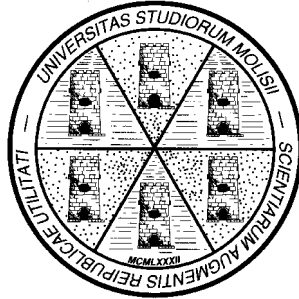


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**Money, Growth and Finite Horizons**

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# Money, Growth and Finite Horizons\*

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## Abstract

The implications of endogenous labor supply for money superneutrality in OLG economies are analyzed. Inflation increases capital and output, while it affects labor ambiguously in a closed economy. Inflation reduces capital and output, but stimulates wealth in an open economy.

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*Keywords:* Inflation; Capital accumulation; Labor supply; Overlapping generations.

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# 1 Introduction and summary

In a context of infinite-lived agents, endogenous capital formation and elastic labor supply, Wang and Yip (1992) discover that inflation is negatively associated with capital stock, labor and output in the long-run. This reversed Tobin effect, which holds qualitatively within various approaches to "money and growth", depends on the labor supply endogeneity.

The implications of endogenous labor supply for money superneutrality have not yet received attention within nonaltruistic life-cycle setups.<sup>1</sup> When labor supply is inelastic, an OLG economy with disconnected generations is characterized by a Tobin effect, i.e. a positive effect of inflation on capital and output (see Drazen, 1981, and Van der Ploeg and Marini, 1988).

The purpose of this paper is to investigate the consequences of anticipated inflation on capital formation and economic development in life-cycle models when labor supply is elastic. We discover that in a closed economy long-run inflation stimulates capital formation and raises output, but moves labor ambiguously. In this case, we depart substantially from the Wang and Yip (1992) results. In a small open economy, instead, anticipated inflation reduces capital, labor and output, but stimulates nonhuman wealth and consumption. Thus, the Wang and Yip (1992) results on the reversed Tobin effect can also be obtained in a life-cycle small open economy.

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<sup>1</sup>See Orphanides and Solow (1990) and Wang and Yip (1992).

## 2 Closed economy

### 2.1 The model

Consider a monetary closed economy, whose demand-side derives from the nonaltruistic OLG approach of Blanchard (1985). Individuals face a constant mortality rate  $\theta$ . Population, composed of the cohorts of different ages, is constant. Labor is elastically supplied and money balances enter the utility function of consumers. If preferences are logarithmic, the aggregate labor supply and money demand are given by

$$1 - L = \frac{\beta C}{\alpha w} \quad (1a)$$

$$M = \frac{\gamma C}{\alpha(r + \pi)}, \quad (1b)$$

where  $L$ ,  $C$ , and  $M$  denote labor hours, consumption and real money balances, respectively;  $w$ ,  $r$  and  $\pi$  define the hourly real wage, the real interest rate and the inflation rate, respectively;  $\alpha$ ,  $\beta$  and  $\gamma$  are preference parameters.

The law of motion of consumption in the Blanchard (1985) setup is given by the following intertemporal arbitrage relationship

$$r = \rho + \frac{\dot{C}}{C} + \alpha\theta(\theta + \rho)\frac{(K + M)}{C}, \quad (1c)$$

where  $K + M$  denotes consumers' nonhuman wealth,  $K$  physical capital, and  $\rho$  the subjective rate of time preference (exogenous).

Output  $Y$  is produced competitively by means of a well-behaved and linearly homogeneous production function:  $Y = F(K, L) = Lf(k)$ , where  $k = \frac{K}{L}$ ,  $f' > 0$  and  $f'' < 0$ . Maximum profit requires

$$f'(k) = r \tag{2a}$$

$$f(k) - kf'(k) = w. \tag{2b}$$

The monetary authority keeps the nominal money supply growth rate,  $\mu$ , constant. Hence real money balances evolve over time as follows:  $\dot{M} = M(\mu - \pi)$ . The government keeps the budget balanced by transferring seignorage to the public in a lump-sum fashion:  $X = \mu M$ , where  $X$  denotes government lump-sum transfers. Finally, the goods market equilibrium requires:

$$Y = Lf(k) = C + \dot{K}. \tag{3}$$

## 2.2 Steady state effects of inflation

The analysis is only concerned with the steady state. Substituting (1b) into (1c) for  $\bar{M}$ , using (2a) and (3), we obtain

$$f(\bar{k}) = \frac{\alpha\theta(\theta + \rho)}{[f'(\bar{k}) - \rho]} \left\{ \bar{k} + \frac{\gamma f(\bar{k})}{\alpha[f'(\bar{k}) + \mu]} \right\}, \tag{4}$$

where overbars denote long-run values and  $\bar{\pi} = \mu$  has been used.

Totally differentiating (4) yields

$$\frac{d\bar{k}}{d\mu} = -\frac{\alpha\theta(\theta + \rho)\bar{M}}{\Delta\bar{L}} > 0,$$

where  $\Delta = (\bar{r} + \mu)ff'' + \theta(\theta + \rho)\left[\alpha\frac{\bar{M}}{\bar{L}}f'' - \gamma\bar{r} - (\bar{r} + \mu)\alpha\frac{(\bar{w}\bar{L} - \bar{r}\bar{M})}{\bar{C}}\right] < 0$ .<sup>2</sup>

An increase in  $\mu$  raises capital intensity in the long-run. This is because higher inflation, which implies greater seignorage distributed by the government to consumers, causes a redistribution of income from the older people (who consume more and save less) to the younger people (who consume less and save more). Aggregate saving is increased. Therefore capital formation is spurred (as capital is the only alternative asset to money, namely the taxed asset) and the capital-labor ratio is increased. This in turn reduces the real interest rate and pulls the wage rate up.

