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Abstract

Deterministic models of conjunctive surface and groundwater management aren't much more complicated than typical groundwater-only management models under simple assumptions. However, when water quality problems exist, the fact that there are two alternative sources of water gains a new significance, as there is no guarantee that they will be of comparable quality. Thus the benefit from using one unit of surface water may not be the same as that of one unit of groundwater. This paper analyses the implications of considering a conjunctive ground and surface water system where water quality varies according to source, with and without uncertainty in hydrological parameters.

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Abstract

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1   Introduction

In most water systems groundwater is not used on its own but rather as a complement of whatever surface water supplies are available (rainfall, stream flows, surface water reservoirs). Accordingly, the literature that analyses management of groundwater stocks has included conjunctive use from the beginning (see the seminal article by Burt [1] and the reviews on the topic by Provencher [10] and Tsur [17]). With a set of simple assumptions, such as that surface water is constant, cheaper, and that surface and groundwater are perfect substitutes, deterministic conjunctive use models are not much more complicated than groundwater-only management models. The main difference is that groundwater is used only after the given endowment of surface water has been exhausted.

A natural extension that brings these models closer to reality is to consider stochastic surface water supplies, highlighting the role played by groundwater in protecting users against uncertainty. Tsur [15] studies the buffer role of groundwater in a static setting and shows that it is positive under standard concavity assumptions of the benefit function, so that groundwater is more highly valued when surface water varies than when it is constant. Tsur and Graham-Tomasi [?] provide a similar analysis for a dynamic setting, although in this case the proof of positive buffer values requires more restrictive assumptions (namely, that marginal benefits are convex). Knapp and Olson [7] also consider surface water variability; their paper analyses decision rules and establishes conditions for convergence of extraction and stock to limiting probability distributions using lattice programming, which is a useful method in problems where the value function associated with the dynamic programming problem is not concave. Provencher and Burt [11] consider a two period model of surface water variability with risk averse firms to identify the risk externality associated with common property situations.

However, there is one aspect of water use that seems to have been somewhat overlooked in typical conjunctive use models. Considering that water quality is a relevant parameter in many regions, the fact that there are two alternative sources of water gains a new significance, since there is no guarantee that both sources will be of comparable quality. Therefore, the benefit for users of using one unit of surface water may not be the same as that of using one unit of groundwater. Yet they should still be regarded as part of one single management system, except in the few extreme situations where only one source of water is explored.

There are some examples of deterministic models of joint quantity-quality management of groundwater in economic literature, but the only case where a conjunctive system is considered is the salinity model of Dinar[2], Dinar and Xepapadeas [?]. There is also a paper on drainage problems by Tsur [16] which briefly touches the issue. Tsur and Zemel's [19] paper on irreversibility has a stochastic element (the size of stock below which groundwater use becomes unfeasible is unknown), but it does not consider conjunctive use. Two other papers that consider uncertainty but not conjunctive use are Fisher and Rubio [5] where the recharge flow is variable and the maximum size of

water stock depends on how much capital is invested, and Rubio and Castro [13], where there is both recharge and demand uncertainty.

This chapter analyses the implications of considering a conjunctive ground and surface water system where water quality varies according to source, with and without uncertainty in hydrological parameters. A simple, static model of conjunctive use is introduced in section 2 to illustrate the issues that ensue from the inclusion of a quality parameter in the water revenue function. Results are compared to those of Tsur [15]. Finally, a dynamic model of groundwater evolution is presented, both in the standard deterministic version and in a stochastic version that uses methods similar to those in Fisher and Rubio [?], Rubio [12].

2 Model (static case)

Users of the water (such as farmers) are assumed to maximize their profit by choosing the amount of water they want to apply. Surface water is exogenous, so that by choosing total water use the amount of groundwater to be pumped is established. There is a fixed unit cost of pumping, z, surface water, s, is provided at no cost, and there is a water revenue function which depends on total water used, w, and on the concentration of some undesirable pollutant in that water: $y(w, C)$. The maximum profit, Π, is:[1]

$$\Pi = \max_{\{w\}} \quad y(w, C) - z(w - s) \tag{2}$$

Under the usual assumptions on y (namely, considering that the derivatives of y have the following properties: $y_w > 0, y_{ww} < 0, y_C < 0, y_{wC} \leq 0$)[2] this

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[1]If s was costly or $C^g - C^s < 0$, then the choice of s might become endogenous, although there would still be an exogenous maximum available amount of surface water (this makes sense for certain types of surface water, such as stream flows and lakes, and not for others, such as precipitation). The water management problem would become:

$$\begin{aligned} \max_{w,s} \quad & y(w, C^s \tfrac{s}{w} + C^g \tfrac{w-s}{w}) - z(w - s) \\ s.t. \quad & s \leq s^{\max} \end{aligned} \tag{1}$$

[2]The expected sign of the second derivative on concentration, y_{CC}, depends on whether additional pollution is more harmful for small values of concentration or for large ones. This is an empirical question. See Letey and Dinar [?], which contains a number of estimated agricultural production functions when the quality problem is salinity.

looks like a simple conjunctive use problem. However, even if the pollutant concentration levels in surface water and groundwater are both exogenous, C will be a weighted average of the two, thus it will be endogenous:

$$C = C^s \frac{s}{w} + C^g \frac{w-s}{w} \qquad (3)$$
$$= C^g - \frac{s(C^g - C^s)}{w}$$

where C^s and C^g are respectively pollutant concentrations in surface and groundwater.

