Stability Analysis of Different Monetary Policy Rules for a Macroeconomic Model with Endogenous Money and Credit Channel

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Abstract: Bofinger (2001) had built a macroeconomic model that tried to integrate the financial intermediation feature of the Bernanke-Blinder model with the "exogenous" character of money market interest rate. In the so-called IS-CM model there are two interest rates: the credit or banking interest rate, determined by the interactions between goods market and credit market clearing conditions, and the money market interest rate, set by monetary authorities. Up to now no efforts are made in the way to analyse the dynamics of Bofinger model under different assumptions about the determination of money market interest rate. The objective of this paper is to analyse the stability properties of Bofinger (2001) model under different monetary policy rules. More specifically we will consider four different cases: (a) strict inflation targeting; (b) strict inflation targeting plus anti-cyclical fiscal policy; (c) strict output targeting and (d) "double mandate" for Central Bank (output and inflation stabilization). As we will see, long-run equilibrium of the model ***can be*** stable only in cases (a) and (d); which means that the existence of *nominal anchor* is of fundamental importance for macroeconomic stability. We also conclude that an anti-cyclical fiscal policy is incompatible with macroeconomic stability under a strict inflation targeting regime.

Key-words: Monetary Policy, Inflation Targeting, Dynamic Stability.

JEL: E50, E31, C62

Resumo: Bofinger (2001) desenvolveu um modelo macroeconômico com o intuito de integrar o arcabouço financeiro da intermediação financeira de Bernanke-Blinder com o caráter exógeno da taxa básica de juros do mercado monetário. Assim, temos o modelo IS-CM onde existem duas taxas de juros, a saber: taxa de juros do crédito bancário, determinada pela interação entre o mercado de bens e o mercado de crédito; e a taxa de juros de mercado, determinada pela autoridade monetária. Até então, nenhum esforço havia sido empregado com o intuito de se analisar a dinâmica do modelo de Bofinger sob diferentes hipóteses acerca dos determinantes da taxa básica de juros. Sendo assim, o objetivo desse artigo é analisar as propriedades da estabilidade dinâmica do modelo de Bofinger sob diferentes regras de política monetária. Mais especificamente, iremos considerar quatro casos distintos: (a) regime de metas de inflação estrito; (b) metas de inflação em conjunto com utilização de políticas fiscais anticíclicas (c) regime de metas de produto estrito; (d) existência de um duplo mandato para o Banco Central em termos de produto e estabilização da inflação. Vamos mostrar, ainda, que a estabilidade dinâmica do modelo só é encontrada nos casos (a) e (d), de tal sorte que a existência de uma âncora nominal é fundamental para a estabilidade macroeconômica. Sendo assim, podemos concluir que políticas fiscais anticíclicas são incompatíveis com a estabilidade macroeconômica sob o arranjo do regime de metas de inflação.

Palavras-chave: Política Monetária, Metas de Inflação, Estabilidade Dinâmica

Área 3: Macroeconomia, economia monetária e finanças.

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1. Introduction.

Since the seminal work of Bernanke (1983), we are told that non-monetary effects related to the workings of the credit markets are of fundamental importance for the explanation of the magnitude and duration of the Great Depression. However, the standard macroeconomic model, the IS-LM, developed by Hicks (1937) and Modigliani (1944), had no room for changes in the conditions of financial intermediation to affect aggregate demand and output. The inclusion of the credit market in the structure of the IS-LM model was made by Bernanke and Blinder (1988). One of the main theoretical results of their model is that money targeting policy rule can be destabilizing if money-demand shocks are more important than credit-demand shocks. In this case, it is recommended a policy that aims to target the level (or the rate of expansion) of credit rather than the level of money stock.

Bernanke and Blinder model, however, has a fundamental flaw for the point of view of operation of monetary policy: it supposes that money market interest rate is an endogenous variable that results from the interaction between the credit market and money market clearing conditions. This is not the way that Central Banks conduct monetary policy. As shown by Romer (2000), money market interest rate is set by monetary authorities in order to achieve the level of real interest rate that is required to produce some target level for inflation rate. This means that money market interest rate should be treated as a policy variable instead of being considered the result of the interactions between credit and money markets in general equilibrium.

