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The Demand for Money Revisited

THE MONEY MARKET IS A CRITICAL COMPONENT of virtually all theories that explain the evolution of aggregate economic activity. More particularly, an accurate understanding and portrayal of this market is essential both to the analysis of past monetary policies and to the formulation of appropriate contemporary policy. This paper focuses on one aspect of the money market, the demand side, and provides an extensive review of the current state of the art concerning the demand for money. The emphasis will be unabashedly empirical, with concentration on the short term, taken here to be quarterly, since this horizon appears to be the most relevant to policy purposes.[1]

There has been a substantial amount of past research on the demand for money and several survey pieces as well.[2] Nevertheless, a number of good reasons argue for embarking on another broad empirical effort. In the first

1. The recent interest within the Federal Reserve System in monthly and even weekly models suggests that an even shorter-run focus might be appropriate.

2. See, for example, David E. W. Laidler, *The Demand for Money: Theories and Evidence* (International Textbook, 1969), and John T. Boorman, "The Evidence on the Demand for Money: Theoretical Formulations and Empirical Results," in John T. Boorman and Thomas M. Havrilesky, *Money Supply, Money Demand, and Macroeconomic Models* (Allyn and Bacon, 1972).

instance, until recently research with quarterly data had not been that extensive. Consequently, most of the received wisdom on the subject stems from empirical work with long-term annual data[3] whose relevance for short-term purposes is questionable.[4]

A second reason for undertaking a broad empirical effort is that much of the existing evidence stems from the work of researchers who have each used a different sample period, measurement method, and estimating technique. There is much to be said for attacking the substantial range of issues that I wish to examine in a homogeneous and consistent manner. This procedure seems all the more desirable since it will permit me to use the latest data uniformly, which seems important in view of the varied behavior of money and interest rates in recent years.[5]

A final motivation for this paper is that recent events have raised the question, in both the popular and the professional press, as to whether the conventional money demand formulation is adequate to explain the monetary experience of the seventies. For example, from early to mid-1971 the money stock rose rapidly but so did short-term interest rates. Over roughly the next half-year money grew at a meager 1 percent rate but interest rates fell below their early 1971 lows. Both during this period and subsequently, observers questioned whether the economy had experienced short-run shifts in the demand for money. More recently, the first half of 1973 saw sharply rising interest rates. But while the money stock rose only marginally in the first quarter, it spurted ahead at the annual rate of 11 percent in the second quarter. Once again the press has referred to the puzzling behavior of the demand for money. The basic issue is whether the demand function for money can be assumed by the policy maker to be essentially stable in the short run. This issue, which has not been examined previously in any great detail, will receive particular emphasis in this paper.

3. This is certainly true of the research reviewed, for example, in Laidler, *Demand for Money*.

4. In fact, much of the short-term analysis seems to contradict many aspects of the received wisdom. For example, the evidence from the annual data tends to favor M_2 over M_1, long-term over short-term interest rates, and wealth over current income. Practitioners working with quarterly data tend to the opposite.

5. Whatever problems it may have caused money holders and policy makers, the behavior of interest rates in the last four years—historic peaks at the end of 1969, followed by pronounced cyclical behavior and ending with current near-record levels—is an econometrician's delight.

Outline

The plan of the paper is as follows. The next section briefly spells out the conventional story on the origins and general nature of the demand for money and then reports estimates of one simple and common version of the money demand function. The estimates are then analyzed with primary focus on the following two questions:

1. Is there any evidence of economies of scale in aggregate money holdings? Is there any indication, as previously has been suggested, that the income elasticity is difficult to pin down from quarterly data?

2. Has the demand function for money remained stable over the postwar period? Put another way, is there any evidence of either systematic long-run shifts or marked short-run instabilities that make historically estimated relationships unsuitable for forecasting purposes?

The results of that section will serve as a rough standard for considering other important issues on the proper specification of the money demand function that are taken up in the third section:

3. What degree of aggregation is appropriate with respect to currency, demand deposits, and time deposits?

4. What sorts of lags appear to be present in the adjustment of money holdings and what rationale can be offered to explain these lags?

5. Is there any evidence that expected rates of inflation measured either directly or indirectly influence the demand for money?

6. Should income, or wealth, or perhaps both, be used in the demand function?

In the fourth section a number of more technical issues are explored:

7. Which interest rates work best in explaining the demand for money?

8. Are estimated demand-for-money functions sensitive to the time unit used to construct the aggregate data?

9. How important are the problems of serial correlation and simultaneous equations bias in the demand for money?

10. Is the demand for money homogeneous with respect to prices or population?

The fifth section examines the problems of disaggregation in somewhat more detail, using the flow of funds data on holdings by type of holder (business and consumers and the rest). The basic question is whether separate analysis of more homogeneous groups of money holders can improve understanding of the money demand process and the ability to forecast the demand for money. The paper concludes with a summary of the main results and an attempt to draw some lessons from them.

As the outline suggests, I shall cover a fairly broad range of issues on the specification and properties of the demand-for-money function. While these questions are clearly interrelated, simultaneous consideration of all of them would be a strategic and expositional monstrosity. Consequently, except where it seems particularly warranted, I shall try to avoid a flood of permutations and alternative specifications. Even so, some may regard the output as a "junior encyclopedia" if not the full-fledged thing.

Some Underpinnings

The conventional textbook formulation of the demand for money typically relates the demand for real money balances—$m = M/P$, assumed to be noninterest bearing[6]—to "the" interest rate, r, and some measure of economic activity such as real GNP—$y = Y/P$, where M = money holdings, P = the price level, and Y = gross national product. Thus

$$m = f(r, y). \tag{1}$$

A variety of stories can explain the origins of equation (1). Perhaps the most satisfying is the transactions view, in which the demand for money evolves from a lack of synchronization between receipts and payments and the existence of a transactions cost in exchanging money for interest-bearing assets (usually taken to be short term).

One example of this approach is the well-known Baumol-Tobin formulation which readily leads to an equation of the form of (1). Its simplest version is the so-called square root law of money holdings,[7]

$$m = ky^{\frac{1}{2}}r^{-\frac{1}{2}}, \tag{1'}$$

6. Although interest payments on demand deposits have been prohibited, the existence of service charges may produce an implicit yield on demand deposits. Some writers have used service charges as a measure of negative interest payment but this practice suffers from rather serious conceptual problems. Recently, Barro and Santomero have constructed an explicit marginal return on deposits based on remission of service charges. Unfortunately, the series is annual and stops in 1968. It does, however, vary substantially in the late 1960s, suggesting that this may be an important omitted variable in demand-for-money equations. See Robert J. Barro and Anthony M. Santomero, "Household Money Holdings and the Demand Deposit Rate," *Journal of Money, Credit and Banking*, Vol. 4 (May 1972), pp. 397–413.

7. One assumption necessary to produce (1′) is that real transactions costs have remained essentially constant. This is an assumption of doubtful validity and also may

where k is related to the transactions cost. This implies that the income elasticity of the demand for money is $1/2$ while the interest elasticity is $-1/2$.

Other analyses of the demand for money emphasize speculative, precautionary, or utility considerations in addition to the transactions motive.[8] These tend to blur the specific predictions of income and interest rate elasticities that emerge from the simple transactions approach, but they are broadly consistent with the general form of equation (1).[9]

At an empirical level such an equation has underpinned estimation in a number of studies of the demand for money. This has typically been the case where annual data are involved. With quarterly data, empirical workers have generally resorted to a more complicated version of (1) involving lagged as well as current variables. At least two motivations—not necessarily conflicting—have been offered for modifying (1) in this way, the partial adjustment mechanism and expectations formation. For the present only the former justification is explored, but the expectational lag will be considered more extensively below.

The ubiquitous partial adjustment assumption usually proceeds by interpreting (1) as setting a "desired" value for money holdings, say m^*, as in

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systematically bias standard estimates of the demand for money. For one lighthearted attempt to correct for this bias, see Saschba Telphlluch, "A Remark on the Transactions Demand for Money," CORE Discussion Paper 7034 (Catholic University of Louvain, Belgium, 1970; processed).

8. See, for example, J. Tobin, "Liquidity Preference as Behavior Towards Risk," *Review of Economic Studies*, Vol. 25 (February 1958), pp. 65–86; and Don Patinkin, *Money, Interest, and Prices: An Integration of Monetary and Value Theory* (2nd ed., Harper and Row, 1965).

9. The ideal would be a theory that simultaneously treats the various considerations cited above. Such a fully general theory has yet to be produced but a number of promising starts have been made. For example, Ando and Shell have recently analyzed a model in which risk and transactions costs are handled simultaneously. They consider three assets: equities, saving deposits, and money. The rate of return on equities and the rate of change of the price level were considered to be random variables while the nominal rates of return on saving deposits and money were taken as known with certainty. Adopting an expected utility framework but allowing for transactions costs, they were able to show that the demand for money becomes a function of the volume of transactions and the interest rate differential between saving deposits and money. Assuming the latter is zero leads to a formulation like (1). In particular, money holdings do not depend on an expected return on equities, on wealth, or on anticipated inflation. I shall return to this below. See Albert Ando and Karl Shell, "Demand for Money in a General Portfolio Model in the Presence of an Asset that Dominates Money," appendix to a paper presented to a Brookings conference on model building, 1972 (June 1972; processed).

$$m^* = f(r, y). \tag{2}$$

Portfolio adjustment costs, both pecuniary and nonpecuniary, are then assumed to prevent a full, immediate, adjustment of actual money holdings to desired levels. Depending upon the functional form of (2), actual money holdings are assumed to adjust linearly or logarithmically to the gap between desired holdings and last period's holdings; that is,

$$m_t - m_{t-1} = \gamma(m_t^* - m_{t-1}), \tag{3}$$

or

$$\ln m_t - \ln m_{t-1} = \gamma(\ln m_t^* - \ln m_{t-1}), \tag{3'}$$

where γ is the coefficient of adjustment. While, as demonstrated below, the partial adjustment model is not without its shortcomings, it seems, in view of its widespread use, a convenient starting point for empirical work.

A CONVENTIONAL EQUATION

The first step is estimating an equation following the format of (3′) and (1′) above. Detailed definitions of the variables are found in the appendix but a few words on the matter are in order here. The narrow money stock (currency plus demand deposits, M_1) is used as the dependent variable; it is measured as a quarterly average of monthly data and deflated by the implicit GNP deflator. Income was defined as real GNP and the interest rate was measured in two ways—by the rate on commercial paper (*RCP*) and by the rate on time deposits (*RTD*). The results obtained with ordinary least squares, using the Cochrane-Orcutt technique to adjust for serial correlation, are given below (the numbers in parentheses here and in following equations are *t*-statistics):

$$\begin{aligned} \ln m = {} & \underset{(2.2)}{0.271} + \underset{(5.3)}{0.193} \ln y + \underset{(11.5)}{0.717} \ln m_{-1} \\ & - \underset{(6.0)}{0.019} \ln RCP - \underset{(4.0)}{0.045} \ln RTD. \end{aligned} \tag{4}$$

$R^2 = 0.995$; $\rho = 0.414$; standard error $= 0.0043$; Durbin-Watson statistic $= 1.73$.
Sample period $= 1952{:}2\text{–}1972{:}4$.[10]

10. This sample period was used in most of the equations that follow, primarily for ease of comparison with equations based on the flow of funds data, which are available

At first glance this equation seems quite reasonable. Both the commercial paper rate and the time deposit rate are significant, with long-run elasticities of 0.07 and 0.16, respectively. The coefficient of adjustment—that is, γ in (3′)—is 0.283 ($= 1 - 0.717$); while this is not dramatically rapid, it is certainly more plausible than the slow 0–10 percent estimates that some writers have reported.[11] The point estimate of the long-run income elasticity is 0.68 and a 95 percent confidence interval for the income elasticity, derived by a method due to Fieller,[12] turns out to be (0.60, 0.82). Consequently, the income elasticity appears to be significantly less than unity.[13]

Besides yielding plausible parameter values, equation (4) also fits the data quite well. This can be seen in Figure 1, which depicts the actual values of the real money stock along with the values predicted by equation (4).

INCOME ELASTICITY: A CLOSER LOOK

While equation (4) seems to be a satisfactory first approximation to a money demand function, the results need closer scrutiny. One aspect that

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only from 1952. Equation (4), run over the longer sample period, 1949:2 to 1973:2, resulted in the following:

$$\ln m = \underset{(3.2)}{0.286} + \underset{(4.9)}{0.179} \ln y + \underset{(12.0)}{0.731} \ln m_{-1} - \underset{(4.6)}{0.020} \ln RCP - \underset{(3.6)}{0.040} \ln RTD.$$

$R^2 = 0.988$; $\rho = 0.217$; standard error $= 0.0073$; Durbin-Watson statistic $= 2.08$.

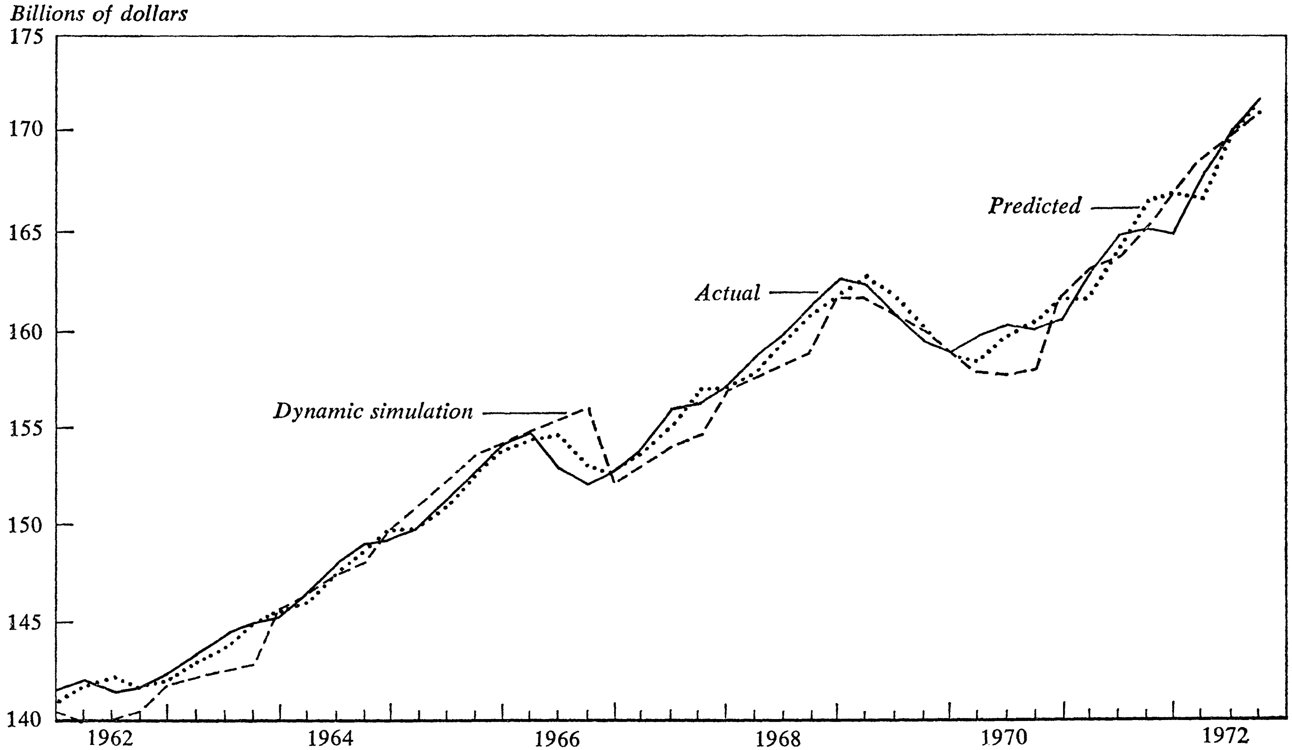
The point estimates in the above equation and in (4) are quite similar although there are some indications of a difference with respect to the error structure (for example, the estimated ρ and standard error).

11. See, for example, the logarithmic specification in Franco Modigliani, Robert Rasche, and J. Philip Cooper, "Central Bank Policy, the Money Supply, and the Short-Term Rate of Interest," *Journal of Money, Credit and Banking*, Vol. 2 (May 1970), pp. 166–218.

12. Fieller's method is needed since the long-run elasticity is a ratio derived from two estimated coefficients. The resulting interval will, in general, not be symmetric around the point estimate. This is true here since the midpoint of the interval is 0.71 while the point estimate of the elasticity is 0.68. Furthermore, in the present context, since the underlying estimates are not unbiased, I have only an approximate confidence interval. For a discussion of the Fieller technique, see Wayne A. Fuller, "Estimating the Reliability of Quantities Derived from Empirical Production Functions," *Journal of Farm Economics*, Vol. 44 (February 1962), pp. 82–99.

13. This is usually an implication of the transactions approach to the demand for money. A problem arises in a concrete application of this approach, however, because it is not clear that real GNP is a good measure of transactions or that real transactions costs are constant.

Figure 1. Actual and Predicted Values of Real Money Stock, Quarterly, 1962–72



Sources: See the appendix for sources of actual data. Predicted values are derived from text equation (4).

deserves additional attention is the estimate of the long-run income elasticity. Judged by the size of the confidence interval reported above, the estimate of this important parameter appears to be fairly precise. On the other hand, William Poole has suggested that the income elasticity estimated from quarterly postwar data really cannot be pinned down accurately.[14] Since it will shed some further light on the quality of the estimates in (4), a brief exploration of Poole's argument will be worthwhile.

