



UNIVERSITÀ DEGLI STUDI ROMA TRE
DIPARTIMENTO DI ECONOMIA

**PERFECT AGGREGATION IN LINEAR MODELS:
A GEOMETRICAL INSIGHT**

S. Terzi

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Perfect aggregation in linear models: a geometrical insight

S. Terzi

1. Introduction

The aggregation problem has been thoroughly studied in econometrics mainly within the context of parameter estimation. In this context the question is: what is the relationship between the parameters (\mathbf{b}) of the aggregate relation and the parameters ($\beta_i, i=1, \dots, m$) of the micro-relations? It is well known that least squares (LS) estimation of the aggregate model leads, in general, to biased estimators. As Theil (1954) shows, in order for the LS estimator to be free of aggregation bias it has to be either:

$$\beta_i = \mathbf{b} \quad \forall i = 1, 2, \dots, m$$

or:

exact linear relations between independent variables of different micro relations¹.

A second issue is concerned with prediction. In this context starting from Grunfeld and Griliches's pioneer work (1960) the focus is on whether to predict the aggregate dependent variable by means of macro or micro equations. These authors – followed successively by others (see for example Sasaki, 1977) introduce a within sample goodness of fit criterion based on the sum of the squared residuals (R^2) and argue that whenever the goodness of fit of the aggregate model is greater than the goodness of fit of the model derived from the micro-equations, there is an aggregation gain.

In this paper we argue that the selection criterion suggested by Grunfeld and Griliches (GG) is biased: the expected goodness of fit of the aggregate model cannot be greater than that of the model derived from the correctly specified micro-relations.

In fact when predicting an aggregate variable ($\mathbf{y}_a = \sum_i \mathbf{y}_i$) by means of a macro relation we are projecting \mathbf{y}_a on the subspace (S_a) spanned by the k aggregate independent variables (in other words, on a

¹ It is also assumed – although not explicitly - that the aggregate independent variables are linearly independent.

subspace whose dimension is at most k). Vice versa when we resort to micro-relations each micro dependent variable y_i is projected on a k -dimensional subspace (S_i); thus – unless the subspaces S_i , $S_{i'}$ are isomorphic $\forall i, i' = 1, \dots, m$ – the sum of these projections can belong to a subspace (T) of greater dimension. Thus the goodness of fit of the model derived from well specified micro-relations will, in general, be greater than that of the aggregate model. Moreover, since $S_a \subseteq T$, prediction by means of the aggregate model will be as good as prediction via the disaggregate model only if the two subspaces, S_a and T , have the same dimension. If we define *perfect aggregation* the equivalence between the two models, a necessary condition would thus be: $\dim(T) = \dim(S_a)$.

As we will see, this condition can be easily reconduced to Theil's *rule of perfection*, derived within the estimation context. Although it may seem an obvious requirement that conditions for perfect aggregation within estimation and prediction contexts be the same, this consistency requirement has rarely been pursued in literature. In fact perfect aggregation within the prediction context is often implicitly defined as non-contradiction between the two models with respect to some goodness of fit criterion. This gives rise to a less restrictive definition of perfection, but also to inconsistent consequences. For example Pesaran, Pierse and Kumar (1989) while seeking a test for perfect aggregation within a prediction approach, explicitly leave aside the case of exact linear relations among variables.

The aim of the present paper is to define a goodness of fit criterion that does not contradict Theil's findings. First of all we will shed a light on the definition of perfect aggregation in order to unify the estimation and prediction approaches. Then we will define an appropriate goodness of fit criterion which – in contrast with GG's findings – leads to an unbiased selection criterion, and prove the non implementability of a test for perfect aggregation suggested by Pesaran, Pierse and Kumar (1989) (PPK).

