

STRATEGIC ASSET ALLOCATION IN
STOCHASTIC ENVIRONMENT AND
INCOMPLETE MARKETS: EVIDENCE
ON HORIZON AND HEDGING EFFECTS

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Abstract

The paper analyzes optimal portfolio choice when the investment opportunity set is driven by multi-stochastic factors, namely; stochastic interest rates, stochastic volatility and stochastic inflation. The analysis is implemented in an incomplete market setting, where the number of risk sources is larger than the number of risky assets. The model segregates the effect of inflation from the other two state variables by deriving the dynamics in real wealth. The derived optimal portfolio shows that inflation plays a significant role in forming investors' hedging demand through the correlation structure between inflation and assets held in the portfolio. Empirically, the paper calibrates the optimal portfolio choice for different classes of investors distinct by the degree of risk tolerance and investment horizon length in an attempt to mimic the popular financial planners' advice. Calibration results show that the joint inclusion of stochastic interest rates, stochastic volatility and stochastic inflation introduces a plausible simultaneous resolution for both Samuelson puzzle and asset allocation puzzle of Canner, Mankiw, and Weil (1997).

التنوع الاستراتيجية للمحافظ المالية في بيئة متغيرة وضمن شروط السوق غير الكاملة:
بعض الأدلة على أثري التحوط وطول الأفق الاستثماري

ملخص

تناقش هذه الورقة الاختيار الأمثل لعناصر المحفظة المالية في الحالة التي يتحدد بها منحى الفرص الاستثمارية بعدد من العوامل غير المستقرة أو عشوائية الحركة. أي في الحالة التي يكون فيها كل من أسعار الأسهم وأسعار الفائدة والذبذبة والتضخم تتحرك بشكل عشوائي. لقد تم التحليل بافتراض شروط السوق غير التامة، والتي يكون فيها عدد مصادر المخاطرة يفوق عدد الأصول المالية الخطرة المكونة للمحفظة. في النموذج المقترح، تم فصل أثر التضخم عن كل من أسعار الأصول الخطرة وأسعار الفائدة من خلال اشتقاق المنظومة الحركية التي يتبعها التغير في مستوى الثروة الحقيقية. لقد أشار التحليل إلى أن للتضخم أثراً مهماً في تحديد الشكل الأمثل للمحفظة فهو يحدد الجزء من المحفظة التي يحملها المستثمرون لغايات التحوط. في الجانب التطبيقي، تم استنباط الاختيار الأمثل للمحفظة المالية لعدد من المستثمرين الذين يتفاوتون في درجة تقبل المخاطرة وطول الأفق الاستثماري، في محاولة لمحاكاة المحافظ الاستثمارية التي يوصي بها في العادة أشهر المخططين الماليين. بشكل عام أشارت النتائج إلى أن افتراض الحركة العشوائية لسعر الفائدة والذبذبة والتضخم (باعتبارها من مصادر المخاطرة) يساهم بشكل جيد في حل كل من معضلي ساملسون (Samuelson Puzzle) ومعضلة تخصيص الأصول (Asset Allocation Puzzle) التي أشار إليها كتر ومانكو وويل (1997).

1. Introduction

There is significant inconsistency among the theory of asset allocation and professional financial planners' advices. Tobin's (1958), Samuelson (1969) and Merton (1969, 1971), conclude that, under the assumption of equally available information and random walk hypothesis (when returns are normally distributed), all investors should hold the same optimal portfolio of risky assets, regardless of the investor's risk tolerance or the length of investment horizon. Accordingly, no investor should alter the relative proportions of the risky assets in his optimal portfolio but he can hold more or less from the risky portfolio combining with the risk free asset.

The professionals' advice, on the other hand, contradicts that by emphasizing the effect of risk tolerance and time horizons in forming portfolios. The typical advice is that high risk tolerance investors and young investors (who have long horizons) should hold portfolios with high stock to bond ratio compared with average investor.

Research in asset allocation and optimal portfolio choice addresses those inconsistencies as investment puzzles. The horizon inconsistency has been addressed first by Samuelson (1963) and it has been known as Samuelson puzzle. Canner, Mankiw and Weil (1997) [CMW there after], point at the risk tolerance argument as the asset allocation puzzle.

Attempts at reconciliation of the above inconsistencies adopt the early Samuelson's (1969), and Merton's (1969, 1971, 1973) insights for intertemporal investors. In which they indicate that optimal portfolio choice for long-term investors is different from short-term (myopic) investors. Merton (1973) and later Cox and Huang (1989) suggest that long-term investors should intertemporarily hedge against changes in the investment opportunity set. Any change in means, variances or covariances of securities' returns is sufficient to generate changes in the investment opportunity set that needs to be hedged. Investors hedge such stochastic variations by including in their portfolios hedge funds equal in number to the state variables that derive the dynamics in returns.

Nielsen and Vassalou (2000) redefine the instantaneous investment opportunity set as the instantaneous capital market line (ICML) rather than all first and the second moments and covariances of instantaneous rates of returns. They argue that investors need to hedge only against random changes in the slope, and the position of the ICML. If the ICML is constant or deterministic, investors do not need to hold any hedge portfolio even if means, variances and covariances of asset returns are changing randomly over time.

This paper uses the general continuous-time modeling framework of Merton (1969, 1971, and 1973) in analyzing rational portfolio choice and hedging demand in a stochastic environment. It derives the optimal asset allocation for a long-lived investor who continuously invests in cash, nominal bonds and stocks and faces a stochastic investment opportunity set. Dynamics in the stochastic investment opportunity set are assumed to be driven by three stochastic elements, the interest rates, volatility of returns and inflation. The model's results are achieved within market incompleteness conditions, where the number of sources of uncertainty is larger than the number of risky assets. This sort of market incompleteness is analyzed by He and Pearson (1991), Karatazas et al (1991) and Nielsen and Vassalou (2000).

In the empirical part, the paper estimates the parameters of the stochastic processes by means of the Spectral GMM estimator. The estimated parameters are used to calibrate the optimal portfolio choice for three different risk-recipients investors with different investment horizons. The calibration aims to mimic observed financial planners' investment advice.

