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Generalization of a nonparametric co-integration analysis for multivariate integrated processes of an integer order

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Abstract

This paper provides a further generalization of co-integration tests in a nonparametric setting. We adopt Bierens'([2]) approach in order to give an extension for processes $I(d)$, with a fixed integer d. A generalized eigenvalue problem is solved, and the test statistics involved are obtained starting from two matrices that are independent on the data generating process. The mathematical tools we adopt are related to the asymptotic theory of the stochastic processes. The key point of our work is linked to the distinguishing between the stationary and non-stationary part of an integrated process.

Keywords: *Multivariate analysis, Nonparametric methods, Co-integration, Asymptotic properties*

JEL Classification: *C14,C32*

1 Introduction

The importance of the co-integration concept in the economic literature is due to the possibility of linking the information about the long run equilibrium, coming from the economic theory, and the statistical evidence of the short-run dynamics in the observed series. Most of the studies have investigated the properties of co-integrated systems of order one, I(1), (Engle and Granger, 1987[3]; Engle and Yoo, 1987[4]; Johansen, 1988[5], 1991[6], 1995[7]; Phillips, 1991[8]; Stock and Watson, 1988[9]). If each element of a vector of time series y_t has a unit root, but a linear combination αy_t exists and is stationary, the time series y_t are said to be co-integrated with co-integrating vector α. A nonparametric approach has recently been proposed to study co-integrated system of order one. Bierens (1997) developed new consistent co-integration tests and estimators of a basis of the space co-integrating vectors that do not depend on the specification of the data-generating process. The tests proposed are conducted analogously to Johansen's (1988, 1991) tests, inclusive of the test for parametric restrictions on the co-integrating vectors. This paper proposes an extension of the Bieren's approach to the so-called $I(d)$ process, that is non-stationary with d order difference. The paper is organized as follows. Section 2 provides the main tools we used. Section 3 describes the data generating process. In section 4, non parametric results via convergence theorems are obtained. Section 5 concludes.

2 Main tools

This section is devoted to the survey of several results and definitions, in order to let this paper be self-containing. First of all, we recall the definition of an integrated process of order d.

Definition 1 *Given $p \in \mathbf{N}$, a discrete time p-variate integrated process of order d with drift μ, $Y_t \sim I(d)$, is defined by*

$$Y_t = \mu + \nabla^{-d}\epsilon_t = \mu + (1-L)^{-d}\epsilon_t, \tag{1}$$

where $Y_t = (Y_t^1, \dots, Y_t^p)$, L is the lag difference operator, i.e. $L\epsilon_t := \epsilon_{t-1}$, and $\epsilon_t = (\epsilon_t^1, \dots, \epsilon_t^p)$ is a zero mean stationary process.

Proposition 2 *Given $d \in \mathbf{N}$, let us consider $Y_t \sim I(d)$ with drift μ. Then $\bar{X}_t := Y_t - Y_0 \sim I(d)$ with drift μ.*

Proof. Immediate. ■

Remark 3 *By Proposition 2, fixed* $d \in \mathbf{N}$*, we don't lose of generality assuming* $Y_t \sim I(d)$ *with drift* μ *such that* $Y_0 = 0$*. We make these assumptions for the rest of the paper. This assumption is due to the fact, that the problem we address in this paper can be solve uniquely by the statement of a null condition on the initial data.*

Proposition 4 *Let us consider* $Y_t \sim I(d)$ *with drift* μ *and let us denote* $\Delta := 1 - L$*. Then* $\Delta Y_t = Y_t - Y_{t-1} \sim I(d-1)$ *with drift* μ*.*

Proof. Immediate. ■

Remark 5 *The following relation between* Y_t *and* ΔY_t *holds:*

$$Y_t = \Delta Y_t + \Delta Y_{t-1} + \ldots + \Delta Y_1. \tag{2}$$

By (2), we have that the t*-th realization of an* $I(d)$ *process can be viewed as the sum of the first* t *realizations of an* $I(d-1)$ *process.*