2. Perfect aggregation

Let us consider the following micro-behavioural equations referring to n observations ($h = 1, 2, \dots, n$) of m micro-units ($i = 1,$

2,...,m) in which a dependent variable Y is expressed as a linear combination of k explanatory variables X_j ($j=1,\dots,k$):

$$Y_{ih} = \sum_j \beta_{ij} X_{ijh} + u_{ih}$$

In matrix notation the same model can be written as:

$$\mathbf{Y}_i = \mathbf{X}_i \boldsymbol{\beta}_i + \mathbf{u}_i \quad i = 1, 2, \dots, m \quad (1)$$

We assume model (1) to be correctly specified, so that $E(\mathbf{Y}_i) = \mathbf{X}_i \boldsymbol{\beta}_i$; $E(\mathbf{u}_i \mathbf{u}_i') = \sigma_i^2 \mathbf{I}$; $E(\mathbf{u}_i \mathbf{u}_j') = \sigma_{ij}^2 \mathbf{I}$, and $\text{rank}(\mathbf{X}_i) = k \quad \forall i = 1, \dots, m$.

From (1), if we define $\mathbf{Y}_a = \sum_i \mathbf{Y}_i$ we can write the *derived aggregate model* H_d (sometimes improperly referred to as the “disaggregate model”):

$$H_d : \mathbf{Y}_a = \sum_i \mathbf{X}_i \boldsymbol{\beta}_i + \sum_i \mathbf{u}_i \quad (2)$$

Alternatively, defining $\mathbf{X}_a = \sum_i \mathbf{X}_i$, we can write the *aggregate model* H_a as:

$$H_a : \mathbf{Y}_a = \mathbf{X}_a \mathbf{b} + \mathbf{v}_a \quad (3)$$

where $\mathbf{v}_a \equiv \sum_i \mathbf{V}_i \boldsymbol{\beta}_i + \sum_i \mathbf{u}_i$ and $\sum_i \mathbf{V}_i \boldsymbol{\beta}_i \equiv \sum_i \mathbf{X}_i \boldsymbol{\beta}_i - \mathbf{X}_a \mathbf{b}$ ².

We denote by k_a the rank of \mathbf{X}_a .

Following Theil, we define perfect aggregation as non-contradiction between the derived aggregate model H_d and the aggregate model H_a .

Model H_d states that \mathbf{Y}_a belongs to the sum of the subspaces spanned by the columns of the \mathbf{X}_i matrices (plus a random disturbance of null expected value). Vice versa, the aggregate model states that \mathbf{Y}_a belongs to the subspace spanned by the columns of the \mathbf{X}_a matrix (plus a random disturbance). Let us call S_i the subspace spanned by \mathbf{X}_i , and define $T = S_1 + \dots + S_m$. Moreover let us call S_a the subspace spanned by the columns of \mathbf{X}_a . Of course S_a is also – by definition – a subspace of T .

Non-contradiction between the two models requires:

$$\dim(T) = \dim(S_a).$$

First of all note that $\dim(T) = \text{rank}(\mathbf{X}_1; \dots; \mathbf{X}_m)$. Thus a necessary condition for perfect aggregation to hold is:

$$\text{rank}(\mathbf{X}_1; \dots; \mathbf{X}_m) = k \quad (c.1)$$

Obviously, given that $\text{rank}(\mathbf{X}_i) = k$, this condition is met if and only if any one of the \mathbf{X}_i matrices spans all the subspaces S_i , $\forall i' = 1, \dots, m$; in other words if and only if the subspaces S_i and $S_{i'}$ are isomorphic $\forall i, i'$.

² The definition of \mathbf{v}_a stems from Theil's auxiliary equation: $\mathbf{X}_i = \mathbf{X}_a \boldsymbol{\Gamma}_i + \mathbf{V}_i$, with: $\sum_i \mathbf{V}_i = \mathbf{0}$ and $\sum_i \boldsymbol{\Gamma}_i = \mathbf{I}$.

In order to derive an equivalent condition, we can resort to a decomposition of the \mathbf{X}_i matrices (the so-called auxiliary equations). In fact, since the \mathbf{X}_i matrices all belong to \mathbb{R}^n , and since they all span k -dimensional subspaces of \mathbb{R}^n , for a given i and for $\forall i'=1, \dots, m$, $\mathbf{X}_{i'}$ can be decomposed in the sum of two matrices, one belonging to S_i , the other belonging to the orthogonal subspace S_i^\perp . Thus we can write:

$$\mathbf{X}_{i'} = \mathbf{X}_i \mathbf{C}_{i'/i} + \mathbf{F}_{i'/i} \quad (4)$$

where $\mathbf{X}_{i'}' \mathbf{F}_{i'/i} = \mathbf{0}$.

