

**THE LUXURY AXIOM, THE WEALTH PARADOX,
AND CHILD LABOR**

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Basu and Van (1998) present a fundamental framework of child labor with two important axioms: *the luxury axiom* and *the substitution axiom*. A number of empirical studies, however, reveal a “wealth paradox”. The current paper has two aims. First, it develops a model that provides an explanation for “the wealth paradox” in light of *the luxury axiom* and *the substitution axiom*. Second, it helps understand the relationship between *the luxury axiom* and *the substitution axiom*.

Keywords: Child Labor, Luxury Axiom, Wealth Paradox, Substitution Axiom

JEL classification: D10, J22

1. INTRODUCTION

Beginning with Basu and Van (1998), the past decade has witnessed a proliferation of the literature on child labor.¹ In their path breaking contribution, Basu and Van (1998) present a fundamental framework of child labor with two important axioms: the luxury axiom and the substitution axiom. Basu and Van (1998, p. 416) define the luxury axiom as “a family will send the children to the labor market only if the family’s income from non-child-labor sources drops very low,” and the substitution axiom as “from a firm’s point of view, child labor and adult labor are substitutes.”

A number of empirical studies, however, reveal some puzzles. For example, Hunt (1973, 1986) and Nardinelli (1990) find that in Britain in the nineteenth century, despite

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¹ For example, see the surveys by Cigno and Rosati (2005) and Edmonds (2008). Indeed, child labor has long been an important social and economic phenomenon. For example, back in 1861, 36.9 percent of boys and 20.5 percent of girls in the 10-14 age group in England and Wales were laborers (Basu, 1999). Child labor is still prevalent in many developing countries of the present time (Basu, 1999). It is reported that many children in rural India and Brazil begin to contribute to farm work at the age of five or six and are sizable contributors by age twelve (Becker, 1991).

the large regional variations in wages, there is no correlation between the general wages of adult males and children's labor market participation rates. Bhalotra and Heady (2003) show that in rural Pakistan and Ghana in the 1990s, the children of land-rich households are often more likely to work than those of land-poor households. As their finding appears to be in contradiction with the luxury axiom, Bhalotra and Heady (2003) name it "the wealth paradox". Moreover, similar empirical findings to "the wealth paradox" are obtained in other developing countries (Kambhampati and Rajan, 2006; Dumas, 2007; and Kruger, 2007).

This paper attempts to develop a simple model that sheds light on "the wealth paradox". We incorporate four crucial ingredients, child leisure, child labor, education, and a household's subsistence constraints, in a unified framework. Indeed, while all of these ingredients are identified as being essential to the understanding of child labor in the literature, to our best knowledge, no attempt has been made to consider them in the same framework. For example, Basu and Van (1998) abstract from the consideration of children's education. Much subsequent literature (e.g., Baland and Robinson, 2000; Ranjan, 2001; Hazan and Berdugo, 2002; Fan, 2004a, 2004b; Doepke and Zilibotti, 2005) examine the relationship between child labor and human capital. However, somewhat surprisingly, none of the literature has considered child leisure in relation to the luxury axiom of Basu and Van (1998). Thus, the current paper attempts to help fill this gap.

The analysis of this paper implies that when adults' wage rate is low so that a typical household's faces a subsistence constraint in consumption, parents' income is the key determinant of child labor. In this case, the luxury axiom holds strictly and children's working time decreases as parents' income rises. On the other hand, when adults' wage rate is relatively high so that the subsistence constraint is not binding, the substitutability between child labor and adult labor may become the major determinant of child labor. In fact, under some circumstances, the analysis implies that the relative productivity of child labor, namely the substitutability between child labor and adult labor, may matter much more than parents' absolute income to children's labor market participation.

Thus, the analysis of this paper provides an explanation for "the wealth paradox" in light of the luxury axiom and the substitution axiom. For example, although the Industrial Revolution greatly increased the average wage rates of adult workers in Britain, the rates of children's labor market participation were much higher when the Industrial Revolution began than prior to the Industrial Revolution (e.g., Deane and Cole, 1967). A number of economic historians explain this phenomenon as that the Industrial Revolution greatly increases the substitutability between child labor and adult labor (e.g., Nardinelli, 1990; Lavalette, 1998; Tuttle, 1999). In particular, based on her extensive empirical research, Tuttle (1999, pp.75-76) concludes that,

"It was demand, not supply, which dramatically increased the employment of children and youths in certain leading industries during the British Industrial Revolution The Industrial Revolution in Great Britain had an impact on the demand

for child labor because several new inventions in the textile industry and innovations in the production process of making cloth and extracting coal increased the productivity of children and youths. As children and youths became more productive, the demand for their services rose.”

