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Private Provision of Public Goods between Families

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Private Provision of Public Goods between Families*

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Abstract

We consider a two-stage voluntary provision model where individuals in a family contribute to a pure public good and/or a household public good, and, at the same time, the parent makes private transfers to her child within the same family. We show not only that Warr's neutrality holds regardless of the different timings of parent-to-child transfers, but also that there is a continuum of Nash equilibria in the sense that individuals' contributions and parental transfers are indeterminate, although the allocation of each's private consumption and total public good provision is uniquely determined. We further show that, even in the presence of impure altruism or productivity difference in supplying public goods, neutrality and uniqueness of the equilibrium allocation may persist.

Key Words: private provision, public good, Nash equilibrium, subgame perfect equilibrium, family
JEL Classification: C72, D64, H41

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1 Introduction

The standard framework for the analysis of private provision of public goods is the Nash-Cournot model in which agents choose their contributions simultaneously and independently. An important assumption of these models is that each contributor is a single individual or else a private organization which behaves as a single player in provision games. In reality, most voluntary public goods are contributed by a variety of groups consisting of heterogenous agents, such as private companies, NPO's, groups of volunteers, families and so on, in the society rather than individuals. Torsvik (1994) recognizes the importance of this observation, and shows how a representative democracy may induce each group to act strategically in the election of representatives who subsequently decide the contribution to a public good when several groups voluntarily contribute towards the public good. In this paper we particularly focus on the contributing behavior of the family to public goods. Each family comprises several heterogeneous agents characterized by different preferences as well as different income sources - for example, a given family may consist of a husband, a wife, children, a grandmother, and so on. Families make significant voluntary contributions to public goods in the real world. Furthermore, members of a given family strategically interact with each other, not only through voluntary contributions to household public goods, but also through voluntary income transfers. Thus their collective contribution decisions to contribute to public goods may be quite different from that of a single agent. Our main task is to clarify the implications for income redistribution policy between different families rather than between members of a given family (see, e.g., Konrad and Lommerud, 1995); in particular, given the complicated collective decisions within a family, we examine whether Warr's neutrality theorem holds or not.

The literature on family economics long ago moved away from modeling a family as a single decision unit. Researchers now routinely make extensive use of game theoretical modelling, using either a cooperative or a non-cooperative approach. Apps and Rees (2009) provide a good survey of the present state of research on the behaviour of multi-person households. The cooperative approach, which assumes efficiency, originated from the Nash bargaining model of Manser and Brown (1980) and McElroy and Horney (1981). Although most subsequent work assumes that households are cooperative, there are at least two potential problems with bargaining models. The first is that the implementation and enforcement of cooperative agreements within marriage requires couples to incur transaction costs. The second is that empirical evidence does not support the hypothesis that households are organized efficiently. This "efficiency" assumption has been relaxed by Lundberg and

Pollak (1993), Konrad and Lommerud (2000) and Chen and Woolley (2001), which consider a Nash bargaining model with non-cooperative Nash equilibrium threat points.[1]

The cooperative bargaining models proposed by Lundberg and Pollak (1983), Konrad and Lommerud (2000) and Chen and Woolley (2001) have some features of the non-cooperative models. However, Konrad and Lommerud (1995) go further and develop a fully non-cooperative model in which two family members non-cooperatively choose his or her time allocation between market work and household production of a household public good without relying on a bargaining process. They find that lump-sum redistribution from one spouse to the other may alter the intrafamily equilibrium outcome. Furthermore, such redistribution may lead to a Pareto improvement. Many decisions within a household can be analyzed in a non-cooperative model. Ashworth and Ulph (1981) analyze the strategic labor supply decisions of family members, while Anderberg (2007) analyzes the mix of government spending. We also employ the fully non-cooperative family game in the spirit of Konrad and Lommerud (1995) in order to avoid transaction costs as well as the unrealistic "efficiency" hypothesis. Moreover, our model departs from their model in two further respects.

The non-cooperative games within the family described so far are in the tradition of the voluntary contributions game analysed by Warr (1983), Cornes and Sandler (1985) and Bergstrom et al. (1986). These models, in common with the core results of the basic public good provision model, predict that household public goods will generally be underprovided. This common feature arises from the presence of a household public good whose benefits are enjoyed by members of the same family. Examples might include housework, a beautiful garden, a clean house, care for sick family members, the well-being of elderly parents and children, and so on. At the same time, households also voluntarily contribute to many public goods whose benefits spill over to members of other families (which we call "interfamily" public goods to distinguish them from "intrafamily" or "household" public goods). Such contributions include donations to charities, and community orchestras, various volunteer activities and so on. Even an attractive garden usually has a spillover effect on neighbors, and thus it can be viewed as generating both intra- and interfamily public goods. Every family member has the oppor-

[1]Lundberg and Pollak (1983) consider a threat point to be a "separate sphere" contribution equilibrium where socially prescribed gender roles assign the primary responsibility for certain public goods to the wife and others to the husband, while Konrad and Lommerud (2000) and Chen and Woolley (2001) define a threat point as the utility levels obtained from the spouses' non-cooperative decisions of human capital investments and contributions to a household public good prior to the bargaining, respectively.

tunity of purchasing "environmentally friendly" goods and services such as hybrid or electric cars, energy-saving electrical appliances. Moreover, they have the opportunity of purchasing green-electricity which is generated with renewable souses of energy such as solar, wind, geothermal, and biomass. Purchases of this provide an environmental public good as a result of a reduction in pollution emissions associated with increased production of green electricity. Hence, a more realistic model allows family members to contribute to both intra- and interfamily public goods in day-to-day life. This extension is not only motivated by theoretical concerns, but also is to reveal new implications for a redistribution policy. Although Konrad and Lommerud (1995) found the well-known non-neutrality of redistribution policy between spouses in the presence of productivity difference between spouses in supplying a household public good, their model contains neither voluntary income transfers nor voluntary contributions to an interfamily public good. We examine an income redistribution policy between different families rather than within family members in a situation where family members are connected through those channels.

The second departure from the non-cooperative family model is that, either because family members care about each other or for other motivations such as self-interested exchange (e.g., Bernheim et al., 1985), they make private income transfers within the family in addition to their contributions to public goods. Consider, for example, alumni giving to private colleges and universities. The parents makes direct income transfers to their children in order to pay their tuition and support their living; at the same time, the parents may give as alumni to these colleges and universities which can be viewed as a voluntary contribution to a public good. Therefore, it is very natural to tie together two different stands in the literature; voluntary provision to public goods and the economics of family which analyzes provision to household public goods and income transfers among family members. Such a hybrid model is consistent with common empirical observations and would provide new theoretical implications to both fields as well.

Our analysis reveals three major findings. First, even if the distribution of income among individuals is fixed, an infinite number of combinations of private contributions to a public good, including a household public good, and intrafamily transfers may be compatible with a unique profile of individual private consumptions and the total supply of public good. In other words, there is a continuum of (subgame perfect) Nash equilibria in the strategy space of individual voluntary contributions. This non-uniqueness property contrasts sharply with the result of Bergstrom et al. (1986) in which, when agents have convex preferences defined over a private good and a normal public good, there exists a unique Nash equilibrium, with a unique set of individual contributions and the total provision of public good.

The source of this non-uniqueness result is straightforward. Suppose a parent simultaneously makes two types of voluntary contributions; private donations to interfamily public goods and parental altruistically motivated transfers to their child. From the viewpoint of the parent, both contributions would be regarded as perfect substitutes for provisions to the same public good, although such private transfers would be indirect contributions to that public good via the well-being of the child. Yet, we shall show that this intuition is not sufficient to account for the non-uniqueness of the Nash equilibrium.

The main focus of our model is on the timing of parental transfers to children in the presence of voluntary contributions to interfamily public goods. In the literature there are two possible timings of parent-to-child transfers; more precisely, the parent makes private transfers to a selfish child either before or after observing the child's action (i.e., ex-ante or ex-post transfers, respectively). Assuming further that the parent both contributes to public goods and also makes parent-to-child transfers (which is the most plausible assumption), we demonstrate the robustness of Warr's (1983) neutrality theorem by considering these different sequential orders of actions chosen by the parent and child. Put differently, the neutrality result is independent of the details of the environment in which strategic behavior associated with the timing of parent's transfers, which is our second major finding.

Bernheim and Bagwell (1988) have argued that the presence of interfamily linkages through a common child produced by originally unrelated individuals may make government policies, such as public transfers, distortionary taxation and so on, neutral in the sense that those policies have no real effect. The neutrality property they found, which is called "cross sectional neutrality", is much stronger than Barro's neutrality which works through altruistically motivated intergenerational transfers. Bernheim and Bagwell's cross sectional neutrality operates through interfamily transfers based on marital connections.[2] In addition to such links provided by altruistically motivated transfers, we introduce an interfamily public good which provides another possible link connecting individuals in different families as well as within the same family. We show not only that the latter link acts as a perfect substitute for the link of operative interfamily transfers assumed in the model of Bernheim and Bagwell (1988), but also that the introduction of the strategic motives for transfers (i.e., different sequential orders of transfers) does not affect the likelihood of Bernheim and Bagwell's cross sectional

[2]However, Laitner (1991) points out that when material connections are explicitly modeled via a market for marriage, this neutrality is not robust.

neutrality. Indeed, the presence of voluntarily supplied public goods will enhance Bernheim and Bagwell's cross sectional neutrality in the sense that even if interfamily transfers are not operative, the redistribution neutrality holds as long as private donations to a public good are operative.

Cornes and Itaya (2010) show that in a one-shot, Nash provision game with many public goods, any income redistribution has no effect on the original equilibrium allocation, as long as that redistribution occurs within a set of *linked individuals* who are eventually connected each other through effective private contributions to many interfamily public goods. They call it "partial neutrality". We shall show that their linkage concept plays a key role in generating the neutrality as well as non-uniqueness of an equilibrium allocation in the present two-stage provision game. In our model we define a "link" as either *positive* parent-to-child transfers or *positive* private donations to interfamily public goods, and then show that when a redistribution of income is undertaken among the *linked individuals* who are eventually connected each other through the latter link, neutrality arises. Moreover, if the number of links is larger than the minimum number of links which connect individuals, the indeterminacy in terms of choice variables at the node where extra links emerge arises.

The third finding is that even if parents or children stop contributing to a public good, the neutrality with respect to a redistribution of income involving those non-contributing agents may remain valid, as long as the parental transfers are operative. In other words, even if we take "*non-contributors*" at face value; namely, the individuals who do not contribute a positive amount to the public good, the redistribution policy either between non-contributors or between contributors and non-contributors may not destroy neutrality, as long as they are connected through operative private transfers. Accordingly, our finding has not been addressed in the literature on private provision of public goods (see, e.g., Bergstrom et al., 1986) in which a redistribution of income among individuals involving non-contributors usually negates neutrality. Although Konrad and Lommerud (1995) demonstrate that the intrafamily equilibrium outcome by lump-sum redistribution from one spouse to the other is not neutral, and, moreover, such redistribution might lead to a Pareto improvement. The presence of a second channel through which individuals are linked – specifically through the possibility of income transfers – has the significant implication that neutrality continues to hold even when those individuals differ with respect to their productivities as public good contributors.

