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On the volatility-volume relationship in energy futures markets using intraday data

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Abstract

This paper investigates the relationship between trading volume and price volatility in the crude oil and natural gas futures markets when using high-frequency data. By regressing various realized volatility measures (with/without jumps) on trading volume and trading frequency, our results feature a contemporaneous and largely positive relationship. Furthermore, we test whether the volatility-volume relationship is symmetric for energy futures by considering positive and negative realized semivariance. We show that *(i)* an asymmetric volatility-volume relationship indeed exists, *(ii)* trading volume and trading frequency significantly affect negative and positive realized semivariance, and *(iii)* the information content of negative realized semivariance is higher than for positive realized semivariance.

JEL Classification: C15, C32, C53, G1, Q4

Keywords: Trading Volume; Price Volatility; Crude Oil Futures; Natural Gas Futures; High-Frequency Data; Realized Volatility; Bipower Variation; Median Realized Volatility; Realised Semivariance; Jump

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1 Introduction

Trading volume and price volatility are heavily studied because their behaviors matter for traders, speculators, and hedgers who need to extract information from these variables to predict future prices.[1] Markets are assumed to contain noise, whose source is the precision of information contained in prices. Since price alone does not allow traders to observe the information signal, or the true value of the asset, volume may yield sufficient additional information for that signal to be observed. From that perspective, different groups of agents are interested in the volatility-volume relationship in futures markets. Volume may be used to forecast future price movements because traders condition their expectations on volumes exchanged, as well as on the actual price level. On the one hand, hedgers typically engage in trading futures contracts to stabilize their future income flows or costs, with the trading volume determined by their expectations about future spot (and futures) price movements. On the other hand, speculators take a position in futures contracts based on their expectations of futures' price variability (Foster (1995)). Furthermore, policymakers follow closely their evolution to assess market activity, and to identify potential regulatory changes (Wang and Yau (2000)).

According to Karpoff (1987), the price-volume relationship constitutes a central question in finance because it relates to the rate of information flow to the market, how the information is disseminated, the extent to which market prices convey the information, the size of the market and the existence of short sales constraints.[2] Indeed, it is likely that simultaneous large volumes and large price changes (either positive or negative) may be traced to their common ties to information flows, or to a directing process that may be interpreted as the flow of information. Since changes in prices and trading volume are both driven by the same directing variable, namely the information flow, it is expected that they will exhibit positive correlation. Trading volume may be typically considered along three dimensions (Jones et al. (1994)): *trading volume* (*i.e.* the number of shares traded), *trading frequency* (*i.e.* the number of trades) and *trade size* (*i.e.* the number of shares per trade). In theory, the volatility-volume relationship may be driven by either one or several components. Hence, to capture the full dynamics behind the volatility-volume relationship, we need to consider the differential roles of these three components.

Previous empirical research has noted that there is generally a strong contemporaneous positive relationship between volume and price volatility in futures markets (Gallant et al. (1992)), despite the coexistence of several theoretical backgrounds (see Huang and Masulis (2003) for a review). On the one hand, based on the insight that larger-sized trades tend to be executed by better-informed investors, Easley et al. (1997) document a positive relationship between trading volume and price volatility. Chiang et al. (2010) provide strong evidence to support the sequential information hypothesis, and demonstrate that it is useful to use lagged values of trading volume to predict return volatility. On the other hand, assuming that informed investors engage in stealth trading by breaking up large trades into many smaller transactions, Kyle (1985) finds a positive relationship between trading frequency and volatility. These results are confirmed by Jones et al. (1994) and Huang and Masulis (2003), among others. Finally, models relying on the mixture of distributions hypothesis (MDH, Harris (1987))[3] ad-

[1] See the discussion in O'Hara (1995), section 6.2.

[2] The relatively large cost of taking a short position provides an explanation for the observation that, in equity markets, the volume associated with a price increase generally exceeds that with an equal price decrease, since costly short sales restrict some investors' abilities to trade on new information.

[3] The MDH for the joint distribution of price changes and volumes is based on two assumptions. First, the joint distribution of price changes and trading volume is bivariate normal conditional upon the arrival of information. Second, the daily number of information events is random, which implies that price increments are generated by the stochastic rate of information arrival.

vocate that trading frequency shall not affect price volatility, and that trading volume is the relevant factor reflecting the arrival of new information. Chen and Daigler (2008) show how these perspectives are complementary rather than competitive in nature, and help explaining the different aspects of the volatility-volume relationship as information passes from one group of agents to another.

In addition, price-volume relationships have significant implications for futures markets, *i.e.* how price volatility affects the volume of trades in futures contracts. Knowledge of price volatility is useful for estimating margin requirements and option prices. In general, the price variability of a futures contract indicates the riskiness of holding the commodity. This information may then be used to set the price in many energy futures contracts for the physical commodity. From that perspective, the volume of trades constitutes an important aspect in influencing price volatility: as it increases, the variability of daily price changes is also likely to increase, and hence both margin requirements and option prices are likely to increase. What concerns energy futures markets, Serletis (1991) and Foster (1995) have studied the volume-volatility relationship for crude oil futures contracts. Fujihara and Mougoué (1997) further confirm that the knowledge of current trading volume improves the ability to forecast petroleum futures prices. Herbert (1995) documents that the volume of trades explains the volatility of natural gas futures contracts.

To enrich the understanding of the trading dynamics behind the volatility-volume relationship in the crude oil and natural gas futures markets, we examine in this paper the role of trading volume and trading frequency on high-frequency measures of volatility. Namely, we investigate the presence of an *asymmetric* volatility-volume relationship in oil and gas futures markets by using high-frequency data. The main contribution is to consider downside *vs.* upside semivariance (Barndorff-Nielsen, Kinnebrock and Shephard, henceforth BNKS (2008)), and to discriminate between them. Broadly speaking, negative (positive) realised semivariance may be defined as measuring the variation of asset price falls (increases). To our best knowledge, its application to the volatility-volume relationship in the context of energy futures markets has not been explored to date. An asymmetric volatility-volume relationship implies that the relation is fundamentally different for positive and negative price changes, *i.e.* the correlations between volume and positive/negative price changes are expected to vary and to have distinct explanatory powers. This view is supported by the empirical observation that trading tends to be higher in bull markets than in bear markets (Foster (1995))[4]. However, this asymmetry is generally not present in futures markets.

The literature on the volatility-volume relationship using high-frequency data is still very sparse. The main idea behind using high-frequency data consists in re-examining the results of previous literature by using a more precise estimator of the unobserved volatility.[5] By constructing the realized volatility measure from the sum of intraday squared returns, Chan and Fong (2006) establish that the number of trades is the dominant factor behind the volatility-volume relation and that, beyond the trading volume or the number of trades, trade size adds very little explanatory power for realized volatility. Giot et al. (2010) further decompose realized volatility into two major components: a continuously

Since price changes and trading volume are assumed to react to information events, their total daily quantities is the cumulative sum of reactions to each news event. The implication of the MDH is that prices and volume have a joint response to information due to their common distribution (Foster (1995)).

[4]Note that explaining what accounts precisely for this asymmetric price-volume relationship goes beyond the scope of this paper. As mentioned by Karpoff (1987), if the key is short sale constraints, then futures market data would reveal no correlation between volumes and price changes. To the extent that organized option trading reduces the cost of taking net short positions, the asymmetry should also be attenuated in price and volume data from optionable securities.

[5]The interest of using realized measures is well illustrated by Avramov et al. (2006), who use the realized volatility estimator as a robustness check. They show that the explanatory power in their regression of the volatility on the volume is two times greater when using intraday data compared to daily data.

varying (persistent) component and a discontinuous (temporary) jump component based on bipower variation (Barndorff-Nielsen and Shephard, henceforth BNS, (2004,2006)). To do so, they distinguish between the *level* of volatility (*i.e.*, low *vs.* high volatility) and the *nature* of volatility (*i.e.*, continuous *vs.* discontinuous volatility). While previous literature has been focusing on the former aspect of the positive volatility-volume relationship, their study aims at characterizing the latter aspect by introducing the concept of jumps for the 100 largest stocks traded on the New York Stock Exchange (NYSE) from January 1995 to September 1999. Giot et al. (2010) find that the number of trades remains the dominant factor, whatever the volatility component considered, except for jumps which are not related in most cases to any trading activity variables.