Since $\text{rank}(\mathbf{X}_{i'}) = \text{rank}(\mathbf{C}_{i'/i}) + \text{rank}(\mathbf{F}_{i'/i})$, in order for $(\mathbf{X}_1 : \mathbf{X}_{i'})$ to have rank k a necessary condition is $\text{rank}(\mathbf{F}_{i'/i}) = 0$ (condition that implies $\mathbf{F}_{i'/i} = \mathbf{0}$ and $\text{rank}(\mathbf{C}_{i'/i}) = k$). Thus, a necessary condition for $\text{dim}(T) = k$ is:

$$\text{rank}(\mathbf{F}_{i'/i}) = 0 \quad \forall i'=1, \dots, m \quad (c.2)$$

When this last condition is met (together with the rank condition for the \mathbf{X}_i matrices) $\mathbf{C}_{i'/i}$ is non singular $\forall i'$ so that all subspaces $S_{i'}$ are isomorphic. Thus (c.1) and (c.2) are equivalent.

From auxiliary equation (4) we can also derive:

$$\mathbf{X}_a = \mathbf{X}_i \sum_{i'} \mathbf{C}_{i'/i} + \sum_{i'} \mathbf{F}_{i'/i} = \mathbf{X}_i \mathbf{C}_i + \mathbf{F}_i \quad (5)$$

thus implying that, whenever conditions (c.1) or (c.2) hold:

$$\mathbf{X}_a = \mathbf{X}_i \mathbf{C}_i \quad (c.3)$$

Condition (c.3) can also be stated as exact linear relations among independent variables. It is easy to see that (c.3) \leftrightarrow (c.1) or (c.2). Thus conditions (c.1), (c.2) and (c.3) are all equivalent.

However, since $\text{rank}(\mathbf{X}_a) = \text{rank}(\mathbf{C}_i) + \text{rank}(\mathbf{F}_i)$, condition (c.3) does not itself guarantee that $\text{rank}(\mathbf{X}_a) = k$; in fact \mathbf{C}_i is the sum of square full-rank matrices $\mathbf{C}_{i'/i}$, but this does not guarantee its non-singularity; moreover there are no "obvious" conditions to be posed in order for this requirement to be fulfilled. Thus in the context of linear prediction, perfect aggregation requires:

$$\text{rank}(\mathbf{X}_1 : \dots : \mathbf{X}_m) = k \quad \text{AND} \quad \text{rank}(\mathbf{X}_a) = k \quad (c.4)$$

This condition is both necessary and sufficient for perfect aggregation.

Of course, whenever the (correctly specified) micro-relations state:

$$\mathbf{Y}_i = \mathbf{X}_i \mathbf{b} + \mathbf{u}_i \quad i = 1, 2, \dots, m$$

the derived aggregate model is equivalent to the aggregate model without further assumptions. Note that in this case condition (c.4) holds without further assumptions. In fact the derived aggregate model is given by:

$$H_d : \mathbf{Y}_a = \sum_i \mathbf{X}_i \mathbf{b} + \sum_i \mathbf{u}_i = \mathbf{X}_a \mathbf{b} + \sum_i \mathbf{u}_i$$

thus stating that that $E(\mathbf{Y}_a)$ belongs to the S_a subspace, spanned by the columns of the \mathbf{X}_a matrix. This subspace is spanned by any one of the \mathbf{X}_i matrices. Thus their isomorphism is implicitly assumed, as well as a rank condition: $\text{rank}(\mathbf{X}_a) = \text{rank}(\mathbf{X}_i)$.

Let us now turn our attention to the estimation context. The conditions stated by Theil are: $\mathbf{X}_i = \mathbf{X}_a \Gamma_i, \forall i = 1, 2, \dots, m$ or $\beta_i = \mathbf{b} \forall i = 1, 2, \dots, m$ (equality of the micro coefficients). It is easy to see that the first of these conditions is equivalent to condition (c.4), where as the assumption of equality of the micro coefficients although sufficient, seems unduly restrictive.

