Bottleneck Congestion and Modal Split Revisited

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Summary

The paper examines the efficiency of alternative road pricing schemes when an alternative railroad service is available. The paper uses a model, developed by Tabuchi (1993), in which road transport presents a bottleneck congestion technology while railroad transport shows economies of scale with respect to the number of train users. The competition between the two modes is assumed to be on cost basis only. It is found that if the railroad fare is set equal to the average cost, the relative efficiency of the regimes depends on parameters' values. The numerical simulation shows that the fine toll regime is generally to be preferred to the alternative regimes but when the fixed railroad cost are large enough so that the inefficient exploitation of the scale economies is less than compensated by the toll revenue.

Non technical summary

Road congestion takes place when the number of users exceed the road capacity. It entails a cost (loss of time) to the car user who enters the road and to all car users who are on the road. Consequently, it represents a private and social cost. Economic theory suggests that excess congestion derives from a lack of perception of the costs involved due to the lack of a price signal. Road pricing would potentially send the right signal to users. Because of practical reasons, road pricing is scarcely implemented and, in particular, it is not implemented as the theory suggests. The toll is not perfectly time-dependent, but it is often fixed or it differentiates between peak and off-peak periods. The paper focuses on the efficiency of such toll systems when an alternative railroad transport mode is available. Railroad transport shows economies of scale.
Such problem has not receive wide attention in the literature. We focus our analysis to the case when rail transport is priced on the basis of average costs in order to cover all costs. The application of a average-cost pricing rule to railroad transport is known as a theoretically sub-optimal pricing rule, but it is interesting to study as second best case which might take place in reality if rail is privatised. On the contrary, Tabuchi (1993) analysed the case when the rail transport fare is set equal to marginal costs.
Tabuchi (1993) concludes that there is a definite ranking of the road toll regimes and shows that the social optimum is reached with the perfectly-time dependent toll system. We find that if the average cost pricing rule is applied to railroad transport the relative efficiency of the regimes is not definite and depends on parameters' values. The crucial variable is not the number of commuters (a proxy of

the city size) but the amount of fixed costs (a proxy of the type of infrastructure). For a large enough value of the railroad fixed cost, the finer the toll system the greater are the total cost of the transport system as a whole because the inefficient exploitation of the scale economies is less than compensated by the toll revenue.

1 *Introduction*

This paper revisits the article by Tabuchi (1993) “Bottleneck Congestion and Modal Split” in which he investigates the optimality and efficiency of several railroad fare and road toll regimes in a model with two competing transport modes (the private car and the railroad) connecting the same origin-destination pair[1]. Road transport presents a bottleneck congestion technology while railroad transport shows economies of scale with respect to the number of commuters due to the fixed infrastructure. Competition between the two modes is assumed to be on cost-basis only, with an inelastic total demand and with the two modes being perfect substitutes.
Tabuchi states that the set of road toll policies is analysed while at the same time setting the railroad fare on a cost basis (i.e., equal to the average cost). But this is not what it is performed in the formal derivation. Formally, Tabuchi applies to railroad transport the marginal cost-pricing rule. This paper follows Tabuchi’s stated proposal and formally derives the modal split and the total cost when applying the average cost pricing rule to railroad transport and applying to road transport four alternative toll regimes: no toll, a fine toll, a coarse toll and a uniform toll. The application of a average-cost pricing rule to railroad transport is known as a theoretically sub-optimal pricing rule, but it is interesting to study as second best case which might take place in reality if rail is privatised.
Tabuchi (1993) concludes that there is a definite ranking of the regimes and shows that the social optimum is reached with the fine toll system[2]. Unlike Tabuchi, we find that if the average cost pricing rule is applied to railroad transport the relative efficiency of the regimes is not definite and depends on parameters' values. The crucial variable is not the number of commuters (a proxy of the city size) but the amount of fixed costs (a proxy of the type of infrastructure). For a large enough value of the railroad fixed cost, the finer the toll system the greater are the total cost of the transport system as a whole because the inefficient exploitation of the scale economies is less than compensated by the toll revenue.
The bottleneck congestion model is briefly presented in Section 1, the interaction between road and railroad transport is modelled in Section 2. Section 3 is devoted to a numerical simulation and to a test of the sensitivity of the results to changes in the parameters’ values. Conclusions and extensions are discussed in Section 4.