The objective of this paper is to address the research question behind the volatility-volume relationship in the context of energy futures markets. We reconsider this issue, studied initially by Foster (1995) and Herbert (1995) for oil and gas respectively, by using high-frequency data to obtain a less noisy measure of volatility. To our best knowledge, this methodology has not been applied yet to energy futures markets. We follow Chan and Fong (2006) by using high-frequency data to investigate the relation between volatility and transaction data, such as trading volume and trading frequency. In addition, we provide new evidence based on filtering jumps from realized volatility measures with bipower variation (BPV, BNS (2004,2006), Andersen, Bollerslev and Diebold, henceforth ABD, (2007)) and median realized volatility (MedRV, Andersen, Dobrev and Schaumburg, henceforth ADS, (2011)) to properly account for the role of volume on the continuous component of volatility, as in Giot et al. (2010).

Our work contributes to the literature in a number of dimensions. First, we study crude oil and natural gas, the two most liquid energy markets in the world, with high-frequency data. Second, we establish that trading volume and trading frequency are significant and positive in explaining various realized volatility measures, which emphasize their central role in shaping the information flow. Overall, we find that the variables for trading volume and trading frequency share the same information content. Thus, they yield to the same qualitative results in the context of energy futures markets. We find that trade size has a limited additional impact compared to these variables. Third, to detect jumps from the continuous component of realized volatility, we use MedRV (in addition to BPV), which has the advantage of being robust to the occurrence of zero-returns and is not upward biased in empirical work. Fourth, we uncover an *asymmetric* volatility-volume relationship by using positive and negative realized semivariance (BNKS (2008)). We show that the volatility-volume relationship holds for this kind of measure of realized volatility. Interestingly, we emphasize that the information content of negative realized semivariance is higher than for positive realized semivariance.

The paper is organized as follows. Section 2 presents the data. Section 3 discusses the econometric approach. Section 4 contains the empirical results. Section 5 develops some robustness checks. Section 6 concludes.

2 Data

The sample for this study is based on the following energy futures contracts. For the New York Light Sweet Crude Oil Futures contract, the sample period starts on January 3, 2007 and ends on December

15, 2010 which is equal to 998 trading days (after cleaning).[6] For New York Natural Gas Futures contract, the sample period starts on September 28, 2006 and ends on January 15, 2010 which is equal to 810 trading days (after cleaning).[7] Both futures contract are traded on the New York Mercantile Exchange (NYMEX), which is now part of the CME Group. We build continuous time series by using front-month contracts, and by switching from one contract to the next as soon as the volume for the next month is higher than the volume for the present contract. These futures were selected primarily because they are the most actively traded futures in their own category, so the problem of infrequent trading is minimized. As an illustration, over the period of interest, the average number of daily ticks for our front-month continuous series is equal to 73,723 and 21,356 for oil and gas, respectively. Similarly, the average number of contracts traded is equal to 153,974 and 45,539.

Note that our data does not contain order imbalances, but this is not likely to be an issue since previous literature (Chan and Fong (2006), Avramov et al. (2006) and Giot et al. (2010) among others) has shown the limited additional explanatory power of order imbalances beyond that of trading activity variables for a wide range of equity and futures markets.

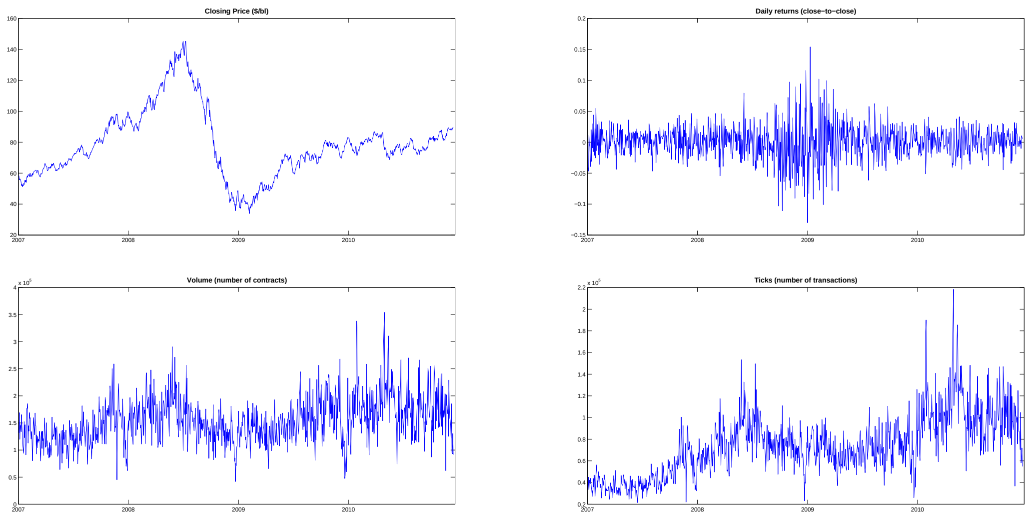

Figure 1
Closing price, returns, volume and ticks for the NYMEX Light Sweet Crude Oil Futures Contract (from top to bottom and left to right) from January 3, 2007 to December 15, 2010

Closing prices and returns for each contract are shown in the first row of Figures 1 and 2. The graphs illustrate several common patterns of crude oil and natural gas futures price behaviors. We notice a sharp break in each price series around mi-2008. This price pattern is explained by the downward revision of expectations about economic perspectives in fast-growing economies. Besides, both contracts exhibit volatility clustering in the series of returns, during the winter 2008/2009 for oil and near the

[6] As detailed in Zivot and Wang (2005), for the WTI futures contract we apply a first filter to remove: (1) transactions outside the official trading period, (2) transactions with a variation of more than 5% in absolute value compared to the previous transaction and (3) transactions not reported in chronological order. The filter has also been applied to the gas futures contract. Then, we apply an oil-specific filter to eliminate days with insufficient trading activity. Namely, we remove days with less than fifty four 5-minute returns, days with more than eight zero-return and days with less than 20,000 transactions. This procedure ensures that our realized volatility estimators are well-behaved, and that consistent estimators are obtained for the latent volatility which is the variable of interest.

[7] The gas-specific filter consists in removing days with less than sixty six 5-minute returns, more than ten zero-return and less than 6,000 transactions.

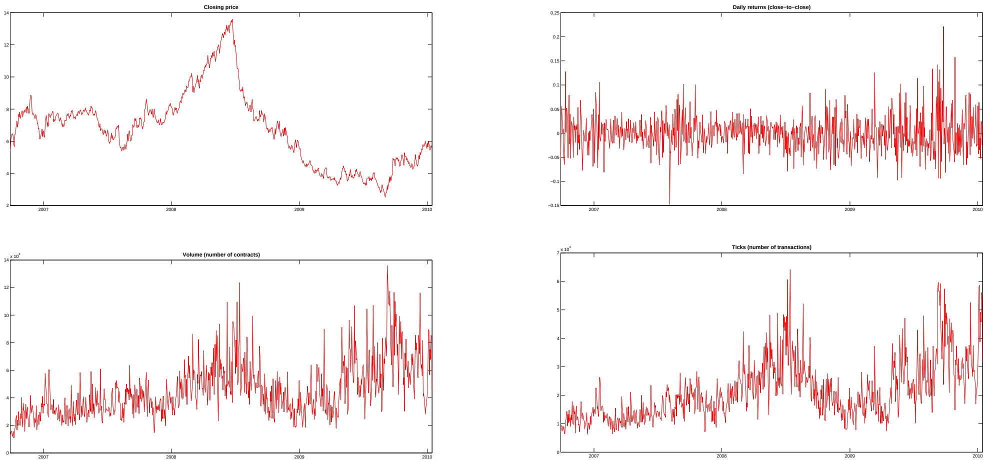

Figure 2
Closing price, returns, volume and ticks for the NYMEX Natural Gas Futures Contract (from top to bottom and left to right) from September 28, 2006 to January 15, 2010

end of the period for gas, which accounts for some but not all of the fat tail effect (or excess kurtosis) typically observed in financial data.[8]

In Figures 1 and 2, the second row presents daily trading volume[9] and daily trading frequency for front-month oil and gas contracts. The information reported here confirms that oil and gas record very high levels of trading volumes and trading frequency among energy futures markets. Indeed, we may notice a large variation in the trading activity for both contracts. This variability appears more pronounced for crude oil, which is reported to be more prone to speculation than gas. Note that the largest trading volumes do not systematically correspond to periods of volatility clustering.

In the top and middle panels of Figures 3 and 4, we may observe the realized volatility (RV) and bipower variation (BPV) measures estimated for each contract. Note that realized volatility is computed from open-to-close returns, and not from overnight returns. The main justification behind this modelling choice is that overnight returns follow different dynamics, and are rather difficult to reconcile with trading volume.