Obviously, in order for perfect aggregation to hold for $\forall h$ condition (c.4) must be satisfied in and out of sample. This extended condition (also known as *compositional stability*) is usually stated as: $\mathbf{X}_a = \mathbf{X}_i \mathbf{C}_i \forall i, \forall h=1, \dots, n, \dots$, and $\text{rank}(\mathbf{X}_a) = k$.

3. Prediction and within sample goodness of fit

The question is whether to predict the aggregate variable \mathbf{Y}_a using model H_d or model H_a . In the first case we predict the aggregate variable \mathbf{Y}_a aggregating the predicted values of the micro-dependent variables $\mathbf{Y}_i, i= 1, \dots, m$; in the second case we predict \mathbf{Y}_a by means of the aggregate independent variables.

Assume we use LS. We can thus define the two predictors: $\hat{\mathbf{y}}_d = \sum_i \mathbf{X}_i \hat{\beta}_i = \sum_i \mathbf{X}_i \beta_i + \sum_i \mathbf{A}_i \mathbf{u}_i$ and $\hat{\mathbf{y}}_a = \mathbf{X}_a \hat{\mathbf{b}} = \mathbf{A}_a \sum_i \mathbf{y}_i$; and the two residuals $\mathbf{e}_d = \mathbf{y}_a - \hat{\mathbf{y}}_d = \sum_i \mathbf{M}_i \mathbf{u}_i, \mathbf{e}_a = \mathbf{y}_a - \hat{\mathbf{y}}_a = \sum_i \mathbf{V}_i \beta_i + \mathbf{M}_a \sum_i \mathbf{u}_i$ where $\mathbf{A}_i = \mathbf{X}_i (\mathbf{X}_i' \mathbf{X}_i)^{-1} \mathbf{X}_i'$, $\mathbf{A}_a = \mathbf{X}_a (\mathbf{X}_a' \mathbf{X}_a)^{-1} \mathbf{X}_a'$, $\mathbf{M}_i = \mathbf{I} - \mathbf{A}_i$ $\mathbf{M}_a = \mathbf{I} - \mathbf{A}_a$.³

It is well known that the predictor $\hat{\mathbf{y}}_a$ is the orthogonal projection of the aggregate dependent variable on the subspace spanned by the columns of the \mathbf{X}_a matrix. Thus it belongs to the k_a dimensional subspace S_a .

³In order for \mathbf{X}_a to be invertible we will assume either: $\text{rank } \mathbf{X}_a = k$, or that \mathbf{X}_a is an $n \times k_a$ full rank matrix.

Vice versa, $\hat{\mathbf{y}}_d$ is the sum of the orthogonal projections of the micro dependent variables on the S_i subspaces, and it belongs to a subspace T. It should be noted that, in general, $\hat{\mathbf{y}}_d$ is not an orthogonal projection of \mathbf{y}_a on the subspace T.

Since $\dim(T) \geq \dim(S_a)$ we should expect $\hat{\mathbf{y}}_d$ to have a better fit than $\hat{\mathbf{y}}_a$; however a goodness of fit measure is needed. Since $\hat{\mathbf{y}}_d$ is not an orthogonal projection of \mathbf{y}_a , for the disaggregate model we cannot resort to the usual definition of R^2 as:

$$R^2 = \frac{\text{dev}(\hat{\mathbf{y}}_d)}{\text{dev}(\mathbf{y}_a)} = 1 - \frac{\mathbf{e}_d' \mathbf{e}_d}{\mathbf{y}_a' \mathbf{y}_a}$$

since the equality does no longer hold. But, instead, we have to choose an appropriate definition among the most frequently used in literature.

