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**The Dynamics of Inflation and Currency Substitution
in a Small Open Economy**

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In this paper, we analyse the relationship between money and inflation in a small open economy where domestic and foreign currencies are perfect substitutes as means of payment. It is shown that, if the path of domestic money supply is such that individuals find it optimal to change the currency in which transactions are settled, there will be an adjustment period during which domestic inflation adjusts so as to equalise the foreign inflation rate. In the case of a disinflation program, it is shown that the foreign currency is not necessarily abandoned as means of payment. The results obtained are consistent with both dollarisation hysteresis and reversibility, without requiring the specification of dollarisation costs.

JEL Classification: E41, E52, F41.
Key words: currency substitution, dollarisation, money-demand and hysteresis.

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1. Introduction

In high inflation countries, the rapid erosion of the value of domestic currency leads agents to substitute it by foreign currency in either any or all of the basic functions of money. This phenomenon is usually called currency substitution, or simply *dollarisation*, and is a matter of concern for policymakers, as it constrains their ability to implement economic policy. In particular, it aggravates the inflationary consequences of seigniorage and reduces the effectiveness of monetary policy, even under flexible exchange rates (for a literature review, see Giovannini and Turtelboom, 1992).

A fact that is hard to explain by conventional money demand theories is that dollarisation exhibits a persistent behaviour unrelated to either current inflation or interest rate differentials. In some countries the implementation of successful anti-inflation programmes was not enough to bring down sharply the demand for dollars (evidence of dollarisation hysteresis was presented by Guidotti and Rodriguez, 1992, for Bolivia, Mexico, Peru and Uruguay, by Kamin and Ericsson, 1993, for Argentina, by Clements and Schwartz, 1993, for Bolivia, by Mueller, 1994, for Lebanon). However, in some other countries, dollarisation was easier to reverse (reversibility patterns were identified in Egypt, Yemen and Chile, by Mueller, 1994, and in some transition countries in East Europe, by Sahay and Végh, 1996).

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This paper provides a theoretical explanation for this phenomenon, using a small open economy model where domestic and foreign currencies are perfect substitutes as means of payment. The assumption of perfect means of payment substitutability (PMS) was previously discussed by Kareken and Wallace (1981) for the case of international currencies, but it has been increasingly used in models analysing the phenomenon of dollarisation hysteresis in less developed countries (e.g., Guidotti and Rodriguez, 1992, Uribe, 1997, and Reding and Morales, 1999). Guidotti and Rodriguez (1992) proposed a PMS model with switching costs, that is costs associated to the *change* of the dollarisation level. These costs result in an inflation band within which agents choose not to switch between currencies. Hence, de-dollarisation may only be achieved if the domestic inflation decreases enough to offset the de-dollarisation costs. The PMS model was extended by Uribe (1997) and Reding and Morales (1999), who stressed the role of network economies, instead of switching costs, as the source of non-linearities in the relationship between the money demand and inflation.

A limitation of Guidotti and Rodriguez (1992), Uribe (1997) and Reding and Morales (1999) is that the inflation rate was assumed exogenous, instead of depending on the money demand. To the extent that changes in the aggregate money demand impact on the inflation rate, the dynamic analysis made by these authors may be misleading. This paper extends the PMS model by allowing the inflation rate to be determined by money demand and supply and by then exploring the implied short-term dynamics. The results obtained are consistent with the evidence of both dollarisation hysteresis and reversibility, as described in the literature, without the

need to introduce dollarisation costs. The model differs from that of Kareken and Wallace (1981) because foreign residents are not allowed to hold the domestic currency (only *asymmetric* currency substitution is considered) and there is a minimum constraint on the amount of domestic money holdings.

The paper is organised as follows. In Section 2, we present the basic model. In Section 3, we discuss the dynamics of inflation and currency substitution. In Section 4 we discuss the indeterminacy associated to a disinflation programme. Conclusions are presented in Section 5.

2. The model

Consider a small open economy that operates under flexible exchange rates and perfect capital mobility. Transactions may be carried out by using both domestic and foreign money, the two currencies being perfect substitutes. The economy is inhabited by a fixed number N of atomistic individuals who are blessed with perfect foresight and infinite life. Foreign residents do not hold domestic money balances. There exists only one international indexed bond and one consumption good. There are neither impediments to trade nor transport costs, so that purchasing power parity holds instantaneously. Time is continuous and in each instant individuals receive a fixed real income and a government transfer. Individuals maximise their lifetime utility function subject to a wealth constraint. Seigniorage is the only source of government revenues and a balanced budget is imposed each period. Foreign

seigniorage revenues are appropriated by the foreign country, so that a trade surplus occurs when domestic residents hold foreign money balances. The complete set-up of the model is presented in Appendix 1.

