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Job Search Mechanism and Individual Behaviour

by

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Abstract

This paper modelles job search mechanism at individual level by a determinstic-stochastic approach in a economy with perfect competion and rational agents. Each single unit, firm or worker, is analyzed over time; aggregate dynamics comes directly from the micro-structure of the economy. We show that the unemployment as well as vacancy rate converge in the long run to an ergodic distribution whose average value lies on the Beverdige curve. Transitional paths are not-monotone and depending on initial conditions. The micro-model is exploited to assess the relationship between job search and social networks (neighborhood effects); results show that, when the network is endogenous, such spillovers affect both transitional paths and steady state in several way, not last in a negative way.

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1 Introduction

Like most other aggregate functions in the macroeconomist's tool kit, the matching function is a black box; we have good intuition about its existence and properties but only some tentative ideas about its microfoundations. .... The most popular microeconomic models, such as the urn-ball game, do not perform as well empirically.[1]

Job search is an intrinsically individual-level issue; it sums up the behaviour of two elementary units of an economy: firm and worker. What we observe in the job market data is the consequence of such elementary processes; unfortunately we have no a clear idea whether the aggregate level is just the mirror, on a larger scale, of the microcosm or only a fuzzy and distorted representation. Microdata shows that job market is characterized by large gross flows of jobs creation and destruction (Davis, Haltiwanger and Schuh 1996) as well as workers' transition among unemployment, employment and out of the labor force status. At an aggregate analysis, the Beveridge curve shows movements rather fuzzy, alternating relative stable patterns to large changes in position and slope, as well as non-monotone transitional paths (see Blanchard and Diamond, 1989, Bleakley and Fuhrer, 1997, Nickell et al. 2002). In other words, data shows that there is a remarkably level of "churning" in the labour market; movements of the Beverdige curve are the economy-wide response to a complex microeconomic framework. The "matching function" is the common thread between these two worlds.

The process characterizing the single match between firm and worker involves a high number of variables which are well reviewed in the Petrongolo and Pissarides, 2000, survey. Under this point of view, the idea of a "simple" description of the microcosm could be frustrated; nevertheless we can not investigate into the labour market without to try to challenge this point. This paper presents a first attempt at providing a simple but reasonably descriptive analytical body for modelling job search at the individual level which is coherent with a simple macroeconomic model. It relies heavily on recent developments of mathematical and statistical research applied to the investigation of the dynamics of epidemic processes.

The simplest way of looking at the job search mechanism is the one described by the Pissarides Mortensen approach (PM hereafter): at every time a firm randomly "contacts" a worker, the match is closed and the unemployed

[1] *Petrongolo and Pissarides, 2000.*

worker changes her status to employed (hence she is removed from the unemployment population), the firm fills the vacancy and it can be removed from the population of vacant firm. Meanwhile, new unemployed are born and new vacancies are opened as the result of an exogenous separation between an existing firm-worker unit. The matching rate, as well as the separation one, can depend both on worker features, notably human capital, and firm characteristics as well as on local spillovers.

However, before of describing the search and matching process, we have to identify properly the "actors" of this play. As pointed out by Blanchard and Diamond, the common view of considering workers either as unemployed or employed is too narrow: *Much of the movement into ad out of employment is from "out of the labor force"*[2] and again "*the relevant pool of workers appears to include some workers classified as being out of the labor force*[3]". "Persons not in the labor force who want a job now" is the standard U.S. Bureau of Labor Statistics definition of this share of workers and in general is a form of hidden unemployment or labour reserve (see Castillo 1998 for a detailed description). According to the US Current Population Survey, in September 2003 the number of workers who were out of workforce not looking for a job but that would like to work was about 4.6 million; it is worth stressing that the larger share of hidden unemployment lies between age 16 and 54, i.e. in the most productive part of the working life. Average duration of hidden unemployment is about 12 months in the CPS; nevertheless this group of workers do not show a higher labour market attachment. Following the Castillo analysis, *only 41 percent of nonparticipants who said they wanted a job in 1994 were in the labor force in 1995* as opposed to the strong labour attachments shown by traditional unemployed job-seekers: 78 percent continued to be labor force participants in 1995[4]. The hidden uemployment is a sort of temporary retirement from the job market and this choice depends on a lot of individual characteristics not least on the "discouraged worker" effect; according to the US BLS: *Discouraged workers are those persons who say that they want a job, were available to work, had searched for a job some time in the previous 12 months but had stopped looking for work because they believed that there were no jobs available for them.*

Once they decide to become job-seekers, persons out of the labor force have more difficulty in finding a job. This introduce a difference in the duration of unemployment between these two groups, which broadly we can define

[2] Blanchard and Diamond, 1989, page 3.

[3] Idem, page 4.

[4] This implies that the flow from job seeker to out of labour force is low and negligible. A point remarkable for our model.

as long and short term unemployment[5]. In general the distinction between attached and unattached workers is crucial for the analysis, as Blanchard and Diamond show, and both theoretical and empirical models should account for it.

Another relevant point concerns quits and layoffs with or without recall. Workers who leave voluntary the job are usually considered as retired from the labour force or not available for a "new hire", at least temporarily. Voluntary quits can also be driven by an employment-to-employment reason but even in this case this flow does not create new hires; Akerlof Rose and Yellen, 1986, calculate that 40 percent of workers quitting their job move directly to a new job without enter the unemployment status. More controversial concerns layoffs: *A worker who is laid off may remain attached to the firm in two distinct senses. One is that the worker is less available for employment elsewhere than the typical unemployed worker. The second is that the worker is available for recall by the firm without the need to post a vacancy*[6]. Moreover, Blanchard and Diamond assume that even the attached fired worker becomes unattached over time if not recalled or hired in another job. These points induce econometricians to exclude job losers on layoff from the stock of unemployment used to estimate the matching function but including persons out of the labor force but that would accept a job if offered, for the reason previously recalled.

Our paper tries to set-up a theoretical model of job search and matching on individual level providing also dynamical equations for the economy as a whole. The questions recalled so far induce to think that the standard dichotomy between employed and unemployed of the standard MP approach is too stringent and it should be relaxed. On the other hand the introduction of either element makes quite hard the development of an analytical body. By the point previously discussed it is possible to infer two questions: the importance of the introduction of "persons not in the labor force who would a job" in the total labour supply and the explanative hierarchy between job-losers and job seeker for the matching process. This twofold remark coming from current literature induce us to consider labour force as being composed by unemployed job-seekers, U, employed, E, and out of the labour force or unattached, R. It is worth stressing that this status can proxy for the long-run unemployment, as stressed by Blanchard and Diamond. Nevertheless we have also to identify how workers change their status; this is particularly important for the probabilistic model we will see in section 5. The previously

[5]Blanchard and Diamond find that long-term unemployment is a good proxy for the pool of workers out of the labor force (page 32).

