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A Reduced Rank Regression Approach to Coincident and Leading Indexes Building

by

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Abstract

This paper proposes a reduced rank regression framework for constructing coincident and leading indexes. Based on a formal definition that requires that the first differences of the leading index are the best linear predictor of the first differences of the coincident index, it is shown that the notion of polynomial serial correlation common features can be used to build these composite variables. Concepts and methods are illustrated by an empirical investigation of the US business cycle indicators.

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Keywords: Coincident and Leading Indexes, Polynomial Serial Correlation Common Feature, Reduced Rank Regression.

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1 Introduction

In a large number of countries coincident and leading indexes are routinely built in order to provide economic analysts with early signals of the broad swings in macroeconomic activity known as the business cycle. These indexes are typically constructed in two steps. The first step aims at identifying groups of variables that move in, before or after the recession (see e.g. Niemera and Klein, 1994). In this paper the focus is on the first two groups of variables, which are respectively defined as the coincident and leading indicators. The second step consists in forming composite indicators, namely the Coincident Index [CI] and Leading Index [LI], in order to extract the relevant business cycle features from the individual indicators.

Among the various statistical methods for constructing such CI and LI, the procedure developed by Stock and Watson (1989, 1991, and 1993) for the NBER has rapidly become a standard reference. But other approaches exist since a while, from the well known principal component and classical linear time series analyses to more complex non-linear methods such as smooth transition regressions, switching regimes and probit models, and nonparametric procedures (see Camacho and Perez-Quiros, 2002, for a comparison of the forecasting performances of some of these procedures).

In a similar spirit as Emerson and Hendry (1996), the viewpoint in this article is that the construction of coincident and leading indexes should be based on a formal statistical analysis of the multivariate time series properties of the data. Hence, a Reduced Rank Regression [RRR] approach is proposed to build a CI&LI from a vector of cointegrated economic indicators. RRR has been extensively analyzed in the statistical and macroeconometric literature (see *inter alia* Anderson, 1984; Velu *et al.,* 1986; Ahn and Reinsel, 1988; Tiao and Tsay 1989; Johansen, 1995) but, to the best of my knowledge, it has not yet been applied for the problem at hand. This seems a promising route to follow since there is convincing evidence (see *inter alia* Reinsel and Ahn,1992; Camba-Mendez *et al.,* 2003) that imposing reduced-rank structure in Vector Auto-Regressive [VAR] models improve in prediction performances.

In particular, the dynamic properties of the data are investigated within the polynomial serial correlation common feature modeling (Cubadda and Hecq, 2001). Similarly as the composite indexes built by The Conference Board (1997), the proposed CI&LI's are obtained as linear combinations of observed variables. However, the weights of the novel indexes are derived such that the changes of the LI are the best linear predictor of the changes of the CI.

Hence, the suggested CI&LI's are constructed in order to satisfy the purpose of documenting and predicting the variations of the overall economic activity.[1] Other relevant characteristics of the new composite indicators are that the existence of such CI&LI is tested and not assumed *a priori*, it is possible to check if the individual indicators significantly enter in the CI and LI, and the multivariate Beveridge-Nelson (1981) cycle of the LI leads that of the CI.

This paper is organized as follows. Section 2 proposes a definition of the CI&LI, and shows how to build such indexes by means of RRR. In Section 3 the conditions for the existence of a long leading index are examined. In Section 4 the methodology is applied to the US business cycles indicators. Section 5 concludes.

2 The statistical methodology

The aim of this section is to present a RRR framework to build the CI&LI from a set of cointegrated time series.

2.1 Preliminaries

Let us start with the VAR(p) model for a n-vector of I(1) time series $\{y_t, t = 1, \ldots, T\}$,

$$A(L)y_t = \varepsilon_t,$$

for fixed values of $y_{-p+1}, ..., y_0$ and where $A(L) \equiv I_n - \sum_{i=1}^{p} A_i L^i$, and ε_t are i.i.d. $N_n(0, \Sigma_\varepsilon)$ errors. To simplify the notation, the deterministic terms are omitted at this stage.

It is further assumed that the process y_t is cointegrated of order (1,1), namely that 1°) $rank(A(1)) = r$, $0 < r < n$, so that $A(1)$ can be expressed as $A(1) = \alpha\beta'$ with α and β both $(n \times r)$ matrices of full column rank r, and 2°) the matrix $\alpha'_\perp A^*(1)\beta_\perp$ has rank equal to $(n - r)$ where $A^*(1)$ denotes the first derivative of $A(z)$ at $z = 1$. The columns of β span the space of cointegrating vectors, and the elements of α are the corresponding adjustment coefficients. In order to rewrite the system in a VECM form we use the identity $A(L) \equiv \Gamma(L)\Delta - A(1)L$ where

[1] Although the proposed CI&LI's are not specifically designed for predicting the business cycle turning points, they may also be used for such purpose along the lines of Wecker (1979), and Hamilton and Perez-Quiros (1996).

$\Gamma(L) = I_n - \sum_{i=1}^{p-1} \Gamma_i L^i$, and $\Gamma_i = -\sum_{j=i+1}^{p} A_j$ for $i = 1, \ldots, p-1$. And finally we obtain

$$\Gamma(L)\Delta y_t = \alpha\beta' y_{t-1} + \varepsilon_t. \tag{1}$$

The stationary process Δy_t admits the following Wold representation

$$\Delta y_t = C(L)\varepsilon_t, \tag{2}$$

with $\sum_{j=1}^{\infty} j\,|C_j| < \infty$, and $C_0 = I_n$.

Based on the polynomial factorization $C(L) = C(1) + \Delta C^*(L)$, where $C_i^* = -\sum_{i+1}^{\infty} C_j$ for $i \geq 0$, we obtain the multivariate Beveridge and Nelson (1981, BN henceforth) representation of the series y_t

$$y_t = \tau_t + \xi_t, \tag{3}$$

where $\xi_t = C^*(L)\varepsilon_t$, and $\Delta\tau_t = C(1)\varepsilon_t$.

The multivariate BN decomposition has a natural interpretation in forecasting terms. Indeed, we easily get from equations (2) and (3) that

$$\lim_{h\to\infty} E(y_{t+h}|\Omega_t) = \tau_t,$$

where Ω_t is the σ-field generated by $\{y_{t-i}; i \geq 0\}$. Based on the popular view that the trend of a non-stationary time series coincides with its infinite-step ahead prediction (see e.g. Harvey, 1990), the processes τ_t and ξ_t are respectively defined as the stochastic trends and cycles of variables y_t. Proietti (1997) and Hecq *et al.* (2000) provided explicit expressions of the components τ_t and ξ_t in terms of the VECM parameters.