The literature that examines optimal asset allocation in dynamic setting is numerous. In a setting without volatility uncertainty, Munk *et al.*(2004), Brennan and Xia (2002) and Campbell and Viceira (2001) solve for the optimal portfolio policy and decompose the portfolio selection into a "speculative term" and a hedging term. Liu (2001) analyzes optimal portfolio choice where the dynamics in the investment opportunity set are derived by the interest rate and volatility uncertainties without considering the inflation effect. Brennan and Xia (2000), and Omberg (2001) consider a similar problem without including inflation and volatility dynamics. Chacko and Viceira (2003b) solve for the optimal asset allocation in an incomplete market setting with volatility as a single source of risk.

The model in this paper is close to the models applied in Munk et al. (2004) Brennan and Xia (2002) and Campbell and Viceira (2001). In these papers, interest rate is described by a Vasicek-model whereas, we use CIR model rather than Vasicek-model. The Vasicek-model allows for a negative interest rate which, cannot be explained in real application. Brennan and Xia (2002) and Campbell and Viceira (2001) describe nominal interest rates by a two-factor model and their model construction allows for complete market conditions by investing in two different nominal bonds.

The model is calibrated in two steps. In the first step the paper estimates the parameters of the model using monthly US data from the April 1953 to September 2001 by means of the Spectral GMM techniques. In the second step, the obtained capital market parameters are used in calibrating the relevant preference parameters by minimizing the sum of squared deviations between the theoretical and observed asset allocation advice across investors with different risk attitudes and investment horizons. With this calibration, the model could provide simultaneous resolution for both the Samuelson and the asset allocation puzzle.

The paper is organized as follows. Section 2 presents the formal model and the solution to the intertemporal portfolio problem. Section 3 introduces estimates for the capital market parameters and, subsequently, calibrates the model to the observed asset allocation advice and section 4 concludes.

2. The Stochastic Environment

Let's consider the investment problem of an investor who has access to capital market and wants to transfer current wealth W_0 into a future terminal wealth at a specific investment horizon. We consider the basic asset allocation problem of how much to invest in a money market bank account (cash), nominal zero coupon bonds, and stocks. Here we assume that nominal bonds differ from cash in that they provide capital gains beside the coupon payments. It is held also as a part of the hedging portfolio not only the speculation portfolio. The duration of the bond reflects the stochastic duration of the portfolio. Investors who hold bonds in their portfolio keep a certain portion of their portfolio in the form of bonds, whenever a bond expires it is replaced by a longer maturity bond.

2.1 The Capital Market Dynamics

The stock index is assumed to evolve according to the stochastic differential equation

$$dS_t = (r_t + \mu_{ts})S_t dt + \sqrt{v_{ts}}S_t dZ_{st} \quad (1)$$

Where r_t is the short nominal interest rate, μ_{ts} is the time varying expected excess return from investing in stocks, and v_{ts} is the time varying stock index volatility. Z_{st} is a Wiener process. Accordingly, the variation in the investment opportunity set is induced by stochastic variation of the short-term interest rate, the expected excess return and the conditional variance of the risky assets. Merton (1980) suggests that risk premium is crucial to portfolio choice, but it is difficult to estimate empirically. Merton proposes that risk premium might depend on the volatility powers. The power could be 0, 1 or 2. Here the assumption is made that the risk premium is proportional to the volatility (i.e. the power 1). By definition, the proportionate factor that connects the expected risk premium μ_{ts} with volatility v_{ts} is the instantaneous market price of risk or instantaneous sharp ratio. Thus $\mu_{ts} = \lambda_s v_s$.

Volatility v_{st} , on the other hand, is assumed to follow a square root diffusion process of Heston (1993):

$$dv_{st} = \kappa_v (\bar{v} - v_{st})dt + \sigma_s \sqrt{v_{st}} dZ_{vt} \quad (2)$$

Where \bar{v} represents the long run mean for volatility, κ_v is the speed of adjustment, and σ_s is constant.

The nominal interest rate dynamics are described by a Cox, Ingersoll and Ross (1985), CIR model,

$$dr_t = \kappa_r (\bar{r} - r_t)dt - \sigma_r \sqrt{v_r} dZ_{rt} \quad (3)$$

Where \bar{r} represents the long run mean of the interest rate, κ_r is the speed of adjustment, and v_r is the interest rate volatility, which is assumed constant for simplicity.

The term structure of interest rates has the same form as in CIR. In particular, the price of a zero-coupon bond with time to maturity τ is given by

$$P(r, t; \tau) = e^{-a(\tau) - b(\tau)r} \quad (4)$$

Where

$$a(\tau) = \frac{2\bar{r}}{\sigma_r^2} \ln \left(\frac{2\Phi \exp((\kappa_r + \lambda_r \sigma_r^2 + \Phi)^{\frac{1}{2}})}{(\kappa_r + \lambda_r \sigma_r^2 + \Phi)(\exp(\Phi \tau) - 1) + 2\Phi} \right)$$

$$b(\tau) = - \left(\frac{2(\exp(\Phi \tau) - 1)}{(\kappa_r + \lambda_r \sigma_r^2 + \Phi)(\exp(\Phi \tau) - 1) + 2\Phi} \right)$$

and $\Phi = \sqrt{(\kappa_r + \lambda_r \sigma_r^2)^2 + 2\sigma_r^2}$ and λ_r is the market price of the interest rate risk.

Bonds are interest rate contingent claims, by Ito's lemma, the dynamics of the bond price B_t can be described by a stochastic differential equation

$$dB_t = (r_t + b(\tau)\lambda_r \sigma_r^2 r_t)B_t dt + b(\tau)\sigma_r \sqrt{r_t} B_t dZ_{rt} \quad (5)$$

$\lambda_r \sigma_r^2$ is the risk premium on the interest rate, so, it represents μ_r .

with $\mu_B = \mu_r \eta(r, t)$, $v_B = v_r \eta(r, t)$ where $\eta = -\frac{\partial B}{\partial r} \frac{1}{B}$ is the elasticity of the bond price with respect to the short interest rate; this elasticity is usually referred to as the stochastic duration of the interest rate contingent claim [Ingersoll, Skeldon and Weil (1978) and Cox, Ingersoll and Ross (1979)]. Here, we assume that the bond available for the investor has a constant duration η . This can be thought of as reflecting the duration of the aggregate portfolio of bonds outstanding, or a bond index, where bonds that expire are always substituted with new longer term bonds. Also, note that the short interest rate and the return on the bonds are perfectly negatively correlated and with covariance rate $v_{Br} = -\eta v_r^2 = -\left(\frac{1}{\eta}\right)v_B^2$.