Suppose an estimating equation takes the form

$$\ln m_t = a + b \ln y_t + c \ln r_t + d \ln m_{t-1}. \tag{5}$$

The short-run income elasticity is b while the long-run elasticity is $b/(1 - d)$. Suppose the long-run elasticity is constrained to be some number e. Equation (5) then becomes

$$\ln m_t - e \ln y_t = a + c \ln r_t + d(\ln m_{t-1} - e \ln y_t), \tag{6}$$

which for a given e could then be simply estimated. A comparison of the properties of (6) for alternative values of e could then be made. Poole tried values of e ranging from 0.5 to 3.0 and emphasized two properties of the resulting estimates. He found that the estimated interest elasticity steadily increased with e, rising to 2.5–2.7 for $e = 3.0$; and the R^2 of the estimated equation was essentially flat for values of e from 1 to 3. It was this latter finding that led Poole to suggest the impossibility of obtaining a firm estimate of the income elasticity.

The equation Poole primarily focused on had one interest rate variable and no lagged dependent variable; it was, that is, like (6) with $d = 0$. It is consequently of some interest to see how equation (4) behaves for alternative values of e. Table 1 reports the relevant results, giving long-run interest elasticities for *RTD* and *RCP*, the speed-of-adjustment parameter, γ, and the R^2 and standard error. The interest elasticities display a clear tendency to increase with e, but the rise is not nearly as pronounced as Poole found.[15] The table also shows a systematic decline in the speed of adjustment as e increases.

As for the relative explanatory power of the equation as e increases, the table points to uniformly high R^2s, which rise steadily with e. That this is

14. William Poole, "Whither Money Demand?" *Brookings Papers on Economic Activity* (3:1970), pp. 485–500.

15. Although it is not indicated, the t-statistic for *RTD* declined to about 0.5 as e increased.

Table 1. Effects of Alternative Long-run Income Elasticity Constraints on Money Demand Equation

	Interest elasticities					Root mean-squared error		
						Sample period		Ex post to 1972,[b] dollar level[a]
Constrained elasticity	*Time deposits*	*Commercial paper*	*Speed-of-adjustment parameter*	R^2	*Standard error*	*Log*	*Dollar level*[a]	
0.6	0.135	0.049	0.325	0.9976	0.00441	0.0105	1.29	3.65
0.68[c]	0.160	0.066	0.283	0.9984	0.00432	0.0093	1.11	1.65
0.8	0.177	0.101	0.192	0.9989	0.00441	0.0099	1.29	2.71
0.9	0.186	0.136	0.140	0.9991	0.00450	0.0106	1.51	5.28
1.0	0.193	0.178	0.107	0.9993	0.00457	0.0112	1.68	7.90
1.1	0.194	0.222	0.086	0.9995	0.00462	0.0128	1.79	10.46
1.5	0.193	0.400	0.046	0.9997	0.00472	0.0150	2.03	19.81
2.0	0.184	0.633	0.028	0.9999	0.00477	0.0154	2.11	29.58

Sources: Derived from equation (4), as discussed in the text. For data sources and definitions, see appendix.

a. In billions of 1958 dollars.

b. Obtained by estimating equation (4) through 1961 and extrapolating forward by dynamic simulation to the end of 1972.

c. Full sample constraint.

misleading, however, is plain in the second row of the table, which was obtained by constraining e to be the value implied by equation (4). This procedure naturally reproduced the results of that equation except for the R^2. The trouble is that the dependent variable in (6) changes as e changes and consequently the R^2 is not strictly comparable across rows of the table.[16] The standard error of the regression, which is comparable, tells a different story. It clearly is lowest for the equation reported in the second row, as it should be. As e rises so does the standard error, although the deterioration is mild.

Another, perhaps more useful, way of looking at the overall performance of equation (4) for alternative values of e relies on dynamic simulation. In a dynamic simulation the lagged values of the dependent variable that are fed into the equation are those that are generated by the equation itself, not the historical values.[17] This is in general a more stringent test of an estimated equation than something like the R^2, and indeed is probably a more relevant test from a forecasting point of view. In this vein, I dynamically simulated the basic equation over the full sample period for each value of e. Table 1 also reports the root mean-squared error (RMSE) of the simulated around the true values. The first RMSE column is in the same units as the standard error while the second converts the logarithmic equation to dollar levels so that the units are in billions of 1958 dollars.[18] Equation (4) (the second row of Table 1) yielded an RMSE of \$1.1 billion. Alternative values of e led to a deterioration of the RMSE much more marked than the corresponding worsening of the standard error of the regression, pointing up the more discriminating nature of this technique.[19]

An even more vivid illustration of this point arises from the ex post performance of the basic equation. The last column of Table 1 reports for alternative values of e the root mean-squared errors obtained from esti-

16. Poole's results partly reflect this R^2 illusion but he has a number of specifications that do not suffer from this difficulty (for example, the one using the interest rate as the dependent variable).

17. Dynamic simulations in the presence of serially correlated errors also involve an additional correction for the lagged disturbance term.

18. The simulated values of the level were obtained simply by taking antilogs. In fact, this is not the best way to obtain them, but rough calculations suggested that the proper correction was small. On this see Arthur S. Goldberger, "The Interpretation and Estimation of Cobb-Douglas Functions," *Econometrica*, Vol. 36 (July–October 1968), pp. 464–72.

19. The RMSE in row 2 of Table 1 is roughly twice the standard error but rises to over three times for $e = 2.0$.

Table 2. Summary Statistics for Money Demand Equation, and Ex Post Forecasts for Alternative Sample Periods Ending with 1961 through 1972

	Coefficient					Root mean-squared error			
		Interest rate							
End point[a]	*Income* (1)	*Time deposits* (2)	*Commercial paper* (3)	*Money lagged* (4)	*Standard error* (5)	*Sample period*[b] (6)	*Four-quarter ex post forecast*[c] (7)	*Four-quarter mean error* (8)	*Full ex post root mean-squared error*[d] (9)
1961	0.216	−0.060	−0.019	0.605	0.0036	0.80	1.42	0.01	5.22
1962	0.183	−0.046	−0.020	0.687	0.0036	0.85	1.60	1.43	4.08
1963	0.196	−0.045	−0.021	0.718	0.0036	0.96	0.65	0.30	1.24
1964	0.203	−0.046	−0.021	0.728	0.0035	0.99	0.88	−0.86	2.57
1965	0.205	−0.047	−0.021	0.719	0.0035	0.93	2.33	−1.69	2.19
1966	0.187	−0.042	−0.022	0.703	0.0038	1.08	1.14	1.04	2.71
1967	0.199	−0.045	−0.022	0.691	0.0037	1.02	1.48	1.24	2.21
1968	0.204	−0.046	−0.022	0.702	0.0037	1.05	0.70	0.27	1.13
1969	0.204	−0.046	−0.023	0.702	0.0037	1.08	2.05	1.84	1.53
1970	0.201	−0.045	−0.022	0.702	0.0038	1.10	0.86	−0.14	1.23
1971	0.193	−0.044	−0.020	0.718	0.0041	1.10	1.10	−0.47	1.10
1972	0.193	−0.045	−0.019	0.717	0.0043	1.12	1.56[e]	−1.57	...

Source: Derived from equation (4), dynamically simulated as described in the text. For data sources and definitions, see appendix.

a. Each sample period begins with 1952:2 and has a terminal point that systematically moves from the end of 1961 to the end of each succeeding year in steps of four quarters.

b. In billions of 1958 dollars.

c. This column gives the RMSE (of the level in billions of 1958 dollars) for the year following the end point in the corresponding row.

d. This column gives the RMSE (of the level in billions of 1958 dollars) for all quarters following the end point up until the end of 1972.

e. For 1972, the RMSE is for the first two quarters of 1973 only instead of the full year following the terminal point.

mating the equation through 1961 and extrapolating forward by dynamic simulation to the end of 1972. The quality of these extrapolations deteriorates dramatically for high values of e.[20]

On balance, then, the specific estimates of equation (4) still seem satisfactory, both in terms of absolute performance and relative to the equations obtained for alternative income elasticities. Taken as a whole, the results seem to suggest that the relevant income elasticity can be pinned down within a reasonable range of accuracy, and that it is significantly less than unity, reflecting economies of scale.

SHORT-TERM INSTABILITIES?

The tentative conclusion just reached—that an equation like (4) does a satisfactory job of tracking money demand—was based on summary statistics derived from the within-sample performance of the equation. However, one of the primary concerns is the potential for short-run instability in the demand function for money. This problem can be attacked in a variety of ways, but one straightforward way is to ascertain the quality of the short-term ex post forecasts generated by this specification. To do this the specification in (4) was estimated over twelve sample periods, each starting in 1952:2 and differing in that the terminal point was systematically moved from the end of 1961 to the end of 1972, in steps of four quarters. Based on the estimates obtained for each sample period, the equation was dynamically simulated for the next four quarters.

A number of features of the estimated equations are contained in columns 1 through 6 of Table 2. Columns 1 through 4 list the individual coefficient estimates, which on casual inspection do appear to shift around somewhat. Columns 5 and 6 give the standard error of the regression and the RMSE (in billions of dollars) from within-sample dynamic simulations. Both these numbers tend to rise as the end point is extended, in part because the mean of the dependent variable is also increasing.

Columns 7 and 8 assess the out-of-sample forecasting performance, giving both the RMSE of a four-quarter forecast and the mean error. The

20. The estimate of the long-run income elasticity obtained from data through 1961 is lower than the full-sample estimate of 0.68. Consequently, a more realistic estimate of an attainable RMSE is higher than the $1.65 billion reported in Table 1 (see Table 2 below). Nevertheless, the more realistic estimate of roughly $2 billion to $5 billion in Table 2 is distinctly lower than all the high e entries in Table 1.

data underlying these calculations are plotted in Figure 1. The four-quarter forecast is for the year *following* the end point for a particular row. For example, the worst forecasting error occurred in 1966 with an RMSE of $2.3 billion and this appears in the 1965 row. In five of the twelve years the ex post forecast was no worse than the within-sample RMSE, which seems a creditable performance. Furthermore, this was true in 1971, a year reputed to be one of instability,[21] as well as in 1972. The forecasts for 1973 appear to be a bit wide of the mark but this judgment is based on only two observations—of preliminary data, at that—so one should not make too much of it.

On the whole, the money demand function does not exhibit marked short-run instability. However, this is only one chapter of the short-term forecasting story. For one thing, the analysis has assumed both known interest rates and real GNP. In addition, it explains money demand in real terms so that to forecast nominal money demand would require a price forecast, which would introduce further error.[22] Given these caveats, however, it is reassuring to find a reasonable degree of short-run stability.

LONG-TERM STABILITY

The companion question to the one just considered is whether the money demand function is stable in the long run. This question is usually addressed with annual data, often covering a span of seventy or so years; sometimes the focus is on whether the same money demand function held both in the 1930s and in the rest of the period.[23] The concern here is solely with whether quarterly data from the postwar period can be used homogeneously in face of a number of institutional developments (such as the certificate of deposit and Eurodollar markets) that at least suggest the possibility of shifts in the demand for money.[24]

Long-run stability can be examined in a variety of ways. The data sample

21. See, for example, the discussion of this issue in Michael J. Hamburger, "The Demand for Money in 1971: Was There a Shift?" *Journal of Money, Credit and Banking*, Vol. 5 (May 1973), pp. 720–25.

22. One other technical point should be noted. Table 2 is based on estimates with the latest and therefore fully revised data (except for 1973). In practice, these data would not be available.

23. See, for example, Laidler, *Demand for Money*.

24. Slovin and Sushka have reported some evidence that the period 1955:1 to 1962:1 may be different from 1962:2 to 1968:4. This interval roughly coincides with the start of the market for certificates of deposit. See M. B. Slovin and M. E. Sushka, "A Financial

can be split up at a priori chosen points[25] and the resulting estimates for the subperiods can be compared, either formally—say, via the Chow test—or informally. One useful informal comparison is to simulate dynamically the equation based on the first part of the period over the second part, thus extending the technique used in the previous section to a longer forecasting period.

The last column of Table 2 reports the root mean-squared errors for a number of such simulations. In each case, the money demand equation was estimated through the indicated end point and simulated from the following quarter through the end of 1972. The RMSEs are thus based on observations over varying periods, the longest being forty-four quarters. As could be expected, these RMSEs are generally larger than the four-quarter RMSEs, although markedly so only for the equations reported in the first two rows of the table. Moreover, these equations display coefficients that differ substantially from subsequent entries. This in turn is consistent with the Slovin-Sushka finding cited earlier and argues for a more careful examination of the pre- and post-1961 periods. Equations (4′) and (4″) report the estimates of equation (4) obtained by breaking the sample at the end of 1961.

$$(4') \quad \ln m = \underset{(1.9)}{0.699} + \underset{(4.6)}{0.216} \ln y + \underset{(6.4)}{0.604} \ln m_{-1} - \underset{(5.4)}{0.019} \ln RCP - \underset{(4.1)}{0.060} \ln RTD$$

$R^2 = 0.978$; standard error $= 0.0036$.
Sample period: 1952:2–1961:4.

$$(4'') \quad \ln m = \underset{(1.8)}{0.657} + \underset{(3.3)}{0.191} \ln y + \underset{(4.8)}{0.632} \ln m_{-1} - \underset{(2.4)}{0.014} \ln RCP - \underset{(0.3)}{0.010} \ln RTD.$$

$R^2 = 0.992$; standard error $= 0.0050$.
Sample period: 1962:1–1972:4.

Market Approach to the Demand for Money and the Implications for Monetary Policy" (Board of Governors of the Federal Reserve System, 1972; processed).

25. Rather than split the sample at some given point, one may use techniques for testing the hypothesis that a split occurred at some arbitrary point in the period. A number of these techniques are described in Stephen M. Goldfeld and Richard E. Quandt, *Nonlinear Methods in Econometrics* (North-Holland, 1972), Chap. 9.

The biggest difference between these two equations appears in the coefficient of *RTD* and it is largely attributable to the sizable jump in *RTD* that occurred precisely at the breaking point.[26] A formal test of stability, carried out by applying a Chow test to this sample split, resulted in an F statistic of 0.84, which does not allow one to reject the hypothesis of stability.[27]

On balance, then, the evidence does not seem to suggest any need to estimate the money demand equation over separate subsamples of the postwar period.

Alternative Specifications of the Basic Equation

Up to this point I have analyzed extensively the properties of essentially one specification—that embodied in equation (4). As the first section made clear, however, many questions concerning specification can only be resolved empirically. The purpose of the present section is to shed some light on these issues.

AGGREGATION AND DISAGGREGATION IN THE DEFINITION OF MONEY

Aggregation. To this point I have used the most common definition of money—M_1, which is the sum of currency and demand deposits. Other writers, however, have preferred a broader definition, such as M_2, which includes time deposits at commercial banks. This choice seems questionable on a variety of grounds since it constrains the specification, including the adjustment pattern, of M_1 and time deposits to be the same. Furthermore, since *RTD* should positively affect time deposit holdings and should negatively influence M_1 holdings, aggregation may badly muddy interest rate

26. The time deposit rate, *RTD*, jumped from 2.9 to 3.5 percent at this point. This was the largest quarterly change in the sample and obviously is an important influence both on the variance of *RTD* and, consequently, on the precision with which its coefficient can be estimated. Extending the sample period in (4′) to include this observation reduces the RMSE corresponding to the last column in Table 2 to 2.8. Furthermore, including this observation in (4″) as well makes the coefficients of *RTD* in the two equations considerably more alike.

27. The Chow test is, strictly speaking, not quite valid here because of the use of the lagged dependent variables and the serial correlation correction. A more appropriate test, at least asymptotically, is the likelihood ratio test. This yielded a χ^2 statistic of 9.3. The appropriate critical value is 12.6 so this gives the same result as the F test.

effects. On the other hand, an argument sometimes advanced in favor of M_2 is that it yields a more stable demand function.[28] In fact, according to evidence developed later, this is definitely not the case.

The tabulation below contains the results of estimating equation (4) with the M_2 definition and with time deposits alone, and, for comparison, repeats the equation (4) estimates:

Definition of money	*Income*	*Money variable lagged*	*RTD*	*RCP*	R^2	*Standard error*
M_2	0.119	0.948	0.006	−0.030	0.9987	0.0044
	(2.6)	(33.4)	(0.8)	(7.7)		
Time deposits	0.255	0.847	0.062	−0.051	0.9997	0.0075
	(3.0)	(18.7)	(4.7)	(7.2)		
M_1	0.193	0.717	−0.045	−0.019	0.9953	0.0043
	(5.3)	(11.5)	(4.0)	(6.0)		

It is evident from these numbers that the use of M_2 produces an equation with properties quite different from those of either of the component equations. First, the speed of adjustment is an unreasonably slow 5 percent per quarter as compared with 15 percent for time deposits and 28 percent for M_1. Second, *RTD*, as expected, has a negligible and insignificant impact on M_2, reflecting the offsetting effects of the component equations. Finally, the long-run income elasticity of M_2 is a huge 2.3, which exceeds both the 1.7 for time deposits and the 0.7 for M_1.