In order to analyze non-contemporaneous short-run comovements, Cubadda and Hecq (2001) have introduced the notion of Polynomial Serial Correlation Common Features [PSCCF] such that

Definition 1 ***Polynomial Serial Correlation Common Features of order m:*** *series Δy_t have s PSCCF of order m, henceforth PSCCF(m), iff there exists a $n \times s$ polynomial matrix $\delta(L) = \delta_0 - \sum_{i=1}^{m} \delta_i L^i$ with $m < (p-1)$ such that the matrix δ_0 is full column rank, $\delta_m \neq 0$, and $\delta(L)'\Delta y_t = \delta_0'\varepsilon_t$.*

Notice that the notion of serial correlation common feature (Engle and Kozicki, 1993) is obtained as a special case of the PSCCF(m) with $m = 0$.

The presence of the PSCCF(m) endows series y_t with several interesting properties. First, the following restrictions on the VECM (1) parameters hold

$$\text{Condition 1.} \qquad \delta_0'\alpha = 0$$

$$\text{Condition 2.} \qquad \delta_0'\Gamma_i = \begin{cases} \delta_i' & if\ i \leq m \\ 0 & if\ i > m \end{cases}$$

Second, variables y_t must share at least one common trend since Condition 1 implies that the matrix α has rank less then n. Third, the multivariate BN cycles ξ_t respect the following condition

$$E(\delta(L)'\xi_{t+h}|\Omega_t) = 0, \quad h \geq m, \tag{4}$$

which is equivalent to say that the process $\delta(L)'\xi_{t+h}$ is a VMA($m-1$) for $m \geq 1$.[2]

2.2 The Coincident and Leading Indexes

Let us assume that the vector of n time series may be partitioned into two subvectors such that $y_t = (z_t', x_t')'$. The first n_1 series z_t are the relevant business cycle indicators whereas the remaining $n_2 = n - n_1$ series x_t must Granger-cause the reference series z_t. Hence, the following notion of coincident and leading indexes is proposed.

Definition 2 ***CI&LI.*** *CI_t and LI_t are respectively the composite coincident and leading indexes iff*

$$E(\Delta CI_{t+1}|\Omega_t) = E(\Delta CI_{t+1}|\Delta LI_t), \tag{5}$$

where CI_t is a linear combinations of the reference series z_t, and LI_t is a linear combinations of series $(y_t', .., y_{t-m+1}')'$.

The above definition can be motivated as follows. In view of the BN decomposition in (3), if the reference series z_t possess some cyclical components, their first differences Δz_t must be autocorrelated. The weights of the suggested CI&LI are simultaneously determined such that the CI exhibits a cyclical behavior but $\Delta CI_{t+1} - E(\Delta CI_{t+1}|\Delta LI_t)$ is an innovation with respect to Ω_t. Hence, the BN cycle of CI_{t+1} is cancelled after removing the influence of LI_t.

[2] When $m = 0$, i.e. $\delta(L) = \delta_0$, equation (4) stands for the common cycle property $\delta_0'\xi_t = 0$.

Notice that Definition 2 involves the differences rather than the levels of the indexes. The reason of this choice is that CI&LI's are conceived as a tool for short-term analysis. Indeed, whether the goal is to monitor and predict the turning points in the business cycle or macroeconomic growth, the changes of the indexes are entailed (see e.g. The Conference Board, 1997, TCB henceforth).

Suppose now that series Δy_t exhibit at least one PSCCF(m) such that $\delta_0' = (\omega_0', 0_{n_2}')$. In view of Definition 1, we have that

$$E(\omega_0' \Delta z_{t+1} | \Omega_t) = \underline{\delta}(L)' \Delta y_t,$$

where $\underline{\delta}(L) = \sum_{i=1}^{m} \delta_i L^{i-1}$. Consequently the coincident and leading indexes are simply given by

$$CI_t = \omega_0' z_t,$$

and

$$LI_t = \underline{\delta}(L)' y_t.$$

It is easy to see that the reverse implication holds as well, i.e. if there exists a pair of CI&LI according to Definition 2 then series Δy_t have at least one PSCCF(m) with $\delta_0' = (\omega_0', 0_{n_2}')$. These results, along with equation (4), imply that the CI&LI have the following important property:

Proposition 3 *Let us define the detrended CI&LI respectively as* $CI_t^{\xi} \equiv (\omega_0', 0_{n_2}')\xi_t$ *and* $LI_t^{\xi} \equiv \underline{\delta}(L)'\xi_t$. *Then we have*

$$E(CI_{t+h}^{\xi} | \Omega_t) = E(LI_{t+h-1}^{\xi} | \Omega_t), \quad h \geq m.$$

The above proposition tells us that the cyclical movements of the LI lead those of the CI when the forecast horizon is not less than the PSCCF order. Hence, the case of the PSCCF(1) with $\delta_0' = (\omega_0', 0_{n_2}')$ is particularly attractive for CI&LI building. In the rest of the paper the focus will be on such particular case.

Based on Cubadda and Hecq (2001), we can make inference on the existence of such CI&LI's by means of the following RRR procedure. We first solve the following canonical correlation program

$$CanCor\left\{\begin{pmatrix}\Delta z_t\\ -\Delta y_{t-1}\end{pmatrix},\begin{pmatrix}\hat{\beta}' y_{t-1}\\ \Delta y_{t-1}\\ \Delta y_{t-2}\\ \vdots\\ \Delta y_{t-p+1}\end{pmatrix}\mid D_t\right\}, \tag{6}$$

where D_t is a vector of deterministic terms, then the LR test statistic for the hypothesis that are at most s CI&LI couples is

$$LR_1 = -T\sum_{i=1}^{s}\ln(1-\widehat{\lambda}_i), \qquad s = 1,\ldots,\min(n_1, n-r) \tag{7}$$

where $\widehat{\lambda}_i$ is the $i-$th smallest squared canonical correlation coming from (6) and the estimates of the parameters $(\omega_0', \delta_1')'$ are the eigenvectors associated with the s smallest eigenvalues $\widehat{\lambda}_1, ..., \widehat{\lambda}_s$.[3] Under the null hypothesis the test statistic (7) is asymptotically distributed as a $\chi^2_{(d_1)}$ with $d_1 = s\times(n(p-3)+r+s+n_2)$.[4]

A relevant feature of the RRR approach is that it is possible to test for linear restrictions on the CI&LI weights. Alike Johansen (1995) in cointegration analysis, these restrictions are expressed as follows

$$\begin{pmatrix}\omega_0\\ \delta_1'\end{pmatrix} = \underbrace{H}_{(n_1+n)\times g}\underbrace{\varphi}_{g\times s} \equiv \begin{pmatrix}\underbrace{H_{11}}_{n_1\times g_1} & \underbrace{H_{12}}_{n_1\times g_2}\\ \underbrace{H_{21}}_{n\times g_1} & \underbrace{H_{22}}_{n\times g_2}\end{pmatrix}\begin{pmatrix}\underbrace{\varphi_0}_{g_1\times s}\\ \underbrace{\varphi_1}_{g_2\times s}\end{pmatrix}, \tag{8}$$

where H is matrix of known elements, the sub-matrix H_{11} has rank equal to g_1, $g = g_1 + g_2$, and φ is a parameter matrix to be estimated.