Indeed, in both historical and contemporary times, much evidence shows that the rise of the substitutability between child labor and adult labor is a major or even the most important cause of child labor.² Thus, this paper complements the existing literature toward a better understanding of the historical and contemporary phenomena of child labor.³

Also, this paper shows that the Luxury Axiom and the Substitution Axiom of Basu and Van (1998) are intrinsically linked. In particular, we show that if the degree of substitution between child labor and adult labor is below a certain threshold level, child labor exists only if adults’s wage rate is low so that the subsistence constraint is binding. Namely, the Luxury Axiom of Basu and Van (1998) holds if and only if their Substitution Axiom holds weakly. Moreover, it shows that the greater is parents’ taste for children’s leisure or the greater is a child’s rate of return from devoting more time to study in the accumulation of human capital, the less likely will parents send children to work. However, a greater rate of return from investing more financial resources on children’s education may increase parents’ incentive to send children to work since child labor increases household income and hence more financial resources on the child’s education.

2. THE MODEL

Our basic analytical framework builds on Basu and Van (1998), Fan (2004a), and other existing literature, with the following extensions: (1) parents care about both children’s leisure and their human capital,⁴ (2) a child’s human capital formation depends on both money input and her time input.

We assume that every individual lives for two periods: childhood and adulthood (i.e., parenthood). Each family has one parent and one child, and the parent is the only

² For example, see Hunt (1973), Nardinelli (1990), Lavalette (1998), and Tuttle (1999) for historical evidence and see Levy (1985), Bonnet (1993), Mehra-Kerpelman (1996), and Nangia (1987) for the evidence in developing countries of modern times.

³ For example, see Basu and Van (1998), Baland and Robinson (2000), Ranjan (2001), Hazan and Berdugo (2002), Fan (2004a, 2004b), and Doepke and Zilibotti (2005).

⁴ The basic setup of the paper is similar to Fan (2004a), and some of the results of the paper (e.g., Proposition 1) are similar to Fan (2004a). But Fan (2004a) does not consider children’s leisure in a household’s utility function. Thus, this paper extends Fan (2004a).

decision maker of a household. A parent cares about the family's consumption, her child's human capital, and her child's leisure. Formally, a parent's (i.e., an adult's) utility function takes the following form

$$V \equiv \ln(c) + \delta \ln(h) + \theta l, \quad (1)$$

where c , h , and l denote the household consumption, the child's human capital, and the child's leisure, respectively. δ and θ are positive parameters that measure the extent to which parents are altruistic. Note that the utility function implies that relative to a household's material consumption and a child's human capital, a child's leisure is a "luxury good" for the parent. So, the formulation of the utility function is consistent with the "Luxury Axiom" of Basu and Van (1998).

Every child is endowed with one unit of time, which divided into three parts: (1) time for work, e , (2) time for study, s , (3) time for leisure, l . So, we have

$$e + s + l = 1. \quad (2)$$

A child's human capital is determined by the financial resources on her education (x) as well as her time of study. Specifically, as in Fan (2004a), we assume that a child's human capital production function takes the following Cobb-Douglas form.

$$h = x^\alpha s^\beta, \quad (3)$$

where α , and β are both positive coefficients. Then, we can rewrite (1) as,

$$V \equiv \ln(c) + \delta \ln(h) + \theta l = \ln(c) + \alpha\delta \ln(x) + \beta\delta \ln(s) + \theta l. \quad (4)$$

We now turn our attention to the demand of labor and the determination of the wage rates. We assume that individuals operate in a small open economy in a one-good world. The production function is,

$$Y = F(K, L) \equiv Lf(k), \quad k \equiv \frac{K}{L}, \quad (5)$$

where Y , K , and L are total output, the quantity of capital and the quantity of labor respectively. Since the economy is perfectly competitive, the interest rate of physical capital, r , and the wage rate of skilled labor, w , are determined as follows,

$$r = f'(k), \quad w = f(k) - kf'(k). \quad (6)$$

Suppose that the world interest rate is constant at \bar{r} . Assuming that the small

economy permits unrestricted international lending and borrowing, its interest rate must also be equal to \bar{r} . Therefore, the ratio between capital and skilled labor in this economy is constant at a level $f'^{-1}(\bar{r}) \equiv \bar{k}$. Thus, the wage rate, w , is constant at the level of

$$f(\bar{k}) - \bar{k}f'(\bar{k}).$$

We assume that every adult is endowed with *one* unit of labor and that every child is endowed with γ unit of labor ($\gamma \geq 0$). Then, an adult's income and a child's wage rate are w and γw respectively. Clearly, γ measures the substitutability between child labor and adult labor (or the *relative* productivity of child labor). First, as shown by Basu and Van (1998), γ is determined by the relative productivity between child labor and adult labor in a competitive labor market. So, as discussed previously, " γ " will be the crucial parameter in this model. Second, γ can also be affected by the implementation of the laws that punish or ban child labor. If child workers are the ones who would be fined if being caught, their expected earnings will clearly decrease. If the fines are on employers, they would have to pay an extra cost (either the possible fines or the cost of bribing policemen) when they employ children. This extra cost will at least partially pass on to the child workers. Thus, in both cases, from the perspective of parents and children, these laws will reduce the returns to child labor. And, the more strictly these laws are enforced, the smaller " γ " will become.