The consumption good may be either purchased with P units of domestic currency (M) or P^* units of foreign currency (F). The liquidity constraint requires purchases to be made with money held at the time of the transaction and broad money velocity is set equal to one. Although individuals are allowed to hold cash in excess of their purchasing needs, as long as they have perfect foresight and an interest-bearing asset dominates money, the cash-in-advance constraint holds as equality[1]:

$$m_t + f_t = c_t, \tag{1}$$

where $m_t = M_t / NP_t$ and $f_t = F_t / NP_t^*$ are the home country per capita holdings of domestic and foreign real money balances and c_t is the per capita real consumption.

To capture the existence of institutional limits to dollarisation (e.g., transactions involving the government or closely monitored by it, which cannot be

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[1] Note that, in terms of the timing of transactions, this cash-in-advance constraint is not equivalent to those of the discrete time models. In continuous time, it is impossible to distinguish between moments in which money is held by consumers from moments in which money is held by firms. Each instant, individuals have to carry an amount of cash equal to their consumption needs and

settled in the foreign currency[2]), it will be assumed that the amount of transactions carried out with domestic currency cannot fall below a minimum level, $\overline{m}$. This assumption, together with (1) and the observation that it does not pay to be short on the foreign currency, implies:

$$m_t \in [\overline{m}, c_t].\qquad(2)$$

With the usual assumption that the rate of time preference is equal to the real interest rate on the indexed bond, the optimal consumption becomes constant over time. Thus, without loss of generality we can normalise it to unity, so as to interpret m as a proportion. Given the liquidity constraint (1) and the assumption of perfect means-of-payment substitutability, the per capita demand for domestic money balances becomes:

$$m_t = \begin{cases} 1 & if \quad \pi < \pi^* \\ any & if \quad \pi = \pi^* , \\ \overline{m} & if \quad \pi > \pi^* \end{cases}\qquad(3)$$

where $\pi_t = \dot{P_t}/P_t$ and $\pi_t^* = \dot{P_t^*}/P_t^*$. This is a common arbitrage condition, stating that the domestic currency holdings of a representative individual will be respectively 1, any allowed amount or $\overline{m}$, as the domestic inflation is lower, equal or higher than

there is no way of avoiding the wealth erosion caused by the inflation taxes. At most, individuals are able to change the currency composition of their cash holdings.

[2] Stubbornness of domestic money was observed, even during extremely high inflation rates, such as the German hyperinflation (Giovannini and Turtelboom, 1994).

the foreign inflation rate. An implication is that, for any currency substitution to occur in this model, the domestic and foreign inflation rates must be equal, because only in that case will individuals hold a positive amount of both currencies.

Assuming that the nominal money supply is a continuous function of time, given by $\mu_t = \dot{M}_t / M_t$, the real money stock evolves according to:

$$\dot{m}_t = (\mu_t - \pi_t) m_t . \tag{4}$$

The equilibrium paths of inflation and real money balances may be defined in the following way:

Definition 1 (equilibrium). A money market equilibrium in this model is given by a time continuous function m_t for the representative consumer, and a function π_t for the whole economy, such that, given the nominal rate of money growth μ_t, the per capita real money stock (4) is consistent with individual money demands (3).

In Definition 1, not only instantaneous money market equilibrium is required, but also continuity of the real value of domestic money balances over time. This is because the nominal money supply is time continuous and jumps in the price level are rule out by the assumptions of perfect foresight and no arbitrage opportunities. From (1), the demand for foreign currency is given by $f_t = 1 - m_t$.

Note that, when agents decide upon their cash holdings, they take the inflation path as given, because individually they cannot change the price level. Due to the

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small country assumption, the domestic demand for foreign currency does not affect the foreign rate of inflation.

In the following discussion, it will be useful to take as a benchmark the trivial solutions corresponding to time-invariant paths of μ.

Definition 2 (trivial solutions). When μ is time-invariant, the equilibrium path of m is: (i) $m_t = \overline{m}$ if $\mu > \pi^*$; (ii) $m_t = 1$ if $\mu < \pi^*$; (iii) $m_t = m$ for any $\overline{m} \le m \le 1$ if $\mu = \pi^*$.