The so-called IS-RM model, developed by Romer, was the first attempt to bring real word monetary policy procedures to macroeconomic modeling. This model, however, had no room at all for financial intermediation to affect aggregate demand and output. Changes in the rate of inflation and in the level of real output are the result of changes in the position of the IS curve (caused by changes in fiscal policy and/or in the expectations of the private sector) or in the position of the MR (monetary policy rule) curve (caused by changes of target level of inflation and/or changes in the weights of inflation and output in the loss function of the Central Bank). Last but not least, the level of money stock has no importance at all for the determination of output or inflation. The monetary policy of the IS-RM model is a policy where money does not matter.

More recently, Bofinger (2001) had built a macroeconomic model that tried to integrate the financial intermediation feature of the Bernanke-Blinder model with the “exogenous” character of money market interest rate. In the so-called IS-CM model there are two interest rates: the credit or banking interest rate, determined by the interactions between goods market and credit market clearing conditions, and the money market interest rate, set by monetary authorities. One of the main theoretical results of the model is that an increase of credit default risk – caused by a decrease of banks´ state of confidence regarding the ability of borrowers to repay their debt – will reduce the supply of banking credit, increasing the credit interest rate (and banking spread), causing a contraction of aggregate demand and real output. As a result of the contraction in the supply of credit, money stock will also fall even if Central Bank does not change the level of money market interest rate. This means that a monetary contraction is compatible with a situation of a constant short-term nominal interest rate.

Up to now no efforts are made in the way to analyse the dynamics of Bofinger model under different assumptions about the determination of money market interest rate. The literature on monetary policy usually treats the issue of money market interest rate determination by means of a *Taylor Rule*, according to which changes in the value of money rare are the result of deviations of inflation from some target long-run level and/or deviations of output from full-employment level. Taylor rule can be considered the synthesis between two alternative policy rules: the *strict inflation targeting rule*, according to which nominal interest rate responds only to deviations of inflation from its long-run target; and *strict output targeting* (or Keynesian monetary policy) according to which nominal interest rate responds only to deviations of output from the full-employment level.

The objective of this paper is to analyse the stability properties of Bofinger model under different monetary policy rules. More specifically we will consider four different cases: (a) strict inflation targeting; (b) strict inflation targeting plus anti-cyclical fiscal policy; (c) strict output targeting and (d) “double mandate” for Central Bank (output and inflation stabilization). As we will see, long-run equilibrium of the model ***can be*** stable only in cases (a) and (d); which means that the existence of *nominal anchor* is of fundamental importance for macroeconomic stability. We also conclude that an anti-cyclical fiscal policy is incompatible with macroeconomic stability under a strict inflation targeting regime.

2. The IS-CM-PC Model

The IS-CM model developed by Bofinger considered inflation to be constant or equal to zero. In this section, we will present a slight modified version of IS-CM model in order to introduce inflation as a third endogenous variable. This new version will be called IS-CM-PC model since we will also add the Phillips Curve (PC) equation within model´s basic structure.

The economy under analysis will be a closed one, where output is homogenous and can be either consumed or invested. There are two interest rates: one for credit operations and other for operations in money market. Money and credit are considered to be perfect substitutes so that credit interest rate is the opportunity cost for holding money and supply of money is determined by the supply of banking credit. We will suppose that credit market is competitive so that banking rate is determined by demand and supply of credit. Since Central Bank set the money market interest rate, banks can get all the reserves they want in the money market at the rate fixed by monetary authorities. In other words, money is a full endogenous variable within the model at hand. This feature of monetary policy procedures gives a high level of elasticity to the supply of banking credit.

Output is demand determined in the short-run which means that goods market is not competitive since there is some level of price stickiness in the system, a feature that requires, at least, imperfect competition in goods market (Gali, 2008, p.5). Inflation is determined by inflation expectations and output gap according to the standard formulation of the Phillips curve equation.