3 Description of the data generating process

In this section we provide a description of the data generating process in the case of $I(d)$ with a drift μ, under some conditions on the process ϵ.
We assume that the hypotheses of the Wold decomposition theorem hold for the process ϵ hold. Due to this fact, fixed $t = 1, \ldots, n$, we can write

$$\epsilon_t = \sum_{j=0}^{\infty} C_j v_{t-j} =: C(L) v_t, \tag{3}$$

where v_t is a p-variate stationary white noise process and $C(L)$ is a p-squared matrix of lag polynomials in the lag operator L.
Let us now state a condition for the matrix $C(L)$, defined in (3).
Assumption I
The process ϵ_t *can be written as (3), where* v_t *are i.i.d. zero mean* p*-variate gaussian variables with variance* I_p*, and there exist* $C_1(L)$ *and* $C_2(L)$*,* p*-squared matrices of lag polynomials in the lag operator* L *such that all the roots of* $\det C_1(L)$ *are outside the complex unit circle and* $C(L) = C_1(L)^{-1} C_2(L)$*.*

The lag polynomial $C(L) - C(1)$ attains value zero at $L = 1$. Thus, there exists a lag polynomial

$$D(L) = \sum_{k=0}^{\infty} D_k L^k$$

such that $C(L) - C(1) = (1 - L)D(L)$. We can write

$$\epsilon_t = C(L)v_t = C(1)v_t + [C(L) - C(1)]v_t = C(1)v_t + D(L)(1 - L)v_t. \quad (4)$$

Let us define $w_t := D(L)v_t$. Then, substituting w_t into (4), we get

$$\epsilon_t = C(1)v_t + w_t - w_{t-1}. \quad (5)$$

(5) implies that, given $Y_t \sim I(d)$ with drift μ, we can write recursively

$$\Delta^{d-1}Y_t = \Delta^{d-1}Y_{t-1} + \epsilon_t + \mu =$$

$$= \Delta^{d-1}Y_{t-1} + C(1)v_t + w_t - w_{t-1} + \mu = \Delta^{d-1}Y_0 + \mu + w_t - w_0 + C(1)\sum_{j=1}^{t} v_j. \quad (6)$$

If $\text{rank}(C(1)) = q - r < q$, then the process $\Delta^{d-1}Y_t$ is co-integrated with r linear independent co-integrating vectors $\gamma_1, \ldots, \gamma_r$.

Remark 6 *By Assumption I, we get that* $C(L)v_t$ *and* $D(L)v_t$ *are well-defined stationary processes and the series*

$$\sum_{k=0}^{\infty} C_k, \qquad \sum_{k=0}^{\infty} C_k C_k^T, \qquad \sum_{k=0}^{\infty} D_k, \qquad \sum_{k=0}^{\infty} D_k D_k^T$$

converge.

Assumption II
Let us consider R_r *the matrix of the eigenvectors of* $C(1)C(1)^T$ *corresponding to the* r *zero eigenvalues. Then the matrix* $R_r^T D(1)D(1)^T R_r$ *is nonsingular.*

4 Convergence properties of random matrices and generalized eigenvalues for $I(d)$ processes, $d > 2$ integer

First of all, we give a further definition of fractionally integrated process of order d, where d is a positive integer.

Definition 7 *Given $p \in \mathbf{N}$, a discrete time p-variate fractionally integrated process of order d, $Y_t \sim I(d)$, is defined by the following property: $\Delta^k Y_t$ is a non-stationary process, for $k = 0, 1, \ldots, d-1$ and $\Delta^d Y_t$ is a stationary process.*

