



Department of Economics

Essays on Asset Pricing, Banking and the Macroeconomy

Afroditi Kero

*Thesis submitted for assessment with a view to obtaining the degree of
Doctor of Economics of the European University Institute*

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Department of Economics

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DEDICATION

...στον πολυαγαπημένο μου Νίκο!

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People usually begin their acknowledgments section by thanking those who have helped them in the scientific part of their thesis. However, in this case I will make an exception and begin by thanking my husband for his endless love and support during these last five years. Niko: thank you for being always next to me, ready to wipe my tears either because I received a poor grade or because I was struggling with my research. But most of all I thank you for teaching me how to be patient, enjoy research and feel joy even for my humble contribution.

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CONTENTS

I	Introduction	v
II	Chapters	viii
1	BANKS RISK TAKING, FINANCIAL INNOVATION AND MACROECONOMIC RISK	1
1.1	Introduction	2
1.2	Literature Review	3
1.3	Empirical Motivation	6
1.4	The model	8
1.4.1	Autarky	9
1.4.2	Financial Innovation	10
1.4.3	Comparative Statics	12
1.4.4	Extension: General Equilibrium	15
1.5	Conclusion	18
1.6	Appendix	24
1.7	Data Appendix	26
1.8	Figures	27
2	GREAT MODERATION OR GREAT MISTAKE: CAN OVERCONFIDENCE IN LOW MACRO-RISK EXPLAIN THE BOOM IN ASSET PRICES?	38
2.1	Introduction	39
2.2	Motivation: The Great Moderation, its uncertain cause and persistence, and the boom in asset prices	43
2.2.1	Asset Prices and the Great Moderation: Stylized Facts	43
2.2.2	Uncertainty about Origin and Persistence of the Great Moderation	45
2.3	The model	46
2.3.1	Preferences	46
2.3.2	The Processes for Consumption and Dividend Growth	47
2.3.3	Full Information Price-Dividend Ratios	48
2.3.4	Learning and Subjective Beliefs	49
2.4	Quantitative Results for the Benchmark Economy	51
2.4.1	The exercise	51

2.4.2	Parameter choice	51
2.4.2.1	Preferences	51
2.4.2.2	The Process for Consumption and Dividends	52
2.4.2.3	Learning Parameters	52
2.4.3	Asset Price Dynamics: Mean and Variance Effects of Learning and Uncertainty about Transition Probabilities	53
2.4.3.1	Sensitivity	56
2.5	Asset Prices under Alternative Learning Schemes	56
2.5.1	"Good policy or good luck": Learning when low-volatility is suspected to be permanent	56
2.5.2	Ad hoc learning	58
2.5.3	Sensitivity	60
2.5.3.1	Alternative prior probabilities of a permanent Great Moder- ation	60
2.5.4	Different assumptions for ad hoc learning rules	61
2.5.5	Lower Risk Aversion and Dividend Volatility	61
2.5.6	Alternative Timing of the Great Moderation	62
2.6	Conclusion	62
2.7	Appendix	69
2.7.1	Data Appendix	69
2.7.2	Appendix B	69
2.7.2.1	Numerical solving for the price when agents use Bayesian learning	69
2.8	Figures	71
3	HOUSING AND THE MACROECONOMY: THE IMPACT OF MORTGAGE MAR- KET INNOVATIONS	82
3.1	Introduction	83
3.2	The model	84
3.2.1	Household's problem	84
3.2.2	The Real Estate Construction Sector	85
3.2.3	Equilibrium	86
3.3	Calibration	86
3.4	Results	88
3.5	Conclusions	89

Part I

Introduction

The work on this thesis began at the same time the global economy was hit by the largest economic crisis since the Great Depression. As a consequence, the thesis was very much influenced by it. In these years of high uncertainty and turbulence many economists have devoted their energies to understanding why the crisis happened and what can and should be done during and after it. This thesis contributes to the first part of the discussion and it tries to explain how changes at the macro level in the financial system before the crisis have shaped the economic environment in which the economic crisis of 2007 took place.

The two principle elements that we are considering are financial innovations and the changes in the level of macroeconomic risk. Hence, in the first chapter, we study the joint effect of credit derivative innovation and macroeconomic risk on banks' portfolio decisions and characteristics. In the second chapter we examine whether the drop in macro risk have influenced the large rise in asset prices observed in the 1990s. Finally, in the third chapter we analyse the effect of mortgage market innovation on consumers' investment decisions. From the modelling point of view, the three chapters are quite different. What is common to all of them, however, is the motivation: the pressing need to understand the pre-existing conditions of sectors that were heavily influenced by the crisis, such as the financial sector, the banking sector, and the mortgage market.