Let us a consider the illustrative example where the reference series z_t are the coincident indicators used by TCB (1997), namely the industrial production, employment, real income, and manufacturing and trade sales, and we wish to test if the reference series do enter in the

[3]Since such eigenvalues and eigenvectors are invariant to non-singular linear transformation of variables $(y_{t-1}'\hat{\beta}, \Delta y_{t-1}', ..., \Delta y_{t-p+1}')'$, inference on the CI&LI's does not depend on the identification of the cointegration vectors β.

[4]Based on Cubadda and Hecq (2001), a test statistic with better small-sample properties can be obtained by applying the scaling factor $(T-n(p-2)-r)/T$ to (7).

LI. Then the matrix H takes the form

$$H = \begin{bmatrix} I_4 & 0_{4\times(n-4)} \\ 0_{n\times 4} & \begin{pmatrix} 0_{4\times(n-4)} \\ I_{n-4} \end{pmatrix} \end{bmatrix}, \tag{9}$$

which means that there are no cross restrictions between ω_0 and δ_1, ω_0 is unrestricted, and $\delta_1' = (0_{s\times 4}, \varphi_1',)$.

We can handle such linear restrictions by means of the following procedure. We first solve the following canonical correlation program

$$CanCor \left\{ H' \begin{pmatrix} \Delta z_t \\ -\Delta y_{t-1} \end{pmatrix}, \begin{pmatrix} \hat{\beta}' y_{t-1} \\ \Delta y_{t-1} \\ \Delta y_{t-2} \\ \vdots \\ \Delta y_{t-p+1} \end{pmatrix} \mid D_t \right\}. \tag{10}$$

Then the LR test statistic is

$$LR_2 = T \sum_{i=1}^{s} \ln \left(\frac{1-\hat{\lambda}_i}{1-\widehat{\eta}_i} \right), \qquad s = 1, \ldots, \min(g_1, n-r), \tag{11}$$

where $\widehat{\eta}_i$ is the i−th smallest squared canonical correlation coming from (10) and the estimates of the parameters $(\varphi_0', \varphi_1')'$ are the eigenvectors associated with the s smallest eigenvalues $\widehat{\eta}_1, ..., \widehat{\eta}_s$. Under the null hypothesis the test statistic (11) follows asymptotically a $\chi^2_{(d_2)}$ distribution where $d_2 = s(n_1 + n - g)$.

Notice that when $s > 1$ there is not necessarily a unique CI&LI pair.[5] In the sequel we consider both the case where several indexes are individually identified and the most usual case where a unique CI&LI pair must be constructed.

[5] A practically relevant case for which $s \leq 1$ is when z_t is formed by a single reference series, such, e.g., the monthly gross domestic product.

2.3 Identifying the CI&LI's

This subsection shows how to identify "structural" pairs of CI&LI's by means of overidentifying restrictions. Coming back the previous illustrative example, we may wish to construct a leading index that does not include the four Conference Board coincident series. In this case, we need to test for zero canonical correlations between $(\Delta z_t', -\Delta x_{t-1}')'$ and the past of y_t.

More generally, suppose that we are willing to consider only composite indexes with weights which obey the linear restrictions (8). Then the LR test statistic for the null hypothesis that there exist s "restricted" CI&LI's against the alternative that no restricted CI&LI's exist is given by

$$LR_3 = -T\sum_{i=1}^{s} \ln(1-\widehat{\eta}_i), \qquad s = 1,\ldots,\min(g_1, n-r). \tag{12}$$

Under the null hypothesis the test statistic (12) is asymptotically distributed as a $\chi^2_{(d_3)}$ with $d_3 = s \times (n(p-1) + r + s - g)$.

2.4 Building the Optimal CI&LI

This subsection shows how to combine several CI&LI's in order to extract the most relevant pair for forecasting purposes. More precisely, the following notion of optimal coincident and leading composite indexes is proposed.

Definition 4 ***Optimal CI & LI.*** *$CI_t^* \equiv \xi^{*\prime} CI_t$ and $LI_t^* \equiv \xi^{*\prime} LI_t$ are respectively the optimal composite coincident and leading indexes iff*

$$\xi^* = \arg\min_{(\xi)} \left\{ \frac{\xi' V(e_t)\xi}{\xi' V(\Delta CI_t)\xi} \right\}, \tag{13}$$

where ξ is a generic s-vector, $e_t \equiv \Delta CI_t - E(\Delta CI_t|\Delta LI_{t-1})$ and $V(\cdot)$ is the covariance matrix of the process in argument.

When several PSCCF vectors exist (i.e., $s > 1$), condition (13) requires that ΔCI_t^* and ΔLI_{t-1}^* are the most correlated among all the linear combinations of Δz_t and Δy_{t-1} that satisfy equation (5). Based on a standard result from canonical correlation theory, equation (13) is solved by $\xi^* = [V(\Delta CI_t)]^{-1/2}\zeta_1$, where ζ_1 is the eigenvector associated to the smallest eigenvalue of the matrix

$$[V(\Delta CI_t)]^{-1/2} V(e_t) [V(\Delta CI_t)]^{-1/2}. \tag{14}$$

Hence, the optimal CI weights are given by $\omega_0^* = \omega_0\xi^*$ and the optimal LI weights are given by $\delta_1^* = \delta_1\xi^*$. We summarize the above results in the following proposition.

Proposition 5 ***Construction of the optimal CI&LI.*** *Suppose that there exist s PSCCF*(1) *vectors such that* $\delta_0' = (\omega_0', 0_{s\times n_2}')$ *and* $\delta_1 \neq 0$. *In this case, the optimal CI and LI are respectively given by* $CI_t^* = \omega_0^{*\prime} z_t$ *and* $LI_t^* = \delta_1^{*\prime} y_t$, *where* $\omega_0^* = \omega_0\xi^*$, $\delta_1^* = \delta_1\xi^*$, $\xi^* = [V(\Delta CI_t)]^{-1/2}\zeta_1$, *and* ζ_1 *is the eigenvector associated to the smallest eigenvalue of the matrix* (14).