Abel and Bernheim (1991) observe that frictions such as impure altruism, incomplete information about others' preferences and egalitarian social norms that constrain parents to divide their transfers evenly among children, may destroy Bernheim and Bagwell's cross sectional neutrality. Nevertheless, we show that even if there are such frictions, neutrality and uniqueness of the Nash equilibrium may persist. This finding makes a sharp contrast with that of Andreoni (1990) in which introducing a "warm glow" (i.e., impure altruism) destroys Warr's neutrality result. Again, the presence of transfers preserves and/or voluntary provision of public goods the neutrality property in the presence of such frictions.

Section 2 presents a basic model. Section 3 considers the two-stage provision game with ex-post transfers to children. Section 4 considers the two-stage provision game with pre-committed transfers to children. Section 5 considers the two-stage provision game where different families adopt the different modes of transfers; one of the families adopt ex-ante transfers, while another family adopts ex-post transfers. Section 6 derives configurations of each family member's contribution and transfer associated with different income distributions under Cobb-Douglas preferences. Section 7 considers the case of impure altruism. Section 7 considers the basic model augmented with the inclusion of a household public good. Section 9 concludes the paper with a discussion of some possible extensions of the model.

2 The Model

In this paper, without loss of generality, we consider an economy of two families, each consisting of a single altruistic parent and a single selfish child. Within family i, whose members are identified by the superscript i, The utility function of the parent, who is altruistic towards her child, is given by

$$U_p^i(c_p^i, G; c_k^i) \equiv u_p^i(c_p^i, G) + \alpha^i u_k^i(c_k^i, G),\ i = 1, 2, \tag{1}$$

where c_p^i and c_k^i are, respectively, the parent's and child's consumptions of the private good, G is an interfamily public good, α^i is the parameter which measures the strength of parent i's altruism towards her child. The utility functions of the parent and child, denoted respectively by u_p^i and u_k^i, are twice-continuously differentiable, strictly quasi-concave, strictly increasing in each argument and $\partial u_h^i/\partial c_h^i \to \infty$ as $c_h^i \to 0$ for $i = 1, 2$ and $h = p, k$.[3] We assume that c_p^i (and c_k^i) and G are normal goods. We further assume that $\alpha^i \in [0, 1]$, which implies that the parent neither cares about her child more than herself nor hates her child. To make the analysis simpler, we here

[3] Although the form of the utility function given by (1) has been commonly used in the literature on family economics, it is easy to show that the results obtained in the present paper remain valid in the more general utility function such as $U_p^i(c_p^i, G, u_k^i(c_k^i, G))$.

omit a purely intrafamily (household) public good, but we will introduce one in Section 8.

The public good G is an interfamily public good whose benefits spill over to members of the other family. Moreover, the public good is entirely supplied by voluntary contributions made by the parent and child, g_p^i and g_k^i of family $i = 1, 2$, respectively. The public good is thus produced according to the following summation technology:

$$G = \sum_{j=1}^{2} \left(g_p^j + g_k^j \right). \tag{2}$$

3 Subgame Perfect Equilibrium with Ex-post Transfers

In this section we consider a two-stage contribution game in which the child and parent play sequentially, and characterize the resulting subgame perfect Nash equilibrium (or simply *SPE*). We shall consider three different timings that the parent and child play. We first investigate the following timing of actions (which we call *Game I*). In stage 1, the child of family i chooses her own consumption, c_k^i, and contribution to the public good, g_k^i. In stage 2, after having observed the contributions made by the children of both families, (g_k^1, g_k^2), the parent of family i chooses c_p^i and g_p^i (or equivalently, π^i and g_p^i) so as to maximize her utility function (1) subject to

$$\begin{aligned} c_p^i + \pi^i + g_p^i &= y_p^i, & (3) \\ c_k^i + g_k^i &= y_k^i + \pi^i, & (4) \end{aligned}$$

where π^i represents the transfer from the parent of family i to her child, and y_p^i and y_k^i are the fixed incomes of the parent and child of family i, respectively.

We use backward induction to solve the parent's optimization problem first. After substitution of (3) and (4) into (1), the parent's problem is:

$$\max_{\{g_p^i, \pi^i\}} U_p^i = u_p^i(y_p^i - \pi^i - g_p^i, G) + \alpha^i u_k^i(y_k^i + \pi^i - g_k^i, G). \tag{5}$$

The first-order conditions characterizing an interior solution are[4]

$$\frac{\partial U_p^i}{\partial g_p^i} = -\frac{\partial u_p^i(y_p^i - \pi^i - g_p^i, G)}{\partial c_p^i} + \frac{\partial u_p^i(.)}{\partial G} + \alpha^i \frac{\partial u_k^i(y_k^i + \pi^i - g_k^i, G)}{\partial G} = 0,\ i = 1, 2, \tag{6}$$

$$\frac{\partial U_p^i}{\partial \pi^i} = -\frac{\partial u_p^i(.)}{\partial c_p^i} + \alpha^i \frac{\partial u_k^i(.)}{\partial c_k^i} = 0,\ i = 1, 2. \tag{7}$$

Since (6) and (7) form the system of equations in g_p^i and π^i, $i = 1, 2$, given g_k^i and g_k^j, by applying the implicit function theorem we can solve it for $\hat{g}_p^i\left(g_k^1, g_k^2\right)$ and $\hat{\pi}^i\left(g_k^1, g_k^2\right)$, $i = 1, 2$. Given the optimal contribution and transfer functions $\hat{g}_p^i\left(g_k^1, g_k^2\right)$ and $\hat{\pi}^i\left(g_k^1, g_k^2\right)$ for $i = 1, 2$, the child's maximizing problem of family i at stage 1 is given by

$$\max_{\{g_k^i\}} U_k^i = u_k^i(y_k^i + \hat{\pi}^i\left(g_k^i, g_k^j\right) - g_k^i, \hat{G}),$$

where $\hat{G} \equiv \hat{g}_p^1\left(g_k^1, g_k^2\right) + \hat{g}_p^2\left(g_k^1, g_k^2\right) + g_k^1 + g_k^2$. The first-order conditions for an interior solution are given by

$$\frac{\partial U_k^i}{\partial g_k^i} = \frac{\partial u_k^i(.)}{\partial c_k^i}\left[\frac{\partial \hat{\pi}^i}{\partial g_k^i} - 1\right] + \frac{\partial u_k^i(.)}{\partial G}\left[\frac{\partial \hat{g}_p^i}{\partial g_k^i} + \frac{\partial \hat{g}_p^j}{\partial g_k^i} + 1\right] = 0,\ i = 1, 2. \tag{8}$$

As shown in Appendix A, it follows from (A2) that

$$\frac{\partial \hat{\pi}^i}{\partial g_k^i} = 1 \text{ and } \frac{\partial \hat{g}_p^i}{\partial g_k^i} + \frac{\partial \hat{g}_p^j}{\partial g_k^i} + 1 = 0,\ i = 1, 2, \tag{9}$$

which automatically leads to the condition that $\partial U_k^i / \partial g_k^i = 0$ for $i = 1, 2$. This implies that the first-order conditions (8) are satisfied for any interior values of g_k^i, $i = 1, 2$, that give rise to interior values of the functions π^i, g_p^i in Stage 1, so that indeterminacy in terms of g_k^i arises.[5]

Next, we investigate how income redistribution affects the equilibrium allocation derived above - that is, whether the so-called neutrality theorem

[4]An interior equilibrium with $(\pi^i, g_p^i, g_k^i) > 0$, $i = 1, 2$, may not *generically* exist, as shown in Cornes and Itaya (2010). We will confirm its existence by examining a specific model with Cobb-Douglas preferences in Section 6.

[5]Since it can be confirmed that child's payoff function $U_k^i(g_p^1, g_k^1, g_p^2, g_k^2)$ in its own contribution is concave (i.e., its second derivative is equal to zero), the second-order condition is certainly satisfied. Moreover, it is well documented that when the strategy set of each player is compact and convex and when each player's payoff function is quasi-concave in its own strategy, there exists a Nash equilibrium in this game (see, e.g., Glicksberg, 1952)

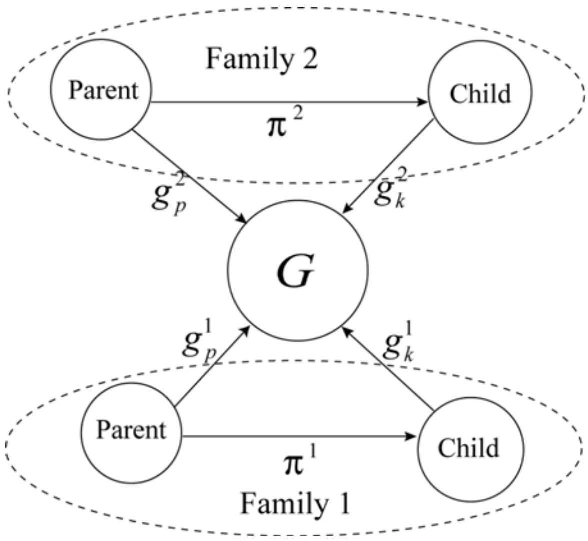

Figure 1: Both parent and child contribute to the public good G.

holds or not. To do this, we assume "small" redistributions of income–that is, no one suffers from a loss in income greater than their initial contributions or the sum of their initial contribution and intrafamily transfer. Given a set of changes in incomes dy_h^i, $i = 1, 2$ and $h = p, k$, such that $dy_p^1 + dy_p^2 + dy_k^1 + dy_k^2 = 0$, consider a set of changes in choices variables by the agents that satisfy

$$dy_p^i = d\pi^i + dg_p^i,\ i = 1{,}2, \tag{10}$$

$$dy_k^i + d\pi^i = dg_k^i,\ i = 1{,}2. \tag{11}$$

We claim that these new choices are an SPE of *Game* I, and since they–together with the pure-redistribution assumption–imply that $dG = 0$ also, this all implies that *Game* I exhibits redistributional neutrality for any such income redistribution.

The key point is that when lump-sum taxes are imposed on, say, the parent of family 1, she can undo their effect either by withdrawing the transfer to her child, or by reducing her public good contribution, or both, in anticipation that the child of family 1 or/and the members of family 2 will receive a lump-sum transfer of the same amount.