Some differences are remarkable between the two volatility series: while $RVOIL$ is characterized by a very high volatility during the winter 2008-2009, $RVGAS$ exhibits less frequent (but larger) spikes and the associated volatility clustering is less pronounced. The bottom panel of Figures 3 and 4 displays the jumps extracted by using the bipower variation measure detected at the 1% significance level. (see Eq. 4 below).

Table 1 reports summary statistics. We observe that the daily realized volatilities present nonzero skewness and excess kurtosis.[10] Table 2 reports the matrix of cross-correlations between endogenous and exogenous variables in Eq. (1) to (5). This table gives us an idea of the relationship between volatility and volume on each energy futures market.

[8] Volatility clustering, or persistence, suggests a time-series model in which successive disturbances, although uncorrelated, are nonetheless serially dependent.

[9] Note that trading volume may be de-trended by using the methodology in Gallant et al. (1992). We choose to follow Giot et al. (2010) by using raw trading volume, thereby not differencing between expected and unexpected returns.

[10] Note for a normally distributed random variable skewness is zero, and kurtosis is three.

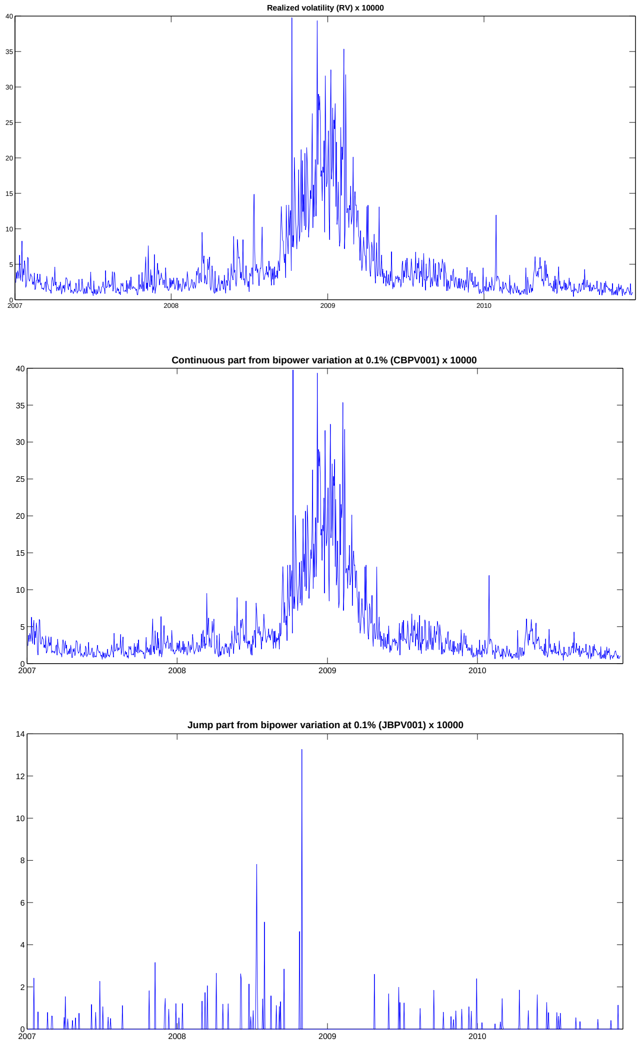

Figure 3
Realized volatility, bipower variation, and the jump component for the NYMEX Light Sweet Crude Oil Futures Contract (from top to bottom) from January 3, 2007 to December 15, 2010

3 Econometric approach

Our research question needs to tackle several methodological issues. First, estimating realized volatility faces the so-called problem of microstructure noise (MN). This phenomenon emerges from market microstructure problems, whose main examples are the existence of a bid-ask spread, non-synchronous trading, etc. When sampling data at a very high frequency, the MN could therefore strongly bias the estimates. To mitigate the MN issue, we provide an analysis of the optimal sampling frequency. Second, because we need to extract jumps for each energy futures contract, we need a jump-robust estimator of realized volatility to disentangle the continuous and jump components. Third, because the standard extraction of jumps keeps the jump component in its quadratic form, it is not possible to assess its sign without relying on deeper methodologies (see Andersen, Bollerslev and Huang, 2011). Thus, we need to consider the fact that jumps volumes are bounded from below, and that most daily

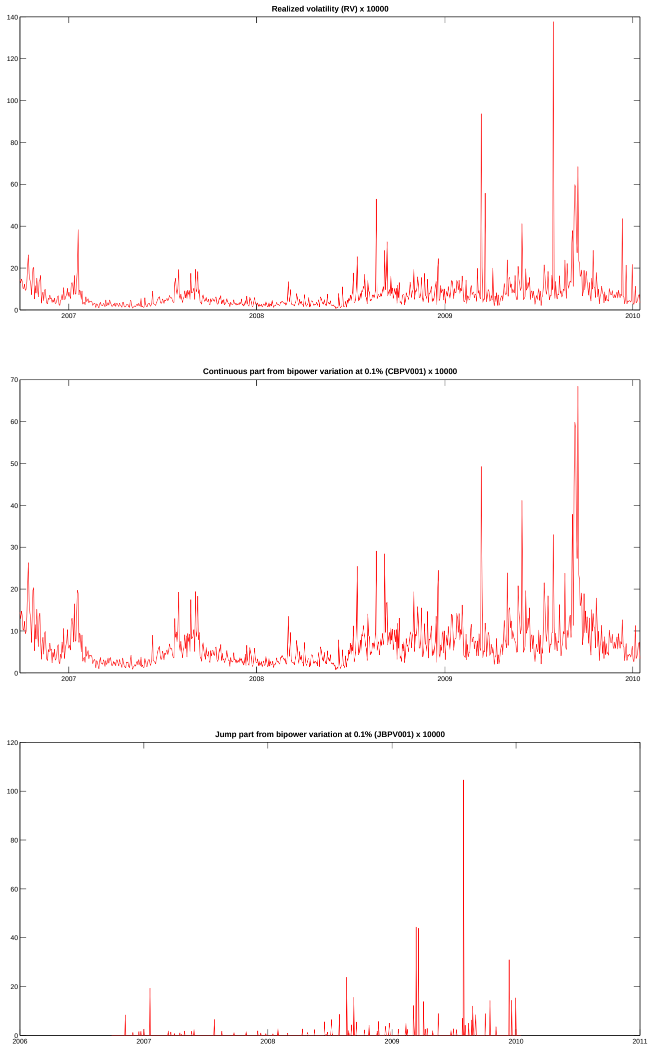

Figure 4
Realized volatility, bipower variation, and the jump component for the NYMEX Natural Gas Futures Contract (from top to bottom) from September 28, 2006 to January 15, 2010

jump estimates are equal to zero. This indicates a TOBIT specification for any regression using the jump component. Fourth, to distinguish between the variability emanating from negative or positive returns (and thereby considering the so-called semivariance concept), we use the new realised semivariance estimator proposed by BNKS (2008). We develop below these issues.

3.1 Optimal sampling frequency

One main problem when computing realized volatility measures is to choose the optimal sampling frequency in order to minimize the effects of microstructure noise. While 5-minute returns are usually retained as a rule-of-thumb for very liquid equities and FX markets, this issue has not been covered in details for energy futures markets like crude oil and natural gas. One way to look at this issue is to look at volatility signature plots, where the realized volatility measure is computed and plotted at

Summary Statistics for Crude Oil Futures

	RV	RS^-	RS^+	$BPV-C$	$BPV-J$	Vol	$Ticks$	ATS
Mean	4.3364	2.2227	2.1136	4.1705	0.1659	153974.4	73722.81	2.2376
Std. Dev.	5.2198	2.8371	2.7499	5.1605	0.7009	42531.00	28556.80	0.5762
Skewness	3.0717	3.0723	3.7675	3.1617	10.0554	0.6829	0.7384	0.9853
Kurtosis	14.1597	14.2048	20.8727	14.8611	150.6915	4.2294	4.1167	2.8640
Obs.	998	998	998	998	998	998	998	998

Summary Statistics for Natural Gas Futures

	RV	RS^-	RS^+	$BPV-C$	$BPV-J$	Vol	$Ticks$	ATS
Mean	7.6647	3.8418	3.8228	6.8470	0.8177	45539.13	21355.80	2.1939
Std. Dev.	8.9084	4.4334	5.9838	6.3219	4.8334	19878.69	10373.80	0.2896
Skewness	6.7476	5.1354	10.1498	4.1167	14.7580	1.1995	1.2370	0.3038
Kurtosis	75.9810	42.0785	148.7313	30.2538	282.3389	4.7264	4.6434	3.5091
Obs.	810	810	810	810	810	810	810	810

Table 1
Summary statistics

Note: RV stands for realized volatility, Vol for trading volume, $Ticks$ for trading frequency, and ATS for average trade size. $BPV-C$ stands for the continuous component of bipower variation, and $BPV-J$ for the jump component of bipower variation both extracted at the 1% level. RS^+ and RS^- stand for, respectively, the positive and negative realized semivariance.