The only redeeming feature of the M_2 equation is that its standard error is only a smidgeon more than the M_1 equation, while that of the time deposit equation alone is substantially higher. This, however, is illusory as can be seen by dynamic simulations. Table 3 reports the results of both four-quarter ex post forecasts and longer-term ex post forecasts obtained by systematically changing the sample period as before. These are in columns 2 and 3 while the within-sample RMSE appears in column 1. Judged on the basis of these results, the equation for M_2 is extremely inadequate. As compared with the results in Table 2, the RMSEs of the four-quarter ex post forecast are both large and variable—ludicrously so in the longer-run extrapolations. From these results one would expect the equation for

28. See Laidler, *Demand for Money*, p. 108.

Table 3. Root Mean-Squared Errors for M_2 and Time Deposits, and Income Coefficients, Alternative Sample Periods Ending with 1961 through 1971

	Root mean-squared error						
	Currency plus demand and time deposits, M_2			*Time deposits*			
		Ex post			*Ex post*		
End point[a]	*Sample period (1)*	*Four-quarter (2)*	*Full (3)*	*Sample period (4)*	*Four-quarter (5)*	*Full (6)*	*Income co-efficient (7)*
1961	1.74	1.54	71.46	0.94	1.52	46.76	−0.011
1962	1.70	5.05	71.55	0.96	3.22	43.37	0.026
1963	2.12	2.30	54.64	1.30	2.52	37.61	0.039
1964	2.39	3.56	39.57	1.60	5.04	28.58	0.072
1965	2.80	3.25	18.71	2.27	0.57	10.86	0.156
1966	2.78	7.84	34.98	2.35	4.18	10.80	0.154
1967	2.76	3.09	18.83	2.52	1.38	4.87	0.169
1968	3.02	7.00	6.81	2.32	3.99	3.68	0.177
1969	3.38	9.81	24.08	2.98	2.93	6.64	0.191[b]
1970	4.14	5.11	4.94	2.29	2.02	1.73	0.248[b]
1971	4.81	1.10	1.10	2.18	2.60	2.60	0.267[b]

Source: Same as Table 2.
a. See Table 2, note *a*.
b. Coefficient significant at 5 percent level.

M_2 to fail any formal test for stability and, indeed, it does. Splitting the sample at the end of 1961 and applying a Chow test yields an F statistic of 3.53; the corresponding likelihood ratio test yields a χ^2 of 18.6. Both of these are significant at the 1 percent level, allowing one easily to reject the hypothesis that the equation for M_2 is stable over the sample period.

Since the M_1 equation was previously found to be stable, the suspicion is that the difficulty lies with the time deposit component, because that component is itself unstable or because of the aggregation process or both. Superficially, the time deposit equation based on the full sample appears quite reasonable. When subjected to the kind of dynamic simulation tests just described, however, this equation also appears questionable. The results are reported in columns 4 through 7 of Table 3. The three sets of RMSEs for time deposits are superior to the corresponding RMSEs for M_2. When judged by an absolute standard, the within-sample and four-quarter RMSEs might be acceptable but the full-period RMSEs remain distinctly unreasonable. The source of the difficulty is indicated in the last column of Table 3, which reports the estimated income coefficient for alternative sample periods. That coefficient rises steadily over the period and does not achieve statistical significance until the sample period runs through 1969. One would expect, as with M_2, that the time deposit equation would fail a formal stability test. The appropriate Chow *F* statistic is 4.25 and the corresponding χ^2 is 22.1, allowing one to reject stability by either test at the 1 percent level.

This finding suggests, at the very least, that the simple specification used for M_1 will not work for time deposits and therefore should not be implicitly so used by estimating a similar equation for M_2.[29] The situation is, however, worse than that, since even given the questionable time deposit equation, the ex post forecasts of M_2 obtained from the aggregate equation are inferior to those obtained from adding together the separate component forecasts, thus suggesting that aggregation is inflicting some positive harm in the present context.[30]

In summary, for both theoretical and empirical reasons, aggregation to the level of M_2 seems to be a distinctly inferior procedure.

Disaggregation. Although these findings confirm that greater aggregation in the estimation of the demand for money is not called for, there remains the question of whether some disaggregation would be appropriate. The most obvious type of disaggregation would be to estimate separate equations for currency and demand deposits,[31] as is done in many macroeconometric models for a variety of reasons. For one, disaggregation permits greater flexibility in the choice of variables and specification of adjustment patterns. Second, and perhaps of more practical importance, currency is needed as an endogenous variable for analyzing monetary policy. In particular, a means of splitting up high-powered money (a variant of which is usually taken as a policy instrument) into reserves and currency may be needed to trace out the money supply mechanism. In any event, there are good precedents for attempting to explain currency and demand deposits separately.

29. I briefly experimented with several other interest rates in both the time deposit and M_2 equations but these never achieved statistical significance.

30. I spare the reader the additional numbers. However, the remark in the text is based on adding together the separate extrapolations for M_1 and time deposits and then comparing the RMSEs with those in Table 3.

31. Disaggregating by type of holder is considered below. To some extent, separation into currency and demand deposits is also a partial step in this direction.

The tabulation below reports the results of estimating separate equations for currency and demand deposits along the lines of equation (4).

Dependent variable	*Income*	*Consumer expenditures*	*Money variable lagged*	*RTD*	*RCP*	R^2	*Standard error*
Demand deposits	0.181 (5.2)	...	0.693 (9.9)	−0.040 (3.7)	−0.021 (6.0)	0.992	0.0049
Currency	0.190 (5.3)	...	0.804 (19.0)	−0.046 (3.9)	−0.007 (2.0)	0.998	0.0042
Currency	...	0.279 (6.2)	0.591 (8.3)	−0.025 (1.7)	−0.001 (0.2)	0.998	0.0043

The first two equations use exactly the same specification and sample period as equation (4). Both seem relatively satisfactory, and surprisingly enough, both interest rate variables show up in the currency equation. The long-run income elasticity of the demand deposit equation is 0.59, while that of the currency equation is 0.97. These bracket the 0.68 elasticity found for M_1. The speed-of-adjustment coefficients also bracket the M_1 result with demand deposits adjusting somewhat more rapidly than currency.

The final row of the tabulation contains the results of one minor modification in the currency equation, the substitution of consumer expenditures for GNP as the transactions variable and the corresponding use of the consumption deflator.[32] This procedure has pronounced effects on the equation: first, it renders both interest variables statistically insignificant; and second, it considerably speeds up the adjustment of currency holdings.[33]

How do the component equations stand up when subjected to dynamic simulation? The relevant results are reported in Table 4. The two versions of the currency equation perform comparably on the four-quarter simulations, producing only small forecasting errors. The demand deposit equation, as expected, yields smaller RMSEs than the aggregate equation

32. This, for example, was used in Modigliani, Rasche, and Cooper, "Central Bank Policy."

33. There is some question, however, about the generality of this second finding. In particular, the lagged stock coefficients with GNP and consumption were virtually identical for all the equations underlying Table 4 below. Only when 1971 (or 1971 and 1972) were included in the sample did the difference cited in the text emerge.

Table 4. Root Mean-Squared Errors for Extrapolations of Demand Deposit and Currency Equations, Sample Periods Ending with 1961 through 1971

	Four-quarter extrapolation			*Full ex post extrapolation*		
		Currency, by transactions variable			*Currency, by transactions variable*	
End point[a]	*Demand deposits*	*Consumer expendi-tures*	*Gross national product*	*Demand deposits*	*Consumer expendi-tures*	*Gross national product*
1961	0.59	0.10	0.09	2.07	2.64	0.45
1962	0.44	0.61	0.68	1.25	0.61	1.59
1963	0.42	0.30	0.05	1.94	3.83	1.17
1964	0.84	0.23	0.22	2.27	1.26	0.77
1965	2.06	0.34	0.45	1.78	1.37	0.87
1966	0.92	0.13	0.25	2.21	0.47	0.69
1967	1.40	0.17	0.34	1.79	0.23	0.84
1968	0.75	0.25	0.10	0.90	0.36	0.43
1969	1.43	0.09	0.23	1.15	0.14	0.29
1970	0.78	0.19	0.22	1.14	0.16	0.20
1971	0.97	0.35	0.37	0.97	0.35	0.37

Source: Same as Table 2.
a. See Table 2, note *a*.

although it still makes a sizable error in forecasting 1966. The long-term extrapolations for all three equations also perform creditably. As between specifications of the currency equation, the GNP formulation does better in the early part of the period but the consumption specification does better at the end of the period.

Comparing the RMSEs in Table 4 with those in Table 2 suggests that extrapolation of M_1 might be accomplished better with the component equations, especially since any offsetting errors in the component equations should help in forecasting. To assess this possibility, I summed the separate forecasts for currency and demand deposits and then computed the appropriate RMSEs. These are reported in Table 5, which also includes for convenience the corresponding results from Table 2 (labeled "aggregate").

On the whole the ex post forecasts from the component equations do extremely well. In particular, they improve markedly the extrapolations of M_1 relatively far into the future (see the first two rows of the table). Overall, the most successful formulation is that which used consumer expenditures as the transactions variable in the currency equation. In the eleven cases

Table 5. Root Mean-Squared Errors for Aggregate and Disaggregate Forecasts of M_1, Sample Periods Ending with 1961 through 1971

	Four-quarter forecast			*Full ex post forecast*		
	Disaggregate, by transactions variable			*Disaggregate, by transactions variable*		
End point[a]	*Gross national product*	*Consumer expenditures*	*Aggregate*	*Gross national product*	*Consumer expenditures*	*Aggregate*
1961	0.64	0.64	1.42	1.87	4.48	5.22
1962	1.13	1.05	1.60	1.56	1.37	4.08
1963	0.41	0.53	0.65	2.97	5.47	1.24
1964	1.04	1.06	0.88	2.95	3.35	2.57
1965	2.49	2.53	2.33	2.48	2.84	2.19
1966	1.10	0.97	1.14	2.82	1.93	2.71
1967	1.72	1.54	1.48	2.50	1.75	2.21
1968	0.73	0.77	0.70	1.04	1.02	1.13
1969	1.64	1.40	2.05	1.28	1.19	1.53
1970	0.88	0.87	0.86	1.15	1.15	1.23
1971	1.12	1.09	1.10	1.12	1.09	1.10

Source: Aggregate columns are from Table 2; disaggregate data are derived from separate forecasts for currency and demand deposits, the components of M_1.
a. See Table 2, note *a*.

considered it yields an RMSE lower than the aggregate equation eight times for long-term extrapolations and six times for short-period projections. This evidence provides some independent support for model builders who choose to use separate currency and demand deposit equations and who include consumption in the currency equation.[34]

On balance, the message of this section should be clear: as far as the money demand equation is concerned, more rather than less disaggregation appears to be desirable.

PARTIAL ADJUSTMENT, EXPECTATIONS, AND LAGS

So far, the analysis has relied on a very simple form of dynamic adjustment, a Koyck-type equation that uses a single lagged dependent variable. While this is a convenient specification, it has the questionable feature of restricting the adjustment pattern of money holdings to be the same with

34. This is the strategy followed in the FMP model. See Modigliani, Rasche, and Cooper, "Central Bank Policy."

respect to both income and interest rates. Careful consideration of the source of lagged adjustments in money holdings is thus in order.

The justification offered above for the form of equation (4) rested on a vague appeal to the partial adjustment mechanism. Despite the superficial plausibility of this mechanism, its theoretical foundation in the context of the demand for money is unclear. For capital stock accumulation the mechanism is satisfactory, but the analogy between money holdings and capital equipment is far from perfect for many reasons. One is that the exact nature of the costs involved is much less clear in adjusting financial portfolios than in the case of adjusting stocks of machinery and plant. Second, the lags that result statistically for money adjustment appear too long to explain on grounds of adjustment costs. Finally, even if the analogy is granted, it does not necessarily imply the simple formulation of (3) or (3′) and indeed does so only under very special assumptions.[35]

This unsatisfactory state of affairs can be partially remedied by reliance on a different rationale for the lagged adjustment. Pushed back one step, the adjustment can be conceived as a slow response of desired stock itself to actual current values of income and interest rates, rather than a gradual shift in money holdings to meet a promptly adopted new level of desired holdings. The response could be slow because of inertia or because individuals respond to expected values that are in turn a function of past values.[36] Of course, expectational and partial adjustment lags may exist in combination.

The workings of a pure expectations influence may be examined in a demand function of the form

$$m = a + by^e + cr^e, \tag{7}$$

35. On this, see J. P. Gould, "Adjustment Costs in the Theory of Investment of the Firm," *Review of Economic Studies*, Vol. 35 (January 1968), pp. 47–55. Another problem with the partial adjustment mechanism in the present context is that the transactions approach may easily lead to "corner" solutions for an individual. That is, he may not respond at all unless some critical condition is met (say, the interest rate changes by more than a certain amount). This suggests the need to pay considerable attention to the details of aggregating over individuals to obtain a macro equation.

For a discussion of this point, see William Breen, "A Note on the Demand for Cash Balances and the Stock-Adjustment Hypothesis," *International Economic Review*, Vol. 12 (February 1971), pp. 147–51.

36. On this, see Franco Modigliani, "The Dynamics of Portfolio Adjustment and the Flow of Savings Through Financial Intermediaries," in Edward M. Gramlich and Dwight M. Jaffee (eds.), *Savings Deposits, Mortgages, and Housing* (Heath, 1972).

where y^e and r^e are expected (or, if one prefers, "permanent") measures.[37] Since y^e and r^e are unobservable, they must be replaced by measured variables. One common device for doing so is to assume that expectations are "adaptive," that is,

$$y_t^e - y_{t-1}^e = \lambda(y_t - y_{t-1}^e) \tag{8}$$

$$r_t^e - r_{t-1}^e = \lambda(r_t - r_{t-1}^e). \tag{9}$$

This device implies that y_t^e is a geometric distributed lag of current and past values of y; that is,

$$y_t^e = \lambda \sum_{i=0}^{\infty} (1 - \lambda)^i y_{t-i}. \tag{8'}$$

Equations (8) and (9) may then be combined with (7) to yield

$$m_t = a + b\lambda y_t + c\lambda r_t + (1 - \lambda)m_{t-1}. \tag{10}$$

Equation (10) obviously has the same form as the equations estimated above, such as (4), but λ has a different interpretation.[38] Equations (8) and (9) have the same λ, implying the restrictive assumption that expectations of y and r are formed analogously. A more natural specification in place of (9) would be

$$r_t^e - r_{-1}^e = \delta(r_t - r_{t-1}^e), \tag{11}$$

where δ may be different from λ. Combining (7), (8), and (11) produces a considerably more complicated estimating equation:

$$m_t = c_0 + c_1 y_t + c_2 y_{t-1} + c_3 r_t + c_4 r_{t-1} + c_5 m_{t-1} + c_6 m_{t-2}, \tag{10'}$$

where the cs are nonlinear functions of the respective original parameters.

This version of the adaptive expectations model leads to a considerably richer lag structure. In fact, even greater generality may be obtained by allowing expectations to adjust in different proportions to two or more of the previously observed forecasting errors, as in[39]

$$y_t^e - y_{t-1}^e = \lambda_1(y_t - y_{t-1}^e) + \lambda_2(y_{t-1} - y_{t-2}^e). \tag{12}$$

37. To simplify notation I have omitted "ln," although the specification continues to be logarithmic.

38. The disturbance term in (10), which is not shown, is actually of a different form from the one implicit in (4).

39. This has been suggested in J. A. Carlson and M. Parkin, "Inflation Expectations" (Purdue University and University of Manchester, May 1973; processed). Using (12) instead of (8) and a corresponding replacement for (9) yields a version of (10′) with three lags for y and r and four lags for m.

Another extension of the formulation is accomplished by combining the adaptive expectations and partial adjustment models. This procedure introduces another lag in all the variables and some further nonlinear restrictions.[40]

Table 6 reports the results of estimating a relatively simple version of these alternatives, equation (10′), as well as two modified versions that either omit the second-order lag in the dependent variable or the lagged variables for income and interest rates. For comparison, equation (4) is reported as regression A in Table 6.

Several features of the results are worth noting. First, the long-run income and interest rate elasticities are virtually identical for all four equations. There are, however, differences in the timing of the responses among the four equations. These differences are illustrated for two of the equations in Table 7, which gives the fraction of the total response to a change in income or interest rates that has occurred after a given number of quarters. For the simple Koyck equation this response is identical for all variables, but this is clearly not the case for the second equation in Table 6.

A second feature is that the three lagged variables for income and interest rates are collectively significant when used without m lagged twice but not when it is included.[41] Finally, m lagged twice appears significant whether or not these other variables are included.[42]

In my judgment, these results leave open the question of whether a specification more complicated than the original Koyck model is appropriate. Clearly, however, satisfactory estimation of equation (10′) is impeded by pronounced multicollinearity. Consequently, unless the nonlinear restrictions underlying such an equation are taken into account properly, it seems pointless to estimate a more sophisticated version.[43] An alternative and potentially more promising route is to rely on Almon distributed lags.

40. It also permits a test of the hypothesis that either the expectations mechanism or the partial adjustment mechanism is absent. See, for example, Edgar L. Feige, "Expectations and Adjustments in the Monetary Sector," in American Economic Association, *Papers and Proceedings of the Seventy-ninth Annual Meeting, 1966* (*American Economic Review*, Vol. 57, May 1967), pp. 462–73.

41. The relevant F statistic for these variables is 3.5, which is significant at the 5 percent level, when the comparison is between B and A in Table 6. The corresponding F statistic for comparing D and C is an insignificant 0.4.

42. The equations reported in Table 6 were all estimated assuming first-order serial correlation. Allowing for second-order effects did not qualitatively change the results.

43. It should be noted that I have ignored such restrictions in estimating (10′). Basically, my energy deteriorated at this point.

