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### Family background, academic ability and associated inequality of opportunity in India

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#### Abstract

The paper adds to the existing work on Inequality of Opportunity in India and using data and techniques proposed in earlier studies, estimates inequality of opportunity in academic ability of Indian males. Taking data from India Human Development Survey (2004-05) and mean log deviation as the inequality measure, the overall observed inequality in academic ability has been decomposed into two components. One of which can be associated to family background (inequality of opportunity) of individuals and another one due to all other factors. The paper finds substantial level of inequality of opportunity in academic ability in India, with the figures relatively higher for the urban regions compared to the rural ones.

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## 1. Introduction

If the recent literature on inequality is analyzed, it can be observed that an increasing number of researchers are focusing on inequality of opportunities rather than inequality of outcomes. Inequality of opportunities is associated with outcome differences that can be accounted by pre-determined circumstances which lie beyond the control of an individual, such as parental education, religion, gender etc. The idea of inequality due to efforts and inequality due to circumstances (inequality of opportunity) has been developed by a number of scholars.<sup>1</sup> It is important to discuss Roemer (2006, 1998 and 1993) because the formalization of the concept of unequal opportunities, suggesting that one should separate the determinants of a person's advantage (i.e., desirable outcomes, such academic ability) into circumstances and efforts was offered by him. The concept is motivated by two principles: the first one, also known as the principle of compensation, states that differences in individual achievements which can be unambiguously attributed to differences in factors beyond the individual responsibility are inequitable and have to be compensated by society. An individual's circumstances such as gender, religion and parental education are outside the control of the individual, for which s/he should not be held responsible. Inequalities due to differences in circumstances often reflect social exclusion arising from weaknesses of the existing systems of property and civil rights, and thus should be addressed through public policy interventions (Ali and Zhuang 2007). On the other hand, the second principle, commonly known as the principle of responsibility, advocates that differences in achievements which can be attributed to factors within the personal responsibility (inequalities due to individual efforts) are equitable and need not be compensated.

There are a number of studies which have obtained inequality of opportunity in earnings (consumption expenditure) of individuals in different country settings (Barros *et al.* 2009; Bourguignon *et al.* 2007; Checchi and Peragine 2010; Checchi, Peragine and Serlenga 2010; Cogneau *et al.* 2006; Ferreira and Gignoux 2008; Lefranc *et al.* 2008; Pistoiesi 2009; Singh 2010; Zhang and Eriksson 2009). There are also a few studies which have investigated inequality of opportunities in health status as well as health expenditures of individuals (Dias 2009; Trannoy *et al.* 2010).<sup>2</sup> But, research on inequality of opportunities in academic ability of individuals is rather missing. Moreover, majority of the above listed studies are either based on Latin America or European countries.

If India which is one of the most diverse nations (given its social division based on caste and religion and regional diversity based on regions and languages) is considered, then studies on inequality of opportunity are absolutely rare. However, there is one study (Singh 2010) which has estimated inequality of opportunity in wage earnings of males (21 – 65 years age group) in urban areas of India. In Singh (2010), family background (circumstances) of an individual is captured by father's education which itself is measured as the number of years of formal schooling completed by the father of the individual. The individuals are grouped into four categories or "types" based on father's education. The overall inequality in wages is decomposed into within groups and between groups components using mean log deviation and the between group component of the overall wage inequality is then taken as inequality of opportunity. The results of Singh (2010) are interesting for reasons explained below: first, earlier studies (see

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<sup>1</sup> See Rawls (1971), Sen (1979), Dworkin (1981), Arneson (1989), Cohen (1989), Roemer (1993, 1998) and Dirk (1993) for theoretical background and a formal discussion.

<sup>2</sup> Please refer to Ferreira and Gignoux (2008) for details of the different approaches used for measuring inequality of opportunity.