We propose a test based on two random matrices, taking into account the stationary and the non-stationary terms of the $I(d)$ process.
We write such matrices as

$$A_m = \sum_{k=1}^{m} a_{n,k} a_{n,k}^T \tag{7}$$

and

$$B_m = \sum_{k=1}^{m} b_{n,k} b_{n,k}^T, \tag{8}$$

where

$$a_{n,k} = \frac{M_n^{Y,\Delta Y,\ldots,\Delta^{d-1}Y} / \sqrt{n}}{\sqrt{\int\int F_k(x)F_k(y)\min\{x,y\}\mathrm{d}x\mathrm{d}y}} \tag{9}$$

and

$$b_{n,k} = \frac{\sqrt{n} M_n^{\Delta^d Y}}{\sqrt{\int F_k(x)^2 \mathrm{d}x}}, \tag{10}$$

where

$$M_n^{Y,\Delta Y,\ldots,\Delta^{d-1}} = \frac{1}{n}\sum_{t=1}^{n} F_k(t/n)\Delta^{d-1}Y_t + \sum_{h=2}^{d}\left[\frac{1}{n^{2+h}}\sum_{t=1}^{n} G_h(k,t/n)\Delta^{d-h}Y_t\right] \tag{11}$$

and

$$M_n^{\Delta^d Y} = \frac{1}{n}\sum_{t=1}^{n} F_k(t/n)\Delta^d Y_t. \tag{12}$$

Remark 8 *A_m and B_m represent, respectively, the random matrices related to the non stationary part of the process and to the stationary one.*

Now we state an important convergence result.

Theorem 9 *Assume that Assumption I and the following properties for the functions F_k and $G_h(k,\cdot)$ hold.*

$$\lim_{n\to+\infty} \frac{1}{n^{h+\frac{3}{2}}}\sum_{t=1}^{n} t^{d-h} G_h(k,t/n) = 0; \qquad h = 2,3,\ldots,d. \tag{13}$$

$$\frac{1}{\sqrt{n}}\sum_{t=1}^{n} F_k(t/n) = o(1); \tag{14}$$

$$\frac{1}{n\sqrt{n}}\sum_{t=1}^{n} tF_k(t/n) = o(1); \tag{15}$$

$$\int\int F_i(x)F_j(y)\min\{x,y\}\mathrm{d}x\mathrm{d}y = 0, \qquad i \neq j; \tag{16}$$

$$\int F_i(x)\int_0^x F_j(y)\mathrm{d}x\mathrm{d}y = 0, \qquad i \neq j; \tag{17}$$

$$\int F_i(x)F_j(x)\mathrm{d}x = 0, \qquad i \neq j. \tag{18}$$

Then we have the following convergence in distribution:

$$\begin{pmatrix} M_n^{Y,\Delta Y,\ldots,\Delta^{d-1}}(F_k,G_k)/\sqrt{n} \\ \\ M_n^{\Delta^d Y}(F_k)\sqrt{n} \end{pmatrix} \to \begin{pmatrix} C(1)\int F_k(x)W(x)\mathrm{d}x \\ \\ C(1)(F_k(1)W(1) - \int f_k(x)W(x)\mathrm{d}x) \end{pmatrix}, \tag{19}$$

where W is a p-variate standard Wiener process and f_k is the derivative of F_k.

Proof. In order to fix ideas, let us consider $d = 3$, i.e. the process $Y \sim I(3)$. By Remark 3, we can write recursively

$$Y_t = \sum_{j=0}^{t}\Delta Y_{t-j} = \sum_{j=0}^{t}\Big[\sum_{i=1}^{t-j}\Delta^2 Y_{t-j-i}\Big] =$$

$$= \sum_{j=0}^{t}\Big[j\Delta^2 Y_{t-j+1}\Big] = \sum_{j=0}^{t}\Big[j\sum_{k=1}^{t-j+1}\epsilon_k\Big]. \tag{20}$$

Thus we have, as $n \to +\infty$,

$$\frac{1}{n^5}\sum_{t=1}^{n} G_3(k,t/n)Y_t \sim \frac{1}{n^5}\epsilon_1\sum_{t=1}^{n} G_3(k,t/n)t^3. \tag{21}$$

