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Abstract

We model a vertically differentiated duopoly with quantity-setting firms as an extended game in which firms noncooperatively choose the timing of moves at the quality stage, to show that at the subgame perfect equilibrium sequential play obtains, with the low-quality firm taking the leader's role.

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1 Introduction

A well established result produced by the theory of vertical differentiation is that the first entrant fills the highest quality niche, letting newcomers locate further down along the quality spectrum. This result is commonly derived under Bertrand competition (see Gabszewicz and Thisse, 1979; Shaked and Sutton, 1982; Aoki and Prusa, 1997; and Lehmann-Grube, 1997, *inter alia*). The opposite may apply if the role of time is accounted for, in such a way that low-quality leadership emerges when the exploitation of *ad interim* monopoly power matters more than skinning the cream (see van Dijk, 1996; and Lambertini and Tedeschi, 2007a,b).

Here we take a different angle to address the same issue, exploiting an idea dating back to d'Aspremont and Gérard-Varet (1980) and then further developed by Hamilton and Slutsky (1990), according to whom a game is Stackelberg-solvable if there exists a Stackelberg equilibrium that Pareto-dominates the Nash solution. We use this approach in a vertically differentiated duopoly in which firms bear a convex cost of quality improvement and then behave *à la* Cournot-Nash. From the analysis of the quality stage, there emerges that, while the high-quality firm's best reply is increasing, that of the low-quality firm is decreasing, and therefore the choice of roles concerning the timing of moves in the quality space univocally selects sequential play with low-quality leadership as part of the subgame perfect equilibrium. This framework may indeed fit real-world episodes in which innovation from below has been observed; one such instance is the introduction of solid state (transistor) circuitry to replace vacuum tube designs in consumer electronics at the turn of the Seventies, with a large production characterised by a comparatively lower quality, higher quality versions being introduced later.

2 The model

We consider a duopoly market for vertically differentiated products supplied by single-product firms. The demand side is modelled *à la* Mussa and Rosen (1978). There is a continuum of consumers whose types are identified by θ, uniformly distributed with density equal to one in the interval $[0, \Theta]$ (so that total demand is equal to Θ). Parameter θ represents the consumers' marginal willingness to pay for quality. Each consumer is assumed to buy at most one unit of the vertically differentiated good in order to maximise the following surplus function:

$$U = \theta q_i - p_i, \tag{1}$$

where $q_i \in [0, Q]$ indicates the quality of the product and p_i is the market price at which that variety is supplied by firm $i = H, L$, with $q_H \geq q_L$. Therefore, the consumer who is indifferent between q_H and q_L is identified by the level of marginal willingness to pay $\widehat{\theta}$ that solves $\widehat{\theta} q_H - p_H = \widehat{\theta} q_L - p_L$, and therefore $\widehat{\theta} = (p_H - p_L) / (q_H - q_L)$. Thus, market demand for the high-quality good is $x_H = \Theta - \widehat{\theta}$. We assume partial market coverage, so that there exists a consumer indifferent between buying q_L or not buying at all, identified by $\widetilde{\theta}$ solving $\widetilde{\theta} q_L - p_L = 0$, whereby $\widetilde{\theta} = p_L / q_L$ and the demand for the inferior variety is $x_L = \widehat{\theta} - \widetilde{\theta}$. This is what one needs to use in order to model Bertrand behaviour, while inverse demands

$$\begin{aligned} p_H &= (\Theta - x_H)\, q_H - q_L x_L \\ p_L &= (\Theta - x_H - x_L)\, q_L \end{aligned} \tag{2}$$

are to be used under Cournot competition.

On the supply side, as in Motta (1993), *inter alia*, firms incur in convex fixed costs of quality improvement $C_i = c q_i^2$, $i = H, L$. Variable costs are assumed away. Hence profit functions are $\pi_H = p_H x_H - c q_H^2$ and $\pi_L = p_L x_L - c q_L^2$. Competition takes place in three stages. In the first, firms choose the timing to be followed in the second stage, where qualities are set, and then in the third stage simultaneous Cournot competition takes place. The solution concept is the subgame perfect equilibrium by backward induction.