[6]Blanchard and Diamond, ibidem, page 18.

cited Castillo analysis, points out the strong attachment of job seeker to labour force and so we can assume that the flow $U \rightarrow R$ is negligible and that the relevant one is $U \rightarrow E$. At the same time the flow $E \rightarrow U$ tends to be of a less importance in analyzing "new hires" because quits or employment-to-employment changes do not involve new vacancy posted and, further, because lay offs , as previously recalled, involve either strong attachment to the firm, for example because of firm or industry specific human capital, and in this case we are close to the employment-to-employment flow, or a low attachment. We follow the Blanchard and Diamond suggests of considering low attachment as a form of long term unemployment as opposed to the short term one characterizing workers in the U state. In order to simplify the model we are going to use the R state both for hidden and long run unemployment. In this way we can model attached workers, U, and non attached, R, or alternatively unemployed job seeker and long run unemployed. For such reason we assume $E \rightarrow R$ and obviously $R \rightarrow U$ since non attached workers become employed by passing through the job seeker status. In other words, a cyclical hierarchy of status will be considered, namely $U \rightarrow E \rightarrow R \rightarrow U$. We are ruling out intermediate steps as $U \rightarrow R$, $E \rightarrow U$ as of minor importance for the job search and matching process, as the cited literature points out. This strict hierarchy of states allows us to exploit a dynamic Monte Carlo technique to build a probabilistic model describing individual behaviour over time; moreover search intensity, driving the transition rates $U \rightarrow E$ and $V \rightarrow F$ - where V stands for the firm vacancy and F for filled - of both firm and worker, is made endogenous by a rational behaviour. From the micro-model we also obtain the differential equations describing average transitional path to the long run equilibrium - i.e. to the Beverdige curve - providing the dynamical analysis over the economy as a whole, an investigation close in spirit to the model developed in Blanchard and Diamond.

Our goal is hence to provide an alternative approach to the job search issue based on a probabilistic law that allows us to analyze coherently both the single unit history, i.e. the micro-dynamics, and on the economy as a whole, undertaking in this case analysis based on aggregate data.

In the last part of the paper we focus on how transition rates are affected by local spillovers or social networks. A part of job matches come from alternative channel than the usual posted vacancy; as recalled in Cahuc and Fontaine, 2002, " *a large proportion of people (about 50% on average) hear about or get their job through friends and relatives*". We will investigate on transitional path and steady state when the worker transition rate $U \rightarrow E$ depends in part on the her own search intensity and in part on her social network. In particular we are going to assume that the transition rate increases according to the number of employed person in a surrounding of the individual. This a typical spillover effect, close in spirit to the literature focussing on local interactions in the accumulation of human capital. It is worth stressing however, that, unlike current literature, our probabilistic model allows us to make endogenous the network structure, in sense that the number of employed workers changes over time according to the matching process of the economy. This has remarkable effects on results; social network can or not increase matching efficiency according to the weights put on the local component of the transition rates, parameters and initial conditions. A reduction in efficiency is possible when the matching process relies heavy on the local component and the social network tends to be emptied over time, i.e. the number of employed individuals reduce as a consequence of an increase in unemployment. This result induce more caution in stressing the importance of local effects on improving job matching than the one now recognized.

The paper is organized in four logical steps: the first one is devoted to the individual analysis where "optimal" transition rates are obtained according to a rational behaviour of firm and worker. The second one shows how obtaining macro equations by averaging over these individual rates. Subsequently the probabilistic model shows how we can obtain the micro-dynamics coherently with the macro one. In the last section of the paper, we show how the probabilistic micro-model allows to take into consideration "local" or "social network" components in the job search mechanism.

2 Single-Worker Analysis

We start by considering the individual working life. As pointed out in the previous section, we are going to divide the total labour supply in job seeker or attached workers, U, employed, E, and unattached or long run unemployed temporary not looking for a job but that would a job if offered, R.
Obviously $L = U + E + R$. Total labour supply L provides the metric for the flows hence $1 = u + e + r$ where $x = X/L$. K is the total number of firms in the economy; when $L = K$ full employment is possible, since we are going to assume, as in the MP spirit, one worker one firm. In such a case $F + V = L$ or $f + v = 1$ where obviously $f = e$.

A worker randomly drawn from the population occupies one of these three status; nevertheless the status will be ordered hierarchically since a worker is employed only after she was a job seeker, i.e. $U \longrightarrow E$. For reasons previously recalled, we assume that once the worker separates from the firm she does not became immediately a job seeker but stays in the state R before of entering again the state U; the average time of occupancy of the status R depends obviously on the transition rate from state R to U. As will be clearer later, this assumption of stages hierarchically ordered allows us to characterize the steady state of the job market as a balancing of flows out and in the U, E and R states; in this way the model fits the official partition of the work force in unemployed, employed and labour reserve.

Under a probabilistic point of view, the simplest way at modelling the single worker lifetime is a "three states process". Worker lifetime is a continuos jump in and out three possible states: U, E and R. Each agent is characterized by an own string of U, E and R recording her history over time; the length of time the individual stays in a given state is a random variable and the Poisson distribution is a good approximation of this occupancy time, as well known.

This process can be easily described in a probabilistic way: if the worker is unemployed but actively engaged in job search at time t, she can change her status to employed at the instantaneous rate α, and the probability that she will be effectively employed in the time interval $t+\Delta t$, is simply $\alpha \Delta t$[7]. Likewise, when the worker is employed, she has a probability $\beta \Delta t$ to be separated during $t+\Delta t$;finally an unattached worker has a probability $\mu \Delta t$ to be a job seeker during $t+\Delta t$. The coefficients α, β and μ are the transition rates; they can depend both on individual and firm characteristic, although we are going to treat them as exogenous parameters along this section.

When the worker changes her status, she stays in the new state for a random time τ; as said, this waiting time is well modelled by a Poisson process. Our worker is a job seeker for a given time T_1^U after then she becomes employed for another random time T_1^E then changes again to unattached for a time T_3^R and finally come back to the status of job seeker and so on. The dynamic of the probability to be in a given state at time t follows the Kolmogorov forward equations (see Cox and Miller, page 172):

$$\begin{aligned} \frac{dP^E(t)}{dt} &= -\beta P^E(t) + \alpha P^U(t) \\ \frac{dP^U(t)}{dt} &= -\alpha P^U(t) + \mu P^R(t) \\ \frac{dP^R(t)}{dt} &= -\mu P^R(t) + \beta P^E(t) \end{aligned} \tag{1}$$

with initial condition and $P^U(0) = 1 - P^E(0) - P^R(0)$ given. The above equations have a prompt interpretation; they describe the evolution of the

[7]More precisely is $\alpha \Delta t + o(\Delta t)$ where $o(\Delta t)$ means a function which tend to zero more rapidly than Δt. Intuitively this means that in a small time interval (ideally when Δt tends to zero) the process can undertake only a single change of state (point process).

probability over time according to a balancing of inflows and outflow. As an example, let us focus on the first one; the probability of finding a worker in the employed status increases with the inflows from unemployed to employed ($\alpha P^U(t)$) and decreases with the separation $\beta P^E(t)$.