This introduces two different sorts of new problems: first, the likelihood of getting a differentiable and concave objective function using only the intuitive assumptions presented above is a lot smaller, so that second order conditions will be more difficult to check. All cases that will be analyzed in this chapter assume that the functional objective is well behaved: concavity is satisfied, and $w > s$ (ensuring differentiability in the relevant range of w). Situations where excess water is harmful, such as floods, although possible, are ruled out. It is considered that the amount of surface water is never too large, so that the last unit of water received is still revenue-increasing.

Second, the optimal choice of water will vary with the amount of available surface water, which is something that did not happen in quantity-only static conjunctive models. To show this, consider the first order condition for problem 2:

$$y_w + y_C C_w = z$$
$$\iff y_w + y_C \frac{s(C^g - C^s)}{w^2} = z \qquad (4)$$

Thus the marginal benefit of pumping has two terms: the first one is the direct impact on production of oumping additional water, and the second is the impact on production through the effect on water quality. Note that this term is positive if groundwater is less contaminated than surface water and negative otherwise.

Equation 4 implicitly defines the optimal water decision, $w^*(s, C^g, C^s, z)$, so that:

$$w_s^* = -\frac{y_{wC} \frac{-(C^g - C^s)}{w} + y_{CC} \frac{-s(C^g - C^s)^2}{w^3} + y_C \frac{(C^g - C^s)}{w^2}}{y_{ww} + 2y_{wC}C_w + y_{CC}(C_w)^2 + y_C C_{ww}} \qquad (5)$$

Note that if $C^g = C^s = C$, $w_s^* = 0$ and the traditional conjunctive use model holds. In that case, if surface water fluctuates, groundwater is

simply pumped so as to keep total water used constant (ie. stabilizing water consumption).[3] If $C^g \neq C^s$, however, optimal water consumption is not stable when s varies. It may increase or decrease, depending on the sign of the numerator in 5 (the denominator is negative by the assumption of concavity).

Example 1 *Suppose s is rainfall; then it should be true that $C^g - C^s > 0$. If $y_{CC} = 0$ and $y_{wC} = 0$, then $w_s^* < 0$. Thus, an increase in rainfall will decrease total water used. The reason is that an increase in s increases the negative impact on concentration of additional pumped units of water (C_w increases), so that if w remained constant the marginal benefit of pumping an extra unit would be lower than the marginal cost. This requires optimal w to decrease. If $y_{CC} > 0$ (ie. benefit is convex in C) the effect of a larger s would be even stronger, and w would decrease even more, whereas if $y_{CC} < 0$ then w would decrease less or even increase. If $y_{wC} < 0$, on the other hand, there is an extra increase in the marginal benefit of using water due to the higher availability of the better quality water (s), so that w_s^* tends to increase, although it may be positive or negative.*

Performing comparative static analysis with the remaining parameters of the model highlights some other interesting properties of the optimal water choice. Denoting $\zeta = y_{ww} + 2y_{wC}C_w + y_{CC}(C_w)^2 + y_C C_{ww}$, and recalling that $\zeta < 0$, the following results are obtained:

$$w_z^* = \frac{1}{\zeta} < 0 \tag{6}$$

$$w_{C^g}^* = -\frac{y_{wC}\frac{(w-s)}{w} + y_{CC}\frac{(w-s)}{w}\frac{s(C^g-C^s)}{w^2} + y_C\frac{s}{w^2}}{\zeta} \tag{7}$$

$$w_{C^s}^* = -\frac{y_{wC}\frac{s}{w} + y_{CC}\frac{s}{w}\frac{s(C^g-C^s)}{w^2} - y_C\frac{s}{w^2}}{\zeta} \tag{8}$$

Note that an increase (decrease) in w^* corresponds to an increase (decrease) in pumped water, since surface water is now being held constant. Thus, equation 6 shows that, as expected, less water is pumped when pumping costs increase. However, equations 7 and 8 are ambiguous. The optimal reaction

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[3]The first order condition for the model without quality is the same as for the model with constant quality $C^g = C^s = C$, ie. $y_w = z$. This expression does not depend on s.

to a higher level of contamination in either type of water is undetermined, depending again on y_{CC}, y_{wC} and $(C^g - C^s)$. If the second order derivatives are zero, then $w^*_{C^g} < 0$ (less groundwater is pumped when its quality deteriorates[4]) and $w^*_{C^s} > 0$ (more groundwater is pumped to compensate a fall in surface water quality). These results seem reasonable, but they do not hold in general. For instance, it is actually possible for more groundwater to be pumped even though its quality has fallen; notice that the numerator of 7 can be written as $\left(\frac{d(y_C)}{dw}\right)\frac{w-s}{w} + y_C\frac{s}{w^2}$. The second term is negative, so that $w^*_{C^g} > 0$ requires $\frac{d(y_C)}{dw} > 0$, which implies that the (negative) impact of quality on revenue will increase (ie. become less negative), increasing the attractiveness of pumping extra water. If this effect is strong enough, more water will be pumped.