The first stage is a pre-play stage *à la* Hamilton and Slutsky (1990), in which, under complete, symmetric and imperfect information, firms play a discrete strategy game represented in Matrix 1.

$$\begin{array}{cc|c|c|} & & \multicolumn{2}{c}{L} \\ & & F & S \\ \cline{3-4} H & F & \pi_H^N; \pi_L^N & \pi_H^{SL}; \pi_L^{SF} \\ \cline{3-4} & S & \pi_H^{SF}; \pi_L^{SL} & \pi_H^N; \pi_L^N \\ \cline{3-4} \end{array}$$

Matrix 1

Actions F and S stand for "first" or "second", and refers to the choice of roles pertaining to the quality stage, while superscripts N, SL, and SF stand for Nash, Stackelberg leader and Stackelberg follower, respectively. If firms select the same strategy - along the main diagonal - then the second-stage quality game is simultaneous. Conversely, along the secondary diagonal, the quality stage is going to be solved *à la* Stackelberg. For future reference, it is worth recalling that the firms' incentives as to the timing of moves is entirely driven by the slope of their best replies (in this case, in the quality space), in such a way that if a firm has a decreasing (resp., increasing) reaction function, it will prefer to move first (resp., second) (see Hamilton and Slutsky, 1990, Theorem V, p. 38).

3 Results

To begin with, we characterise optimal outputs for any given quality pair:

$$x_H^N = \frac{\Theta(2q_H - q_L)}{4q_H - q_L};\ x_L^N = \frac{\Theta q_H}{4q_H - q_L} \tag{3}$$

where superscript N stands for Nash equilibrium. The explicit derivation of the Cournot equilibrium is omitted as it is known from Motta (1993).

We now turn to the second stage where the quality game takes place. The relevant profit functions are:

$$\begin{aligned} \pi_H &= \frac{q_H\left[\Theta^2\left(2q_H-q_L\right)^2-cq_H\left(4q_H-q_L\right)^2\right]}{\left(4q_H-q_L\right)^2} \\ \pi_L &= \frac{q_L\left[\Theta^2 q_H^2-cq_L\left(4q_H-q_L\right)^2\right]}{\left(4q_H-q_L\right)^2} \end{aligned} \tag{4}$$

The first order conditions for non cooperative profit maximisation are:

$$\frac{\partial \pi_H}{\partial q_H}=\frac{\Theta^2\left(16q_H^3-12q_H^2q_L+4q_Hq_L^2-q_L^3\right)-2cq_H\left(4q_H-q_L\right)^3}{\left(4q_H-q_L\right)^3}=0, \tag{5}$$

$$\frac{\partial \pi_L}{\partial q_L}=\frac{\Theta^2 q_H^2\left(4q_H+q_L\right)-2c\left(4q_H-q_L\right)^3}{\left(4q_H-q_L\right)^3}=0. \tag{6}$$

Given that the above FOCs do not allow for a fully analytical characterisation of Nash and Stackelberg equilibria, we investigate the solution of the quality stage by studying the map of the reaction functions, implicitly revealed by (5-6). In particular, following Bulow *et al.* (1985), we know that the nature of strategic interaction is entirely determined by the sign of the partial derivatives of FOCs with respect to the competitor's quality, which ultimately indicate the slopes of reaction functions $q_i^*\left(q_j\right)$, $i,j=H,L$, $i\neq j$. These derivatives are:

$$\frac{\partial q_H^*\left(q_L\right)}{\partial q_L}\propto\frac{\partial^2\pi_H}{\partial q_H\partial q_L}=\frac{8\Theta^2 q_Hq_L\left(q_H-q_L\right)}{\left(4q_H-q_L\right)^4}>0, \tag{7}$$

$$\frac{\partial q_L^*\left(q_H\right)}{\partial q_H}\propto\frac{\partial^2\pi_L}{\partial q_L\partial q_H}=-\frac{2\Theta^2 q_Hq_L\left(8q_H+q_L\right)}{\left(4q_H-q_L\right)^4}<0. \tag{8}$$