We assume that a household cannot borrow in the capital market. Also, for simplicity, this paper abstracts from the consideration of bequest from parents to children.⁵ Then, an adult's budget constraint is

$$c + x = w + e\gamma w. \quad (7)$$

Plugging (2) into (7), we can rearrange (7) as

$$c + x + \gamma ws + \gamma wl = w + \gamma w. \quad (8)$$

Finally, we consider that a household faces a subsistence constraint, whose simplest form can be described as

$$c \geq \Phi, \quad (9)$$

where Φ is the minimum level of consumption for the subsistence of both the parent and the child of a household.

⁵ Baland and Robinson (2000) present an analysis that considers both child labor and bequest.

First, we consider the intuition in which the subsistence constraint, (9), is not binding. In this case, a parent maximizes her utility “ V ” subject to (8). Then, the Langragian can be written as

$$L = \ln(c) + \alpha\delta \ln(x) + \beta\delta \ln(s) + \theta l + \lambda(w + \gamma w - c - x - \gamma ws - \gamma wl).$$

So, the first order conditions are

$$\frac{\partial L}{\partial c} = \frac{1}{c} - \lambda = 0, \quad (10)$$

$$\frac{\partial L}{\partial x} = \frac{\alpha\delta}{x} - \lambda = 0, \quad (11)$$

$$\frac{\partial L}{\partial s} = \frac{\beta\delta}{s} - \lambda\gamma w = 0, \quad (12)$$

$$\frac{\partial L}{\partial l} = \theta - \lambda\gamma w \leq 0, \text{ (with strict equality holds if } l > 0\text{)}. \quad (13)$$

Then, we have the following lemma.

Lemma 1: When the subsistence constraint is not binding, we have the following results.

(1) If $\beta\delta > \theta$, then

Condition	Result
$\gamma < \frac{\theta}{1 + \alpha\delta + \beta\delta - \theta}$	$e = 0, \quad l > 0$
$\frac{\theta}{1 + \alpha\delta + \beta\delta - \theta} < \gamma < \frac{\theta}{1 + \alpha\delta}$	$e = 0, \quad l = 0$
$\gamma > \frac{\theta}{1 + \alpha\delta}$	$e > 0, \quad l = 0$

(2) If $\beta\delta < \theta$, then

Condition	Result
$\gamma < \frac{\beta\delta}{1 + \alpha\delta}$	$e = 0, l > 0$
$\frac{\beta\delta}{1 + \alpha\delta} < \gamma < \frac{\theta}{1 + \alpha\delta + \beta\delta - \theta}$	$e > 0, l > 0$
$\gamma > \frac{\theta}{1 + \alpha\delta + \beta\delta - \theta}$	$e > 0, l = 0$

Proof. See Appendix. ■

Lemma 1 implies that when the subsistence constraint is not binding, a key determinant of a child's working time and leisure time is the relative productivity of child labor, γ . As child labor productivity rises, a child tends to work more and enjoys less leisure.

Next, the following lemma characterizes the conditions for that the subsistence constraint is and is not binding.

Lemma 2:

(1) If $\gamma \leq \frac{\theta}{1 + \alpha\delta + \beta\delta - \theta}$, the subsistence constraint is binding if and only if

$$w < \frac{\theta}{\gamma} \Phi.$$

(2) If $\gamma > \frac{\theta}{1 + \alpha\delta + \beta\delta - \theta}$, the subsistence constraint is binding if and only if

$$w < \frac{1 + \alpha\delta + \beta\delta}{1 + \gamma} \Phi.$$

(3) The subsistence constraint is not binding if

$$w > \max\left(\frac{\theta}{\gamma} \Phi, \frac{1 + \alpha\delta + \beta\delta}{1 + \gamma} \Phi\right).$$

Proof. See Appendix. ■

If the subsistence constraint is binding, then $c = \Phi$. In this case, the Langragian can be written as

$$L = \ln \Phi + \alpha \delta \ln(x) + \beta \delta \ln(s) + \theta l + \lambda(w + \gamma w - c - x - \gamma w s - \gamma w l).$$

So, the first order conditions are

$$\frac{\partial L}{\partial x} = \frac{\alpha \delta}{x} - \lambda = 0, \quad (14)$$

$$\frac{\partial L}{\partial s} = \frac{\beta \delta}{s} - \lambda \gamma w = 0, \quad (15)$$

$$\frac{\partial L}{\partial l} = \theta - \lambda \gamma w \leq 0, \text{ (with strict equality holds if } l > 0 \text{)}. \quad (16)$$

Then, we have the following proposition.