2.1 Surface water variability

In many geographical regions surface water supplies fluctuate greatly between periods. In Portugal, for example, the available water supplies in a dry year can be as little as one third of their average values [**?**]. It has already been remarked in the previous section that the optimal consumption of water in a simple model, without quality considerations, is invariant with surface water. Accordingly, in the presence of a stochastic surface water supply, groundwater will be used to complement surface water so that total water use remains constant. When there are quality differences between the two types of water, this result no longer holds, and groundwater use may fluctuate more or less than surface water. Moreover, the impact on groundwater use will depend on whether pumping decisions are made before (ex ante) or after (ex post) the exact realization of surface water is known. The latter is the more realistic assumption for most systems, thus it will be the one pursued here.

Surface water variability is introduced into a static conjunctive use model in Tsur [15]. As he notes, in a static model where decisions are made ex post "the uncertainty of water supplies is really an instability". He compares the value of groundwater when s is a random variable to its value when s is fixed at the mean ($\bar{s}$), naming the difference the buffer value of groundwater. He

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[4]If only groundwater is used ($s = 0$), this result also holds, as expected.

shows that the buffer value is positive as long as the water benefit function is concave. In this section the same concept is applied to the case where there are quality differences. To ensure differentiability for any s, it must be assumed that desired water use will be greater than the highest admissible value for s.

By definition, the buffer value of groundwater is given by:

$$BV = E\left\{y(w^*(s), C(w^*(s), s)) - y(s, C^s) - z(w^*(s) - s)\right\}$$
$$- \left[y(w^*(\overline{s}), C(w^*(\overline{s}), \overline{s})) - y(\overline{s}, C^s) - z(w^*(\overline{s}) - \overline{s})\right]$$
$$= \underbrace{y(\overline{s}, C^s) - E\left\{y(s, C^s)\right\}}_{1} + \underbrace{E\left\{\Pi(s)\right\} - \Pi(\overline{s})}_{2} \tag{9}$$

By Jensen's inequality, the first term is positive under simple concavity of y in w. In fact, in Tsur's model the buffer value is exactly equal to this term,[5] since other terms are zero when w^* is independent of surface water. Thus he concludes that the buffer value is always positive. In our case, to ascertain the sign of the buffer value the curvature properties of Π have to be investigated. Using the envelope theorem and recalling expressions 2 and 3:

$$\Pi_s = y_C \frac{-(C^g - C^s)}{w} + z \tag{10}$$

The sign of Π_s depends on which source of water is more contaminated, with $\Pi_s > 0$ whenever surface water is the relatively cleaner source (ie. $(C^g - C^s) > 0$), and $\Pi_s < 0$ when surface water is the relatively more polluted source. It should be stressed that the increase in maximum profit depends only on the relative contamination of surface water, not on its absolute value.[6] As for second order conditions, differentiating 10 yields:

$$\Pi_{ss} = \left(y_{CC} \frac{-(C^g - C^s)}{w} + y_{wC} w_s^*\right) \frac{-(C^g - C^s)}{w} + y_C \frac{(C^g - C^s)}{w^2} w_s^* \tag{11}$$

If $\Pi_{ss} > 0$, then the buffer value is always positive and it is greater than in the no quality model. Otherwise its sign is undetermined. Although the

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[5]Note that $y(\mu, C^s) - E\left\{y(s, C^s)\right\} = y(\mu, 0) - E\left\{y(s, 0)\right\}$; since only surface water is being used in either case, the $y(.)$ simply shifts down when $C^s > 0$.

[6]As for $\Pi_{C^g} = y_C \frac{w-s}{w}$ and $\Pi_{C^s} = y_C \frac{s}{w}$, they are both negative, as expected.

sign of Π_{ss} cannot always be ascertained, it is possible to check that it is positive for the case of $y_{CC} = 0$. In this case, taking into account that w_s^* is given by 5, expression 11 can then be rewritten as:

$$\Pi_{ss} = -\frac{1}{\zeta} \left[\left(\frac{y_C}{w} - y_{wC} \right) \frac{(C^g - C^s)}{w} \right]^2 > 0$$

On the other hand, if $\Pi_{ss} < 0$, then the buffer value is lower than in the no quality case, and it cannot be guaranteed that its sign will be positive. Thus the incorporation of quality differences raises new questions on the buffer role of groundwater.

3 Dynamic water stock evolution

3.1 Optimal choices under certainty

Considering that the groundwater stock is not constant implies that pumping cost is not constant either. Moreover, when taking aquifer dynamics into account all users of the aquifer system must be considered simultaneously. It is assumed that there are M identical agents exploiting a single stock of groundwater, which contains G_t units of recoverable water and is characterised by a flat bottom and perpendicular sides. The aquifer receives a constant recharge, R. The unit cost of groundwater extraction, denoted by $z(G_t)$, depends negatively on the size of the groundwater stock and the cost increase per unit depleted is higher the lower the remaining stock (i.e. $z(G_t)$ is decreasing and convex). A percentage α of the applied water returns to the aquifer, so that G evolves according to:

$$\dot{G} = M \left[-(w - s) + \alpha w \right] + R \tag{12}$$

Depending on the source of surface water being considered, it would also be possible for its amount and quality to be stock variables (surface water reservoirs, lakes). However, that would bring additional complexity to the model without bringing new insights, so in this chapter surface water is always considered a flow variable in the sense that it is used up immediately.[7] s, C^s and C^g are known, constant values.

