly worse for the rich.

Conversely, if the rich are in the majority, they must do at least as well under decentralization as under uniformity. But this preference may be weak; with $\bar{\lambda} > 1/2$ uniform provision yields the same outcome as a pooling equilibrium under decentralization. What benefits the rich may not necessarily harm the poor here. With $\bar{\lambda} > 1/2$, the poor may be better off, or worse off,

⁷In Lee (1993), spending limits harm bureaucrats, but only benefit voters if they are sufficiently large. However, unlike the case considered here, in Lee (1993) small spending limits make everyone (voters and bureaucrats) strictly worse off.

in a separating equilibrium under decentralization than under uniform central provision. Figure 6 illustrates the poor benefiting from decentralization, and Figure 5 the poor being hurt.

7 Transfers among jurisdictions

The model here may also explain the popularity of transfers from higher levels of government to low-income communities. If local government policies are decentralized, and if residents are mobile, these transfers may benefit both the rich and the poor.

That is, consider again a prior constitutional stage, at which a national legislature decides whether to provide transfers to communities with a large proportion of poor inhabitants. These transfers may be financed directly by payments from richer communities, or indirectly through a national income tax.⁸ In either case, the rich will be making transfers to any of the poor people who choose to live in homogeneous communities. But the rich may still benefit from such a transfer policy, through its effect on the location decisions of poor people. Transferring resources to Poorville make that poor-only community more attractive. Doing so means that Richville does not have to distort its public expenditure decision as much, in order to keep out the poor. In some case, introducing these transfers may induce a Pareto-preferred separating equilibrium under decentralization.

In particular, suppose that peer-group benefits are a normal good, with the poor valuing them little, and the rich valuing them much. Suppose as well that the equilibrium under decentralization is a separating equilibrium. Transfers to any some community in which λ is below some (low) threshold would make a poor community more attractive, thereby allowing the rich to reduce spending on g in rich jurisdictions, without inducing the poor to migrate. As an extreme example, suppose a poor person pays almost no taxes. An increase in g in a rich community will not prevent migration. But a subsidy to a poor community can deter migration.

Notice that subsidies here are directed not at poor people, but at communities inhabited exclusively by poor people. Indeed, because an increase in a poor person's income increases his willingness to pay for g and his willingness to pay for peer-group benefits, a direct subsidy hurts the rich. Instead, the rich favor a subsidy or transfer that is contingent on the individual residing

⁸The latter case seems more common in practice.

in a jurisdiction populated by the poor. A subsidy to the jurisdiction itself accomplishes the goal.

8 Appendix: Analytical solution

8.1 Assumptions

This Appendix formalizes the results given above.

DEFINITION A population distribution $\{(L^1, \lambda^1), (L^2, \lambda^2), \dots, (L^N, \lambda^N)\}$ of total populations L^i for each jurisdiction, and proportion of rich people λ^i in each jurisdiction is defined to be consistent with the menu (g^1, g^2, \dots, g^N) of public output levels if the following conditions apply

- $\sum_{n=1}^N \lambda^n L^n = \bar{R}$
- $\sum_{n=1}^N (1 - \lambda^n) L^n = \bar{P}$
- If $\lambda^m L^m > 0$, then $V_R(g^m, \lambda^m) \geq V_R(g^n, \lambda^n)$ for any $n \neq m$.
- If $(1 - \lambda^m) L^m > 0$, then $V_P(g^m, \lambda^m) \geq V_P(g^n, \lambda^n)$ for any $n \neq m$.
- If $g^m > g^n$, then $\lambda^m \geq \lambda^n$
- If $U_P(g^m, 0) \geq U_P(g^n, 1)$ then $\lambda^n = 1$
- If $L^m > 0$ and $\lambda^m > 0$, then $\lambda^n = 1$ for $g^n > g^m$.

This definition says that individuals do not behave strategically here, that the population composition reflects their optimizing choices, and that they perceive correctly the income composition of each jurisdiction.

The final three items are needed only to account for completely empty jurisdictions. If L^m and L^n are both positive, then the single-crossing assumption ensures that $\lambda^m \geq \lambda^n$ if $g^m > g^n$. As well, if $U_P(g^m, 0) \geq U_R(g^n, 1)$, then no poor person would ever choose to live in jurisdiction n , so that λ^n must equal 1 if $L^n > 0$. But if $L^n = 0$, then the fraction rich in jurisdiction n is arbitrary. It seems a natural restriction on people's beliefs that they infer that higher-public-output jurisdictions are more likely to attract richer residents. Absent some restrictions on the fraction λ^n attributed to unpopulated

jurisdictions, there would be far too many consistent population distributions for any given menu of output levels.⁹

At the risk of terminological overkill, we introduce a further restriction on population distributions. Without this restriction, equilibrium may fail to exist.

DEFINITION: A population distribution $\{(L^1, \lambda^1), (L^2, \lambda^2), \dots, (L^N, \lambda^N)\}$ is said to **correspond** to a menu (g^1, g^2, \dots, g^N) of public output levels if

- The population distribution is consistent with the menu (g^1, g^2, \dots, g^N)
- No other population distribution consistent with that menu is weakly preferred by both income classes, and is strictly preferred by at least one income class.

This restriction can be motivated by some possibility of communication among residents: certainly they have an incentive to coordinate on a population which they all prefer.¹⁰

8.1.1 Definition of equilibrium

DEFINITION A menu (g^1, g^2, \dots, g^N) , and a population distribution

⁹For example, suppose that there are two jurisdictions, with g^1 and g^2 close to each other in value, and with $g_P^*(\bar{\lambda}) < g^1 < g^2 < g_R^*(\bar{\lambda})$. Then if the final restriction on unpopulated jurisdictions were not used, there would be at least two consistent population distributions: one in which everybody chose to live in jurisdiction 1, and attributed a value of $\lambda^2 = 0$ to jurisdiction 2, and one in which everyone chose to live in jurisdiction 2 and attributed a value of $\lambda^1 = 0$ to jurisdiction 1. Only the second distribution is consistent, when the second-last restriction in the definition above is imposed.

¹⁰As an example a consistent population distribution which does not correspond to the menu of public output levels, consider two jurisdictions, with $g^1 = g_R^*(\bar{\lambda})$ and $g^2 > g^1$ such that $U_R(g^2, 1) = U_R(g_R^*(\bar{\lambda}), \bar{\lambda}) - \epsilon$ where ϵ is very small but positive.