The steady state solution is :

$$\begin{aligned} P^E &= \frac{\mu\alpha}{\beta\alpha + \beta\mu + \alpha\mu} \\ P^U &= \frac{\beta\mu}{\beta\alpha + \beta\mu + \alpha\mu} \\ P^R &= \frac{\beta\alpha}{\beta\alpha + \beta\mu + \alpha\mu} \end{aligned} \tag{2}$$

The steady state is a stable node and so the long run distribution does not depend on initial conditions; this means that if we observe a single worker job history over a sufficient long horizon, then the percentage of time spent in a given state, E , U, or R is close to the theoretical distribution. The strong convergence and the independence of initial conditions are valuable properties for empirical analysis.

From here we have the following lemma:

Lemma 1 *Over a sufficiently long time, the proportion of time spent in any state $i \in \{U, E, R\}$ converges in probability to P^i, where P^i is given by eqs. 2.*

Proof. See Cox and Miller, 1972, pages 172-175. ■

This lemma is particular useful to our scope; it says that if the worker lifetime is sufficiently long, as we are going to assume, then the individual knows the expected time of being employed, job seeker or out of work force. This property will be exploited to analyze individual behaviour.

If the worker lifetime would consist of two states only, U and E, then $P^E = \frac{\alpha}{\alpha+\beta}$ and $P^U = \frac{\beta}{\alpha+\beta}$; Blanchard and Diamond, page 7, call P^E as c, for cycle, in sense that it measures the degree of aggregate activity in steady state and $s = \beta c$, where s stands for shift, as an index of the intensity of reallocation in the economy.

3 Individual Choice

In the previous section we have modelled the transition probability for individuals as driven by exogenous parameters α, β.and μ. However human capital literature stresses the role of the investment in education in affecting these transition rates. In particular we focus on α, leaving β and μ exogenous. The transition probability from job seeker to employed is positively affected by the educational level: higher education induces higher transition rates. However the investment in education is performed before the entry in the job market (we leave aside on the job human capital accumulation) and so the simplest way at modelling the individual worker lifetime is to divide it in two periods, chronologically ordered. During the first one, the individual perform her choice about education. The educational choice is conditional on the expected return of joining the market; moreover human capital is irreversible and indivisible hence the choice can not be renegotiated further over time. It is worth stressing that the individual choice is not conditioned on the status of being unemployed, as happens in MP, but on the expected return of joining the job market.

As previously stressed, once joined the labour market, the individual lifetime is a continuos change over the three admissible status; Lemma 1 says that the total expected time spent in each state is given by the 2. Before joining the job market the individual has to invest in education; in order to simplify the model, we are going to assume that education is offered to individuals as a continuum of "school-packages" indexed by the amount of human capital α embodied in each package; α is also the cost of each package in effort term. In this way the worker must buy a given package before joining the labour market and spending the acquired human capital. The length of this first period is instantaneous; once bought the optimal package the individual enters the job market immediately. When employed the worker earns w per unit of time, while as job-seeker she incurs in a instantaneous search-cost $C(\alpha)$ proportional to the acquired human capital; the higher α, the higher the transition probability from U to E but heavier is the suffered loss. Finally we assume that when the worker is out of the work-force she earns neither w nor pays for the search cost $C(\alpha)$; in particular we simplify assuming zero value for this state.

By these assumptions, the optimal package α is chosen in order to maximize the discounted expected income stream over the working life:

$$\underset{\alpha_e}{Max} \int_0^\infty e^{-\rho t} \left[(P^E w - P^U C(\alpha_e) \right]^\gamma dt = \underset{\alpha_e}{Max} \frac{\left[(P^E w - P^U C(\alpha_e) \right]^\gamma}{\rho}$$

where probabilities are given in the 2 and $\gamma < 1$. By assuming $C(\alpha) = \alpha^2$, simple static maximization leads to:

$$\alpha_e^* = \frac{\sqrt{\mu\left(\beta^2\mu + w(\beta+\mu)\right)} - \beta\mu}{\beta+\mu} \tag{3}$$

Optimal educational effort is increasing in w. The supply curve in the economy is upward sloping: the average number of matches is $\alpha_\epsilon^* U$ and is increasing in w.

As far as firms is concerned, they have to choose the optimal search intensity α_f, i.e. the transition probability per unit of time of jumping from vacant to filled. In this case the MP approach applies: the arbitrage equations for, respectively, the filled V^F and vacant V^V status are:

$$\begin{aligned} rV^F &= y - w + \beta(V^V - V^F) + \dot{V}^F \\ rV^V &= -c(\alpha^f) - \alpha^f(V^V - V^F) + \dot{V}^V \end{aligned}$$

Perfect competition requires $\dot{V}^V = V^V = 0$ hence $V^F = \frac{c(\alpha^f)}{\alpha^f}$. By assuming a quadratic search cost we have that the firm value when filled coincides with the search intensity $V^F = \alpha^f$. As in Pissarides (pages 28 and 29) the only rational expectation equilibrium requires $\dot{V}^F = 0$ and this brings immediately to:

$$\alpha_f = \frac{y-w}{r+\beta} \tag{4}$$

the higher the wage the lower the search intensity. In this case the demand curve is downward sloping.

Wage determination comes from a market clearing condition instead of a bargaining process, as in MP; instantaneous market equilibrium requires that same number of matches comes form both sides of the market, i.e. $\alpha^* V = \alpha_e^* U$ or $\alpha^* v = \alpha_e^* u$ at the same wage w. By solving 3 and 4 jointly with the market clearing condition $\alpha^* v = \alpha_e^* u$, we obtain:

$$\alpha_e^* = \frac{\sqrt{\phi_1 u^2 + \phi_2 uv + \phi_3 v^2} - \mu\left(2\beta v + u(r+\beta)\right)}{2v(\mu+\beta)} \tag{5a}$$

where $\phi_1 = \mu^2(r^2 + 2r + \beta^2)$, $\phi_2 = 4\mu^2\beta(r+\beta)$, $\phi_3 = 4\mu(\beta^2\mu + \mu + \beta)$. It is straightforward to see that α_e^* is always positive.