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[7] Note that the one stock/one flow model can also be used in the absence of groundwater, whenever there are alternative sources of surface water of which at least one is a stock.

Optimal use of the aquifer requires (let $M = 1$ for the moment since it does not affect optimal choices):

$$\max_{\{w_t\}} \quad \int_0^\infty \left[y(w_t, C_t) - z(G_t)(w_t - s) \right] e^{-\rho t} dt \tag{13}$$

subject to equation 12, and to non-negativity restrictions on w_t and G_t, as well as an initial condition $G_0 = \overline{G}$. The current value Hamiltonian for this problem is:[8]

$$H = \left[y(w, C^g - \frac{s(C^g - C^s)}{w}) - z(G)(w - s) \right] + \lambda \left(\dot{G} \right)$$

Letting π denote instantaneous profit, $\pi_w = y_w + y_C \frac{s(C^g - C^s)}{w^2} - z(G)$ and first order conditions for interior solutions can be stated as:

$$\pi_w = \lambda(1 - \alpha) \tag{14}$$

$$\dot{\lambda} = \rho\lambda + z_G(w - s) \tag{15}$$

$$\dot{G} = -(1 - \alpha)w + s + R \tag{16}$$

From conditions 14 to 16, the behaviour of w along the optimal path can be derived:

$$\dot{w} = \frac{\rho\pi_w + z_G(\alpha s + R)}{\pi_{ww}} \tag{17}$$

Considering cost function properties and concavity of $y(.)$, the $\dot{w} = 0$ locus has a positive slope:

$$w_G|_{\dot{w}=0} = -\frac{-\rho z_G + z_{GG}(\alpha s + R)}{\rho\pi_{ww}} > 0$$

Thus the steady state will be a saddle point. The $\dot{w} = 0$ locus may be convex or concave, depending on the signs of π_{www} and z_{GGG}, since:

$$w_{GG}|_{\dot{w}=0} = -\frac{[-\rho z_{GG} + z_{GGG}(\alpha s + R)]\rho\pi_{ww} - \rho\pi_{www}w_G[-\rho z_G + z_{GG}(\alpha s + R)]}{(\rho\pi_{ww})^2}$$

The case of linear pumping costs and convex marginal benefits for water use ($\pi_{www} > 0$) provides an example of a convex $\dot{w} = 0$ locus. A phase diagram of the system might look like that of Figure 1. There is a stable arm that leads to the steady state equilibrium. For a given G_0, the chosen level of w_0 must be on that stable arm, so that the optimal path will converge to the steady state.

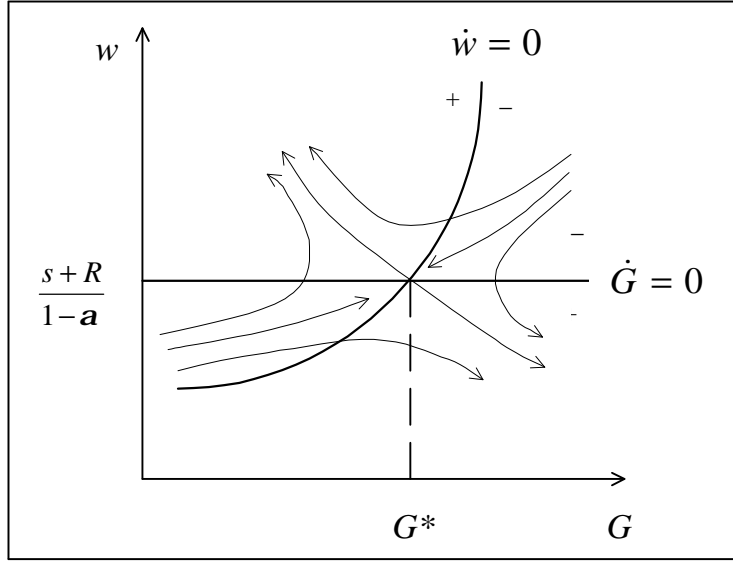[8]t subscripts have been dropped for ease of exposition.

Figure 1: Possible phase diagram for $w_{GG} > 0$

3.2 Uncertainty in hydrological parameters

There are several ways in which uncertainty could affect the problem of groundwater extraction. One of them, as noted in section 2.1, is through surface water variability. When surface water and groundwater have different quality levels, these can also fluctuate, depending on weather conditions or imperfectly known pollution processes (some references to stochastic pollution processes can be found in Kampas and White [6], Shortle and Dunn [14], Xepapadeas [20]). Hence, none of the three hydrological parameters, s, C^s and C^g will generally be known with certainty, so that a stochastic setting in the decision problem may be more adequate.

It is assumed that the current realization of all parameters is known, although their future increments are stochastic, according to the following:

$$ds = \sigma_1 s d\omega_1 \tag{18}$$

$$dC^s = \sigma_2 C^s d\omega_2 \tag{19}$$

$$dC^g = \sigma_3 C^g d\omega_3 \tag{20}$$

where ω_1, ω_2, ω_3 are brownian motions with correlation coefficients given

by ρ_{12}, ρ_{13} and ρ_{23}, respectively.[9] The specific type of stochastic behaviour chosen for the hydrological parameters (as geometric brownian motions without drift) implies that each of them is lognormally distributed, taking only positive values and with constant expected value (equal to its initial value). Similar assumptions are used in Fisher and Rubio [5] for a hydrological parameter like s. If a drift component was relevant in shaping the behaviour of s, C^s or C^g, it would have to be incorporated in the above equations (see also Appendix A.1).