Furthermore, it follows from the budget constraints (3) and (4), in conjunction with the invariance of c_k^i and c_p^i, that the resulting indeterminacy of g_k^i entails the indeterminacy of π^i and g_p^i as well, although the values of $\pi^i + g_p^i$ and thus $g_p^i + g_k^i$ are uniquely determined for each i. In other words, an infinite number of combinations of g_k^i and π^i (or equivalently, combinations of g_p^i and π^i) are consistent with a unique profile of private consumptions and the total provision of public good in a *SPE*.[6]

As for non-interior solutions, there are several patterns of family members' contributions and transfers, as illustrated in Figures 2-4. We first consider the case where only the children of both families are contributors (i.e., $g_p^i = 0$, $i = 1, 2$), which corresponds to Figure 2. In this case we consider the following hypothetical changes in the contributions of children and the parental transfers in both families in response to an income redistribution that satisfies $dy_p^1 + dy_p^2 + dy_k^1 + dy_k^2 = 0$:

$$\begin{aligned} dy_p^i &= d\pi^i,\ i = 1{,}2, &(12)\\ dy_k^i + d\pi^i &= dg_k^i,\ i = 1{,}2. &(13)\end{aligned}$$

These hypothetical responses, in conjunction with the budget constraints (3) and (4), result in the constant private consumption of every individual and

$$\begin{aligned} dG &= dg_k^1 + dg_k^2,\\ &= (dy_k^1 + d\pi^1) + (dy_k^2 + d\pi^2),\\ &= dy_k^1 + dy_p^1 + dy_k^2 + dy_p^2 = 0,\end{aligned}$$

where the second equality follows from (13), while the third equality follows from (12). In addition, the constancy of the variables c_p^i, c_k^i and G leaves the first-order conditions (6), (7) and (8) intact as well. Taken together, the hypothetical responses prescribed by (12) and (13) are fulfilled at equilibrium, thus leading to the neutrality of income redistribution policy. Both π^i and thus g_k^i are uniquely determined from (3) and (4), respectively, for π^i is a sole decision variable confronted by the parent.[7]

[6] When the child has another choice variable such as a labor supply (i.e., 'a lazy kid' in the sense of Bergstrom; 1989), the optimizing problem of the child becomes

$$\max_{\{g_k^i\}} U_k^i = u_k^i(y_k^i(a^i) + \hat{\pi}^i(g_k^i, g_k^j, a^i) - g_k^i, \hat{G}, a^i),$$

where $y_k^i(a^i)$ represents the labor income earned by the child which depends positively on the child's labor supply a^i, and is assumed to be a C^2-class function. The optimal transfer function $\hat{\pi}^i(g_k^i, g_k^j, a^i)$ has been obtained by solving the first-order condition for the parent's optimizing problem given by (6) and (7), given that g_k^i, g_k^j and a^i have been determined at stage 1. Assuming (10) and (11), we can show that neutrality as well as indeterminacy hold true in a similar way.

[7] When all individuals make positive contributions to the public good but the parents stop the transfers to their children, the resulting case precisely corresponds to the standard Nash provision game (see, e.g., Bergstrom et al., 1986).

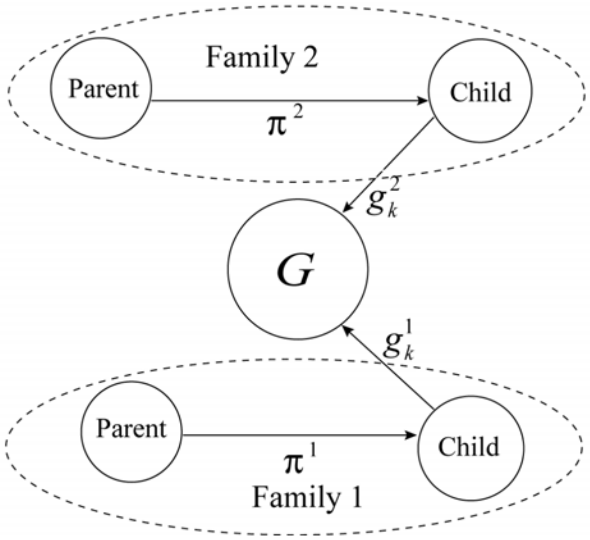

Figure 2: Only the child contributes to the public good G.

Next, consider the case where only the parents of both families are contributors (i.e., $g_k^i = 0$, $i = 1, 2$), as illustrated in Figure 3. As before, we consider the following hypothetical changes in the children's contributions and the parental transfers in response to the income redistribution:

$$dy_p^i = d\pi^i + dg_p^i,\ i = 1{,}2, \tag{14}$$

$$dy_k^i + d\pi^i = 0,\ i = 1{,}2. \tag{15}$$

In an analogous manner, we can confirm that the hypothetical responses are fulfilled thus resulting in the validity of neutrality, and that π^i and g_p^i are uniquely determined from the budget constraint (3) and (4), respectively.

Finally, consider the case depicted in Figure 4 (i.e., $g_k^1 = 0$ and $g_p^2 = 0$). In this case we assume the following responses of private contributions as well as parental transfers to the income redistribution:

$$dy_p^1 = d\pi^1 + dg_p^1 \text{ and } dy_k^1 + d\pi^1 = 0, \tag{16}$$

$$dy_p^2 = d\pi^2 \text{ and } dy_k^2 + d\pi^2 = dg_k^2, \tag{17}$$

which ensures Warr's neutrality theorem as well as the uniqueness of π^1, π^2, g_p^1 and g_k^2 as before.

To sum up:

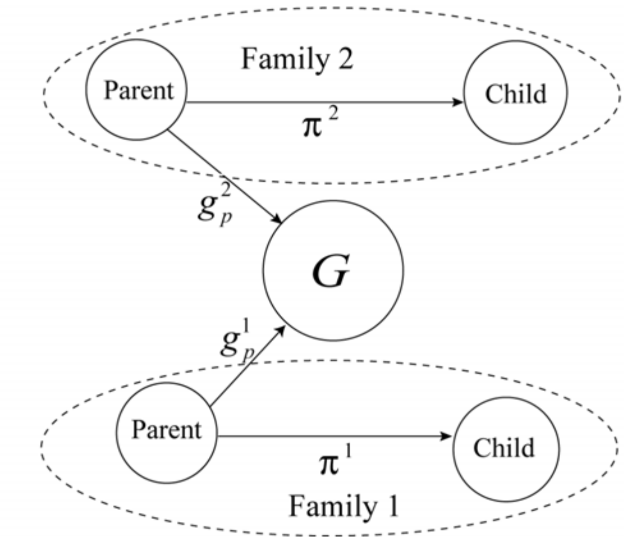

Figure 3: Only the parent contributes to the public good G.

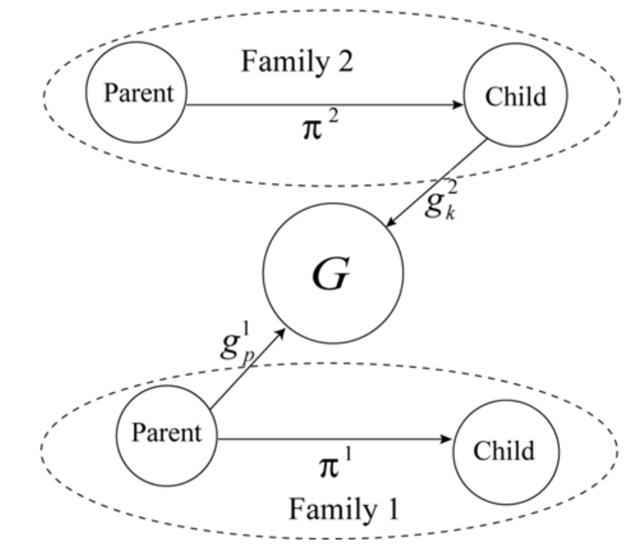

Figure 4: Only the parent contributes to the public good G in family 1, while only the child contributes to the public good G in family 2.

Proposition 1 *Consider a two-family, two-stage contribution game with the preferences in (1), where the parents decide how much to make transfers to the children after having observed the contributions of all children. Assume "small"redistributions of income. Then, we have*
(i) *Neutrality in terms of a redistribution of income as well as indeterminacy in terms of the private contributions of all family members as well as the transfers made by the parents hold, provided the contribution of every individual and transfers made by the parents are positive;*
(ii) *If only the parents (only children) of both families are contributors, then the SPE is unique, but neutrality remains valid, provided that the parents of both families make positive transfers to their children, and;*
(iii) *If only the child is a contributor in one family while only the parent is a contributor in another family, then the SPE is unique, but neutrality remains valid, provided that the parents of both families make positive transfers to their children.*

Varian (1994) investigates a private provision model in which two agents sequentially choose voluntary contributions to a public good, and finds that in a Stackelberg equilibrium where both agents contribute a positive amount to the public good the neutrality result holds, which is consistent with our result. Nevertheless, his model does not entail a continuum of Nash equilibria. This implies that the concept of a Stackelberg equilibrium itself will cause redistributional income neutrality but does not suffice to generate indeterminacy.

Statements (ii) and (iii) in Proposition 1 have not been addressed in any of the literature on private provision of public goods. These properties appear to be similar to Proposition 3 in the multiple-public good model of Cornes and Itaya (2010) in which, even if two individuals do not contribute to the same public good, a redistribution of income that is restricted to a set of "*linked individuals*", who are eventually connected each other through *effective* (positive) private contributions to voluntarily provided public goods, has no effect on the original equilibrium allocation (they call it "partial neutrality"). However, their analysis is limited to a one-shot, simultaneous Nash provision game. In light of Proposition 1, it is easy to show that "partial neutrality" holds true in our model with the sequential order of actions associated with the Stackelberg equilibrium concept.

In order to clarify the relationship between our current treatment and the "partial neutrality" of Cornes and Itaya, we have to slightly modify their definition of *linked individuals* as follows:

Definition 1 *Individuals i and i' are **linked** at an equilibrium if at least* *one of the following conditions holds: (i) there are positive transfers between i and i': (ii) they contribute to the same public good.*

Clearly, if individuals i and i' are linked, regardless of whether they belong to the same family or not, so too are any two individuals belonging to the chain, which may consist of operative private transfers within a family and/or positive private contributions to public goods whose benefits spill over to members of other families, that links them.[8] Using this definition, Proposition 1 can be restated in the following corollary:

Corollary 1 *Consider a two-family, two-stage contribution game with the preferences in (1), where the parents decide how much to make transfers to the children after having observed the contributions of all children. An income redistribution that is restricted to a set of linked individuals, that maintains the links between them, has no effect on the original equilibrium allocation. Moreover, if every individual is linked through* ***at least*** *one link supported by positive transfers to her child or positive private contributions to the public good, neutrality holds. In particular, if every individual is linked through* ***only*** *one link (we call such links "the minimum set of links"), the uniqueness of the associated equilibrium profile emerges.*

Stated differently, if there are *double* links at some node associated with a particular individual; for instance, the parents simultaneously make both positive transfers to their children and private contributions to the public good, indeterminacy as to the associated choice variables emerges.[9] In Figure 1 the double links occur at the nodes labeled by *Parent* in both families and thus indeterminacy between the parents' choice variables such as g_p^i and π^i

[8] As evident in Definition 1, the concept of linked individuals is not restricted in the case involving a single public good and two families, each consisting of a single parent and a single child. It is straightforward to demonstrate the validity of partial neutrality in the context of many public goods, many families and many children.

[9] The concept of "double link" can more carefully be defined using the terminology of graph theory or the analysis of networks as follows (see, e.g., Jackson , 2008). Since all nodes including the node labeled G in Figures 1-4 are tied by several links, all graphs or networks depicted in Figures 1-4 are termed as "the network or graph is connected" since every two nodes in the network are connected by some path (i.e., a sequence of links) in the network. Hence, if the network is connected, neutrality emerges as shown in Proposition 1. Moreover, if one of those links is to be dropped in Figures 2-4, then the network becomes "disconnected", then the original network is called a "tree"; alternatively, if a connected network consisting of n nodes has $n-1$ links, it is a tree, Therefore, if the network is a tree, these links conform "the minimum set of links" (which is called as "minimality of a network" in the analysis of networks), indeterminacy never arises. Thus the concept of "double link" means that if there are more links than the number of the minimum set of links, there exist extra links at some node and we call them "double links".

arises, while in Figures 2-4 there is no double link so that indeterminacy does not arise. Since in all diagrams all nodes are connected, neutrality arises.