We could define, as GG do, $R^2 = 1 - \frac{\mathbf{e}' \mathbf{e}}{\mathbf{y}' \mathbf{y}}$. However, since we are

interested in a projection problem, the most appropriate goodness of fit criterion seems to be the closeness between predicted and observed values, as measured by the square of the cosine of their angle. We thus define:

$$R^{*2} = \cos^2(\hat{\mathbf{y}}, \mathbf{y}) = \frac{(\mathbf{y}' \hat{\mathbf{y}})^2}{(\hat{\mathbf{y}}' \hat{\mathbf{y}})(\mathbf{y}' \mathbf{y})}$$

It can be easily seen that for the aggregate model $\mathbf{y}_a' \hat{\mathbf{y}}_a = \hat{\mathbf{y}}_a' \hat{\mathbf{y}}_a$, thus:

$$R_a^{*2} = \cos^2(\hat{\mathbf{y}}_a, \mathbf{y}_a) = \frac{(\hat{\mathbf{y}}_a' \hat{\mathbf{y}}_a)}{(\mathbf{y}_a' \mathbf{y}_a)} = R_a^2$$

where as for the derived model H_d :

$$R_d^{*2} = \cos^2(\hat{\mathbf{y}}_d, \mathbf{y}_a) = \frac{(\mathbf{y}_a' \hat{\mathbf{y}}_d)^2}{(\hat{\mathbf{y}}_d' \hat{\mathbf{y}}_d)(\mathbf{y}_a' \mathbf{y}_a)} = \frac{(\mathbf{e}_d' \hat{\mathbf{y}}_d)^2}{(\mathbf{y}_a' \mathbf{y}_a)(\hat{\mathbf{y}}_d' \hat{\mathbf{y}}_d)} + R_d^2$$

Thus, in general $R_d^{*2} \geq R_d^2$. Moreover $R_d^{*2} = R_d^2$ if and only if \mathbf{e}_d and $\hat{\mathbf{y}}_d$ are orthogonal.

We now want to show that defining goodness of fit as R^{*2} leads, on average, to select the model H_d unless perfect aggregation holds (in which case the two models are equivalent).

In other words the selection criterion we introduce – unlike the criterion based on $(R_a^2 - R_d^2)$ – is unbiased.

It can be easily seen that:

$$(R_d^{*2} - R_a^{*2}) \mathbf{y}_a' \mathbf{y}_a = \mathbf{e}_a' \mathbf{e}_a + \left(\frac{(\mathbf{e}_d' \hat{\mathbf{y}}_d)^2}{\hat{\mathbf{y}}_d' \hat{\mathbf{y}}_d} - \mathbf{e}_d' \mathbf{e}_d \right)$$

All terms that appear in this expression are ≥ 0 ; however

$\frac{(\mathbf{e}_d' \hat{\mathbf{y}}_d)^2}{\hat{\mathbf{y}}_d' \hat{\mathbf{y}}_d} \leq \mathbf{e}_d' \mathbf{e}_d$. Thus $(R_d^{*2} - R_a^{*2})$ attains its minimum when

$\frac{(\mathbf{e}_d' \hat{\mathbf{y}}_d)^2}{\hat{\mathbf{y}}_d' \hat{\mathbf{y}}_d} = 0$, in other words when $\hat{\mathbf{y}}_d$ is an orthogonal projection of \mathbf{y}_a ⁴.

However, when $\mathbf{e}_d' \hat{\mathbf{y}}_d = 0$,

$$\mathbf{e}_d' \mathbf{e}_d = \mathbf{e}_d' \mathbf{y}_a = \sum_i \sum_j \mathbf{y}_i' \mathbf{M}_i \mathbf{y}_j = \sum_i \sum_j \beta_i' \mathbf{X}_i' \mathbf{M}_j \mathbf{u}_j + \sum_i \sum_j \mathbf{u}_i' \mathbf{M}_j \mathbf{u}_j.$$

Moreover, since it is always $\mathbf{V}_i' \mathbf{X}_a = \mathbf{0}$ ⁵:

$$\mathbf{e}_a' \mathbf{e}_a = \sum_i \sum_j \mathbf{u}_i' \mathbf{M}_a \mathbf{u}_j + \sum_i \sum_j \beta_i' \mathbf{V}_i' \mathbf{M}_a \mathbf{V}_j \beta_j = \sum_i \sum_j \mathbf{u}_i' \mathbf{M}_a \mathbf{u}_j + \sum_i \sum_j \beta_i' \mathbf{V}_i' \mathbf{V}_j \beta_j.$$