In order to investigate what happens to labor hours, we substitute consumption from (3) into (1a) and, after using (2), obtain

$$\bar{L} = \frac{\alpha s(\bar{k})}{[\alpha s(\bar{k}) + \beta]}, \quad (5)$$

where  $s(\bar{k}) = \frac{[f(\bar{k}) - \bar{k}f'(\bar{k})]}{f(\bar{k})}$  represents the labor share of national income. According to (5), there exists a positive relationship between  $\bar{L}$  and  $s(\bar{k})$ ; the labor share in turn depends on capital intensity. The sign of the functional relationship between  $s$  and  $\bar{k}$  depends on the elasticity of factor substitution  $\sigma = -\frac{\bar{w}f'(\bar{k})}{\bar{k}f(\bar{k})f''(\bar{k})}$  as  $s' = -\frac{\bar{k}f''(\bar{k})f(\bar{k})}{f(\bar{k})^2}(1 - \sigma)$ . If  $\sigma > 1$ , the increase in capital intensity determined by a rise in  $\mu$  results in a reduction of  $s(\bar{k})$  and

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<sup>2</sup> $\Delta < 0$  by saddle-point stability of the steady state.

hence labor hours. In the case of a Cobb-Douglas technology, i.e.  $\sigma = 1$ , the labor share of national income is independent of  $\bar{k}$  implying that labor remains unaffected by a change in  $\mu$ . Finally if  $\sigma < 1$ , the higher  $\mu$ , by raising  $s(\bar{k})$ , results in a rise of manhours.

Output and consumption are increased. The nominal interest rate is driven up, while real money balances may rise or fall. Seignorage is unambiguously increased.

### 3 Small open economy

#### 3.1 The model

Extend the analysis to a small open economy facing a perfect world capital market.<sup>3</sup> Since  $r$  is fixed by the exogenously given world interest rate  $r^*$ , the input demand system (2) implies that capital intensity and the wage rate are fixed; that is,  $\frac{K}{L} = \kappa^*$  and  $w = \omega^*$ , where  $\kappa^* = f'^{-1}(r^*) > 0$  and  $\omega^* = f(\kappa^*) - \kappa^* f'(\kappa^*)$ . The household asset menu is now composed of physical capital, real money balances, and net foreign assets,  $B$ . The current account gives the rate of accumulation of net foreign assets

$$\dot{B} = Y - C - \dot{K} + r^* B. \quad (6)$$

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<sup>3</sup>Moreover, this economy produces a single tradable good, which is perfectly substitutable with the foreign-produced good, and operates in a flexible exchange-rate system.

### 3.2 Steady state effects of inflation

The long-run economy is described by the system

$$\bar{M} = \frac{\gamma \bar{C}}{\alpha(r^* + \mu)} \quad (7a)$$

$$\bar{C} = \frac{\alpha\theta(\theta + \rho)}{(r^* - \rho)} (\bar{K} + \bar{M} + \bar{B}) \quad (7b)$$

$$\bar{K} = \kappa^* \left(1 - \frac{\beta \bar{C}}{\alpha\omega^*}\right) \quad (7c)$$

$$\bar{C} = \frac{\alpha [\omega^* + r^*(\bar{K} + \bar{B})]}{(\alpha + \beta)}, \quad (7d)$$

where (1a) has been used. Lump-sum transfers are solved residually.<sup>4</sup>

By substituting  $\bar{M}$  from (7a) into (7b) and solving for  $\bar{K} + \bar{B}$ , we get

$$\bar{K} + \bar{B} = \frac{(r^* - \rho)\Psi(\mu)}{\alpha\theta(\theta + \rho)} \bar{C}, \quad (8)$$

where  $\Psi(\mu) = 1 - \frac{\gamma\theta(\theta + \rho)}{(r^* - \rho)(r^* + \mu)} > 0$  and  $\Psi' > 0$ . By plugging (8) into (7d) for  $\bar{K} + \bar{B}$  and totally differentiating, we obtain<sup>5</sup>

$$\frac{d\bar{C}}{d\mu} = \frac{r^*(r^* - \rho)\Psi' \bar{C}}{[(\alpha + \beta)\theta(\theta + \rho) - r^*(r^* - \rho)\Psi(\mu)]} > 0.$$

An increase in  $\mu$  expands consumption and, through (7b), nonhuman wealth. The higher consumption implies a reduction of labor and, through

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<sup>4</sup>As the stock of nonhuman wealth  $\bar{K} + \bar{M} + \bar{B}$  is assumed to be strictly positive, i.e. if  $B$  is negative it is not too negative, the condition  $r^* > \rho$  is assumed to be satisfied.

<sup>5</sup>Saddle-point stability requires:  $(\alpha + \beta)\theta(\theta + \rho) > r^*(r^* - \rho)\Psi(\mu)$ .



(7c), capital stock. Real money balances fall and  $\mu \bar{M}$  increases. Moreover, since nonmonetary wealth is increased and  $\bar{K}$  and  $\bar{M}$  are reduced, an increase of net foreign assets takes place.

In this small open economy, a Tobin effect on nonhuman wealth is accompanied by a reversed Tobin effect on capital and output. The Tobin effect on nonhuman wealth derives from the saving-promoting redistribution of resources across generations caused by government transfers. The reversed Tobin effect on capital is due to consumption, which, once pulled up by the rise in nonhuman wealth, drives manhours and capital down.<sup>6</sup>

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<sup>6</sup>When labor supply is inelastic, capital stock is fixed by  $r^*$ . A rise in  $\mu$  does not change capital and output, but increases foreign assets and consumption.

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