Nevertheless, it is still unclear whether or not the property of indeterminacy is robust with respect to the different timings of the sequential ordering of actions. To verify this, we need to further investigate the model with alternative timing of parental transfers and private donations, and the hybrid model where different families adopt different timings of transfers and private donations in the following sections, respectively.[10]

4 Subgame Perfect Equilibrium with Pre-committed Transfers

In this section we suppose that the parent pre-commits to a fixed transfer before the child chooses her public good contribution. Given the pre-committed transfer π^i, at stage 2 the child chooses her contribution to maximize the utility function (which we call *Game II*). More formally, given the transfers (π^1, π^2) and the contributions made by the parents (g_p^1, g_p^2), after substituting the budget constraint (4), the child's decision problem in family i at stage 2 is to maximize her own utility function:

$$\max_{\{g_k^i\}} U_k^i = u_k^i(y_k^i + \pi^i - g_k^i, G).$$

The first-order conditions characterizing an interior solution for the respective families are given by

$$\frac{\partial u_k^i\left(y_k^i + \pi^i - g_k^i, G\right)}{\partial c_k^i} = \frac{\partial u_k^i\left(y_k^i + \pi^i - g_k^i, G\right)}{\partial G},\ i = 1, 2. \tag{18}$$

Since the first-order conditions (18) constitute the system of equations in g_k^i for $i = 1, 2$, we can solve it for $\hat{g}_k^i\left(\pi^1, \pi^2, g_p^1, g_p^2\right)$ for i = 1,2. After substitution of the above optimal consumption and contribution functions into the

[10] We can address the efficiency problem. To do this, combining (6) and (7) and rearranging yields

$$\frac{\partial u_p^i(c_p^i, G)/\partial G}{\partial u_p^i(c_p^i, G)/\partial c_p^i} + \frac{\partial u_k^i(c_k^i, G)/\partial G}{\partial u_k^i(c_k^i, G)/\partial c_k^i} = 1,\ i = 1, 2.$$

This implies that the Samuelson's efficiency condition for provision holds within a family. This is because by making use of the ex-post transfers the parent of each family can internalize the externality of the public good so that the Rotten-kid theorem (see, e.g., Bruce and Waldman, 1990) in terms of private provision of public goods holds true. However, it should be noted that it is not fully Pareto-efficient in the whole society.

parent's utility function, the parent's decision problem in family i at stage 1 is to maximize her utility function:

$$\max_{\{g_p^i,\pi^i\}} U_p^i = u_p^i(y_p^i - \pi^i - g_p^i, \hat{G}) + \alpha^i u_k^i(y_k^i + \pi^i - \hat{g}_k^i\left(\pi^1,\pi^2,g_p^1,g_p^2\right), \hat{G}),$$

where $\hat{G} \equiv g_p^i + g_p^j + \hat{g}_k^i\left(\pi^1,\pi^2,g_p^1,g_p^2\right) + \hat{g}_k^j\left(\pi^1,\pi^2,g_p^1,g_p^2\right)$, $i = 1,2$.

The first-order conditions characterizing an interior solution for $i = 1,2$ are

$$\frac{\partial U_p^i}{\partial g_p^i} = -\frac{\partial u_p^i}{\partial c_p^i} - \alpha^i \frac{\partial u_k^i}{\partial c_k^i}\frac{\partial \hat{g}_k^i}{\partial g_p^i} + \left[\frac{\partial u_p^i}{\partial G} + \alpha^i \frac{\partial u_k^i}{\partial G}\right]\left[\frac{\partial \hat{g}_k^i}{\partial g_p^i} + \frac{\partial \hat{g}_k^j}{\partial g_p^i} + 1\right] = 0, \quad (19)$$

$$\frac{\partial U_p^i}{\partial \pi^i} = -\frac{\partial u_p^i}{\partial c_p^i} + \alpha^i \frac{\partial u_k^i}{\partial c_k^i}\left[1 - \frac{\partial \hat{g}_k^i}{\partial \pi^i}\right] + \left[\frac{\partial u_p^i}{\partial G} + \alpha^i \frac{\partial u_k^i}{\partial G}\right]\left[\frac{\partial \hat{g}_k^i}{\partial \pi^i} + \frac{\partial \hat{g}_k^j}{\partial \pi^i}\right] = 0. \quad (20)$$

We first show that the neutrality property holds. We once again consider hypothetical changes in family member's private contributions and parental transfers prescribed by (10) and (11) in response to the redistribution of income. As before, it is straightforward to show that *constant* private consumptions of parents and children, as well as *constant* total provision of public good, are consistent with the budget constraints (3) and (4), as well as the first-order conditions (18), (19) and (20). This entails redistributional income neutrality.

Furthermore, it follows from (B9) and (B10) in Appendix B that the first-order conditions (19) and (20) boil down to a single equation so that there are too few equations to determine the unknown variables π^i and g_p^i, $i = 1,2$; consequently, indeterminacy arises, although the values of $g_p^i + \pi^i$ and thus $g_p^i + g_k^i$ are uniquely determined.

As for non-interior solutions, we have to consider the three cases associated with Figures 2-4. Applying the hypothetical changes in individual contributions and parental private transfers prescribed by either (12) and (13), (14) and (15), or (16) and (17) to the respective cases leads to both the validity of neutrality and the failure of indeterminacy as before.

In summary;

Proposition 2 *Consider a two-family, two-stage contribution game with the preferences in (1), where the children decide how much to make their contributions to the public good after having observed the transfers made by the parents to the children as well as contributions of the parents. Assume "small" redistributions of income. Then, statements (i), (ii) and (iii) in Proposition 1 hold true.*

Hence, Corollary 1 holds as well.

Cornes and Silva (1999) obtain a result similar to ours in that within a single family framework the voluntary contributions to a household public good and the transfers to children are indeterminate, when contributions to the family public good are perfect substitutes. Consequently, there is a continuum of Nash equilibria. Unfortunately, according to Chiappori and Werning (2002), Cornes and Silva's finding is not robust in the sense that an interior Nash equilibrium where every child makes a positive contribution is *non-generic*.

In contrast, our indeterminacy result holds regardless of the forms of utility functions which satisfy the standard properties. The key reason underlying our non-uniqueness result is the presence of an additional choice confronted by the parents, in conjunction with the well-established neutrality property in the literature on voluntary provision of public goods independently of the form of utility functions. Suppose that, initially, there is a minimal set of links. In view of Corollary 1, this means that the original graph would become disconnected if any one edge – or link – were dropped. Then adding a link to the original graph (see footnote 9) generates an indeterminacy of the associated additional choice variable confronted by the parents.

5 Subgame Perfect Equilibrium with the Mix of Ex-post and Pre-committed Transfers

So far we have investigated homogeneous economies in the sense that there is only one type of family: either all families make ex-post transfers or all families make pre-committed transfers. We now relax this setting to allow for the coexistence of those two types of families in the economy. Suppose, without loss of generality, that the parent of family 1 makes ex-post transfers, while that of family 2 makes pre-committed transfers (which we call *Game III*).

In stage 2, after having observed $(g_k^1,\ \pi^2,\ g_p^2)$, the parent of family 1 chooses π^1 and g_p^1, while the child of family 2 chooses g_k^2. Assuming an interior solution, the first-order conditions for the family 1's parent and the family 2's child are respectively given by

$$\frac{\partial U_p^1}{\partial g_p^1} = -\frac{\partial u_p^1(y_p^1 - \pi^1 - g_p^1, G)}{\partial c_p^1} + \frac{\partial u_p^1(.)}{\partial G} + \alpha^1 \frac{\partial u_k^1(y_k^1 + \pi^1 - g_k^1, G)}{\partial G} = 0, \quad (21)$$

$$\frac{\partial U_p^1}{\partial \pi^1} = -\frac{\partial u_p^1(.)}{\partial c_p^1} + \alpha^1 \frac{\partial u_k^1(.)}{\partial c_k^1} = 0, \quad (22)$$

$$\frac{\partial U_k^2}{\partial g_k^2} = -\frac{\partial u_k^2\left(y_k^2+\pi^2-g_k^2, G\right)}{\partial c_k^2} + \frac{\partial u_k^2\left(.\right)}{\partial G} = 0, \tag{23}$$

which form the system of equations in g_p^1, g_k^2 and π^1, given g_k^1, π^2 and g_p^2. Solving this, we can get $\hat{g}_p^1\left(g_k^1,\ \pi^2,\ g_p^2\right),\ \hat{g}_k^2\left(g_k^1,\ \pi^2,\ g_p^2\right)$ and $\hat{\pi}^1\left(g_k^1,\ \pi^2,\ g_p^2\right)$.

Given those functions, the child's maximizing problem in family 1 and the parent's decision problem in family 2 at stage 1 are respectively given by

$$\max_{\{g_k^1\}} U_k^1 = u_k^1(y_k^1+\hat{\pi}^1\left(g_k^1,\ \pi^2,\ g_p^2\right)-g_k^1,\hat{G}),$$

and

$$\max_{\{g_p^2,\pi^2\}} U_p^2 = u_p^2(y_p^2-\pi^2-g_p^2,\hat{G})+\alpha^2 u_k^2(y_k^2+\pi^2-\hat{g}_k^2\left(g_k^1,\ \pi^2,\ g_p^2\right),\hat{G}),$$

where $\hat{G} \equiv \hat{g}_p^1\left(g_k^1,\pi^2,g_p^2\right)+g_p^2+g_k^1+\hat{g}_k^2\left(g_k^1,\pi^2,g_p^2\right)$. The first-order conditions for an interior solution are respectively given by

$$\frac{\partial U_k^1}{\partial g_k^1} = \frac{\partial u_k^1}{\partial c_k^1}\left[\frac{\partial \hat{\pi}^1}{\partial g_k^1}-1\right]+\frac{\partial u_k^1}{\partial G}\left[\frac{\partial \hat{g}_p^1}{\partial g_k^1}+\frac{\partial \hat{g}_k^2}{\partial g_k^1}+1\right]=0, \tag{24}$$

$$\frac{\partial U_p^2}{\partial g_p^2} = -\frac{\partial u_p^2}{\partial c_p^2}-\alpha^2\frac{\partial u_k^2}{\partial c_k^2}\frac{\partial \hat{g}_k^2}{\partial g_p^2}+\left[\frac{\partial u_p^2}{\partial G}+\alpha^2\frac{\partial u_k^2}{\partial G}\right]\left[\frac{\partial \hat{g}_p^1}{\partial g_p^2}+\frac{\partial \hat{g}_k^2}{\partial g_p^2}+1\right]=0, \tag{25}$$

$$\frac{\partial U_p^2}{\partial \pi^2} = -\frac{\partial u_p^2}{\partial c_p^2}+\alpha^2\frac{\partial u_k^2}{\partial c_k^2}\left[1-\frac{\partial \hat{g}_k^2}{\partial \pi^2}\right]+\left[\frac{\partial u_p^2}{\partial G}+\alpha^2\frac{\partial u_k^2}{\partial G}\right]\left[\frac{\partial \hat{g}_p^1}{\partial \pi^2}+\frac{\partial \hat{g}_k^2}{\partial \pi^2}\right]=0. \tag{26}$$

It follows from (C2) in Appendix C that[11]

$$\frac{\partial \hat{\pi}^1}{\partial g_k^1} = 1 \text{ and } \frac{\partial \hat{g}_p^1}{\partial g_k^1}+\frac{\partial \hat{g}_k^2}{\partial g_k^1}+1=0, \tag{27}$$

$$\frac{\partial \hat{g}_k^2}{\partial g_p^2} = -1+\frac{\partial \hat{g}_k^2}{\partial \pi^2} \text{ and } \frac{\partial \hat{g}_p^1}{\partial g_p^2}+\frac{\partial \hat{g}_k^2}{\partial g_p^2}+1=\frac{\partial \hat{g}_p^1}{\partial \pi^2}+\frac{\partial \hat{g}_k^2}{\partial \pi^2}. \tag{28}$$

Property (27) ensures that $\partial U_k^1/\partial g_k^1 = 0$ in (24) always holds, which implies that values of g_k^1 are indeterminate. Moreover, owing to property (28), both (25) and (26) boil down to a single equation, implying that the division between g_p^2 and π^2 is also indeterminate. Furthermore, assuming the hypothetical responses prescribed by (10) and (11), we can easily verify that neutrality holds.