2 *The bottleneck congestion model*

The bottleneck congestion model has been formally developed by various authors (Newell, 1987; Braid, 1989; Arnott, de Palma and Lindsey (hereafter ADL), 1990b, 1993) along the lines traced by Vickery (1969). Several model extensions have been studied (ADL 1990a, 1991, 1992, 1994). The essential feature of the bottleneck congestion model is that it allows for endogenous scheduling so that each individual can choose, for example, whether to enter a queue (incurring in high travel costs) or to postpone his departure (incurring in high schedule-delay costs) depending on his preferences. The model then becomes dynamic in nature with novel and important implications for optimal pricing policies since the information provided by a system of congestion charges may alter queue formation and reduce congestion costs (which are proven to be a pure deadweight loss).

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[1] The interaction between the private car and public transport has long been debated in the literature. In the 70's, Sherman (1971, 1972), Baum (1973) and Jackson (1975) discussed the opportunity of partially or totally subsidising public transport to reduce road congestion. Dorling *et al.* (1974) instead advocated the symmetrical policy issue of using road tolls for reversing the decline of public transport without presenting a formal model. Small (1983) and Viton (1983, 1986) studied the design of optimal tolls on an urban highway jointly used by an express bus service. Glaister and Lewis (1978) and De Borger *et al.* (1986) developed an urban passenger transport model to calculate optimal tolls taking into consideration three modes (the private car, the bus and the rail-service), three external cost sources (congestion, environmental degradation and accidents), distributional aspects and pricing constraints.
[2] He also discusses the opportunity of having a railroad system with different city sizes.

The bottleneck model is mainly focused on the journey-to-work trip and assumes that N individuals commute to work from a residential area to the Central Business District along the same road. Travel is uncongested but at a single bottleneck through which at most s cars can pass per unit of time; if the arrival rate at the bottleneck exceeds s, a queue develops. Congestion technology is therefore of the bottleneck type opposed to a flow congestion technology[3]. Referring to ADL (1993) for a detailed derivation of the model, we just recall that four types of equilibria can be derived: a no-toll equilibrium, a uniform-toll equilibrium, a coarse-toll equilibrium and fine-toll equilibrium. Let us recall the meaning of each equilibrium and the main results.
With no toll, ADL (1993) show that total travel cost TC, which is the sum of total travel time cost TTC and schedule delay cost SDC, is equal to

(2.1) $$TC^e = d\left(\frac{N^2}{s}\right)$$

where the superscript e denotes the no-toll equilibrium and

(2.2) $$d \equiv \frac{bg}{b+g}$$

(2.3) $$TTC^e = SDC^e = \frac{d}{2}\left(\frac{N^2}{s}\right)$$

Average total travel cost ATC and marginal social cost MSC are, respectively, equal to

(2.4) $$ATC^e = \frac{TC^e}{N} = \frac{dN}{s}$$

(2.5) $$MSC^e = \frac{\partial TC^e}{\partial N} = \frac{2dN}{s} = 2ATC^e$$

Note that queuing time in this model is pure deadweight loss. As ADL explain "*If the departure rate were set at s between* t_q *and* $t_{q'}$*, queuing would be eliminated. Furthermore, since the time pattern of arrivals would be the same as in the no-toll equilibrium, total schedule delay cost would be unchanged. Thus, the social saving from the change would equal total travel time cost in the no-toll equilibrium, which equals one-half of total travel cost*". This example highlights the potential improvements in welfare that could be made by using a toll to induce a change in the time pattern of departures.
As a matter of fact, if we impose a fine toll (i.e. a time-varying toll) a social optimum can be reached with no queue and therefore zero travel time cost. Total cost is then equal to

(2.6) $$TC^o = SDC^o = \frac{d}{2}\left(\frac{N^2}{s}\right)$$

where the superscript o denotes the social optimum. Average total travel cost ATC and marginal social cost MSC are, respectively, equal to:

(2.7) $$ATC^o = \frac{TC^o}{N} = \frac{dN}{2s}$$

[3] The latter type of congestion technology assumes that there is congestion outside the bottleneck as well and is related to the number of car that depart or arrive together (see Chu (1995) for a comparison of the two approaches).

(2.8) $MSC^{o}=\frac{\partial TC^{o}}{\partial N}=\frac{dN}{s}=2ATC^{o}$

The toll starts from zero, then it increases linearly from t_q (the most recent time at which there is no queue) to t^* (the desired time of arrival at work) and decreases linearly from t^* to $t_{q'}$ (the end of the rush hour). The average toll equals the average travel cost[4].