Table 6. Estimates of Alternative Forms of Lagged Adjustments in Demand-for-Money Equations

			Interest rate			Interest rate lagged				
Regression	*Income*	*Money lagged*	*Commercial paper*	*Time deposits*	*Income lagged*	*Commercial paper*	*Time deposits*	*Money lagged twice*	ρ	R^2
A	0.193	0.717	−0.019	−0.045	...	...	...	...	0.41	0.995
	(5.3)	(11.5)	(6.0)	(4.0)					(4.1)	
B	0.168	0.657	−0.011	−0.029	0.064	−0.014	−0.025	...	0.47	0.996
	(3.0)	(8.9)	(2.5)	(1.8)	(1.0)	(2.9)	(1.5)		(4.8)	
C	0.181	1.105	−0.015	−0.044	...	...	...	−0.382	0.09	0.996
	(6.2)	(12.4)	(6.6)	(4.9)				(4.8)	(0.8)	
D	0.178	0.977	−0.013	−0.036	0.028	−0.005	−0.014	−0.289	0.20	0.996
	(3.2)	(9.2)	(3.1)	(2.2)	(0.4)	(1.0)	(0.8)	(2.9)	(1.9)	

Source: Derived from equations (4) and (10′) and two modifications of (10′), discussed in the text. For data sources and definitions, see appendix. The numbers in parentheses here and in all the following tables are t-statistics.

Table 7. Comparison of Cumulative Percentage Responses of Regressions A and B of Table 6 after Selected Numbers of Quarters

		Regression B (text equation 10′)		
Number of quarters	*Regression A (text equation 4)*	*Income*	*Interest rate on time deposits*	*Interest rate on commercial paper*
1	28.3	24.7	18.1	15.7
2	48.6	50.3	45.6	45.7
3	63.0	67.2	63.8	65.7
4	73.5	78.2	75.6	78.6
7	90.2	93.2	91.3	96.6

Source: Same as Table 6.

The basic estimating equation for this technique is given by

$$(13)\quad \ln m_t = c + \sum_{i=0}^{n_1} w_i \ln y_{t-i} + \sum_{i=0}^{n_2} w_i' \ln RTD_{t-i} + \sum_{i=0}^{n_3} w_i'' \ln RCP_{t-i}.$$

This equation can be rationalized in a number of ways.[44] For example, the form of its composite variables is simply a generalization of equation (8′) with a finite horizon. Alternatively, one may simply regard (13) as a convenient and flexible equation for approximating a rather complicated underlying process.

A number of a priori expectations surround the coefficients in (13). The w_is, representing the lag distribution for income, should all be positive and should probably decline monotonically. The corresponding interest rate coefficients should be negative; they might well exhibit a humped pattern, especially for *RTD*, because *RCP* is likely to affect primarily large transactors, who are less subject to a learning delay.[45]

Equation (13) was estimated over the same sample period as equation (4)—1952:2 through 1972:4—by the Almon technique, with an adjustment for serial correlation. The individual lag coefficients were assumed to lie on a third-degree polynomial and no end-point constraints were imposed. The length of each lag (n_1, n_2, and n_3) was determined empirically with the

44. See, for example, Harold D. Dickson and Dennis R. Starleaf, "Polynomial Distributed Lag Structures in the Demand Function for Money," *Journal of Finance*, Vol. 27 (December 1972), pp. 1035–43, or Modigliani, Rasche, and Cooper, "Central Bank Policy."

45. On this, see ibid.

Table 8. Estimates of Income and Interest Elasticity Coefficients Using Almon Distributed Lags in the Demand-for-Money Equations[a]

Dependent					
Currency plus demand deposits, M_1			*Demand deposits*		
Income	*RTD*	*RCP*	*Income*	*RTD*	*RCP*
0.146	−0.028	−0.014	0.131	−0.024	−0.014
(3.6)	(2.1)	(3.7)	(3.1)	(1.67)	(3.4)
0.119	−0.033	−0.014	0.105	−0.031	−0.014
(4.8)	(4.9)	(6.7)	(4.3)	(4.7)	(7.2)
0.094	−0.034	−0.012	0.082	−0.032	−0.013
(6.9)	(4.7)	(5.7)	(6.5)	(4.2)	(6.1)
0.073	−0.030	−0.011	0.063	−0.028	−0.012
(6.5)	(3.7)	(3.9)	(5.9)	(3.2)	(4.2)
0.056	−0.021	−0.009	0.047	−0.019	−0.010
(3.7)	(3.2)	(3.0)	(3.0)	(2.6)	(3.3)
0.041	−0.009	−0.006	0.034	−0.004	−0.007
(2.2)	(1.3)	(2.0)	(1.7)	(0.6)	(2.2)
0.030	0.009	−0.003	0.024	0.017	−0.003
(1.5)	(0.6)	(0.7)	(1.2)	(1.1)	(0.7)
0.022	...	...	0.018	...	...
(1.2)			(0.9)		
0.017	...	...	0.015	...	...
(1.1)			(0.9)		
0.016	...	...	0.015	...	...
(1.2)			(1.2)		
0.018	...	...	0.019	...	...
(1.1)			(1.2)		
0.023	...	...	0.025	...	...
(0.9)			(0.9)		
...	...	...	...	...	...
...	...	...	...	...	...
$\Sigma = 0.656$	$\Sigma = -0.145$	$\Sigma = -0.068$	$\Sigma = 0.577$	$\Sigma = -0.121$	$\Sigma = -0.073$
(17.3)	(8.8)	(5.4)	(19.4)	(10.3)	(5.8)
$R^2 = 0.995$, standard error $= 0.0046$, $\rho = 0.82$			$R^2 = 0.992$, standard error $= 0.0051$, $\rho = 0.69$		

Source: Derived from text equation (13). The sample period is 1952:2 through 1972:4. For data sources and definitions, see appendix.

RTD and *RCP* are the interest rates on time deposits and commercial paper, respectively.

a. The summations are calculated from data before rounding.

rough aid of the information on speed of adjustment from the stock adjustment equations.

The results reported in Table 8 agree remarkably well with those obtained earlier. For the long-run income and interest elasticities, which are reported in Table 9, the Koyck and Almon estimates do not differ by more than 0.02. The equations do differ, of course, in the pattern of lagged response, and on this score, the results of equation (13) seem sensible. The length of the lag on income is substantially longer than that of the corresponding lag for interest rates, a finding that is roughly supported by some *variable*

Currency with income as transactions variable			*Currency with consumer expenditures as transactions variable*		
Income	*RTD*	*RCP*	*Consumer expenditures*	*RTD*	*RCP*
0.153	−0.020	−0.0045	0.247	−0.028	−0.0050
(5.4)	(1.3)	(1.2)	(3.8)	(1.6)	(1.1)
0.126	−0.018	−0.0047	0.261	−0.012	−0.0036
(6.2)	(2.0)	(2.1)	(5.4)	(0.9)	(1.3)
0.103	−0.017	−0.0047	0.191	−0.009	−0.0023
(7.2)	(1.5)	(2.0)	(3.9)	(0.7)	(0.8)
0.082	−0.016	−0.0044	0.036	−0.020	−0.0010
(7.4)	(1.8)	(2.1)	(0.5)	(1.1)	(0.2)
0.065	−0.016	−0.0039			
(6.1)	(1.0)	(1.1)			
0.051	...	...	...	...	...
(4.4)					
0.041	...	...	...	...	...
(3.2)					
0.033	...	...	...	...	...
(2.5)					
0.029	...	...	...	...	...
(2.3)					
0.028	...	...	...	...	...
(2.3)					
0.031	...	...	...	...	...
(2.6)					
0.037	...	...	...	...	...
(2.8)					
0.046	...	...	...	...	...
(2.6)					
0.058	...	...	...	...	...
(2.4)					
Σ = 0.883	Σ = −0.086	Σ = −0.022	Σ = 0.734	Σ = −0.069	Σ = −0.012
(13.4)	(2.8)	(2.7)	(12.5)	(2.4)	(1.5)
R^2 = 0.998, standard error = 0.0045, ρ = 0.97			R^2 = 0.997, standard error = 0.0048, ρ = 0.97		

previous work.[46] Furthermore, the peak impact of *RTD* occurs after two quarters, so the interest rate response does exhibit the humped pattern posited above.

More details on the exact timing of responses are given in Table 10, which reports the fraction of the total response to changes in income and interest rates that has occurred after a given number of quarters.[47] The

46. See, for example, A. A. Shapiro, "Inflation, Lags, and the Demand for Money," *International Economic Review*, Vol. 14 (February 1973), pp. 81–96, and Feige, "Expectations and Adjustments."

47. It should be recalled that the dependent variable is measured in logarithms.

Table 9. Comparison of Long-run Income and Interest Elasticities from Koyck and Almon Estimates, for Money and Components

Dependent variable	Koyck			Almon		
	Income	RTD[a]	RCP[a]	Income	RTD[a]	RCP[a]
Money, M_1	0.68	0.16	0.07	0.66	0.15	0.07
Demand deposits	0.59	0.13	0.07	0.58	0.12	0.07
Currency	0.97	0.23	0.04	0.88	0.09	0.02
Currency[b]	0.68	0.06	0.00	0.73	0.07	0.01

Sources: Koyck, equation (4); Almon, equation (13). *RTD* and *RCP* are the interest rates on time deposits and commercial paper, respectively.
a. All interest elasticities are negative.
b. Currency equation using consumer expenditures as a transactions variable.

Table 10. Comparison of Cumulative Percentage Responses, after Selected Numbers of Quarters, of Koyck and Almon Equations, for Money and Components

Number of quarters	Dependent variable: Currency plus demand deposits, M_1				Dependent variable: Demand deposits			
	Koyck	Almon: Income	Almon: RTD	Almon: RCP	Koyck	Almon: Income	Almon: RTD	Almon: RCP
1	28.3	22.2	19.3	20.5	30.7	22.7	19.8	19.2
2	48.6	40.3	42.1	41.2	51.9	40.9	45.4	38.4
3	63.0	54.6	65.5	58.8	66.7	55.1	71.9	56.2
4	73.5	65.7	86.2	75.0	76.9	66.0	95.0	72.6
7	90.2	85.1	100.0	100.0	92.3	84.2	100.0	100.0
10	96.3	93.5	100.0	100.0	97.3	92.5	100.0	100.0

Number of quarters	Dependent variable: Currency with income as transactions variable				Dependent variable: Currency with consumer expenditures as transactions variable			
	Koyck	Almon: Income	Almon: RTD	Almon: RCP	Koyck	Almon: Consumer expenditures	Almon: RTD	Almon: RCP
1	19.6	17.3	23.2	20.5	40.9	33.7	40.6	41.7
2	35.3	31.6	44.2	41.8	65.1	69.2	58.0	71.7
3	48.0	43.3	70.0	62.2	79.4	95.2	71.0	90.9
4	58.2	52.6	82.6	82.2	87.4	100.0	100.0	100.0
7	78.2	70.4	100.0	100.0	97.5	100.0	100.0	100.0
10	88.7	80.6	100.0	100.0	99.5	100.0	100.0	100.0

Sources: Same as Table 9. *RTD* and *RCP* are the interest rates on time deposits and commercial paper, respectively.

Koyck version—equation (4)—necessarily has only one pattern of response while the Almon equation has three separate patterns. The Almon responses to income changes are uniformly slower than the Koyck responses. For interest rates, the Almon response is slower for several quarters but then overtakes the Koyck response. Evidently, constraining all the responses to the same shape in the Koyck version produces an inappropriate average response which masks individual differences.

In addition to the results for M_1, Tables 8, 9, and 10 report the findings of Almon versions of separate equations for estimated demand deposits and currency. The results for demand deposits are, not surprisingly, quite comparable to those for M_1 both in terms of absolute performance and in comparison with the Koyck version presented on page 596. Somewhat larger differences emerge between the Koyck and Almon versions of the two currency equations; but on the whole the Almon currency equation performs creditably.

In summary, a modest amount of evidence suggests that the Koyck formulation of equation (4) is a bit too restrictive.[48] The price paid for this simplification does not seem severe but it deserves additional research—for example, to examine the comparative performance of alternative lag structures in such simulation experiments as those reported earlier.

INFLATIONARY EXPECTATIONS

The discussion of lags and expectation formation in the previous section was restricted to income and interest rate variables. This section explores another variable—prices—and particularly investigates whether inflationary expectations have an independent role to play in the demand-for-money function.

Even at the theoretical level, this question is controversial. On a strict transactions view of the demand for money, a variable measuring anticipated inflation seems to have no place.[49] On the other hand, in theoretical

48. There appears to be some serial correlation left in the Almon equations even after correcting for first-order correlation. However, Dickson and Starleaf, in "Polynomial Distributed Lag Structures," perform a second-order correction in a somewhat analogous M_1 equation and get essentially the same kind of results as I did. For example, their income elasticity is identical to the one in Table 9 and the interest elasticities are quite close.

49. Under suitable assumptions this can be formally shown, as in Ando and Shell, "Demand for Money." Inflationary expectations will be reflected to some extent in nominal interest rates and thus will indirectly affect the demand for money.

writings on demand-for-money functions in the Chicago tradition, money serves as an alternative for physical goods, and the expected rate of price change is given a prominent role.[50] This approach has been buttressed by empirical evidence from hyperinflations abroad. In view of these latter findings, Harry Johnson calls the absence of "American evidence that the expected rate of change of prices enters the demand for money function ... something of a puzzle."[51] He tentatively attributes it to the relative mildness of U.S. inflations and to the possible presence of threshold effects.[52]

In the spirit of empiricism of this paper and in light of the divergence of opinion just cited, the performance of expected inflation variables in money demand equations will be given a brief look. Following one of many possible routes, I shall modify equation (7) to include an expected rate of inflation, ρ^e:

$$m = a + by^e + cr^e + d\rho^e. \tag{7'}$$

If the expected rate of inflation is defined by an adaptive expectations mechanism as in (8) or (9), the resulting equation takes the form[53]

$$\ln m_t = a + b \ln y_t + c \ln m_{t-1} + d \ln RTD_t + e \ln RCP_t + f \ln (P_t/P_{t-1}). \tag{14}$$

The results of estimating equation (14) are given in the first row of Table 11. The price variable is quite significantly negative and its inclusion raises the elasticity for income and lowers the speed of adjustment as compared with equation (4). The elasticities for the interest rate variables remain virtually the same. (The measures in rows 2 and 3 are considered after the discussion of Table 12.)

50. See, for example, the various studies in Milton Friedman (ed.), *Studies in the Quantity Theory of Money* (University of Chicago Press, 1956).

51. Harry G. Johnson, *Macroeconomics and Monetary Theory* (Aldine, 1972), p. 127.

52. Also relevant here is the notion of Allais that people will pay more attention to current and less to past events the more rapidly the current situation is changing. This suggests that one needs more than a simple distributed lag of past rates of inflation to measure expected inflation. See Maurice Allais, "A Restatement of the Quantity Theory of Money," *American Economic Review*, Vol. 56 (December 1966), pp. 1123–57.

53. The functional form for the expected inflation term in (14) is equivalent to using $\Delta P_t/P_{t-1}$ directly without logarithms. This is so since

$$\ln (P_t/P_{t-1}) = \ln\left(1 + \frac{\Delta P_t}{P_{t-1}}\right) \doteq \Delta P_t/P_{t-1}.$$

The regressions reported below were, in fact, estimated both as shown and with $\Delta P_t/P_{t-1}$ as a variable and the results were identical to three decimal places.

Table 11. Coefficients of Variables in Demand-for-Money Equations, for Three Measures of Price Expectations

			Interest rate				
Measure	*Income*	*Money lagged*	*Time deposits*	*Commercial paper*	*Price variable*	R^2	ρ
Equation (14)	0.166 (4.9)	0.782 (13.1)	−0.038 (3.6)	−0.015 (5.0)	−0.657 (4.2)	0.996	0.46
de Menil I	0.200 (5.6)	0.698 (11.3)	−0.046 (4.1)	−0.016 (4.9)	−0.143 (1.9)	0.996	0.41
de Menil II	0.200 (5.6)	0.693 (11.1)	−0.044 (4.0)	−0.016 (4.8)	−0.211 (1.8)	0.996	0.41

Sources: Row 1 gives the results of estimating equation (14) as derived in the text, defining the expected rate of inflation by an adaptive expectations mechanism. In rows 2 and 3, direct measures of price expectations from series constructed from surveys of expected price performance are substituted in equation (14). The series are from G. de Menil, "Rationality in Popular Price Expectations" (Princeton University, August 1973; processed). For other data sources, see appendix.

The impact of the price variable on the money demand equation can be assessed by a simple conceptual experiment. In an equilibrium situation that has persisted long enough, and in which interest rates are constant, real income is growing at 4 percent and the actual rate of inflation is 2 percent, equation (14) states that real money stock should grow at 3 percent and the nominal money stock at 5 percent. Now imagine a once-and-for-all change in the rate of inflation from 2 percent to 6 percent that leaves interest rates and the rate of growth of real GNP unchanged. In the long run, the rate of growth of the real money stock will remain 3 percent, though the nominal money stock will grow at 9 percent. In the short run, however, substantial deviations from these rates of growth will occur if income growth and interest rates are to remain unchanged. I used the estimates of equation (14) to compute these short-run deviations, with the results reported in Table 12.

The largest effect occurs in the initial quarter and after eight quarters the growth rates have nearly reached their equilibrium values. At that point the real money stock is 2½ percent below where it would have been had the rate of inflation remained unchanged. The nominal money stock (given the assumed super-accommodating behavior of the Federal Reserve) is 5½ percent higher.