Bourguignon et al. 2007, Checchi and Peragine 2005, Ferreira and Gignoux 2008, and larger literature on intergenerational mobility) clearly single out parental education as the single most influential circumstance variable as far as inequality of opportunity is concerned. Second, there is a consensus among demographers about the use of parental education as an appropriate variable for capturing family background as far as impact of family circumstances on an individual is concerned (see Davis-Kean 2005, and Eccles and Davis-Kean 2005). Finally, the survey on which Singh (2010) is based was conducted in 2004-05 and the individuals included in the sample are more than 20 years of age (i.e. born before 1984). Even if 18 years (as the lower limit) is taken as the age of fathers at the time of the birth of individuals, the fathers would have been born in or before 1965. The strong correlation between the caste status (or religion) of individuals and their educational attainment has been established beyond doubt in the literature on social inequalities in India (see Anitha 2000, Desai and Kulkarni 2008, Deshpande 2001, and Dreze and Sen 1995). The severity of social inequality in educational attainment increases as we go back in time. It can therefore safely be argued that in 1965 or before that, the educational attainment was lowest for individuals belonging to the historically disadvantaged castes (“Scheduled castes and Scheduled tribes”), followed by the individuals belonging to “Other Backward Castes” and highest for individuals belonging to “Other Castes” (forward castes which have been historically advantaged) categories (Singh 2010, p. 237). Similarly the educational attainment for individuals belonging to “hindu” religion (majority) was higher than those of “muslims” which are in minority in India. Given the above context, the study’s choice of father’s education as a variable to capture the effect of family background (circumstances) on wages seems justified.

The present paper builds on the above study and using the framework proposed in it, estimates inequality of opportunity in academic ability of males (in the same age group, 21-65 years) of urban as well as rural areas of India. The data for the analysis also comes from the same survey which has been used in Singh (2010). The academic ability of the individuals has been obtained in terms of total percentage of marks obtained by the individuals in the secondary school (standard tenth) examination.<sup>3</sup> As in Singh (2010), this study also uses father’s education (and same categorization of father’s completed years of schooling) as the measure of family background (circumstances) to capture inequality of opportunity in academic abilities. When it comes to educational attainment (academic abilities) and the factors influencing it, the role of parental education is well established in literature. The extant literature on intergenerational mobility and parental influence on children’s ability provides the evidence.<sup>4</sup> It would have been desirable to include both father’s as well as mother’s education for the analysis but due to the absence of information on mother’s education for the majority of individuals in the survey, it became impossible to include it in the study. Within this context, the paper finds substantial level of inequality of opportunity in academic ability of males in both urban as well as rural areas, with the figures considerably higher for urban areas than rural ones. The details of the data and methods which have been used in the analysis have been provided in the next section.

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<sup>3</sup> The information on the survey and other details has been provided in the next section.

<sup>4</sup> The start of prominent work on Intergenerational mobility can be traced back to Bowles (1972). Solon (1999) provides an excellent survey of studies based on intergenerational mobility. The association between parental education and children’s academic outcomes (educational attainment) has also been established in studies like Davis-Kean (2005), Eccles and Davis-Kean (2005) and Checchi and Flabbi (2007).

## 2. Data, Methods and Summary Statistics

The data for the present study has been taken from India Human Development Survey (IHDS), 2004-05, conducted by National Council of Applied Economic Research (NCAER), New Delhi, India, in collaboration with the University of Maryland. This is a micro unit recorded, nationally representative survey based on a stratified multistage sampling procedure. It covers 26,734 households (143,374 individuals) and 14,820 households (72,380 individuals) in rural and urban areas respectively. The survey contains information on a person's family background including parental education and occupation and other demographic details like age, sex, caste, religion etc. An additional advantage of the survey is the reporting of divisions obtained by the individuals in secondary (standard 10<sup>th</sup>) board examination. The divisions obtained are in three categories namely: third division (33% to less than 45%, aggregate of all subjects); second division (45% to less than 60%, aggregate of all subjects); and first division (60% and above, aggregate of all subjects). This categorization of the overall percentage of marks (aggregate of all subjects) is a standard practice in Indian education system. Also, secondary board examination is the first examination (in the schooling process) which is conducted on state or national level and individual schools don't have any say in it. Moreover, secondary school board examination has an increased importance in India because students are allocated different streams (example sciences, social sciences, commerce etc.) based on their choices and subject to obtaining some minimum percentage of marks which is required to join in a particular stream.