On the contrary a uniform toll adds a constant fee to each trip and therefore does not alter the departure pattern. The total travel cost is related to N and s in the same way as in the no-toll equilibrium

(2.9) $TC^{u}=d\left(\frac{N^{2}}{s}\right)$

and similarly

(2.10) $MSC^{u}=2ATC^{u}$

For efficiency, the price of a trip should be equal to marginal social price thus

(2.11) $p^{u}=MSC^{u}=C^{u}+t^{u}$

(2.12) $t^{u}=ATC^{u}$

A coarse toll (e.g. a toll with a peak and an off-peak fare) is more efficient that a uniform toll since it alters the departure time, but it does not completely eliminate queuing. ADL (1990) show that

(2.13) $TC^{c}=\frac{d}{4}\left[3-\frac{(g-a)b}{(b+g)(a+b)}\right]\frac{N^{2}}{s}=\frac{ydN^{2}}{s}$

with $y=\frac{1}{4}\left[3-\frac{(g-a)b}{(b+g)(a+b)}\right]$ and where the superscript c denotes the coarse toll.

To summarise, the supply function in the four regimes would then be equal to

(2.14) $p^{j}(N,s)=\frac{2\Gamma dN}{s}$

with

[4] In fact, for efficiency each traveller should pay marginal social cost which equals twice the average cost. In the absence of the queue the person who arrives on time faces zero travel cost and should therefore pay the maximum toll which equals twice the average travel cost. On the contrary the persons who depart first and last incur twice the average schedule delay and should therefore pay a no toll.

$$\Gamma^u = 1$$

(2.15) $$\Gamma^o = \Gamma^e = \frac{1}{2}$$

$$\Gamma^c = \frac{1}{4}\left[3 - \frac{(g-a)b}{(b+g)(a+b)}\right]$$

Let now the demand function be

(2.16) $$N = N(p) \qquad \frac{dN}{dp} < 0$$

Equation (2.15) and (2.16) imply that (Fig. 1)

(2.17) $$\hat{p}^u(s) > \hat{p}^c(s) > \hat{p}^o(s) = \hat{p}^e(s)$$

which, with variable demand as in (2.16), in turn imply

(2.18) $$\hat{N}^u(s) < \hat{N}^c(s) < \hat{N}^o(s) = \hat{N}^e(s)$$

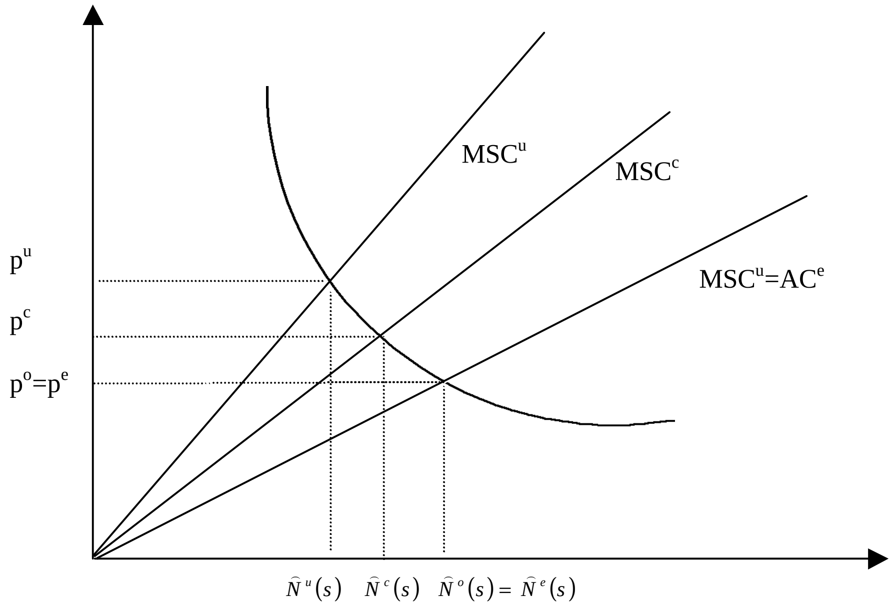

Fig. 1 – Equilibria with alternative pricing regimes in a bottleneck congestion model.