Given the individual transition probability as in the 5a, the average instantaneous matching function for the entire economy is $m = \alpha_e^* u$, i.e.:

$$m \equiv \alpha_e^* u \equiv \alpha^* v = \frac{\sqrt{\phi_1 u^2 + \phi_2 uv + \phi_3 v^2} - \mu\left(2\beta v + u(r+\beta)\right)}{2(\mu+\beta)} \frac{u}{v} = m(u,v) \tag{6}$$

Although a bit cumbersome, the matching function is increasing, concave and linearly homogenous of degree one in u and v, as in the spirit of the MP approach.

4 Macro-Dynamic Analysis

The model is essentially dynamic in its nature and in this section we are going to obtain the differential equations characterizing the average dynamics; given an initial condition, the job market evolves towards the steady state represented by a point on the Beverdige Curve.

Given the instantaneous transition rates 5a we derive the differential equations for the economy by averaging over firms and workers. For a job seeker, the average probability, per unit of time, of changing status from unemployed to hired ($U \to E$) is α_e^* times the probability of finding an individual in the unemployment status, P_U, i.e. $\alpha_e^* P_U$, where $P_U = U/L$. In the time interval Δt, corresponding to L elementary steps[8], the average number of unemployed being employed is $\alpha_e^* P_U L \Delta t$; hence, the number of unemployed decreases at the rate $\Delta U/\Delta t = -\ \alpha_e^* P_U L = -\alpha_e^* uL$. The number of unemployed instead increases by μR. Likewise the number of vacancy decreases at the rate $\Delta V/\Delta t = -\ \alpha^* P_V L = -\alpha_e^* \frac{u}{v} P_V L = -\ -\ \alpha_e^* uL$. Clearly the average matching function m must be the same both for unemployed and vacant firm since that, once a match is closed, both variables decrease by one unit as we assume, as in MP, one worker - one firm. Conversely the number of vacancies increases, in the unit of time, when a separation occurs, i.e. $\beta(1-u-r)$. Finally the number of unattached workers increases by $\beta(1-u-r)$ and decreases by μR.

Summing up, we obtain the following dynamical equations:

[8]For computational purposes we normalize the number of elementary units to the size of the economy.

$$
\begin{aligned}
\frac{du}{dt} &= -m(u,v) + \mu r \\
\frac{dv}{dt} &= -m(u,v) + \beta(1-u-r) \\
\frac{dr}{dt} &= \frac{dv}{dt} - \frac{du}{dt} = \beta(1-u-r) - \mu r
\end{aligned} \tag{7}
$$

$$
u(t) + e(t) + r(t) = 1 \tag{8}
$$

$$
e(t) + v(t) = k \tag{9}
$$

By imposing the steady state condition $\frac{dv}{dt} = 0$ in the second equation and substituting from the third we obtain the long run relationship between vacancy and unemployment, i.e. the Beveridge curve.

The model has three equations in two unknowns, since $r(t)$ is obtained by the previous two, leaving the dynamics essentially driven by $u(t)$ and $v(t)$; the determinant of the Jacobian matrix of eqs. 7 is zero and there are two complex roots with a negative real part while the third is zero. In such a situation, all solutions converge to a three-dimensional manifold with the consequence that the steady state, as well as the transition path, depends on initial conditions (see for example Kamien and Schwartz page 347). The Beveridge curve is the projection of such a manifold in the bidimensional plane u, v. This makes impossible to analyze dynamics by the standard phase-diagram technique and numerical solutions must be carried out.

In order to calibrate parameters, we are going to use data by the US Labour Bureau, and in particular by the recent monthly Job Openings and Labor Turnover Survey (JOLTS)[9]; unfortunately the survey does not cover period before December 2000. Data on unemployment comes from the national unemployment rate seasonally adjusted. Figure 1 plots the monthly series trough January 2001 and November 2003 jointly with and estimated tendency line.

[9]See Monthly Labor Review, December 2001, for details.

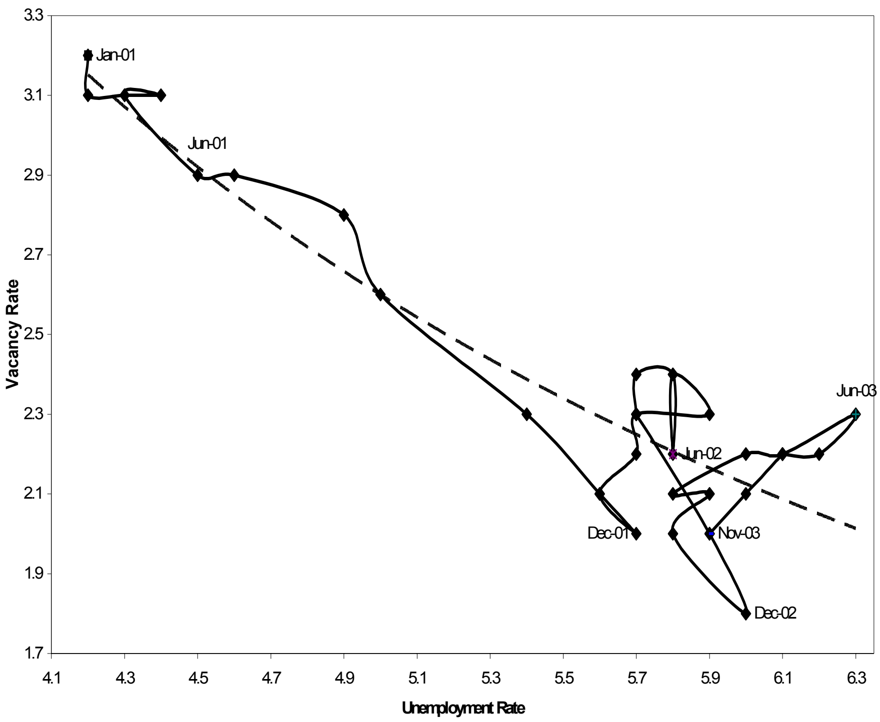

Figure 1: U.S. Beveridge Curve

Although the figure is quite fuzzy, during 2001 the inverse relationship between v and u is rather stable. From January 2002 to November 2003 the curve undertake some rapid fluctuactions around the tendency line.

We calibrate parameters on the estimated beveridge curve of Figure 1 where the unemployment rate spans between 4.2 and 6.3 percent while the vacancy rate between 3.2 and 2.0 percent. Figure 2 shows the calibrated Beveridge Curve and some transitional paths.