The expected present value of total discounted profit is similar to that of problem 13. Thus the optimal value function will be:

$$V(G, s, C^s, C^g) = \max_{\{w_t\}} \quad E_0 \int_0^\infty \left[y(w_t, C_t) - z(G_t)(w_t - s) \right] e^{-\rho t} dt \quad (21)$$

The associated Bellman equation is:[10]

$$\rho V(.) = \max_{\{w\}} \quad y(w, C) - z(G)(w - s) + \frac{1}{dt} E dV \quad (22)$$

Label as x the set of variables $x = \{G, s, C^s, C^g\}$, and define the Jacobian as V_x, the Hessian as V_{xx}, the transition vector as $T_x = (dG, ds, dC^s, dC^g)$, as well as $\sigma = (\sigma_1 s, \sigma_2 C^s, \sigma_3 C^g)$. Then, using Ito's Lemma:

$$dV = V_x T_x^T + \frac{1}{2} T_x V_{xx} T_x^T \quad (23)$$

Expanding terms and taking the expected value of dV as $dt \to 0$ yields:[11]

$$\frac{1}{dt} E dV = V_G \left(-(1-\alpha)w + s + R \right) + \frac{1}{2} \left[\sigma_1^2 s^2 V_{ss} + \sigma_2^2 \left(C^s \right)^2 V_{C^s C^s} + \sigma_3^2 \left(C^g \right)^2 V_{C^g C^g} \right] +$$

$$\sigma_1 \sigma_2 s C^s V_{s C^s} \rho_{12} + \sigma_1 \sigma_3 s C^g V_{s C^g} \rho_{13} + \sigma_2 \sigma_3 C^s C^g V_{C^s C^g} \rho_{23} \quad (24)$$

Or, in more compact notation, with $\widehat{V_{yy}} = [\rho_{ij} V_{ij}]$ for $i, j = s, C^s, C^g$ (ie. y refers to elements of x except G):[12]

$$\frac{1}{dt} E dV = V_G \left(-(1-\alpha)w + s + R \right) + \frac{1}{2} \sigma \widehat{V_{yy}} \sigma^T \quad (25)$$

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[9]The increments of brownian motions have mean zero and variance dt (thus $E(d\omega_i) = 0$, $E(d\omega_i)^2 = dt$, $E(d\omega_i d\omega_j) = \rho_{ij} dt$, $i, j = 1, 2, 3$). For an introduction to stochastic processes, see Dixit and Pindyck [4, cp.3].

[10]Unless otherwise specified, the operator E refers to the expected value at moment t. For an introductory reference to stochastic dynamic programming, see [4, cp.4].

[11]In going from 23 to 25, all terms in $d\omega_i$ disappear as their expected value is zero. Terms of order dt are kept, whereas terms in dt of any order higher than one go to zero.

[12]Note that second derivatives with respect to G are absent, since the transition for stock does not have a variance term. Also note that $\rho_{ii} = 1$.

Replacing expression 25 in 22 and undertaking the maximization yields:

$$\pi_w = (1-\alpha)V_G \tag{26}$$

Differentiating 22 with respect to G at the optimal value of w :

$$\rho V_G = -z_G(w-s) + V_{GG}dG + \frac{1}{2}\left[\sigma_1^2 s^2 V_{ssG} + \sigma_2^2 \left(C^s\right)^2 V_{C^s C^s G} + \sigma_3^2 \left(C^g\right)^2 V_{C^g C^g G}\right] +$$
$$\sigma_1\sigma_2 sC^s V_{sC^s G}\rho_{12} + \sigma_1\sigma_3 sC^g V_{sC^g G}\rho_{13} + \sigma_2\sigma_3 C^s C^g V_{C^s C^g G}\rho_{23} \tag{27}$$

Now, considering that $V_G = V_G(G, s, C^s, C^g)$, and using Ito's lemma to obtain $\frac{1}{dt}EdV_G$, this expression reduces to:

$$\rho V_G = -z_G(w-s) + \frac{1}{dt}EdV_G \tag{28}$$

Note that equations 26 and 28 are the counterparts for the stochastic problem of equations 14 and 15, and they can be used to find the stochastic equivalent of 17. Using similar notation as above, but defining $X = \{w, x\}$ (ie. w and all elements of x), and noting that $\pi_w = \pi_w(w, G, s, C^s, C^g)$:

$$d\pi_w = \pi_{wX}T_X^T + \frac{1}{2}D_X\pi_{wXX}D_X^T \tag{29}$$

Before expanding equation 29, the expressions for dw and $(dw)^2$ must be developed. Along the optimal path, $w = w(G, s, C^s, C^g)$, so that:

$$dw = w_x T_x^T + \frac{1}{2}T_x w_{xx} T_x^T \tag{30}$$

As for $(dw)^2$, it is greatly simplified by recalling that all terms of order higher than dt can be discarded, leaving:

$$(dw)^2 = T_x w_x^T w_x T_x^T \tag{31}$$

Equation 29 can now be rewritten, in expected value form (considering infinitesimal dt):[13]

$$\frac{1}{dt}Ed\pi_w = \pi_{ww}\frac{1}{dt}Edw + \pi_{wG}\frac{dG}{dt} + \frac{1}{2}\left\{\pi_{www}\left[\widehat{\sigma w_y^T w_y}\sigma^T\right] + \sigma\widehat{\pi_{wyy}}\sigma^T\right\}$$
$$+ \pi_{wwy}\left[\widehat{w_y\sigma^T\sigma}\right] \tag{32}$$