Proposition 1:

(1) When the subsistence constraint is binding, child labor decreases as her parent's income increases if it exists, that is,

$$\frac{de}{dw} < 0.$$

(2) When $w < \Phi$, child labor always exists and

$$\frac{de}{d\gamma} < 0.$$

Proof. See Appendix. ■

Proposition 1 discusses the determinants of child labor when the subsistence constraint is binding. In this case, because people are very poor, child labor is mainly caused by the subsistence need of a household. So, a child's working time tends to increase as her parent's income decreases. Meanwhile, if the substitutability between child labor and adult labor, γ , rises, a child needs to work less to meet her family's subsistence level of consumption. Consequently, in this case, as γ increases, a child's working time tends to decrease.

Proposition 2: When $w > \max\left(\frac{\theta}{\gamma}\Phi, \frac{1 + \alpha\delta + \beta\delta}{1 + \gamma}\Phi\right)$ such that the subsistence constraint is not binding, we have the following results:

(1) Child labor exists if and only if

$$\gamma > \frac{\min(\beta\delta, \theta)}{1 + \alpha\delta}.$$

(2) Suppose that child labor exists, then, a child's working time will increase if the relative productivity of child labor, γ , rises.

(3) A child's working time is independent of her parents' income, that is,

$$\frac{de}{dw} = 0.$$

Proof. See Appendix. ■

Firstly, child labor will exist if and only if the relative productivity of child labor is above a certain threshold level; secondly, when child labor exists, children's labor market participation increases as child labor productivity rises. Thus, the first two parts of Proposition 2 complements the existing literature in explaining the empirical observations that a major determinant of child labor is the substitutability between child labor and adult labor.

The third part of Proposition 1 is a somewhat surprising result. Its intuition is as follows. On one hand, as an adult's wage rate, " w ", rises, household wealth will increase and the "income effect" will increase children's study time and reduce their time of working. On the other hand, holding " γ " constant, a child's wage rate, " γw ", will increase as " w " rises. So, a child's opportunity cost of study increases and the "substitution effect" increases child labor. In this model, the "substitution effect" and the "income effect" exactly offset each other, which implies that a child's working time is independent of her parents' income. Thus, when the subsistence constraint is not binding, this model illustrates that under reasonable conditions, child labor may not mainly be determined by parents' income when holding γ constant.

Proposition 2 implies that the substitutability between child labor and adult labor, γ , is a major determinant of child labor. Further, when the income of a typical household is beyond the subsistence level, Proposition 2 implies that the relative productivity of child labor, namely the substitutability between child labor and adult labor, may matter much more than adults' absolute wage rate to children's labor market participation. Thus, our theoretical analysis sheds light on "the wealth paradox" and other empirical findings discussed in the introduction. For example, it provides an explanation for economic

historians' findings that in Britain in the nineteenth century, parents' wages have no impacts on children's labor market participation. Also, note that land and labor are complementary in agricultural production, which implies that an increase in land raises the wage rates for both child labor and adult labor (e.g., Bar and Basu, 2009; Basu *et al.*, 2010). Meanwhile, an increase in family land may raise the relative wage of child labor due to the imperfection of labor market (e.g., Dumas, 2007), which from Proposition 2 implies we know that it will increase children's labor market participation. Thus, this proposition helps us understand "the wealth paradox" revealed by Bhalotra and Heady (2003) and other empirical studies discussed in the introduction.

An implicit assumption of this paper is that when people are poor, children's education may be partially funded by child labor. While to my knowledge there is no such a direct empirical test of this assumption, there is some indirect empirical evidence that appears to support it. For example, Banerjee and Duflo (2007) show that in contemporary world, many people still live under one U.S. dollar per day (at purchasing power parity), and they spend little on their children's education. In this case, children's earnings would clearly help finance children's education, such as buying textbooks and some basic educational materials. Also, a child's human capital clearly includes her health. Some empirical studies (e.g., Horrell *et al.*, 2001; and Park, 2010) show that a household's wealth was a very important factor that determines children's health. Moreover, Weil (2007) and Venkataramani *et al.* (2010) show that health is an important determinant of economic growth and production efficiency.

Next, this analysis aims to better understand the interactions between the Luxury Axiom and the Substitution Axiom of Basu and Van (1998). These two axioms, which serve as the foundation of Basu and Van (1998) and contribute greatly to the recent expansion of the economic literature on child labor, are stated as follows.

The Luxury Axiom: A family will send the children to the labor market only if the family's income from non-child-labor sources drops very low.

The Substitution Axiom: From a firm's point of view, child labor and adult labor are substitutes.

Clearly, the relative productivity of child labor, γ , measures the degree of substitution between child labor and adult labor. Meanwhile, the following corollary shows that *the Luxury Axiom* and *the Substitution Axiom* of Basu and Van (1998) are intrinsically linked.