The paper uses the above described reporting of division obtained by individuals as the measure of their academic ability. Henceforth, the division obtained by an individual in his/her secondary board examination will be referred to as his/her academic score. For the purpose of analysis, the academic scores have been recoded into following four categories: 1 – didn't complete secondary school; 2 – passed secondary school with 3<sup>rd</sup> division; 3 – passed secondary school with 2<sup>nd</sup> division; and 4 – passed secondary school with 1<sup>st</sup> division.<sup>5</sup>

Before proceeding to the details of the framework used to estimate the opportunity share of inequality in academic ability, it is important to mention that the study is limited to individuals between the age group 21 to 65 years. The lower limit is taken as 21 years because it can be safely assumed that by this age an individual would have completed standard tenth unless s/he has not been able to pass the exam or didn't enroll in school or dropped out of the school due to other reasons. The study is further restricted to males only, because in the survey (in the eligible age group), the information on father's education is not available for 90% (94%) of the females residing in urban (rural) areas.

The analysis is carried out separately for urban and rural areas. This is because the schooling conditions (including kinds of schools) are very different in urban and rural areas. There are totally 18,302 males in urban and 32,692 males in rural samples respectively. The samples in urban and rural areas are further divided into different age based cohorts: 21 years (yrs) to 30yrs (first cohort), 31yrs to 40yrs (second cohort), 41yrs to 50yrs (third cohort) and 51yrs to 65yrs (fourth cohort). For each cohort of the two areas, the analysis has been performed separately. This allows not only to measure the role of inequality of opportunities in shaping the

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<sup>5</sup> There can be some heterogeneity across different state (or state and national) boards but in India they are considered as homogenous as far as marks obtained in different boards are considered. Therefore, we will assume that x% in one board is same as x% in another board. Also, please note that the divisions comprise of a wide range of percentage of marks, therefore the variation in division due to being in different boards will be low.

inequality of academic ability at a point in time, but also to study how this role may vary across cohorts.

Though this study has chosen father's education as the circumstance variable based on reasons already explained, but its suitability as a circumstance variable still needs to be verified from the data itself. One possible check is to look for whether the academic scores of individuals systematically increase with the increase in their father's education. Table 1 reports the tenth standard scores of individuals by their father's education for urban and rural cohorts.

**Table 1**

Mean tenth standard scores of individuals by father's education for urban and rural cohorts (IHDS, 2004-05)

	Urban Cohorts					Rural Cohorts				
	First	Second	Third	Fourth	Total	First	Second	Third	Fourth	Total
Father's education										
No formal education	1.49	1.47	1.56	1.58	1.52	1.34	1.27	1.18	1.17	1.24
	0.89	0.90	0.97	0.99	0.94	0.76	0.68	0.57	0.57	0.66
	1658	1945	1657	1444	6704	5234	5492	4847	4724	20297
1-5 years of schooling	1.70	1.79	1.88	1.81	1.79	1.54	1.50	1.42	1.35	1.47
	1.01	1.07	1.09	1.07	1.06	0.90	0.85	0.81	0.78	0.85
	1159	1116	958	836	4069	2551	2055	1416	1299	7321
6-10 years of schooling	2.29	2.34	2.38	2.40	2.34	2.01	1.91	1.84	1.76	1.94
	1.12	1.12	1.11	1.16	1.12	1.05	1.03	1.03	1.00	1.04
	2049	1448	1027	766	5290	2391	1186	566	465	4608
> 10 years of schooling	3.02	3.01	2.93	3.12	3.01	2.51	2.48	2.38	2.43	2.49
	0.97	0.95	0.97	0.95	0.96	1.08	1.00	1.03	1.01	1.05
	1083	641	348	276	2348	607	252	97	61	1017
Total	2.08	1.98	1.97	1.95	2.00	1.60	1.44	1.30	1.26	1.43
	1.15	1.14	1.13	1.15	1.14	0.95	0.83	0.72	0.69	0.84
	5949	5150	3990	3322	18411	10783	8985	6926	6549	33243

Notes: 1. Academic Scores: 1- didn't complete secondary school; 2 – passed secondary school with 3<sup>rd</sup> division; 3 – passed secondary school with 2<sup>nd</sup> division; 4 – passed secondary school with 1<sup>st</sup> division.