One consistent allocation is to have all the people, rich and poor, locate in jurisdiction 1. But if a few people from jurisdiction 1 were to move to jurisdiction 2, then the utility of the remaining residents of jurisdiction 1 would fall. Let $\lambda' < \bar{\lambda}$ be the proportion such that $U_R(g^1, \lambda') = U_R(g^2, 1)$. Then if R' rich people moved from jurisdiction 1 to jurisdiction 2, where $\lambda'(\bar{R} - R' + \bar{P}) = \bar{R} - R'$, there would be another consistent allocation, in which $(L^1, \lambda^1), (L^2, \lambda^2) = (L^1 - R', \lambda'), (R', 1)$. This second allocation is Pareto-dominated by the first, since the fall in λ^1 lowers the utility they both attain in jurisdiction 1, which is still inhabited by both income classes. Therefore, this second allocation does not correspond to the output menu (g^1, g^2) .

$\{(L^1, \lambda^1), (L^2, \lambda^2), \dots, (L^N, \lambda^N)\}$ corresponding to it is said to be an equilibrium if there are no distinct new levels of public output $(g^{N+1}, g^{N+2}, \dots, g^{N+M}) \notin \{g^1, g^2, \dots, g^N\}$ such that there exists a population distribution corresponding to $(g^1, g^2, \dots, g^N, g^{N+1}, \dots, g^{N+M})$ for which $L^m > 0 \quad m = 1, 2, \dots, M$.

Note that to upset an existing population distribution, an entrant may need to open several new jurisdictions.

8.2 Some Properties of Equilibrium

Lemma 1 *There cannot be two jurisdictions m and n with distinct levels of local public output g^m and g^n such that both jurisdictions get positive numbers of both type of resident in any consistent population distribution.*

Proof This result follows from the single-crossing assumption. If $g^m > g^n$, and $U_P(g^m, \lambda_m) = U_P(g^n, \lambda_n)$, then rich people must prefer jurisdiction n strictly, so that $\lambda^m = 0$. •

Lemma 2 *There cannot be an equilibrium in which jurisdiction n has positive population, and in which $\bar{\lambda} < \lambda^n < 1$.*

Proof Suppose that $L^n > 0$ and that $\bar{\lambda} < \lambda^n < 1$.

Lemma ?? implies that no jurisdiction other than jurisdiction n has positive numbers of both people. If $\lambda^n > \bar{\lambda}$ then some poor people do not reside in jurisdiction n . So that implies that in the population distribution corresponding to the given local output levels some other jurisdiction (call it P) attracts some poor people (and no rich people).

If $g^P \neq g_P^*(0)$ then a new entrant could offer the local public output level $g_P^*(0)$ and attract all the poor people (who would get strictly higher utility). So if the menu of local public outputs is an equilibrium, then some jurisdiction provides the local public output level $g_1^*(0)$, and has some poor (but no rich) residents.

That means that $U_P(g^n, \lambda^n) = U_P(g_1^*(0), 0)$, since some poor people choose to live in each jurisdiction.

Now suppose that some new city manager enters, and offers the local public output level g_1^E , defined above as the output level for which $U_P(g_1^*(0), 0) = U_P(g_1^E, 1)$. Therefore, $U_P(g_1^E, 1)$ also equals $U_P(g^n, \lambda^n)$.

The single-crossing property then implies that $U_P(g_1^E, 1) > U_P(g^n, \lambda)$, for any $\lambda \leq \lambda^n$.

Therefore there exists a population distribution consistent with the menu $(g^n, g_1^*(0), g_1^E)$ for which all poor people choose the jurisdiction providing $g_1^*(0)$ and all rich people choose the jurisdiction providing g_1^E . This distribution is not Pareto dominated by any other consistent distribution for these three output levels.

So either a new city manager can enter, providing g_1^E , and attracting a positive number of rich people, or an existing jurisdiction already is providing g_1^E (or something better). In the first case, the menu is not an equilibrium; in the second case the assumption that jurisdiction n had positive population and $1 > \lambda^n > \bar{\lambda}$ is violated. •

Lemma 3 *There cannot be an equilibrium in which jurisdiction n has positive population, and in which $\bar{\lambda} > \lambda^n > 0$.*

Proof Suppose that $L^n > 0$ and that $\bar{\lambda} > \lambda^n > 1$.

Lemma ?? implies that no jurisdiction other than jurisdiction n has positive numbers of both people. If $\lambda^n < \bar{\lambda}$ then not all rich people reside in jurisdiction n . So that implies that in the population distribution corresponding to the given local output levels there is some other jurisdiction (call it R) which attracts some rich people (and no poor people).

The rich people must be indifferent between jurisdictions n and R .

Now suppose that some new city manager enters, with a public output level g' which is very close to g^n , close enough that

$$U_R(g', \bar{\lambda}) > U_R(g^n, \lambda^n).$$

Such a public output level must exist, because of the continuity of preferences, because preferences are monotonic in λ , and because $\lambda^n < \bar{\lambda}$. Furthermore, if g' is sufficiently close to g^n , it will be true that

$$U_P(g', \bar{\lambda}) > U_P(g^n, \lambda^n).$$

So a population distribution exists which is consistent with the new menu (after the entry of the new city manager) in which the new entrant attracts all the people (of all income classes): just assign the existing jurisdictions the same level of λ^m which they had in the previous population distribution (before the entry). With this assignment, people of both classes prefer strictly

the new entrant's jurisdiction to any of the existing jurisdictions. Therefore, the new population distribution corresponds to the new menu of local public output levels, and the new entrant attracts a positive population, proving the lemma. •

The lemmata above imply that in any equilibrium, at most three distinct jurisdictions will be populated (if we treat two or more jurisdictions providing the same level of public output as equivalent to a single jurisdiction doing so).

But the nature of equilibrium can be reduced further.

Proposition 1 *An equilibrium can take one of two forms. It can be a pooling equilibrium, with all people living in identical jurisdictions in which $\lambda = \bar{\lambda}$. Or it can be a separating equilibrium in which all rich people live in (identical) homogeneous jurisdictions and all poor people live in (identical) homogeneous jurisdictions.*

Proof Given the previous lemmata, what must be ruled out is an equilibrium in which $L^1 > 0, \lambda^1 = 0, L^2 > 0, \lambda^2 = \bar{\lambda}$ and $L^3 > 0, \lambda^3 = 1$.