More explicitly, in the first paper we show how financial innovation in combination with the fall of macroeconomic risk can explain the strong growth of the primary and secondary credit markets in the U.S. economy. We document empirically the fall in macroeconomic risk, the expansion of the prime and secondary credit market and we provide evidence that changes in macroeconomic risk are closely related to the evolution of the prime market. In the theoretical part of the paper we study in a simple portfolio optimization framework the effect of financial innovation and macroeconomic risk on banks' risk taking. The results of the model show that financial innovation increases bank appetite for risky investment both in the prime and secondary markets and that this effect is stronger in environments with low aggregate macroeconomic risk. In addition the banking system becomes less stable because of the portfolio risk of each individual bank increases.

In the second paper we introduce learning about the persistence of volatility regimes into a standard asset-pricing model. The results show that the fall in US macroeconomic volatility since the mid-1980s only leads to a relatively small increase in asset prices when investors have full information about the highly persistent, but not permanent, nature of low volatility regimes. When investors infer the persistence of low volatility from empirical evidence, however, the model can deliver a much stronger rise in asset prices similar to that observed in the data. Moreover, depending on the learning scheme, the end of the low

vii volatility period leads to a strong and sudden crash in prices. Ultimately in the third paper we employ the model of Jeske and Krueger (2007) in order to study the impact of mortgage market innovations on consumers' investment decisions and their distributional effects. The results show that the elimination of the foreclosure costs have a big impact on the total amount of mortgages, but no influence on the aggregate amount of real estate in the economy. On the other hand, the cancellation of the mortgage administration fees does not have any quantitative effect on the equilibrium.

Part II

Chapters

CHAPTER 1

BANKS RISK TAKING, FINANCIAL INNOVATION AND MACROECONOMIC RISK

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ABSTRACT

This paper shows how financial innovation in combination with the fall of macroeconomic risk can explain the strong growth of the primary and secondary credit markets in the U.S. economy. We document empirically the fall in macroeconomic risk, the expansion of the prime and secondary credit market, and we provide evidence that changes in macroeconomic risk are closely related to the evolution of the prime market. In the theoretical part of the paper we study in a simple portfolio optimization framework the effect of financial innovation and macroeconomic risk on banks' risk taking. The results of the model show that financial innovation increases bank appetite for risky investment both in the prime and secondary markets and that this effect is stronger in environments with low aggregate macroeconomic risk. In addition the banking system becomes less stable because of the portfolio risk of each individual bank increases.

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1.1 Introduction

The global financial and economic crisis of 2007 turned attentions to many issues economists had not paid special attention to previously. One of these was the large credit expansion that characterized the U.S. financial market the last twenty years (Figure 1). How can such a credit boom be justified? In section two we present a literature review on the different reasons that the existing literature provides as potential motives for the increase in the aggregate credit volume. This paper adds to the literature by offering a complementary explanation for the credit boom. The rationale for this study is that the credit boom was created as a combination of macroeconomic and financial effects that changed banks' perception about risk and which led to a big increase in the total supply of credit in the financial system. More specifically, during the last 20 years, the financial system was characterized by a strong innovation in the secondary markets, which gave birth to new financial products that the financial intermediaries could use to hedge the idiosyncratic, regional risk. The improved risk diversification ultimately led to credit expansion because the credit suppliers were facing less risk. However, an interesting observation arises from the study of the evolution of both prime and the secondary markets for risk and the history of financial innovation: even though many of these new financial products were available from the 1970s, it was almost two decades later, in the 1990s, that both the prime and the secondary market for risk expanded substantially. In the meantime, in the mid-1980s, the macroeconomy was characterized by a large drop in macroeconomic risk which marked the start of the period known as the Great Moderation. We believe that the decrease in aggregate risk played an important role in the credit expansion not only through the direct decrease of the total risk that banks were facing, but also through its effect on the use and development of new financial products in the secondary markets. In environments with lower macroeconomic risk, the weight of idiosyncratic risk is higher and therefore the role of financial innovation on portfolio decisions is more important. Hence the purpose of this paper is to understand how the fall of macroeconomic risk in combination with financial innovation has contributed to the credit boom.

Our main investigation contains two parts. In the first part, we present the empirical motivation for our study by providing stylized facts on our variables of interest. We characterize the evolution of the prime and secondary market for credit and identify the decline in macroeconomic volatility by estimating the standard deviation of the real GDP growth rate. Consistent with the literature, the estimation produces evidence of a substantial downward shift of the aggregate volatility after the mid 1980s. Finally we also examine empirically the correlation of the macroeconomic risk with the prime market for risk. The empirical results