The nominal price of the real consumption good in the economy at time t is Ω_t . Thus the real price of any asset in the economy is deflated by the price index Ω_t . For instance, the real price of the stock is S_t / Ω_t and the real price for the bond is B_t / Ω_t .

The dynamics of the nominal price of consumption goods are given by the following diffusion processes:

$$d\Omega_t = \varphi_t \Omega_t dt + \sqrt{v_\Omega} \Omega_t dZ_{\Omega t} \quad (6)$$

Whereas the expected inflation rate φ_t follows mean reverting Orestein-Uhlenbeck process:

$$d\varphi_t = \kappa_\varphi (\bar{\varphi} - \varphi_t) dt + v_\varphi dZ_{\varphi t} \quad (7)$$

Where φ_t is the expected rate of inflation, $\bar{\varphi}$ is the long-run mean of the rate of inflation, κ_φ is the speed of adjustment. v_Ω and v_φ are the volatilities of the price index and

the inflation rate respectively. v_Ω determines the magnitude of the unexpected short-run inflation in the economy.

Changes in the nominal price index and the inflation rate are correlated with the stock index return and interest rates. The covariance rate between the return on the stock index and the price level is $v_{s\Omega} = \rho_{s\Omega} v_s v_\Omega$, the covariance between the return on the stock index and the inflation rate is $v_{s\varphi} = \rho_{s\varphi} v_s v_\varphi$. The correlation between stock returns and the real interest rate is $\rho_{s(r-\varphi)}$.

By Ito's lemma we can find the processes for real stocks and bonds RS_t and RB_t to be in the following forms:

$$\frac{dRS_t}{RS_t} = ((r_t + \mu_s - \varphi_t) + v_\Omega + v_{s\Omega})dt + \sqrt{(v_s + v_\Omega - 2v_{s\Omega})} dZ_{RS} \quad (8)$$

$$\frac{dRB_t}{RB_t} = ((r_t + \mu_B - \varphi_t) + v_\Omega + v_{B\Omega})dt + \sqrt{(v_B + v_\Omega - 2v_{B\Omega})} dZ_{RB} \quad (9)$$

Accordingly, we find that the dynamics of the real wealth follows the following process:

$$dW_t = \mu_W W dt + v_W W_t dZ_{W_t} \quad (10)$$

Where

$$\mu_W = r + \mathbf{\alpha}' \boldsymbol{\mu} - \varphi_t + v_\Omega - \mathbf{\alpha}' \begin{bmatrix} v_{s\Omega} \\ v_{B\Omega} \end{bmatrix}, \quad v_W^2 = \mathbf{\alpha}' \boldsymbol{\Sigma} \mathbf{\alpha} + v_\Omega - 2\mathbf{\alpha}' \begin{bmatrix} v_{s\Omega} \\ v_{B\Omega} \end{bmatrix}$$

$$\mathbf{\alpha} = \begin{bmatrix} \alpha_s \\ \alpha_B \end{bmatrix}, \quad \boldsymbol{\mu} = \begin{bmatrix} \mu_s \\ \mu_B \end{bmatrix}, \quad \text{and } \boldsymbol{\Sigma} \text{ is the variance covariance matrix} = \begin{bmatrix} v_s^2 & v_{sB} \\ v_{sB} & v_B^2 \end{bmatrix},$$

Where $v_{sB} = -\eta v_{sr}$

2.2 Preferences

The investor chooses a dynamic portfolio strategy in order to maximize the expected utility of terminal wealth at a specific horizon τ . We formulate the optimal portfolio problem for a long-term investor who invests in stocks, bonds, and cash.

Defining the value function or indirect utility function $J(W, r, \varphi, v_s, \tau)$ for an investor with τ periods investment horizon. The value function must satisfy the boundary condition $J(W, r, \varphi, v_s, 0) = U(W)$. Bellman principles assume that the value function at the maximum is martingale.

$$MaxE[dJ] = 0 \quad (11)$$

$$\begin{aligned} Max_{\alpha=(\alpha_s, \alpha_B)} 0 = & \left\{ \mu_W WJ_W + \kappa_r (\bar{r} - r) J_r + \kappa_\varphi (\bar{\varphi} - \varphi) J_\varphi + \kappa_v (\bar{v} - v) J_v + \frac{1}{2} v_W^2 W^2 J_{WW} \right. \\ & + \frac{1}{2} v_r^2 J_{rr} + \frac{1}{2} v_\varphi^2 J_{\varphi\varphi} + \frac{1}{2} \sigma^2 v_v J_{vv} + v_{Wr} WJ_{Wr} + v_{W\varphi} WJ_{W\varphi} + v_{Wv} WJ_{Wv} + v_{r\varphi} J_{r\varphi} \\ & \left. + v_{rv} J_{rv} + v_{v\varphi} J_{v\varphi} + J_t \right\} \quad (12) \end{aligned}$$

$$\text{Where } v_{Wv} = \boldsymbol{\alpha}' \begin{bmatrix} v_{sv} \\ v_{Bv} \end{bmatrix} - v_{\Omega v}, \quad v_{Wr} = \boldsymbol{\alpha}' \begin{bmatrix} v_{sr} \\ v_{Br} \end{bmatrix} - v_{\Omega r}, \quad v_{W\varphi} = \boldsymbol{\alpha}' \begin{bmatrix} v_{s\varphi} \\ v_{B\varphi} \end{bmatrix} - v_{\Omega\varphi},$$