So suppose that there is a menu in which the population distribution is the one just described.

Since poor people reside in jurisdictions 1 and 2,

$$U_P(g^1, 0) = U_P(g^2, \bar{\lambda}).$$

And since rich people reside in jurisdictions 2 and 3,

$$U_R(g^3, 1) = U_R(g^2, \bar{\lambda}).$$

Single crossing therefore requires that $g^3 > g^2 > g^1$.

If the distribution corresponds to an equilibrium, no city manager can enter and attract away the poor people by providing their most-preferred level of public output, so that

$$g^1 = g_P^*(0).$$

Single-crossing also requires that

$$U_P(g^3, 1) < U_P(g^2, \bar{\lambda}) = U_P(g_P^*(0), 0),$$

so that $g^3 > g_1^E$.

Notice as well that g^3 cannot equal $g_R^*(1)$, since monotonicity of preferences in λ imply that $U_R(g_R^*(1), 1) > U_R(g^2, \bar{\lambda})$.

Suppose then that a new city manager entered, providing a local public good level of $g^4 = \max(g_1^E, g_R^*(1))$. Then the population distribution in which all poor people move to jurisdiction 1, and all rich people move to jurisdiction 4 corresponds to the new menu (g^1, g^2, g^3, g^4) . (This population allocation cannot be dominated since it is constrained Pareto optimal.¹¹)

So the menu and corresponding population distribution cannot be an equilibrium, since the new entrant, jurisdiction 4, attracts a positive population in the corresponding distribution. •

8.3 Separating Equilibrium

A separating equilibrium exists if

$$U_R(g_R^*(\bar{\lambda}), \bar{\lambda}) < U_R(g_1^S, 1). \quad (3)$$

Proposition 2 *If $U_R(g_R^*(\bar{\lambda}), \bar{\lambda}) < U_R(g_1^S, 1)$, then the only equilibria are separating equilibria, in which all rich people live in jurisdictions providing public output level g_1^S , and all poor people live in jurisdictions providing public output level $g_1^*(0)$. (And such separating equilibria must exist.)*

Proof If the “right” levels of output are not being provided, then a new entrant can create two new jurisdictions, one providing the public output level $g_P^*(0)$ and the other providing the public output level g_1^S .

To demonstrate the “only” part of the Proposition, it suffices to show that for some population distribution corresponding to $(g^1, g^2, \dots, g^N, g_P^*(0), g_1^S)$ all the rich people choose the jurisdiction providing g_1^S and all the poor people choose the jurisdiction providing $g_P^*(0)$ — for any public output levels (g^1, g^2, \dots, g^N) .

Set $\lambda^n = 0$ for each $i = 1, 2, \dots, N$ with $g < g_1^E$, and $\lambda^n = 1$ for (only) those existing jurisdictions for which $g > g_1^E$, which is consistent with the definition of a consistent population distribution, provided $L^n = 0$. Since $U_P(g_P^*(0), 0) \geq U_P(g, 0)$ for any g , and since $U_P(g_P^*(0), 0) \geq U_P(g_1^S, 1) >$

¹¹See the appendix for a definition of constrained Pareto optimality, and proof that the separating distribution just described is constrained Pareto optimal

$U_P(g', 1)$ for any $g' > g_1^E$ by definition, the poor will be willing to locate in the jurisdiction to which they have been allocated.

Single-crossing implies that no g satisfies $U_R(g_1^E, 1) < U_R(g, 0)$: if there were such a g then the two types' indifference curves through $(g_1^E, 1)$ must cross twice.

The population distribution just described cannot be Pareto-dominated by any feasible population distribution, since it is constrained Pareto optimal.

Therefore, any outcome — other than the proposed equilibrium — can be upset by a city manager entering and providing jurisdictions with $g_P^*(0)$ and g_1^S as output levels.

It remains to prove that an outcome in which all rich people live in the jurisdiction with g_1^S , and all poor people in the jurisdiction with $g_P^*(0)$, is not susceptible to entry.

The poor will never choose to live in the jurisdiction providing g_1^S (since they have their own jurisdiction). Therefore, after entry, the rich must attain utility of at least $U_R(g_1^S, 1)$. (This conclusion depends on the belief restriction, that requires λ to equal 1 in an unpopulated jurisdiction providing g_1^E or more of the public output.)

If the rich get utility of $U_R(g_1^S, 1)$ or more in some new jurisdiction n , then it must be true that $\lambda^n > \bar{\lambda}$, since condition (??) says that their indifference curve through $(g_1^S, 1)$ lies above the pooling line $\lambda = \bar{\lambda}$. If $\lambda^n > 0$, then some of the poor must reside in an all-poor jurisdiction, since all mixed jurisdictions have more rich people than the population average.

So (g^n, λ^n) must lie on the indifference curve through $(g_1^*(0), 0)$ of the poor people, since some poor people live in each jurisdiction. But single-crossing, and the definition of g_1^E , show that all points along this indifference curve offer a rich person lower utility than $U_R(g_1^S, 1)$, meaning the rich will locate only in “their” jurisdiction, which means that the rich cannot be induced to move by any new entry. That means that the poor all reside in jurisdictions in which $\lambda = 0$, which means that they will choose to stay in the best of those jurisdictions, the one in which $g_P^*(0)$ is provided.

Thus the separating equilibrium cannot be upset by any entry, completing the proof of the theorem. ●

8.4 Pooling Equilibrium

If condition (??) does not hold, then a separating equilibrium does not exist. This non-existence is again a straightforward analogue to the results

of Rothschild and Stiglitz. A city manager can enter, and provide the local output level $g_R^*(\bar{\lambda})$.

If the indifference curve of the rich people through $(g_1^S, 1)$ cuts the pooling line $\lambda = \bar{\lambda}$, then $U_R(g_R^*(\bar{\lambda}), \bar{\lambda}) > U_R(g_1^S, 1)$. Single-crossing, and the fact that $g_R^*(\bar{\lambda}) < g_1^E$, imply that $U_P(g_R^*(\bar{\lambda}), \bar{\lambda}) > U_P(g_1^E, 1) = U_P(g_P^*(0), 0)$ as well.

So a consistent population distribution exists in which all the mobile people, rich and poor, prefer strictly to move to the new entrant's jurisdiction.

Therefore the original outcome, in which all jurisdictions provided local outputs of $g_P^*(0)$ or g_1^S , and attracted only one income class, cannot be an equilibrium, since entry by a city manager offering a distinct public output level can attract positive population.