Correlations for Crude Oil Futures

	RV	RS^-	RS^+	$BPV-C$	$BPV-J$	Vol	$Ticks$	ATS
RV	1							
RS^-	0.9363	1						
RS^+	0.9321	0.7456	1					
$BPV-C$	0.9909	0.9330	0.9183	1				
$BPV-J$	0.1512	0.1039	0.1799	0.0171	1			
Vol	-0.0146	-0.0170	-0.0102	-0.0261	0.0834	1		
$Ticks$	0.0969	0.0933	0.0876	0.0890	0.0662	0.8509	1	
ATS	-0.2509	-0.2435	-0.2250	-0.2512	-0.0184	-0.2311	-0.6686	1

Correlations for Natural Gas Futures

	RV	RS^-	RS^+	$BPV-C$	$BPV-J$	Vol	$Ticks$	ATS
RV	1							
RS^-	0.8002	1						
RS^+	0.8958	0.4504	1					
$BPV-C$	0.8519	0.7462	0.7154	1				
$BPV-J$	0.7287	0.4987	0.7153	0.2623	1			
Vol	0.4656	0.3905	0.4038	0.4943	0.2115	1		
$Ticks$	0.4315	0.3865	0.3559	0.4698	0.1808	0.9601	1	
ATS	-0.1297	-0.1772	-0.0618	-0.1810	-0.0022	-0.2087	-0.4377	1

Table 2
Correlations

Note: RV stands for realized volatility, Vol for trading volume, $Ticks$ for trading frequency, and ATS for average trade size. $BPV-C$ stands for the continuous component of bipower variation, and $BPV-J$ for the jump component of bipower variation both extracted at the 1% level. RS^+ and RS^- stand for, respectively, the positive and negative realized semivariance.

different sampling frequencies (Andersen, Bollerslev, Diebold and Labys, henceforth ABDL (2003), ABD (2007)). Figure 5 plots the volatility signature plots for oil and gas over the period of interest. We may observe visually that the 5-minute sampling interval appears as a reasonable choice to preserve the information in intraday data, while minimizing the impact of MN. This result is not surprising in light of the daily trading activity for both contracts.

In addition, we apply a rolling version of the ZT test by Awartani et al. (2009), which has been developed to detect statistically the optimal sampling frequency in presence of MN. Our results, available upon request, confirm statistically that the high liquidity of both energy futures contracts allows to

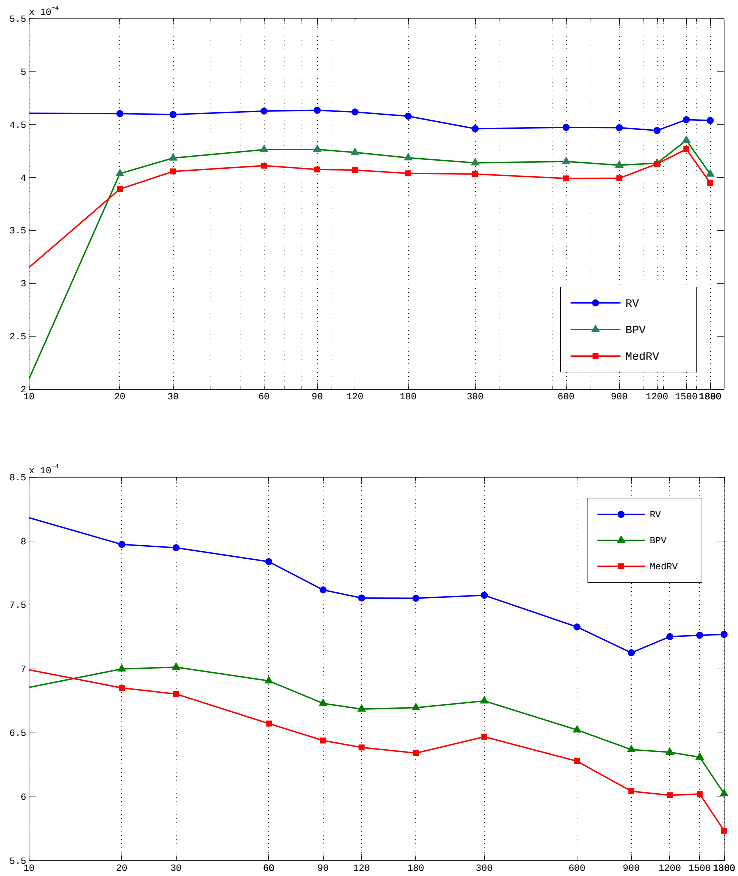

Figure 5
Volatility signature plot for the oil (top panel) and gas (bottom panel) futures contract using front month rollover and the realized volatility, bipower variation and median realized estimators (2006-2010).

sample every 5 minutes while limiting satisfactorily the impact of MN.

3.2 Extracting jumps

Jump detection is a statistical assessment of whether the difference between a jump-robust measure of realized volatility is statistically different from a ‘naive’ measure, such as RV. For the discretely sampled Δ-period returns denoted by $r_{t,\Delta} \equiv p_t - p_{t-\Delta}$, recall that the daily realized volatility is defined by the summation of the corresponding $1/\Delta$ high-frequency intraday squared returns (ABDL (2001), BNS (2002)):

$$RV_{t+1}(\Delta) \equiv \sum_{j=1}^{1/\Delta} r^2_{t+j.\Delta,\Delta} \tag{1}$$

where, without loss of generality, $1/\Delta$ denotes an integer.

Thus, we need two elements: *(i)* a jump-robust measure of realized volatility, and *(ii)* a test for the difference between this measure and the RV measure (and additionally a threshold for the test). We first follow the bulk of the literature by using BPV as developed by BNS (2004,2006):

$$BPV_{t+1}(\Delta) \equiv \mu_1^{-2} \sum_{j=2}^{1/\Delta} \mid r_{t+j.\Delta,\Delta} \mid\mid r_{t+(j-1).\Delta,\Delta} \mid \tag{2}$$

with $\mu_1 \equiv \sqrt{(2/\pi)} = E(Z)$ being the mean of the absolute value of a standard, normally distributed random variable Z.[11] The jump component may be evaluated as:

$$J_{t+1}^{BPV}(\Delta) \equiv max[RV_{t+1}(\Delta) - BPV_{t+1}(\Delta), 0] \tag{3}$$

where, to ensure the non-negativity of daily estimates, the actual empirical measurement is truncated at zero. Hence, an abnormally large value of this standardized difference between $RV_{t+1}(\Delta)$ and $BPV_{t+1}(\Delta)$ may be interpreted as evidence in favor of a significant jump over the $[t, t+1]$ time interval.[12]

Furthermore, this difference may be statistically tested in a more formal analysis. Huang and Tauchen (2005), BNS (2006) and ABD (2007) propose a ratio test statistic which is asymptotically normally distributed:

$$J(t,\Delta) = \frac{\left(1 - \frac{BPV_t(\Delta)}{RV_t(\Delta)}\right)}{\sqrt{(\mu_1^{-4} + 2\mu_1^{-2} - 5)\frac{1}{1/\Delta}\max(1, TQ_t(\Delta)/BPV_t^2(\Delta))}} \tag{4}$$

for day t and $1/\Delta$ the number of squared intraday returns.

Of course, the integrated quarticity that appears in the denominator needs to be estimated in order to implement this statistic. To this end, ABD (2007) define the standardized realized tripower quarticity as:

$$TQ_{t+1}(\Delta) = \Delta^{-1}\mu_{4/3}^{-3} \sum_{j=3}^{1/\Delta} |r_{t+j.\Delta,\Delta}|^{4/3} |r_{t+(j-1).\Delta,\Delta}|^{4/3} |r_{t+(j-2).\Delta,\Delta}|^{4/3} \tag{5}$$

where $\mu_{4/3} \equiv 2^{2/3} \cdot \Gamma(7/6) \cdot \Gamma(1/2)^{-1} = E(|Z|^{4/3})$.

Huang and Tauchen (2005) highlight the interesting small-sample properties of this ratio-statistic in comparison with more simple tests. By using this statistic, we are in position to assess the significance of the difference between BPV and RV, and thus the presence of a jump component in the RV measure. Once the presence of a jump component has been identified, the continuous component may be readily computed as the difference between the realized volatility and the jump component.

[11] Precisely, we use the staggered version of BPV to reinforce the robustness against MN (see BNS (2006)).