The first order condition of the problem in (12) gives the following characterization of the optimal risky asset proportions $\boldsymbol{\alpha}$:

$$\boldsymbol{\alpha} = \frac{-J_W}{WJ_{WW}} \boldsymbol{\Sigma}^{-1} \left\{ \boldsymbol{\mu} + \frac{J_{Wr}}{J_W} \begin{bmatrix} v_{sr} \\ v_{Br} \end{bmatrix} + \frac{J_{Wv}}{J_W} \begin{bmatrix} v_{sv} \\ v_{Bv} \end{bmatrix} - \left(1 + \frac{WJ_{WW}}{J_W} \right) \begin{bmatrix} v_{s\Omega} \\ v_{B\Omega} \end{bmatrix} + \frac{J_{W\varphi}}{J_W} \begin{bmatrix} v_{s\varphi} \\ v_{B\varphi} \end{bmatrix} \right\} \quad (13)$$

$$\begin{aligned} \boldsymbol{\alpha} = & \frac{-J_W}{WJ_{WW}} \boldsymbol{\Sigma}^{-1} \left\{ \boldsymbol{\mu} + \frac{\partial \ln(J_w)}{\partial r} \begin{bmatrix} v_{sr} \\ v_{Br} \end{bmatrix} + \frac{\partial \ln(J_w)}{\partial v} \begin{bmatrix} v_{sv} \\ v_{Bv} \end{bmatrix} - \left(1 + W \frac{\partial(J_W)}{\partial W} \right) \begin{bmatrix} v_{s\Omega} \\ v_{B\Omega} \end{bmatrix} + \frac{\partial \ln(J_w)}{\partial \varphi} \right. \\ & \left. \begin{bmatrix} v_{s\varphi} \\ v_{B\varphi} \end{bmatrix} \right\} \quad (14) \end{aligned}$$

Equations (13) and (14) gives a general characterization of the optimal portfolio weights in the specific market setting. The first term in those equations is the usual speculative portfolio that is optimal for an investor with short horizon or log utility. The other four terms describe how the investor optimally hedges changes in the opportunity set. The second term describes the hedge against the nominal interest rate. The factor $\frac{\partial \ln(J_w)}{\partial r}$ in (14) measures the sensitivity of the logarithm of the marginal utility of wealth to changes in the interest rate and it summarizes the investor's attitude towards changes in the interest rate. The third term describes the hedge against the stochastic changes in interest rate, and it is expressed in a form of logarithm of marginal utility change with respect to volatility in the same way for interest rate. Using the fact that

$$\begin{aligned} \begin{bmatrix} v_{Sr} \\ v_{Br} \end{bmatrix} &= -\frac{1}{\eta} \begin{bmatrix} v_{SB} \\ v_B \end{bmatrix} = -\frac{1}{\eta} \boldsymbol{\Sigma} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ \begin{bmatrix} v_{sv} \\ v_{Bv} \end{bmatrix} &= \sigma_s \begin{bmatrix} v_s \\ v_{SB} \end{bmatrix} = \sigma_s \boldsymbol{\Sigma} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{aligned}$$

we can rewrite the above relation in the form

$$\mathbf{\alpha} = \frac{-J_w}{WJ_{ww}} \left\{ \Sigma^{-1} \boldsymbol{\mu} - \frac{\partial \ln(J_w)}{\partial r} \frac{1}{b} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \frac{\partial \ln(J_w)}{\partial V} \sigma_s \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \left(1 + W \frac{\partial(J_w)}{\partial W} \right) \Sigma^{-1} \begin{bmatrix} v_{s\Omega} \\ v_{B\Omega} \end{bmatrix} + \frac{\partial \ln(J_w)}{\partial \varphi} \Sigma^{-1} \begin{bmatrix} v_{s\varphi} \\ v_{B\varphi} \end{bmatrix} \right\} \quad (15)$$

Hence, the optimal hedge against changes in the interest rate is obtained by investing entirely in the bond; this is similar to the optimal strategy in the complete market Vasicek settings of Brennan and Xia (2000). As a matter of fact, the first two terms in the hedging component are quite similar to what Liu (2001) has.

The last two terms in (13) describe how the investor hedges against short-run unexpected inflation and changes in future inflation rates, respectively. Munk et al (2004) derive in a Vasicek settings similar components for asset allocation. Equations (13)-(15) do not give an explicit solution for the optimal asset allocation. To get an explicit solution, we have to solve the partial differential equation (PDE) in the Bellman principle, equation (12) that is highly non-linear.

The first step to solve this equation, is to conjecture a solution in the form $\frac{W^{1-\gamma}}{1-\gamma} f^{1-\gamma}$, and $f(r, \varphi, v_s, \tau)$ takes the form illustrated in (16) below:

$$J(W, r, \varphi, v_s, \tau) = \frac{(We^{a(\tau)+b(\tau)r-c(\tau)V-q(\tau)\varphi})^{1-\gamma}}{1-\gamma} \quad (16)$$

Where $a(0) = 0$, $b(0) = 0$, $c(0) = 0$, and $q(0) = 0$ which satisfy the boundary condition for the value function $J(W, r, \varphi, v_s, 0) = U(W)$. Now substitute the conjecture in the first order in equation (15) and substitute back the conjecture in equation (16) condition with the first order condition PDE in equation (12). After doing that we would get four ordinary differential equations that can be solved to get the values for $a(\tau)$, $b(\tau)$, $c(\tau)$, and $q(\tau)$. Re-substitute the solution for the ODE,s in the first order condition we get the solution of the vector of risky assets $\mathbf{\alpha}$ at time t where there are τ periods for the horizon:

$$\mathbf{\alpha} = \left(\frac{1}{\gamma} \right) \Sigma^{-1} \boldsymbol{\mu} + \left(1 - \frac{1}{\gamma} \right) \left(\frac{b(\tau)}{\eta} \right) \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \left(1 - \frac{1}{\gamma} \right) c(\tau) \sigma_s \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \left(1 - \frac{1}{\gamma} \right) \Sigma^{-1} \left(q(\tau) \begin{bmatrix} v_{s\varphi} \\ v_{B\varphi} \end{bmatrix} + \begin{bmatrix} v_{s\Omega} \\ v_{B\Omega} \end{bmatrix} \right) \quad (17)$$

The residual $\alpha_r = 1 - \mathbf{1}'\mathbf{\alpha} = 1 - \alpha_s - \alpha_B$ is invested in the cash or bank account.

Equation (17) shows that the optimal portfolio weights for CRRA investors are linear combinations of the speculative portfolio (the first term of equation 17) and the different hedge portfolios. In particular, for investors with the same investment horizon τ the optimal portfolios are linear combinations of the speculative portfolio and a single hedge portfolio; the relative risk tolerance $1/\gamma$, represents the weights on the two relevant portfolios.