Proposition 3 *If $U_R(g_1^S, 1) < U_R(g_R^*(\bar{\lambda}), \bar{\lambda})$, then there exists an equilibrium in which all mobile people locate in a jurisdiction offering $(g_R^*(\bar{\lambda}), \bar{\lambda})$. Under these conditions, in any equilibrium all the mobile people locate in jurisdictions offering $(g_R^*(\bar{\lambda}), \bar{\lambda})$.*

Proof

We first show that the only possible equilibrium is the one described.

It suffices to show that, if $g^1 = g_R^*(\bar{\lambda})$ is one of the public output levels provided by city managers, then a population distribution corresponding to the menu of output levels is the one in which everyone locates in jurisdiction 1.

The hypotheses of the proposition imply that $U_R(g^1, \bar{\lambda}) > U_R(g, 1)$ for any $g \geq g_1^E$, and that $U^P(g^1, \bar{\lambda}) > U_P(g_1^E, 1) = U(g_1^*(0), 0) \geq U(g, 0)$ for any g .

As well, single-crossing implies that $U_i(g_R^*(\bar{\lambda}), \bar{\lambda}) > U_i(g, \bar{\lambda})$ for any $i \in \{P, R\}$ and any $g > g_R^*(\bar{\lambda})$.

So a population distribution in which jurisdiction 1 gets all the people, and in which all other jurisdictions in which $g \geq g_1^E$ are unpopulated and have $\lambda = 1$, all other jurisdictions in which $g_R^*(\bar{\lambda}) < g < g_1^E$ are unpopulated and have $\lambda = \bar{\lambda}$, and all jurisdictions in which $g < g_R^*(\bar{\lambda})$ have $\lambda = 0$ is consistent with any menu in which $g^1 = g_R^*(\bar{\lambda})$. This population distribution also cannot be Pareto dominated, since it is constrained Pareto optimal.

Therefore any other menu of public output levels (in which no jurisdiction has $g = g_R^*(\bar{\lambda})$) is susceptible to entry and cannot be an equilibrium.

To show that there is an equilibrium in which $g^1 = g_R^*(\bar{\lambda})$ and in which everyone locates in jurisdiction 1, consider the population distributions when new entrants offer some other levels of public output g^2, g^3, \dots, g^N .

Could the new distribution offer a higher utility to rich people (than the utility $U_R(g_R^*(\bar{\lambda}), \bar{\lambda})$ which they got in the original equilibrium)? If so, then the rich people must all live in jurisdictions in which $\lambda > \bar{\lambda}$, since the indifference curve through $(g_R^*(\bar{\lambda}), \bar{\lambda})$ is horizontal there.

But single crossing implies that all (g, λ) combinations which rich people prefer to $(g_R^*(\bar{\lambda}), \bar{\lambda})$ are preferred by poor people to $g_P^*(0), 0$.

So if the rich people get higher utility in the new distribution, then we must have the poor people all choosing to live in heterogeneous communities, so that everyone lives in a jurisdiction in which $\lambda > \bar{\lambda}$ which is impossible (since $\bar{\lambda}$ must be the weighted average of all inhabited jurisdictions' fraction of rich people).

Next, could the new distribution offer a higher utility to the poor people (than the utility $U_P(g_R^*(\bar{\lambda}), \bar{\lambda})$ which they got in the original equilibrium)? If so, the poor people must all reside in heterogeneous jurisdictions: $U_P(g_R^*(\bar{\lambda}), \bar{\lambda}) > U_P(g_P^*(0), 0)$ so that they must be worse off if any of them choose to live in exclusively poor jurisdictions.

Could jurisdiction 1 be inhabited exclusively by rich people? Then both groups would be better off than they were in the original distribution, which cannot happen, since the payoffs in the original distribution were constrained optimal.

What if jurisdiction 1 had positive numbers of both groups in the new equilibrium? Since only one (type of) jurisdiction can have positive numbers of both types in equilibrium, and since poor people do not live in homogeneous distributions, this would imply $\lambda^1 < \bar{\lambda}$ in the new distribution, since jurisdiction 1 would have all the poor people and only some of the rich people.

But this contradicts the assumption that the poor people have higher utility in the new equilibrium.

Hence the only possible distributions in which the poor are better off are those in which jurisdiction 1 is uninhabited. The indifference curve of the poor through $(g_R^*(\bar{\lambda}), \bar{\lambda})$ slopes up at $(g_R^*(\bar{\lambda}), \bar{\lambda})$, and is everywhere above the line $\lambda = 0$. So if the poor are better off in the new distribution, then they must reside in some mixed jurisdiction for which $0 < \lambda^n < \bar{\lambda}$ and for which $g^m < g_R^*(\bar{\lambda})$. But the restrictions on consistent population distributions say that if jurisdiction 1 is uninhabited, and $g^1 > g^m$, and $\lambda^m > 0$, then $\lambda^1 = 1$, which means that the rich would prefer strictly to reside in the existing jurisdiction 1 to jurisdiction m , contradicting the assumption that jurisdiction m has $\lambda^m > 0$.

Therefore there can be no new population distribution in which either

the rich or the poor are better off. Therefore, no new entry can yield a corresponding population distribution in which the new entrant's jurisdiction attracts any residents, establishing that the original pooling distribution is an equilibrium. •

8.5 Uniformity versus Decentralization

Proposition 4 *If the poor are in the national majority, then the poor are strictly better off under uniform centralization, and the rich are strictly better off under complete decentralization.*

Proof Under uniform centralization, with policy determined by the poor, the outcome is $g_P^*(\bar{\lambda}, \bar{\lambda})$. The assumption that each group's preferred public output level is unique, and the assumption that the rich prefer more public output than the poor, imply that $g_P(g_P^*(\bar{\lambda}), \bar{\lambda}) > g_P(g_R^*(\bar{\lambda}), \bar{\lambda})$. The assumption that utility increases with λ implies that $U_P(g_P^*(\bar{\lambda}), \bar{\lambda}) > U_P(g_P^*(0), 0)$, so that the poor do strictly better under uniform centralization than they do in a separating equilibrium or in a pooling equilibrium.