2. First cohort: 21- 30 yrs; second cohort: 31 – 40 yrs; third cohort: 41 – 50 yrs; fourth cohort: 51 – 65 yrs.

3. First row – mean score; second row – standard deviation; third row – number of observations.

It can be seen from the table that for every cohort (each cohort should be seen separately) the mean academic score of individuals is increasing with the increase in the level of father's education.

Since, the present study uses the framework of Singh (2010) which itself is based on Checchi and Peragine (2010), only the main intuition is presented here (for greater details readers may refer to these studies). The approach is conceptually simple and is as follows. First, a suitable variable (father's education) related to circumstances exogenous to the individuals is identified. Then the sample is partitioned (in each age cohort) into groups or "cells," such that all individuals in any given cell have exactly the same set of circumstances. The resulting subgroups are known as "types." These "types" or cells (each containing the academic scores of individuals with similar circumstances) are then compared with one another. The difference in academic scores between cells can be attributed to inequality of opportunity, while the differences within cells can be considered as the result of effort or luck. To be precise, in this study, for each cohort, the sample is partitioned into four groups or cells based on father's education, that is, individuals whose fathers are uneducated (*type 1*), educated but up to primary school (0 – 5 years of schooling, *type 2*), educated more than primary but up to a maximum of secondary school (6 – 10 years of schooling, *type 3*) and educated more than secondary school (more than 10 years of schooling, *type 4*). Each cell or *type* contains the academic scores of the individuals belonging to that *type*. Then the overall inequality in academic scores is decomposed into within-*type* (within-group) and between-*type* (between-group) using mean log deviation (also used in Checchi and Peragine 2010 as well as Singh 2010). Mean log deviation is chosen because it is the only measure which satisfies six axioms or properties which comprise of the four standard axioms of (i) anonymity or symmetry; (ii) population replication or replication invariance; (iii) mean independence or scale invariance; (iv) Pigou -Dalton principle of transfers and the additional axioms of (v) additive subgroup decomposability and (vi) path independence. The additional properties of additive subgroup decomposability and path independence are particularly important for the present study. The additive subgroup decomposability is important because the study primarily decomposes the total academic scores inequality into within-group and between-group components. Since the interest is in between-group component, the property of path independence is also required in the sense that the decomposition must yield the same result or the decomposition is invariant to whether within group inequality is eliminated first and the between group component computed second, or the reverse.<sup>6</sup> The results of the decomposition of the overall academic scores inequality into within-group component and between-group component (inequality of opportunity) are presented in next section on "results and conclusion".<sup>7</sup>

### 3. Results and Conclusion

Table 2 reports the decomposition of overall academic ability inequality into inequality of opportunity and inequality due to efforts for the urban cohorts. The inequality of opportunity in academic ability ranges from 14% to 21% across different age cohorts. The simple average across cohorts is 18%.

<sup>6</sup> Refer to Ferreira and Gignoux (2008), Shorrocks (1980), Foster and Shneyerov (2000), and Shorrocks and Wan (2005) for a detailed discussion on these properties.

<sup>7</sup> For interested readers, the decomposition procedure using mean log deviation as the inequality measure has been presented in Appendix 1.

**Table 2**

Total observed academic ability inequality decomposition (between types and within types) by cohorts – Urban (using Mean Log Deviation)

	Opportunity inequality (Between – types)	Effort inequality (Within – types)	Total observed academic scores Inequality	Opportunity share of total observed academic scores inequality (%)
First Cohort (21yrs to 30 yrs)	0.034	0.130	0.164	20.7
Second Cohort (31yrs to 40 yrs)	0.033	0.134	0.167	19.9
Third Cohort (41yrs to 50 yrs)	0.024	0.141	0.165	14.3
Fourth Cohort (51yrs to 65 yrs)	0.026	0.145	0.171	15.2

If the results of inequality of opportunity in academic ability in urban areas are compared to the inequality of opportunity in wages in urban areas (11- 17% across same cohorts, simple average across cohorts being 15%) reported by Singh (2010), then it can be said that the inequality of opportunity in academic scores is more than the inequality of opportunity in wages.

Table 3 presents the findings for the rural regions and it can be observed that for every cohort, the inequality of opportunity in academic ability is lower in rural areas than the corresponding cohort in the urban areas.