The optimal CI&LI weights can be estimated as follows. Compute the RRR estimates $(\widehat{\omega}_0', \widehat{\delta}_1')$ of the CI&LI's weights and fix $(\omega_0', \delta_1') = (\widehat{\omega}_0', \widehat{\delta}_1')$. Then obtain $\widehat{\xi}^*$ by solving equation (13) where $V(\Delta CI_t)$ and $V(e_t)$ are respectively substituted with the sample covariance matrices of $\widehat{\omega}_0'\Delta z_t$ and $(\widehat{\omega}_0'\Delta z_t - \widehat{\delta}_1'\Delta y_{t-1})$. Finally, the point estimates of ω_0^* and δ_1^* are respectively given by $\widehat{\omega}_0^* = \widehat{\omega}_0\widehat{\xi}^*$ and $\widehat{\delta}_1^* = \widehat{\delta}_1\widehat{\xi}^*$.

Linear restrictions on ω_0^* and δ_1^* may be tested by a linear switching algorithm similar as the one proposed by Johansen (1995) in cointegration analysis. In particular, let us consider the following system of hypothesis:

$$\mathrm{H}_0 : \delta^* \equiv (\omega_0^{*\prime}, \delta_1^{*\prime})' = H^*\varphi^* \equiv (\underbrace{H_0^{*\prime}}_{g\times n}, \underbrace{H_1^{*\prime}}_{g\times n})'\varphi^* \text{ vs } \mathrm{H}_1 : \delta^* \text{ is unrestricted,}$$

where H^* is a matrix of known elements and φ^* is a $g \times 1$ parameter matrix.

Let us then write $\delta^{\#} = (\omega_0', \delta_1')'\xi^{\#}$, where $\xi^{\#} = [V(\Delta CI_t)]^{-1/2}\overline{\zeta}_1$, and $\overline{\zeta}_1$ is the matrix of the $(s-1)$ eigenvectors associated to the $(s-1)$ largest eigenvalues of the matrix (14). Thus the iterative procedure goes as follows

1. Estimate $\delta^{\#}$ unrestricted by $\widehat{\delta}^{\#} = (\widehat{\omega}_0', \widehat{\delta}_1')'\widehat{\xi}^{\#}$.

2. For fixed $\delta^{\#} = \widehat{\delta}^{\#}$, obtain $\widehat{\varphi}$ as the eigenvector associated with the smallest eigenvalue coming from the solution of

$$CanCor\left\{ H^{*\prime}\begin{pmatrix} \Delta z_t \\ -\Delta y_{t-1} \end{pmatrix}, \begin{pmatrix} \hat{\beta}' y_{t-1} \\ \Delta y_{t-1} \\ \Delta y_{t-2} \\ \vdots \\ \Delta y_{t-p+1} \end{pmatrix} \mid \delta^{\#\prime}\begin{pmatrix} \Delta z_t \\ -\Delta y_{t-1} \end{pmatrix}, D_t \right\}. \qquad (15)$$

3. For fixed $\delta^* = H^*\widehat{\varphi}^*$, obtain $\widehat{\delta}^{\#} = \delta^*_{\perp}\widehat{\phi}_{(s-1)}$, where $\widehat{\phi}_{(s-1)}$ are the eigenvectors associated with the $(s-1)$ smallest eigenvalues coming from the solution of

$$CanCor\left\{\delta_{\perp}^{*\prime}\begin{pmatrix}\Delta z_t\\ -\Delta y_{t-1}\end{pmatrix},\begin{pmatrix}\hat{\beta}' y_{t-1}\\ \Delta y_{t-1}\\ \Delta y_{t-2}\\ \vdots\\ \Delta y_{t-p+1}\end{pmatrix} \mid \delta^{*\prime}\begin{pmatrix}\Delta z_t\\ -\Delta y_{t-1}\end{pmatrix}, D_t\right\}. \tag{16}$$

4. Continue with 2. and 3. until numerical convergence.

The LR test statistic is

$$LR_4 = T\left[\sum_{i=1}^{s}\ln(1-\hat{\lambda}_i) - \ln(1-\widehat{\rho}_1) - \sum_{i=1}^{s-1}\ln(1-\widehat{\upsilon}_i)\right], \qquad s = 1,\ldots\min(n_1, n-r), \tag{17}$$

where $\widehat{\rho}_i$ and $\widehat{\upsilon}_i$ are the $i-$th smallest squared canonical correlations respectively coming from (15) and (16). The test statistic (17) follows asymptotically a $\chi^2_{(d_4)}$ distribution where $d_4 = (n_1 + n - g)$.

3 The Long Leading Indicator

We have so far focused on building CI&LI's when the time delay is one period only. However, it is often desirable to anticipate the state of economic activity with a larger advance. Hence, the properties of the composite indexes must be evaluated also when the forecast horizon is larger than one. By construction of the CI&LI we get

$$\Delta CI_t = \Delta LI_{t-1} + e_t, \tag{18}$$

where $e_t = (\omega_0', 0_{n_2}')\varepsilon_t$.

Equation (18) implies in turn that

$$E(\Delta CI_{t+h}|\Omega_t) = E(\Delta LI_{t+h-1}|\Omega_t), \tag{19}$$

for $h \geq 2$. Hence, the h-step ahead forecasts of the first differences of CI are given by the $(h-1)$-step ahead forecasts of the first differences of LI.

Notice that the left hand side of equation (19) is generally not a function of ΔLI_t only. For instance, for $h = 2$ we get

$$E(\Delta LI_{t+1}|\Omega_t) = \delta_1'(\alpha\beta^{'} y_t + \sum_{i=0}^{p-2} \Gamma_i \Delta y_{t-i}),$$

which is generally different from $E(\Delta LI_{t+1}|\Delta LI_t)$.

Based on equation (19), the h-step ahead leading index LI_t^h is defined as follows

$$\Delta LI_t^h = \delta_0' E(\Delta y_{t+h}|\Omega_t). \tag{20}$$

In order to build such h-step ahead leading index we may follow two different approaches.

The first approach requires to derive the h-step ahead forecasts of series y_t and combine them with the estimated CI weights $\widehat{\delta}_0$. A possible way to incorporate the CI&LI's restrictions within the VECM is to rely on the following common factor representation

$$(I_n - \Gamma_1 L)\Delta y_t = \Lambda F_{t-1} + \varepsilon_t, \tag{21}$$

where Λ is a full-rank $n \times (n-s)$ matrix such that $(\omega_0', 0_{n_2}')\Lambda = 0$,

$$F_{t-1} = \widetilde{\alpha}\beta' y_{t-1} + \sum_{i=2}^{p-1} \widetilde{\Gamma}_i' \Delta y_{t-i},$$

$\widetilde{\alpha}$ is a $(n-s) \times r$ matrix, and $\widetilde{\Gamma}_i$ is $n \times (n-s)$ matrix for $i = 2, , ..., p-1$. Efficient estimates of the parameters $[\widetilde{\alpha}, \widetilde{\Gamma}_1', \widetilde{\Gamma}_2', ..., \widetilde{\Gamma}_{p-1}']$ are provided by the canonical variates coefficients of $(y_{t-1}'\beta, \Delta y_{t-1}', ..., \Delta y_{t-p+1}')'$ associated to the $(n-s)$ largest eigenvalues $\hat{\lambda}_{s+1}, ..., \hat{\lambda}_n$. Finally, the remaining parameters of model (21) are easily estimated by a regression of Δy_t on $(\Delta y_{t-1}', F_{t-1}')'$.