To see this, we first note that any path with $\dot{m} \ne 0$ cannot be an equilibrium. Indeed, given the money demand (3), any non-horizontal path of *m* would only be possible if $\pi = \pi^*$. However, this cannot happen in case (iii) – because the equality of inflation rates requires an horizontal money demand path - or in case (i) [(ii)], in which this would require the money demand to grow [decline] exponentially forever, and bubbles are ruled out by (2). Hence, only horizontal money demand paths are allowed, and in these cases we know by (4) that the domestic inflation rate is equal to the rate of money creation.

Note that, in cases (i) and (ii) individual optimisation prevents individuals from abandoning their horizontal money demand paths, whereas in case (iii) any level of *m* is optimal from the individual point of view. In this case the two currencies are perfect substitutes both on the demand and the supply side and the price level becomes undetermined, as in Kareken and Wallace (1981). However, only horizontal paths of *m* are equilibria, because otherwise domestic and foreign inflation rates

would not be equal, causing the existing stock of cash holdings to be inconsistent with individual money demands.

3. The dynamics of inflation and currency substitution

In this section we explore the adjustment dynamics of the model when the money supply path is such that individuals find it optimal to alter their cash-holdings.

Proposition 1 (the dynamics of inflation and currency substitution). If the domestic rate of money growth increases linearly over time, according to $\mu_t = \mu + \sigma t$, with $\sigma > 0$ and $0 < \mu < \pi^*$, then the equilibrium paths of m and π are:

$$
m_t = \begin{cases} 1 & t < \tilde{t} \\ \overline{m}\exp\left[\dfrac{\sigma}{2}t^2 - \left(\pi^* - \mu\right)t + \dfrac{\left(\pi^* - \mu\right)^2}{2\sigma}\right] & \tilde{t} \le t < \overline{t} \\ \overline{m} & t \ge \overline{t} \end{cases} \tag{5}
$$

and

$$
\pi_t = \begin{cases} \mu_t < \pi^* & t < \tilde{t} \\ \pi^* & \tilde{t} < t \le \overline{t} \\ \mu_t > \pi^* & t > \overline{t} \end{cases}, \tag{6}
$$

where $\overline{t} = \dfrac{\pi^* - \mu}{\sigma}$ and $\tilde{t} = \overline{t} - \sqrt{\dfrac{-2\ln \overline{m}}{\sigma}}$.

Proof: See Appendix 2.

To interpret Proposition 1 we refer to Figure 1. We depict in the bottom diagram the inflation rate and the rate of money growth and in the upper diagram the money demand. Given the path of inflation, (6), individuals decide to use only domestic currency in the first segment and the maximum allowed level of foreign currency in the third segment. In the intermediate segment, individuals are adjusting their cash holdings between these two corner solutions. Since this requires a reduction in real money balances, the domestic inflation rate rises relative to the rate of money creation, μ. For this adjustment path to be equilibrium, however, domestic and foreign inflation rates must be equal, because only in this case will individuals accept to hold a positive amount of both currencies. If the aggregate money demand declined slower or faster, the inflation differential would not be zero, and no continuous path, such that individual money demands and supply were continuously satisfied, could be found.

An interesting property of this equilibrium is that individuals have to wait for the right moment to start adjusting their cash holdings. This property results from the existence of an upper limit in the individual cash holdings. In general, observing the partial derivatives of t_A with respect to the relevant parameters, the following corollary holds:

Corollary 1 (on the timing of the inflation adjustment). Under the conditions of Proposition 1, the domestic inflation adjusts earlier: (a) the lower the foreign inflation rate; (b) the higher the autonomous money growth, μ; (c) the lower the

minimum allowed proportion of domestic money balances, $\overline{m}$. The time dependent term of the money growth rule, σ, has an ambiguous effect on the timing of the inflation adjustment.

Lines (a) and (b) say that, the earlier the moment $\overline{t}$ (when the currency substitution process is complete), the earlier agents will start adjusting their cash holdings, for a given adjustment speed. Line (c) says that the greater the size of the allowed currency substitution, the sooner the adjustment starts. According to this, as $\overline{m}$ approaches zero, $\tilde{t}$ moves backward, because more time is necessary to switch between currencies. In the limit case in which the minimum allowed amount of cash holdings is zero, actual holdings will be zero from time minus infinity, because agents know that at moment $\overline{t}$ the domestic currency will not have any value. Finally, the time dependent term of the money growth rule affects both moment $\overline{t}$ and the speed of the adjustment process, so that it has an ambiguous effect on the timing of the inflation overshooting.