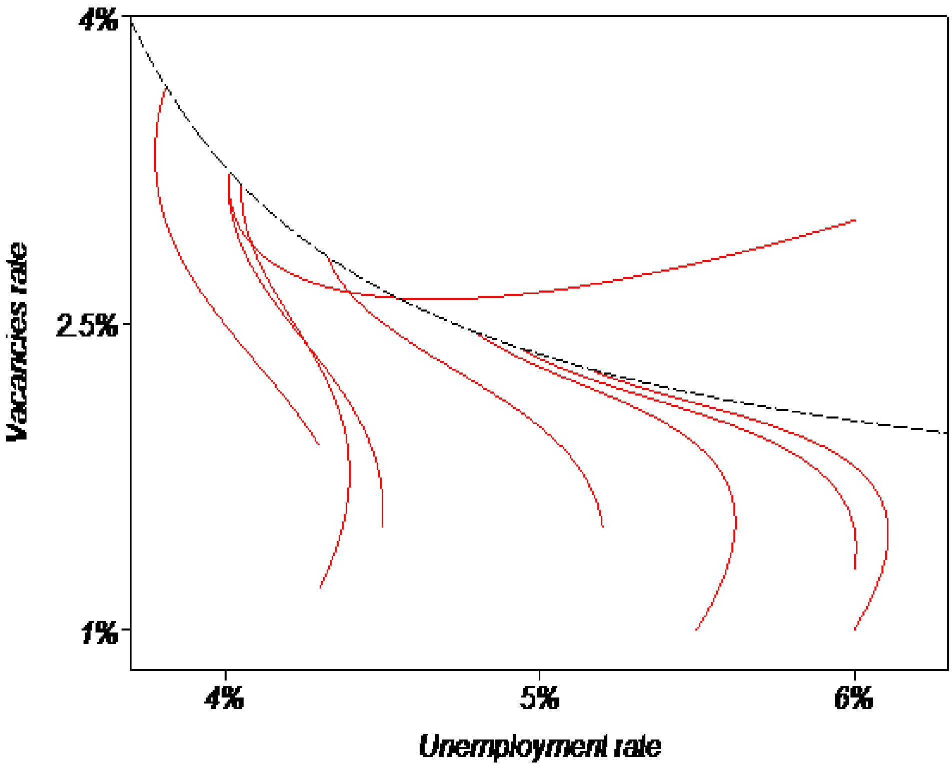

Figure 2: Calibrated Analysis

Transitional paths can be either clockwise or anti-clockwise depending on initial conditions; in general starting from a vacancy rate below 1.9% and unemployment higher than 5.4% involves anti-clockwise dynamics. Vacancy higher than 1.9% involve clockwise dynamics whatever the initial unemployment rate. There is an intermediate parameters region, broadly $v \leq 1.9\%$ and $u \leq 0.54$ where the transition path follows a "S" path.

This sort of "path-dependency" of the system is interesting in the comparison of the working of the job market and equilibrium unemployment across countries: long run differences are induced not only by a different institutional setting or search mechanism but by different initial conditions as well. It is worth stressing that a similar idea can be found in Blanchard and Diamond, although they do not formalize it; in particular they stress that in a real job market movements into and out of unemployment are more complex than the standard job search model, since these flow not necessarily involve posted vacancies (this is the case of strongly attached workers). In this case: "*what happens to vacancies and unemployment after a shock will depend on the initial stocks of attached and unattached workers, which themselves will depend on the history of the shocks..... Whether aggregate ac-*

tivity shocks generate counterclockwise movements in the Beveridge space is much more ambiguous[10]." Although these authors points out the shock response of the Beverdige curve with the consequent transitional paths, they observe that the distinction between attached and unattached workers make the dynamics dependent on past history. The same holds for our model; the type of transitional path is strongly affected by the initial condition on the r status, i.e. the one modelling unattached worker as opposed to the strong attachment shown empirically by job-seekers.

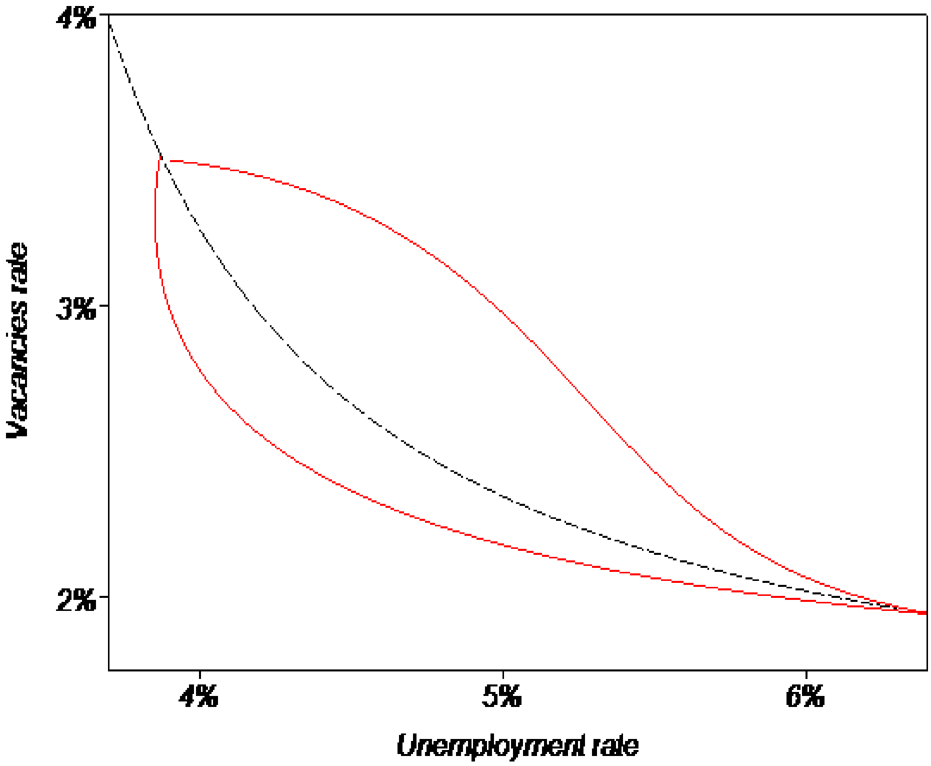

Figure 3: Spurious Cycle

Although the dynamical model does not allow to obtain endogenous cycles around the Beverdige Curve, it is possible identify proper initial conditions and parameters producing dynamical paths that behave much like as cycle. Figure 3 shows this type of spurious cycle; the dependence on initial conditions makes hence rather hard to distinguish, in real data, a genuine cycle from a shift in parameters.