2.1 Building blocks of the model

2.1.1 Equilibrium in credit market

Demand for credit (equal to demand for money) is considered to be a positive function of output, due to transaction motive for money demand, and a negative function of the credit rate of interest, since the alternative of holding money is to pay debts. This means that demand for credit can be expressed by the following equation:

$$c_r^d = \mu + \gamma Y - \alpha i_c \quad (1)$$

*Wh*ere: c_r^d is the demand for credit; Y is real output or income; i_c is the credit interest rate.

Supply of credit is considered to be a positive function of the banking mark-up (i.e the difference between the credit rate of interest and the money market rate), a positive function of real output and a negative function of the bank´s state of confidence about debtor´s ability to repay their debt. Following Bofinger (2001), we will suppose that credit supply is given by:

$$c_r^s = \frac{i_c - i}{\beta} nY \quad (2)$$ [1]

Where: c_r^s is the supply of credit, n is the number of banks in the banking sector, *i* is the money market interest rate, i_c is the credit rate of interest and β is the bank´s state of confidence. This equation indicates that banks will offer loans only if there is a positive interest rate margin. This results is due to the assumption that

[1] For a detailed derivation of this equation from first principles see Bofinger (2001, ch.3).

form the perspective of a individual bank both, volume of deposits and the deposit rate have no impact on the optimal supply of loans (Bofinger, 2001).

Finally, credit market clearing condition implies that[2]:

$$Y = \frac{2\beta(\alpha i_c - \mu)}{2\beta\gamma - n(i_c - i)} \quad (3)$$

Equation (3) determines the CM loci; that is the loci of all possible combinations between output and credit rate of interest for which credit market clears.

In order to determine the slope of CM loci we will take the first derivative of equation (3) in relation to Y and i_c. We get the following expression:

$$\frac{\partial Y}{\partial i_c} = 2\beta\left\{\frac{2\beta\alpha\gamma + n(\alpha i_c - \mu)}{\left(2\beta\gamma - n(i_c - i)\right)^2}\right\} > 0 \quad (4)$$

From (3), it is easy to check that: $\frac{\partial i_c}{\partial i} > 0$ and $\frac{\partial i_c}{\partial \beta} > 0$, that is: an increase in the money market interest rate produces an increase in the level of interest rate that clears the credit market; an increase in the bank´s subjective evaluations about debtor´s risk of default will also increase the banking rate of interest.

2.1.2 Equilibrium in the goods market

We will consider a closed economy with government activities. Consumption expenditures are supposed to be a function of disposable income only, so that there are no interest or wealth effects in the consumption function. Investment expenditures are supposed to be a function of (real) credit interest rate, since investment expenditures are financed by bank loans. For sake of simplicity, government expenditures (G_0) are supposed to be autonomous and taxes (T) are lump-sum.

Equilibrium in goods market is given by:

$$Y = kA_0 - kp_s - k\alpha(i_c - \pi^e) \qquad (5)$$

Where: Y is the current level of output, $k = \frac{1}{1-c}$ is the *autonomous expenditure multiplier*, $A_0 = C_0 + I_o - (1-c)T$, $P_s = T - G_0$ is the *primary fiscal surplus*, π^e is the expected rate of inflation.

II.1.3 Inflation and output: the Phillips curve.

[2] In order to assure a positive equilibrium value for the real output level we have to suppose $(\alpha i_c - \mu) > 0$

Inflation is supposed to be a function of expected rate of inflation and output gap, defined as the difference between current level of output and its "full-employment" level. We have the following equation for inflation determination:

$$\pi = \pi^e + h(Y - \bar{Y}) \quad (6)$$

Where: π is the current level of inflation, $\bar{Y}$ is the "full-employment" output.

2.2 Temporary General Equilibrium and Comparative Statics.

Following Hicks (1946), we will analyse first the temporary general equilibrium[3] of the model; i.e. that state of the economy where goods, credit and money markets clear given expectations regarding future rate of inflation. In the temporary general equilibrium, policy variables (money market interest rate and primary surplus) are supposed to be constant, since changes in monetary and fiscal policy are induced by the state of the economy; which means that some time is required for a policy change to be induced by a particular state of the economy at hand.