[12] While BNS (2004,2006) mainly consider the case of large significant jumps, their test is adapted to the detection of several jumps in a single day (see Andersen et al. (2010)).

3.3 Realised semivariance

BNKS (2008) define downside realised semivariances, noted RS^-, as follows:

$$RS^- = \sum_{j=1}^{t_j \leq 1} (r_{t_j} - r_{t_{j-1}})^2 1_{(r_{t_j} - r_{t_{j-1}} \leq 0)} \tag{6}$$

where $1_{(.)}$ is the indicator function taking the value 1 when the argument in r is true. BNKS (2008) advocate that RS^- provides a new source of information, which parallels the so-called semivariance widely used in investment theory.[13]

The corresponding upside realised semivariance, noted RS^+, may be written as:

$$RS^+ = \sum_{j=1}^{t_j \leq 1} (r_{t_j} - r_{t_{j-1}})^2 1_{(r_{t_j} - r_{t_{j-1}} \geq 0)} \tag{7}$$

with $RV = RS^- + RS^+$. Hence, the innovation lies in bringing high-frequency analysis to measure upside and downside risk. In what follows, we use realised semivariance as an additional instrument to test the volatility-volume relationship in energy futures data.

3.4 Modeling the volatility-volume relationship

We study the volatility-volume relationship for both oil and gas futures contracts. Following Chan and Fong (2006) and Giot et al. (2010), we estimate this relation by OLS with Newey-West standard errors considering that the errors could be autocorrelated and/or heteroscedastic.

We also include a dummy variable (Wednesday for oil, Thursday for gas) as well as various lags (twelve in all regressions, except for the jump component which is not serially correlated) to account for news impacts and for some dynamics in the conditional volatilities.

The general form of the regression of a given realized measure $RM = \{RV, BPV{-}C, BPV{-}J, RS^+, RS^-\}$ on a measure of trading activity $TA = \{Vol, Ticks, ATS\}$ reads as follows:

$$RM_{i,t} = \alpha_i + \delta_i DUMMY_{i,t} + \sum_{j=1}^{12} \beta_{ij} RM_{i,t-j} + \rho_i TA_{i,t} + \epsilon_{i,t} \tag{8}$$

where RV stands for realized volatility, $BPV - C$ for the continuous component of bipower variation, $BPV - J$ for the jump component of bipower variation (both extracted at the 1% level), RS^+ for positive realized semivariance, RS^- for negative realized semivariance, Vol for trading volume (the number of contracts), $Ticks$ for trading frequency (the number of trades), ATS for average trade size, and $\epsilon_{i,t}$ for the error term. All realized volatility measures have been defined previously in Section 3.

[13] In the same spirit, the interested reader may refer to Babsiria and Zakoian (2001) for applications to ARCH variance models using positive and negative daily returns, or to Chen and Ghysels (2011) for applications to news impact curves using semi-parametric MIDAS regressions.

To estimate the relationship between jumps and trading activity, we follow Giot et al. (2010) by running TOBIT regressions[14] and correcting for heteroskedasticity by allowing for GARCH effects.[15]

Furthermore, notice that, compared to previous literature, testing for positive and negative realized semivariance allows us to investigate the presence of an *asymmetric* volatility-volume relationship in energy futures markets.

For the sake of brevity, we do not consider here realized volatility computed with overnight returns, but it may be shown that they have no additional explanatory power.[16] Finally, note that we do not identify any disturbing near-maturity effect which should be considered in the modeling of the dynamics of the conditional volatility above.[17]

Next, we presents the results obtained with this econometric strategy.

4 Empirical results

The empirical results for oil and gas are reported in Tables 3 and 4. For all regressions, we report the coefficients and heteroskedasticity-robust (Newey-West) standard errors. Several diagnostic tests (such as the Ljung-Box-Pierce test) confirm that the residuals are not autocorrelated.[18]

The variables of primary interest are the coefficients on the impact of trading volume (Vol) and trading frequency ($Ticks$) on realized volatility measures. The results presented in Tables 3 and 4 demonstrate that in nearly all cases the variables are significant, thereby indicating a contemporaneous relationship between trading volume and volatility.

As in Giot et al. (2010), we select an AR(12) to take into account the strong autocorrelation (persistence) of the realized volatility measures[19]. Besides, we include a dummy variable on Wednesday for oil, and on Thursday for gas to take into account the impact of weekly news releases on volatility for each market. As is standard, the dummy variables are statistically significant in nearly all regressions.[20]

The first step of our estimation strategy consists in introducing separately trading volume and trading frequency as exogenous variables. In Table 3, regression (1) shows the positive ans statistically significant effect (at 1% level) of trading volume (Vol) on the realized volatility measure for the oil market. The same result may be noted for the gas market (Table 4, regression (16)). This first and important set of results show that there exists a statistical link between trading volume and volatility on both energy futures markets.

Moving to regression (2) (Table 3) for oil and regression (17) (Table 4) for gas, we observe the same kind of statistically significant effect (at 1% level) between trading frequency ($Ticks$) and realized volatility. This second set of results illustrates that, when taken in isolation, trading frequency also impacts positively and significantly (at the 1% level) realized volatility for oil and gas futures. However, the effect

[14] Indeed, the population distribution of jumps is spread over a large range of positive values, with a concentration around zero.
[15] We thereby allow for autocorrelation in the volatility of volatility (see Corsi et al., 2008).
[16] These results are available upon request to the authors.
[17] This is in line with the results in Duong and Kalev (2008) about the absence of a 'Samuelson effect' for the crude oil market in the NYMEX. As is well-known, oil is essentially a 'world' market, thereby explaining the absence of a maturity effect.
[18] These results are not reported here to conserve space, and may be obtained upon request.
[19] To conserve space, the coefficient estimates of the AR part are not shown. They are generally significant at common statistical thresholds.
[20] Similarly, these results are not shown here for the sake of brevity.

Oil		RV	
	(1)	(2)	(3)
Vol	0.000018***		0.000024***
	(0.000002)		(0.000003)
Ticks		0.000021***	-0.000010**
		(0.000003)	(0.000004)
Adj. R-Squ.	0.7596	0.7512	0.7603

Oil		BPV-C			BPV-J	
	(4)	(5)	(6)	(7)	(8)	(9)
Vol	0.000017***		0.000022***	0.000008***		0.000018***
	(0.000002)		(0.000003)	(0.000003)		(0.000005)
Ticks		0.000019***	-0.000008**		0.000005	-0.000017**
		(0.000003)	(0.000004)		(0.000004)	(0.000008)
Adj. R-Squ.	0.7669	0.7599	0.7673	NA	NA	NA

Oil		RS^+			RS^-	
	(10)	(11)	(12)	(13)	(14)	(15)
Vol	0.000009***		0.000011***	0.000010***		0.000013***
	(0.000001)		(0.000002)	(0.000001)		(0.000002)
Ticks		0.000010***	-0.000004*		0.000011***	-0.000005*
		(0.000002)	(0.000002)		(0.000002)	(0.000003)
Adj. R-Squ.	0.6088	0.6026	0.6092	0.7121	0.7040	0.7127

Table 3
Regression results for oil

Note: Standard errors in parenthesis. *** denotes significance at 1%, ** at 5% and * at 10%. RV stands for realized volatility, Vol for trading frequency, and Ticks for trade size. BPV-C stands for the continuous component of bipower variation, and BPV-J for the jump component of bipower variation extracted at the 1% level. RS^+ and RS^- stand for, respectively, the positive and negative realized semivariance. Adj. R-Squ. stands for Adjusted R-Squared. The models estimated are summarized in Eq. (1) to (5).

of trading volume seem to be stronger than that of trading frequency. To evaluate the role of trading frequency, we may compare the adjusted R^2 across the different regressions. In regression (1) (Table 3) and regression (16) (Table 4) where trading volume is used, the adjusted R^2 is equal to 0.7596 for oil and 0.3780 for gas. In regression (2) (Table 3) and regression (17) (Table 4) where trading frequency is used, the adjusted R^2 is lower at 0.7512 for oil and 0.3489 for gas. Therefore, both trading volume and trading frequency appear to play important roles behind the volatility-volume relationship. Yet, the volatility impacts seem slightly greater for trading volume compared to trading frequency.

Next, we regress realized volatility against both trading volume and trading frequency. It is particularly interesting to notice that both variables are highly significant when considered jointly as shown in regressions (3) and (18).[21] Going from regressions (1) and (16) to regressions (3) and (18) for, respectively, oil and gas, we observe that the statistically significant (at the 1% level) effect of trading frequency remains. This finding is consistent with Giot et al. (2010), who find that the volatility volume relationship is driven mainly by trading frequency. Besides verifying the robustness of our previous estimates, the results indicate that considering both variables jointly has a limited additional impact on the adjusted R^2 (0.7603 for oil and 0.3898 for gas). This strengthens our analysis of the volatility-volume relationship on such energy markets, as Vol and $Ticks$ appear to share the same information content, and may be considered separately. In other words, observing trading volume or trading frequency yields to the same qualitative (and almost quantitative) results in our regression framework.

