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SPATIAL COURNOT COMPETITION AND CONSUMERS' HETEROGENEITY: A NOTE

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Spatial Cournot competition and consumers' heterogeneity: a note

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Abstract We consider the standard model of spatial Cournot competition and show that a necessary condition for dispersion equilibria is that the distribution be not unimodal. *JEL Classification no.s*: D31, D40

Keywords Spatial Cournot competition, consumers' distribution.

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1 Introduction

Spatial Cournot competition with linear demand at all addresses has traditionally been studied within a framework where the consumers' distribution across markets is uniform: a framework where, starting from the work by Hamilton *et al.* (1989) and Anderson and Neven (1991), the standard result is that firms agglomerate in the middle of the linear city. As is well known, Gupta *et al.* (1997) showed how the features of the consumers' distribution crucially affect such a result – agglomeration turns out to be conditional on the density not being 'too thin' around the firms' equilibrium location. They also enquired about dispersion equilibria in the case of symmetric densities by presenting some examples, all of which use (roughly speaking) U-shaped distributions.

These results suggest that whether agglomeration or dispersion obtains in equilibrium should depend on how consumers are distributed across markets.[1] In this paper we build upon the work by Gupta *et al.* (1997) to derive necessary conditions for a pair of locations to be an equilibrium in the duopoly case with a generic density. In particular, this enables us to show that a necessary condition for dispersion to be an equilibrium is that the distribution be not unimodal. Our results are derived under assumptions about parameters which ensure that all markets are served in equilibrium – which, as is well known, amounts to transportation costs being low enough relative to the consumers' reservation price.

The paper is organized as follows. In the next section we recall the basic model and characterize its equilibrium locations as the solution to a two-equation system. Section 3 derives non-unimodality as a necessary condition for dispersion to be an equilibrium. Section 4 gathers some concluding remarks.

2 Equilibrium

In this section we review the basic duopoly model with Cournot spatial competition and give a general formulation of its equilibrium conditions for a generic consumer distribution.

Two firms located on the unit interval sell a homogeneous product to be

[1] Notice however that switching to the circular road model may yield dispersion (Pal, 1998), while Benassi *et al.* (2007) show that one other key variable is the level of transportation costs (relative to the consumers' reservation price): dispersion is the unique equilibrium in the linear city with a uniform distribution when such costs are high enough ($A < 1$ in our notation).

delivered in spatially diverse locations. Firm $i = 1, 2$, located in $x_i \in [0, 1]$ bears a linear transportation cost $t\,|x - x_i|$, $t > 0$, to deliver its product at location $x \in [0, 1]$; production takes place at constant marginal costs (standardly normalized at zero). Consumers are distributed as $F : [0, 1] \to [0, 1]$ over the same interval, F differentiable with mean μ; the (inverse) market demand at location x is linear and given by

$$p_x = a - bq_x \tag{1}$$

where $q_x \geq 0$ is the total quantity supplied at x, and a and b are strictly positive. We study the perfect equilibrium of the two stage game where firms choose their locations in the first stage, and the quantities they produce in the second stage of the game. By backward induction we first characterize the second stage Cournot equilibrium for given locations. To do so, we assume that $A = a/t > 2$ to ensure that all markets are covered at equilibrium (*e.g.*, Gupta *et al.*, 1997, p.264), and note that the assumption of constant marginal costs allows to look at the quantity equilibrium as a set of independent Cournot equilibria, one for each location $x \in [0, 1]$.

The profit firm $i = 1, 2$ $(i \neq j = 1, 2)$, located at x_i, obtains from the market located in x is given by

$$\pi_i(x_i, x_j; x) = (a - bq_x - t\,|x - x_i|)\, q_x^i(x_i, x_j)$$

where $q_x^i(x_i, x_j)$ is the output it sells at x and $q_x = q_x^1(x_1, x_2) + q_x^2(x_1, x_2)$ is the total output available at location x. At the unique Cournot equilibrium, firm i's profits are given by

$$\widehat{\pi}_i(x_i, x_j; x) = \tfrac{1}{9b}\left(a + t\,|x - x_j| - 2t\,|x - x_i|\right)^2 \tag{2}$$

which depend on both firms' locations, and are to be used to determine the perfect equilibrium locations in the first stage. To do so, we write the first stage profit function for firm i as

$$\Pi_i(x_i, x_j) = \int_0^1 \widehat{\pi}_i(x_i, x_j; x) f(x) dx \tag{3}$$

where we integrate over all markets and (letting primes denote derivatives) $f(x) = F'(x)$ is the (strictly positive) continuous density describing the consumers' distribution across locations.