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[13]Note that π_{www} is a scalar, whereas π_{wwy} is a 3×1 vector and π_{wyy} is a 3×3 matrix.

where, as before, a matrix with a hat means that each of its elements appears multiplied by the appropriate correlation coefficient. Define:

$$A = \frac{1}{2} \left\{ \pi_{www} \left[\sigma \widehat{w_y^T w_y} \sigma^T \right] + \sigma \widehat{\pi_{wyy}} \sigma^T \right\} + \pi_{wwy} \left[w_y \widehat{\sigma^T \sigma} \right] \qquad (33)$$

Using conditions 26 and 28:

$$\rho \frac{\pi_w}{1 - \alpha} = -z_G(w - s) + \frac{\frac{1}{dt} Ed\pi_w}{1 - \alpha} \qquad (34)$$

Replacing $\frac{1}{dt} Ed\pi_w$ with the expression obtained in equation 32, substituting $\pi_{wG} dG$ and reorganizing terms:

$$\frac{1}{dt} Edw = \frac{\rho \pi_w + z_G(\alpha s + R) - A}{\pi_{ww}} \qquad (35)$$

This expression can be compared with 17. The sign of A will determine whether expected steady state water stock is greater or smaller than in the optimal case.[14] If $A > 0$ then the $\frac{1}{dt} Edw = 0$ locus is below the $\dot{w} = 0$ locus and expected water stock is greater with uncertainty, as can be seen in the phase diagram of Figure 2. If $A < 0$ the opposite occurs. Unlike Fisher and Rubio [5], it is not sufficient to have convex marginal benefits to ensure a clear result, since A has a number of additional terms with generally unknown signs. Note that the term $\frac{1}{2} \left\{ \pi_{www} \left[\sigma \widehat{w_y^T w_y} \sigma^T \right] \right\}$ is positive if $\pi_{www} > 0$ because all terms in $\sigma \widehat{w_y^T w_y} \sigma^T$ are positive.

3.2.1 Surface water variability

Since the derivation of equation 35 in the general uncertain case above is rather abstract, it might be useful to look at the case of only one uncertain variable so that the meaning of those extra terms in A is clarified. When surface water is variable (with increments described by equation 18 as before), the expanded version of 25 is simply:

$$\frac{1}{dt} EdV = V_G \left(-(1 - \alpha)w + s + R \right) + \frac{1}{2} \sigma_1^2 s^2 V_{ss} \qquad (36)$$

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[14]The stochastic steady state equilibrium, if it exists in the sense of convergence to a distribution for w and G, will satisfy $\frac{1}{dt} Edw = \frac{1}{dt} EdG = 0$.
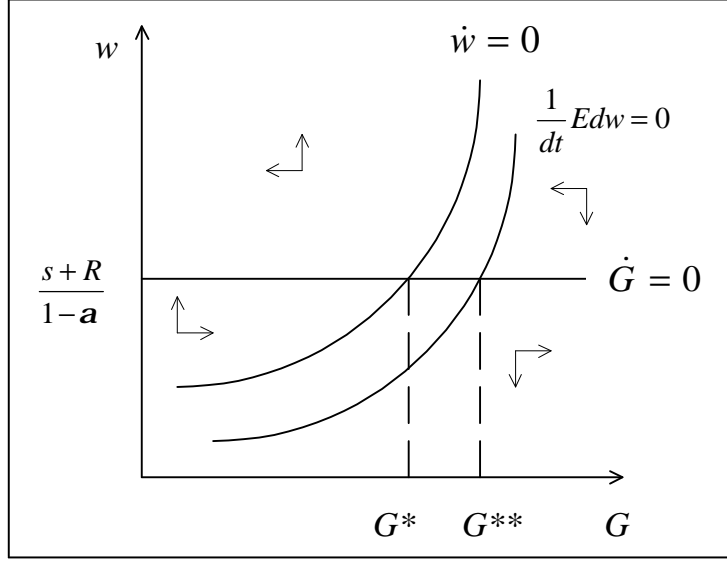
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Figure 2: Phase diagram for $A > 0$

As for $d\pi_w$ and terms in dw, expressions 29, 30 and 31 reduce to:

$$d\pi_w = \pi_{ww}dw + \pi_{wG}dG + \pi_{ws}ds$$
$$+ \frac{1}{2}\left[\pi_{www}(dw)^2 + \pi_{wss}(ds)^2\right] + \pi_{wws}dwds \tag{37}$$

$$dw = w_s ds + w_G dG + \frac{1}{2}w_{ss}(ds)^2 \tag{38}$$

$$(dw)^2 = (w_{ss})^2 \sigma_1^2 s^2 dt \tag{39}$$

So that the expression for the optimal expected motion of water is:

$$\frac{1}{dt}Edw = \frac{\rho\pi_w + z_G(\alpha s + R) - \sigma_1^2 s^2 \left\{\frac{1}{2}\left[\pi_{www}(w_s)^2 + \pi_{wss}\right] + \pi_{wws}w_s\right\}}{\pi_{ww}} \tag{40}$$

This equation corresponds to equation 35 except shocks exist only in s. It is clear now that the additional terms in A result directly from the inclusion of surface water in the production function through weighted-average concentration, since instantaneous profit is no longer linear in s. Thus the cross-derivatives of π_w with respect to s do not disappear.