Corollary 1:

If,

$$\gamma < \frac{\min(\beta\delta, \theta)}{1 + \alpha\delta}, \quad (17)$$

child labor exists only if adults' wage rate is low so that the subsistence constraint is binding. Namely, if (17) is satisfied, the Luxury Axiom of Basu and Van (1998) holds.

Proof. See Appendix. ■

This corollary indicates that if the degree of the substitution between child labor and adult labor is below a certain threshold level, child labor is purely a phenomenon of poverty. Thus, the analysis implies that there is a close relationship between the two axioms of Basu and Van (1998). Namely, *the Luxury Axiom* of Basu and Van (1998) holds if and only if their *Substitution Axiom* holds weakly.

Also, we can see that the threshold level increases with β and θ , but decreases with α . The intuitions are as follows. First, as β increases, the child's rate of return from devoting more time to study in the accumulation of human capital increases, which tends to reduce parents' incentive in sending children to work. Second, θ is an indicator of parents' taste for children's leisure. Thus, when θ increases, parents are less likely to send children to work. Third, when α increases, the rate of return from investing more financial resources on the child's education increases. Since child labor increases household income, which lead to more financial resources on the child's education, an increase in α may induce parents to send children to work.⁶

Moreover, from Corollary 1, we can see that even if parents have strong preference for children's leisure, child labor may still exist. In relation to the model specification, a strong preference for children's leisure is interpreted as θ being large. When θ is sufficiently large, we will have $\min(\beta\delta, \theta) = \beta\delta$. Then, if the relative productivity of child labor, γ , is large enough, we will have $\gamma > \beta\delta$, which implies that child labor exists in this case. With reference to Basu and Van (1998), our analysis implies that *the Luxury Axiom* holds only when the substitution between child labor and adult labor is sufficiently weak.

3. CONCLUSION

In their path breaking contribution, Basu and Van (1998) present a fundamental framework of child labor with two important axioms: *the luxury axiom* and *the*

⁶ In a poor economy, a household's wealth is often essential for children's schooling. As pointed out by Ranjan (2001), children in poor countries often face credit constraints in their schooling. So, in a sense, the increase in child labor productivity and child labor increases the wealth of a poor household and consequently reduces the problem of credit constraint. For example, as household income increases, parents may buy more textbooks and other essential study equipment for the children, which is of great help to children's human capital formation particularly in a poor economy.

substitution axiom. A number of empirical studies, however, reveal a “wealth paradox”, which seems to contradict *the luxury axiom*. The current paper has two aims. First, it helps explain the “wealth paradox”. Second, it attempts to better understand the relationship between *the luxury axiom* and *the substitution axiom*.

It develops a simple model of child labor by incorporating four crucial ingredients, child leisure, child labor, education, and a household’s subsistence constraints, in a unified framework. The analysis of this paper implies that when adults’ wage rate is low so that a typical household’s faces a subsistence constraint in consumption, parents’ income is the key determinant of child labor. In this case, *the luxury axiom* strictly holds and children’s working time decreases as parents’ income rises. On the other hand, when adults’ wage rate is relatively high so that the subsistence constraint is not binding, the substitutability between child labor and adult labor may matter much more than parents’ income to children’s labor market participation. Thus, this analysis provides an explanation for “the wealth paradox” in light of *the luxury axiom* and *the substitution axiom*.

Also, this paper shows that *the Luxury Axiom* and *the Substitution Axiom* of Basu and Van (1998) are intrinsically linked. In particular, we show that if the degree of substitution between child labor and adult labor is below a certain threshold level, child labor exists only if adults’ wage rate is low so that the subsistence constraint is binding. Namely, *the Luxury Axiom* of Basu and Van (1998) holds if and only if their *Substitution Axiom* holds weakly. Moreover, it shows that the greater is parents’ taste for children’s leisure or the greater is a child’s rate of return from devoting more time to study in the accumulation of human capital, the less likely will parents send children to work. However, a greater rate of return from investing more financial resources on children’s education may increase parents’ incentive to send children to work since child labor increases household income and hence more financial resources on the child’s education.

APPENDIX

Proof of Lemma 1: First, we try to solve a parent’s optimization problem when the subsistence constraint is not binding. The analysis is divided into 2 cases.

Case 1: (13) holds with strict equality

In this case, from (10) and (13), we get

$$c = \frac{\gamma w}{\theta}. \tag{A1}$$

From (11) and (13), we get

$$x = \frac{\alpha\delta\gamma w}{\theta}. \quad (\text{A2})$$

From (12) and (13), we get

$$s = \frac{\beta\delta}{\theta}. \quad (\text{A3})$$

Then, plugging (A1), (A2) and (A3) into (8), we get

$$\gamma w l = w + \gamma w - c - x - \gamma w s = w + \gamma w - \frac{\gamma w}{\theta} - \frac{\alpha\delta\gamma w}{\theta} - \gamma w \frac{\beta\delta}{\theta}.$$

So,

$$l = \frac{1}{\gamma} + 1 - \frac{1 + \alpha\delta + \beta\delta}{\theta}. \quad (\text{A4})$$

Thus, from (2), we have

$$e = 1 - s - l = \frac{1 + \alpha\delta}{\theta} - \frac{1}{\gamma}. \quad (\text{A5})$$

Case 2: (13) holds with strict inequality

In this case, from (13), we have $l = 0$.