Munk et al (2004) show that an expression such as the one in (17), without the hedging component against volatility dynamics, describes the demand for hedging against changes in the nominal interest rate and inflation. Such expression consists of mixed positions in the stocks and bonds and it can explain the bonds/stocks puzzle pointed out by Canner, Mankiw and Weil (1997). The first hedge term shows that the interest rate hedge involves bonds. The last two terms explain the inflation hedge that involves stocks. Liu (2001) shows also that an expression like the first two components in the hedging portfolio with a mix of stocks and bonds can explain the asset allocation puzzle. Chacko and Viceira (2003b) indicate that hedging against volatility dynamics in an all stock portfolio give results that are consistent with asset allocation puzzle. Brennan and Xia (2000) show that with an all bonds portfolio that hedging interest rate dynamics gives results also consistent with the financial planners advice.

In this setting with interest rate hedging, volatility hedging and inflation hedging, we get more emphasis on both the asset allocation puzzle and the horizon puzzle. The terms $c(\tau)$ and σ_s involve both hedging effects and asset allocation effects. A particular notice in equation (17) is that the amount of hedge portfolio holding or risky assets depends on the correlation structure between the inflation rate and the assets held in the portfolio. Additionally, equation (17) shows as well that the duration of the bond holding plays a significant role in determining the total bond holding, while the volatility of the volatility σ_s plays an identified role in determining the ultimate holdings of the hedging funds. Correlations among risky assets seem to have a dominant role in the speculating portfolio. If κ_ϕ is small, changes in the expected inflation rate are relatively permanent, and horizon effects may be significant. However, whether this horizon effect implies more or fewer stocks for the long-term investor depends on the precise correlation structure. It depends as well on whether the stock serves as a relatively good substitute for the real bond that should ideally be used for hedging changes in real rates in a complete market setting.

3. Model Estimations and Calibrations

Here we first introduce the estimation technique spectral GMM of Chacko and Viceira (2003a) and Singleton (2001). Then it calibrates the asset price and inflation parameters of the capital market model. In the third section, the calibrated parameters are used in a calibration exercise where the subjective risk aversion parameter and time horizon parameter are fitted to match observed asset allocation advice for different investor groups.

3.1 Model Estimation: Spectral GMM

A spectral GMM approach is adopted to estimate the parameters of the processes in the model Pennacchi (1991), Campbell and Viceira (2001) and Brennan and Xia (2002) have used Kalman filtering in contexts similar to this paper. Chacko and Viceira (2003a&b) use the spectral GMM in stochastic volatility context. One advantage of the spectral GMM over the Kalman filtering is that the spectral GMM does not require the discretization of the stochastic process. It only requires knowledge of its conditional characteristic function. Once we know this function, we can integrate the stochastic interest rate and inflation out and obtain the characteristic function of next period's stock price and commodity price level conditional only on the prior period's prices. Chacko and Viceira (2003a) call this estimation

method Spectral GMM because it uses the generalized method of moments (GMM) to estimate the parameters of the model directly off this conditional characteristic function.

To calculate the conditional characteristic function, we have to transform the process to an exponential affine process as in Chacko and Das (2002). We transform the stock price process into a form of the log stock price,

$$d \ln S_t = (r_t + \lambda_t v_t - \frac{1}{2} v_t) dt + \sqrt{v_t} dZ_s \quad (18)$$

$$dv_t = \kappa_v (\bar{v} - v_t) dt + \sigma_s \sqrt{v_t} dZ_v$$

Now assume the characteristic function $\phi(\ln S_t, v_t, r_t, \varphi_t, \tau)$ satisfy the following PDE:

$$E[d\phi(\ln S_t, v_t, r_t, \varphi_t, \tau)] = 0 \quad (19)$$

Assume $\ln S_t = X_t$

By Ito's Lemma,

$$0 = \left\{ \begin{aligned} &\mu_x \phi_x + \kappa_r (\bar{r} - r) \phi_r + \kappa_\varphi (\bar{\varphi} - \varphi) \phi_\varphi + \kappa_v (\bar{v} - v) \phi_v + \frac{1}{2} v_x^2 \phi_{xx} \\ &+ \frac{1}{2} v_r^2 \phi_{rr} + \frac{1}{2} v_\varphi^2 \phi_{\varphi\varphi} + \frac{1}{2} \sigma^2 v_v \phi_{vv} + v_{xr} \phi_{xr} + v_{x\varphi} \phi_{x\varphi} + v_{xv} \phi_{xv} + v_{r\varphi} \phi_{r\varphi} + v_{rv} \phi_{rv} + v_{v\varphi} \phi_{v\varphi} + \phi_t \end{aligned} \right\} \quad (20)$$

Equation (20) is known as Kolomogorov Backward Equation (KBE) or Fokker-Plank Forward Equation (FPFE). And the characteristic function is the solution for this equation given that

$$\phi(X_t, v_t, r_t, \varphi_t, \omega, 0) = \exp(i\omega X) \quad (21)$$

To solve explicitly for the characteristic function, we conjecture a solution for the KBE in the form of

$$\phi(X_t, v_t, r_t, \varphi_t, \tau) = \exp(i\omega X + A(\tau) + B(\tau)r + C(\tau)v + D(\tau)\varphi) \quad (22)$$

Now substituting the conjecture into the KBE we get four ODEs. By solving those four ODEs, we get the values for the parameters $A(\tau), B(\tau), C(\tau),$ and $D(\tau)$.

Next we integrate the unobserved variable (volatility) out in the same way in Chacko and Viceira (2003a). Here, we get the number of moments that are necessary for the GMM by substituting the number of parameters that are needed to be estimated with ω .

3.2 Estimation of The Processes Parameters

The system is estimated using monthly US data with almost 50 years period from April 1953 until September 2001. Data on eight constant maturity yields are used; the times to maturities are 3 months, 1-year, 2 years, 3 years, 5 years, 10 years, 20 years and 30 years. Unavailable yields are calculated using the simple bootstrap method. Cumulative dividend stock returns data available in Robert Shiller's web site are used for the purpose of estimating the stock returns process. Table 1 shows the data used and the sources of these data.