The effects of increasing surface water variability (ie. increasing σ_1) on stock size can also be derived analytically, for a given level of surface water.

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At the steady state, $\frac{1}{dt}Edw = \frac{1}{dt}EdG = 0$, so that $0 = Es + R - (1-\alpha)Ew$, which implies:

$$\overline{w} \equiv Ew = \frac{\overline{s} + R}{1 - \alpha} \tag{41}$$

Furthermore, evaluating all derivatives at $\overline{w}$ and $\overline{s}$:

$$\rho\pi_w + z_G(\alpha\overline{s} + R) - \sigma_1^2\overline{s}^2 \left\{ \frac{1}{2}\left[\pi_{www}(w_s)^2 + \pi_{wss}\right] + \pi_{wws}w_s \right\} = 0 \tag{42}$$

None of the terms in A depends on G, so the total differential of 42 is:

$$[-\rho z_G + z_{GG}(\alpha\overline{s} + R)]\, dG - \overline{s}^2 \left\{ \frac{1}{2}\left[\pi_{www}(w_s)^2 + \pi_{wss}\right] + \pi_{wws}w_s \right\} d\sigma_1^2 = 0 \tag{43}$$

From this expression it is clear that:

$$\frac{dG}{d\sigma_1^2} = \frac{\overline{s}^2 \left\{ \frac{1}{2}\left[\pi_{www}(w_s)^2 + \pi_{wss}\right] + \pi_{wws}w_s \right\}}{[-\rho z_G + z_{GG}(\alpha\overline{s} + R)]} \tag{44}$$

The denominator is positive under cost convexity, so that the sign of $\frac{dG}{d\sigma_1^2}$ will be the same as the sign of A. If $A > 0$, increasing the variance results in a higher groundwater stock, which is consistent with the result shown in Figure 2, since in that case variance is going from zero to a positive value.

4 Conclusion

A truly integrated approach to water management must embody not only quantity-quality interactions but also conjunctive use of surface water and groundwater. This paper is an attempt at analysing optimal choices when both aspects of water systems are considered, emphasizing the economic implications of conjunctive use when the quality of the water varies according to the source.

Water productivity depends on its quality. When different types of water are mixed, the relevant pollutant concentration is a weighted average of individual concentration levels. This simple fact alters a well established result in the conjunctive use literature, which was that for different levels of surface water endowments, groundwater would be pumped so as to keep a given optimal level of total water consumption, as that level was invariant

with respect to surface water realizations. Now the optimal level of water consumption will no longer remain the same, and performing comparative statics shows that its reaction to parameter variations will always depend on the difference between surface water quality and groundwater quality.

Another aspect that has rightly received attention in the water management literature regards the effect of uncertainty on optimal choices. In this chapter uncertainty in hydrological parameters was modelled in a dynamic setting through their description as geometric brownian motions, and the impact of such uncertainty on the evolution of water consumption and on optimal steady state stock was described, although no general results can be obtained without specifying a production function.

There are two aspects that have been treated in the literature and were not incorporated in the present analysis. One deals with the choice of optimal storage capacity in the context of ground or surface water stocks (see Tsur [15], Fisher and Rubio [5]). Another deals with models where surface water does not have to be entirely consumed, so that surface water used and groundwater pumped are actually two different, albeit related, choices. With uncertainty in quality parameters, each type of water has different risk and return. The conjunctive use problem could then be viewed as a choice of optimal portfolio mix. These are areas for future research.

A   General dynamic case

In this appendix it is acknowledged that both the amount and the quality of available groundwater are stock variables whose dynamics are affected by the agents' choices. Pollutant concentration is, as before, a weighted average of surface and groundwater. However, groundwater quality evolves according to the pollutant concentration in the contaminant load (emissions) and a natural regeneration rate:

$$\dot{C}^g = e(w, \gamma) - \delta C^g \tag{45}$$

where γ refers to the amount of polluting input used in the productive process, as in chapters 1 and 2.[15]

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[15] A more realistic equation for groundwater quality's evolution could take into consideration the relative weights of emissions quality, recharge quality and existing groundwater

If there was no uncertainty, the solution would result from the following problem:

$$\max_{\{w_t, \gamma_t\}} \quad \int_0^\infty \left[y(w_t, \gamma_t, C_t) - z(G_t)(w_t - s) \right] e^{-\rho t} dt$$

subject to equations 12 and 45, initial conditions and non-negativity constraints.