Then, from (10), (11), (12), and (8), we can get

$$c = \frac{w + \gamma w}{1 + \alpha\delta + \beta\delta}, \quad (\text{A6})$$

$$x = \frac{\alpha\delta(w + \gamma w)}{(1 + \alpha\delta + \beta\delta)}. \quad (\text{A7})$$

And

$$s = \frac{\beta\delta(1 + \gamma)}{\gamma(1 + \alpha\delta + \beta\delta)}. \quad (\text{A8})$$

So, when $l = 0$, we have

$$e = 1 - s - l = \frac{\gamma + \alpha\gamma\delta - \beta\delta}{\gamma(1 + \alpha\delta + \beta\delta)}. \quad (\text{A9})$$

Now, we check under conditions, (13) holds with strict equality (i.e., $l > 0$). Note that from (A4), $l > 0$ if and only if

$$\frac{1}{\gamma} + 1 - \frac{1 + \alpha\delta + \beta\delta}{\theta} > 0,$$

namely

$$\gamma < \frac{\theta}{1 + \alpha\delta + \beta\delta - \theta}. \quad (\text{A10})$$

Meanwhile, in this case, from (A5), $e > 0$ if and only if

$$\frac{1 + \alpha\delta}{\theta} - \frac{1}{\gamma} > 0,$$

namely

$$\gamma > \frac{\theta}{1 + \alpha\delta}. \quad (\text{A11})$$

Also, note that if $\beta\delta > \theta$, then

$$\frac{\theta}{1 + \alpha\delta + \beta\delta - \theta} < \frac{\theta}{1 + \alpha\delta}.$$

So, if $\beta\delta > \theta$,

Condition	Result
$\gamma < \frac{\theta}{1 + \alpha\delta + \beta\delta - \theta}$	$e = 0, \quad l > 0$
$\frac{\theta}{1 + \alpha\delta + \beta\delta - \theta} < \gamma < \frac{\theta}{1 + \alpha\delta}$	$e = 0, \quad l = 0$
$\gamma > \frac{\theta}{1 + \alpha\delta}$	$e > 0, \quad l = 0$

Next, we consider the case that (13) holds with strict inequality. From above, we know $l=0$ implies

$$\gamma \geq \frac{\theta}{1 + \alpha\delta + \beta\delta - \theta}. \quad (\text{A12})$$

Meanwhile, when (13) holds with strict inequality, from (A9), $e > 0$ if and only if

$$\frac{\gamma + \alpha\gamma\delta - \beta\delta}{\gamma(1 + \alpha\delta + \beta\delta)} > 0,$$

namely

$$\gamma > \frac{\beta\delta}{1 + \alpha\delta}. \quad (\text{A13})$$

Also, note that if $\beta\delta < \theta$, then

$$\frac{\theta}{1 + \alpha\delta + \beta\delta - \theta} > \frac{\theta}{1 + \alpha\delta} > \frac{\beta\delta}{1 + \alpha\delta}.$$

So, if $\beta\delta < \theta$,

Condition	Result
$\gamma < \frac{\beta\delta}{1 + \alpha\delta}$	$e = 0, \quad l > 0$
$\frac{\beta\delta}{1 + \alpha\delta} < \gamma < \frac{\theta}{1 + \alpha\delta + \beta\delta - \theta}$	$e > 0, \quad l > 0$
$\gamma > \frac{\theta}{1 + \alpha\delta + \beta\delta - \theta}$	$e > 0, \quad l = 0$

■

Proof of Lemma2: Suppose that c^* is the optimal solution without the subsistence constraint. Then, If the subsistence constraint is binding if and only if $c^* < \Phi$.

(1) If $\gamma < \frac{\theta}{1 + \alpha\delta + \beta\delta - \theta}$, from (A1), the subsistence constraint is binding if and only if

$$c^* = \frac{\gamma w}{\theta} < \Phi ,$$

namely

$$w < \frac{\theta}{\gamma} \Phi .$$

(2) If $\gamma > \frac{\theta}{1 + \alpha\delta + \beta\delta - \theta}$, from (A6), the subsistence constraint is binding if and only if

$$c^* = \frac{w + \gamma w}{1 + \alpha\delta + \beta\delta} < \Phi ,$$

namely

$$w < \frac{1 + \alpha\delta + \beta\delta}{1 + \gamma} \Phi .$$

(3) It's obvious from (1) & (2). ■

Proof of Proposition 1: Similar to the proof of Lemma 1, the analysis is divided into 2 cases.