[11]Note that the two expressions in (28) simplify to $\partial \hat{g}_p^1/\partial g_p^2 = \partial \hat{g}_p^1/\partial \pi^2$.

As for non-interior solutions, we have to consider the following four cases; the first three cases correspond to the ones depicted in Figures 2-4, and the last case to the one opposed to Figure 4, where the only child in family 1 is a contributor while the only parent of family 2 is a contributor. Assuming (12) and (13), (14) and (15), as well as (16) and (17) to the first three cases of *Game III* and the following hypothetical changes:

$$dy_p^1 = d\pi^1 \text{ and } dy_k^1 + d\pi^1 = dg_k^1,$$
$$dy_p^2 = d\pi^2 + dg_p^2 \text{ and } dy_k^2 + d\pi^2 = 0,$$

to the last case of *Game III*, respectively, we can establish the validity of neutrality as well as the uniqueness of the equilibrium profile of private contributions and parental transfers in either case. Since the links appearing in Figures 2-4 and the last case to the one opposed to Figure 4 consist of the *minimum set of links*, these results are compatible with Corollary 1. In summary:

Proposition 3 *Consider a two-family, two-stage contribution game with the preferences in (1), where the parent of one family makes ex-post transfers to the child and the parent of the other family makes pre-committed transfers to the child. Assume "small" redistributions of income. Then, statements (i), (ii) and (iii) in Proposition 1 hold true.*

6 Cobb-Douglas Preferences

This section presents examples of the foregoing arguments. A special example may help to show algebraically that Warr's neutrality theorem holds, and, in particular, enables us to identify the range of income distributions that are consistent with the existence of an interior equilibrium. To this end, let us assume the following particular utility functions for the parent and child, respectively:

$$u_p^i(c_p^i, G) \equiv \ln c_p^i + \gamma_p \ln G,\ i = 1, 2, \tag{29}$$
$$u_k^i(c_k^i, G) \equiv \ln c_k^i + \gamma_k \ln G,\ i = 1, 2, \tag{30}$$

while the rest of the model is the same as before. To avoid unnecessary complications, we assume that the degree of parent's altruism is common, i.e., $\alpha = \alpha^i$ for $i = 1, 2$.

6.1 Ex-post Transfers

We first consider the case of ex-post transfers. Assuming an interior solution and given g_k^i for $i = 1,2$, at stage 2 the parent's first-order conditions of family i (6) and (7) are respectively expressed by

$$\frac{\partial U_p^i}{\partial \pi^i} = \frac{-1}{y_p^i - \pi^i - g_p^i} + \frac{\alpha}{y_k^i + \pi^i - g_k^i} = 0,\ i = 1, 2, \tag{31}$$

$$\frac{\partial U_p^i}{\partial g_p^i} = \frac{-1}{y_p^i - \pi^i - g_p^i} + \frac{\gamma_p + \alpha\gamma_k}{G} = 0,\ i = 1, 2. \tag{32}$$

Solving (31) for π^i yields

$$\hat{\pi}^i = \frac{1}{1+\alpha}\left[\alpha\left(y_p^i - g_p^i\right) - y_k^i + g_k^i\right],\ i = 1, 2. \tag{33}$$

Substituting (33) into π^i in (32) for each i and forming the system of equations in g_p^1 and g_p^2, we solve for:

$$\hat{g}_p^i + g_k^i = \frac{\left(1 + \alpha + \gamma_p + \alpha\gamma_k\right)\left(y_p^i + y_k^i\right) - (1+\alpha)\left(y_p^j + y_k^j\right)}{2(1+\alpha) + \gamma_p + \alpha\gamma_k},\ i, j = 1, 2, i \neq j, \tag{34}$$

which implies that there exists a certain range of income distributions among individuals for which an interior solution occurs, as illustrated in Figure 5. Summing (34) over i to get:

$$\hat{G} \equiv \hat{g}_p^1 + \hat{g}_p^2 + g_k^1 + g_k^2 = \frac{\left(\gamma_p + \alpha\gamma_k\right)Y}{2(1+\alpha) + \gamma_p + \alpha\gamma_k}, \tag{35}$$

where $Y \equiv y_p^1 + y_k^1 + y_p^2 + y_k^2$. Therefore, neutrality holds.

Given (35), the child's first-order condition at stage 1 is given by

$$\frac{\partial U_k^i}{\partial g_k^i} = \frac{1}{y_k^i + \hat{\pi}^i - g_k^i}\left(\frac{\partial \hat{\pi}^i}{\partial g_k^i} - 1\right) = 0, i = 1, 2. \tag{36}$$

Differentiating (33) with respect to g_k^i gives rise to

$$\frac{\partial \hat{\pi}^i}{\partial g_k^i} = \frac{1}{1+\alpha}\left(-\alpha\frac{\partial \hat{g}_p^i}{\partial g_k^i} + 1\right) = 1,\ i = 1, 2, \tag{37}$$

since it follows from (34) that $\partial \hat{g}_p^i / \partial g_k^i = -1$ for $i = 1, 2$. Substituting (37) into (36) results in that $\partial U_k^i / \partial g_k^i = 0$ for any value of g_k^i, which implies that the optimal values of g_k^i are indeterminate. Moreover, since the values of

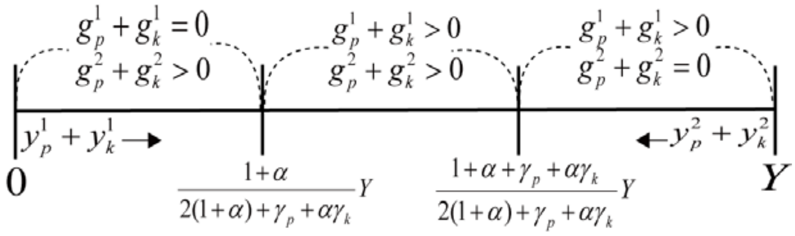

Figure 5: The pattern of family members' contributions to the public good in the ex-post case

$\hat{g}_p^i + g_k^i$, $i = 1, 2$, are uniquely determined from (34), g_p^i and thus π^i turn out to be indeterminate.

Figure 5 illustrates the behavior of family members' contributions to the public good when the income distribution varies, where the interval $[0, Y]$ represents the sum of the incomes of family 1 and 2. The distance from the extreme left point labelled by 0 in Figure 5 represents the total income of family 1, while the rest of the interval $[0, Y]$ does the total income of family 2. If the total income of family 1 is less than $(1+\alpha)Y/(2+2\alpha+\gamma_p+\alpha\gamma_k)$, then none of the members of family 1 contributes to the public good, while the only members of family 2 contribute. If the total income of family 1 is located in the interval $\left[(1+\alpha)Y/(2+2\alpha+\gamma_p+\alpha\gamma_k), (1+\alpha+\gamma_p+\alpha\gamma_k)Y/(2+2\alpha+\gamma_p+\alpha\gamma_k)\right]$, the members of both families contribute. If the total income of family 1 is larger than $(1+\alpha+\gamma_p+\alpha\gamma_k)Y/(2+2\alpha+\gamma_p+\alpha\gamma_k)$, then the only members of family 2 contribute. Note further that the division between g_p^i and g_k^i is indeterminate as long as $g_p^i + g_k^i > 0$.

6.2 Pre-committed Transfers

In this subsection we consider the pre-committed case. Assuming that every family member makes a positive contribution to the public good, the child's first-order conditions of the respective families (18) at stage 2 are given by

$$\frac{\partial U_k^i}{\partial g_k^i} = \frac{-1}{y_k^i + \pi^i - g_k^i} + \frac{\gamma_k}{G} = 0, \; i = 1, 2,$$

which are rearranged as follows:

$$\begin{aligned} (1+\gamma_k)\, g_k^1 + g_k^2 &= \gamma_k \left(y_k^1 + \pi^1\right) - g_p^1 - g_p^2, \\ g_k^1 + (1+\gamma_k)\, g_k^2 &= \gamma_k \left(y_k^2 + \pi^2\right) - g_p^1 - g_p^2. \end{aligned}$$

Solution of this pair of equations yields

$$\hat{g}_k^i = \frac{(1+\gamma_k)\left(y_k^i + \pi^i\right) - \left(g_p^i + g_p^j + y_k^j + \pi^j\right)}{2+\gamma_k}, \; i,j = 1,2, i \neq j. \tag{38}$$

Summing over i yields

$$\hat{G} \equiv g_p^1 + g_p^2 + \hat{g}_k^1 + \hat{g}_k^2 = \frac{\gamma_k\left(g_p^1 + g_p^2 + y_k^1 + \pi^1 + y_k^2 + \pi^2\right)}{2+\gamma_k}. \tag{39}$$

Given (38) and (39), the parent's first-order conditions of the respective families at stage 1, (19) and (20) are, for $i = 1,2$,

$$\frac{\partial U_p^i}{\partial g_p^i} = -\frac{1}{y_p^i - \pi^i - g_p^i} - \frac{\alpha}{y_k^i + \pi^i - \hat{g}_k^i}\frac{\partial \hat{g}_k^i}{\partial g_p^i} + \frac{\gamma_p + \alpha\gamma_k}{\hat{G}}\frac{\partial \hat{G}}{\partial g_p^i} = 0, \tag{40}$$

$$\frac{\partial U_p^i}{\partial \pi^i} = -\frac{1}{y_p^i - \pi^i - g_p^i} + \frac{\alpha}{y_k^i + \pi^i - \hat{g}_k^i}\left(1 - \frac{\partial \hat{g}_k^i}{\partial \pi^i}\right) + \frac{\gamma_p + \alpha\gamma_k}{\hat{G}}\frac{\partial \hat{G}}{\partial \pi^i} = 0. \tag{41}$$

Differentiating (38) with respect to g_p^i and π^i for $i = 1,2$ yields

$$\frac{\partial \hat{g}_k^i}{\partial g_p^i} = -\frac{1}{2+\gamma_k} \text{ and } \frac{\partial \hat{g}_k^i}{\partial \pi^i} = \frac{1+\gamma_k}{2+\gamma_k}, \; i = 1,2, \tag{42}$$

which together implies that $\partial \hat{g}_k^i / \partial g_p^i = 1 - (\partial \hat{g}_k^i / \partial \pi^i)$. Using this fact, we can confirm that (40) and (41) coincide. Hence, the division between g_p^i and π^i is indeterminate.