A perfect Nash equilibrium in locations is then given by the pair (x_1^*, x_2^*) such that $\partial\Pi_i(x_i, x_j)/\partial x_i = 0$ $(i \neq j = 1, 2)$, and each firm's profit is maximized. Straightforward calculations impose that the second order conditions to be satisfied at an interior equilibrium are such that

$$f(x_i^*)(A + x_2^* - x_1^*) \geq 1 \tag{4}$$

for $i = 1, 2$, where w.l.o.g. we are assuming that $x_2 \geq x_1$, so that $x_2^* - x_1^* = \delta^*$ is the non negative equilibrium distance between the firms.

The firms' equilibrium locations are identified by the following result, derived for a generic consumer density:

Proposition 1 *Let* $(x_1^*, x_2^*) = (x^*, x^* + \delta^*)$, $\delta^* \geq 0$, *be the equilibrium locations of firms* $i = 1, 2$. *Then the pair* (x^*, δ^*) *is a solution to the system*

$$\delta^* = \frac{1}{1 + 2F(x^*)} \left(2 \int_{x^*}^{x^* + \delta^*} F(z) dz + x^* - \mu + [1 - 2F(x^*)] A \right) \quad (5.a)$$

$$F(x^* + \delta^*) = \frac{3}{2} \frac{\delta^*}{(A + \delta^*)} + F(x^*) \quad (5.b)$$

Proof. See Appendix. ■

Proposition 1, which follows directly from equations (4.4) and (4.5) of Gupta *et al.* (1997), amounts to identifying the pair (x_1^*, x_2^*) at which the first order conditions for both firms' profit maximization problems are satisfied, and accordingly provides necessary conditions for a perfect equilibrium in (quantities and) locations. As per (4), sufficient conditions also require that the second order constraints $f(x_i^*)(A + \delta^*) \geq 1$, $i = 1, 2$, hold, while we know from Gupta *et al.* (1997, p.265) that no firm's equilibrium location will ever be 0 or 1.[2]

We report in Figure 1 an example of equilibrium dispersion with a symmetric density, based on Gupta *et al.* (1997, p.280): we plot the mappings $x^a(\delta)$ (dotted) and $x^b(\delta)$ (solid) which solve equations (5.a,b), such that $(x_1^*, x_2^*) = (0.4, 0.6)$ and hence $\delta^* = 0.2$.[3]

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[2] These authors' proof relies on differentiating each firm's profit function (3) with respect to its own location, to get that $\lim_{x_1 \to 0^+} \partial \Pi_1 / \partial x_1 > 0$ and $\lim_{x_2 \to 1^-} \partial \Pi_2 / \partial x_2 > 0$.

[3] This is Gupta *et al.*'s example 7 (see also their f.note 6, p.280), where the density is $f(x) = 1/2 + 6(x - 1/2)^2$, symmetric with mean $\mu = 1/2$, and $A = 2.681$; the second order bounds on the density are satisfied at (x_1^*, x_2^*). Notice that in general $x^b(\delta)$ is not a one-to-one function, since clearly equation (5.b) may have more than one solution. Also, symmetry implies that $x^b(\cdot)$ be vertical at δ^*.

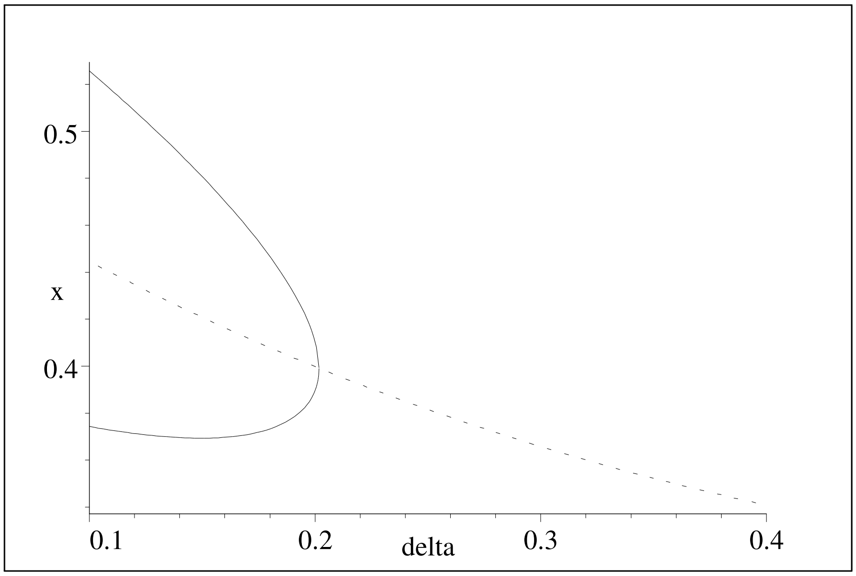

Figure 1. Equilibrium dispersion

We now turn to discussing dispersion equilibria.