We will assume that conditions required by Brower´s fixed point theorem[4] are met so that a temporary general equilibrium exists. A visual representation of the model at equilibrium can be done by Figure 1 above.

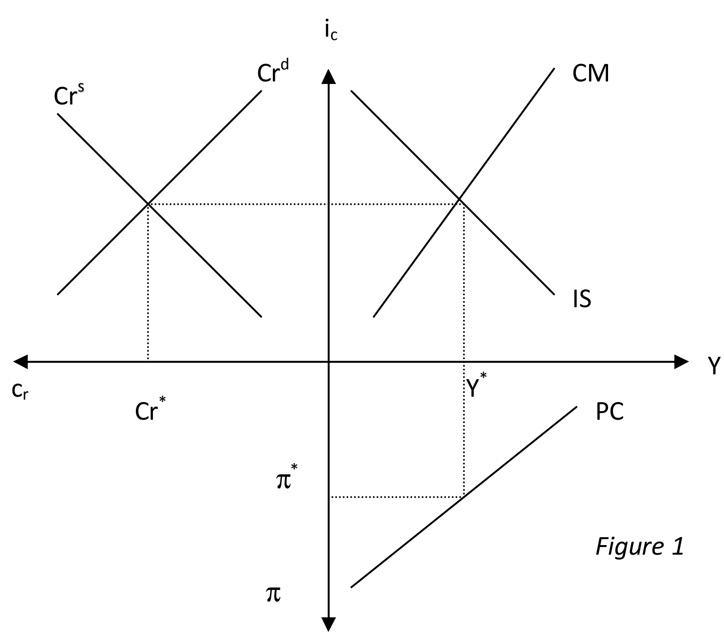

Figure 1

[3] The temporary general equilibrium model was developed by Hicks (1946) in order to represent a sequential economy where transactions are made at each point of time. Markets are supposed to be incomplete, since it is impossible to trade contingent claims over all possible state of nature. In other words, there is no such a thing as Arrow´s sucurities.
[4] See Varian (1992, p. 320).

For the comparative statics of the model, we will write equations (3), (5) and (6) in implicit form:

$$Y = g_0(i_c, i, \beta) \quad \text{(7a)}$$

$$Y = g_1(ps, i_c, \pi^e) \text{ (7b)}$$

$$\pi = g_2(\pi^e, Y) \quad \text{(7c)}$$

Taking total derivatives of the system of equations (7ª)-(7c) we get:

$$dY = \frac{\partial g_0}{\partial i_c} di_c + \frac{\partial g_1}{\partial i} di + \frac{\partial g_2}{\partial \beta} d\beta \quad \text{(8ª)}$$

$$dY = \frac{\partial g_1}{\partial ps} dps + \frac{\partial g_1}{\partial i_c} di_c + \frac{\partial g_1}{\partial \pi^e} d\pi^e \text{ (8b)}$$

$$d\pi = \frac{\partial g_2}{\partial \pi^e} d\pi^e + \frac{\partial g_2}{\partial y} dY \quad \text{(8c)}$$

Putting (8a) in (8c) we arrive at the following expression:

$$di_c = \frac{\left(\frac{\partial g_1}{\partial ps}\right)}{\left[\left(\frac{\partial g_0}{\partial i_c}\right)-\left(\frac{\partial g_1}{\partial i_c}\right)\right]} dps + \frac{\left(\frac{\partial g_1}{\partial \pi^e}\right)}{\left[\left(\frac{\partial g_0}{\partial i_c}\right)-\left(\frac{\partial g_1}{\partial i_c}\right)\right]} d\pi^e - \frac{\left(\frac{\partial g_0}{\partial i}\right)}{\left[\left(\frac{\partial g_0}{\partial i_c}\right)-\left(\frac{\partial g_1}{\partial i_c}\right)\right]} di - \frac{\left(\frac{\partial g_0}{\partial \beta}\right)}{\left[\left(\frac{\partial g_0}{\partial i_c}\right)-\left(\frac{\partial g_1}{\partial i_c}\right)\right]} d\beta \text{ (9)}$$