Analogously, we have

$$\frac{1}{n^4}\sum_{t=1}^{n} G_2(k,t/n)\Delta Y_t \sim \frac{1}{n^4}\epsilon_1\sum_{t=1}^{n} G_2(k,t/n)t^2. \tag{22}$$

It is easy to provide a generalization of this argument. Let us consider $Y \sim I(d)$, with $d > 3$ integer. For each $h \in \{2, \dots, d\}$ we can write

$$\frac{1}{n^{2+h}} \sum_{t=1}^{n} G_h(k, t/n) \Delta^{d-h} Y_t \sim \frac{1}{n^{2+h}} \epsilon_1 \sum_{t=1}^{n} G_h(k, t/n) t^{d-h}. \tag{23}$$

Using the definition of the p-variate normal random variable ϵ_t and the i.i.d. property, we get

$$\sqrt{n} \cdot M_n^{Y, \Delta Y, \dots, \Delta^{d-1}} = \sqrt{n} \cdot \Big\{ \frac{1}{n} \sum_{t=1}^{n} F_k(t/n) \Delta^{d-1} Y_t + \sum_{h=2}^{d} \Big[\frac{1}{n^{2+h}} \sum_{t=1}^{n} G_h(k, t/n) \Delta^{d-h} Y_t \Big] \Big\}. \tag{24}$$

By hypothesis (13), by Proposition 4, by the hypotheses (14), (15), (16), (17), (18) and using [2], we get the thesis. ■
¿From Theorem 9 the following result holds.

Theorem 10 *Assume that the hypotheses of Theorem 9 hold.*
Then we have the following convergence in distribution:

$$\begin{pmatrix} M_n^{Y, \Delta Y, \dots, \Delta^{d-1}}(F_k, G_k)/\sqrt{n} \\ \\ M_n^{\Delta^d Y}(F_k)\sqrt{n} \end{pmatrix} \to \begin{pmatrix} C(1) X_k \sqrt{\int \int F_k(x) F_k(y) \min\{x, y\} \mathrm{d}x \mathrm{d}y} \\ \\ C(1) Y_k \sqrt{\int F_k(x)^2 \mathrm{d}x} \end{pmatrix}, \tag{25}$$

for each k, where X_k and Y_k are independent p-variate standard normally distributed random vectors such that

$$X_k = \frac{\int F_k(x) W(x) \mathrm{d}x}{\sqrt{\int \int F_k(x) F_k(y) \min\{x, y\} \mathrm{d}x \mathrm{d}y}}, \tag{26}$$

$$Y_k = \frac{F_k(1) W(1) - \int f_k(x) W(x) \mathrm{d}x}{\int F_k(x)^2 \mathrm{d}x}. \tag{27}$$

The interaction between the hypotheses of the previous theorem and the Assumption II bring to the following result.

Theorem 11 *Assume that the hypotheses of Theorem 9 hold and there exist r linear independent co-integrating vectors (thus, we get the existence of the matrix R_r defined as in Assumption II).*
Then we have the following joint in $k = 1, \dots, n$ convergence in distribution:

$$\begin{pmatrix} R_r^T M_n^{Y, \Delta Y, \dots, \Delta^{d-1}}(F_k, G_k)\sqrt{n} \\ \\ R_r^T M_n^{\Delta^d Y}(F_k) n \end{pmatrix} \to \begin{pmatrix} R_r^T D(1) Y_k \sqrt{\int F_k(x)^2 \mathrm{d}x} \\ \\ F_k(1) R_r^T Z \end{pmatrix}, \tag{28}$$

where the Y_k's and Z are independent p-variate standard gaussian, Z independent on F_k and Y_k defined as in Theorem 10.

The weight functions defining the matrices A_m and B_m exist.
By a simple computation we get the following result:

Proposition 12 *Let us consider, for each k,*

- $F_k(x) = cos(2k\pi x)$
- *for each $h = 1, \ldots, d-1$, it results*

$$G_h(k,x) = \sum_{j=1}^{N} a_j x^{\alpha_j},$$

for each $N \in \mathbf{N}$, $a_j, \alpha_j \in \mathbf{R}$, $\forall\, j \in \{1, \ldots, N\}$.