The explanation of the first inequality is as follows. For a given number of travellers, the price of a trip is lowest for the no-toll equilibrium (since no toll is charged) and the fine-toll equilibrium (since travel on the road is efficient)[5], intermediate for the coarse-toll equilibrium and highest for the uniform-toll equilibrium. For a decreasing demand function the inequality (2.18) follows.

3 *Congestion and public transport*

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[5] The coincidence of the no-toll and fine-toll supply functions reflects the particular congestion technology assumed.

Let us now suppose that a railroad connecting the residential area to the CBD is available as an alternative commuting mode. The N commuters could then be divided among automobile commuters N_a and train commuters N_b. The sum of the two is fixed (N) and commuting is a must. Following Tabuchi (1993), let us assume that the average cost per train commuter is given by

(3.1) $$C_b = c + \frac{F}{Nb}$$

where c is the marginal cost per commuter, which might include the increase in operative costs and the marginal boarding and alighting time costs , and F is the fixed cost. As the number of train commuters increase C_b decreases. Let us assume the modal equilibrium to be characterised by

(3.2a) $$C_a = C_b \qquad for\ N_b > 0$$

(3.2b) $$C_a < C_b \qquad for\ N_b = 0$$

when (3.2a) holds both modes are in use, when (3.2b) holds the car is the only commuting mode.
Such an equilibrium condition is rather crude. Being based on costs only, it implies that the cross-elasticity of demand between transport and other goods is negligible[6] and that there is perfect substitution among road and rail transport. Such bimodal equilibrium has been termed in the literature as a Wardrop Modal Equilibrium (Williams, 1998; and Mackie and Bonsall, 1989). It has been shown to be a limiting case of the general multimodal equilibrium and to allow for the Downs-Thomson-Mogridge paradox when highway capacity is expanded[7].
Moreover, it does not consider the time cost involved in waiting on riding the train as in Mohring (1972).
Under such hypothesis, total social cost of the transport system is then

(3.3a) $$TC = C_a N_a + C_b N_b \qquad for\ N_b > 0$$

(3.3b) $$TC = C_a N_a \qquad for\ N_b = 0$$

We now derive the equilibrium conditions using equation (3.2) and estimate the total social cost using equation (3.3) in four different regimes: a regime of average cost pricing (cost-based fare) for train commuting and no toll for auto commuting (subsection 3.1); a regime of average cost pricing and fine toll (subsection 3.2); a regime of average cost pricing and coarse toll (subsection 3.3); and a regime of average cost pricing and uniform toll (subsection 3.4).
It seem interesting to analyse a situation in which the congestion externality is either not internalised, perfectly internalised or imperfectly internalised while the optimal marginal cost pricing rule in not applied in the railroad mode. It corresponds to sub-optimal situation when public transport is deregulated or privatised.

3.1 *Average cost pricing with no road toll*

If the railroad fare p^a is set equal to its average cost and there is no road toll, the equilibrium numbers of respective user is

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[6] As Tabuchi (1993, p. 418) underlines, although total demand $N=N_a + N_b$ is fixed, modal demand is not and is endogenously determined.

$$N_b = N - sp/d$$

where p is the railroad fare, which might be equal to the average cost on a zero loss basis or below the average cost if a loss is acceptable.

[7] If the Wardrop Modal Equilibrium applies, an highway expansion meant to relief congestion leads paradoxically to increase both congestion and public transport costs (Downs, 1962; Thomson, 1977, Mogridge, 1990).

$$(3.4)\quad \left(N_a^a, N_b^a\right) = \left(\frac{N}{2} + \frac{cs}{2d} - \sqrt{\left(\frac{N}{2} - \frac{cs}{2d}\right)^2 - \frac{sF}{d}}, \frac{N}{2} - \frac{cs}{2d} + \sqrt{\left(\frac{N}{2} - \frac{cs}{2d}\right)^2 - \frac{sF}{d}} \right) \qquad for\ N > \hat{N}$$

$$= (N,0) \qquad for\ N \leq \hat{N}$$

where $\hat{N} = 2\sqrt{sF} + cs/d$ is the value which equals the square root to zero.
Below $\hat{N}$ only road is in use. Above $\hat{N}$ both road and railroad are in use and the costs of building and operating the railroad are covered. $\hat{N}$ could be interpreted as a (city or corridor) threshold above which it makes economic sense to build a railroad. Note that N_b cannot be small in equilibrium because the fixed cost F in the railroad sector would not be covered. Therefore, the distribution is discontinuous at $N = \hat{N}$. As N gets larger, the distribution remains *(N,0)* for small *N*, but jumps to $\left(N_a^a, N_b^a\right)$ at $N = \hat{N}$.
Since auto commuting has external diseconomies while train commuting has external economies, it is well known that this distribution does not achieve a social optimum.
Substituting (3.4) into (3.3), the total cost is expressed as