3 Dispersion

Let $h(x, A) = x - \mu + [1 - 2F(x)]\,A$, and let some $\widehat{x} \in (0, 1)$ be a solution to $h(\cdot, A) = 0$. It is then obvious by inspection that $(x^*, \delta^*) = (\widehat{x}, 0)$ is a solution to (5.a,b): i.e., agglomeration is a (candidate) equilibrium – actually, one such always exists whenever the density is unimodal,[4] and indeed Gupta *et al.* (1997) show that in general agglomeration can be an equilibrium only if the density is 'thick' enough around the firms' location. However, their discussion about dispersion relies mainly on examples of symmetric densities, all of which are (roughly speaking) U-shaped. This suggests that non unimodality be somehow crucial for the existence of dispersion equilibria. In this section we show that this is indeed the case.

Two general observations should be made as preliminary remarks. First, independently of the actual shape of the distribution, firms cannot locate too far apart from each other, as the following Lemma applies

Lemma 1 *Let $(x_1^*, x_2^*) = (x^*, x^* + \delta^*)$, $\delta^* > 0$, be a pair of dispersed equilibrium locations. Then $F(x_2^*) - F(x_1^*) < \frac{1}{2}$.*

[4]This is an implication of Gupta *et al.* (1997, p.269), as unimodality implies the existence of a 'modal interval' $I \subset (0, 1)$ such that $f(x) > 1$ for all $x \in I$. Notice that $h(0, A) > 0 > h(1, A)$, so that $h(\cdot, A) = 0$ has always a solution.

Proof. Contrary to the result, suppose $F(x_2^*) - F(x_1^*) \geq \frac{1}{2}$. Then equation (5.b) implies $2\delta^* \geq A$: since $A > 2$, this implies $\delta^* = x_2^* - x_1^* > 1$, a contradiction. ■

Secondly, Proposition 1 suggests that the existence of a dispersion equilibrium places some restrictions on the concavity of the consumers' density. Indeed, letting $F(x^* + \delta^*) - F(x^*) = \Delta F^*$, equation (5.b) can be written as $\Delta F^*/\delta^* = 3/2\,(A + \delta^*)$: since by the mean value theorem there trivially exists some $z \in (x^*, x + \delta^*)$ such that $f(z) = \Delta F^*/\delta^*$, at a dispersed equilibrium one must have $\frac{2}{3}f(z) = 1/\,(A + \delta^*) \leq f(x_i^*)$, $i = 1, 2$: i.e., the second order conditions for maximum profits (see (4)) set limits on how thick the density between locations can be, which actually vindicates Gupta *et al.*'s (1997, p.277) emphasis on densities that are not unimodal.

Together with Lemma 1, this intuition accounts for the following result:

Proposition 2 *Let* $(x_1^*, x_2^*) = (x^*, x^* + \delta^*)$*,* $\delta^* > 0$*, be a pair of dispersed equilibrium locations. Then the density* $f(x)$ *is not unimodal.*

Proof. See Appendix ■

Clearly, a natural implication of Proposition 2 is that non-unimodality is required for the co-existence of equilibria with and without dispersion.

4 Concluding remarks

In this note we have considered the standard model of spatial Cournot competition with two firms, to set up a general expression for identifying the firms' equilibrium locations. This allows to establish that non-unimodality is a necessary condition for the existence of dispersion equilibria.

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Appendix

Proof of Proposition 1

The first order conditions for profit maximization, given definition (2) and solving integrals, reduce to

$$A[1-2F(x_1)] - [1+2F(x_1)]\,\delta + x_1 - \mu + 2I(x_1,x_2) = 0 \qquad \text{(A.1.a)}$$
$$A[1-2F(x_2)] + [1-2F(x_2)]\,\delta + x_2 - \mu + 2I(x_1,x_2) = 0 \qquad \text{(A.1.b)}$$

where to ease notation we let $\delta = x_2 - x_1 \geq 0$ and $I(x_1,x_2) = \int_{x_1}^{x_2} F(x)dx$. Now we may sum and subtract over (A.1) to get

$$A\Delta F - \frac{3}{2}\delta + \delta\Delta F = 0 \qquad \text{(A.2.a)}$$

$$A\,[1-\Sigma F] + \frac{x_1+x_2}{2} - \delta\Sigma F - \mu + 2I(x_1,x_2) = 0 \qquad \text{(A.2.b)}$$

where $\Sigma F = F(x_1) + F(x_2)$ and $\Delta F = F(x_2) - F(x_1) \geq 0$. By summing over again we obtain

$$\delta = \frac{1}{1+2F(x_1)}\left(2I(x_1,x_2) + x_1 - \mu + [1-2F(x_1)]\,A\right) \qquad \text{(A.3)}$$

which, letting equilibrium values $x_1^* = x^*$ and $x_2^* = x^* + \delta^*$, amounts to equation (5.a).