Note that, as in Giot et al. (2010), we could not find any statistically significant effect of average trade

[21] Note that the sign of $Ticks$ becomes negative in that specification.

Gas		RV	
	(16)	(17)	(18)
Vol	0.000160***		0.000329***
	(0.000040)		(0.000091)
Ticks		0.000261***	-0.000337***
		(0.000069)	(0.000120)
Adj. R-Squ.	0.3780	0.3489	0.3898

Gas		BPV-C			BPV-J	
	(19)	(20)	(21)	(22)	(23)	(24)
Vol	0.000104***		0.000200***	0.000125*		0.000490**
	(0.000019)		(0.000044)	(0.000067)		(0.000240)
Ticks		0.000172***	-0.000190***		0.000181*	-0.000737**
		(0.000035)	(0.000060)		(0.000110)	(0.000385)
Adj. R-Squ.	0.5835	0.5607	0.5911	NA	NA	NA

Gas		RS^+			RS^-	
	(25)	(26)	(27)	(28)	(29)	(30)
Vol	0.000106***		0.000259***	0.000063***		0.000079***
	(0.000030)		(0.000078)	(0.000014)		(0.000028)
Ticks		0.000165***	-0.000305***		0.000111***	-0.000033
		(0.000051)	(0.000104)		(0.000029)	(0.000052)
Adj. R-Squ.	0.2291	0.1946	0.2506	0.3225	0.3135	0.3230

Table 4
Regression results for gas

Note: Standard errors in parenthesis. *** denotes significance at 1%, ** at 5% and * at 10%. RV stands for realized volatility, Vol for trading frequency, and Ticks for trade size. BPV-C stands for the continuous component of bipower variation, and BPV-J for the jump component of bipower variation extracted at the 1% level. RS^+ and RS^- stand for, respectively, the positive and negative realized semivariance. Adj. R-Squ. stands for Adjusted R-Squared. The models estimated are summarized in eq. (1) to (5).

size (ATS) on realized volatility measures for oil and gas. Hence, these results are not reported here. This comment applies in the remainder of the paper.

Let us now consider the effects of trading volume and trading frequency on the continuous component of bipower variation ($BPV - C$, extracted at 1% level) as the dependent variable (Eq. (2)). The results obtained are qualitatively unchanged (Table 3, regressions (4) to (6) for oil and Table 4, regressions (19) to (21) for gas). Nevertheless, the explanatory power is much higher in the case of gas (around 58% now *vs.* 38% before), which highlights the noisy impact of jumps on the volatility-volume relationship. The dummy variables, however, are not significant anymore which suggests that the reaction to news is mainly explained by jumps (as explained below) and *not* by the continuous component of bipower variation. This set of regression results therefore points out likely differences in the behavior of energy futures contract, once the influence of jumps has been removed.

This comment leads us to study the effects of trading volume and trading frequency on the jump component of bipower variation ($BPV - J$), as detailed in Eq. (3), using TOBIT regressions to account for large numbers of zeros in the data (see Giot et al. (2010)). We remark that trading volume remains statistically significant and positive (regressions (7) and (22) for oil and gas, respectively). As stated earlier, the dummy variables are also significant. Next, trading frequency is significant at the 10% level for gas (regression (23)), while it does not appear significant for oil (regression (8)). Again, when considering both variables in the same regression ((9) for oil and (24) for gas), our econometric strategy shows that both trading volume and trading frequency are statistically significant in explaining the jump component of bipower variation. By comparing the coefficients obtained, one may also cautiously conclude that trading volume is more relevant than trading frequency to analyze the volatility-volume relationship in oil and gas futures markets when considering $BPV - J$. Taken together, these effects are quite new, since bipower variation allows us to distinguish between the continuous and the discontinuous *nature* of volatility. To our best knowledge, they were only noticed by Giot et al. (2010) previously in the context of the 100 largest stocks traded on the NYSE. Therefore, we provide the first application of this methodology to energy futures markets.

The last step of our estimation strategy consists in identifying the impacts of trading volume and trading frequency on the negative and positive realized semivariances. As explained in Section 3, this methodology allows us to investigate the presence of an *asymmetric* volatility-volume relationship. The relation is thus expected to be fundamentally different for positive and negative price changes.

Concerning RS^+ and RS^-, we observe roughly the same patterns as those highlighted for the realized volatility measure (regressions (10) to (15) for oil and regressions (25) to (30) for gas). In regressions (10) and (11) (Table 3) for oil and in regressions (25) and (26) (Table 4) for gas, we uncover statistically significant (at the 1% level) and positive effects of trading volume and trading frequency, taken separately as exogenous regressors, on positive realized semivariance. In regressions (13) and (14) (Table 3) for oil and in regressions (28) and (29) (Table 4) for gas, we notice that Vol and $Ticks$ have a statistically significant (at the 1% level) impact on negative realized semivariance when considered separately. Furthermore, the explanatory power of trading activity variables is found to vary between positive and negative semivariances. Indeed, for oil, the adjusted R^2 is around 60% for RS^+ *vs.* 71% for RS^-. For gas, the picture is even more striking, as the adjusted R^2 goes from 20% for RS^+ to 32% for RS^-. For both energy markets, the adjusted R^2 for RS^- is almost equal to that of RV. This result demonstrates the superior information content of RS^- compared to RS^+. This finding is also in line with Patton and Sheppard (2011), where RS^- has a better forecasting power to predict RV than RS^+.

For RS^+, regressions (12) and (27) reveal that Vol and $Ticks$ are statistically significant when considered together for both markets. However, the regression results differ from previously when we regress RS^- on Vol and $Ticks$ simultaneously. While trading volume remains significant (at the 1% level) for both variables, no statistically significant effect may be detected for trading frequency in the gas market (regression (30)). Hence, we uncover dramatically different behaviors for the two energy futures contracts when considering the positive and negative realized semivariance. In the case of oil, both trading volume and trading frequency are found to play a significant role. In the case of gas, trading volume is found to be the main driving force behind the volatility-volume relationship.

Besides, we are able to uncover an *asymmetric* volatility-volume relationship in energy futures based on RS^+ and RS^-. Note that the dummy variable is statistically significant for RS^- (but not for RS^+), which suggests that news are more related to negative volatility than to positive volatility. The latter result may be explained by the fact that positive volatility is more prone to speculative activity and noise trading (under the form of positive feedback trading). Therefore, it may be less sensitive to fundamental news compared to negative volatility.

For nearly all regressions, the coefficient estimates are statistically significant. Consequently, the volatility-volume relationship - which is well-documented with daily data - seems to hold for the crude oil and natural gas markets when using high-frequency data.

Although the above results indicate that this relationship holds regardless of whether trading volume or trading frequency is used, it seems that trading volume explains realized volatility measures better than trading frequency does. It is clear from Tables 3 and 4 that trading frequency does not have as much impact on energy futures volatility as trading volume. Indeed, when Vol and $Ticks$ are jointly significant, the coefficient for trading frequency is slightly smaller than that for trading volume.

Furthermore, the above evidence indicates that trading frequency has different volatility impacts depending on the volatility measure (with/without jumps) and the market considered. The significance of trading frequency is consistent with the presence of stealth trading, as informed investors break up large trades into many small ones to hide their private signals. Finally, our results confirm the findings by Giot et al. (2010), who conclude that trade size plays a very secondary role to trading frequency in explaining realized volatility.

5 Robustness checks

In this section, we provide additional empirical results to check the robustness of our findings. Namely, we use *(i)* different jump detection thresholds for BPV, *(ii)* MedRV instead of BPV, *(iii)* different jumps detection thresholds for MedRV, and *(iv)* a microstructure noise-robust estimator for RV.

5.1 Jump detection thresholds for BPV

In section 4, we have followed the bulk (see ABD (2007) among others) of the previous literature by using a detection threshold for jumps of 1%. In Table 5, we consider three alternative jump detection thresholds (i.e. 5%, 0.1% and 0.5%), and investigate the sensitivity of the results obtained. Note that detection thresholds below 1% are rather conservative. For instance, they may be used to detect only very significant jumps as in Andersen et al. (2010) when distributional properties are under scrutiny.