$$(3.5)\quad \begin{aligned} TC^e &= cN + \frac{N}{N_b^e}F && for\ N > \hat{N}^u \\ TC^e &= \frac{dN^2}{s} && for\ N < \hat{N}^u \end{aligned}$$

3.2 *Average cost pricing with a fine road toll*

If the railroad fare p^a is set equal to its average cost and there is a fine road toll, which on average equals the ATC, the equilibrium numbers of respective user is equal to the previous case. So is the total cost gross of the toll. The total cost net of the toll is

$$(3.6)\quad \begin{aligned} TC^f &= cN + \frac{N}{N_b^f}F - \frac{d\left(N_a^f\right)^2}{2s} && for\ N > \hat{N}^f \\ TC^f &= \frac{dN^2}{s} && for\ N < \hat{N}^f \end{aligned}$$

Such derivation and those in subsection 3.3 and 3.4 are the main differences with Tabuchi (1993, p. 425-7). In fact, Tabuchi obtains the socially optimum number of road users equating the MSC of road users to the MC of train users, although he states that he wants to analyse the same problem. On the contrary, we obtained the modal split whereby users are comparing the MSC of using the road with the AC of using the train (railway fares are assumed to be cost-based), therefore obtaining a different set of TC formulations and different conclusions[8].

3.3 *Average cost pricing with a coarse road toll*

If the railroad fare p^a is set equal to its average cost and there is a coarse toll on road users, the equilibrium numbers of respective user is

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[8] Furthermore, Tabuchi (1993, p. 425) mistakenly considers only half of the MSC the auto users face (the toll is not included).

(3.7)

$$\left(N_a^c, N_b^c\right) = \left(\frac{N}{2} + \frac{cs}{4yd} - \sqrt{\left(\frac{N}{2} - \frac{cs}{4yd}\right)^2 - \frac{sF}{2yd}}, \frac{N}{2} - \frac{cs}{4yd} + \sqrt{\left(\frac{N}{2} - \frac{cs}{4yd}\right)^2 - \frac{sF}{2yd}}\right) \qquad \text{for } N > \hat{N}^c$$

$$= \left(N, 0\right) \qquad \text{for } N \le \hat{N}^c$$

where $\hat{N}^c = 2\sqrt{\frac{sF}{2yd}} + \frac{cs}{2yd}$ is the value which equals the square root to zero. The total cost net of the toll is

(3.8)

$$TC^c = cN + \frac{N}{N_b^c}F - \frac{yd\left(N_a^c\right)^2}{s} \qquad \text{for } N > \hat{N}^c$$

$$TC^c = \frac{dN^2}{s} \qquad \text{for } N < \hat{N}^c$$

3.4 *Average cost pricing with a uniform road toll*

If the railroad fare p^a is set equal to its average cost and there is a uniform toll on road users, the equilibrium numbers of respective user is

(3.9)

$$\left(N_a^c, N_b^c\right) = \left(\frac{N}{2} + \frac{cs}{4d} - \sqrt{\left(\frac{N}{2} - \frac{cs}{4d}\right)^2 - \frac{sF}{2d}}, \frac{N}{2} - \frac{cs}{4d} + \sqrt{\left(\frac{N}{2} - \frac{cs}{4d}\right)^2 - \frac{sF}{2d}}\right) \qquad \text{for } N > \hat{N}^u$$

$$= \left(N, 0\right) \qquad \text{for } N \le \hat{N}^u$$

where $\hat{N}^c = 2\sqrt{\frac{sF}{2d}} + \frac{cs}{2d}$ is the value which equals the square root to zero. The total cost net of the toll is

(3.10)

$$TC^u = cN + \frac{N}{N_b^u}F - \frac{d\left(N_a^u\right)^2}{s} \qquad \text{for } N > \hat{N}^u$$

$$TC^u = \frac{dN^2}{s} \qquad \text{for } N < \hat{N}^u$$

3.5 *Comparison*

When N is fixed the inequalities (2.25) for auto commuters

$$\hat{N}_a^u(s) < \hat{N}_a^c(s) < \hat{N}_a^o(s) = \hat{N}_a^e(s)$$

imply the opposite inequalities for train commuters (Fig. 2)