By subtracting (A.2.b) from (A.2.a) we now get

$$[1-2F(x_2)]\,\delta = -\left\{2I(x_1,x_2) + x_2 - \mu + [1-2F(x_2)]\,A\right\}$$

which, solving for δ from (A.3) and letting $x_2 = x_1 + \delta$ gives, after semplifications, the equilibrium value

$$\delta^* = \frac{2A\Delta F}{3 - 2\Delta F} \tag{A.4}$$

which can be solved for $\Delta F = F(x^* + \delta^*) - F(x^*)$ to give equation (5.b).■

Proof of Proposition 2

If $(x^*, x^* + \delta^*)$ is a pair of equilibrium dispersed locations, $\delta^* \in (0, 1 - x^*)$ must satisfy the twin conditions

$$\kappa(\delta^*) = 0 \tag{A.5.a}$$
$$\lambda(\delta^*) = 0 \tag{A.5.b}$$

where, using (5.a,b), the following definitions apply:

$$\kappa(\delta) = \delta[1 + 2F(x^*)] - 2\int_{x^*}^{x^*+\delta} F(z)dz - h(x^*, A) \tag{A.6.a}$$
$$\lambda(\delta) = \Delta(\delta) - \frac{3}{2}\frac{\delta}{A + \delta} \tag{A.6.b}$$

here $h(x^*, A) = x^* - \mu + [1 - 2F(x^*)]\,A$ and, to ease notation, $\Delta(\delta) = F(x^* + \delta) - F(x^*)$, an increasing function. Both κ and λ are continuous functions. We now proceed in three steps:

(i) The function $\kappa(\delta)$ is strictly concave for any $\delta \in [0, 1 - x^*]$, increasing at least up to some $\widetilde{\delta} \in (\delta^*, 1 - x^*]$, and such that $\kappa(0) = -h(x^*, A) < 0$ and $\kappa'(\delta^*) < 1$. Indeed, differentiation shows that $\kappa'(\delta) = 1 - 2\Delta(\delta)$, clearly decreasing and positive for $\delta \le \delta^*$, as by Lemma 1 $\Delta(\delta^*) < 1/2$: hence, $\kappa(0) = -h(x^*, A) < 0$ and that $\kappa'(\delta^*) > 0$. Since (A.5.a) is a necessary condition for equilibrium, this ensures that for the given x^* there is only one δ^* such that $(x^*, x^* + \delta^*)$ is an equilibrium location pair.

(ii) The function $\lambda(\delta)$ is such that:

(a) $\lambda(1 - x^*) > 0$: from (i) it must be $h(x^*, A) > 0$: this directly implies that

$$1 - F(x^*) > \frac{1}{2}\left(1 - \frac{x^*}{A}\right) \tag{A.7}$$

Using (A.6.b) and noting that $\Delta(1 - x^*) = 1 - F(x^*)$, from (A.7) $\lambda(1 - x^*) > \frac{1}{2}\left(1 - \frac{x^*}{A}\right) - \frac{3}{2}\frac{1-x^*}{A+1-x^*}$, so that $\lambda(1 - x^*) > 0$ if

$$\frac{1}{2}\left(1 - \frac{x^*}{A}\right) - \frac{3}{2}\frac{1 - x^*}{A + 1 - x^*} > 0 \tag{A.8}$$

which amounts to $(A-x^*)(A+1-x^*)-3A(1-x^*)>0$: this is surely true for any $x^*\in[0,1]$ and $A>2$.