Case 1: (16) holds with strict equality

In this case, from (14), and (16), we get

$$x = \frac{\alpha\delta\gamma w}{\theta} . \tag{A14}$$

From (15) and (16), we get

$$s = \frac{\beta\delta}{\theta} . \tag{A15}$$

Then, plugging (A14), and (A15), into (6), we get

$$\gamma w l = w + \gamma w - c - x - \gamma w s = w + \gamma w - \Phi - \frac{\alpha \delta \gamma w}{\theta} - \gamma w \frac{\beta \delta}{\theta}.$$

So,

$$l = \frac{1}{\gamma} + 1 - \frac{\alpha \delta + \beta \delta}{\theta} - \frac{\Phi}{\gamma w}. \quad (\text{A16})$$

Thus, from (2), we have

$$e = 1 - s - l = \frac{\alpha \delta}{\theta} + \frac{\Phi}{\gamma w} - \frac{1}{\gamma}. \quad (\text{A17})$$

Clearly,

$$\frac{de}{dw} = -\frac{\Phi}{\gamma w^2} < 0. \quad (\text{A18})$$

Meanwhile, from (A17), $e > 0$ if and only if

$$\frac{\alpha \delta}{\theta} + \frac{\Phi}{\gamma w} - \frac{1}{\gamma} > 0,$$

namely

$$\frac{\gamma \alpha \delta}{\theta} + \frac{\Phi}{w} > 1. \quad (\text{A19})$$

So, clearly, if $w < \Phi$, we have $e > 0$. Meanwhile, from (A17)

$$\frac{de}{d\gamma} = -\frac{w - \Phi}{w \gamma^2} < 0.$$

Case 2: (16) holds with strict inequality

In this case, from (16), we have $l = 0$.

Then, from (14), (15), and (8), we can get

$$x = \frac{\alpha}{\alpha + \beta} (w + \gamma w - \Phi). \quad (\text{A20})$$

And

$$s = \frac{\beta}{\alpha + \beta} \frac{w + \gamma w - \Phi}{\gamma w}. \quad (\text{A21})$$

So, when $l = 0$, we have

$$e = 1 - \frac{\beta}{\alpha + \beta} \frac{w + \gamma w - \Phi}{\gamma w} = \frac{\alpha}{\alpha + \beta} + \left(\frac{\beta}{\alpha + \beta} \right) \left(\frac{1}{\gamma} \right) \left(\frac{\Phi}{w} - 1 \right). \quad (\text{A22})$$

From (A22), $e > 0$ if $w < \Phi$. And, if $w < \Phi$, we have

$$\frac{de}{dw} = - \left(\frac{\beta}{\alpha + \beta} \right) \frac{\Phi}{\gamma w^2} < 0, \quad (\text{A23})$$

$$\frac{de}{d\gamma} = \left(\frac{\beta}{\alpha + \beta} \right) \frac{w - \Phi}{\gamma^2 w} < 0. \quad (\text{A24})$$

■

Proof of Proposition 2:

(1) From Lemma 1 and its proof, we can see that if $\beta\delta > \theta$, then child labor exists if and only if

$$\gamma > \frac{\theta}{1 + \alpha\delta} = \frac{\min(\beta\delta, \theta)}{1 + \alpha\delta}.$$

And, if $\beta\delta < \theta$, then child labor exists if and only if

$$\gamma > \frac{\beta\delta}{1 + \alpha\delta} = \frac{\min(\beta\delta, \theta)}{1 + \alpha\delta}.$$

Thus, child labor exists if and only if

$$\gamma > \frac{\min(\beta\delta, \theta)}{1 + \alpha\delta}.$$

(2) From (A5), we have

$$\frac{de}{d\gamma} = \frac{1}{\gamma^2} > 0.$$

From (A9), we have

$$\frac{de}{d\gamma} = \frac{\beta\delta}{\gamma^2(1+n\alpha\delta+n\beta\delta)} > 0.$$

Thus, child labor increases with γ if it exists.