The second approach consists in estimating the h-step ahead leading index by a single-equation method. In particular, for fixed $(\delta_0', \delta_1') = (\widehat{\delta}_0', \widehat{\delta}_1')$, the General Method of Moments can be used to estimate the equation

$$\Delta CI_{t+h} = \gamma^h \Delta LI_{t+h-1} + e_{t+h}^h \tag{22}$$

using series $(y_t'\hat{\beta}, \Delta y_t', ..., \Delta y_{t-p+2}')'$ as instruments, where CI_t and LI_t are a generic CI&LI pair, γ^h is a scalar, and e_t^h is a MA$(h-1)$ error. Clearly, ΔLI_t^h is then obtained by subtracting the residuals $\widehat{e}_{t+h-1}^h$ to the observed values of ΔCI_{t+h}.

Although the second approach may be preferred for its simplicity, one should keep in mind that statistical inference on (22) is conditional on the estimated CI&LI's weights and hence their sample variability is ignored.

An interesting question to be posed is if one can build an optimal CI&LI pair such that LI_t^* is a valid leading indicator for any forecast horizon of CI_t^*. Such CI&LI should satisfy, along with condition (13), the following equation

$$E(\Delta CI_{t+h}^*|\Omega_t) = E(\Delta LI_{t+h-1}^*|\Delta LI_t^*), \tag{23}$$

for any $h \geq 1$.

In view of equation (19) and keeping in mind that ΔCI_t^* and ΔLI_t^* are stationary ARMA processes, we see that equation (23) is satisfied when

$$\Delta LI_t^* = \rho \Delta LI_{t-1}^* + \nu_t, \tag{24}$$

where $\rho \neq 0$, $|\rho| < 1$, and $\nu_t = \delta_1^{*\prime}\varepsilon_t$.

Equation (24) implies that the optimal leading index is an ARIMA(1,1,0) process. But we need a stronger requirement that the error term of this ARIMA process is an innovation with respect Ω_{t-1}. In the terminology of Granger and Yoon (2001), the optimal LI must be a self-generating variable.

By comparing equation (18) with equation (24) and in view of Proposition 5, we conclude that condition (23) holds when $\delta_1^{*\prime} = \rho(\omega_0^{*\prime}, 0_{1\times n_2}')$. These non-linear restrictions on the optimal CI&LI weights can be tested and possibly imposed in estimation by means of a grid search of the likelihood function over different values of ρ.

4 Coincident and Leading Indexes for the US Economy

This section illustrates the use of the RRR framework for constructing coincident and leading indicators for the US economy. The aim of this empirical analysis is twofold. First, the historical components of the new indexes are extracted and compared with those proposed by Stock and

Watson (1989, SW henceforth) and TCB (1997). Second, an out-of-sample forecasting exercise is performed to asses the predictive performances of the RRR procedure.

4.1 Variable Definitions and Description

For the empirical analysis we consider the monthly Business Cycle Indicators [BCI] that TCB used to build their own indexes. The first two columns of Table 1 report the variables of interest along with their BCI code. With the exception of the stock prices index and consumer expectations index, the data are seasonally adjusted[6] and span the period 1959.01 to 2002.12. The sub-sample 1959.01-1999.12 is used to build the new CI&LI and the remaining observations are left for an out-of-sample forecasting exercise.

The fourth column of Table 1 reports the results of the ADF unit root tests on the BCI indicators that have been transformed as indicated in the third column. Only the vendor performance, the interest rate spread, and the building permits series appear to be I(0). In order to build the new CI&LI, these stationary indicators are integrated, namely their cumulative sum are taken in the analysis (see e.g. Rahbek and Mosconi, 1999). This operation allows to include in the first differences of the CI&LI also the I(0) variables that otherwise would have been annihilated by Condition 1 of Definition 1. Notice that the cumulated I(0) series do not posses an exact unit root by construction. Indeed, from Table 2 we see that the ADF tests indicate the presence of significant but not exact unit roots in such cumulated series. Finally, the volatility of all the transformed indicators has been adjusted as TCB suggests, that is all the first differences of these series have unitary standard deviations and are now all expressed in comparable scale.

SW (1989) impose a single dynamic common factor for summarizing the information contained in the past. Since such assumption is not formulated in the RRR approach to CI&LI building, the new procedure is prone a dimensionality problem. Consequently it is required to rely on the following step-wise procedure based on the minimization of an information criterion such as the Bayes Information Criterion [BIC]. Let us start with the four BCI coincident variables $z_t = (z_{1t}, z_{2t}, z_{3t}, z_{4t})'$ and select the VAR order that minimizes the BIC from 0 up to $p^{\max}$. Then, we add separately in the right hand side the lags of each of the ten TCB leading

[6] Although seasonal filtering poses problems for common features analysis (see e.g. Cubadda, 1999), the purpose of comparing the new CI&LI with the existing ones imposes to follow the usual practice of using seasonally adjusted data.

indicators and we estimate all the VARX models with order from 0 up to $p^{\max}$. Finally, we compare the smallest BIC of these $10 \times p^{\max}$ VARX models with the BIC of the previously selected VAR model. If the value of the BIC is smallest for the VAR model, we keep only the reference series in the analysis. Otherwise, the leading indicator associated to the VARX model with the smallest BIC is retained as exogenous variable. In the second round, each of the nine remaining TCB indicators is included as an additional exogenous variable and the BIC is computed for all the $9 \times p^{\max}$ VARX models. Again, we compare the smallest BIC of these $9 \times p^{\max}$ VARX models with the BIC of the previously selected VARX model. The procedure stops when it is not possible to find a better VARX model according to the BIC. The outcome is that the selected series are the average weekly hours, vendor performances building permits, and interest rate spread. These four series respectively comprise the leading indicators vector $x_t = (x_{1t}, x_{2t}, x_{3t}, x_{4t})'$ in the subsequent analysis.

4.2 Building the RRR-based CI&LI

Table 3 reports both the asymptotic and the small-sample corrected versions of the Johansen trace statistics in a VAR(3). We can not reject the presence of three cointegrating vectors and then five common trends. A graphical inspection of the cointegrating vector confirms the outcome of the formal analysis. Hence, we fix at three the number of cointegrating vectors and we pursue the CI&LI analysis.