Corollary 2 (on the length of the adjustment period). Under the conditions of Proposition 1, the length of the adjustment period, $\lambda = \overline{t} - \widetilde{t}$, is: (a) unaffected by the difference $\left(\pi^* - \mu\right)$; (b) larger, the lower the minimum allowed proportion of domestic balances, $\overline{m}$; (c) larger, the lower the rate of monetary expansion, σ.

These results follow straightforwardly from the partial derivatives of $\lambda = \left(-2\ln\overline{m}/\sigma\right)^{1/2}$ with respect to the relevant parameters. Line (a) says that, although different levels of μ and π^* affect the absolute position of $\overline{t}$, they do not

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change the length of the adjustment period. Line (b) says that the length of the adjustment period depends on the allowed level of currency substitution. When $\overline{m}=1$, there is no place for currency substitution, so that the length of the adjustment period is zero. As $\overline{m}$ approaches zero, $\hat{t}_0$ moves backward (relative to $\bar{t}$), as more time is necessary to adjust the cash holdings. Although the rate of monetary expansion has an ambiguous effect on the absolute timing of the overshooting (Corollary 1) line (c) of Corollary 2 says that the overshooting starts earlier relative to moment $\bar{t}$, the lower the rate of money growth.

4. The dynamics of a disinflation programme

To investigate the dynamics of a disinflation programme, assume that initially the domestic inflation rate is higher than the foreign inflation rate and that individuals start holding $m=\overline{m}$. The anti-inflation programme consists reducing the rate of domestic money growth until it reaches the foreign inflation rate, remaining at this level thereafter. When this is so, the following proposition holds:

Proposition 2 (the dynamics of a disinflation programme). If the domestic nominal money growth follows $\mu_t = \begin{cases} \mu - \sigma t & t < \bar{t} \\ \pi^* & t \geq \bar{t} \end{cases}$, *where* $\mu > \pi^*$ *and* $\bar{t} = \dfrac{\mu - \pi^*}{\sigma}$, *then there exist an infinity of equilibrium paths for m and* π, *given by:*

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$$
m_t = \begin{cases} \overline{m} & t < \tilde{t} \\ m(\overline{t})\exp\left[-\dfrac{\sigma}{2}t^2 + \left(\mu - \pi^*\right)t - \dfrac{\left(\mu - \pi^*\right)^2}{2\sigma}\right] & \tilde{t} \leq t < \overline{t} \\ m(\overline{t}) & t \geq \overline{t} \end{cases}
\tag{7}
$$

and

$$
\pi_t = \begin{cases} \mu - \sigma t > \pi^* & t < \tilde{t} \\ \pi^* & t \geq \tilde{t} \end{cases},
\tag{8}
$$

where $m(\overline{t})$ *is any of those in (2) and* $\tilde{t} = \overline{t} - \sqrt{\dfrac{-2\ln(\overline{m}/m(\overline{t}))}{\sigma}}$.

Proof: Same as Proposition 1, except that in this case any level of m is equilibrium after $\overline{t}$.

This case has multiple equilibria because after the disinflation programme the two currencies become perfect substitutes on the supply side, as in the flat equilibrium (iii) of Definition 2. Hence, there is an adjustment path for each allowed level of dollarisation in the new steady state.

To illustrate Proposition 2 we refer to Figure 2. In panel (a) we depict two possible paths where the disinflation programme leads to a decline in the level of dollarisation. The thick line depicts the case in which the adjustment programme is successful in driving out the foreign currency, that is, $m(\overline{t}) = 1$. The thin line represents a case in which some dollarisation remains at the end of the adjustment programme. In both cases the demand for domestic money rises along the adjustment period, implying an inflation rate lower than the rate of money creation and equal to

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the foreign inflation rate, as required by the non-arbitrage condition. Since in the thin line case the cash switching is smaller, the inflation adjustment starts later and the adjustment period is shorter ($\tilde{t}' > \tilde{t}$) [3].

In panel (b) we depict the case in which the disinflation programme does not at all reduce the dollarisation level. In this case, $m(\bar{t}) = \bar{m}$, implying $\bar{t} = \tilde{t}$ and the inflation rate is always equal to the rate of money growth.