Under complete decentralization, the utility of the rich is either $U_R(g_R^*(\bar{\lambda}), \bar{\lambda})$ (in a pooling equilibrium), or else greater than or equal to $U_R(g_R^*(\bar{\lambda}), \bar{\lambda})$ (in a separating equilibrium). Since $U_R(g_R^*(\bar{\lambda}), \bar{\lambda}) > U_R(g_P^*(\bar{\lambda}), \bar{\lambda})$, the rich must be strictly better off under complete decentralization. •

If the rich are in a majority nationally, then they weakly prefer decentralization. With the rich in a majority, uniform centralization and the pooling equilibrium under complete decentralization give the same outcome. If there is a separating equilibrium, then the rich must prefer their outcome $(g_1^E, 1)$ strictly to the outcome $g_R^*(\bar{\lambda}, \bar{\lambda})$ of uniform centralization.

However, the conflict between rich and poor over the form of organization may be less clear if the rich are in a majority. If the outcome under complete separation is a pooling equilibrium, then there is no difference between uniform centralization and complete decentralization (if the rich are in a majority). But if the outcome under complete separation is a separating equilibrium, then the poor may be better or worse off than under uniform centralization.

Proposition 5 *If the rich are in the majority, they will prefer weakly complete decentralization to uniform centralization. The poor may be better, or worse off, in a separating equilibrium than under complete centralization with the rich in a majority.*

Proof What remains to be shown is that there are separating equilibria in which $U_P(g_P^*(0), 0) < U_P(g_R^*(\bar{\lambda}), \bar{\lambda})$ and separating equilibria in which $U_P(g_P^*(0), 0) > U_P(g_R^*(\bar{\lambda}), \bar{\lambda})$. Figure 5 illustrates the first case, figure 6 the second.

Underlying the figures are the following preferences. In each case, $U_R(g, \lambda) \equiv (48 - g)g + 500\lambda$. In figure 5, $U_P(g, \lambda) \equiv (36 - g)g + 144\lambda$, whereas in figure 6 $U_P(g, \lambda) \equiv (36 - g)g + 40\lambda$. Therefore $g_R^*(\lambda) = 24$ and $g_P^*(\lambda) = 18$ for any λ in either figure, $g_1^E = 30$ in Figure 5, $g_1^E = 24.3246$ in Figure 6, $\bar{\lambda} = 0.54$ in each case, and in each case $U_R(g_1^E, 1) > U_R(g_R^*(\bar{\lambda}), \bar{\lambda})$ so that the equilibrium under complete decentralization is separating. •

9 Appendix: Constrained Pareto Optimality

A menu of local public output levels (g^1, g^2, \dots, g^N) , and a population distribution consistent with it, are **constrained Pareto optimal** if there is no other menu of output levels and consistent population distribution which makes one income class better off without making the other income class worse off.

The use of this concept here is that, if a menu of output levels, and a population distribution consistent with it are constrained Pareto optimal, then the population distribution must correspond to the menu of output levels: by definition, no other consistent population distribution can Pareto dominate it.

Lemma 4 *If condition (??) holds, then the candidate separating equilibrium, in which all the rich people live in jurisdictions providing g_1^S and all the poor people lives in jurisdictions providing $g_0^*(0)$, must be constrained Pareto optimal.*

Proof First, the case in which the selection constraint is not binding, $g_R^*(1) = g_1^S > g_1^E$ is easy: the highest possible utility for the rich to get, even ignoring the location decisions of the poor, is the utility $U_R(g_R^*(1), 1)$ which they get in this situation.

Suppose then that $g_1^S = g_1^E \geq g_R^*(1)$, so the selection constraint is binding.

If some new menu and population increased the utility of the poor, then the poor must all live in mixed jurisdictions in the new situation: previously the utility $U_P(g_P^*(0), 0)$ was the highest possible for them in a poor-only jurisdiction. So making the poor strictly better off means locating them in jurisdictions in which $\lambda > 0$, and which are above their (dashed blue) indifference curve through $(g_1^*(1), 1)$.

But some of the poor must live in jurisdictions in which $\lambda \leq \bar{\lambda}$, the population average. And condition (??) implies the rich are made strictly worse off (from the initial separating situation) if they must locate in a jurisdiction in which $\lambda \leq \bar{\lambda}$. (The rich people's (solid red) indifference curve through $(g_R^*(1), 1)$ in figure 1 lies strictly above the pooling line.) So it is impossible to make the poor better off without making the rich worse off.

And any new situation which makes the rich better off must make the poor better off: if a (g, λ) combination is above the rich people's (solid red) indifference curve in figure 1, it must be above the poor people's indifference curve.

So there is no way of making one group better off than they are in the candidate separating equilibrium without making the other group worse off.

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Lemma 5 *If condition (??) does not hold, then the candidate pooling equilibrium, in which everyone lives in a jurisdiction providing a public output level of $g_R^*(\bar{\lambda})$, must be constrained Pareto optimal.*

Proof Note that in these circumstances (as in figure 2), the poor get higher utility in the candidate equilibrium than they would in any situation in which some of them lived in a homogeneous community: the indifference curve of the poor through $(g_R^*(\bar{\lambda}), \bar{\lambda})$ must lie above the line $\lambda = 0$ if condition (??) does not hold.

So making the poor strictly better off again implies that all the poor have to live in mixed communities. And adding-up then implies that some populated jurisdiction in the new situation has $\lambda \leq \bar{\lambda}$. And that means that the rich must be strictly worse off, as there is no point in figure 2 which is on or below the pooling line $\lambda = \bar{\lambda}$ which gives the rich a utility level as high as $U_R(g_R^*(\bar{\lambda}), \bar{\lambda})$, except the original candidate pooling equilibrium itself.

So there is no way of making the poor better off without making the rich strictly worse off.

Making the rich strictly better off means giving the rich a (g, λ) combination which is above their (solid red) indifference curve through $(g_R^*(\bar{\lambda}), \bar{\lambda})$ in figure 2. But that means that the poor must get strictly more utility than they would in any homogeneous poor-only community. So all the poor would have to live in mixed jurisdictions. And it was just demonstrated (in the first part of the proof) that putting all the poor in mixed jurisdictions must make the rich worse off.