The next step is testing whether there exists a PSCCF vector such that the CI is formed by the four TCB coincident series only. We use the test statistic (7), both in the asymptotic and the small-sample corrected version. From Table 4 we see that one cannot reject the presence of a single CI&LI at the 5% confidence level. The weights of such CI&LI are also reported in the same Table.[7]

It is also possible to evaluate additional restrictions on the individual indicator coefficients. As a result of a general to specific testing procedure, we cannot reject the null hypothesis that sales (z_{4t}) do not enter in both the CI and the LI. In particular, the p-value associated with the test statistic (11) for these joint restrictions on the CI&LI's weights is equal to 0.312.[8] Table 4

[7] Notice that such coefficients are normalized such that the sum of the absolute value of the CI weights is equal to one.

[8] A possible explanation of this result is that the growth rates of sales display very little autocorrelation. This implies that z_{4t} has a negligible BN cyclical component.

also reports the value of the test statistics (12) and the associated coefficients of such restricted CI&LI. In the sequel, we will refer to these restricted CI and LI respectively as the RRR_CI and RRR_LI.

4.3 Comparison with Other Coincident Indices

In this sub-section the RRR_CI is compared with two other composite indicators, namely TCB [TCB_CI] and SW [SW_CI] coincident indicators. The levels of the three series, rebased to average 100 in 1995, are graphed in Figure 2. Visual inspection suggests that these indexes provide a rather similar picture of the business cycle.

Table 5 reports the cross-correlation functions between the monthly growth rates of the various CI's. It is apparent that the three indexes are clearly synchronous and highly cross-correlated. Moreover, Table 6 shows the average spectral coherency of the alternative CI's growth rates in the 3-9 year period band. It emerges that these indexes are almost perfectly coherent at the business cycle frequencies.

Table 7 compares the recessions determined by each index with the NBER official chronology. To facilitate the comparison, the following set of dummy variable were created

$$d_t = \begin{cases} 1, & \text{if there was a recession at date } t \text{ according to NBER;} \\ 0, & \text{otherwise.} \end{cases}$$

$$d_{i,t} = \begin{cases} 1, & \text{if there was a recession at date } t \text{ according to index } i; \\ 0, & \text{otherwise.} \end{cases}$$

for $i =$ RRR, TCB, SW, and for each $d_{i,t}$ its average squared deviation from d_t was computed:

$$TP_i = T^{-1} \sum_{t=1}^{T} (d_{i,t} - d_t)^2. \tag{25}$$

We see that TCB_CI captures the NBER reference series best, but the new index perform very similarly. The SW_CI exhibits the same value of the above index as the RRR_CI.

4.4 Comparison with Other Leading Indices

So far, the one-month ahead leading index [RRR_LI] was obtained. However, SW (1989) and TCB built their leading indicators, denoted respectively by SW_LLI and TCB_LLI, in order to foresee the business cycles about six months in advance. Hence, also the six-month ahead Long Leading Index [RRR_LLI] was constructed using equation (22). Similarly as in the case of TCB, the growth rates of RRR_LLI were adjusted in order to have the same variability as those of RRR_CI. Moreover, the levels of RRR_LLI were computed using the values of RRR_CI at 1959.7-8 as starting values.

The levels of the indexes RRR_LLI, SW_LLI, TCB_LLI, rebased to average 100 in 1995, are plotted in Figure 2. The graphical comparison indicates that RRR_LLI is smoother than its competitors, providing so a clearer picture of the business cycle.

Table 8 reports the correlations of each CI's monthly growth rates with the lags of the associated LI's growth rates. We see that RRR_LI forecasts its CI changes best for shorter lags, namely one and two, whereas RRR_LLI performs better from three up to twelve periods in advance. One may observe that this is an unfair way of comparing the in-sample forecasting performances of the alternative LI's because the RRR-based LI's are explicitly designed for predicting the associated CI's growth rates. Hence, Table 9 shows the correlations of the alternative CI's j-month growth rates with the j-th lags of the associated LI's j-month growth rates for $j = 1, 2, ..., 12$. We see that RRR_LI again forecasts best for $j = 1, 2$, RRR_LLI performs better for $j = 3, ...6$, whereas TCB_LLI is superior to its competitors for longer lags.

4.5 Out-of-Sample Forecasting Exercise

In this sub-section we wish to evaluate the out-of-sample performances of the new CI&LI. Hence, the weights estimated using the sub-sample from 1959.01 to 1999.11 are kept fixed in the forecasting period 2000.1-2002.12.

Let us preliminary compare the properties of the RRR_CI with those of SW_CI and TCB_CI, which are instead built using the full sample. In Table 10 we see the cross-correlation functions of the alternative CI's growth rates for the period 2000.01-2002.12. We notice that the various CI's clearly exhibit positive contemporaneous comovements, even if the evidence is less strong than within the sample. Table 11 shows the recessions determined by each index and the NBER official chronology. The index (25) indicates that the RRR_CI accords with

NBER chronology quite well, since the index assume an intermediate value with respect to those associated with TCB_CI and SW_LI.

In order to check for possible structural breaks in the forecasting period, the Chow tests for parameter stability were applied. For the unrestricted VECM, the value of the $\chi^2(288)$ test statistic is 264.69 that corresponds to a p-value equal to 0.834. After imposing the CI&LI's restrictions through the common factor representation (21), the value of the test statistic becomes 252.01 and the associated p-value increases to 0.938.

Finally, the forecasting performances of ΔLI_t^h built according to equation (22) are contrasted with those of an unrestricted h-step ahead forecasts of ΔCI_{t+h}. The latter forecasts are obtained by estimating with Generalized Least Squares the equation

$$\Delta CI_{t+h} = \gamma_0^{h\prime}\beta' y_t + \sum_{i=0}^{p-2} \gamma_i^{h\prime} \Delta y_{t-i} + e_{t+h}^h, \qquad h = 1, \ldots 6, \tag{26}$$

where γ_0^h is a r-vector, and γ_i^h is a n-vector for $i = 1, 2, ..., p-2$, and e_t^h is a MA$(h-1)$ error.

Table 12 shows the tests proposed by Diebold and Mariano (1995) and modified by Harvey *et al.* (1997) for the equality of the Mean Square Forecasting Errors [MSFE] of equations (22) and (26) for $h = 1, ..., 6$. The third column reports the p-values for the alternative hypothesis that the former equation has a smaller MSFE than the latter, and the p-values for the opposite inequality are the complements to one of the third column elements. It emerges that ΔLI_t^h forecasts significantly better than equation (26) for $h = 1$ at the 5% level, and $h = 2$ at the 10% level, whereas none of the two predictors has a significantly smaller MSFE for larger forecasting horizons at the 10% level.