The current value Hamiltonian for this problem is:

$$H = \left[y(w, \gamma, C^g - \frac{s(C^g - C^s)}{w}) - z(G)(w - s) \right] + \lambda \left(\dot{G} \right) + \beta \left(\dot{C}^g \right)$$

First order conditions:

$$\pi_w = \lambda(1 - \alpha) - \beta e_w \tag{46}$$

$$\pi_\gamma = -\beta e_\gamma \tag{47}$$

$$\dot{\lambda} = \rho\lambda + z_G(w - s) \tag{48}$$

$$\dot{\beta} = (\rho + \delta)\beta - \left(y_C \frac{w - s}{w} \right) \tag{49}$$

Conditions 46 to 49 can also be used for the case of dynamic quantity-quality evolution when only groundwater is used (see chapter 2); in that case $s = 0$ and $C = C^g$, simplifying 46, and $\frac{w-s}{w} = 1$ in 49.

Unfortunately, this model is too general to produce tractable expressions for $\dot{w}$ and $\dot{\gamma}$, which are the relevant decision variables.[16] However, for the purpose of analysing the impact of shocks on water choices while considering a dynamic quality evolution, it will be assumed that the amount of polluting input is exogenous and that emissions vary linearly with water use, ie. $e(w, \gamma) = \epsilon w \gamma$. Under these circumstances:

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quality, making $e(.)$ a function of G, R, C^R and C.

[16]Equations 46 and 47 imply $\beta = -\frac{\pi_\gamma}{e_\gamma} \equiv \Phi$ and $\lambda = \frac{\pi_w + \frac{\pi_\gamma}{e_\gamma} e_w}{(1-\alpha)} \equiv \Upsilon$, which are both non-linear with respect to w, γ, and $C(w, C^g)$. Even if it is assumed that $\pi_{\gamma C} = 0$, calculating their time derivatives and using the remaining first order conditions yields:

$$\dot{w} = \frac{(1 - \alpha) \left[\rho\Upsilon - \Upsilon_{C^g}\dot{C}^g - \frac{\Upsilon_\gamma}{\Phi_\gamma} \left((\rho + \delta) \Phi - y_C \frac{w-s}{w} \right) \right] + z_G(\alpha s + R)}{(1 - \alpha) \left[\Upsilon_w - \Upsilon_\gamma \frac{\Phi_w}{\Phi_\gamma} \right]}$$

$$\dot{w} = \frac{\rho\pi_w + z_G\left(\alpha s + R\right) - \pi_{wC^g}\left(\epsilon w\gamma - \delta C^g\right) - \epsilon\gamma\left(\delta\beta + y_C\frac{w-s}{w}\right)}{\pi_{ww}} \qquad (50)$$

The new terms in this expression, as compared to equation 17, appear because water profitability varies with pollutant concentration in groundwater, which is now endogenous. The fact that there is a single control variable and two state variables implies that the term in the costate β cannot be fully eliminated.

A.1 Stochastic shocks in water quality

The three types of hydrological shocks described in section 3.2 could be introduced into the general dynamic model developed above. However, one of them must be treated in a different manner now that groundwater pollutant concentration is not exogenous, since the stochastic increments in C^g can no longer be modeled as a simple geometric brownian motion without drift, like in equation 20. Therefore, to avoid the cumbersome notation of the multiple-shock case, this section considers that surface water and its pollutant concentration are constant, so that only C^g is prone to stochastic shocks.

The increments in groundwater pollutant concentration are assumed to follow:

$$dC^g = \left(\epsilon w\gamma - \delta C^g\right)dt + \sigma_4 d\omega_4 \qquad (51)$$

where ω_4 is a brownian motion.

The Bellman equation for the problem becomes:

$$\rho V(.) = \max_w \left[\begin{array}{c} y(w, C; \gamma) - z(G)(w - s) + V_G\left(-(1-\alpha)w + s + R\right) \\ + V_{C^g}\left(\epsilon w\gamma - \delta C^g\right) + \frac{1}{2}V_{C^g C^g}\sigma_4^2 \end{array} \right] \qquad (52)$$

Using similar methods as in section 3.2, the stochastic analogues of equations 46, 48 and 49 can be derived:

$$\pi_w = V_G(1-\alpha) - \epsilon\gamma V_{C^g} \qquad (53)$$

$$\frac{1}{dt}EdV_G = \rho V_G + z_G(w-s) \qquad (54)$$

$$\frac{1}{dt}EdV_{C^g} = (\rho + \delta)V_{C^g} - \left(y_C\frac{w-s}{w}\right) \qquad (55)$$

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From 53:

$$\frac{1}{dt}Ed\pi_w = \rho\pi_w + (1-\alpha)z_G(w-s) - \delta\epsilon\gamma V_{C^g} - \epsilon\gamma y_C\frac{w-s}{w} \qquad (56)$$

Using Ito's Lemma to obtain $\frac{1}{dt}Ed\pi_w$ from $\pi_w = \pi_w(w, C^g, G)$, replacing that expression in the right hand side of 56, and rearranging terms:

$$\pi_{ww}\frac{1}{dt}Edw = \rho\pi_w + z_G(\alpha s + R) - \epsilon\gamma\left(\delta V_{C^g} + y_C\frac{w-s}{w}\right) - \pi_{wC^g}\left(e\alpha w - \delta C^g\right)$$

$$-\sigma_4^2\left\{\frac{1}{2}\left[\pi_{www}(w_{C^g})^2 + \pi_{wC^gC^g}\right] + \pi_{wwC^g}w_{C^g}\right\} \qquad (57)$$

This expression depends explicitly on V_{C^g}, which cannot be eliminated. However, it can be seen that the impact of uncertainty is similar to the case where quality was exogenous.

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