Substituting (42) into (40) and (41), and rearranging yields

$$\begin{aligned} \left[1 + \gamma_p + (1+\gamma_k)\alpha\right]\left(\pi^1 + g_p^1\right) + \left(\pi^2 + g_p^2\right) &= B_1, \\ \left(\pi^1 + g_p^1\right) + \left[1 + \gamma_p + (1+\gamma_k)\alpha\right]\left(\pi^2 + g_p^2\right) &= B_2, \end{aligned} \tag{43}$$

where $B_i \equiv \left[\gamma_p + (1+\gamma_k)\alpha\right] y_p^i - y_k^i - y_k^j$ for $i,j = 1,2, i \neq j$.

Solving (43) for $\pi^i + g_p^i$ $(i = 1,2)$ yields

$$\pi^i + g_p^i = \frac{\left[1 + \gamma_p + (1+\gamma_k)\alpha\right] y_p^i - y_k^i - y_p^j - y_k^j}{2 + \gamma_p + (1+\gamma_k)\alpha}, \; i,j = 1,2, i \neq j. \tag{44}$$

Substituting (44) into $\pi^i + g_p^i$ $(i = 1,2)$ in (39) results in

$$\hat{G} = \Delta^{-1}\gamma_k(\alpha + \gamma_p + \alpha\gamma_k)Y, \tag{45}$$

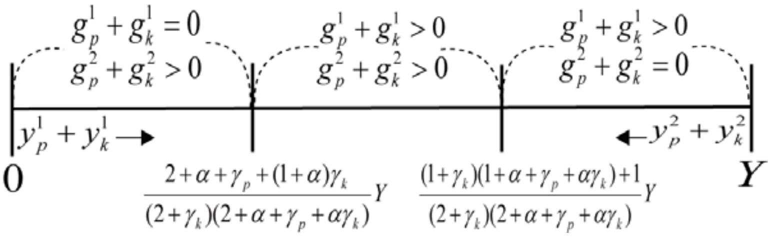

Figure 6: The pattern of family members' contributions to the public good in the pre-committed case.

where $\Delta \equiv (2+\gamma_k)\left[2+\gamma_p+(1+\gamma_k)\alpha\right]$. These results establish neutrality. Furthermore, substituting (44) into (38) for $i=1,2$ yields

$$\begin{aligned}\hat{g}_k^i + g_p^i = \Delta^{-1} & \left[\left\{(1+\gamma_k)\left(1+\alpha+\gamma_p+\alpha\gamma_k\right)+1\right\}(y_p^i+y_k^i)\right. \\ & \left. -\left\{2+\alpha+\gamma_p+(1+\alpha)\gamma_k\right\}(y_p^j+y_k^j)\right],\ i,j=1,2, i\neq j. \qquad (46)\end{aligned}$$

Using (46), we can draw Figure 6 which shows the behavior of family members' contributions to the public good when the income distribution varies. Figure 6 exhibits the same features as those in Figure 6 except for the critical levels of income at which the respective family's members alternate their contributions between a positive amount and a zero.

6.3 Mix of Ex-post and Pre-committed Transfers

Given (g_k^1, π^2, g_p^2) and assuming interior solutions for $\hat{g}_p^1$, $\hat{\pi}^1$ and $\hat{g}_k^2$, at stage 2 we can solve (21), (22) and (23) to get

$$\hat{g}_p^1 + g_k^1 = \frac{(1+\gamma_k)(\gamma_p+\alpha\gamma_k)(y_k^1+y_p^1)-(1+\alpha)\gamma_k(y_k^2+g_p^2+\pi^2)}{\gamma_k(1+\alpha)+(1+\gamma_k)(\gamma_p+\alpha\gamma_k)}, \qquad (47)$$

$$\hat{\pi}^1 = \frac{\alpha\gamma_k(y_p^1+y_k^2+g_p^2+\pi^2)-[\gamma_k+(1+\gamma_k)(\gamma_p+\alpha\gamma_k)]y_k^1}{\gamma_k(1+\alpha)+(1+\gamma_k)(\gamma_p+\alpha\gamma_k)}+g_k^1, \qquad (48)$$

$$\hat{g}_k^2 = \frac{\gamma_k(1+\alpha+\gamma_p+\alpha\gamma_k)(y_k^2+\pi^2)-(\gamma_p+\alpha\gamma_k)(y_p^1+y_k^1+g_p^2)}{\gamma_k(1+\alpha)+(1+\gamma_k)(\gamma_p+\alpha\gamma_k)}. \qquad (49)$$

Manipulating (47) and (49), and adding g_p^2 yields

$$\hat{G} = \hat{g}_p^1 + g_k^1 + g_p^2 + \hat{g}_k^2 = \frac{\gamma_k(\gamma_p+\alpha\gamma_k)(y_p^1+y_k^1+y_k^2+\pi^2+g_p^2)}{\gamma_k(1+\alpha)+(1+\gamma_k)(\gamma_p+\alpha\gamma_k)}. \qquad (50)$$

Given the optimal contribution functions $\hat{g}_p^1$, $\hat{\pi}^1$, $\hat{g}_k^2$ in (47)-(49) and the total provision of $\hat{G}$ in (50), the parent's decision problem of family 2 at stage 1 gives the following first-order conditions for an interior solution:

$$\frac{\partial U_p^2}{\partial g_p^2} = -\frac{1}{y_p^2 - \pi^2 - g_p^2} - \frac{\alpha}{y_k^2 + \pi^2 - \hat{g}_k^2}\frac{\partial \hat{g}_k^2}{\partial g_p^2} + \frac{\gamma_p + \alpha\gamma_k}{\hat{G}} \cdot \frac{\gamma_k(\gamma_p + \alpha\gamma_k)}{\gamma_k(1+\alpha) + (1+\gamma_k)(\gamma_p + \alpha\gamma_k)} = 0, \tag{51}$$

$$\frac{\partial U_p^2}{\partial \pi^2} = -\frac{1}{y_p^2 - \pi^2 - g_p^2} + \frac{\alpha}{y_k^2 + \pi^2 - \hat{g}_k^2}\left(1 - \frac{\partial \hat{g}_k^2}{\partial \pi^2}\right) + \frac{\gamma_p + \alpha\gamma_k}{\hat{G}} \cdot \frac{\gamma_k(\gamma_p + \alpha\gamma_k)}{\gamma_k(1+\alpha) + (1+\gamma_k)(\gamma_p + \alpha\gamma_k)} = 0. \tag{52}$$

Differentiating $\hat{g}_k^2$ given by (49) with respect to g_p^2 and π^2, respectively, yields

$$-\frac{\partial \hat{g}_k^2}{\partial g_p^2} = \frac{\gamma_p + \alpha\gamma_k}{\gamma_k(1+\alpha) + (1+\gamma_k)(\gamma_p + \alpha\gamma_k)} = 1 - \frac{\partial \hat{g}_k^2}{\partial \pi^2},$$

which renders the parent's first-order conditions (51) and (52) become identical. As a result, the division between π^2 and g_p^2 is *indeterminate*, although $\pi^2 + g_p^2$ is uniquely given by:

$$\pi^2 + g_p^2 = \frac{\alpha\left(\gamma_p + \gamma_k + \alpha\gamma_k\right) y_p^2 - \gamma_k\left(y_p^1 + y_k^1 + y_k^2\right)}{\alpha\left(\gamma_p + \gamma_k + \alpha\gamma_k\right) + \gamma_k}. \tag{53}$$

Further substitution of (53) into (50) yields

$$\hat{G} = \Gamma^{-1}\alpha\gamma_k\left(\gamma_p + \gamma_k + \alpha\gamma_k\right) Y, \tag{54}$$

where $\Gamma \equiv \left[\gamma_k(1+\alpha) + (1+\gamma_k)(\gamma_p + \alpha\gamma_k)\right]\left[\alpha\left(\gamma_p + \gamma_k + \alpha\gamma_k\right) + \gamma_k\right]$. These results establish neutrality.[12]

Finally, substituting (53) into (47) and (49), respectively, and rearranging yields

$$\hat{g}_p^1 + g_k^1 = \Gamma^{-1}\left[(1+\gamma_k)(\gamma_p + \alpha\gamma_k)\alpha\left(\gamma_p + \gamma_k + \alpha\gamma_k\right) + \left\{(1+\gamma_k)(\gamma_p + \alpha\gamma_k) + (1+\alpha)\gamma_k\right\}\gamma_k(y_k^1 + y_p^1) - \alpha\gamma_k(1+\alpha)\left(\gamma_p + \gamma_k + \alpha\gamma_k\right)(y_p^2 + y_k^2)\right], \tag{55}$$

[12] Although we can verify that the total provision of the public good in the ex-post case, (35), is larger than that at the pre-committed case, (45), it is not possible to rank between either of them and the total provision level in the mixed case, (54).

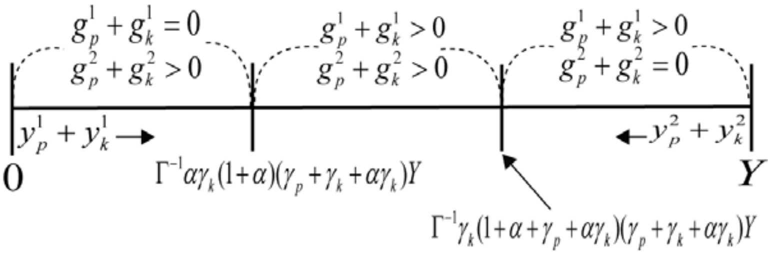

Figure 7: The pattern of family members' contributions to the public good in the mixed case.

and

$$\begin{aligned}\hat{g}_k^2 + g_p^2 = \Gamma^{-1}\big[&\alpha\gamma_k(1+\alpha+\gamma_p+\alpha\gamma_k)\left(\gamma_p+\gamma_k+\alpha\gamma_k\right)(y_p^2+y_k^2) - \\ &\left\{(\gamma_p+\alpha\gamma_k)\left[\alpha\left(\gamma_p+\gamma_k+\alpha\gamma_k\right)+\gamma_k(1+\gamma_k)\right]+\gamma_k^2(1+\alpha)\right\}(y_p^1+y_k^1)\big].\end{aligned} \tag{56}$$

Using (55) and (56), we can draw Figure 7 which shows the behavior of family members' contributions to the public good when the income distribution varies. Figure 7 exhibits the same features as those in Figures 5 and 6 except for the critical levels of income at which the respective family's members alternate their contributions between a positive amount and a zero.

7 Impure Altruism

In this section we consider the following parent's utility functions:

$$U_p^i(c_p^i, G, g_p^i; c_k^i) \equiv u_p^i(c_p^i, G, g_p^i) + \alpha^i u_k^i(c_k^i, G),\ i = 1, 2, \tag{57}$$

or

$$U_p^i(c_p^i, G, \pi^i; c_k^i) \equiv u_p^i(c_p^i, G, \pi^i) + \alpha^i u_k^i(c_k^i, G),\ i = 1, 2, \tag{58}$$

while the rest of the model is the same as before.