From (9) we can conclude that in temporary general equilibrium it is true that: $\frac{\partial i_c}{\partial ps} < 0$; i.e. an increase in the primary surplus will decrease banking rate of interest; $\frac{\partial i_c}{\partial \pi^e} > 0$, i.e. an increase in the expected rate of inflation will increase the banking rate of interest; $\frac{\partial i_c}{\partial i} > 0$, i.e. an increase in the money market rate of interest will increase the banking rate and $\frac{\partial i_c}{\partial \beta} > 0$, i.e. an increase in the bank´s subjective evaluation about borrowers´ default risk will also increase the banking rate of interest.

Writing (7a) in the inverse form we have:

$$i_c = i_c(Y, i, \beta) \quad \text{(7a´)}$$

Putting (7a´) in (7b) and taking the total derivative of the resulting expression we get:

$$dY = \frac{\left(\frac{\partial g_1}{\partial ps}\right)}{\left[1-\left(\frac{\partial g_1}{\partial i_c}\right)\left(\frac{\partial i_c}{\partial Y}\right)\right]} dps + \frac{\left(\frac{\partial g_1}{\partial \pi^e}\right)}{\left[1-\left(\frac{\partial g_1}{\partial i_c}\right)\left(\frac{\partial i_c}{\partial Y}\right)\right]} d\pi^e - \frac{\left(\frac{\partial g_1}{\partial i_c}\right)\left(\frac{\partial i_c}{\partial i}\right)}{\left[1-\left(\frac{\partial g_1}{\partial i_c}\right)\left(\frac{\partial i_c}{\partial Y}\right)\right]} di - \frac{\left(\frac{\partial g_1}{\partial i_c}\right)\left(\frac{\partial i_c}{\partial \beta}\right)}{\left[1-\left(\frac{\partial g_1}{\partial i_c}\right)\left(\frac{\partial i_c}{\partial Y}\right)\right]} d\beta \quad \text{(10)}$$

From (10) we can conclude that: $\frac{\partial Y}{\partial ps} < 0$, $\frac{\partial Y}{\partial i} < 0$, $\frac{\delta Y}{\partial \pi^e} > 0$, $\frac{\partial Y}{\partial \beta} < 0$.

3 – Stability Analysis of Different Monetary Policy Rules.

In the temporary general equilibrium, both inflation expectations and policy variables are kept constant. We will now turn our attention to the "long-run" solution of the model where both type of variables adjusts to the state of the economy.

In order to do that, we have to specify the monetary policy rules followed by monetary authorities. We will consider three cases: (a) strict inflation targeting, (b) active monetary policy rule and (c) Taylor rule. As it will be show, in case (a) the system can be dynamically stable; i.e. converge to a steady state or long-run equilibrium position. This will happen if the speed of adjustment of money market interest rate to divergences between current inflation and the long-run target for this variable is higher than some critical level. Case (c) can also be dynamically stable if the ratio between weight of inflation and weight of output in the monetary policy rule is higher than some critical level. In case (b), where monetary policy aims only to stabilize the level of output around its "full-employment" value, the system will be dynamically unstable for all possible values of the parameters of the dynamic equations. **These results show that a "conservative" monetary policy is required by macroeconomic stability**. Finally, we will analyze the joint interaction of a strict inflation targeting and an anti-cyclical fiscal policy where primary surplus is reduced (increased) whenever output is lower (higher) than "full-employment" output. As we will see, the system will be also dynamically unstable, which means that fiscal policy should not be used for stabilizing the economy.

3.1.1 Adaptive expectations and strict inflation targeting.

Let us consider a dynamical system where the time path of inflation expectations and money market interest rate are given by:

$$\frac{d\pi^e}{dt} = \vartheta_0(\pi - \pi^e) \quad (11)$$

$$\frac{di}{dt} - \theta_1(\pi - \bar{\pi}) \quad (12)$$

Equation (11) shows that inflation expectations are adaptive in the sense that they are revised according to the expectations errors committed by economic agents. It is important to notice that equation (11) will be compatible with rational expectations hypothesis (identical to perfect foresight in a deterministic world) if $\theta_0 \to \infty$.