(3.11) $\hat{N}_b^u(s) > \hat{N}_b^c(s) > \hat{N}_b^o(s) = \hat{N}_b^e(s)$

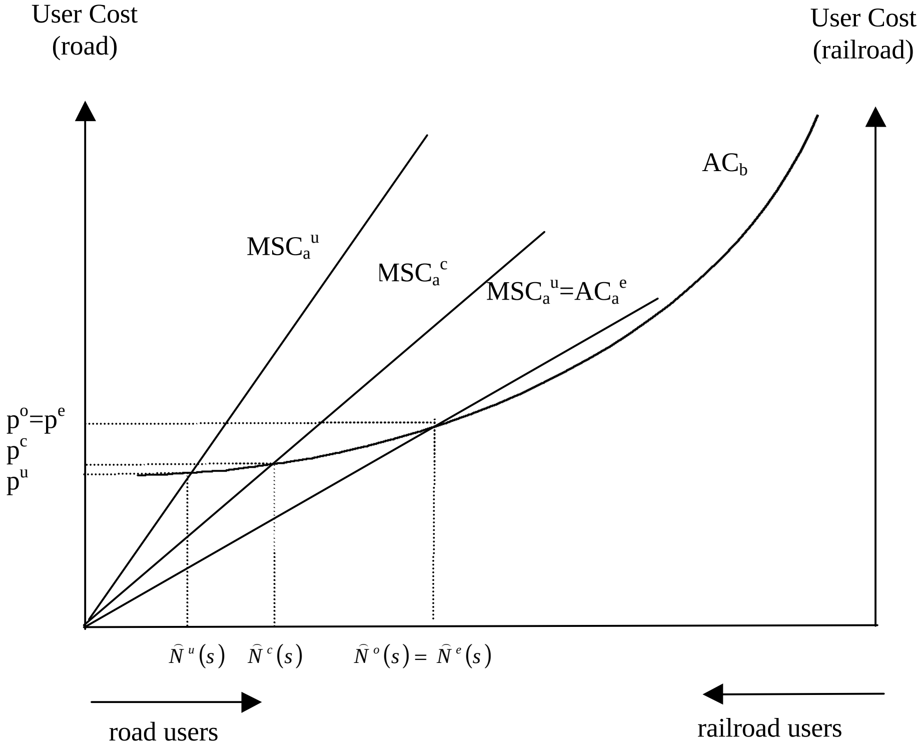

Fig. 2 – Bimodal equilibria with alternative road pricing schemes.

It follows from (3.35), (3.36), (3.37) and (3.38) that the TC gross of toll revenue (i.e., the initial cost to commuters) is certainly

(3.12) $TCg_b^u(s) < TCg_b^c(s) < TCg_b^o(s) = TCg_b^e(s)$

This result deserves some discussion. It appears that the finer the toll system the higher will be the total cost (gross of toll revenue) of the transport system as a whole. ADL (1991), considering a road-only system, arrive at an opposite conclusion, that is, the finer the toll system the lower are the total cost of the (road-only) system.
The reason is that a finer toll system, accommodating more auto commuters on the road, causes a lower exploitation of the economies of scale inherent to railroad technology. But if one considers the TC net of toll revenue (after the revenue is invested), the ranking is not certain because the inefficient exploitation of the economies of scale due to the finer toll system might be offset by a larger revenue generation (higher toll along with a larger number of auto commuters).
In order to study the relative efficiency of the four regimes we use the following index

(3.13) $eff^j = \dfrac{TC^e - TC^j}{TC^e - TC^o} \qquad for\ j = u, c.)$

4 *A numerical simulation*

As a numerical example we have assumed for α, β and γ the values proposed in ADL (1993). N is set equal to 20,000 commuters/hr and road capacity s to 8000 vehicles/hr so that rush hour lasts 2.5 hours. The marginal cost of train is assumed to be 1.46$ and the fixed cost for capital 0.2$ per rider as in Straszheim (1979) calculation for Philadelphia.