Abel and Bernheim (1991) show that, under the utility function (58) with the preference toward the size of the transfers the parent makes rather than the total provision of the public good, Bernheim and Bagwell's cross sectional neutrality fails to hold, while Andreoni (1990), and Cornes and Sandler (1984) show that under the utility function (57), without the preference toward the welfare of the child, Warr's neutrality theorem fails. We first examine the neutrality issue under the utility function (58). Assuming the ex-post transfers made by the parents of both families, the first-order conditions for an interior solution are given by (6), which is modified by adding the variable π^i, and

$$\frac{\partial U_p^i}{\partial \pi^i} = -\frac{\partial u_p^i(y_p^i - \pi^i - g_p^i, \pi^i, G)}{\partial c_p^i} + \frac{\partial u_p^i(.)}{\partial \pi^i} + \alpha^i \frac{\partial u_k^i(y_k^i + \pi^i - g_k^i, \pi^i, G)}{\partial c_k^i} = 0,\ i = 1, 2. \tag{59}$$

Under the hypothetical changes prescribed by (10) and (11), the above first-order conditions (6) and (59) are not satisfied at the original values of c_p^i, c_k^i and G, because π^i is free to change. Instead, assuming π^i to be fixed at the original value, the hypothetical changes are amended as follows; $dy_p^i = dg_p^i$ and $dy_k^i = dg_k^i$, i = 1,2. It is easy to see that under these hypothetical changes the allocation of resource allocation remains unchanged and thus neutrality holds, yet the indeterminacy between g_p^i and π^i clearly disappears. Similarly, as long as an interior equilibrium is assumed, the neutrality and uniqueness of an equilibrium allocation are robust regardless of the different timings of parental transfers.

Under the utility function (57), on the other hand, we consider the hypothetical changes given by $dy_p^i = d\pi^i$ and $dy_k^i + d\pi^i = dg_k^i$, i = 1,2, keeping g_p^i unchanged. In a similar way, we can show that the neutrality and uniqueness of the equilibrium allocation hold.

In either case, since one of the parent's choice variables such as π^i or g_p^i does not appear in the utility function as a single element and thus the first-order condition, that variable is able to freely change so as to neutralize the effect of income redistribution policy. Nevertheless, the presence of imperfect altruism will eliminate the non-uniqueness of a Nash equilibrium allocation even in an interior equilibrium, because one of the variables g_p^i and π^i should be kept at the original value in order to realize neutrality; consequently, it should be uniquely determined so as to satisfy the relevant first-order condition. Moreover, when both types of impure altruism are simultaneously present; that is, the parent's (or child's) utility function depends directly and individually both on her own transfers as well as on her own private contribution, even neutrality no longer holds true.[13]

[13] Note, however, that if the utility function is given by $U_p^i(c_p^i, G, g_p^i; c_k^i) \equiv u_p^i(c_p^i, G, g_p^i + \pi^i) + \alpha^i u_k^i(c_k^i, G)$, the property of indeterminacy recovers because the first-order conditions for an interior solution in terms of g_p^i and π^i become identical due to perfect substitutes between g_p^i and π^i.

8 Household Public Goods

In this last section we consider a two-stage contribution game where family members contribute to both a household, or intrafamily, public good and an interfamily public good. Due to space limitations, we shall consider only the case of ex-post transfers. The utility function of the parent of family i is given by

$$U_p^i(c_p^i, H^i, G; c_k^i) \equiv u_p^i(c_p^i, H^i, G) + \alpha^i u_k^i(c_k^i, H^i, G),\ i = 1, 2, \tag{60}$$

where H^i represents a household public good which benefits only the members of family i, and the other notations are the same as before. The budget constraint of the parent and child of family i are respectively given by

$$\begin{aligned} c_p^i + \pi^i + h_p^i + g_p^i &= y_p^i, & (61) \\ c_k^i + h_k^i + g_k^i &= y_k^i + \pi^i, & (62) \end{aligned}$$

where h_p^i and h_k^i represent the contributions made by the parent and child of family i to the household public good, respectively, and $H^i = h_p^i + h_k^i$.

We use backward induction to solve the parent's optimization problem first. After substitution of (61) and (62) into (60), the parent's problem is:

$$\max_{\{g_p^i, \pi^i\}} U_p^i = u_p^i(y_p^i - \pi^i - g_p^i - h_p^i, H^i, G) + \alpha^i u_k^i(y_k^i + \pi^i - h_k^i - g_k^i, H^i, G). \tag{63}$$

The first-order conditions characterizing an interior solution are

$$\begin{aligned} \frac{\partial U_p^i}{\partial g_p^i} &= -\frac{\partial u_p^i(y_p^i - \pi^i - h_p^i - g_p^i, H^i, G)}{\partial c_p^i} + \frac{\partial u_p^i(.)}{\partial G} \\ &+ \alpha^i \frac{\partial u_k^i(y_k^i + \pi^i - h_k^i - g_k^i, H^i, G)}{\partial G} = 0,\ i = 1, 2, \end{aligned} \tag{64}$$

$$\frac{\partial U_p^i}{\partial h_p^i} = -\frac{\partial u_p^i(,)}{\partial c_p^i} + \frac{\partial u_p^i(.)}{\partial H^i} + \alpha^i \frac{\partial u_k^i(.)}{\partial H^i} = 0,\ i = 1, 2, \tag{65}$$

$$\frac{\partial U_p^i}{\partial \pi^i} = -\frac{\partial u_p^i(.)}{\partial c_p^i} + \alpha^i \frac{\partial u_k^i(.)}{\partial c_k^i} = 0,\ i = 1, 2. \tag{66}$$

Since (64), (65) and (66) form the system of equations in g_p^i, h_p^i and π^i, $i = 1, 2$, given g_k^i , g_k^j, h_k^i and h_k^j we can solve it for $\hat{g}_p^i\,(g_k^1, g_k^2, h_k^1, h_k^2)$, $\hat{h}_p^i\,(g_k^1, g_k^2, h_k^1, h_k^2)$ and $\hat{\pi}^i\,(g_k^1, g_k^2, h_k^1, h_k^2)$, $i = 1, 2$. Given these functions, the child's maximizing problem of family i at stage 1 is given by

$$\max_{\{g_k^i\}} U_k^i = u_k^i(y_k^i + \hat{\pi}^i\left(g_k^1, g_k^2, h_k^1, h_k^2\right) - h_k^i - g_k^i, h_p^i\left(g_k^1, g_k^2, h_k^1, h_k^2\right) + h_k^i, \hat{G}).$$

The first-order conditions for an interior solution are given by

$$\begin{aligned}\frac{\partial U_k^i}{\partial g_k^i} &= \frac{\partial u_k^i(y_k^i + \hat{\pi}^i - h_k^i - g_k^i, H^i, \hat{G})}{\partial c_k^i}\left[\frac{\partial \hat{\pi}^i}{\partial g_k^i} - 1\right] + \frac{\partial u_k^i(.)}{\partial H^i}\frac{\partial \hat{h}_p^i}{\partial g_k^i} \\ &+ \frac{\partial u_k^i(.)}{\partial G}\left[\frac{\partial \hat{g}_p^i}{\partial g_k^i} + \frac{\partial \hat{g}_p^j}{\partial g_k^i} + 1\right] = 0,\ i = 1, 2. \end{aligned} \tag{67}$$

$$\begin{aligned}\frac{\partial U_k^i}{\partial h_k^i} &= \frac{\partial u_k^i(.)}{\partial c_k^i}\left[\frac{\partial \hat{\pi}^i}{\partial h_k^i} - 1\right] \\ &+ \frac{\partial u_k^i(.)}{\partial H^i}\left[\frac{\partial \hat{h}_p^i}{\partial h_k^i} + 1\right] + \frac{\partial u_k^i(.)}{\partial G}\left[\frac{\partial \hat{g}_p^i}{\partial h_k^i} + \frac{\partial \hat{g}_p^j}{\partial h_k^i}\right] = 0,\ i = 1, 2. \end{aligned} \tag{68}$$

In order to prove the validity of the neutrality property in terms of income redistribution, given a set of changes in incomes dy_h^i, $i = 1, 2$ and $h = p, k$, such that $dy_p^1 + dy_p^2 + dy_k^1 + dy_k^2 = 0$, we consider a set of changes in choice variables by the agents that satisfy

$$\begin{aligned} dy_p^i &= d\pi^i + dg_p^i + dh_p^i,\ i = 1,2, \\ dy_k^i + d\pi^i &= dg_k^i + dh_k^i,\ i = 1,2. \\ dh_k^i + dh_p^i &= 0,\ i = 1,2. \end{aligned}$$

These hypothetical responses, in conjunction with the budget constraints (61) and (62), result in the constant private consumption of every individual and

$$\begin{aligned} dG &= dg_k^1 + dg_k^2 + dg_p^1 + dg_p^2, \\ &= (dy_k^1 + d\pi^1 - dh_k^1) + (dy_k^2 + d\pi^2 - dh_k^2) \\ &\quad +(dy_p^i - d\pi^i - dh_p^1) + (dy_p^2 - d\pi^2 - dh_p^2), \\ &= dy_k^1 + dy_p^1 + dy_k^2 + dy_p^2 = 0. \end{aligned}$$

In addition, the constancy of the variables c_p^i, c_k^i, H^i and G leaves the first-order conditions (64), (65), (66), (67) and (68) intact as well.

Two remarks are in order. First, an infinite number of combinations of $d\pi^i$, dg_p^i and dh_p^i hold under the above hypothetical changes, thus leading to the indeterminacy property. Second, even if due to the traditional gender roles of husband and wife – emphasized by Lundberg and Pollak (1993) – or productivity difference in supplying household public goods – emphasized by Konrad and Lommerud (1995) – either the parent or child is at a corner solution as a contributor to the intrafamily public good, income redistribution does not distort the real allocation as long as the channels of voluntary income transfers and/or contributions to the interfamily public good are operative, that is, neutrality as well as indeterminacy continue to hold in our augmented model. This outcome stems from the property uncovered by Corollary 1, i.e., if everyone is *linked*, neutrality remains effective. Although this finding seems to be inconsistent with the result of Konrad and Lommerud (1995) where the productivity difference between spouses undermines the neutrality property, it is not. This difference stems from the fact that their model contains neither the contributions to interfamily public goods nor voluntary income transfers.

9 Concluding Remarks

In this paper we have shown that when parents voluntarily and simultaneously make both private donations to public goods and transfers to children in an interior equilibrium, the contributions made by parents are indeterminate. In other words, in a two-stage Nash provision game an infinite number of combinations of individual contributions and transfers are sustained as a (subgame perfect) Nash equilibrium with a unique profile of every individual's consumption as well as the total supply of public good, regardless of whether the parents decide to make parent-to-child transfers before or after having observed the contributions made by the children. However, the introduction of some friction such as impure altruism with respect to either her own private donations or her own intrafamily transfers eliminates such indeterminacy, although neutrality may still remain valid.

The neutrality result established in this paper has two important policy implications. First, Bergstrom et al. (1986) point out that if the redistribution of income from non-contributors to contributors takes place, not only neutrality fails, but also total provision of the public good will increase. In contrast to their claim, in our multi-family setting there is the possibility that neutrality continues to hold under such a redistribution, as long as intrafamily transfers are operative. On the other hand, even if most interfamily links through transfers are not operative, Bernheim and Bagwell's cross sectional neutrality may survive because the private donations to interfamily public goods are effectively perfect substitutes for interfamily private transfers.