Equation (12) shows that monetary authorities will change the money market interest rate only as a reaction against deviations of current inflation to the inflation target, which is exogenously given. This means that monetary authorities follow a monetary rule that can be named “strict inflation targeting”, since the only concern of monetary policy is to stabilize the rate of inflation around its long-run target.

We will consider the dynamical stability of the system around its long-run equilibrium position. In order to that, we will take a linear approximation of the system around its equilibrium position by means of the first element of the Taylor expansion. After that, we get the following expression in matrix form:

$$\begin{bmatrix} \frac{d\pi^e}{dt} \\ \frac{di}{dt} \end{bmatrix} = \begin{bmatrix} \theta_0\left(\frac{\partial\pi}{\partial Y}\frac{\partial Y}{\partial\pi^e} + \left(\frac{\partial\pi}{\partial\pi^e} - 1\right)\right) & \theta_0\frac{\partial\pi}{\partial Y}\frac{\partial Y}{\partial i} \\ \theta_1\left(\frac{\partial\pi}{\partial Y}\frac{\partial Y}{\partial\pi^e} + \frac{\partial\pi}{\partial\pi^e}\right) & \theta_1\frac{\partial\pi}{\partial Y}\frac{\partial Y}{\partial i} \end{bmatrix} \begin{bmatrix} \pi^e - \pi_0^e \\ i - i_0 \end{bmatrix} \quad (13)$$

Dynamical stability of the system requires that trace and determinant of the Jacobian matrix are negative and positive, respectively. From (13) we get:

$$DET\ |J| = -\theta_0\theta_1\frac{\partial\pi}{\partial\pi^e}\frac{\partial Y}{\partial i} > 0 \ (14)$$

$$Trace\ |J| = \left[\theta_0\left(\frac{\partial\pi}{\partial Y}\frac{\partial Y}{\partial\pi^e} + \left(\frac{\partial\pi}{\partial\pi^e} - 1\right)\right) + \theta_1\frac{\partial\pi}{\partial Y}\frac{\partial Y}{\partial i}\right] \quad (15)$$

In equation (15) we can see that the trace of Jacobian matrix can be positive or negative depending on the magnitudes of the first and the second element of the expression at hand. This means that speed of adjustment of money market interest rate will be of fundamental importance for the stability of the system. In fact, Trace will be positive if and only if: $\theta_1 > -\frac{\left(\frac{\partial\pi}{\partial Y}\frac{\partial Y}{\partial\pi^e} + \left(\frac{\partial\pi}{\partial\pi^e} - 1\right)\right)}{\left(\frac{\partial\pi\partial Y}{\partial Y\partial i}\right)}\theta_0 = \varphi$; i.e. the speed of adjustment of money market interest rate must be higher than some critical level φ in order for the system to converge to its long-run equilibrium.

The problem with this result is that stylized facts about monetary policy and central banking shows the existence of a phenomenon known as “interest rate smoothing”; i.e. the empirical behavior of money market interest rates exhibits a very high level of inertia in the time series of this variable (Barbosa, 2004). This means that the observed stability of inflation targeting economies cannot be attributed to this kind of response of short-term interest rate. Observe that this result is close to that one found in Gali (2008). Note that in contrast with the “contemporaneous” rule, determinacy of equilibrium requires that the Central Bank reacts neither too strongly nor too weekly to deviations of inflation

3.1.2 Adaptive expectations, strict inflation targeting and anti-cyclical fiscal policy.

As we had seen above, the adoption of a monetary policy rule based in a strict inflation targeting is compatible with dynamical stability of the economy at hand. However, this result requires a high speed of adjustment of money market interest rates which is in contradiction to the empirical behavior of these variables.