(b) there are at least two values of δ in the interval $(0,1-x^*)$ such that the derivative of $\lambda(\delta)$ vanishes. To see this, notice that λ cannot be monotone as $\lambda(0)=\lambda(\delta^*)=0$. Also, $\lambda(1-x^*)>0$ implies the existence of some $\widehat{\delta}\in(0,1-x^*)$ such that $\widehat{\delta}\geq\delta^*$, with $\lambda(\widehat{\delta})=0<\lambda'(\widehat{\delta})$ and $\lambda(\delta)>0$ for all $\delta\in(\widehat{\delta},1-x^*)$. Let now $\Gamma(\delta)=\kappa'(\delta)\lambda(\delta)+\kappa(\delta)$, such that $\Gamma(0)=\Gamma(\delta^*)=0$: there has to be some $\overline{\delta}\in(0,\delta^*)$ such that $\Gamma'(\overline{\delta})=\kappa''(\overline{\delta})\lambda(\overline{\delta})+\left[\lambda'(\overline{\delta})+1\right]\kappa'(\overline{\delta})=0$ i.e. $\kappa''(\overline{\delta})\lambda(\overline{\delta})=-\left[\lambda'(\overline{\delta})+1\right]\kappa'(\overline{\delta})<0$: since $\lambda'(\delta)+1=f(x+\delta)+\frac{2(A+\delta)^2-3A}{2(A+\delta)^2}>0$ for all $\delta\in[0,1-x^*]$ and κ is concave, it must be $\lambda(\overline{\delta})>0$. All of which implies that $\lambda'(\delta)$ changes sign at least twice over $[0,\widehat{\delta}]$: there is at least a pair (δ_1,δ_2), $\delta_1<\delta_2$, say, $\delta_i\in(0,\widehat{\delta})$ for $i=1,2$, such that $\lambda'(\delta_i)=0$.

(*iii*) By (A.6.b), it is now easily seen that $f(x^*+\delta_1)>f(x^*+\delta_2)$, since $\lambda'(\delta_i)=0$ is equivalent to $f(x^*+\delta_i)=\frac{3}{2}\frac{A}{(A+\delta_i)^2}$ and $\delta_1<\delta_2$. Hence, unimodality is ruled out if one can find some $\delta_3>\delta_2$ such that $f(x^*+\delta_3)>f(x^*+\delta_2)$. To do so, consider the function $\theta(\delta)=\frac{1-F(x^*)}{1-x^*}\delta-\frac{3}{2}\frac{\delta}{A+\delta}$; this is positive for any $\delta\in(0,1-x^*]$: indeed, $\theta(1-x^*)=\lambda(1-x^*)>0=\theta(0)$, while $\theta'(\delta)=\frac{1-F(x^*)}{1-x^*}-\frac{3}{2}\frac{A}{(A+\delta)^2}>0$: as θ is strictly convex, $\theta'(\delta)>0$ if $\theta'(0)\geq0$, i.e. $\frac{1-F(x^*)}{1-x^*}\geq\frac{3}{2A}$, which is true for $A>2$. To see this, notice that $h(x^*,A)>0$ implies $\frac{1-F(x^*)}{1-x^*}>\frac{A-x^*+\mu}{2A(1-x^*)}$, so that $\theta'(0)>0$ if $A+2x^*+\mu-3\geq0$. That the latter is verified in equilibrium can be seen as follows:

(a') $h(x+\delta^*,A+\delta^*)=x^*+\delta^*-\mu+\left[1-2F(x^*+\delta^*)\right](A+\delta^*)<0$: this follows from (A.5) and the definitions (A.6), by noting that $\delta^*\left[1+2F(x^*)\right]>h(x^*,A)$ and substituting for $F(x^*)$ from (A.5b); there follows $\mu>x^*+\delta^*+\left[1-2F(x^*+\delta^*)\right](A+\delta^*)$ and $A+2x^*+\mu-3>2A-3(1-x^*)+2\delta^*-2(A+\delta^*)F(x^*+\delta^*)$;

(b') by substituting back $F(x^*+\delta^*)(A+\delta^*)$ from (A5.b) we get $A+2x^*+\mu-3>2A-3(1-x^*)-\delta^*+2(A+\delta^*)F(x^*)>0$: which holds as $3(1-x^*)+\delta^*<4$, while $2A>4$.

Now notice that by construction $\widehat{\delta}>\delta_2>\delta_1$ and $\theta(\widehat{\delta})=\frac{1-F(x^*)}{1-x^*}\widehat{\delta}-\Delta(\widehat{\delta})>0$. Since $\Delta(1-x^*)=1-F(x^*)$, it is true at $\widehat{\delta}$ that $\frac{\Delta(1-x^*)}{1-x^*}>\frac{\Delta(\widehat{\delta})}{\widehat{\delta}}$, so that there exists some $\delta_3\in[\widehat{\delta},1-x^*]$ such that $\frac{\Delta(1-x^*)}{1-x^*}<\Delta'(\delta_3)=f(x+\delta_3)$; but $0<\theta'(\delta_2)=\frac{\Delta(1-x^*)}{1-x^*}-f(x^*+\delta_2)$: hence $f(x^*+\delta_2)<\frac{\Delta(1-x^*)}{1-x^*}<f(x+\delta_3)$.■