Since the seminal paper of Konrad and Lommerud (1995), most household economists tend to believe that the redistribution policy between family members is not neutral, and, moreover, there is the possibility of Pareto improvement. However, our paper casts doubt on the robustness of this non-neutrality result at least from a theoretical viewpoint, since there are various channels between family members which may undo the effectiveness of redistribution policy.

There are extensions in several directions. First, a natural extension is to combine the present non-cooperative multi-family model with the cooperative or Nash bargaining (single) family model such as Lundberg and Pollak (1993), Konrad and Lommerud (2000), Chen and Woolley (2001). This extension would further provide richer implications. Our analysis suggests that the threat points, and the cooperative Nash bargaining solutions of the models of Lundberg and Pollak, Konrad and Lommerud are highly likely to be independent of the non-wage incomes. This conjecture may not hold in the bargaining model of Chen and Woolley, where bargaining is over income transfers rather than private consumption allocations. Therefore, these conjectures call for further examination of their models under our setting. Second, it is also interesting to investigate the case of several children, possibly coupled with the endogenous fertility decision of the parent. It would be interesting to introduce intragenerational conflict among children, e.g., children's transfer seeking activities proposed by Chang (2009) into the present model. Although the analysis will be complicated, we expect that our basic results would be still valid.

Appendix A

In order to prove that (9) holds true, we totally differentiate (6) and (7) to yield

$$
\begin{bmatrix}
u^{1p}_{cc} - u^{1p}_{Gc} + \alpha^1 u^{1k}_{Gc} & 0 & u^{1p}_{cc} - 2u^{1p}_{Gc} + u^{1p}_{GG} + \alpha^1 u^{1k}_{GG} & -u^{1p}_{cG} + u^{1p}_{GG} + \alpha^1 u^{1k}_{GG} \\
0 & u^{2p}_{cc} - u^{2p}_{Gc} + \alpha^2 u^{2k}_{Gc} & -u^{2p}_{cG} + u^{2p}_{GG} + \alpha^2 u^{2k}_{GG} & u^{2p}_{cc} - 2u^{2p}_{Gc} + u^{2p}_{GG} + \alpha^2 u^{2k}_{GG} \\
u^{1p}_{cc} + \alpha^1 u^{1k}_{cc} & 0 & u^{1p}_{cc} - u^{1p}_{cG} + \alpha^1 u^{1k}_{cG} & -u^{1p}_{cG} + \alpha^1 u^{1k}_{cG} \\
0 & u^{2p}_{cc} + \alpha^2 u^{2k}_{cc} & -u^{2p}_{cG} + \alpha^2 u^{2k}_{cG} & u^{2p}_{cc} + u^{2p}_{cG} - \alpha^2 u^{2k}_{cG}
\end{bmatrix}
\begin{bmatrix}
d\pi^1 \\ d\pi^2 \\ dg^1_p \\ dg^2_p
\end{bmatrix} =
$$

$$\begin{bmatrix} \left(\alpha^1 u_{Gc}^{1k} + u_{cG}^{1p} - u_{GG}^{1p} - \alpha^1 u_{GG}^{1k}\right) dg_k^1 + \left(u_{cG}^{1p} - u_{GG}^{1p} - \alpha^1 u_{GG}^{1k}\right) dg_k^2 \\ \left(\alpha u_{Gc}^{2k} + u_{cG}^{2p} - u_{GG}^{2p} - \alpha^2 u_{GG}^{2k}\right) dg_k^2 + \left(u_{cG}^{2p} - u_{GG}^{2p} - \alpha^2 u_{GG}^{2k}\right) dg_k^1 \\ \left(\alpha^1 u_{cc}^{1k} + u_{cG}^{1p} - \alpha^1 u_{cG}^{1k}\right) dg_k^1 + \left(u_{cG}^{1p} - \alpha^1 u_{cG}^{1k}\right) dg_k^2 \\ \left(u_{cG}^{2p} - \alpha^2 u_{cG}^{2k}\right) dg_k^1 + \left(\alpha^2 u_{cc}^{2k} + u_{cG}^{2p} - \alpha^2 u_{cG}^{2k}\right) dg_k^2 \end{bmatrix} \tag{A1}$$

Applying Cramer's rule, we can get the following comparative statics results:

$$\frac{d\hat{\pi}^i}{dg_k^i} = 1;\ \frac{d\hat{g}_p^i}{dg_k^i} = -1;\ \frac{d\hat{g}_p^j}{dg_k^i} = 0,\ i, j = 1, 2,\ i \neq j. \tag{A2}$$

Appendix B

In order to show that the first-order conditions (19) and (20) reduce to a single equation, we totally differentiate (18) to yield

$$\begin{bmatrix} -u_{cc}^{1k} + 2u_{Gc}^{1k} - u_{GG}^{1k} & u_{cG}^{1k} - u_{GG}^{1k} \\ u_{cG}^{2k} - u_{GG}^{2k} & -u_{cc}^{2k} + 2u_{Gc}^{2k} - u_{GG}^{2k} \end{bmatrix} \begin{bmatrix} dg_k^1 \\ dg_k^2 \end{bmatrix} =$$
$$\begin{bmatrix} \left(u_{Gc}^{1k} - u_{cc}^{1k}\right) d\pi^1 + \left(u_{GG}^{1k} - u_{cG}^{1k}\right) dg_p^1 + \left(u_{GG}^{1k} - u_{cG}^{1k}\right) dg_p^2 \\ \left(u_{Gc}^{2k} - u_{cc}^{2k}\right) d\pi^2 + \left(u_{GG}^{2k} - u_{cG}^{2k}\right) dg_p^2 + \left(u_{GG}^{2k} - u_{cG}^{2k}\right) dg_p^1 \end{bmatrix}. \tag{B1}$$

Application of Cramer's rule yields

$$\frac{d\hat{g}_k^i}{dg_p^i} = |B|^{-1} \left(u_{GG}^{ik} - u_{cG}^{ik}\right) (-u_{cc}^{jk} + u_{Gc}^{jk}), \tag{B2}$$

$$\frac{d\hat{g}_k^j}{dg_p^i} = |B|^{-1} (u_{cG}^{jk} - u_{GG}^{jk})(-u_{cc}^{ik} + u_{Gc}^{ik}), \tag{B3}$$

$$\frac{d\hat{g}_k^i}{d\pi^i} = |B|^{-1} \left(u_{Gc}^{ik} - u_{cc}^{ik}\right) (-u_{cc}^{jk} + u_{Gc}^{jk} + u_{cG}^{jk} - u_{GG}^{jk}), \tag{B4}$$

$$\frac{d\hat{g}_k^j}{d\pi^i} = |B|^{-1} \left(u_{Gc}^{ik} - u_{cc}^{ik}\right) (u_{cG}^{jk} - u_{GG}^{jk}), \tag{B5}$$

where $|B|$ represents the determinant of the matrix appearing on the left-hand side of (B1).

Furthermore, by straightforward calculation, we have

$$1-\frac{\partial \hat{g}_k^i}{\partial \pi^i} = |B|^{-1}\left(u_{cG}^{ik}-u_{GG}^{ik}\right)\left(-u_{cc}^{jk}+u_{Gc}^{jk}\right),\ i=1,2, \tag{B6}$$

$$\frac{\partial \hat{g}_k^i}{\partial g_p^i}+\frac{\partial \hat{g}_k^j}{\partial g_p^i}+1 = |B|^{-1}\left(u_{Gc}^{ik}-u_{cc}^{ik}\right)\left(3u_{cG}^{jk}-2u_{GG}^{jk}-u_{cc}^{jk}\right),\ i,j=1,2,\ i\neq j, \tag{B7}$$

$$\frac{\partial \hat{g}_k^i}{\partial \pi^i}+\frac{\partial \hat{g}_k^j}{\partial \pi^i} = |B|^{-1}\left(u_{Gc}^{ik}-u_{cc}^{ik}\right)\left(-u_{cc}^{jk}+3u_{Gc}^{jk}-2u_{GG}^{jk}\right),\ i,j=1,2,\ i\neq j. \tag{B8}$$

Comparison of (B6) with (B2) and (B7) with (B8), respectively, reveals that

$$1-\frac{\partial \hat{g}_k^i}{\partial \pi^i} = -\frac{d\hat{g}_k^i}{dg_p^i},\ i=1,2, \tag{B9}$$

$$\frac{\partial \hat{g}_k^i}{\partial g_p^i}+\frac{\partial \hat{g}_k^j}{\partial g_p^i}+1 = \frac{\partial \hat{g}_k^i}{\partial \pi^i}+\frac{\partial \hat{g}_k^j}{\partial \pi^i},\ i,j=1,2,\ i\neq j. \tag{B10}$$

Appendix C

In order to prove that (27) and (28) holds true, we totally differentiate (??), (22) and (23) to yield

$$\begin{bmatrix} u_{cc}^{1p}-u_{Gc}^{1p}+\alpha^1 u_{Gc}^{1k} & u_{cc}^{1p}-2u_{Gc}^{1p}+u_{GG}^{1p}+\alpha^1 u_{GG}^{1k} & -u_{cG}^{1p}+u_{GG}^{1p}+\alpha^1 u_{GG}^{1k} \\ u_{cc}^{1p}+\alpha^1 u_{cc}^{1k} & u_{cc}^{1p}-u_{cG}^{1p}+\alpha^1 u_{cG}^{1k} & -u_{cG}^{1p}+\alpha^1 u_{cG}^{1k} \\ 0 & u_{cG}^{2k}-u_{GG}^{2k} & -u_{cc}^{2k}+u_{cG}^{2k}+\alpha^2 u_{Gc}^{2k}-u_{GG}^{2k} \end{bmatrix}\begin{bmatrix} d\pi^1 \\ dg_p^1 \\ dg_k^2 \end{bmatrix} =$$

$$\begin{bmatrix} \left(u_{cG}^{1p}-u_{GG}^{1p}+\alpha^1 u_{Gc}^{1k}-\alpha^1 u_{GG}^{1k}\right)dg_k^1+\left(u_{cG}^{1p}-u_{GG}^{1p}-\alpha^1 u_{GG}^{1k}\right)dg_p^2 \\ \left(u_{cG}^{1p}+\alpha^1 u_{cc}^{1k}-\alpha^1 u_{cG}^{1k}\right)dg_k^1+\left(u_{cG}^{1p}-\alpha^1 u_{cG}^{1k}\right)dg_p^2 \\ \left(u_{cG}^{2k}+u_{GG}^{2k}\right)dg_k^1+\left(-u_{cc}^{2k}+u_{Gc}^{2k}\right)d\pi^2+\left(-u_{cG}^{2k}+u_{GG}^{2k}\right)dg_p^2 \end{bmatrix}. \tag{C1}$$

Application of Cramer's rule yields

$$\frac{\partial \hat{\pi}^1}{\partial g_k^1}=1;\ \frac{\partial \hat{g}_p^1}{\partial g_k^1}=-1;\ \frac{\partial \hat{g}_k^2}{\partial g_k^1}=0;\ \frac{\partial \hat{g}_k^2}{\partial g_p^2}=-1+\frac{\partial \hat{g}_k^2}{\partial \pi^2};\ \frac{\partial \hat{g}_p^1}{\partial g_p^2}=\frac{\partial \hat{g}_p^1}{\partial \pi^2}. \tag{C2}$$

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