For oil, the results shown in Table 5 (regressions (31) to (39)) are broadly similar to the results obtained in Table 3 (regressions (7) to (9)). When taken in isolation, Vol is shown to have a significant explanatory power for the jump component of bipower variation, while $Ticks$ is not significant. By using both variables, $Ticks$ becomes significant (at 10% level) in addition to Vol only for $BPV - J - 0.5\%$ (regression (36)).

Compared to Table 4 (regressions (22) to (24)), the results for gas shown in Table 5 (regressions (40) to (48)) are also consistent. Indeed, both variables exhibit explanatory power when considered separately. Besides, we find that the combination of Vol and $Ticks$ is only significant for $BPV - J - 0.5\%$ (regression (45)).

The main results shown in Tables 3 and 4 therefore appear robust to the variation in the jump detection threshold for BPV.

Next, we consider an alternative estimator of the continuous component of volatility.

5.2 Median realized volatility

As soon as the sampling frequency does not tend to infinity, which is obviously the case in empirical work, the bipower variation estimator is upward biased. This is due to possible large jumps which are not fully eliminated when multiplied by an adjacent not infinitely small return. To gauge the robustness of our results against the possible upward bias in BPV, we rely on an alternative estimator, namely

Oil	BPV-J-5%		
	(31)	(32)	(33)
Vol	0.000004*		0.000008**
	(0.000002)		(0.000004)
Ticks		0.000003	-0.000007
		(0.000003)	(0.000006)

Oil	BPV-J-0.5%			BPV-J-0.1%		
	(34)	(35)	(36)	(37)	(38)	(39)
Vol	0.000008**		0.000018***	0.000011**		0.000018**
	(0.000003)		(0.000006)	(0.000005)		(0.000008)
Ticks		0.000006	-0.000016*		0.000010	-0.000012
		(0.000005)	(0.000009)		(0.000007)	(0.000012)

Gas	BPV-J-5%		
	(40)	(41)	(42)
Vol	0.000099**		0.000295*
	(0.000050)		(0.000162)
Ticks		0.000154*	-0.000394
		(0.000082)	(0.000251)

Gas	BPV-J-0.5%			BPV-J-0.1%		
	(43)	(44)	(45)	(46)	(47)	(48)
Vol	0.000140*		0.000533**	0.000174*		0.000525*
	(0.000075)		(0.000266)	(0.000096)		(0.000308)
Ticks		0.000206*	-0.000794*		0.000274*	-0.000710
		(0.000124)	(0.000428)		(0.000163)	(0.000489)

Table 5
Robustness checks: Jump detection thresholds for BPV

Note: Standard errors in parenthesis. *** denotes significance at 1%, ** at 5% and * at 10%. BPV-J stands for for the jump component of bipower variation extracted at the 5%, 0.5% and 0.1% levels, Vol for trading frequency, and Ticks for trade size. The models estimated are summarized in eq. (1) to (5).

the median realized volatility estimator introduced by ADS (2011):

$$MedRV_{t+1}(\Delta) = \frac{\pi}{6-4\sqrt{3}+\pi}\left(\frac{1/\Delta}{1/\Delta-2}\right)\sum_{j=2}^{1/\Delta-1} \text{med}(|r_{t+(j-1).\Delta,\Delta}|, |r_{t+j.\Delta,\Delta}|, |r_{t+(j+1).\Delta,\Delta}|)^2 \quad (9)$$

where r_{t_i} denotes the i^{th} intraday return on day t for $i = 1, 2, 3, \dots, 1/\Delta$.

If jumps are rare, which is the case in most financial series, their impact is null when the median (and not the product) of adjacent returns is chosen.[22] An additional advantage of MedRV is its robustness to the presence of zero-returns which is also a drawback of BPV.[23] Other estimators have been proposed recently (see Boudt et al. (2010), ADS (2011) among others) but in light of the analysis in Theodossiou and Žikeš (2010), the MedRV estimator exhibits very interesting empirical properties.

This estimator also allows to disentangle jumps from the diffusive component, while ensuring the non-negativity of daily estimates, by using and adapted version of the Huang and Tauchen's (2005) ratio test:

[22] In case of two consecutive jumps, the impact on the MedRV is significant but dramatic for the BPV estimator.
[23] A thorough analysis of the robustness of realized estimators to the presence of noise and/or jumps is provided in Theodossiou and Žikeš (2010). The interested reader may refer to this paper for alternative estimators and their empirical properties.

$$J(t,\Delta) = \frac{\left(1 - \frac{MedRV_t(\Delta)}{RV_t(\Delta)}\right)}{\sqrt{0.96 \frac{1}{1/\Delta} \max(1, MedRQ_t(\Delta)/MedRV_t^2(\Delta))}} \tag{10}$$

where the consistent estimator of the integrated quarticity, $MedRQ_{t+1}(\Delta)$, is given by:

$$MedRQ_{t+1}(\Delta) = 1/\Delta \frac{3\pi}{9\pi + 72 - 52\sqrt{3}} \left(\frac{1/\Delta}{1/\Delta - 2}\right) \sum_{j=2}^{1/\Delta - 1} \text{med}(|r_{t+(j-1).\Delta,\Delta}|, |r_{t+j.\Delta,\Delta}|, |r_{t+(j+1).\Delta,\Delta}|)^4 \tag{11}$$

When significant, the jump component may be defined similarly to BPV:

$$J_{t+1}^{MedRV}(\Delta) \equiv max[RV_{t+1}(\Delta) - MedRV_{t+1}(\Delta), 0] \tag{12}$$

Again, once the jump component has been identified, the continuous component may be inferred as the difference between the realized volatility and the jump component.

As shown in Table 6 (regressions (49) to (60) for oil and regressions (61) to (72) for gas), the results obtained with the median realized volatility are consistent with those obtained for the continuous component of bipower variation (Table 3, regressions (4) to (6) for oil and Table 4, regressions (19) to (21) for gas). Both variables are statistically significant across all regressions (when considered either separately or jointly), thereby highlighting the usefulness of MedRV to strenghten our results.

Oil	MedRV-1%			MedRV-5%		
	(49)	(50)	(51)	(52)	(53)	(54)
Vol	0.000017***		0.000021***	0.000017***		0.000021***
	(0.000002)		(0.000003)	(0.000002)		(0.000003)
Ticks		0.000020***	-0.000008*		0.000019***	-0.000008*
		(0.000003)	(0.000004)		(0.000003)	(0.000004)
Adj. R-Squ.	0.7519	0.7446	0.7523	0.7493	0.7420	0.7498

Oil	MedRV-0.5%			MedRV-0.1%		
	(55)	(56)	(57)	(58)	(59)	(60)
Vol	0.000017***		0.000022***	0.000017***		0.000022***
	(0.000002)		(0.000003)	(0.000002)		(0.000003)
Ticks		0.000020***	-0.000008*		0.000020***	-0.000008*
		(0.000003)	(0.000004)		(0.000003)	(0.000004)
Adj. R-Squ.	0.7513	0.7439	0.7518	0.7518	0.7444	0.7523

Gas	MedRV-1%			MedRV-5%		
	(61)	(62)	(63)	(64)	(65)	(66)
Vol	0.000089***		0.000174***	0.000088***		0.000167***
	(0.000016)		(0.000040)	(0.000016)		(0.000036)
Ticks		0.000146***	-0.000169***		0.000145***	-0.000158***
		(0.000028)	(0.000055)		(0.000029)	(0.000048)
Adj. R-Squ.	0.6027	0.5819	0.6100	0.5992	0.5779	0.6062

Gas	MedRV-0.5%			MedRV-0.1%		
	(67)	(68)	(69)	(70)	(71)	(72)
Vol	0.000091***		0.000179***	0.000090***		0.000182***
	(0.000016)		(0.000041)	(0.000016)		(0.000040)
Ticks		0.000147***	-0.000177***		0.000147***	-0.000183***
		(0.000029)	(0.000056)		(0.000028)	(0.000058)
Adj. R-Squ.	0.6024	0.5812	0.6103	0.6106	0.5889	0.6190

Table 6
Robustness checks: MedRV

Note: Standard errors in parenthesis. *** denotes significance at 1%, ** at 5% and * at 10%. MedRV stands for for the median realized volatility extracted at the 1%, 5%, 0.5% and 0.1% levels, Vol for trading frequency, and Ticks for trade size. Adj. R-Squ. stands for Adjusted R-Squared. The models estimated are summarized in eq. (1) to (5).

5.3 Jump detection thresholds for MedRV

Similarly to $BPV - J$, we consider here several jump detection thresholds for $MedRV - J$.