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Figure 1 : Separating Equilibrium

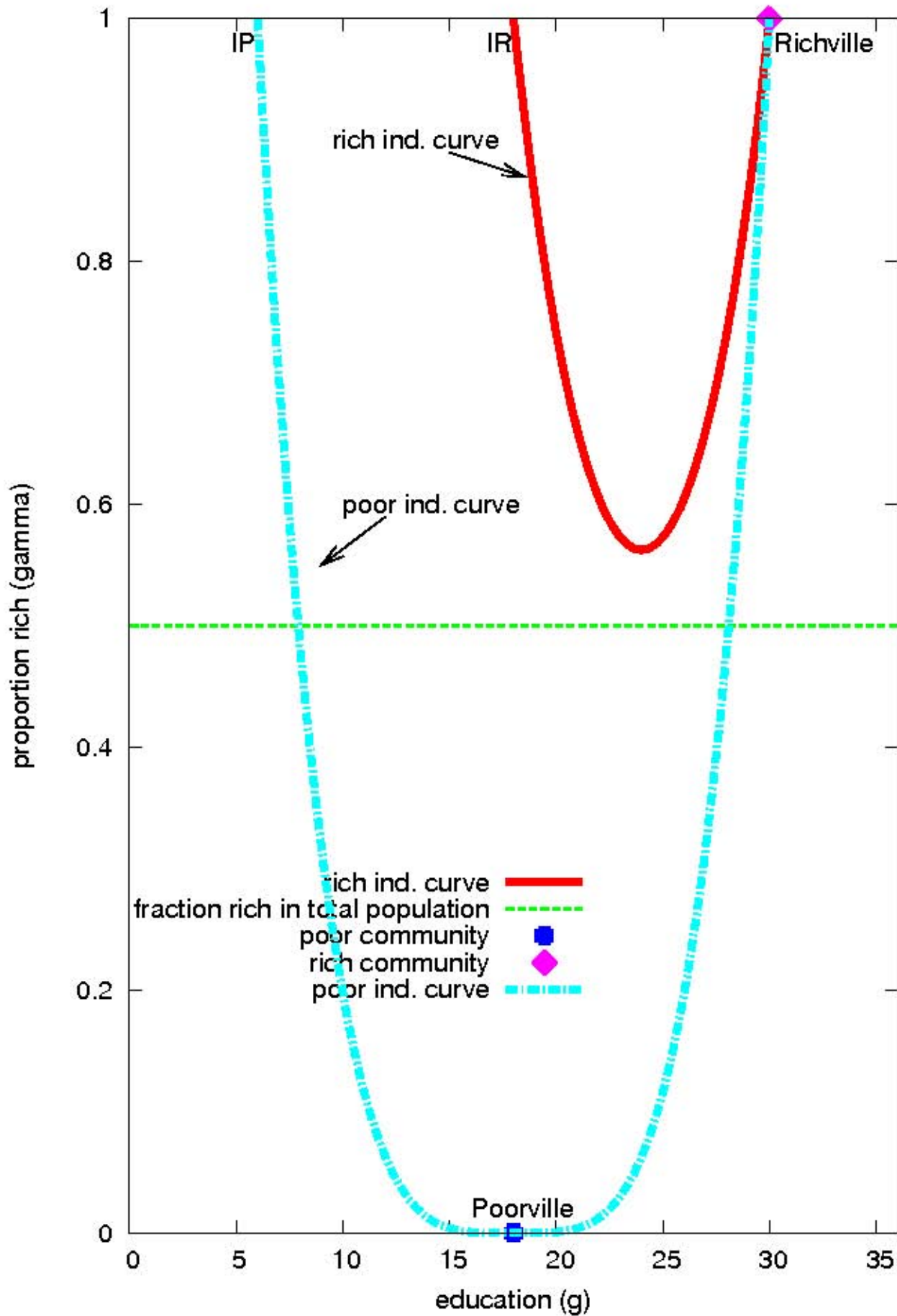


Figure 2 : Pooling Equilibrium