5 Conclusions

This paper has presented a new method to build a CI and a LI from a set of cointegrated time series y_t. Based on the notion of PSCCF (Cubadda and Hecq, 2001), the CI and LI are respectively obtained as linear combinations of the reference series z_t and y_{t-1} such that the changes of the LI are the best linear predictors of the changes of the CI. The proposed methodology covers also additional aspects of composite indicators building such as testing on the CI&LI weights and the construction of long leading indicators. Finally, concepts and methods have been illustrated by an empirical application with the US business cycle indicators.

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Table 1

	ADF unit root tests		
BCI code	Variable	Transf.	t-ADF
	Potential Coincident Indicators		
BCI-041	Employees on non-agricultural payrolls	log level	-2.83
BCI-051	Personal income less transfer payments	log level§	-2.41
BCI-047	Industrial production	log level	-2.15
BCI-057	Manufacturing and trade sales	log level	-2.98
	Potential Leading Indicators		
BCI-001	Average weekly hours, manufacturing	log level	-3.34
BCI-005	Average weekly initial claims for unemployment insurance	log level	-2.30
BCI-008	Mfrs' new orders, consumer goods and materials	log level	-3.24
BCI-032	Vendor performance, slower deliveries diffusion index	level	-5.63**
BCI-027	Mfrs' new orders, nondefense capital goods	log level	-2.69
BCI-029	Building permits for new private housing units	log level	-3.61*
BCI-019	Index of stock prices, 500 common stocks	log level	-0.43
BCI-106	Money supply, M2	log level	-2.85
BCI-129	Interest rate spread, 10-year Treasury bond less fed. funds	level	-4.25**
BCI-083	Univ. of Michigan Index of consumer expectations	level	-2.65

§ Two additive outliers corresponding to 1992.12 and 1993.12 were removed.

* (**) Insignificant at the 5% (10%) confidence level

Table 2

	ADF unit root tests		
BCI code	Variable	Transf.	t-ADF
BCI-032	Vendor performance, slower deliveries diffusion index	Σ level	-1.43
BCI-029	Building permits for new private housing units	Σ log level	-2.87
BCI-129	Interest rate spread, 10-year Treasury bond less fed. funds	Σ level	-1.49

Table 3

Johansen's cointegration tests		
	Trace	Trace[§]
$r = 0$	295.0**	280.6**
$r \leq 1$	171.0**	162.6**
$r \leq 2$	119.3**	113.5**
$r \leq 3$	73.86*	70.23*
$r \leq 4$	40.20	38.22
$r \leq 5$	18.13	17.24
$r \leq 6$	8.069	7.673
$r \leq 7$	0.029	0.028

[§] Small-sample corrected test statistics

* (**) Insignificant at the 5% (10%) confidence level

Table 4

CI&LI's tests					
Unrestricted CI&LI's			Restricted CI&LI's		
	LR_1	$LR_1^{\S}$		LR_3	$LR_3^{\S}$
$s \leq 1$	15.05*	14.71*	$s \leq 1$	17.37*	16.98*
$s \leq 2$	36.72	35.90	$s \leq 2$	39.87	38.97
$s \leq 3$	100.1	97.80	$s \leq 3$	164.1	160.4
$s \leq 4$	232.0	226.8			

CI&LI's weights											
Unrestricted CI&LI ($s = 1$)						Restricted CI&LI ($s = 1$)					
z_{1t}	0.268	z_{1t-1}	0.075	x_{1t-1}	-0.077	z_{1t}	0.295	z_{1t-1}	0.072	x_{1t-1}	-0.095
z_{2t}	0.132	z_{2t-1}	0.059	x_{2t-1}	0.103	z_{2t}	0.208	z_{2t-1}	0.091	x_{2t-1}	0.115
z_{3t}	0.486	z_{3t-1}	0.186	x_{3t-1}	0.122	z_{3t}	0.497	z_{3t-1}	0.227	x_{3t-1}	0.155
z_{4t}	-0.115	z_{4t-1}	0.002	x_{4t-1}	0.132	z_{4t}	0.000	z_{4t-1}	0.000	x_{4t-1}	0.168

[§] Small-sample corrected test statistics

* (**) Insignificant at the 5% (10%) confidence level

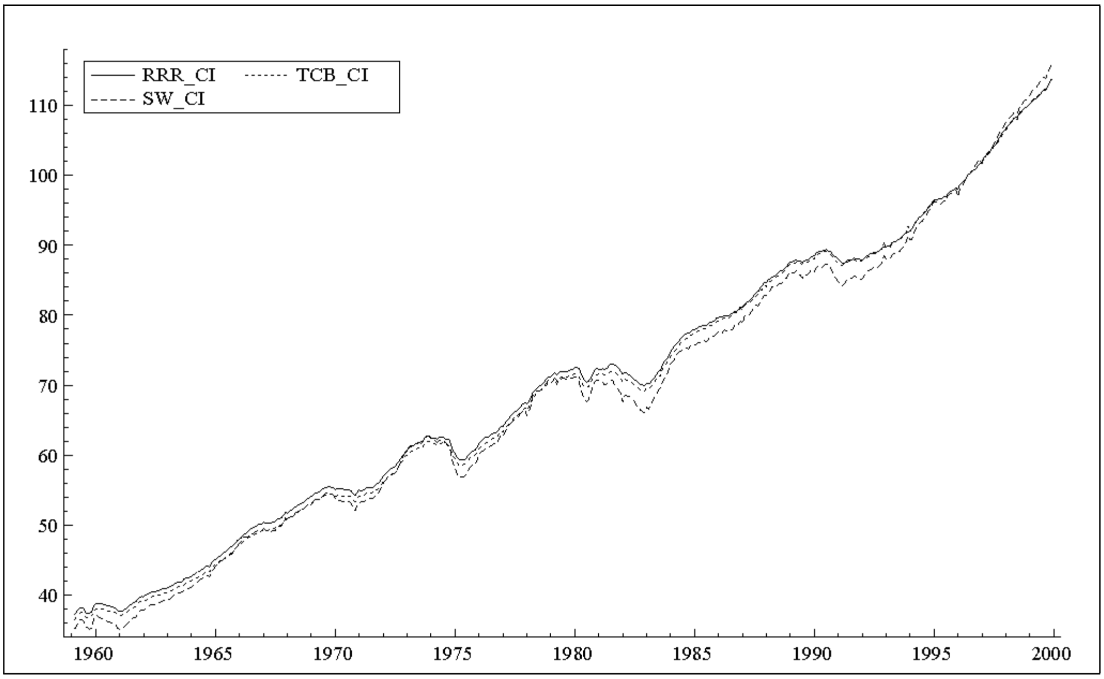

Figure 1: RRR, TCB, and SW coincident indexes

TABLE 5

Cross-correlation functions of different CI's growth rates													
Lag	−6	−5	−4	−3	−2	1	0	1	2	3	4	5	6
RRR vs TCB	.101	.086	.186	.248	.302	.467	.910	.457	.324	.262	.204	.010	.129
RRR vs SW	.046	.020	.080	.169	.281	.479	.926	.437	.326	.232	.178	.080	.122
TCB vs SW	.051	.032	.095	.170	.260	.426	.916	.376	.269	.224	.155	.078	.098