In Table 2, from the Spectral GMM estimates, we find that the market price of risk on the stock index is estimated to be 0.0762 and the long-term volatility mean is 12.1%. Table 2 shows also that the estimated long term mean of the nominal interest rate is 4.42%, and the volatility of the interest rate is 2.12%. With the estimated value of the interest rate risk premium λ_r to be around 1%, we can compute the expected excess return, and the volatility of bonds with any horizon to maturity. For instance, based on CIR (1985), the duration on a pure discount bond with 3 years to maturity $\eta(0,3)$ is $b(3) = 2.873$ years. Its volatility (v_B) would be $\eta(0,3) \times v_r = 4.11\%$. The expected excess return on that bond would be trivial according to those estimations. The estimation shows also that long term expected rate of inflation is 3.64%, which leaves a long-term positive real interest rate. Generally, Munk et al (2004), Campbell and Viceira (2001) and Brennan and Xia (2002) report similar estimates.

The estimates of correlations coefficients show that the stock index is negatively correlated with the nominal interest rate, the commodities price index and the inflation rate. Additionally, estimates show that the stock return index has also negative correlation with real interest rates. This significantly different from what Munk et al (2004) got. Munk et al. (2004) report some positive correlation and that required them to allow for the parameters to move two standard deviations around the estimate in order to get some non-positive correlation and then to be able to mimic the financial planners advice. Getting such negative correlation makes our task much easier in calibrations. This negative correlation between the stock index and the real interest rate means that stocks can be used instead of long term real bonds as hedging instruments against changes in real interest rates induced when there is significant inflation rates, and that is what Campbell and Viceira (2001) basically report.

Our estimates differ quantitatively and sometimes qualitatively from Munk's et al. (2001) estimates. One reason might be besides the sample differences and the estimation techniques (they use Kalman Filtering rather than Spectral GMM) be the inclusion of the stochastic volatility in estimating the stock return process and using the CIR model for interest rate rather than the Vasicek model.

3.3 Calibration to The Professional Financial Planners' Advice

Following Munk et al, (2004) exercise in matching the financial planners' advice. However, we construct the matching data in a different way. Additionally, the differences in estimates between our work and their work allow for some flexibility in the calibration, thus we do not impose all the restrictions they impose.

Table 3 tabulates the asset allocation recommendations as considered the match data. These recommendations are generated from the advice of the four financial planners and their three classic portfolios and rank them by their market risk: low risk, medium risk, and high risk. These portfolios are tabulated in Canner, Mankiw and Weil (1997) as for "conservative," "moderate," and "aggressive" investors. Those portfolios are constructed in a way that the most conservative advice is assigned to the short horizon conservative investor, and the least conservative advice is assigned to the long horizon conservative investor. On the other hand, the most aggressive advice is assigned to the long run aggressive investor, and the least aggressive advice is assigned to the short-term aggressive investor. So, the short run horizon takes the most conservative or the least aggressive of all advices, and the long run horizon takes the least conservative or the most aggressive advices. The medium horizon takes the medium of the all advices.

The recommendations in Table 3 are in accordance with the popular advice that investors with a long horizon should invest a higher fraction of wealth in stocks. Also, the investment recommendations are in accordance with the pattern pointed out by Canner, Mankiw and Weil (1997) and, in fact, for any investment horizon the bond to stock ratio is increasing with risk aversion.

We calibrate parameters so as to minimize the sum of squared deviations between the asset allocation recommendations in Table 3 and the optimal asset allocations in the economic modeling framework in section 2. The summation of squared deviations that will be minimized is taken over all portfolio weights for the three horizons (short, medium, long), the three degrees of risk aversion (conservative, moderate, aggressive), as well as the allocations into stocks, bonds, and cash. This makes a total of 27 (= 3 x 3 x 3) squared deviations in the summation.

In calibrating the model, we vary three risk aversion parameters: $\gamma_{con} > \gamma_{mod} > \gamma_{agg} > 0$. Likewise, we vary three investment horizon parameters: $0 < H_{short} < H_{med} < H_{long} < 35$ years. These parameters are meant to represent the relative risk aversion parameters of "conservative," "moderate," and "aggressive" investors as well as the investment horizon of investors with short, medium, and long horizons, respectively.

Furthermore, we allow investors with different investment horizons to use bonds that differ in duration. The individual investor can thus invest in cash, stocks and a bond with a duration that depends on the investment horizon. Without loss of generality the bond can be thought of as a zero coupon bond and when we refer to the duration of the bond in the following, we are in fact referring to the time to maturity on the relevant zero coupon bond. This duration concept is known as the stochastic duration as shown by Ingersoll, Skeldon and Weil (1978) and Cox, Ingersoll and Ross (1979). We calibrate the stochastic

durations as part of the problem and we impose the intuitive restriction that investors with longer investment horizons should not use shorter duration bonds and the restriction that the duration on the bond is between 5 years and 15 years so that it could represent a realistic aggregate bond index; i.e. in the calibration we have the restriction: $5 \leq \eta_{short} \leq \eta_{med} \leq \eta_{short} \leq 15$.

We perform one calibration by varying only the risk attitude parameters, investment horizons, and relevant durations subject to the above restrictions. The point estimates of the asset price, interest rate, and inflation dynamics in Table 2 are applied in generating the optimal theoretical asset allocations as we derived in equation 17.

It can be observed that the calibrated model asset allocation in Table 4 conforms to the advice that longer term investors should invest a higher fraction of wealth in stocks. It confirms also the advice that aggressive investors should allocate more stocks in their portfolios as compared with bonds. The trend in table 4 is quite obvious. The short horizon-conservative investors allocate 1.47 % in bonds relative to stocks, which almost mimic the financial planner advice for the short-term – conservative investors. However, our model could not mimic exactly the advice for the long-term- aggressive investor. According to our calibrations, this investor's bonds to stock ratio is 7%, where it is zero for the match data. Generally speaking, the model and the estimates could mimic closely the financial planner advice.

The representative investment horizons calibrated in Table 4 - B seems to be reasonable. Specifically, investor with a short investment horizon has an investment horizon of 5.69 years while a long-term investor acts so as to maximize utility of wealth at a thirty five year horizon. On the other hand, the calibrated relative risk aversion parameters are seemed to be unreasonably high. For example, an "aggressive" investor has a relative risk tolerance of 0.32 (= 1/3.12). Hence, this investor will only allocate 32% of wealth to the speculative portfolio while the remaining 68% is allocated to the hedge portfolio. Also, the "conservative" investors are very cautious in the sense that they will only allocate 3.5% (= 1/28.26) of wealth to the speculative portfolio while 96.5% are invested in a hedge portfolio. Munk et al. (2004) report similar unreasonable numbers for hedging portfolio and speculation portfolios.