This richness in transitional dynamics holds also in response to a parameter shock; Figure 4 shows a simulation where β increases by 20%. As

[10]Blanchard and Diamond, 1989, pages 19-20.

expected the Beverdige Curve shifts to right reducing matching efficiency; the adjustment is counterclockwise but other paths are possible according to the initial conditions, as previously stressed. This type of exercise is close in spirit to Blanchard and Diamond figure 2; an increase in β in our model corresponds to a shock on the shift s parameter measuring the intensity of reallocation. However, unlike these authors, our model provides more rich dynamics and, as said, counterclockwise adjustment is possible but it is not the only case.

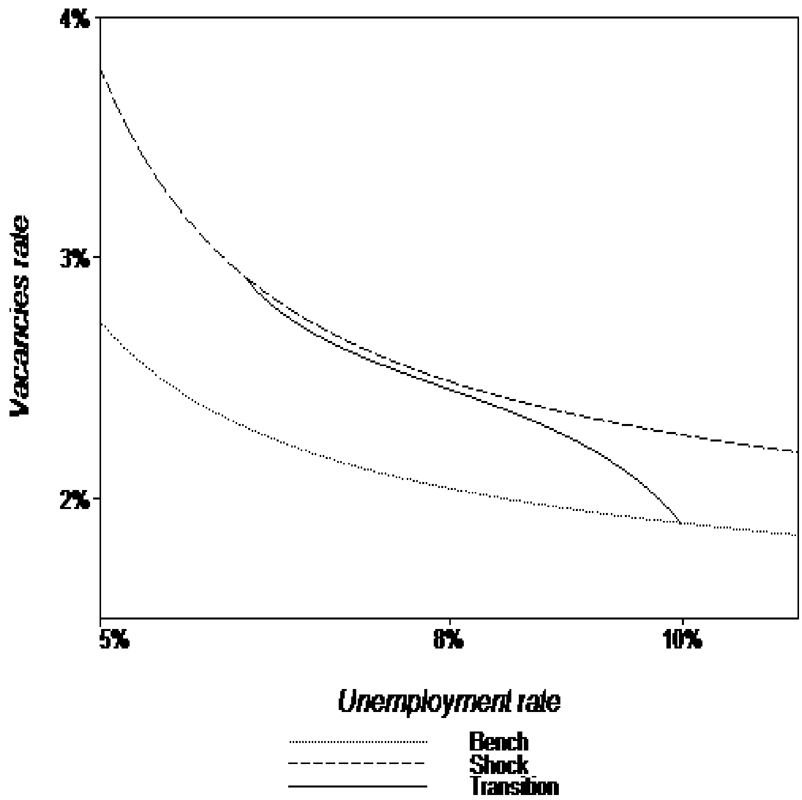

Figure 4: Shock Response

More complex transitional paths can be obtained adding to the matching function a temporal trend and/or some trigonometric function capturing waves and cycles[11]; nevertheless this is out of our scope and we do not go further on this.

[11]We do not show such simulations for brevity.

5 Micro-Dynamic Analysis

In the previous section we have obtained the dynamical equations describing the behaviour of the averages (macro-variables) over time. In this section we show the microeconomic behaviour driving the micro one. In other words, we are going to set-up a microeconomic world whose averages behave exactly as the equations obtained in the previous section; if both ways of obtaining the average dynamics produce same results then we conclude that the macro and micro-analysis are coherent to each other; in this case we can indifferently investigate both the macro-world and the life history of a single agent, depending on our research goal.

Probabilistic approach at modelling our problem involves, theoretically, the analysis of the Kolmogorov equations for the entire economy, as we did for a single agent. Unfortunately the dependence of the transition rates on u and v makes very hard or impossible managing it. This type of problems are common to other disciplines making use of a probabilistic approach to systems characterized by a large number of states. Recently epidemiologists working on the spreading of diseases across individuals have exploited a dynamic Monte Carlo algorithm which can be fruitfully applied both to the numerical solutions of the probabilistic models used to forecast the evolution over time of diseases and to the estimation of individual transition rates; a good description of these methods is in Gilks et al., 1996. In particular we are going to follow the Monte Carlo algorithm presented by Aiello et al., 2000 and 2001. For a technical and detailed description of the algorithm and its probabilistic features the reader should refer to the original papers; here we present the main steps of the methodology applied to our problem of job search.

We sum up the model basic assumption:

1. Workers amount to L and firm to K and the technology is 1:1, so full employment is theoretically possible when $L = K$. At the beginning, workers are divided in three sub-populations: employed E_0, unemployed looking for a job U_0 and long run unemployed or unattached workers, R_0. Firms are divided in vacant V_0 and filled F_0. If $L = K$ then $U_0 + E_0 + R_0 = L = V_0 + F_0$ and $F_0 = E_0$.

2. Workers and entrepreneurs are completely separated and there is not flow from one to other; we can assume that this partition of the population is based on different abilities, managerial skills, business risk aversion and so on. The population is stationary.

3. When an unemployed worker meets successfully a vacant firm they

become a productive unit labelled by the status M. Matched units can separate at the exogenous rate β. After separation, the worker enter the state R - she comes out from this state as a new seeker at the exogenous rate μ.− and the firm becomes V. By doing so, workers have three possible stages hierarchically ordered, i.e. $U \rightarrow M \rightarrow R$ whilst the firm has only two $V \rightarrow M$.

4. As in MP, firms and workers do accept the match, no matter who is the proponent; we are interested in counting the number of matches and separations over time, and updating the set $\{U, M, R, V\}$ consequently, and it does not matter who is "driving" the match. There is a strict duality between workers and firms spaces.

5. Transition rates from U to M and from V to M are given by eqs 3 and 4. From M to R and M to V by the exogenous separation rate β. Finally the exogenous parameter μ is the transition rate from R to U.

The DMC algorithm allows us to investigate this structure over time. As stressed at point 4, it does not matter who is leading the matching process; what is important is that when a match occurs in the economy, the sub-populations are adequately updated. Since we are interested in analyzing the search mechanism on the side of the individual worker, we are going to focus on this population. Workers are distributed into three sub-populations initially randomly allocated in a one-dimensional vector; workers are hence represented as a random collection of indexed individuals $U, U, M, R, M, U...$ on a one dimensional space. The dual vector representing firms is $V, M, M, V, V, M....$.