These observations made us to investigate if dynamical stability can be achieved by the addition of an anti-cyclical fiscal policy in the framework at hand. In this setting, monetary policy would be responsible for the stabilization of inflation around its long-run target whereas fiscal policy would be responsible for the stabilization of output around its "full-employment" level. This framework of monetary and fiscal policy should then be considered a synthesis between "Monetarist" and "Keynesian" principles of economic policy.

The dynamical system of equations is given by:

$$\frac{d\pi^e}{dt} = \theta_0(\pi - \pi^e) \quad (11)$$

$$\frac{di}{dt} = O_1(\pi - \bar{\pi}) \quad (12)$$

$$\frac{dps}{dt} = \theta_2(Y - \bar{Y}) \quad (16)$$

In equation (16) we can see that primary surplus will increase (decrease) whenever actual output is above (below) "full-employment" level. This is the essence of Keynesian demand management policies.

Taking a linear approximation of the system around its equilibrium position by means of the first element of the Taylor expansion, and writing the resulting system in matrix form we get:

$$\begin{bmatrix} \frac{d\pi^e}{dt} \\ \frac{di}{dt} \\ \frac{dps}{dt} \end{bmatrix} = \begin{bmatrix} O_0\left\{\frac{\partial\pi}{\partial Y}\frac{\partial Y}{\partial\pi^e} + \left(\frac{\partial\pi}{\partial\pi^e} - 1\right)\right\} & O_0\left\{\frac{\partial\pi}{\partial Y}\frac{\partial Y}{\partial i}\right\} & O_0\left\{\frac{\partial\pi}{\partial Y}\frac{\partial Y}{\partial ps}\right\} \\ \theta_1\left\{\frac{\partial\pi}{\partial Y}\frac{\partial Y}{\partial\pi^e} + \left(\frac{\partial\pi}{\partial\pi^e}\right)\right\} & \theta_1\left\{\frac{\partial\pi}{\partial Y}\frac{\partial Y}{\partial i}\right\} & \theta_1\left\{\frac{\partial\pi}{\partial Y}\frac{\partial Y}{\partial ps}\right\} \\ \theta_2\frac{\partial Y}{\partial\pi^e} & \theta_2\frac{\partial Y}{\partial i} & \theta_2\frac{\partial Y}{\partial ps} \end{bmatrix} \begin{bmatrix} \pi^e - \pi_0^e \\ i - i_0 \\ ps - ps_0 \end{bmatrix} \quad (17)$$

Theorem 1. (Routh-Hurwitz Criteria). Given the polynomial, $P(\mu) = \mu^n + a_1\mu^{n-1} + \cdots + a_{n-1}\mu + a_n = 0$ where the coefficients a_i are real constants, $i = 1, \ldots, n$, define the n Hurwitz matrices using the coefficients a_i the characteristic polynomial:

$$H_1 = (a_1),\ H_2 = \begin{pmatrix} a_1 & 1 \\ a_3 & a_2 \end{pmatrix},\ H_2 = \begin{pmatrix} a_1 & 1 & 0 \\ a_3 & a_2 & a_1 \\ a_5 & a_4 & a_3 \end{pmatrix}$$

and $H_n = \begin{pmatrix} a_1 & 1 & 0 & 0 & \cdots & 0 \\ a_3 & a_2 & a_1 & 0 & \cdots & 0 \\ a_5 & a_4 & a_3 & a_2 & & 0 \\ & \vdots & & & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & a_n \end{pmatrix}$, where $a_j = 0 \; if \; j > n$. All the roots of the polynomial $P(\mu)$ are negative or have negative real part if the determinants of all Hurwitz matrices are positive: $\det H_j > 0\,, j = 1, \ldots, n$.

For a proff of the Routh-Hurwitz criteria, see Gantmacher (1964).

Corollary: Suppose the coefficients of the polynomial are real. If all the roots of the characteristic polynomial $P(\mu) = \mu^n + a_1\mu^{n-1} + \cdots + a_{n-1}\mu + a_n = 0$ are negative or have negative real parts, then the coefficients $a_i > 0$ for $i = 1,2, \ldots n$.