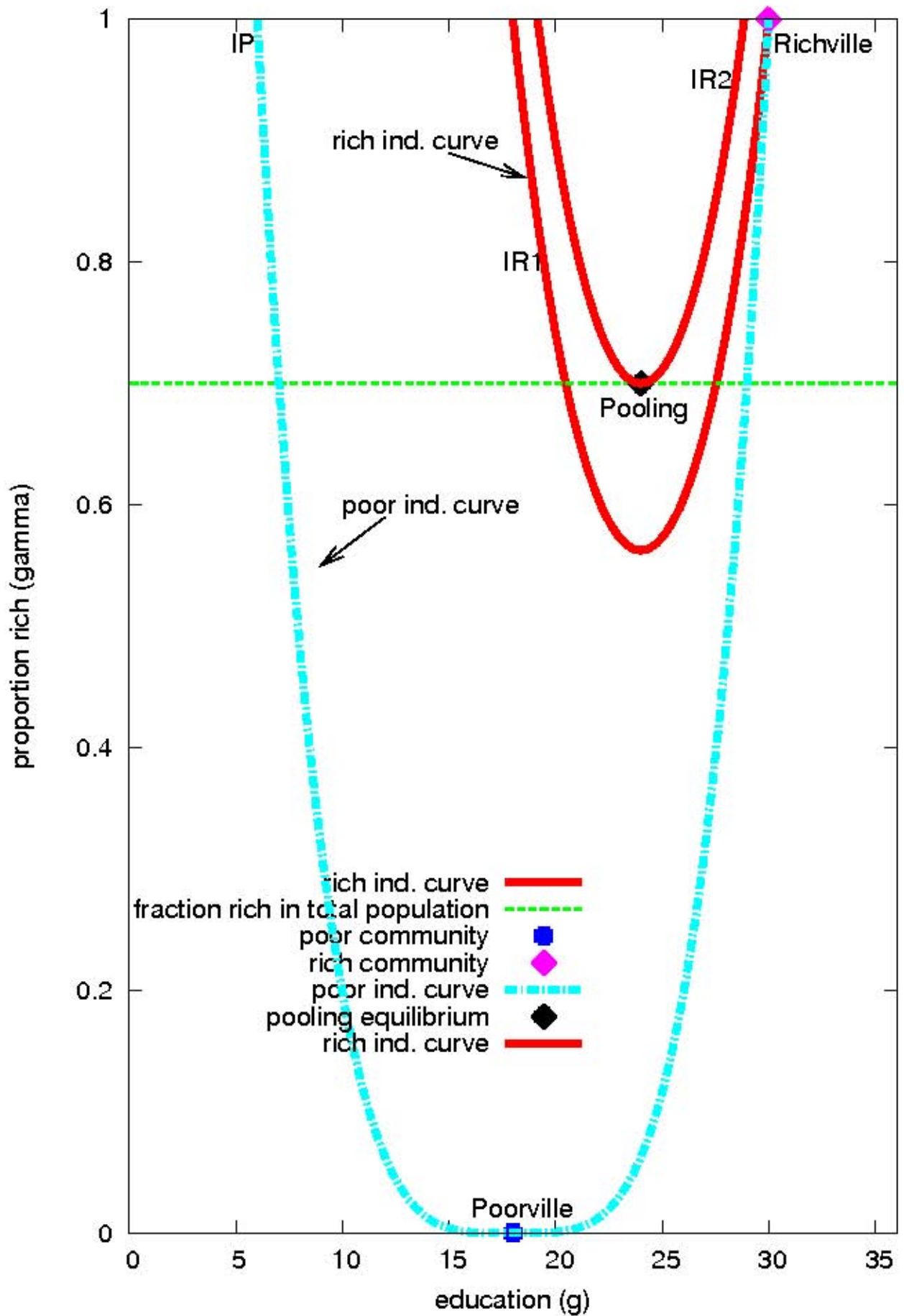


Figure 3 : Spending Limits Help No-one

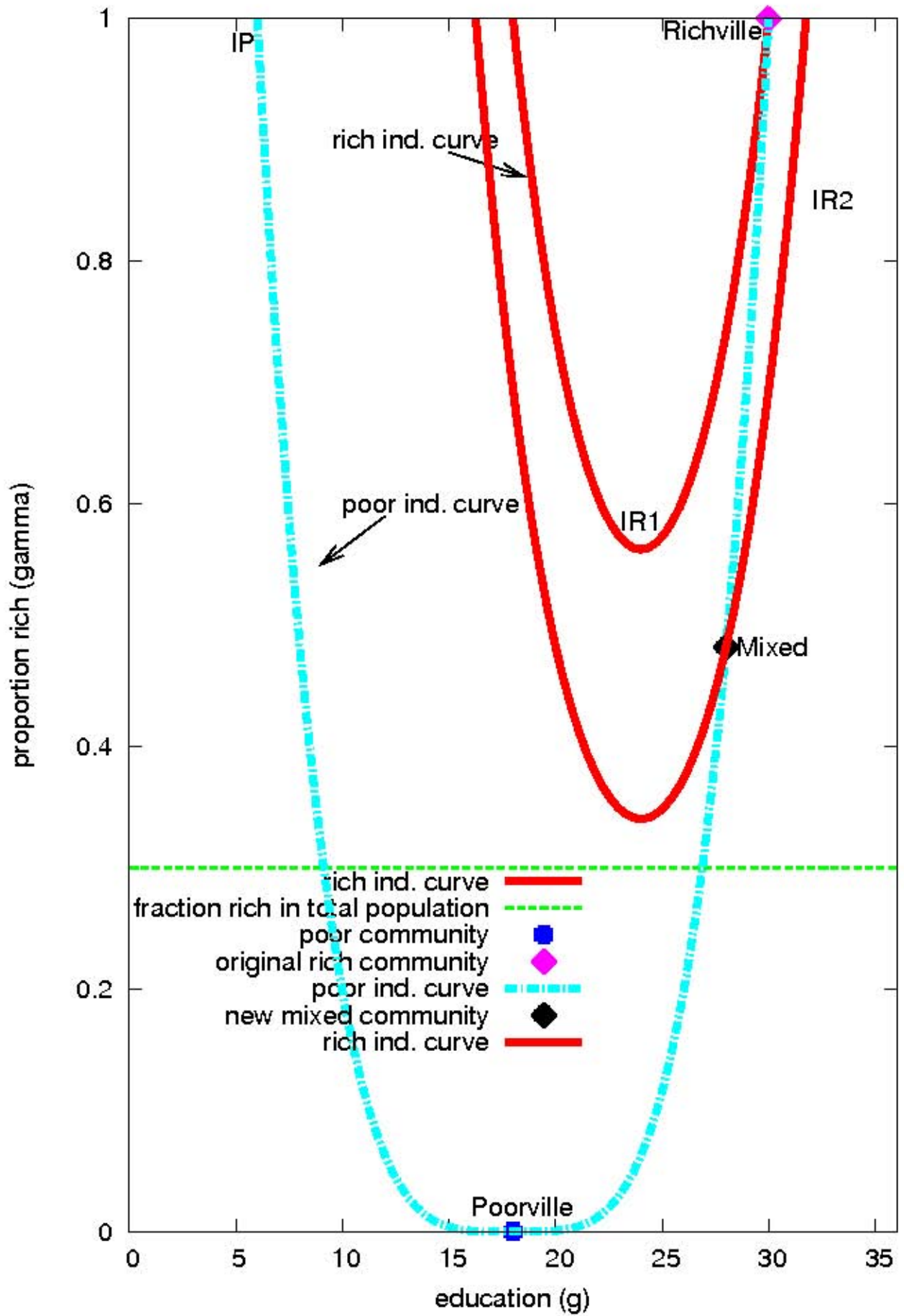


Figure 4 : Spending Limits Start to Help Poor