4. Conclusion

In this paper, we solve for the optimal asset allocation for an investor who faces a stochastic investment opportunity set in an incomplete market setting. Incomplete markets in this sense imply that the resources of uncertainties are larger than the number of risky assets held in the portfolio. We allow for four sources of uncertainty, in addition to stock returns. We assume stochastic volatility, stochastic interest rates, stochastic expected inflation and a stochastic consumer price index. Theoretical results show that the correlation between inflation and risky assets held in the portfolio plays a significant role in constructing the hedging portfolios. It has no role in determining the portion of the portfolio held for speculation. On the other hand, the correlation structure between risky assets held in the portfolio seem to be more important in determining the size of funds held in the portfolio for speculation purposes.

Introducing volatility as a risk factor enables us to mimic the popular advice of financial planners. Including all of the uncertainty factors that affect the instantaneous slope and instantaneous intercept of the instantaneous capital market line seems to introduce a simultaneous resolution for both the Samuelson puzzle as well as the asset allocation puzzle pointed out by Canner, Mankiw and Weil (1997).

References

- Brennan, M. J. & Xia, Y. (2000). "Stochastic Interest Rates and Bond-Stock Mix". *European Finance Review*, 4, 197-210.
- Brennan, M. J. & Xia, Y. (2002). "Dynamic Asset Allocation under Inflation". *Journal of Finance*, 57, 1201-1238.
- Campbell, J. Y. & Viceira, L. M. (2001). "Who Should Buy Long-Term Bonds?" *American Economic Review*, 91, 99-127.
- Canner, N., Mankiw, N. G., & Weil, D. N. (1997). "An Asset Allocation Puzzle". *American Economic Review*, 87 181-191.
- Chacko, G.,& Viceira L. M. (2003a). "Spectral GMM Estimation of Continuous Time Processes". *Journal of Econometrics*, Vol. 116, 259-292.
- Chacko, G.,& Viceira L. M. (2003b). "Dynamic Consumption and Portfolio Choice with Stochastic Volatility in Incomplete Markets". Working Paper, Harvard University.
- Cox, J. C. & Huang, C.F. (1989). "Optimal Consumption and Portfolio Policies when Asset Prices follow a Diffusion Process". *Journal of Economic Theory*, 49, 33-83.
- Cox, J. C. & Huang, C.F. (1991). "A Variational Problem Arising in Financial Economics". *Journal of Mathematical Economics*, 20, 465-487.
- Cox, J. C. Ingersoll, Jr., J. E., & Ross, S. A. (1979). "Duration and the Measurement of Basis Risk". *Journal of Business*, 52, 51-61.
- Elton, E. J. & Gruber, M. J. (2000). "The Rationality of Asset Allocation Recommendations". *Journal of Financial and Quantitative Analysis*, 35(1), 27-42.

- Ingersoll, Jr., J. E.; Skelton, J., & Weil, R. L. (1978). "Duration Forty Years Later". *Journal of Financial and Quantitative Analysis*, 13(4), 627-650.
- Liu, J., (2001), "Portfolio Selection in Stochastic Environments," Working Paper UCLA.
- Markowitz, H. (1959). "*Portfolio Selection: Efficient Diversification of Investment*". John Wiley and Sons.
- McCulloch, J. H. (1990). "U.S. Term Structure Data, 1946-87". *Handbook of Monetary Economics*, Volume I, 672-715.
- Merton, R. C., (1969). "Lifetime Portfolio Selection Under Uncertainty: The Continuous-Time Case". *Review of Economics and Statistics*, 51, 247-257.
- Merton, R. C. (1971). "Optimum Consumption and Portfolio Rules in a Continuous-Time Model". *Journal of Economic Theory*, 3, 373-413.
- Merton, R. C. (1973). "An Intertemporal Capital Asset Pricing Model". *Econometrica*, 41, 867-887.
- Munk, C., C. Sorensen, and T. Vinther, (2004). "Dynamic Asset Allocation under Mean-reverting Returns, Stochastic Interest Rates and Inflation Uncertainty". *International Review of Economics & Finance*, 13, 141-166.
- Omberg, E. (2001). "Non-Myopic Asset-Allocation with Stochastic Interest Rates". Working paper, San Diego State University.
- Pennacchi, G. G. (1991). "Identifying the Dynamics of Real Interest Rates and Inflation: Evidence Using Survey Data". *Review of Financial Studies*, 4(1), 53-86.
- Samuelson, P. A. (1963). "Risk and Uncertainty: A Fallacy of Large Numbers". *Scientia*, 98, 108-113.
- Samuelson, P. A. (1989). "The Judgment of Economic Science on Rational Portfolio Management: Indexing, Timing, and Long-Horizon Effects". *Journal of Portfolio Management*, 16, 4-12.

Samuelson, P. A. (1994). "The Long-Term Case for Equities and How it can be Oversold". *Journal of Portfolio Management*, 21, 15-24.

Singleton, K. J.(2001). "Estimation of Affine Pricing Models Using Empirical Characteristic Function". *Journal of Econometrics*, 102, 111-141.

Tobin, J., (1958). "Liquidity Preference as Behavior towards Risk". *Review of Economic Studies*, 25, 68-85.

Vasicek, O. A. (1977). "An Equilibrium Characterization of the Term Structure". *Journal of Financial Economics*, 5,177-188.