(3) From (A5) and (A9), we know

$$\frac{de}{dw} = 0. \blacksquare$$

Proof of Corollary1: It's obvious from Part 1 of Proposition 1 and Part 1 of Proposition 2. \blacksquare

REFERENCES

- Baland, J.M., and J.A. Robinson (2000), "Is Child Labor Inefficient?" *Journal of Political Economy*, 108(4), 663-679.
- Banerjee, A., and E. Duflo (2007), "The Economic Lives of the Poor," *Journal of Economic Perspectives*, 21(1), 141-167.
- Bar, T., and K. Basu (2009), "Children, Education, Labor and Land: In the Long Run and Short Run," *Journal of the European Economic Association*, 7(2-3), 487-497.
- Basu, K. (1999), "Child Labor: Cause, Consequence, and Cure, with Remarks on International Labor Standards," *Journal of Economic Literature*, 37(3), 1083-1119.
- Basu, K., S. Das, and B. Dutta (2010), "Child Labor and Household Wealth: Theory and Empirical Evidence of an Inverted-U," *Journal of Development Economics*, 91(1), 8-14.
- Basu, K., and P.H. Van (1998), "The Economics of Child Labor," *American Economic Review*, 88(3), 412-427.
- Becker, G. (1991), *A Treatise of the Family*, Cambridge, M.A.: Harvard University Press.
- Bhalotra, S., and C. Heady (2003), "Child Farm Labor: The Wealth Paradox," *World Bank Economic Review*, 17(2), 197-227.
- Bonnet, M. (1993), "Child Labour in Africa," *International Labour Review*, 132(3), 371-389.

- Cigno, A., and F.C. Rosati (2005), *The Economics of Child Labour*, Cambridge, M.A.: Oxford University Press.
- Deane, P., and W.A. Cole (1967), *British Economic Growth, 1688-1959: Trends and Structure*, London, Cambridge U.P.
- Doepke, M., and F. Zilibotti (2005), "The Macroeconomics of Child Labor Regulation," *American Economic Review*, 95(5), 1492-1524.
- Dumas, C. (2007), "Why Do Parents Make Their Children Work? A Test of the Poverty Hypothesis in Rural Areas of Burkina Faso," *Oxford Economic Papers*, 59(2), 301-329.
- Edmonds, E.V. (2008), "Child Labor," in T.P. Schultz and J. Strauss, eds., *Handbook of Development Economics*, 4, Elsevier Science, Amsterdam, North-Holland.
- Fan, C.S. (2004a), "Relative Wage, Child Labor and Human Capital," *Oxford Economic Papers*, 56(4), 687-700.
- ____ (2004b), "Child Labor and the Interaction between the Quantity and Quality of Children," *Southern Economic Journal*, 71(1), 21-35.
- Hazan, M., and B. Berdugo (2002), "Child Labor, Fertility, and Economic Growth," *Economic Journal*, 112(482), 810-828.
- Horrell, S., J. Humphries, and H.-J. Voth (2001), "Destined for Deprivation: Human Capital Formation and Intergenerational Poverty in Nineteenth-Century England," *Explorations in Economic History*, 38(3), 339-365.
- Hunt, E.H.(1973), *Regional Wage Variations in Britain 1850-1914*, Oxford: Clarendon Press.
- ____ (1986), "Industrialization and Regional Inequality: Wages in Britain, 1760-1914," *Journal of Economic History*, 935-966.
- Kambhampati, U.S., and R. Rajan (2006), "Economic Growth: A Panacea for Child Labor?" *World Development*, 34(3), 426-445.
- Kruger, D.I. (2007), "Coffee Production Effects on Child Labor and Schooling in Rural Brazil," *Journal of Development Economics*, 82(2), 448-463.
- Lavalette, M. (1998), "Child Labour: Historical, Legislative and Policy Context," in B. Pettitt, eds., *Children and Work in the UK : Reassessing the Issues*, London: Child Poverty Action Group.
- Levy, V. (1985), "Cropping Pattern, Mechanization, Child Labor, and Fertility Behavior in a Farming Economy: Rural Egypt," *Economic Development and Cultural Change*, 33(4), 777-791.
- Mantoux, P. (1983), *The Industrial Revolution in the Eighteenth Century*, Chicago: University of Chicago Press.
- Mehra-Kerpelman, K. (1996), "Children at Work: How Many and Where?" *World of Work*, 15, 8-9.
- Nangia, P. (1987), *Child Labour: Cause-Effect Syndrome*, New Delhi: Janak Publishers.
- Nardinelli, C. (1990), *Child Labor and the Industrial Revolution*, Bloomington: Indiana University Press.
- Park, C. (2010), "Children's Health Gradient in Developing Countries: Evidence from

- Indonesia,” *Journal of Economic Development*, 35(4), 25-44.
- Ranjan, P. (2001), “Credit Constraint and the Phenomenon of Child Labor,” *Journal of Development Economics*, 64, 81-102.
- Tuttle, C. (1999), *Hard at Work in Factories and Mines: The Economics of Child Labor during the British Industrial Revolution*, Boulder and Oxford: West-view Press.
- Venkataramani, A.S., K.R. Shanmugam, and J.P. Ruger (2010), “Health, Technical Efficiency, and Agricultural Production in Indian Districts,” *Journal of Economic Development*, 35(4), 1-22.
- Weil, D.N. (2007), “Accounting for The Effect of Health on Economic Growth,” *Quarterly Journal of Economics*, 122(3), 1265-1306.

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