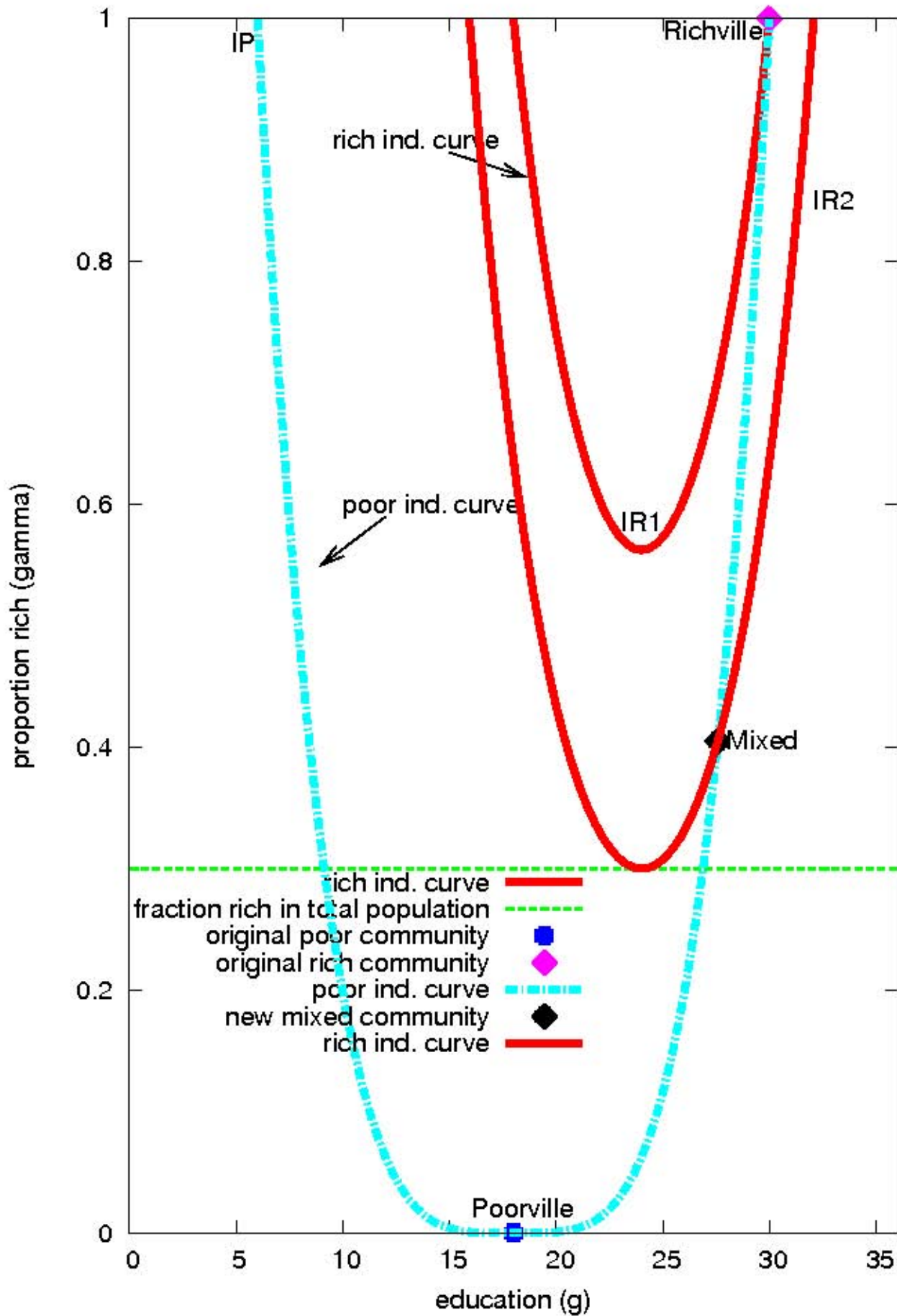


Figure 5 : Poor Prefer Uniform Provision

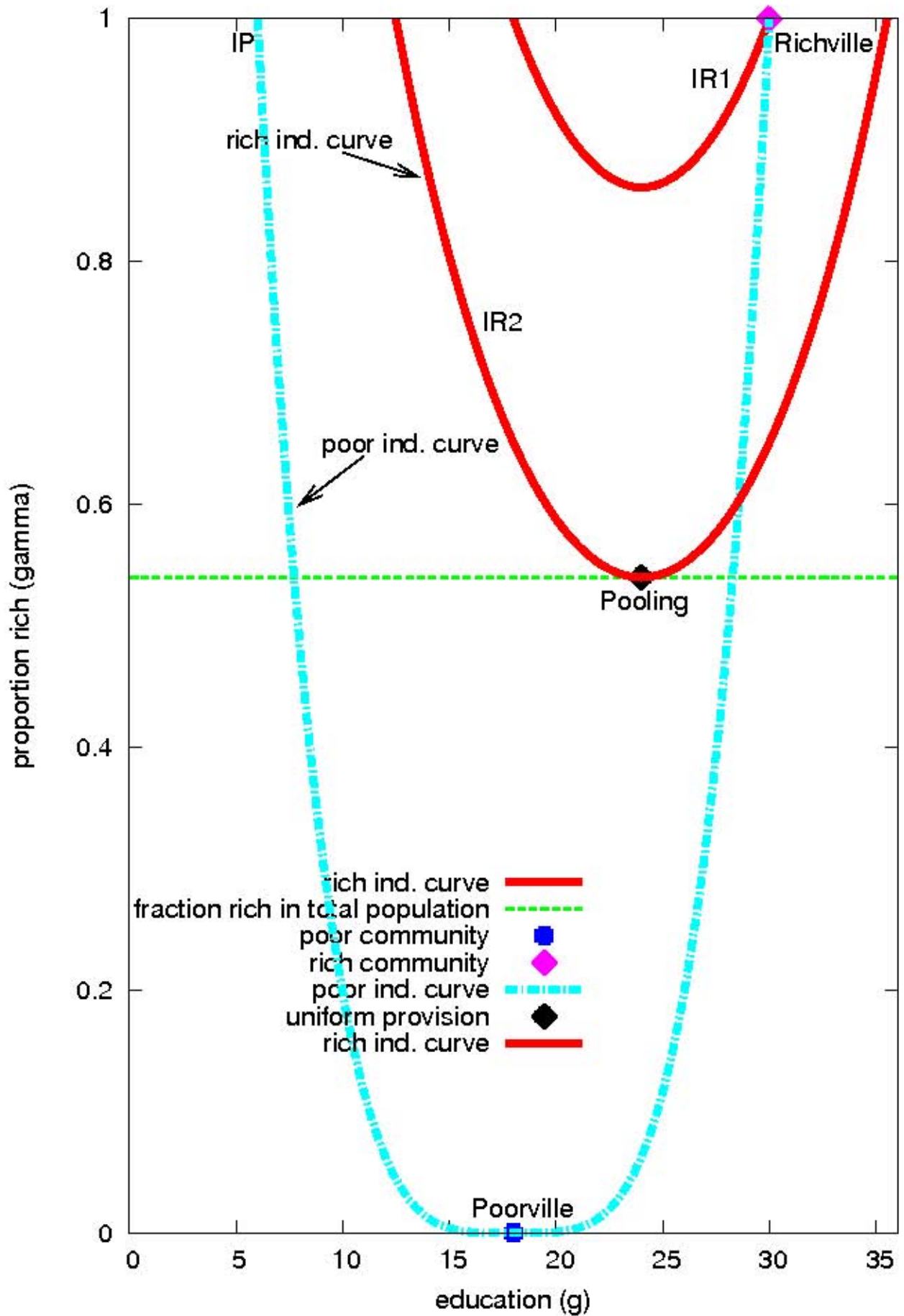


Figure 6 : Poor Prefer Decentralization