It is, of course, unrealistic to assume that nominal interest rates will be unchanged in the face of this higher rate of inflation. For illustrative purposes, assume that *RTD* would rise from 5 percent to 6 percent and *RCP* from 6 percent to 9 percent as a result of the higher inflation. The resulting

Table 12. Short-run Rates of Growth of the Money Stock in Transition from 2 Percent to 6 Percent Inflation, with Fixed and Variable Interest Rates

Percent

	Interest rates fixed		*Interest rates variable*	
Quarter	*Real*	*Nominal*	*Real*	*Nominal*
1	0.4	6.4	−5.7	0.3
2	1.0	7.0	−3.8	2.2
3	1.4	7.4	−2.3	3.7
4	1.8	7.8	−1.1	4.9
5	2.1	8.1	−0.2	5.8
6	2.3	8.3	0.5	6.5
7	2.5	8.5	1.0	7.0
8	2.6	8.6	1.5	7.5

Source: Computed from estimates of equation (14). For the variable interest rate columns, the interest rate on time deposits is assumed to rise from 5 percent to 6 percent, and that on commercial paper from 6 percent to 9 percent.

money growth rates are given in the final two columns of Table 12, and reveal more dramatic variations. At the end of eight quarters the real money stock is about 8½ percent lower than it otherwise would have been and the nominal money stock is ½ percent lower.

While the specific inflationary assumptions and calculations are unrealistic, the results in Table 12 indicate that substantial short-run variations in the growth of money demand may accompany changes in inflationary expectations and these in turn may immensely complicate the job of the monetary authorities.

It is also possible to interpret equation (14) as arising from a partial adjustment model rather than from expectational lags. To do this requires modifying the equation defining the desired stock of money—for example, (2) above—to include the anticipated rate of inflation:

$$m^* = f(y, r, \rho^e). \tag{2'}$$

Under this interpretation, however, equation (14) results only in the unlikely event that expectations are perfectly accurate—that is, only if $\rho^e = \Delta P_t/P_{t-1}$. Fortunately, some alternative measures for ρ^e yield a more satisfactory interpretation. In particular, George de Menil has constructed two series of expected price performance from the annual surveys of inflationary expectations conducted by the Survey Research Center of the University of Michigan, that can be used to give a direct measure of expectations.[54] Substituting these in equation (14) leads to substantially smaller price coefficients (see Table 11), which barely border on statistical significance and do not provide strong support for the anticipated inflation variable.

In fact, an alternative view of the stock adjustment process suggests that (14) is misspecified regardless of how ρ^e is measured. In particular, it may be more plausible to combine (2) or (2′) with an adjustment equation specified in nominal terms:

$$(3'') \qquad \ln M_t - \ln M_{t-1} = \gamma \, (\ln M_t^* - \ln M_{t-1}),$$

where $M_t^* = P_t m_t^*$. If this is done the following equation results:

$$(15) \quad \ln (M_t/P_t) = a + b \ln y_t + c \ln (M_{t-1}/P_t) + d \ln RTD_t + e \ln RCP_t + f\rho^e.$$

The major difference between (14) and (15) is the deflator for the lagged nominal stock of money. Equation (15) uses the current price level while (14) uses the lagged price level. Within the context of the stock adjustment model, equation (14) thus implies that any reduction of the real value of the lagged nominal money stock due to rising prices is subject to immediate adjustment, while equation (15) views it as subject to partial or lagged adjustment.

When (15) was estimated with each of the three possible measures for ρ^e, it never yielded a significant coefficient for the ρ^e.[55] At least under the stock adjustment interpretation, then, this suggests that misspecification of equation (14) led to a spurious effect of ρ^e.[56] Under the expectational lag hypothesis, (14) is the proper specification.[57]

The expectational version can be investigated further with Almon distributed lags. Among other things, this technique has the virtue of getting

54. The series are denoted de Menil I and de Menil II here, and their construction is described in G. de Menil, "Rationality in Popular Price Expectations" (Princeton University, August 1973; processed).

55. Equation (15) without ρ^e yielded essentially the same results as equation (4).

56. If the correct hypothesis is (15) with $f = 0$ and one estimates (14), one would expect to find the coefficient f in (14) to be roughly equal and opposite in magnitude to the coefficient c. This is so since $c \ln M_{t-1}/P_t = c \ln M_{t-1}/P_{t-1} - c \ln P_t/P_{t-1}$. Row 1 in Table 11 suggests that this is indeed the case.

57. One finding that is invariant to whether (14) or (15) is correct is the result contained in Table 12. While the numbers are slightly different for (15), the basic story told by that table holds.

Table 13. Coefficients Showing Effect on Equation (16), of Three Alternative Measures of Price Expectations

Measure	Income	Interest rate: Time deposits	Interest rate: Commercial paper	Price level	R^2	Standard error	ρ
Equation (17)	0.693 (16.7)	−0.157 (8.9)	−0.062 (4.8)	−1.911[a] (2.1)	0.996	0.0044	0.84
de Menil I	0.652 (17.6)	−0.144 (9.1)	−0.066 (5.1)	−0.088 (1.1)	0.995	0.0046	0.81
de Menil II	0.641 (17.9)	−0.138 (8.9)	−0.064 (5.2)	−0.257 (2.1)	0.995	0.0045	0.80

Sources: Row 1 gives the results of estimating equation (16), as derived in the text, with Almon distributed lags (equation 17). In rows 2 and 3, direct measures of price expectations from de Menil (cited in Table 11), are substituted in equation (16). For other data sources and definitions, see appendix.

a. Individual coefficients are as follows:

	Lag 0	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
Coefficient	−0.607 (3.2)	−0.440 (2.2)	−0.311 (1.4)	−0.222 (1.0)	−0.172 (1.0)	−0.160 (1.1)

the lagged money stock out of the equation and removing the possible statistical artifact just cited. The relevant estimating equation is

$$(16)\quad \ln m_t = k + \sum_{i=0}^{n_1} w_i \ln y_{t-i} + \sum_{i=0}^{n_2} w_i' \ln RTD_{t-i} + \sum_{i=0}^{n_3} w_i'' \ln RCP_{t-i} + b\rho^e,$$

in which expected inflation can be expressed by either of the de Menil measures or by

$$(17)\quad \rho^e = \sum_{i=0}^{n_4} w_i''' \ln (P_{t-i}/P_{t-i-1}).$$

The various results are given in Table 13.[58] Only one of the two equations using the direct measures has a statistically significant price effect, and even that effect is much smaller than that yielded by the distributed lag proxy—that is, equation (17). This latter variable seems to work reasonably well; it produces a sensible dynamic adjustment pattern (shown in

58. To conserve space, except for the price variable from (17), I have reported only the sum of the lag coefficients. The individual coefficients, however, were extremely close to those reported in Table 8. The same lag lengths and polynomial degrees were used in Tables 8 and 13.

Table 13, note *a*) in which the length of the lag for past rates of inflation is slightly shorter than that for interest rates and considerably shorter than the income lag.[59]

Taken together, these results are a mixed bag. Under the expectations view, some case emerges for including a measure of expected inflation in the demand for money. On the other hand the partial adjustment view, at least as amended, suggests that this case may rest merely on a statistical curiosity. The reader should feel free to indulge his own prejudices.

THE APPROPRIATE SCALE VARIABLE: INCOME OR WEALTH?

An issue that has been extensively examined in the literature is whether income or wealth (or perhaps permanent income) is the appropriate scale variable. Laidler has reviewed this literature and concludes that the evidence favors wealth. Citing work of Meltzer, he suggests that once wealth is included, income has little to explain. Furthermore, he reports work of Brunner and Meltzer that suggests that the wealth variable has superior predictive ability.[60] Nonetheless, numerous writers continue to follow the transactions approach, which focuses on income as the primary scale variable.

The evidence cited by Laidler is based on long-term annual data while recent writings following the transactions approach have tended to be concerned with a shorter term. Whatever the merits of Laidler's evidence in the long-term context, the conclusions do not necessarily apply in explaining the short-run demand for money with quarterly data, and their robustness should be examined.

While the transactions approach emphasizes income, it allows room for a wealth variable since some transactions are obviously associated with portfolio shifts related to total wealth. Unfortunately, a good measure of such transactions is difficult to obtain. An attempt to use the value of stock (equity) transactions had only limited success.[61] Another possibility is to add the change in net worth to the variables in the demand for money,

59. At least one vaguely similar equation that has been reported in the literature has the same feature. See Shapiro, "Inflation, Lags, and the Demand for Money."

60. See Laidler, *Demand for Money*, Chap. 8, and the references cited therein, pp. 121, 123.

61. See Modigliani, Rasche, and Cooper, "Central Bank Policy."

Table 14. Estimates of the Money Demand Equation with Alternative Wealth and Income Variables

Equation (4) variant	Money lagged	Interest rate: Time deposits	Interest rate: Commercial paper	Income	Wealth	Change in wealth	ρ	R^2
1	0.920 (25.4)	−0.027 (2.5)	−0.015 (4.2)	...	0.104 (3.9)	...	0.52	0.995
2	0.986 (30.7)	−0.005 (0.5)	−0.010 (2.9)	...	0.040 (1.5)	0.201 (3.2)	0.39	0.995
3	0.801 (12.5)	−0.031 (2.7)	−0.014 (4.1)	0.139 (3.6)	...	0.160 (2.9)	0.35	0.996
4	0.729 (11.4)	−0.049 (4.2)	−0.018 (5.7)	0.165 (3.8)	0.032 (1.1)	...	0.43	0.995
5	0.801 (12.5)	−0.031 (2.5)	−0.014 (4.1)	0.140 (3.3)	−0.001 (0.04)	0.161 (2.7)	0.35	0.996

Source: Derived from variants of the basic money demand equation (4). For data sources and definitions, see appendix.

thus allowing money holdings to absorb an arbitrary fraction of initial allocations of new wealth.[62]

Table 14 reports the results of estimating several variants of the basic equation. The first substitutes a measure of net worth for the income variable while the second uses both net worth and its change. The next two equations use the income variable and one of the net worth measures while the last equation utilizes all three. Several findings are worth emphasizing.

First, without an income variable the speed of adjustment becomes unreasonably low. Second, income and the change in net worth both achieve statistical significance when they appear in the same equation, suggesting that transactions on wealth account may well be important. Finally, unlike the results cited above, the level of net worth is unimportant when used with income alone while the latter retains its significance. When all three variables are used, the level effect of net worth is obliterated.

The predictive ability of the various equations is reported in Table 15,

62. This suggestion has been made by William C. Brainard and James Tobin, "Pitfalls in Financial Model Building," in American Economic Association, *Papers and Proceedings of the Eightieth Annual Meeting, 1967* (*American Economic Review*, Vol. 58, May 1968), pp. 99–122. Brainard and Tobin actually specify a demand-for-money equation in the context of a complete balance sheet, so they express money as a fraction of wealth as a function of interest rates and income. Furthermore, they suggest decomposing the change in net worth into new saving and capital gains since the source of the change in wealth may affect asset choice.

Table 15. Root Mean-Squared Errors for Extrapolations with Wealth Variables, Alternative Sample Periods Ending with 1961 through 1971

	Income and change in wealth			*Wealth only*		
	Ex post			*Ex post*		
End point[a]	*Full*	*Four-quarter*	*Sample period*	*Full*	*Four-quarter*	*Sample period*
1961	9.84	2.75	0.49	17.57	3.53	0.93
1962	5.89	0.96	0.85	16.38	2.42	1.40
1963	2.68	0.84	0.95	11.00	1.17	1.95
1964	1.53	0.72	0.99	8.21	0.65	2.16
1965	1.67	1.10	0.94	5.47	0.73	2.24
1966	1.95	0.41	1.02	7.10	2.36	2.19
1967	2.35	0.94	1.03	5.77	2.16	2.32
1968	1.67	1.04	1.00	3.49	0.50	2.34
1969	1.81	2.35	1.03	4.67	4.57	2.38
1970	1.81	0.90	1.08	1.57	0.81	2.50
1971	1.62	1.62	1.10	1.30	1.30	2.53

Source: Equations 1 and 3 of Table 14, dynamically simulated.
a. See Table 2, note *a*.

reflecting the results of dynamic simulations of the specifications embodied in equations (1) and (3) of Table 14. The results of using the wealth variable alone in level form are distinctly inferior to the original equation (4) (see Table 2) both for extrapolations and within the sample period. When the variable reflecting change in wealth is added in equation (4), the results are somewhat more mixed, but the original equation is still to be preferred on its ex post performance.

On balance, then, at least for quarterly data, use of an income variable in the demand-for-money equation seems eminently sensible. A variable reflecting the change in wealth slightly improves the explanatory power of the equation but slightly worsens its predictive ability.[63]

63. This conclusion should be tempered for two reasons. For one, the quality of the quarterly net worth data is suspect. In addition, as defined, net worth includes capital gains on equities that should probably be excluded or at least separated out. Along these lines Bosworth and Duesenberry have successfully used a variable defined as net acquisition of financial assets in equations explaining household liquid assets of various types. See Barry Bosworth and James Duesenberry, "A Flow-of-Funds Model and Its Implications," in Federal Reserve Bank of Boston, *Proceedings of the Monetary Conference, 1973* (FRBB, 1973).

Table 16. Coefficients for Alternative Interest Rate Variables in the Money Demand Equation

			Interest rate						
Equation	*Income*	*Money lagged*	*Time deposits*	*Commercial paper*	*Treasury bills*	*Saving, weighted average*	*Corporate bonds*	ρ	R^2
1	0.200	0.691	−0.049	...	−0.012	...	...	0.51	0.995
	(5.1)	(10.1)	(4.0)		(4.3)			(5.4)	
2	0.193	0.717	−0.045	−0.019	...	...	...	0.41	0.995
	(5.3)	(11.5)	(4.0)	(6.0)				(4.1)	
3	0.192	0.756	...	−0.020	...	−0.068	...	0.39	0.995
	(5.5)	(14.5)		(6.5)		(4.1)		(3.8)	
4	0.214	0.713	...	...	−0.014	−0.080	...	0.49	0.995
	(5.3)	(11.9)			(4.9)	(4.2)		(5.1)	
5	0.049	0.929	...	...	−0.012	...	...	0.49	0.994
	(4.2)	(26.4)			(3.8)			(5.2)	
6	0.053	0.942	...	−0.018	...	...	...	0.41	0.994
	(5.4)	(32.5)		(5.4)				(4.1)	
7	0.189	0.663	−0.051	...	...	...	...	0.69	0.994
	(4.4)	(8.6)	(3.7)					(8.6)	
8	0.210	0.656	...	...	...	−0.086	...	0.74	0.994
	(4.4)	(8.6)				(3.6)		(9.9)	
9	0.061	0.901	...	...	...	...	−0.021	0.63	0.993
	(2.8)	(18.9)					(1.7)	(7.4)	
10	0.205	0.674	−0.049	...	...	...	−0.017	0.66	0.994
	(4.6)	(8.9)	(3.6)				(1.4)	(8.1)	

Source: Derived by estimating equations in log linear form over the sample period 1952:2 to 1972:4 by the Cochrane-Orcutt technique. For specific definitions of the variables, see appendix.

Some Econometric Issues

The previous two sections considered a number of basic problems in the specification of the money demand function. The present section focuses on a somewhat narrower and more technical set of issues and considers in sequence questions (7) through (10) posed at the beginning of the paper.

ALTERNATIVE INTEREST RATES

The original debate over interest rates initially centered on whether *any* interest rate mattered. In more recent years, with this question settled, discussion has turned to the appropriate rate or rates to include in the money demand function.[64] The major dispute has concerned short rates (on commercial paper, Treasury bills, and the like) versus longer rates[65] (on corporate bonds, U.S. government obligations, or even equities), although the importance of various types of saving deposit rates (at savings and loan associations, mutual savings banks, and commercial banks) has also been an issue. Most researchers do not confront the question directly, however; they simply use whatever set of interest rates is consistent with the rationale offered for the demand for money. In the context of the transactions approach such a set typically means something like the two rates (*RCP* and *RTD*) used in equation (4) but there are other choices. Table 16 reports the results of some alternative specifications.

The rates considered, in addition to *RCP* and *RTD*, were the Treasury bill rate, *RTB*, a weighted-average saving rate, *RAVG* (combining *RTD*, a savings and loan, and a mutual savings bank rate), and, for completeness, the corporate bond rate (*RCB*). Generally speaking, *RCP* and *RTB* appear interchangeable as do *RTD* and *RAVG*. Although the results are not shown in the table, a weighted average of the savings and loan and mutual savings bank rates, and a separate rate on certificates of deposit, were also tried.

64. See, for example, Laidler, *Demand for Money;* Tong H. Lee, "Alternative Interest Rates and the Demand for Money: The Empirical Evidence," *American Economic Review*, Vol. 57 (December 1967), pp. 1168–81; and comments by Harvey Galper and Michael J. Hamburger, *American Economic Review*, Vol. 59 (June 1969), pp. 401–07, and 407–12, respectively.

65. Hamburger, in "Demand for Money in 1971," has been one main proponent of the longer rates, while Laidler, in *Demand for Money*, has suggested that the appropriate rate may depend on the definition of money.

Table 17. Coefficients for Alternative Measures of the Money Stock in the Money Demand Equation

			Interest rate				
Measure	*Income*	*Money lagged*	*Time deposits*	*Commercial paper*	R^2	*Standard error*	ρ
Last month of quarter	0.207	0.706	−0.047	−0.023	0.996	0.0043	0.38
	(5.6)	(11.4)	(4.1)	(7.6)			
Two-month average centered on end of quarter	0.226	0.677	−0.051	−0.024	0.996	0.0044	0.36
	(6.0)	(10.9)	(4.5)	(7.8)			
Point estimate (flow of funds)	0.224	0.690	−0.047	−0.022	0.992	0.0068	−0.12
	(4.7)	(9.3)	(3.5)	(7.2)			
Quarterly average (equation 4)	0.193	0.717	−0.045	−0.019	0.995	0.0043	0.41
	(5.3)	(11.5)	(4.0)	(6.0)			

Source: Derived from estimating equation (4) with each of the alternative measures.