Note that in the extreme case depicted by the thick line in panel (a) domestic inflation adjusts once-and-for-all at moment $\hat{t}$, while the demand for foreign currency declines smoothly until moment $\bar{t}$. In all the other cases some dollarisation remains, despite the success of the anti-inflation policy. In all cases the demand for foreign currency fails to respond linearly to changes in the inflation rate, which is in accordance to the empirical evidence.

A planner would, if possible, choose the equilibrium path leading to *m=1*, because in that case all seigniorage revenues would accrue to the domestic economy. However, in a decentralised economy, there is no way of assuring that this equilibrium will occur[4].

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[3] Obviously, results similar to those discussed in corollaries 1 and 2 apply for each equilibrium path of Proposition 2.

[4] Obviously, any small barrier to change, such a lump sum switching cost, would favour the non-reversibility outcome, depicted in panel (b).

Obviously, the indeterminacy holds only when the disinflation programme reduces the rate of money growth exactly to the level of foreign inflation. However, in Appendix 3 it is shown that, with network externalities, such indeterminacy is extended to a range of money supply paths.

5. Conclusions

In this paper we analyse the behaviour of money demand and inflation in a small open economy where a domestic and a foreign currency are perfect substitutes as means of payment. It is shown that, if the domestic money growth is such that currency substitution becomes optimal from the individual point of view, there will be an adjustment period during which the domestic inflation adjusts to equal the foreign inflation rate. The length of the adjustment period depends on the rate of money growth and the minimum allowed level of domestic money holdings. In the case of a disinflation programme, it is shown that the economy does not necessarily return to the single-currency regime, despite that being the superior outcome in terms of aggregate welfare. The model captures the different patterns of dollarisation identified in the literature, namely reversibility in some cases and non-reversibility in others, without requiring the specification of dollarisation costs.

Appendix 1

The lifetime utility of the representative individual is given by

$$U = \int_0^\infty u(c_t)e^{-rt}dt ,$$ (a1)

where c denotes real consumption, r is the subjective rate of discount, and u is a well-behaved utility function.

Individuals hold real wealth in the format of one international indexed bond, b, yielding a real return r and real money balances, m and f. In each moment in time individuals receive an exogenous real income, y, and a government transfer, τ. The change in real wealth is given by

$$\dot{w} = rb - \pi_t m_t - \pi^* f_t + (y_t + \tau_t - c_t) ,$$ (a2)

where $w_t = b_t + m_t + f_t$. The foreign inflation rate and the real interest rate on the indexed bond are exogenous and assumed constant. Using (1), integrating over time and imposing the no Ponzi game condition $\lim_{t \to \infty} w_t e^{-rt} = 0,$ we obtain the consumer's lifetime budget constraint:

$$w_0 + \int_0^\infty \left[y_t + \tau_t - c_t \left(1 + r + \pi^* \right) \right] e^{-rt} dt + \int_0^\infty \left[m_t \left(\pi^* - \pi \right) \right] e^{-rt} dt = 0.$$ (a3)

The last term of (a3) shows that the individual real wealth depends on how much she gains (or looses) by shifting from the foreign inflation tax to the domestic

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inflation tax. It is also clear that the money demand path that achieves the highest consumption level is the one that maximises the last term, implying that the choice between currencies is separable from the saving-consumption decision.

Each consumer maximises his utility function (a1) subject to (1), (2) and (a2), taking the inflation paths and government transfers as given. One first-order condition leads to:

$$u'(c_t) = \lambda_t [1 + r + \pi^*] \tag{a4}$$

and the co-state dynamics is:

$$\dot{\lambda}_t = 0. \tag{a5}$$

Equation (a4) is the usual condition stating that the marginal utility of consumption divided by the shadow price of wealth is equal to the price of consumption, which is one plus the opportunity cost of money. The latter is measured in terms of the foreign currency, being the gains or losses associated to the use of domestic currency captured by the wealth effect in (a3). Since the real interest rate is equal to the rate of time preference, both the shadow price of wealth and the consumption level will be constant over time.

Since the individual objective function is linear in *m* and (2) defines a closed control set, the resulting money demand for domestic currency is given by (3).

To close the model, we assume that government consumption is zero and that seigniorage is the only source of revenue. We also impose a balanced budget constraint in each period. In real per capita terms,

$$\tau_t = \frac{N \dot{M}_t}{NP_t} = \mu_t m_t. \tag{a6}$$

This restriction assures that domestic seigniorage has no income effects. However, when individuals make their decisions they take government transfers as given.