Oil	MedRV-J-1%			MedRV-J-5%		
	(73)	(74)	(75)	(76)	(77)	(78)
Vol	0.000006		0.000015**	0.000004*		0.000008**
	(0.000003)		(0.000007)	(0.000002)		(0.000004)
Ticks		0.000002	-0.000016*		0.000003	-0.000007
		(0.000005)	(0.000009)		(0.000003)	(0.000006)

Oil	MedRV-J-0.5%			MedRV-J-0.1%		
	(79)	(80)	(81)	(82)	(83)	(84)
Vol	0.000005		0.0000011	0.000008		0.000015
	(0.000004)		(0.000008)	(0.000005)		(0.000011)
Ticks		0.000003	-0.000011		0.000006	-0.000012
		(0.000006)	(0.000011)		(0.000008)	(0.000015)

Gas	MedRV-J-1%			MedRV-J-5%		
	(85)	(86)	(87)	(88)	(89)	(90)
Vol	0.000143**		0.000422*	0.000125		0.000360**
	(0.000072)		(0.000236)	(0.000053)		(0.000171)
Ticks		0.000221*	-0.000560		0.000197	-0.000470*
		(0.000120)	(0.000378)		(0.000088)	(0.000264)

Gas	MedRV-J-0.5%			MedRV-J-0.1%		
	(91)	(92)	(93)	(94)	(95)	(96)
Vol	0.000157*		0.000465*	0.000207*		0.000622*
	(0.000082)		(0.000275)	(0.000108)		(0.000361)
Ticks		0.000240*	-0.000616		0.000327*	-0.000834
		(0.000138)	(0.000447)		(0.000184)	(0.000585)

Table 7
Robustness checks: Jump detection thresholds for MedRV

Note: Standard errors in parenthesis. *** denotes significance at 1%, ** at 5% and * at 10%. MedRV-J stands for for the jump component of median realized volatility extracted at the 1% 5%, 0.5% and 0.1% levels, Vol for trading frequency, and Ticks for trade size. The models estimated are summarized in Eq. (1) to (5).

In Table 7 (regressions (73) to (84) for oil and regressions (85) to (96) for gas), the results need to be compared with the jump component of bipower variation (in Table 3- regressions (7) to (9) for oil, Table 4- regressions (22) to (24) for gas, and section 5.1). For oil, there are only a few significant results to report (in regressions (75), (76) and (78)). For gas, Vol is nearly always significant, while $Ticks$ is mostly significant when considered independently. The only significant combination of Vol and $Ticks$ is achieved in regression (90), *i.e.* for $MedRV - J - 5\%$.

Overall, we reach the conclusion that the jump component of MedRV has *less* informational content than that of BPV to capture the volatility-volume relationship. This conclusion may arise as a consequence of the upward bias in BPV, as explained previously.

5.4 Addressing microstructure noise with two-scale RV

When sampling at $1/\Delta$ frequency (with 5-minute returns for instance to deal with MN), a significant share of the data is simply ignored. In addition, there are many possible 'grids' of $1/\Delta$ on which the estimates could be computed. Zhang et al. (2005) propose a K-subsampling methodology to address

MN, called the 'two-scale' estimator of realized volatility (TSRV):

$$TSRV_K = \frac{1}{K}\sum_{j=1}^{K} RV_{\mathcal{G}_{K_j}} - 2\overline{n}_K \frac{RV_{\mathcal{G}}}{2n} \tag{13}$$

with n the number of transactions leading to price changes, $\overline{n}_K = \frac{n-K+1}{K}$ and the grid $\mathcal{G} \equiv \{t_0, t_2, \ldots, t_n\}$ of (non-overlapping) subgrids:

$$\mathcal{G}_{K_j} = \{t_{j-1}, t_{j-1+K}, \ldots, t_{j-1+c_jK}\} \tag{14}$$

for $j = 1, \ldots, K$ where $c_j \equiv \lfloor \frac{n-j+1}{K} \rfloor$.

Hence, the idea behind 'sub-sampling' is to compute the chosen estimator over different 'grids', and then to average them to increase the statistical robustness by decreasing the variance of the estimate.

Oil		TSRV	
	(97)	(98)	(99)
Vol	0.000018***		0.000023***
	(0.000002)		(0.000003)
Ticks		0.000020***	-0.000009**
		(0.000003)	(0.000004)
Adj. R-Squ.	0.7852	0.7776	0.7858

Gas		TSRV	
	(100)	(101)	(102)
Vol	0.000123***		0.000256***
	(0.000022)		(0.000057)
Ticks		0.000200***	-0.000266***
		(0.000040)	(0.000083)
Adj. R-Squ.	0.5174	0.4895	0.5291

Table 8
Robustness checks: Regression results with TSRV

Note: Standard errors in parenthesis. *** denotes significance at 1%, ** at 5% and * at 10%. TSRV stands for the two-scale estimator of realized volatility, Vol for trading frequency, and Ticks for trade size. Adj. R-Squ. stands for Adjusted R-Squared. The models estimated are summarized in Eq. (1) to (5).

Results for TSRV are shown in Table 8 (regressions (97) to (99) for oil and regressions (100) to (102) for gas). Compared to Table 3 (regressions (1) to (3)) for oil and Table 4 (regressions (16) to (18)) for gas, we obtain consistent results at each stage of our econometric strategy. Indeed, both Vol and $Ticks$ exhibit significant explanatory power. The most striking result is obtained for gas, where the adjusted R^2 jumps from 37% for RV to more than 50% for TSRV. As the gas market is generally less liquid than oil, it may be more sensitive to MN. Hence, the use of the TSRV estimator seems strongly justified in this case.

6 Conclusion

This paper deals with the relationship between price volatility and trading volume in energy futures markets, which is found to be largely significant and positive. In undertaking this analysis, this paper provides further contributions by using empirical techniques which allow volume and volatility to be modeled at a high-frequency, a practice not so frequent in the current literature.

The main contributions of the paper may be summarized as follows. First, we study the crude oil and natural gas markets, which are the world's most important energy futures markets in terms of size and liquidity. Overall, we find that trading volume and trading frequency have a statistically significant impact on various realized volatility measures, and that they essentially share the same information content. Second, we examine the impact of both trading activity variables on the continuous and the discontinuous (jump) components of realized volatility, by using BPV as in Giot et al. (2010). In addition, we detect jumps from the continuous component of realized volatility with the MedRV estimator (ADS (2011), which is robust to the occurrence of zero-returns and is not upward biased is empirical applications. Third, we consider the impact of trading volume and trading frequency on positive and negative realized semivariances (BNKS (2008)), so that the volatility-volume relationship may be *asymmetric*. Across our various regressions, the explanatory power of trading activity measures is found to be different depending on positive or negative semivariances. For both oil and gas futures, we find indeed that *negative* realized semivariance has a superior information content. Hence, contrary to Foster (1995) for crude oil, this paper finds that the magnitude of trading volume and the dispersion of price changes are rather asymmetric. This asymmetry is indicative of markets having a nonlinear reaction to price changes depending on their sign. Finally, we find that trade size has no significant additional information content beyond that of trading volume and trading frequency in explaining the volatility-volume relationship on energy futures markets. As generic robustness checks, we have verified that the volatility-volume relationship studied in this paper is not qualitatively sensitive to the choice of the realized volatility estimator (as the 'naive' and 'two-scale' estimators provide coherent support to our results in light of the microstructure noise issue), or to the jump detection thresholds chosen for BPV and MedRV.

Another finding of our analysis concerns the quality of the information provided by trading activity in energy futures markets. As noted in Blume et al. (1994), trading volume provides information about the *quality of information signals* rather than the *information signal* itself. This subtle difference in the role of volume would explain its statistical significance without requiring to explain price volatility. Hence, following Blume et al. (1994), trading volume may be viewed as an inappropriate surrogate for the rate of information arrival. In the case of crude oil and natural gas futures, in light of the high explanatory power of our numerous regressions, we may conclude that the quality of information signals is high compared to other markets. Indeed, the level of the adjusted R^2 identified in this paper is much higher than in previous work when using realized volatility as a proxy for the latent volatility variable (see Giot et al. (2010)).

Our results have important implications for empirical work in explaining the volatility-volume relationship. Possible extensions in energy futures markets include studying the relationship between volatility and maturity by unit of volume (volume time), or by number of transactions (transaction time). This will allow to assess whether volatility truly increases when maturity approaches. Another more theoretical extension may be to test some causal relationship between some volatility measure and trading volume, as in Lee and Rui (2002) among others, but considering properly the long memory feature of these time-series. Indeed, the possibility of long memory or local-to-unity processes strongly biases standard statistical (causality) tests, and any conclusion from a standard analysis (using, say, OLS) are not reliable. Recently, Bauer and Maynard (2008) proposed an econometric approach to deal with this issue.

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