Proof. The corollary is a direct consequence of the Routh-Hurwitz criteria. The characteristic equation can be factored into the form:

$$(\mu + r_1)\ldots(\mu + r_{k_1})(\mu^2 + 2c_1\mu + c_1^2 + d_1^2)\ldots(\mu^2 + 2c_{k_2}\mu + c_{k_2}^2 + d_{k_2}^2) = 0$$

Where the roots are $-r_i < 0 \; for \; i = 1, \ldots, k_1$ and the complex roots are $-c_j \pm d_j i \;\; for \; j = 1, \ldots, k_2$ and $k_1 + 2k_2 = n$. If all of the roots are either negative or have negative real part, then $r_i > 0$ and $c_j > 0$ **for all** i and j. Thus, all the coefficients in the factored characteristic equation are positives.

Remark. Routh's stability criteria describe how many roots have positive real part. The remaining roots have negative or zero real part. A necessary and sufficient condition for all roots to have strictly negative real part is that all coefficients in the polynomial $P(\mu)$ are strictly positive, i.e. no zero coefficients.

Stability analysis of a 3x3 system of linear differential equations is made by means of Routh-Hurwitz theorem (Takayama, 1993). The characteristic equation of the system (17) is given by:

$$A\mu^3 + B\mu^2 + C\mu + D = 0 \quad (18)$$

Where:

$A = -1 < 0$

$$B = \left\{\theta_0 \left[\frac{\partial \pi}{\partial Y}\frac{\partial \pi}{\partial \pi^e} + \left(\frac{\partial \pi}{\partial \pi^e} - 1\right)\right] + \theta_1 \left(\frac{\partial \pi}{\partial Y}\frac{\partial Y}{\partial i}\right) + \theta_2 \frac{\partial Y}{\partial ps}\right\} = ?$$

$$C = \theta_1\theta_2\left(\frac{\partial\pi}{\partial Y}\frac{\partial Y}{\partial i}\right)\frac{\partial Y}{\partial ps} + \theta_0\theta_2\left[\frac{\partial\pi}{\partial Y}\frac{\partial\pi}{\partial\pi^e} + \left(\frac{\partial\pi}{\partial\pi^e} - 1\right)\right]\frac{\partial Y}{\partial ps} + \theta_0\theta_1\left[\frac{\partial\pi}{\partial Y}\frac{\partial\pi}{\partial\pi^e} + \left(\frac{\partial\pi}{\partial\pi^e} - 1\right)\right]\left(\frac{\partial\pi}{\partial Y}\frac{\partial Y}{\partial i}\right) + \theta_0\vartheta_1\left(\frac{\partial\pi}{\partial Y}\frac{\partial Y}{\partial i}\right)\left[\frac{\partial\pi}{\partial Y}\frac{\partial\pi}{\partial\pi^e} + \frac{\partial\pi}{\partial\pi^e}\right] < 0$$

$$D = 0$$

Since D=0, the conditions required for the stability of the system are not satisfied, for the results of the *corollary 1*, the long-run equilibrium is dynamically unstable. This means that an anti-cyclical fiscal policy is incompatible with dynamic stability under a strict inflation targeting regime.

4 – Summary of the results

In the present paper we have analysed the stability properties of Bofinger (2001) model under different monetary policy rules. We have considered four different cases: (a) strict inflation targeting; (b) strict inflation targeting plus anti-cyclical fiscal policy; (c) strict output targeting and (d) "double mandate" for Central Bank (output and inflation stabilization). As it was shown by stability analysis of the system of differential equations, long-run equilibrium of the model ***can be*** stable only in cases (a) and (d); which means that the existence of *nominal anchor* such a certain target for inflation rate is of fundamental importance for macroeconomic stability. However, it is also true that a "Keynesian activist monetary policy" is not incompatible with macroeconomic stability. We also conclude that an anti-cyclical fiscal policy is incompatible with macroeconomic stability under a strict inflation targeting regime. The stabilization of output should be a task for monetary policy that operates under a "double mandate" for Central Bank. Fiscal policy must be concerned with other issues such as, for example, stabilization of public debt.

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