**Table 3**

Total observed academic scores inequality decomposition (between types and within types) by cohorts – Rural (using Mean Log Deviation)

	Opportunity inequality (Between – types)	Effort inequality (Within – types)	Total observed academic scores Inequality	Opportunity share of total observed academic scores inequality (%)
First Cohort (21yrs to 30 yrs)	0.021	0.121	0.141	14.6
Second Cohort (31yrs to 40 yrs)	0.016	0.104	0.120	13.2
Third Cohort (41yrs to 50 yrs)	0.013	0.083	0.095	13.4
Fourth Cohort (51yrs to 65 yrs)	0.009	0.079	0.088	10.6

The opportunity share of overall academic ability inequality in rural areas varies from 11% to 15% across different age cohorts. The simple average across cohorts is 13%. The possible reason for lower inequality of opportunity in rural areas may lie in the constraints in educational (and related) facilities in rural areas which limit the choices available to fathers regarding education decisions about their children, decisions which have the potential of affecting the academic abilities of their children. For example if parents take decisions on their children's schooling which may later affect their academic ability, then in the absence of choices in availability of schools, a father with ten years of schooling will be forced to send his/her child to the same (and only) village school where a father with five years of schooling is sending his/her child. Therefore higher father's education will not translate into better schooling for their children which in turn will not translate into better academic abilities.

Though the study performs the analysis separately for each cohort, it must be noted that the variation of inequality of opportunity estimates across cohorts should not be considered as variation over time. This is because they are measured at the same point in time. Moreover, it is impossible to disentangle period, age and cohort effects.

It can also be noted that the estimation may get affected by number of groups or *types* (4 in present case) as there is evidence in existing literature that between-group inequality increases with the number of groups. Combining this with the conservative choice of circumstance variables (only father's education) for the analysis, the analysis identifies the lower bound estimates of inequality of opportunity. In cases where the number of groups can increase (by finer division of *types* or increase of circumstance variables) the between group inequality (inequality of opportunity) might increase. A subdivision of groups into finer groups will not lower the between-group inequality and unless the subdivision is due to the addition of another circumstance variable which is orthogonal to the measure of outcome, will raise it.<sup>8</sup>

As a concluding remark, it can be mentioned that this study presents the first evidence of existence of substantial inequality of opportunity in academic ability as far as India is concerned. Given the very conservative choice of circumstance variables for the analysis, if other factors like caste, religion, gender and region of birth are also included, then the true opportunity share of the academic ability inequality may be even higher.

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<sup>8</sup> See Ferreira and Gignoux (2008, p.13); Barros *et al.* (2009, p.127); Checchi *et al.* (2010, p.12) and Shorrocks and Wan (2005) for a detailed discussion on the point.



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### Appendix 1

For every cohort, the decomposition of overall academic ability inequality into within-group and between-group (the groups or “types” based on father’s education) has been carried out separately using mean-log deviation. The exact decomposition procedure is as follows:

Let the index (mean log deviation) be represented by  $M$ , and suppose that the individuals (in any cohort),  $N$ , are partitioned into  $m$  proper subgroups  $N_k$  ( $k = 1, 2, \dots, m$ ) based on their father’s education, with respective academic score vectors  $y^k$ , mean scores  $\mu_k$ , population sizes  $n_k$ , and population shares  $v_k = \frac{n_k}{n}$ . Also, let  $\bar{y}^k$  denote the distribution obtained by replacing each score in the vector  $y^k$  with the subgroup mean,  $\mu_k$ . Then,

$$\begin{aligned} M(y) &= M(y^1, y^2, \dots, y^m) = \frac{1}{n} \sum_{k=1}^m \sum_{i \in N_k} \ln \frac{\mu}{y_i} \\ &= \sum_{k=1}^m \frac{n_k}{n} \frac{1}{n_k} \sum_{i \in N_k} \ln \frac{\mu_k}{y_i} + \frac{1}{n} \sum_{k=1}^m \sum_{i \in N_k} \ln \frac{\mu}{\mu_k} \\ &= \sum_{k=1}^m v_k M(y^k) + \sum_{k=1}^m v_k \ln \frac{\mu}{\mu_k} \\ &= W + B \end{aligned}$$

where  $W$  is the within group inequality and  $B$  represents the between group (inequality of opportunity) component.