The former worked, although it did not do as well as *RTD* or *RAVG* and was not significant when used in conjunction with *RTD*. The certificate rate was quite insignificant.

Table 16 makes clear that including a saving deposit rate of any sort increases the speed of adjustment, from much less than 10 percent per quarter to about 20 percent. The corporate bond rate does not work nearly as well as these others, never achieving statistical significance and in some unreported combinations actually yielding a positive coefficient.

On balance then, the specification in (4) seems to work about as well as any other. One potential problem with this for extrapolation purposes is that *RTD* (or *RAVG*) has become more difficult to measure in view of the widespread importance of consumer-type certificates.

TIME UNIT OF MEASUREMENT

The quarterly money series used thus far was obtained by averaging the officially reported monthly data for the three months of the quarter. These monthly data are in turn produced by averaging daily data. Gibson has argued that this procedure is the proper way of characterizing the behavior of the money series over a quarter, and that it provides a reasonable correspondence with the GNP data from the national income accounts.[66] But the money stock series has been measured in many other ways in empirical research on the demand for money: by an average of two months' data centered on the end of the quarter, by data for the last month of the quarter, and by end-of-quarter point estimates (for example, from call report data).

Would substituting one of these definitions change any of the basic results? This question is of particular interest, because Gibson has found that, for the early postwar period, the time unit of measurement may have a pronounced impact on the coefficient of the speed of adjustment.[67]

The results obtained from estimating equation (4) with each of the three alternative measures just noted are reported in Table 17. (The point estimate of the money stock is taken from the flow of funds data, which will be utilized more extensively below.)

66. W. E. Gibson, "Demand and Supply Functions for Money in the United States: Theory and Measurement," *Econometrica*, Vol. 40 (March 1972), pp. 361–70.
67. Ibid.

Table 18. Effect of Alternative Estimating Techniques on Basic Money Demand Equation

			Interest rate				
Estimation technique	*Income*	*Money lagged*	*Time deposits*	*Commercial paper*	R^2	*Standard error*	ρ
Ordinary least squares	0.177	0.751	−0.041	−0.019	0.994	0.0048	...
	(5.4)	(13.6)	(4.1)	(8.2)			
Two-stage least squares	0.216	0.690	−0.052	−0.021	0.994	0.0048	...
	(4.7)	(9.1)	(3.7)	(8.0)			
Two-stage corrected least squares	0.350	0.466	−0.094	−0.021	0.994	0.0048	0.51
	(4.4)	(3.6)	(3.7)	(4.4)			

Source: Same as Table 17.

The three results reported in Table 17 and the results of the original equation (4) are obviously all quite similar to one another.[68] Consequently, Gibson's finding that use of quarterly average data led to a much more rapid speed of adjustment is not borne out when the sample period is extended to the later part of the postwar period.

SERIAL CORRELATION AND SIMULTANEITY

All of the estimates reported so far have been obtained by applying the Cochrane-Orcutt technique for correction of serial correlation in conjunction with ordinary least squares. Thus, problems of simultaneous equation bias have been ignored. In the absence of a complete model, the choice of means to carry out simultaneous equation estimation is somewhat arbitrary. Moreover, a casual interpretation of the evidence suggested that simultaneity bias was not likely to be important but that serial correlation was. These rough impressions were checked by choosing a plausible set of instruments and reestimating equation (4) by ordinary least squares (OLSQ) and by two-stage least squares, both corrected (TSCORC) and not corrected (TSLS) for serial correlation.[69] The results are reported in Table 18.

The results obtained by OLSQ and TSLS are fairly similar to each other and to the estimates given in equation (4). Correcting for both simultaneity and serial correlation (TSCORC) yields a considerably faster speed of adjustment but the long-run elasticities are essentially the same as in equation (4).[70] To see whether this faster speed of adjustment would improve the tracking ability of the equation, I performed the standard set of dynamic simulations described above. I also computed these simulations based on the estimates obtained by OLSQ, with the results reported in Table 19.

68. The only difference of any note is that the residuals based on the point estimate definition do not seem to be serially correlated (all the other estimates of ρ are statistically significantly different from zero).

69. In carrying out the two-stage procedures, income and both interest rate variables were treated as endogenous. For TSLS the instruments used were population, the discount rate, state and local government spending, and the lagged money stock. To ensure consistency for TSCORC, four additional instruments were used—income lagged, both interest rates lagged, and money lagged twice.

70. The original estimates of elasticities for income and for interest rates on time deposits and commercial paper were 0.68, 0.16, and 0.07, respectively, while they are 0.66, 0.18, and 0.04 for the TSCORC equation.

Table 19. Root Mean-Squared Errors for Ordinary and Two-Stage Corrected Least Squares Estimating Techniques, Alternative Sample Periods Ending with 1961 through 1971

	Ordinary least squares			*Two-stage corrected least squares*		
		Ex post			*Ex post*	
End points[a]	*Sample period*	*Four-quarter*	*Full*	*Sample period*	*Four-quarter*	*Full*
1961	1.12	1.33	6.37	0.81	1.87	3.32
1962	1.19	1.98	5.39	0.86	1.21	1.57
1963	1.35	1.86	2.12	0.91	0.45	2.61
1964	1.46	0.90	1.61	0.92	1.54	3.25
1965	1.44	1.92	1.48	0.85	2.16	1.95
1966	1.50	0.60	2.33	1.11	1.20	2.93
1967	1.47	1.85	2.61	0.99	1.44	2.22
1968	1.46	1.36	1.84	0.99	0.79	1.32
1969	1.46	1.48	1.77	1.03	2.18	1.61
1970	1.48	1.21	0.91	1.11	1.08	1.44
1971	1.52	0.60	0.60	1.11	1.38	1.38

Source: Derived from dynamic simulations using the techniques of Table 18.
a. See Table 2, note *a*.

For OLSQ, the within-sample RMSEs are all about 40 percent larger than the corresponding results in Table 2, thus pointing up the benefits of correcting for serial correlation.[71] The ex post results also favor the original estimates, but by a smaller margin. In six out of the eleven cases, the within-sample results with TSCORC are actually better than the original. However, the ex post extrapolations distinctly favor the original estimates on balance.

An alternative specification of the money demand function also sheds some light on the simultaneity question. The money demand function can be inverted to put an interest rate on the left-hand side, relegating the money variable to the right-hand side.[72] Among the interest rates used

71. A comparison of equation (4) to the OLSQ result in Table 18 indicates that the standard errors tend to be understated if serial correlation is ignored. The original standard errors are themselves somewhat understated since I have not accounted for the presence of the lagged dependent variable. See J. P. Cooper, "Asymptotic Covariance Matrix of Procedures for Linear Regression in the Presence of First-Order Autoregressive Disturbances," *Econometrica*, Vol. 40 (March 1972), pp. 305–10. Cooper presents some formulas for making the appropriate correction, which for equation (4) yields roughly a 30 percent increase in all standard errors.

72. This procedure was used in Poole, "Whither Money Demand?"

above, the commercial paper rate seems to be the more natural candidate. The result of inverting equation (4) and estimating by ordinary least squares corrected for serial correlation is as follows:

$$(18)\quad \ln RCP = \underset{(1.1)}{-6.75} - \underset{(3.5)}{9.128} \ln m + \underset{(3.0)}{7.319} \ln m_{-1} + \underset{(2.5)}{2.761} \ln y - \underset{(0.8)}{0.280} \ln RTD.$$

$$R^2 = 0.928;\ \text{standard error} = 0.116;\ \rho = 0.82.$$

If one reinverts (18), an equation rather different from (4) results.[73] In particular, the elasticities are 1.53 for income, 0.55 for *RCP*, and 0.15 for *RTD*. Only the last estimate is even close to what was previously obtained.

Surprisingly enough, given the results of Table 18, the source of the discrepancy turns out to be the existence of rather strong simultaneous equations bias in (18). That this is the case can be seen by reestimating (18) by TSCORC, which yields[74]

$$(19)\quad \ln RCP = \underset{(2.1)}{17.77} - \underset{(4.7)}{30.338} \ln m + \underset{(2.2)}{12.329} \ln m_{-1} + \underset{(3.7)}{12.197} \ln y - \underset{(3.0)}{3.135} \ln RTD.$$

$$R^2 = 0.853;\ \text{standard error} = 0.165;\ \rho = 0.54.$$

The results of (19) are dramatically different from (18) and in fact much more in line with (4). The implied elasticities are 0.68 for income, 0.17 for *RTD*, and 0.06 for *RCP*, virtually identical to those obtained initially. The major difference between (19) and (4) is that the speed of adjustment of (19) is considerably faster—roughly 60 percent per quarter, which is even faster than the corresponding result in Table 18.

On balance, it appears important to correct for serial correlation and probably for simultaneous equations bias as well, especially if an interest rate is the dependent variable. One virtue of the TSCORC estimates is that they produce substantially faster speeds of adjustment. On a partial adjustment view, this result seems desirable, but it did not appear particularly to improve the tracking ability of the equation. It might, however, make a greater difference in the context of a complete econometric model. Finally,

73. Poole, ibid., found a similar discrepancy.

74. The instruments are the same as described in note 69 above.

while it might have been desirable to use simultaneous equations techniques throughout this analysis, the generally comparable performance of the original and TSCORC estimates suggests that the results would not be qualitatively affected by such a procedure.

HOMOGENEITY WITH RESPECT TO PRICES AND POPULATION

In the demand functions considered throughout this paper real money holdings have been assumed to be a function of real GNP. Although some writers have used nominal magnitudes, the specification in real terms is the most common form used in empirical research and is the one suggested by economic theory. For example, under the simplest Baumol-Tobin formulation, money holdings are given by

$$M = (kY/2r)^{\frac{1}{2}}, \tag{20}$$

where k is a fixed charge per transaction. Dividing both sides of (20) by the price level yields

$$\frac{M}{P} = \left[\frac{k}{P} \cdot \frac{Y}{P}/2r\right]^{\frac{1}{2}}, \tag{21}$$

or

$$m = (k'y/2r)^{\frac{1}{2}}, \tag{22}$$

where k' is a transactions cost in real terms. Assuming k' is constant yields the type of specification employed in this paper—that is, an equation of the form

$$\ln m = a + b \ln y + c \ln r. \tag{23}$$

While (20) implies that the appropriate specification is in real terms, it says less about whether deflation by population is required. Indeed, strictly speaking, one cannot aggregate (20) in any simple way. Rather, the distribution of income needs to be taken into account, which suggests that some features of the income distribution might be important variables in the money demand function and that in the aggregate either real income or real income per capita may not be strictly appropriate variables. However, in the simplified situation in which each individual has the same income, aggregation of (20) is possible, and that case does imply that real per capita money holdings become a function of real per capita income.

As an empirical matter, the appropriateness of each type of deflation can be tested simply. For prices one should estimate an equation of the form

$$\ln m = a + b \ln y + c \ln r + d \ln P, \tag{24}$$

and test the hypothesis that $d = 0$. For population, one estimates

$$\ln m = a + b \ln y + c \ln r + d \ln (POP). \tag{25}$$

If per capita deflation is appropriate one should be able to accept the hypothesis that $\hat{d} = 1 - \hat{b}$ and that $\hat{d}$ is significantly different from zero. It should be noted that these latter tests ignore the problem of income distribution and simply compare the merits of two approximations—using real income or real income per capita.

Versions of equations (24) and (25) based on the detailed specification of equation (4) are reported below.

$$\begin{aligned} \ln m = \underset{(1.6)}{0.272} &+ \underset{(5.3)}{0.193} \ln y - \underset{(5.9)}{0.019} \ln RCP - \underset{(3.8)}{0.045} \ln RTD \\ &+ \underset{(11.0)}{0.717} \ln m_{-1} + \underset{(0.008)}{0.0017} \ln P \end{aligned} \tag{24'}$$

$$\begin{aligned} \ln m = \underset{(1.8)}{0.820} &+ \underset{(5.1)}{0.222} \ln y - \underset{(6.2)}{0.019} \ln RCP - \underset{(2.3)}{0.033} \ln RTD \\ &+ \underset{(11.3)}{0.707} \ln m_{-1} - \underset{(1.2)}{0.133} \ln (POP). \end{aligned} \tag{25'}$$

In (24′) the coefficient of $\ln P$ is insignificantly different from zero so that one cannot reject the hypothesis of unitary price elasticity.[75] In equation (25′), the coefficient of population is insignificantly different from zero. Consequently, unlike deflation by the price level, deflation by population does not seem to be called for.[76]

75. There is an alternative but not quite identical test which involves estimating (24) in nominal terms and testing whether $\hat{d} = 1 - \hat{b}$. When this was done, the hypothesis was accepted, confirming the appropriateness of deflating by the price level.

76. In addition to statistical insignificance of the coefficient of population, one can reject the hypothesis that its coefficient is equal to unity less the coefficient of y and the coefficient of m_{-1}. This latter test is the analog of the test for $\hat{d} = 1 - \hat{b}$ referred to in the text.

Disaggregation Using Flow of Funds Data

A number of results reported in the last two sections suggested the desirability of greater disaggregation of money holdings by type of asset (such as currency and demand deposits). I shall now explore disaggregation with respect to type of holder, using flow of funds data, compiled by the Federal Reserve, that disaggregate money holdings into the following broad categories: households; business; state and local governments; financial sectors; rest of the world; and mail float.

Ideally, each of these components should be analyzed in the context of a complete model of the determination of assets and liabilities for each type of holder, so as to yield a clear picture of the appropriate explanatory variables and permit systematic use of balance sheet constraints. This, however, is a task for an army of econometricians (one has already been mobilized, in fact). Within the scope of this paper, it is possible merely to explore component money holdings with some rough and ready adhockery.

The nature of the venture is clarified by the basic data on money holdings at the end of 1972:[77]

Sector	*Dollars (billions)*	*Percent of total*	*Percent change, 1952–72*
Business (including float)	72.3	27.0	36.6
Household	156.5	58.3	152.3
State and local government	14.6	5.4	102.8
Financial	17.0	6.3	151.5
Rest of the world	7.8	2.9	309.0
All sectors	268.3	100.0	105.0

At that time, 15 percent of money holdings were accounted for by groups other than business and households; for these groups the variables conventionally used in money demand equations may not be appropriate. Furthermore, the composition by sector has changed greatly in the past

77. Source: Board of Governors of the Federal Reserve System, *Flow of Funds Accounts, 1945–1972* (August 1973) (see appendix at the end of this paper). Figures are based on data before rounding.

twenty years. In particular, the share of business holdings of money has declined steadily from 40 percent in 1952, while households have steadily increased their share from 48 percent. The remaining components have risen in the aggregate but have also exhibited substantial fluctuations.

Money holdings of the different sectors have also moved in diverse ways in the short run as evidenced by the very low and frequently negative simple correlation coefficients for the seasonally adjusted quarterly flows of the different sectors over the period 1952:2 to 1972:4:

Business	*Household*	*State and local government*	*Financial*	*Rest of the world*
	−0.11	−0.03	−0.06	−0.09
		−0.14	0.25	−0.05
			−0.19	0.09
				0.06

This result again suggests that disaggregating by holder should pay off.

Disaggregation will not be a simple matter, however. The first problem lies in the quality of the data. In recent years the Federal Reserve has conducted a survey on the ownership of demand deposits by type of holder.[78] Attempts to reconcile these data with the flow of funds data have revealed a number of discrepancies that raise serious questions about the quality of the flow of funds data in general and the allocation between business and households in particular. Judging by the survey, the flow of funds data understate business holdings and overstate household holdings of money.

Even taking the data at face value, a number of other clues warn that the analysis of sectoral money holdings may be complicated. When the total percentage growth in the various components from the end of 1952 to the end of 1972 is compared with the growth in the transactions variables relevant for each sector, some marked differences emerge. For example, business transactions are nearly three times their 1972 level (if measured by business sales) or three and one-half times (if measured by business output), but business money holdings have increased by less than one-half.

78. For a good description of the survey and a reconciliation with both the conventional money stock data and the flow of funds data, see "Survey of Demand Deposit Ownership," *Federal Reserve Bulletin*, Vol. 57 (June 1971), pp. 456–67.

Table 20. Coefficients for Household Demand for Money[a]

	Interest rate							
Lagged money	*Time deposits*	*Commercial paper*	*GNP*	*Consumer expenditures*	*Change in net worth*	R^2	*Standard error*	ρ
0.736	−0.055	−0.025	0.312	...	...	0.991	0.013	−0.20
(11.5)	(3.3)	(4.5)	(4.4)					
0.784	−0.044	−0.017	0.251	...	0.230	0.992	0.012	−0.27
(12.6)	(2.7)	(2.7)	(3.6)		(2.0)			
0.796	−0.045	−0.021	...	0.249	...	0.992	0.013	−0 15
(12.3)	(2.5)	(3.7)		(3.5)				
0.844	−0.033	−0.013	...	0.187	0.260	0.993	0.013	−0.24
(13.8)	(2.0)	(2.1)		(3.7)	(2.2)			

Sources: Based on flow of funds data from the Board of Governors of the Federal Reserve System. See the appendix for specific information on data used.

a. The period of fit is 1952:2 to 1972:4. The equations are estimated in logarithmic form by ordinary least squares, with a correction for serial correlation, and use the Koyck lag specification.