Then the conditions (13), (14), (15), (16), (17) and (18) hold.

¿From Theorems 10 and 11, we have

Theorem 13 *Let us assume* $\mathrm{rank} C(1) = p - r$. *The following convergence in distribution results are satisfied:*

$$\begin{pmatrix} I_{p-r} & 0 \\ 0 & nI_r \end{pmatrix} R^T A_m R \begin{pmatrix} I_{p-r} & 0 \\ 0 & nI_r \end{pmatrix} = \begin{pmatrix} R_{p-r}^T A_m R_{p-r} & nR_{p-r}^T A_m R_r \\ nR_r^T A_m R_{p-r} & n^2 R_r^T A_m R_r \end{pmatrix} \rightarrow$$

$$\rightarrow \begin{pmatrix} R_{p-r}^T C(1) \sum_{k=1}^m X_k X_k^T C(1)^T R_{p-r} & R_{p-r}^T C(1) \sum_{k=1}^m \gamma_k X_k Y_k^T D(1)^T R_r \\ R_r^T D(1) \sum_{k=1}^m \gamma_k Y_k X_k^T C(1)^T R_{p-r} & R_r^T D(1) \sum_{k=1}^m \gamma_k^2 Y_k Y_k^T D(1)^T R_r \end{pmatrix} \tag{29}$$

and

$$\begin{pmatrix} I_{p-r} & 0 \\ 0 & \sqrt{n} I_r \end{pmatrix} R^T B_m R \begin{pmatrix} I_{p-r} & 0 \\ 0 & \sqrt{n} I_r \end{pmatrix} = \begin{pmatrix} R_{p-r}^T B_m R_{p-r} & \sqrt{n} R_{p-r}^T B_m R_r \\ \sqrt{n} R_r^T B_m R_{p-r} & n R_r^T B_m R_r \end{pmatrix} \rightarrow$$

$$\rightarrow \begin{pmatrix} R_{p-r}^T C(1) \sum_{k=1}^m Y_k Y_k^T C(1)^T R_{p-r} & R_{p-r}^T C(1) \sum_{k=1}^m \delta_k Y_k Z^T D_*^T R_r \\ R_r^T D_* \sum_{k=1}^m \delta_k Z Y_k^T C(1)^T R_{p-r} & R_r^T D_* \sum_{k=1}^m \delta_k^2 Z Z^T D_*^T R_r \end{pmatrix} \tag{30}$$

where X_j, Y_i and Z are the same defined in Theorems 10 and 11. Furthermore, the following convergence in distribution holds:

$$\frac{R^T A_m^{-1} R}{n^2} \to \begin{pmatrix} 0 & 0 \\ 0 & V_{r,m}^{-1} \end{pmatrix} \tag{31}$$

with

$$V_{r,m} = R_r^T D(1) \sum_{k=1}^{m} \gamma_k^2 Y_k Y_k^T D(1)^T R_r - \Big(R_r^T D(1) \sum_{k=1}^{m} \gamma_k Y_k X_k^T C(1)^T R_{p-r}\Big) \cdot$$

$$\cdot \Big(R_{p-r}^T C(1) \sum_{k=1}^{m} X_k X_k^T C(1)^T R_{p-r}\Big)^{-1} \cdot \Big(R_{p-r}^T C(1) \sum_{k=1}^{m} \gamma_k X_k Y_k^T D(1)^T R_r\Big).$$

Due to Assumption II, we deduce immediately that the matrix $V_{r,m}$ is not singular.
Let us denote

$$X_k^* = \Big(R_{p-r}^T C(1) C(1)^T R_{p-r}\Big)^{\frac{1}{2}} R_{p-r}^T C(1) X_k,$$

$$Y_k^* = \Big(R_{p-r}^T C(1) C(1)^T R_{p-r}\Big)^{\frac{1}{2}} R_{p-r}^T C(1) Y_k. \tag{32}$$

By [1], [2] and using Theorem 13, we prove the following result.