The concavity/convexity of best replies is determined by the following derivatives:

$$\frac{\partial^3\pi_H}{\partial q_H\partial q_L^2}=-\frac{8\Theta^2\left(4q_H^2-5q_Hq_L-2q_L^2\right)}{\left(4q_H-q_L\right)^5}, \tag{9}$$

which is positive for all $q_H \in \left(0, \frac{5+\sqrt{57}}{8} q_L\right)$ and negative for all $q_H > \frac{5+\sqrt{57}}{8} q_L$; and

$$\frac{\partial^3 \pi_L}{\partial q_L \partial q_H^2} = \frac{2\Theta^2 q_L \left(64 q_H^2 + 28 q_H q_L + q_L^2\right)}{\left(4q_H - q_L\right)^5} > 0. \tag{10}$$

Hence $q_H^*(q_L)$ is always increasing and convex for all $q_H \in \left(0, \frac{5+\sqrt{57}}{8} q_L\right)$ and concave for all $q_H > \frac{5+\sqrt{57}}{8} q_L$. On the other hand, $q_L^*(q_H)$ is everywhere decreasing and convex.

A Nash equilibium exists if the vertical intercept of $q_H^*(q_L)$ is lower than the vertical intercept of $q_L^*(q_H)$ in the space $\{q_L, q_H\}$. We can see that:

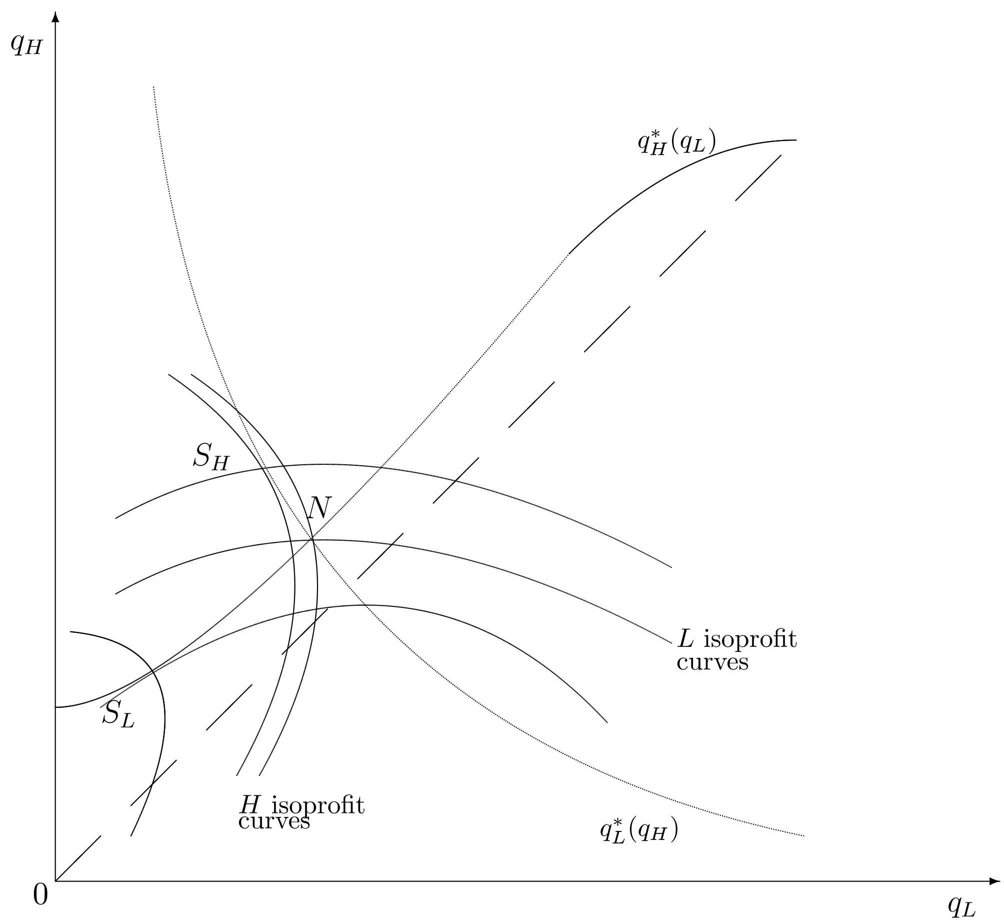
$$\left.\frac{\partial \pi_H}{\partial q_H}\right|_{q_L=0} = -\frac{\Theta^2 - 8cq_H}{4} = 0, \tag{11}$$