Similarly, state and local government spending is ten times what it was twenty years earlier, while money holdings have just doubled. For households, transactions as measured by consumption have quadrupled and money holdings are two and one-half times the earlier level. At the very least these numbers suggest that "income" elasticities are dramatically different across sectors. In principle, allowing for such differences is one of the virtues of disaggregating. More importantly, however, this evidence suggests that a simple transactions model (especially if couched in real terms) will have a hard time explaining money holding by business and by state and local governments. With these caveats, I turn to some results.

The sample period for all the estimates to be presented is identical to the one used above—1952:2 to 1972:4. All estimates were obtained by ordinary least squares with a correction for serial correlation, although this was not much of a problem. The equations were estimated in logarithmic form and the lag specification was limited to the Koyck form.[79]

HOUSEHOLD SECTOR

The household sector has the largest share of total money holdings and in many ways is the easiest to explain. Essentially the same type of specification used for aggregate money demand works equally well for the household sector. Some representative results are contained in Table 20.

The first equation is identical in specification to equation (4). Both in-

79. Estimation of Almon distributed lags would have required sacrificing a substantial number of initial observations and would have made comparisons with earlier results difficult.

come and interest elasticities for the household sector exceed those found for total money holdings, with the long-run income elasticity exceeding unity. The change in net worth, as before, also achieves statistical significance. An equally sensible equation is obtained if one substitutes consumption for GNP as the scale variable.[80] On the whole, the results for the household sector are reasonable.

BUSINESS SECTOR

The business sector comes next in the size order of money holdings but here, as anticipated, I met with considerably less success. One typically unsatisfactory result follows:

$$\text{(26)} \quad \ln mb = \underset{(1.4)}{0.359} + \underset{(0.5)}{0.010} \ln SALE + \underset{(18.9)}{0.905} \ln mb_{-1} - \underset{(2.3)}{0.016} \ln RCP.$$

$R^2 = 0.948$; standard error $= 0.014$; $\rho = 0.02$.

This equation is in real terms, deflated by the business deflator of the national income accounts; *SALE* is manufacturing and trade sales.

Unfortunately, the equation produces a transactions variable that is not significant and a speed of adjustment that is unreasonably slow. A number of attempts were made to improve this equation. In particular, I tried a business GNP measure, a certificate of deposit rate, a measure of cash flow, and inventory investment, but none of these variables achieved statistical significance. I also tried a linear functional form but this did not help either. On balance I can only conclude either that the quality of the data makes this a futile exercise or that considerably more ingenuity is needed to explain aggregate business money holdings.[81]

FINANCIAL SECTOR

Next in importance in volume of money holdings comes the financial sector (savings and loan associations, mutual savings banks, and so on).

80. In this instance the consumption deflator is used to deflate all nominal magnitudes.

81. One direction for possible improvement would be to integrate the money holding and trade credit variables. Another problem of unknown proportions is created by the absence of any reliable information on compensating balances. Development of such information would also be a step in the right direction. Finally, the mail float item is included with the business sector. While this is approximately correct, some further work could be done. See the article cited in note 78 above.

Table 21. Coefficients for Financial Sector Demand for Money[a]

Lagged money	*Treasury bill rate*	*Deposits*	*Change in deposits*	*Proxy for outflow of deposits*[b]	R^2	*Standard error*	ρ
0.698	−0.014	0.154	0.514	...	0.995	0.018	0.12
(8.5)	(1.6)	(4.0)	(1.6)				
0.659	−0.016	0.179	1.429	0.066	0.995	0.018	0.04
(8.1)	(1.9)	(4.5)	(2.9)	(2.2)			

a. See sources and note for Table 20. The equations are in undeflated form.
b. Ratio of the Treasury bill rate to the saving deposit rate after 1968:3, and zero before.

For this sector, the appropriate scale variable is a measure of deposit activity. The level of deposits and the change in deposits used jointly worked relatively well. Two such equations using these variables are given in Table 21. The first employs these variables in conjunction with the Treasury bill rate while the second adds a variable designed to capture the anticipated outflow of deposits due to disintermediation. This variable is defined as the ratio of the Treasury bill rate to the saving deposit rate after 1968:3 and zero before. The higher this variable the more financial institutions expect to lose funds through disintermediation and the more liquid they therefore wish to be. This variable obtains the expected positive sign and is statistically significant at the 5 percent level. On the whole, then, money holdings by the financial sector appear to lend themselves to a reasonably straightforward explanation.[82]

STATE AND LOCAL GOVERNMENT SECTOR

The final equation to be considered is for the state and local government sector. From initial examination of the data, this was expected to be a troublesome sector, and indeed it was. As with the business sector, a number of specifications were tried, including several interest rate variables and a budget surplus variable. No fully satisfactory equation ever emerged. A typical equation is

$$(27)\quad \underset{}{\ln msl} = \underset{(0.6)}{0.092} + \underset{(22.1)}{0.946} \ln msl_{-1} + \underset{(0.4)}{0.011} \ln gsl - \underset{(1.0)}{0.022} \ln RCP,$$

$$R^2 = 0.867;\ \text{standard error} = 0.047;\ \rho = -0.10.$$

82. The equations in Table 21 were run in undeflated form.

Table 22. Root Mean-Squared Errors for Aggregate and Component Money Holding Equations, Alternative Sample Periods Ending with 1961 through 1971

	Aggregate equation			*Component equations*		
	Ex post			*Ex post*		
End point[a]	*Full*	*Four-quarter*	*Within-sample*	*Full*	*Four-quarter*	*Within-sample*
1961	7.21	2.07	0.73	18.71	2.60	1.19
1962	4.54	1.28	0.87	17.41	1.03	1.20
1963	2.33	2.04	0.95	14.95	3.03	1.32
1964	3.70	0.67	1.06	3.59	1.03	1.66
1965	3.50	2.93	0.99	2.96	1.15	1.60
1966	2.15	1.08	1.15	3.44	2.03	1.65
1967	2.23	1.80	1.09	4.51	1.40	1.55
1968	2.82	1.59	1.12	6.77	1.98	1.51
1969	2.94	1.86	1.13	5.11	1.54	1.69
1970	3.83	1.99	1.17	6.29	3.20	1.69
1971	3.30	3.30	1.18	3.84	3.84	1.89

Sources: See sources and note for Table 20. The aggregate equation corresponds to the aggregate flow of funds equation in Table 17. The component equations used are (26) for the business sector, (27) for the state and local government sector, the first equation in Table 21 for the financial sector, and the fourth in Table 20 for the household sector. Money holdings for the rest of the world were considered exogenous.
a. See Table 2, note *a*.

where *gsl* is state and local government expenditures.[83] Quite evidently I am unable to provide anything close to a satisfactory explanation for this sector.

OVERVIEW

Taken as a whole, the batting average on disaggregation by holder is .500. Two of the four categories, households and the financial sector, are reasonably well explained; two others, business and state and local government, are not. One would expect that the first two would behave reasonably well in the simulation exercises I have performed, and—sparing the reader the details—this was the case. The remaining two sectors, not surprisingly, did relatively poorly. Table 22 summarizes the performance in the aggregate with the relevant root mean-squared errors for total money holdings. The

83. Equation (27) is in real terms, all nominal variables having been deflated by the state and local expenditure deflator. Estimation without deflation produced slightly better but still unsatisfactory results.

left half of the table corresponds to the aggregate flow of funds equation reported in Table 17, while the right half extrapolates aggregate money holdings based on the equations for the individual components.[84] While the aggregate equation does not uniformly dominate the individual equations, one plainly would not forecast total money holdings by separate use of this particular set of component equations. Nevertheless, despite this tentatively pessimistic finding, the results hold enough promise to warrant withholding a final verdict on this issue. Further research along these lines is clearly in order.

Concluding Remarks

In the process of sequentially examining each of the questions set forth at the beginning of this paper, a considerable amount of information has emerged concerning the nature of the demand for money. This section enumerates the highlights of the findings, attempts to illuminate them by examining velocity, both actual and simulated, under a variety of assumptions, and briefly assesses the demand for money through 1974.

Perhaps most interesting is the apparent sturdiness of a quite conventional formulation of the money demand function, however scrutinized. More particularly, such a function yields sensible interest and income elasticities. The income elasticity appears to be significantly less than unity and can be pinned down reasonably well on the basis of quarterly data. In addition, the conventional equation exhibits no marked instabilities, in either the short run or the long run. Finally, the conventional equation yields a reasonable speed of adjustment to changes in income or interest rates, with patterns and magnitudes of adjustment that are generally similar in the Koyck and Almon specifications.

While the conventional equation performs well, it is nevertheless possible to improve on it in a number of ways. In the first instance disaggregation of M_1 into currency and demand deposits appears desirable from both a structural and a forecasting point of view. Aggregation to the level of M_2,

84. The equations used in these calculations were (26) and (27), the first equation in Table 21, and the fourth in Table 20. Money holdings of the rest of the world were taken as exogenous. Furthermore, the results were made comparable with those reported earlier by reinflating the component forecasts (where necessary) and then deflating by the GNP deflator.

however, is definitely counterproductive. Furthermore, the addition of a number of variables appears to improve the performance of the standard formulation. These include the change in wealth and, possibly, a variable measuring inflation expectations. On the other hand, substitution of wealth for income imposes a marked deterioration in the performance of the equation.

Finally, while the diverse sectoral pattern of movements in money holdings exhibited by the flow of funds data implied some payoff to greater disaggregation, efforts in this direction were only partially successful. The tentative nature of the results suggests that this remains an open issue.

THE BEHAVIOR OF VELOCITY

An empirical money demand function has implications about the behavior of the income velocity of money, $v = y/m$. One important implication, long debated by economists, concerns the sensitivity of v to interest rate changes, which is simply the other side of the coin of the debate concerning the interest elasticity of the demand for money. The results here have reconfirmed the importance of interest rate variables in explaining the demand for money, and their implications for the behavior of velocity help to put their importance in perspective.

The basic money demand function estimated above can be written (in nonlogarithmic form) as

$$m = Ay^a r^b,$$

which yields

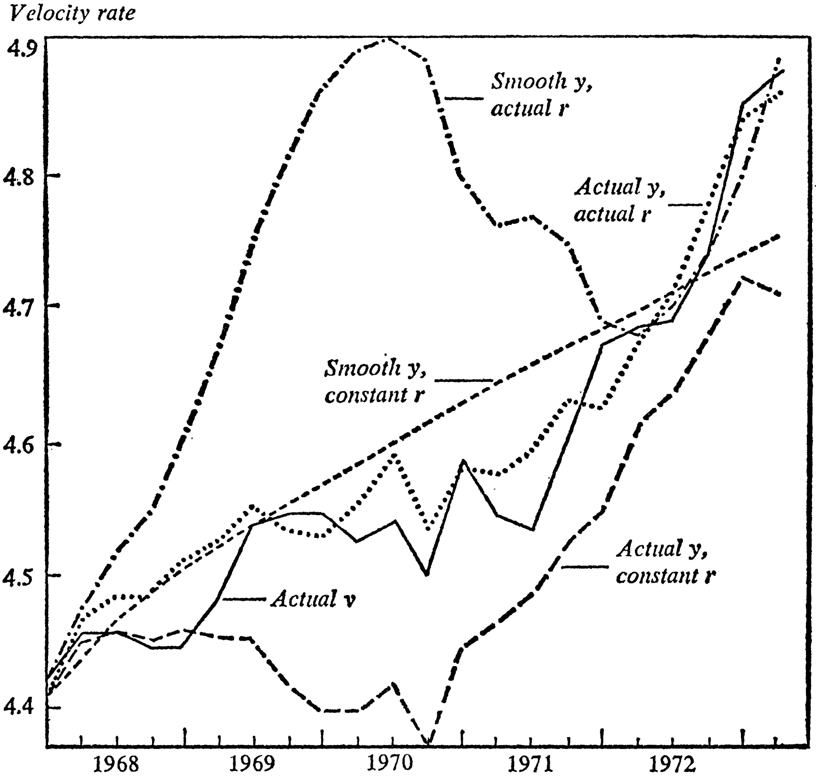
$$v = y/m = y^{1-a}/Ar^b.$$

This equation implies that, with a constant interest rate, velocity will increase at the fraction $(1 - a)$ of the growth rate of y. With a value of a of about 0.7, annual growth in y of 4 percent would lead to a 1.2 percent growth in v. Since 1952, v has actually increased at about 2½ percent per year; the excess over 1.2 reflects the upward trend in interest rates.[85]

While velocity has trended upward, its path has hardly been steady, as the series labeled "actual v" in Figure 2 readily demonstrates for the period 1968:1 to 1973:2. To assess the sensitivity of velocity to alternative paths

85. Velocity (defined on the basis of M_1) has risen from about 2¾ in 1952 to nearly 5 in 1973.

Figure 2. Actual and Simulated Income Velocity of Money under Four Assumptions, Quarterly, 1968–73



Sources: See appendix.

for income and interest rates, I dynamically simulated a version of equation (4) estimated through 1973:2 with four alternative paths. These four paths resulted from combining two assumptions for interest rates with two assumptions for GNP. In particular, interest rates were either assumed to take on their historical values ("actual *r*" in Figure 2) or to remain constant at their 1967:4 values ("constant *r*"). Similarly, for income I either used actual values ("actual *y*") or let it grow smoothly ("smooth *y*") over the period (ending up at the actual value in 1973:2). In each case the dynamic simulation was started in 1968:1.

In comparison with the series labeled "actual *v*," the one labeled "actual *y*, actual *r*" in Figure 2 indicates the tracking ability of the equation with historical data. On the whole, that ability is reasonably satisfactory, but the pattern of errors is interesting.[86] When actual velocity is below that predicted by the equation, some interest rates must be higher and GNP lower than would otherwise be the case, given the money supply. The precise impact on interest rates and GNP depends on the relationship of investment and consumer demand to interest rates and income—the shape of the *IS* curve, a set of vital issues outside the scope of this paper. Nonetheless, to some extent, unusually low velocity is a drag on aggregate economic activity, while unusually high velocity is a stimulus, so long as the Federal Reserve does not fully offset the surprise by changing the stock of money.

By this standard, the low value of velocity early in 1969, a period of excess demand, exerted some anti-inflationary influence. On the other hand, the low velocity readings in the second and third quarters of 1971 could be a factor in the rather weak start of the economic recovery after the 1969–70 recession. In 1971:4 and 1972:1, velocity swung sharply upward and crossed its predicted value; that movement may have reinforced the acceleration of economic activity.

The series "smooth *y*, constant *r*" demonstrates, as indicated above, that velocity will steadily increase with continued growth in income. This latter curve can be compared with the remaining two curves—"smooth *y*, actual *r*" and "actual *y*, constant *r*"—to isolate the impact of fluctuations in interest rates and income, respectively. The historical movement of interest rates produces a strikingly different pattern from the constant interest rate assumption. Similarly, the actual pattern of income yields a velocity series that is markedly different from the steady growth assumption.

In short, velocity can be extremely variable in the short run (quite apart from the residuals in the money demand function) and any policy prescription that does not take this into account may be very misleading.

SOME ILLUSTRATIVE PROJECTIONS

In earlier parts of this paper, I made extensive use of dynamic simulations to examine the forecasting performance of various specifications. In

86. The residuals depend on the starting point of the dynamic simulations, and hence the pattern discussed here could be different for other starting points.

Table 23. Real and Nominal Monetary Growth Required under Three Patterns for Interest Rates on Commercial Paper, 1973:3 through 1974:4

	Pattern 1: moderate decline in interest rates			*Pattern 2: mild decline in interest rates*			*Pattern 3: substantial decline in interest rates*		
		Monetary growth rate (annual percentage)			*Monetary growth rate (annual percentage)*			*Monetary growth rate (annual percentage)*	
Year and quarter	*Interest rate (percent)*	*Real*	*Nominal*	*Interest rate (percent)*	*Real*	*Nominal*	*Interest rate (percent)*	*Real*	*Nominal*
1973:3	9.6	−2.3	5.1	9.6	−2.3	5.1	9.6	−2.3	5.1
4	9.0	−0.2	7.1	9.3	−0.4	6.9	9.0	−0.2	7.1
1974:1	8.5	0.8	7.0	9.0	0.4	6.6	8.3	0.9	7.1
2	8.0	1.3	6.4	8.7	0.8	5.9	7.6	1.6	6.7
3	7.5	1.7	6.6	8.4	1.2	6.1	6.9	2.2	7.1
4	7.0	2.4	7.0	8.1	1.7	6.3	6.2	3.0	7.6

Sources: Extrapolated from a version of text equation (4). The time deposit rate is held constant at its 1973:3 level. See the appendix for specific sources.

view of the reasonably good performance, what can be said about the future behavior of money demand in 1974? One sensible way to approach this problem is to take as given forecasts for real GNP and for the GNP deflator and examine the behavior of money demand for alternative patterns of interest rates. For this purpose I chose the forecast produced by the Michigan quarterly model, which foresees real growth of about 2½ percent and price inflation of 6 percent for the year 1974.[87]

Table 23 sets out the annual percentage rates of growth of the money stock (both in nominal and in real terms) consistent with three alternative patterns for the commercial paper rate.[88] The first panel shows a moderate decline in interest rates, the next a very mild decline in short rates, and the last a more substantial decline. It should be emphasized that these are not forecasts of actual money growth, but rather of the rates of monetary expansion consistent with the assumed values for interest rates.[89] Over the six quarters taken as a whole the three interest rate patterns imply nominal monetary growth rates of from 6 to 6¾ percent, with not much quarter-to-quarter variability after 1973:3.[90] While this finding in no way offers a prescription for monetary policy, it does suggest that extremely low nominal growth rates are not consistent with any plausible expansion of the economy through 1974.