Theorem 14 *Let us consider $\hat{\lambda}_{1,m} \geq \ldots \geq \hat{\lambda}_{p,m}$ the ordered solutions of the generalized eigenvalue problem*

$$\det\Big[A_m - \lambda(B_m + n^{-2} A_m^{-1})\Big] = 0, \tag{33}$$

and let us consider $\lambda_{1,m} \geq \ldots \geq \lambda_{p-r,m}$ the ordered solutions of the generalized eigenvalue problem

$$\det\Big[\sum_{k=1}^{M} X_k^* X_k^{*T} - \lambda \sum_{k=1}^{M} Y_k^* Y_k^{*T}\Big] = 0, \tag{34}$$

where the X_i^*'s and Y_j^*'s are i.i.d. random variables following a $N_{p-r}(0, I_{p-r})$ distribution. If z_t is co-integrated with r linear independent co-integrating vectors, then Assumptions I and II assure that we have the following convergence in distribution:

$$(\hat{\lambda}_{1,m}, \ldots, \hat{\lambda}_{p,m}) \to (\lambda_{1,m}, \ldots, \lambda_{p-r,m}, 0, \ldots, 0)$$

Let us define now

$$Y_k^{**} = (R_r^T D(1) D(1)^T R_r)^{-\frac{1}{2}} R_r^T D(1) Y_k. \tag{35}$$

We have that $Y_k^{**} \sim N_r(0, I_r)$.
Moreover, we can write the matrix $V_{r,m}$ as

$$V_{r,m} = (R_r^T D(1) D(1)^T R_r)^{\frac{1}{2}} V_{r,m}^* (R_r^T D(1) D(1)^T R_r)^{\frac{1}{2}}, \tag{36}$$

where

$$V_{r,m}^* = \Big(\sum_{k=1}^{m} \gamma_k^2 Y_k^{**} Y_k^{**T} \Big) - \Big(\sum_{k=1}^{m} \gamma_k Y_k^{**} X_k^{*T} \Big) \Big(\sum_{k=1}^{m} X_k^* X_k^{*T} \Big) \Big(\sum_{k=1}^{m} \gamma_k X_k^* Y_k^{**T} \Big). \tag{37}$$

So, we get the following result

Theorem 15 *Let us consider $\lambda_{1,m}^* \geq \ldots \geq \hat{\lambda}_{r,m}^*$ the ordered solutions of the generalized eigenvalue problem*

$$\det \Big[V_{r,m}^* - \lambda (R_r^T D(1) D(1)^T R_r)^{-1} \Big] = 0, \tag{38}$$

with $V_{r,m}^*$ defined as in (37) and the X_i^*'s and Y_j^{**}'s are i.i.d. random variables following respectively a $N_{p-r}(0, I_{p-r})$ and $N_r(0, I_r)$ distribution.
Under the hypotheses of Theorem 14, we have the following convergence in distribution

$$n^2 (\hat{\lambda}_{p-r+1,m}, \ldots, \hat{\lambda}_{p,m}) \to (\lambda_{1,m}^{*2}, \ldots, \lambda_{r,m}^{*2})$$

5 Conclusions

In this work, a nonparametric co-integration approach for I(1) process developed by Bierens (1997) is extended to the I(d) process. The approach followed by Bierens is linked to the construction of two matrices taking into account the stationary and non-stationary part of the data generating process. Via some convergence results, the author provided the solution of a generalized eigenvalue problems, and thus the construction of random matrices independent of the process.
In our work we adopt the same strategies and techniques. We focus our attention on writing a pair of data generating process matrices, showing the distinction between the stationarity and non-stationarity part of the data generator. Moreover, we check that the weight we use in order to construct the model does exist. By imposing asymptotic conditions on the model's parameters, non parametric results are obtained.

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