Note: 95% significance is .089

TABLE 6

Average spectral coherency of different CI's growth rates at the business cycle frequencies (3-9 year periods)		
RRR vs TCB	RRR vs SW	TCB vs SW
0.997	0.983	0.987

Note: spectra are estimated by a rectangular spectral window with width = 45

TABLE 7

Recession periods determined by alternative indexes			
NBER	RRR	TCB	SW
1960.04-1961.02	1960.04-1961.02	1960.04-1961.02	1960.02-1961.02
1969.12-1970.11	1969.10-1970.11	1969.12-1970.11	1969.10-1970.11
1973.11-1975.03	1973.12-1975.04	1973.12-1975.04	1973.11-1975.05
1980.01-1980.07	1980.02-1980.07	1980.02-1980.07	1980.01-1980.07
1981.07-1982.11	1981.08-1982.12	1981.08-1982.12	1981.07-1982.12
1990.07-1991.03	1990.07-1991.04	1990.07-1991.03	1990.08-1991.03
TP index	0.0171	0.0107	0.0171

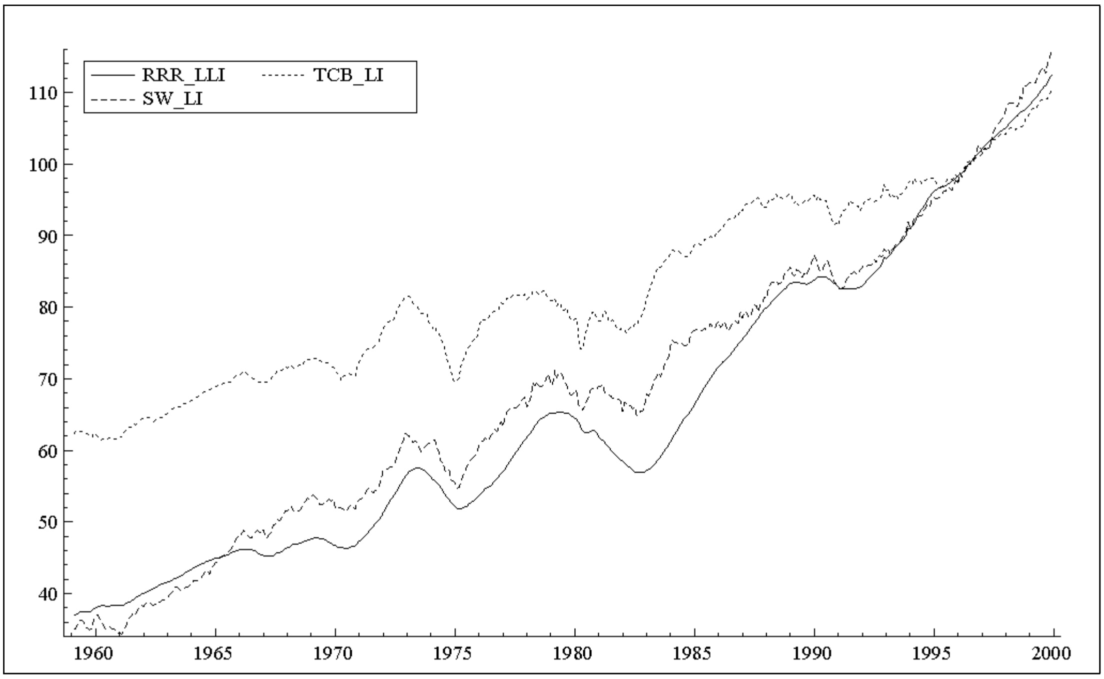

Figure 2: RRR, TCB, and SW (long) leading indexes

Table 8

Correlations of CI's growth rates with past LI's growth rates for alternative indexes				
Lag	RRR_LI	RRR_LLI	TCB_LLI	SW_LLI
1	0.587	0.481	0.195	0.221
2	0.481	0.456	0.318	0.233
3	0.424	0.438	0.293	0.152
4	0.333	0.427	0.317	0.165
5	0.255	0.405	0.183	0.121
6	0.272	0.383	0.214	0.167
7	0.283	0.369	0.172	0.188
8	0.253	0.364	0.224	0.206
9	0.228	0.338	0.206	0.172
10	0.183	0.316	0.257	0.106
11	0.101	0.269	0.190	0.090
12	0.035	0.234	0.209	-0.028

Table 9

Correlations of CI's j-month growth rates with j-th lags of LI's j-month growth rates for alternative indexes				
j	RRR_LI	RRR_LLI	TCB_LLI	SW_LLI
1	0.587	0.481	0.195	0.221
2	0.601	0.543	0.416	0.330
3	0.572	0.579	0.516	0.360
4	0.552	0.605	0.556	0.429
5	0.540	0.619	0.572	0.515
6	0.514	0.615	0.607	0.573
7	0.476	0.604	0.639	0.585
8	0.426	0.583	0.656	0.558
9	0.377	0.557	0.652	0.511
10	0.330	0.526	0.639	0.462
11	0.285	0.492	0.626	0.422
12	0.244	0.456	0.611	0.394

Table 10

Cross-correlation functions of different CI's growth rates (2000.01-2002.12)													
Lag	-6	-5	-4	-3	-2	1	0	1	2	3	4	5	6
RRR vs TCB	.020	.224	.264	.342	.431	.548	.754	.585	.531	.343	.201	.238	.154
RRR vs SW	-.083	.112	.170	.275	.367	.501	.798	.637	.569	.345	.271	.308	.170
TCB vs SW	.002	.183	-.012	.334	.263	.385	.884	.308	.492	.328	.137	.410	.082

Note: 95% significance is .3267

Table 11

Recession periods determined by alternative indexes (2000.01-2002.12)			
NBER	RRR	TCB	SW
2001.03-2001.11	2000.12-2002.01	2001.01-2001.12	1999.10-2002.01
TP index	0.1389	0.0833	0.1944

Table 12

Modified Diebold-Mariano tests		
Lead	Statistic	P-value
1	-2.344	0.0125
2	-1.672	0.0517
3	-0.174	0.4316
4	0.066	0.5261
5	-0.036	0.4856
6	1.119	0.8646