1. At each elementary time step l one individual is randomly drawn from the population; let us suppose an unemployed worker. This agent could change her status from unemployed to matched according to α_e^* but from a probabilistic point of view there can be other agents in the economy whose probability of changing the status is higher than the one characterizing our individual. So in deciding whether effectively the drawn individual will undertake a change, we have to compare her transition rate with the higher one in the population, i.e. $p_U = \alpha_e^*/W_{Max}$ where $W_{Max} = \sup P, .P = \{\alpha_e^*, \beta, \mu\}$ The method suggested in Aiello et al., 2000, is to compare p_U against a random number r in the closed unitary interval taken from a uniform distribution; the uniform assumption models a situation where every agent has the same a-priori probability of a jump; this means that when $p_U < r$ the real situation is less informative than the one characterized by a total ignorance

about individual transition rates. Hence, if $p_U > r$ the drawn individual changes her status otherwise she stays in the original one. When the individual changes the status the sub-populations U, V, M, R are consequently updated.

2. The waiting time between two consecutive jumps follows a Poisson process at rate $r = \alpha_e^* U + \beta M + \mu R$.

3. After the delay time, a new draw is performed and the simulation restarts.

The result of this Markov Chain[12] is the convergence of the sub-populations to a long run probability measure whose first moment is given by the deterministic equations 7. In the following section we show some simulations.

6 Results

We have performed 20 replications of the Monte Carlo method described in the previous section; we have chosen 20 runs because it is enough to obtain a smooth curve to be compared to the deterministic solution. Figure 5 shows the adjustment path obtained by the average equations and the average of 20 simulations obtained by the micro-model[13]. As the reader can see, the fit is satisfactory[14].

[12]The dual DMC is the one where we analyze firms instead of workers but the two chains lead obviously to same results.

[13]We plots only one transitional path for brevity, being the numerical computation highly consuming in time. For this reason we graph a transition path broadly following US data in 2001.

[14]A closer and smoother curve can be obtained by averaging over a higher number of simulations. Nevertheless this is very time-consuming.

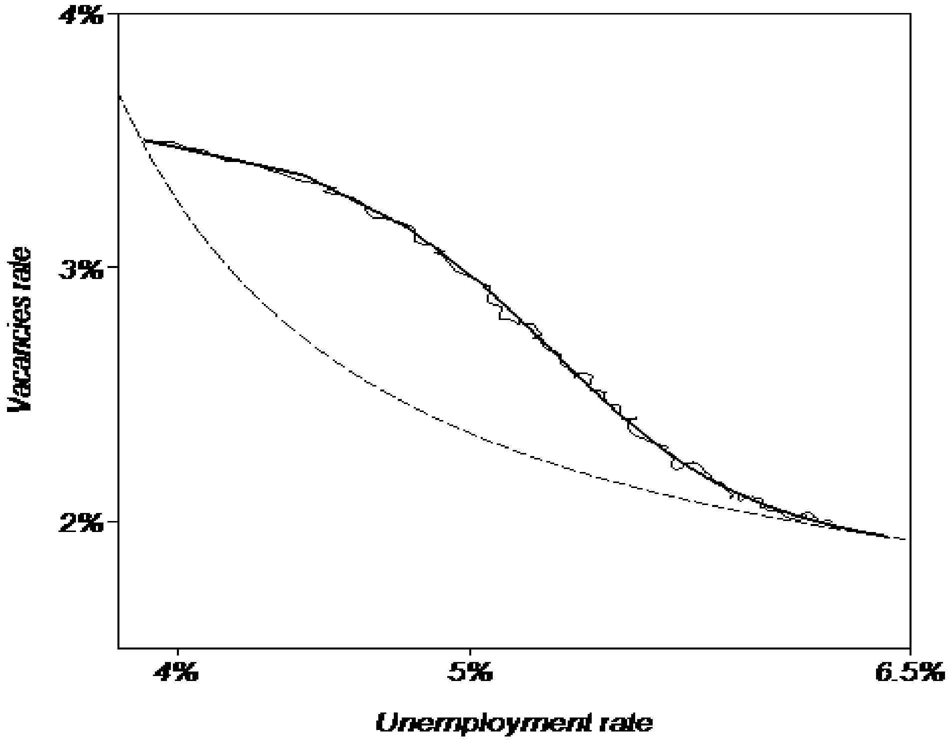

Figure 5: Stochastic vs Deterministic Solution

By doing so, we can describe the adjustment path to the Beveridge curve indifferently by the macro-equations and the microeconomic model; both analytical approach provides same results. Under this point of view, the macro model is well founded in the microstructure. The probabilistic law characterizing the micro-behaviour is working properly: it does provide same dynamic path of the deterministic rules driving the first moment of the distribution.

7 Local Effects and Individual Behaviour

Once built a suitable analytical tool that allows us to obtain average dynamics from the micro-structure, it can be used in a very flexible way to modify the individual behaviour. In this section we show how we can account for local interactions in the search mechanism. It is not hard to assume that matching can depends on "local" variables. Local interactions, or social networks, are often invoked in the human capital literature for explaining "club convergence" and inter-generational inequality persistence. In the search model social network are analyzed in Calvo-Armengol and Zenou 2001, Cahuc and Fontaine, 2002 where the social network is used to spread the job offer into the network, reducing the individual searching cost.

We are going to assume a rather simple local effect; individual matching rates depends both on aggregate variables and on the number of workers in a neighboring of the unemployed individual. The higher the number of working agent in a surrounding of the unemployed worker, the higher the matching rate. We can think to a sort of positive spillover or complementarity effect. Let us define p as the probability that an unemployed worker becomes working due to the presence of a worker in the neighbor and so $(1-p)^n$ is the probability, per unit of time, for a unemployed not to become working if she has n working neighbors hence $(1-(1-p)^n)$ is the probability, per unit of time, for a unemployed to be hired if she has n working neighbors. When n is large the local term tends to 1 whilst when $n=0$ the local term is zero (no-spillover); an increase in n makes higher the transition rate from U to M. The individual transitions rates modify in:

$$U \rightarrow M \quad : W_U = \Gamma\alpha^*(u,v) + \Lambda(1-(1-p)^n)$$

where $\Gamma + \Lambda = 1$ in order to balance and local effects. Under this point of view, homogeneity of degree one for the matching function involves only the microfounded term since the local effect is a pure externality. It is worth stressing that, unlike Calvo-Armengol and Zenou 2001, Cahuc and Fontaine, 2002[15], in our approach n varies across workers and over time; this implies that there is a distribution of matching rates at each time t. This characteristic is allowed by the probabilistic algorithm developed in the previous section and it represents a novelty in the job-search literature.