Table 1 – Base case

Parameter values : α= $5/hr, β=$3.05/hr, γ=$11.88/hr, c=1.46, s=8000, F=440, N=20,000 commuters				
	No toll	Fine toll	Coarse toll	Uniform toll
Auto users	4908	4908	3468	2447
Train users	15091	15091	16531	17552
Average social cost, average railroad cost	1.489	1.489	1.486	1.485
Average toll	0	0.744	0.743	0.742
Gross total cost	29,783	29,783	29,732	29,701
Total cost net of toll revenue	29,783	26.128	27153	27,883

On the basis of these values the ranking of TCg of equation (3.12) is confirmed, while the ranking of the TC net of toll revenue is as follows

(4.1) $TCn_b^e(s) >> TCn_b^u(s) > TCn_b^c(s) > TCn_b^o(s)$

The toll regimes have a lower TC with respect to the untolled one and the finer the regime the lower the net total cost. It is interesting now to test the stability of the inequality (4.1) to a change in the number of commuters (as a proxy of the city size) and to a change in the amount of fixed costs (as a proxy of the type of infrastructure).

Varying the number of commuters N

When the number of commuters N is increased the relative efficiency indicator (3.13) behaves as in fig. 3.

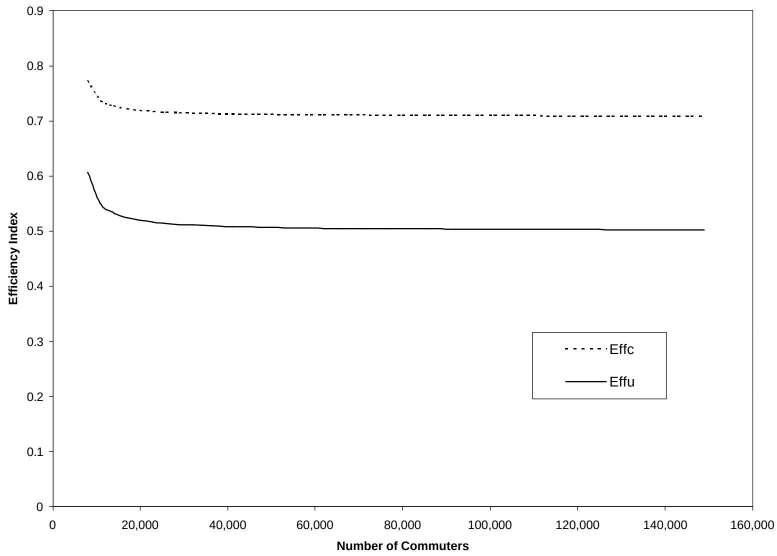

Fig. 3 – Efficiency of alternative regimes with an increasing number of commuters N.

If we start from a situation in which $TC_f<TC_c<TC_u$, for increasing values of N the following could be observed:

- the efficiency of the uniform-toll regime is always lower than that of the coarse-toll one, which in turn is always lower than that of the fine-toll regime. N_b increases and N_a decreases due to the effect of the economies of scale so that the modal split shifts in favour of public transport. The equilibrium average cost decreases. There are no forces at work for an inversion in the ranking;
- the relative efficiency of the untolled and coarse-toll regime with respect to the fine-toll one decreases because the railroad average cost curve shows a greater elasticity when it crosses the fine toll cost curve than when it crosses the coarse-toll or uniform toll curves. The greater elasticity can be interpreted as a stronger ability of the signal provided by the fine toll to induce a mode shift with respect to the other two charging systems. Moreover, such superior ability is more evident as N starts increasing since the reduction in the average costs is decreasing with N.

Varying the amount of fixed capital F

When the amount of fixed capital F is increased from the starting value of 440 to 15000 the TC curves assume the values presented in fig. 4.