The foreign seigniorage revenues are assumed to be appropriated by the foreign country. Using (4), (a2) and (a6) and integrating over time, we obtain the economy's inter-temporal resource constraint in real per capita terms:

$$b_0 + f_0 + \frac{y - c}{r} = \int_0^\infty \left(r + \pi^* \right) f_t e^{-rt} dt, \tag{a7}$$

where b_0 and f_0 denote the initial real asset holdings. Equation (a7) says that, as long as domestic residents hold foreign money balances, they suffer from wealth erosion proportional to the amount of foreign currency being held. This implies a reduction in individual consumption and generates a current account imbalance. However, this imbalance only affects the consumption level, which remains constant over time, so that the results obtained do not depend on this wealth effect.

Appendix 2

To prove Proposition 1, let us first consider the problem starting at moment $\bar{t}$. Since $\mu_t > \pi^*$, $\forall t > \bar{t}$, by the same argument used for time-invariant paths of μ in Definition 2, the unique solution of this problem is $m = \overline{m}$. This gives the third line of (5). For the problem starting at moment zero, however, the solution cannot be $m = \overline{m}$ up to $\bar{t}$ for that would imply $\pi < \pi^*$, which in turn would be inconsistent with the money demand. Hence, the solution up to time $\bar{t}$ involves currency substitution. To find the equilibrium path of domestic money balances, we use the non-arbitrage condition $\pi = \pi^*$ in (4) and solve for m, obtaining

$$m(t) = m(\bar{t})\exp\left\{ \left(\pi^* - \mu\right)\left(\bar{t} - t\right) - \frac{\sigma}{2}\left(\bar{t}^2 - t^2\right)\right\}. \tag{a8}$$

Using $m(\bar{t}) = \overline{m}$ and the definition of $\bar{t}$ in (a8), we obtain the second line of (5). To find $\tilde{t}$ we simple observe that $m(\tilde{t}) = 1$ and the result is straightforward[5]. Finally, using (4) and (5), the inflation path given by (6) results immediately, implying that both money demand and supply are satisfied.

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[5] Note that (5) is in accordance with both requirements of Definition 1. Indeed, it can be shown that $\lim_{t \to \tilde{t}^-} m_t = 1 = m_{\tilde{t}}$ and $\lim_{t \to \bar{t}^-} m_t = \overline{m} = m_{\bar{t}}$, so that (5) is continuous over time. This fact, together with the observation that m decreases monotonically along the intermediate segment, assures the verification of (2).

Appendix 3

To show that how the indeterminacy is extended to a range of money supply paths, we introduce transaction costs associated to the level of dollarisation. Transactions involving agents using different currencies require some support activities, such as those that are normally required for international trade (for example, the effort involved in obtaining information on the relevant exchange rate, in dealing with foreign exchange traders and managing risk). These costs depend on the level of dollarisation and do not disappear with learning.

In order to keep this note simple we postulate an *ad hoc* dollarisation costs function, g, with decreasing marginal costs[6]:

$$g(f) = \delta f(1-f). \qquad \text{(a9)}$$

The marginal cost of dollarisation, $\delta(1-2f)$, decreases with f and becomes negative after *f>1/2* (that is when the foreign currency becomes more liquid than the domestic currency, the cost of reducing the use of the domestic currency is offset by

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[6] Dollarisation cost functions with different shapes were specified, for example, by Sturzenegger (1992), Guidotti and Rodriguez (1992), Chang (1993), Dowd and Greenaway (1993) and Uribe (1997). Reding and Morales (1999), on the other hand, incorporated an explicit model for the network effect.

the advantage of rising the proportion of the more popular currency)[7]. Dollarisation costs are assumed to be paid by consumers in the proportion of their holdings of foreign currency (as a brokerage fee). Thus, the *private cost* will be the dollarisation cost per unit of foreign currency, denoted by α [8]:

$$\alpha_t = \delta m_t. \tag{a10}$$

Since each agent takes α as given, the individual money demand becomes:

$$m_t = \begin{cases} \overline{m} & if \quad \pi_t > \pi^* + \alpha_t \\ any & if \quad \pi_t = \pi^* + \alpha_t \\ 1 & if \quad \pi_t < \pi^* + \alpha_t \end{cases}. \tag{a11}$$

An immediate implication of (a11) is that there exists a minimum inflation differential, α, below which dollarisation does not occur.