so that $\left. q_H^*(q_L)\right|_{q_L=0} = \Theta^2/(8c)$, while there are no solutions w.r.t. q_H to $\left.\partial \pi_L / \partial q_L\right|_{q_L=0} = 0$. Therefore, defining $\widehat{q}_H \equiv \left(q_L^*(q_H)\right)^{-1}$, we have $\lim_{q_L \to 0} \widehat{q}_H = +\infty$.

Figure 1 shows the map of the best replies and the respective isoprofit curves. The Nash equilibrium (point N) yields a lower profit both to firm L and to firm H as compared to the Stackelberg equilibrium where L acts as the leader (point S_L). As to the Stackelberg solution with firm H leading, the map of isoprofit curves reveals that the high-quality firm is better off w.r.t. the Nash equilibrium, while the opposite applies to the low-quality firm. Overall, the following chains of inequalities hold:

$$\begin{aligned} &\pi_H^{SF} > \pi_H^{SL} > \pi_H^{N} \\ &\pi_L^{SL} > \pi_L^{N} > \pi_L^{SF} \end{aligned} \tag{12}$$

Figure 1. The map of best replies with the equilibrium points.



Using these properties, one can go back to Matrix 1 and observe that the first (pre-play) stage can be solved by applying iterated dominance, since (i) firm L drops strategy S as it is strictly dominated by F; then, (ii) firm H drops what remains of strategy F, and therefore (iii) the matrix reduces to the cell (S, F). This discussion allows us to formulate

Proposition 1 *The three-stage game has a unique subgame perfect equilibrium in pure strategies, where firm L takes the lead in the quality stage.*

To this regard, it is worth remarking that the slopes of reaction functions and the ranking of payoffs in (12) are the two sides of the same coin, the first dating back to Hamilton and Slutsky (1990), the second to d'Aspremont and Gérard-Varet (1980). Taken together, these two features of the second stage of the game allow players to select the timing of moves so as to attain at equilibrium the Pareto-efficient outcome.

As a last remark, observe that, as switching from simultaneous to sequantial play with firm L leading involves a decrease in both quality levels. This surely has negative consequences on consumer surplus

$$CS = \int_{\widetilde{\theta}}^{\widehat{\theta}} (kq_L - p_L)dk + \int_{\widehat{\theta}}^{\Theta} (zq_H - p_H)dz, \tag{13}$$

as can be ascertained from the following partial derivatives:

$$\frac{\partial CS}{\partial q_H} = \frac{\Theta^2\left[4q_H^2\left(4q_H - 3q_L\right) + q_L^2\left(2q_H + q_L\right)\right]}{2\left(4q_H - q_L\right)^3}; \; \frac{\partial CS}{\partial q_L} = \frac{\Theta^2 q_H^2\left(12q_H - 7q_L\right)}{2\left(4q_H - q_L\right)^3} \tag{14}$$

which are both positive. Hence, the balance between the increase in industry profits and the decrease in consumer surplus is ambigous.

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