Appendix

Table (1)
Sources of the Monthly Data Used in The Spectral GMM Estimation

| The Series | Dates | Source |
|--|--|--|
| 3-Month Treasury Constant Maturity Rate | April 1953-Dec. 1981 Jan. 1982-Sep.2001 | McCulloch (1990) Federal Reserve Board. |
| 1-year Treasury Constant Maturity | April 1953-Sep.2001 | Federal Reserve Board. |
| 2-year Treasury Constant Maturity | April 1953-May 1976 June 1976-Sep.2001 | McCulloch (1990) Federal Reserve Board. |
| 3-year Treasury Constant Maturity | April 1953-Sep.2001 | Federal Reserve Board. |
| 5-year Treasury Constant Maturity | April 1953-Sep.2001 | Federal Reserve Board. |
| 10-year Treasury Constant Maturity | April 1953-Sep.2001 | Federal Reserve Board. |
| 20-Year Treasury Constant Maturity Rate | April 1953-Sep.2001 | Federal Reserve Board. |
| CPI | April 1953-Sep.2001 | Bureau of Labor Statistics |
| Cum Dividend Stock Returns | April 1953-Sep.2001 | Robert Shiller Home Page |

Table (2)
Estimation of the General Investment Opportunity Set Parameters,
Using Spectral GMM, (April 1953-Sep.2001)

| Parameter | Estimate | Std. Error |
|---|----------|------------|
| Panel A: Stock Returns Volatility Process | | |
| $dS_t = (r_t + \lambda_s v_s)S_t dt + \sqrt{v_{ts}} S_t dZ_{st}$ | | |
| $dv_t = \kappa_v (\bar{v} - v_t) dt - \sigma_s \sqrt{v_t} dZ_{vt}$ | | |
| λ_s | 0.0761 | 0.0063 |
| κ_v | 0.0963 | 0.0521 |
| \bar{v} | 0.121 | 0.0056 |
| σ_s | 0.106 | 0.0106 |
| Panel B: Interest rate Process | | |
| $dr_t = \kappa_r (\bar{r} - r_t) dt - \sigma_r \sqrt{v_r} dZ_{rt}$ | | |
| κ_r | 0.0235 | 0.0452 |
| \bar{r} | 0.0442 | 0.0487 |
| σ_r | 0.0143 | 0.0214 |
| v_r | 0.0212 | 0.0002 |
| λ_r | 0.0093 | 0.0625 |
| Panel C: Commodity Prices Process | | |
| $d\Omega_t = \varphi_t \Omega_t dt + v_\Omega \Omega_t dZ_{\Omega t}$ | | |
| v_Ω | 0.0211 | 0.0003 |
| Panel D: Inflation Process | | |
| $d\varphi_t = \kappa_\varphi (\bar{\varphi} - \varphi_t) dt + v_\varphi dZ_{\varphi t}$ | | |
| κ_φ | 0.2345 | 0.0652 |
| $\bar{\varphi}$ | 0.0364 | 0.0213 |
| v_φ | 0.0321 | 0.00414 |
| Panel E: Correlation Coefficients | | |
| ρ_{sv} | -0.0041 | 0.0714 |
| ρ_{sr} | -0.00212 | 0.0231 |
| $\rho_{s\Omega}$ | -0.0211 | 0.0474 |
| $\rho_{s\varphi}$ | -0.0142 | 0.0524 |
| $\rho_{r\Omega}$ | 0.00124 | 0.0214 |
| $\rho_{\Omega\varphi}$ | 0.0412 | 0.0143 |
| $\rho_{r\varphi}$ | 0.8014 | 0.0014 |
| $\rho_{s(r-\varphi)}$ | -0.0184 | 0.0214 |

Table (3)
Asset Allocation Advices Used for Calibration*

| Horizon | Risk Tolerance | Cash% | Stocks% | Bonds% | Bonds/Stocks |
|----------------|-----------------------|--------------|----------------|---------------|---------------------|
| Short | Conservative | 50 | 20 | 30 | 1.50 |
| | Moderate | 20 | 40 | 40 | 1.00 |
| | Aggressive | 5 | 65 | 30 | 0.46 |
| Medium | Conservative | 20 | 40 | 40 | 1.00 |
| | Moderate | 10 | 50 | 40 | 0.80 |
| | Aggressive | 0 | 80 | 20 | 0.25 |
| Long | Conservative | 20 | 45 | 35 | 0.78 |
| | Moderate | 10 | 60 | 30 | 0.50 |
| | Aggressive | 0 | 100 | 0 | 0.00 |

* The table constructed from the four recommendations reported by Canner, Mankiw and Weil (1997). The most conservative advice is assigned to the short horizon conservative investor, and the least conservative advice is assigned to the long horizon conservative investor. On the other hand, the most aggressive advice is assigned to the long run aggressive investor, and the least aggressive advice is assigned to the short term aggressive investor. So, the short run horizon takes the most conservative or the least aggressive of all advices, and the long run horizon takes the least conservative or the most aggressive advices. The medium horizon takes the medium of the all advices.

Table (4)
Calibrated Asset Allocation, Investor Risk Parameters, Horizon and Duration Length

| Panel A: Calibrated Optimal Portfolio Choice | | | | | |
|--|-----------------------|-----------------|-----------------|---------------|---------------------|
| Horizon | Risk Tolerance | Cash% | Stocks% | Bonds% | Bonds/Stocks |
| Short | Conservative | 51.1 | 19.8 | 29.1 | 1.47 |
| | Moderate | 22.2 | 48.1 | 29.7* | 0.62 |
| | Aggressive | 6.1 | 67.4 | 26.5 | 0.39 |
| Medium | Conservative | 15.3 | 43.2 | 41.5 | 0.96 |
| | Moderate | 10.1 | 62.6 | 27.3* | 0.44 |
| | Aggressive | -1.1 | 83.3 | 17.7 | 0.21 |
| Long | Conservative | 20.3 | 42.8 | 36.9 | 0.86 |
| | Moderate | 8.1 | 66.1 | 25.8 | 0.39 |
| | Aggressive | -2.6 | 96.1 | 6.5 | 0.07 |
| Panel B: Calibrated Investor's Risk Parameters, Horizon and Duration length | | | | | |
| | Parameter | Estimate | Boundary | | |
| Attitude toward risk | γ_{con} | 28.26 | no | | |
| | γ_{mod} | 8.13 | no | | |
| | γ_{agg} | 3.12 | no | | |
| Investment Horizons | H_{short} | 5.69 | no | | |
| | H_{med} | 10.31 | no | | |
| | H_{long} | 35.00 | upper | | |
| Optimal Duration of Bonds | η_{short} | 4.99 | lower | | |
| | η_{med} | 5.714 | upper | | |
| | η_{longt} | 5.714 | Lower | | |
| Value of the object function | | 0.1658 | | | |

* Diverge from the recommendation by more than 10%.

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