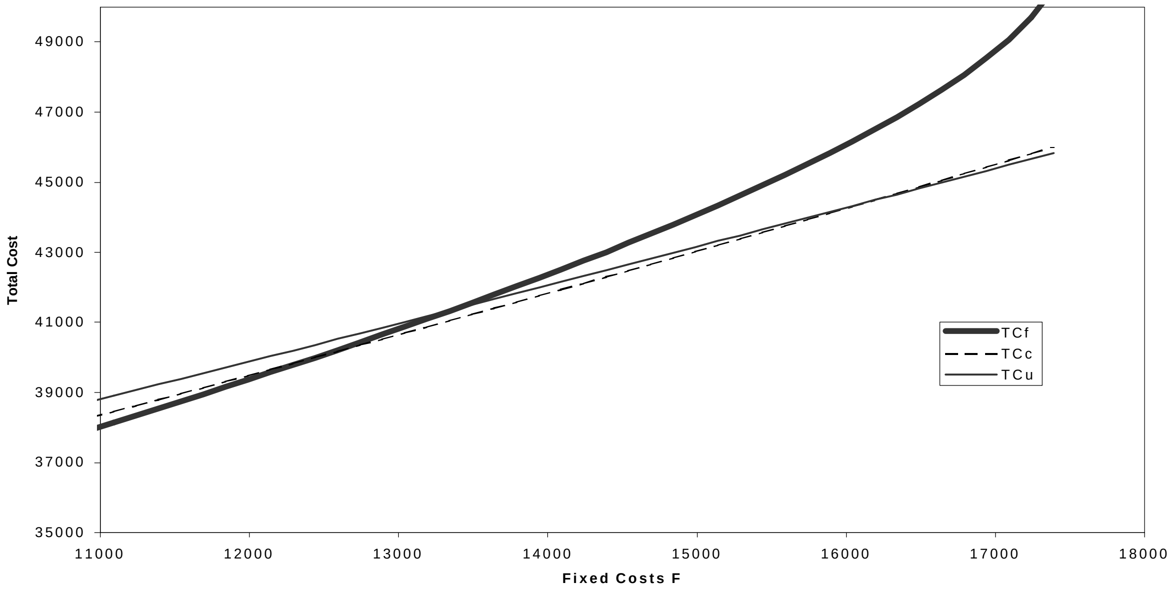

Fig. 4 – Efficiency of alternative regimes with increasing fixed costs F.

The most noticeable result is that, for sufficiently high values of F, the starting TC ranking is reversed. It can be observed that

(4.2) $TCn_b^u(s) < TCn_b^c(s) < TCn_b^o(s)$

The reason is that, while for low values of F the lower exploitation of the economies of scale in fine-toll regime with respect to the coarse-toll and uniform-toll ones is more than compensated by the greater amount of revenues generated, whereas for sufficiently high values of F the opposite is true. The high or low values of F might be interpreted as two types of rail infrastructure, e.g., a light rail train or an underground train. The inversion takes place if there are substantial economies of scale to be gained which might suggest an over-investment in the infrastructure. To summarise, the conclusion is that the finer the toll systems the lower the total net cost, but for a high enough value of F. In such a case, relieving congestion through a fine toll leads to higher (average and total) costs compared to using a rough uniform toll.
Such result is similar to Williams's statement (Williams, 1998) that the Downs-Thomson-Mogridge paradox (higher road and public transport costs as a result of an highway expansion meant to relief congestion) takes place when the economies of scale are large (i.e., F is large)[9]. The difference being that our result is determined both by the economies of scale and by the toll revenue.

Conclusions and extensions

Refining the model by Tabuchi (1993), this paper examined the efficiency properties of alternative road pricing schemes when a competing railroad service is available. It is found that, opposite to what Tabuchi concludes, when the railroad fare is set equal to the average cost pricing there is no definite ranking of the regimes. It cannot be stated a priori that a fine-toll regime is preferable to a coarse-toll or to a uniform-toll one on the basis of the total costs of the transport system as a whole, as it could be stated when only a road transport system is considered (ADL, 1990).

[9] Williams (1997) show also that when the economies of scale are very high (when the elasticity of the road cost curve is lower in absolute value than the public cost curve) the opposite is true.

The crucial variable appears to be not the number of commuters N (the city size) but the cost of the fixed infrastructure because of the presence of economies of scale. For sufficiently high values of F and assuming a bottleneck congestion technology, a finer toll system performs worse than the alternative rougher charging systems. This is because it allows more auto users to stay on the road and consequently does not sufficiently exploit the economies of scale inherent to the railroad, notwithstanding the larger toll revenue generated. Such case might be seen in practise when a very costly railroad infrastructure has been chosen either because no alternative was available or because of a wrong investment decision.

The result rests on a sort of Wardrop modal equilibrium hypothesis which could be refined by accounting for the railroad users' time cost and for their trade-off between riding packed trains in rush hour times or waiting for the next train. In a general equilibrium setting with variable demand and positive own- and cross-elasticities it would be possible to capture further relevant factors, though at a cost of adding a lot of complexity to the model.

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