Thus:

$$\mathbf{e}_a' \mathbf{e}_a - \mathbf{e}_d' \mathbf{e}_d = \sum_i \sum_j \mathbf{u}_i' (\mathbf{M}_a - \mathbf{M}_j) \mathbf{u}_j + \sum_i \sum_j \beta_i' \mathbf{V}_i' \mathbf{V}_j \beta_j - \sum_i \sum_j \beta_i' \mathbf{X}_i' \mathbf{M}_j \mathbf{u}_j$$

and:

$$E(\mathbf{e}_a' \mathbf{e}_a - \mathbf{e}_d' \mathbf{e}_d) = (k - k_a) \sum_i \sum_j \sigma_{ij} + \sum_i \sum_j \beta_i' \mathbf{V}_i' \mathbf{V}_j \beta_j \geq 0$$

so that our selection criterion is unbiased.

It can easily be seen that, whenever condition (c.4) holds $\mathbf{e}_d' \hat{\mathbf{y}}_d = 0$,

$k_a = k$ and $\mathbf{V}_i = \mathbf{0} \forall i$. Thus, in this case $R_d^{*2} = R_a^2$.

⁴ Of course perfect aggregation is a sufficient condition for $\hat{\mathbf{y}}_d$ to be the orthogonal projection of \mathbf{y}_a .

⁵ Like the other auxiliary equations we introduced, the auxiliary equation: $\mathbf{X}_i = \mathbf{X}_a \Gamma_i + \mathbf{V}_i$, assumes $\mathbf{V}_i \in S_a^c$ and thus: $\mathbf{V}_i' \mathbf{X}_a = \mathbf{0}$.

4. Concluding remarks

GG base their selection criterion on $E(\mathbf{e}'_a \mathbf{e}_a - \mathbf{e}'_d \mathbf{e}_d)$ and consequently define perfect aggregation as $\sum_i \sum_j \beta'_i \mathbf{V}'_i \mathbf{V}_j \beta_j = 0$. It is easy to see that $\sum_i \sum_j \beta'_i \mathbf{V}'_i \mathbf{V}_j \beta_j = 0$ if

a) $\beta_i = \mathbf{b} \quad \forall i = 1, \dots, m$

or:

b) $\mathbf{V}_i = \mathbf{0} \quad \forall i = 1, \dots, m \quad \Leftrightarrow \quad \mathbf{F}_{i'/i} = \mathbf{0} \quad \forall i' = 1, \dots, m \text{ AND}$
 $\text{rank}(\mathbf{C}_i) = k, \quad \forall i = 1, \dots, m$

These properties suggest that a perfect aggregation test could be based on the statistic $\hat{\xi} = \sum_i \mathbf{X}_i \hat{\beta}_i - \mathbf{X}_a \hat{\mathbf{b}}$. In fact PPK (1989) show that, under the assumption that u is normally distributed with mean zero and known covariance matrix, when $\sum_i \sum_j \beta'_i \mathbf{V}'_i \mathbf{V}_j \beta_j = 0$:

$$m^{-1}(\hat{\xi}' \psi_m^{-1} \hat{\xi}) \sim \chi^2_v$$

where:

$$\psi_m = m^{-1} \sum_{i,j=1}^m \sigma_{ij} H_i H_j;$$

$$\mathbf{H}_i = (\mathbf{A}_i - \mathbf{A}_a)$$

$$v = \text{rank}(\psi_m).$$

They also derive a sufficient (but not necessary) condition for ψ_m to have full rank.

However, it is easy to see that when condition (c.2) holds:

$$\mathbf{M}_a - \mathbf{M}_j = \mathbf{A}_j - \mathbf{A}_a = \mathbf{X}_j (\mathbf{X}'_j \mathbf{X}_j)^{-1} \mathbf{X}'_j - \mathbf{X}_j \mathbf{C}_j (\mathbf{X}'_a \mathbf{X}_a)^{-1} \mathbf{C}'_j \mathbf{X}'_j$$

is a symmetric and idempotent matrix; thus it is positive semi definite with rank $k - \text{rank}(\mathbf{C}_j)$. However when $\mathbf{V}_i = \mathbf{0}$, $\text{rank}(\mathbf{C}_i) = k \quad \forall i$, so that, in fact, under perfect aggregation the test is not implementable.

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