The effect of the local spillover is summarized in figure 6; this shows the adjustment path without local variables (the dashed line), the path arising by assuming $p=0.5$ (the lowest one) and the one resulting by assuming $p=0.1$ (the highest one). As far as Γ, this has been held constant on 0.2 in order to put more weight on the local term. The network consists of 12 individuals centered on the $i-th$ worker; six on the left and the remaining six on the right.

$$\underbrace{\overbrace{(i-th)-6;(i-th)-5;........}^{Left}(i-th).\overbrace{........(i-th)+5;(i-th)+6}^{Right}}_{Social\ Network}$$

In other words, we have tried to compare the benchmark solution with a two opposite situations: in the first one, the local term is rather strong since

[15]These authors assume that each worker is re-matched at random with n workers each time a job offer arrives.

the probability $p = 0.5$ is relatively high. This corresponds to an individual for whom the social network is important for the search mechanism. On the contrary, when $p = 0.1$ the network is weak and this induces a negative effect on matching efficiency, as the figure shows.

Whether the social networks increases or not the match efficiency depends on parameters; when p is sufficiently high the local network increase individual probability to be hired, employment increases hence n grows making the local component more powerful. This virtuoso circle increases matching efficiency respect to the case without spillovers. But when p is not sufficient to start the positive feedback, it works in a reverse way, reducing n. The final result is that, for suitable parameters Γ and p, the social network reduces matching efficiency both along the transitional path and the steady state equilibrium.

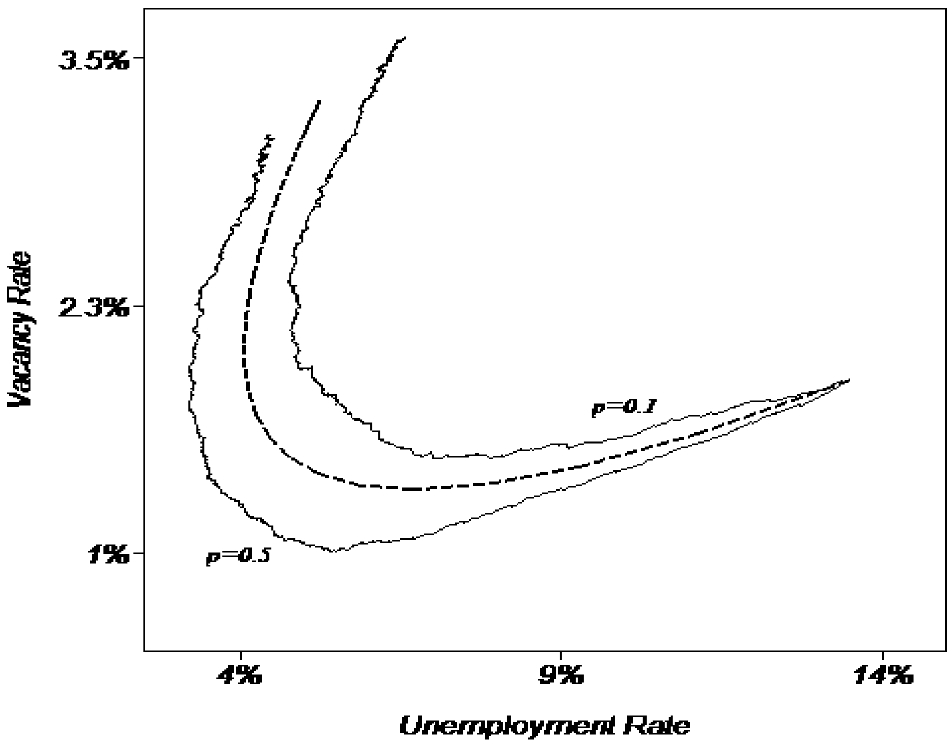

Figure 6: Social Network Effect

This result depends on the endogeneity of the social network. This is made clearer by comparing the distribution of the network at the steady state when $p = 0.5$ and $p = 0.1$; results are in figure 7. It is rather clear that the distribution when p=0.5 is more concentrated in the right part, at meaning the highest weight that the network has in this situation

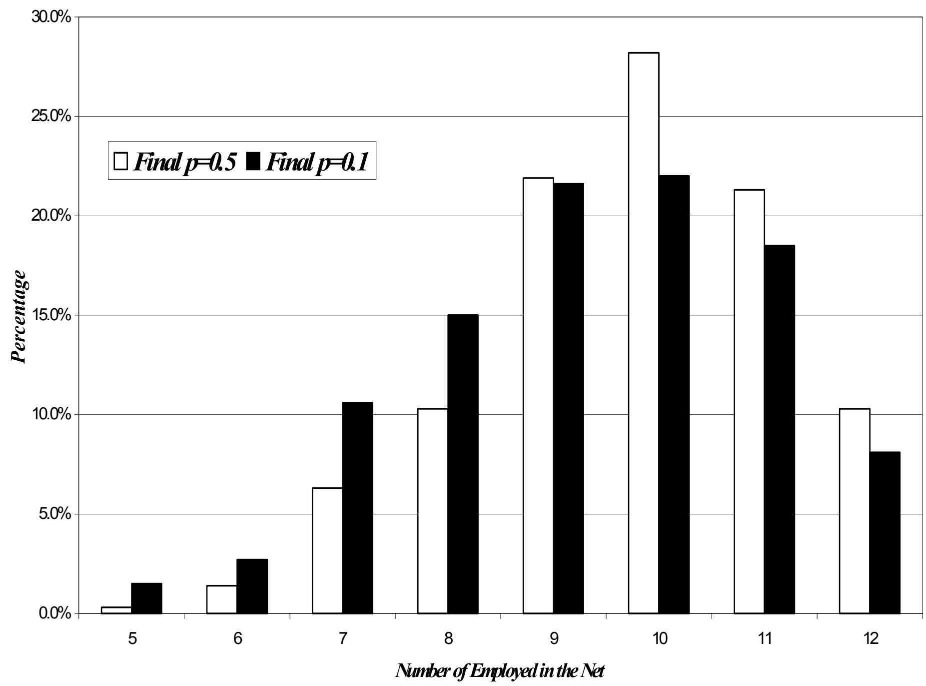

Figure 7: Network Distribution

Concluding, when we account for endogeneity of the social network composition, it is not so clear whether local effects are undoubtedly positive for the matching mechanism but in order to investigate such a possibility we have to focus on the single worker lifetime with proper tools. This paper is a first answer to the question.

8 Conclusions

This first attempt at modelling job search at individual level has interesting potentialities that the macro-dynamics can not capture. The debate on investment in human capital and the effects on job search and quitting probabilities can only be faced by analyzing the single worker; in particular, neighborhood effects can have important effects on the dynamics of the job market. The approach we are developing fits particularly well in coping with these spillover effects. Individual transition rates can depend on the social and economic neighbor of the individual and in general there is heterogeneity in such rates; this brings to multiple equilibria and to dynamical path which are quite different by the ones we can describe by the aggregate equations. In a more mature stage of our work we will exploit such interesting opportunity.

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