Given (a11), for time-invariant paths of μ, the equilibrium paths of *m* are:

$$m=1 \text{ if } \mu < \pi^* + \delta\overline{m}; \tag{a12}$$

$$m = \overline{m} \text{ if } \mu > \pi^* + \delta; \tag{a13}$$

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[7] This is the standard natural monopoly argument. An equivalent assumption is of identical and concave *total* transaction costs functions per currency, that is, $h(m) = h(f)$ with $h' > 0$ and $h'' < 0$. Since $f = 1 - m$, the relevant *extra* cost due to dollarisation would be equivalent to (a9).

[8] Since this cost is not defined at the origin, we set it equal to its limit, which is the marginal cost (individuals perceive the cost of an incipient dollarisation).

$$m = \begin{cases} 1 \\ \overline{m} \\ \dfrac{\pi - \pi^*}{\delta} \end{cases} \quad \rangle \quad if \quad \pi^* + \delta\overline{m} \le \mu \le \pi^* + \delta . \qquad \text{(a14)}$$

Cases (a12)-(a14) are depicted in Figure A1. In case (a12), the domestic inflation is lower than the user cost associated to an incipient amount of foreign currency, which is ($\pi^* + \delta$). Hence, $m=1$ is an equilibrium solution. Furthermore, since $\pi < \pi^* + \delta\overline{m}$ no other dollarisation level satisfies the money demand. Similar comments are valid for case (a13).

Case (a14) compares to line (iii) of Definition 2. With dollarisation costs, however, there is an entire range of steady state money supply paths ($\pi^* + \delta\overline{m} \le \mu \le \pi^* + \delta$) for which the indeterminacy occurs.

The effects of unexpected shifts on the monetary policy are straightforward in light of Figure 3[9]. For example, if initially $m=1$ and then the rate of money growth unexpectedly shifts once-and-for-all from $\mu \le \pi^* + \delta$ to $\mu > \pi^* + \delta$, then the price

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[9] Obviously, with dollarisation costs, the perfect foresight dynamics is more complex. Since the dynamics properties obtained in Section 3 depend only on the PMS assumption, they should not be affected by the introduction of costs associated to the dollarisation level. The only predictable difference is that the equality between opportunity costs during the adjustment period would be consistent with an inflation differential. Since there are no reasons to expect any important novelties in the non-linear case, we avoid the mathematical complications by discussing only equilibria with time invariant paths of μ.

level will jump so as to reduce the real money balances from $m=1$ to $m=\overline{m}$. On the other hand, if the rate of money growth unexpectedly shifts once-and-for-all from $\mu > \pi^* + \delta$, to $\pi^* + \delta\overline{m} \leq \mu \leq \pi^* + \delta$, then the price level becomes undetermined and any dollarisation levels may occur in the steady state. Thus, a rate of money creation equal to the one verified previously to dollarisation is not a sufficient condition for the return to a single currency regime, despite that being the superior outcome in terms of welfare (note that, with dollarisation costs, the welfare loss associated to non-reversibility is higher).