87. See Saul H. Hymans and H. T. Shapiro, "The Economic Outlook at Mid-Year" (University of Michigan, August 1973; processed). This forecast preceded the Arab oil embargo. The quarterly pattern of changes (at annual rates) produced by the model is as follows:

	Quarter					
	1973:3	*1973:4*	*1974:1*	*1974:2*	*1974:3*	*1974:4*
Real GNP	5.0	2.7	1.8	1.3	1.6	3.1
GNP deflator	7.4	7.3	6.2	5.1	4.9	4.6

88. The results are based on extrapolating a version of equation (4) estimated through 1973:2 where the time deposit rate is held at its level for 1973:3 through all succeeding quarters. While this is a convenient assumption, it is also in the nature of a forecast consistent with the assumed patterns of behavior for the commercial paper rate and the impact of interest rate ceilings.

89. The Michigan forecast of interest rates is essentially like the first panel in Table 23. Consequently, any major deviations from this pattern will not be consistent (as far as the Michigan model is concerned) with the assumed price and output behavior.

90. Extrapolations based on the equation including the price expectation term show roughly the same growth over the period as a whole but more quarter-to-quarter variability. For example, the growth rates of money corresponding to panel 1 in the table (in real terms) are as follows: −3.5; −0.8; 0.8; 1.9; 2.3; and 3.0.

APPENDIX

Data Sources

ALL DOLLAR DATA used in this paper are in billions and are seasonally adjusted. The flow data are at annual rates. The interest rate variables are in percentage points and are not seasonally adjusted. Gross national product and related variables are based on the July 1973 revisions of the national income accounts (published in the *Survey of Current Business*) while the flow of funds data are based on the August 1973 revisions. Although readily available in published sources, many of the series used were actually taken from the data deck of the Federal Reserve-MIT-Pennsylvania (FMP) Econometric Model. This was generously supplied by Jared Enzler of the Board of Governors, Federal Reserve System. The flow of funds data needed for this study and helpful comments about their use were supplied by Stephen T. Taylor, also of the Board of Governors.

Currency, demand deposits, time deposits. Taken primarily from the February 1973 issue of the *Federal Reserve Bulletin*. The time deposits series excludes large negotiable certificates of deposit. Except as noted in the text these were measured as quarterly averages of monthly data.

Interest rates. The rates on Treasury bills, commercial paper, corporate bonds, time deposits, savings and loan deposits, and mutual savings bank shares were all taken from the FMP deck. The latter three variables were combined into the averaged variable used in Table 16 by weighting the individual rates. The weights summed to one and are proportional to the quantity of deposits associated with each rate in the previous period. These quantity variables were also used directly in Table 21.

Price indexes. Unless indicated, the nominal money stock is put in real terms by use of the implicit GNP deflator. The exceptions are in the cases where consumption is the scale variable, when the consumption deflator is used; in equation (26), when the business output deflator is used; and (27), where the implicit deflator for state and local government spending is used.

Flow of funds. Seasonally adjusted quarterly flows were cumulated (both forward and backward) starting from the 1970 stock data, to yield adjusted series for the various stocks used.

Comments and Discussion

James Duesenberry: Stephen Goldfeld has written a very fine paper, a thorough piece of work that really moves us ahead in the field. I do have several comments, though they should not be classified as criticisms.

First, I hope that the econometricians among us will note Goldfeld's method of choosing among equations. He does not rely primarily on small differences in R^2s or on the t-statistics of additional variables; instead, he compares the success in forecasting money demand of simulations run on alternate equations. While I know of no formal theory that tells us how to assess such evidence, I feel that the use of this technique is one of the merits of Goldfeld's work. On the whole, Goldfeld has gone about as far as possible in extracting information from this body of aggregate data, short of taking it down to the cellar and beating it with a rubber hose. The next major steps in research in this area should probably try to incorporate information from sources other than time series on the structure of money demand, and to employ Bayesian methods to evaluate the time series data.

With regard to the substance of the paper, I was somewhat disturbed by the results on the business demand for money. The data base used in these disaggregated equations is weak: serious measurement errors arise in the attempt to break down ownership of demand deposits and currency into household and business categories. Moreover, some confusion may arise from the effects of compensating balances, which are included in the measurement of business holdings. I think this is an area in which micro-level data might be used to refine the aggregate equation. However, the success of the demand equation with household data leads one to believe that the failure of the business demand equation may be due to a basic difference in the response of business demand to changes in income and interest rates. That difference may show up in the results of the aggregate equation and may account for the estimated long-run income elasticity of money holdings of 0.68, against a value near unity for the household sector.

The difference between household and business responses may involve threshold effects in businessmen's decisions to economize on cash balances. Short-run interest rates may have to rise considerably in order to induce businesses to incur the set-up costs, in employees and perhaps computer facilities, of working to economize on cash balances. The extent of the business response is also dependent on the size of the money holdings (and therefore of the potential interest gain) of any firm. Similar threshold effects may exist on the down side: once the initial overhead cost has been incurred, businessmen may continue to hold smaller transactions balances even when the interest rate falls slightly. The aggregate equation is looking for prompt responses to changes in interest rates and it is likely to miss some of these delayed business responses. It may hence understate the interest elasticity of the demand for money, and, since the effects of interest and income run in opposite directions, also understate the response to changes in income.

I found the performance of the price expectations variable in the money demand equation puzzling, since it appears to contradict some of the results I have obtained in measuring the effects of expected inflation on saving. Goldfeld obtained significant coefficients for these variables in his equations, though a comparison of the abilities of the equations with and without this variable to simulate the demand for money in future periods resulted in a tie. The belief that price expectations influence the demand for money is based on the assumption that, if prices are expected to rise, people will trade their money holdings for physical assets. Except perhaps in the most recent period, saving typically responds to inflationary expectations in the opposite way: people tend to save more in anticipation of higher prices in order to prevent a deterioration in their living standards.

Since Goldfeld referred to the Bosworth-Duesenberry model in his section on the use of wealth as a scale variable, I might discuss our results briefly. We found that the net acquisition of financial assets (from the flow of funds data) does help to explain portfolio behavior. This variable essentially represents the change in wealth due to accumulation without regard to the change in the price of assets, and it might be substituted for the full change-in-wealth variable, part of which is due to capital gains.

From a theoretical viewpoint, a consideration of cross-sections suggests that wealth should play a role in the demand equation. People with a small net worth face a liquidity constraint because much of their wealth is tied up in physical assets. After making their downpayments on homes, cars, and so forth, they have little liquidity. Wealthier people, having taken care of these needs, are in a better position to trade off between more and less liquid financial assets. But the greater the collinearity between income and wealth, the less important the choice between the two scale variables. Moreover, threshold effects may be involved, in that some increases in wealth, such as those resulting from a rise in the stock market, may not affect the liquidity actions of very wealthy people on the margin.

Finally, the success of the currency equation bothers me. Regressions of the stock of currency on interest rates frequently get significant results, but they do not explain the reasons for the large volume of currency holdings. The denominations of these holdings are not those needed to run a news stand. Much of large-denomination currency must be associated with hoarding and illegal activities. Historically, the volume of currency jumped suddenly in the Second World War and again during the Korean War. Then it remained flat until a renewed rise began in the early sixties; during the fifties, the wartime accumulation of currency gradually came out into the open. At any rate, I tend to be suspicious of dandy equations about currency—and this makes me suspicious of dandy equations about other things, including aggregate money holdings.

As a final point, I would like to see a few results on first differences from the simulations. Though the correction for serial correlation probably introduces technical problems here, the first differences could throw additional light on how precisely these equations are predicting money demand.

William Poole: This paper is a very useful study of a large number of the unsettled issues in this field. Instead of paying all of the compliments that Goldfeld deserves, I would like to focus my discussion on the few areas in which I have reservations about the methodology.

First, with regard to the question of pinpointing the income elasticity of the money stock, Goldfeld is quite right in citing my error in using R^2s to compare equations with different dependent variables. However, I do not think that a comparison of the standard errors of the equations shown in Table 1 justifies his confidence in the accuracy of his income elasticity estimate. The equation with the elasticity of income constrained to 1.0 has a standard error only about 6 percent greater than that of the equation with the elasticity constrained to his estimated value of 0.68. This indicates that the use of a Bayesian technique of combining the sample evidence with a very diffuse prior distribution centered on unity would result in a flat distribution of income elasticity estimates over a very wide range.

A second point concerns the choice of the dependent variable. Several arguments favor the use of the interest rate, rather than money, on the left-hand side. The most widely recognized of these is that the interest rate is "more endogenous" than the money stock. I would stress that the presence of errors in variables may provide an even more compelling reason for running the equation in this form. Aside from the obvious problem of measurement error, there may be errors in the interest rate variables that arise from their use as proxies for other interest rates, since nobody knows which rate affects behavior the most. I won't care about the commercial paper rate if I am never going to buy commercial paper, but I may respond to changes in this variable if it is highly correlated with a rate I do care about. As a technical aside, I would point out that serious statistical problems could result from the correction for serial correlation in the presence of measurement errors in the variables.

I have my doubts about the inclusion of the change in prices in the money demand equation. I would argue that the cost of holding cash balances is equal to the real rate of interest plus the rate of inflation (or of expected inflation), and the sum of the two components is presumably measured by the nominal interest rate. The only situation in which the nominal interest rate would fail to measure this cost would be a severe deflation, when the nominal rate of interest could not become negative whatever the expected rate of deflation.

On the choice of the appropriate interest rate for the money equation, I would like to make one point that favors the long-term rate. Since the difference in goodness of fit between the equations is small, and since the R^2s are very high in both, using the long rate may be preferable because it is more appropriate in the context of a complete econometric model. The long-term rate enters directly into the investment equation, so its use in the money demand equation may eliminate the need for the model to grapple with the term structure of interest rates.

The disaggregated equations for currency and demand deposit holdings raise many questions in my mind. The results of the two equations with income as a scale variable show the income elasticity of currency holdings to be much higher than that of demand deposit holdings. Given the long-run increase in income, the currency-deposit ratio should therefore have risen, and indeed it did display a gradual updrift between 1920 and 1950. Thereafter, the ratio does not exhibit the continued rise that would be expected on the basis of the strong increase in real income in the postwar period. The equations attribute the puzzle to increases in the interest rate, since holdings of currency show a higher interest elasticity than do demand deposits. But that difference in interest elasticities is not a plausible result.

The separate equations for currency and demand deposits also raise an issue about the strategy of monetary policy. They imply that the Federal Reserve might just as well pursue its monetary policy objectives by setting a target for the quantity of currency and then adjusting the reserve base as necessary to provide whatever volume of demand deposits the public desired in conjunction with the targeted volume of currency. I doubt, however, that any economist would believe that such control through currency alone would represent an adequate way for monetary policy to influence GNP.

My final point concerns the flow of funds data. The measurement errors in the flow of funds accounts may be quite large. Estimates of demand deposits and currency held by the various sectors on December 31, 1972, were about 6 percent less than the estimate of the sum of currency outstanding and the deposit liabilities of the commercial banks. The discrepancy represents an unknown combination of measurement error and mail float. Also, the results obtained by disaggregating holdings of households, corporations, and so forth differ considerably from the results of the new survey of demand deposit ownership. For these reasons, I doubt that work with the disaggregated data can prove fruitful; the data are simply not yet robust enough to stand up to statistical regression techniques.

General Discussion

Several participants commented on the statistical problems of Goldfeld's estimate of the demand for money that uses the stock of money as the dependent variable. William Gibson remarked that a complete analysis of the behavior of the money stock should deal explicitly with responses of the money supply through the actions of the Federal Reserve. He advocated using simultaneous estimation techniques to separate the effect of interest rates on money supply and demand. On some assumptions about the determination of the money supply, Robert Hall argued, the estimation of the demand function by ordinary least squares could give seriously misleading results. In support of Hall's contention, Franco Modigliani cited the results he obtained from estimating the demand equation as part of a system of simultaneous equations. These estimates show a substantially larger interest elasticity of demand and a faster speed of adjustment to changes in income and interest rates than estimates obtained by ordinary least squares. Though Goldfeld's results using two-stage least squares move in the direction of the results from simultaneous equations, Modigliani felt that the issue had not been completely resolved.

Hall saw a possibility of obtaining reliable estimates of the relationship among income, interest rates, and the money supply by ordinary least squares if the money supply could indeed be considered exogenous to the model. In that case, income or interest rates, rather than the money stock, would appear on the left side in an equation like Goldfeld's. Hall pointed to work by Christopher Sims which supports the view that the money supply can indeed be considered exogenous. Thomas Sargent agreed that Sims' exogeneity tests show that it is inappropriate to treat money as endogenous and income as exogenous in estimating a demand schedule for money. Lawrence Klein, on the other hand, held that tests that rely on lagging and leading correlations of money and income are insufficient to determine the exogeneity of the money stock. In particular, Klein argued, the actual money supply resulting in the very short run depends not only on Federal Reserve actions in targeting the money supply but also on income and interest rate conditions. Stephen Magee added that international money flows present a further argument in favor of considering the money stock endogenous. Sargent maintained that Sims' theorems prove that his exogeneity test is valid, and that the mere fact that the Fed has pursued an interest rate target is insufficient to show that the money stock was not statistically exogenous, since the Fed revised its target interest rate quite often and in a complicated way.

In response to these criticisms, Goldfeld agreed that the issue was unresolved. But he emphasized that his results with two-stage estimation using the interest rate on the left-hand side essentially duplicated those of the equation with money on the left side, and also agreed with the ordinary least squares estimates using money as dependent. The ordinary least squares estimates with the interest rate dependent differed sharply from the other three sets. He saw this 3-to-1 "vote" as offering some support for his approach.

Modigliani expressed his admiration for the thoroughness of the tests Goldfeld made. He was reassured that the results generally confirmed sensible theoretical views—in particular, that income is the proper transactions variable, that the demand for money is stable and homogeneous in prices, and that the income elasticity of the demand for money is less than one. In addition, he approved the use of the short-term rate rather than the long-term rate; in response to a suggestion by Poole that firms' decisions to set up cash management programs might depend on anticipations of the short rate over a longer horizon, Modigliani suggested that the most relevant rate might be a combination of the short-term rate and an interest expectations variable.

On the question of the nature of the lags in the equation, Modigliani emphasized the need to distinguish between the partial adjustment and the expectations arguments for the use of distributed lags. He held that the lagged adjustment to the interest rate reflects the process of learning to economize on cash balances in the face of higher interest rates. It then takes time to adjust cash balances to the desired level. For this reason, Modigliani did expect the differences in lag structures and adjustment speeds for the interest rate and income variables that Goldfeld obtained in his Almon equations.

Interpreting the lag as a partial adjustment resolved the mystery of the large impact of price change on the demand for money, in Modigliani's judgment. The desired stock of money is determined in real terms, but the gradual adjustment applies to the nominal money stock. That adjustment will take time when prices rise as well as when real incomes or interest rates change. Temporarily—but only temporarily—rising prices can hold down the real demand for money through the partial adjustment. This view is confirmed by Goldfeld's finding that the deflation of lagged money by current (rather than lagged) prices eliminates the significance of the inflation variables.

Modigliani was puzzled by the results of Goldfeld's test of the effects of population growth on money demand. The economies of scale in the transactions model should apply to income growth *per capita;* that portion of aggregate income growth that reflects just keeping up with population should raise the demand for money proportionately. Goldfeld's empirical finding disagrees—his only counter-theoretical result, in Modigliani's judgment. Arthur Okun suggested that the proper population variable to capture the economies of scale might be the number of households rather than of persons, and that the latter may be a poor proxy for the former during the postwar period.

Several participants commented on the price expectations variable in the money demand equation. Gibson did not see why the expected rate of inflation should influence the demand for money except through its influence on the nominal rate of interest. Okun agreed with Gibson's remarks (and with Poole's earlier comments) insofar as they applied to an Irving Fisher world in which the full effect of anticipated inflation would be reflected in nominal interest rates. But Okun and William Brainard suggested that direct substitution of goods for money would alter the story. As Brainard explained, some people might be more concerned with the allocation of their wealth between physical and financial assets than with allocation among various forms of financial assets. If this were the case, the rate of increase of commodity prices would affect the demand for money, whether or not the nominal interest rate captured fully the effects of price expectations. Moreover, even with no direct substitution between money and goods, the demand for money could be influenced by price expectations if for any reason nominal interest rates did not adjust fully to changes in price expectations.

David Fand noted that the extent to which disaggregation is desirable depends on the reasons for wanting to know a particular monetary total. Though it might be possible to get a better estimate of the sum of currency and demand deposit holdings by estimating these holdings separately, someone interested in a different monetary total might want to disaggregate differently, or not at all. Poole questioned the conclusiveness of Goldfeld's evidence in favor of the disaggregation of money into currency and demand deposits, pointing out that the disaggregated equations using GNP showed lower ex post mean-squared errors in only five of the eleven cases.

Brainard offered a final comment about Goldfeld's tests. He applauded Goldfeld's use of out-of-sample forecasts, but he thought the ex post forecasts should give more weight to the accuracy of the estimates for the early quarters after the sample period—say, the first four—than to later ones. He also wondered how well the equations performed in the ex post simulations as compared to simple naive autoregressive models.