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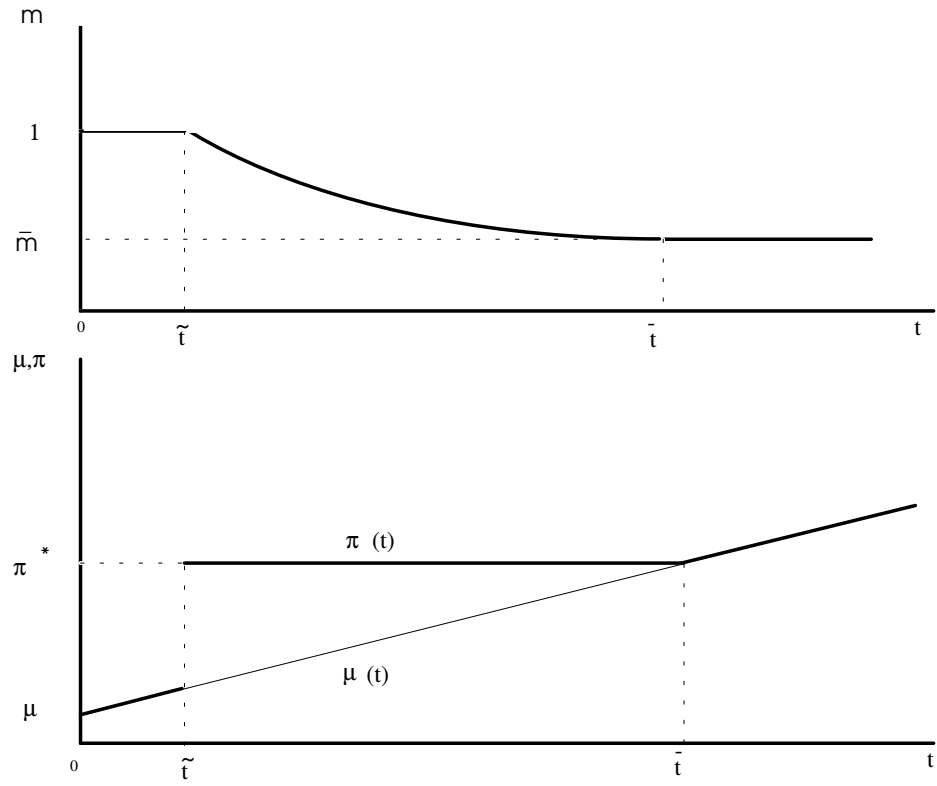*Figure 1: The dynamics of inflation and currency substitution*
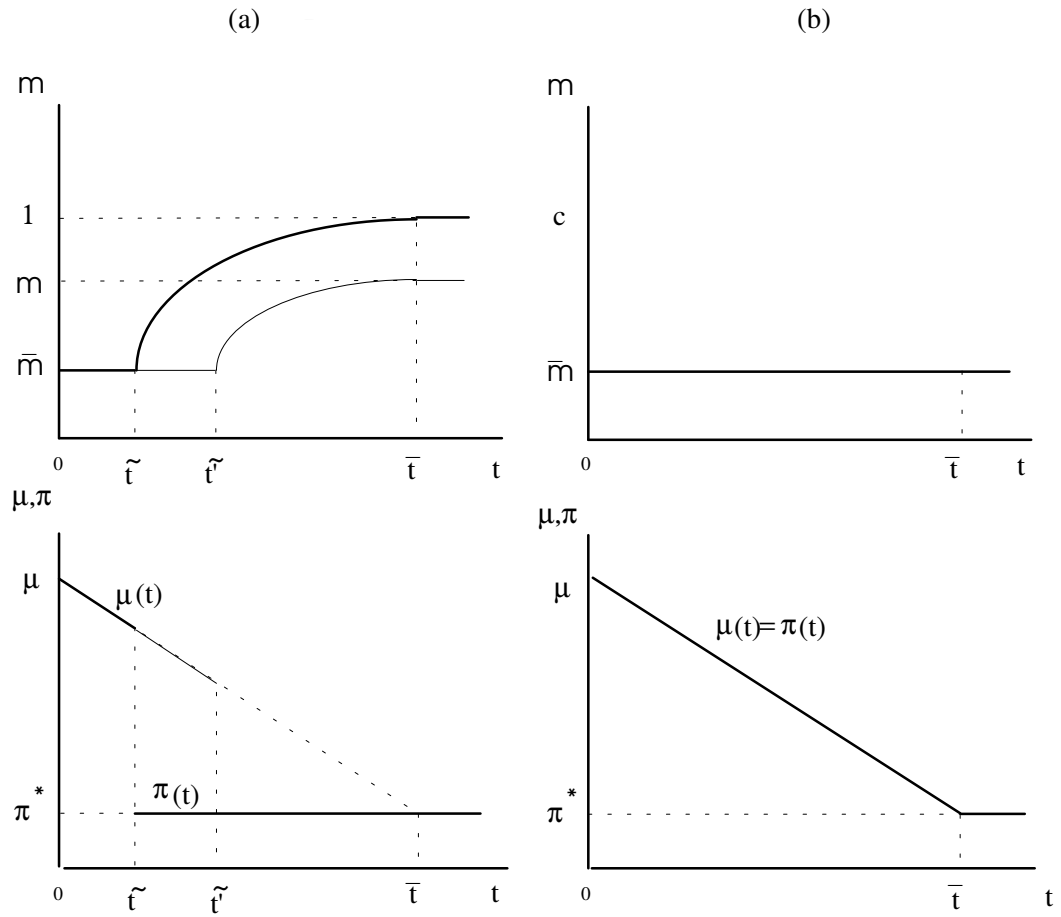
Figure 2: A disinflation programme

Figure A1: An inflation band with dollarisation costs