show that changes in macroeconomic conditions are closely related to the evolution of the prime market for risk. In the second part, we put together these different elements and show their interconnection in a simple portfolio optimization model. The theoretical part of our study explores the portfolio optimization problem of a bank under two different scenarios; (i) without a secondary market for risk ("autarky") and (ii) with a secondary market for risk ("financial innovation"). The banking system is segmented and the banks are identical except the fact that they face different idiosyncratic return risk. In the "financial innovation" scenario banks can use credit derivatives in order to hedge its idiosyncratic risk. By using the credit derivatives, the banking sector becomes more homogeneous, more integrated, given that the regional differences between the different banks decrease. We solve the model and compute the optimal portfolio choices of the bank under the two different scenarios. The CARA-Normal specification of the model permits the generation of closed-form expressions for the demand of risky assets and for the demand of credit derivatives. Pursuing a comparative statics analysis we show that the use of the credit derivatives induces banks to invest more in risky assets and in credit derivatives. The portfolio variance of the banks increases because even though credit derivatives help to hedge the idiosyncratic risk, they induce the banks to acquire more risky assets that embody also unhedgeable aggregate risk. Therefore the total variance increases. The results also highlight the interesting nonlinear effects that arise between financial innovation and macroeconomic risk. The strength of the effect of the financial innovation on the banking sector is stronger in environments with low aggregate macroeconomic risk. The reason is that when aggregate risk decreases, the importance of the idiosyncratic risk on the bank's portfolio choices is bigger; as a result, the effect of financial innovation is more powerful. Finally we also extend the model in general equilibrium and we study the effect of financial innovation and aggregate risk on prices. Consistent with the literature, the results show that the decrease in aggregate risk and the increase in the degree of financial innovation decrease the equity premium.

The rest of this paper is organized as follows. Section 2 reviews the literature and Section 3 presents the empirical motivation of our study. Section 4 presents the model and discusses the results. Section 5 concludes.

1.2 Literature Review

This paper presents a very simple model that provides a rationale for the credit boom, which characterized the financial markets before the 2007 crisis. We support that financial innovation, in combination with changes in macroeconomic risk, have significantly affected

the intermediaries incentives for risk and as a consequence the final volume of credit in the economy. This analysis is therefore related to other studies, both in the macroeconomic and in the finance literature that investigate the factors that led to the extraordinary credit expansion observed the last two decades.

Both, practitioners (Cantor (2008), Cantor and Hu (2007)) and academics in finance (Bolton, Freixas and Shapiro (2009), Pagano and Volpin (2010)) share the view that financial innovation on the one hand was facilitating risk diversification and risk hedging, but on the other hand its opaque and complex nature led to an overexpansion of credit. In these papers, agency problems lead to a credit expansion because financial institutions, which were trading securities and derivatives, were able to increase their profits by not disclosing their true risk. In addition Shin (2009) underlines that the securitization process increased the leverage ratio of the financial institutions and as a consequence lending standard were decreased and credit was extended also to low quality borrowers. In our model we completely abstract from agency problems and all information is common knowledge and symmetric. However, we can still generate an expansion of credit beyond the level that is generated purely from diversification and hedging because we take into account the specific macroeconomic conditions that the economy experienced during the last two decades before the 2007 crisis. In this way we want to stress that in order to fully understand the effect of financial innovation on credit expansion, except of the microeconomic issues, we should take into account also the macroeconomic environment in which these securities were issued and traded. More specifically the macroeconomic element that we consider is the big drop in macroeconomic risk which is known as the phenomenon of the Great Moderation.

What is special about the Great Moderation? There is now broad consensus among macroeconomists of a widespread and persistent decline in the volatility of real macroeconomic activity after the mid 1980s. Kim and Nelson (1999) and McConnell and Perez-Quiros (2000) were the first to formally identify structural change in the volatility of U.S. GDP growth, occurring sometime around the first quarter of 1984. Blanchard and Simon (2001), using a different set of econometric tools, also found a large decline in output volatility over the last 20 years. Following this work, Stock and Watson (2002) subject a large number of macroeconomic time series to an exhaustive battery of statistical tests for volatility change. They conclude that the decline in volatility has occurred broadly across sectors of the aggregate economy. It appears in employment growth, consumption growth, inflation and sectoral output growth, as well as in GDP growth. Therefore, the fall in macroeconomic risk is important because the drop was large, persistent and identified in many macro aggregates. As a result two new strands of literature were developed: the macroeconomic literature that

investigates the cause of this sustained volatility decline (see Cecchetti, Flores-Lagunes and Krause (2006) for a review of this literature), and a more recent literature that studies the effect of uncertainty on macroeconomic outcomes (e.g, Lettau, Ludvigson and Wachter (2009) that studies the effect of macroeconomic risk on the equity premium and Fogli and Perri (2009) studies how changes in macroeconomic volatility have affected external imbalances). This paper is related to the second strand of literature because we take the Great Moderation as an exogenous effect and we examine its consequences on the expansion of the prime and secondary U.S. credit market.

Regulatory changes in the U.S. banking system like the Riegle–Neal Interstate Banking and Branching Efficiency Act of 1994 have also contributed to banks' ability to extend credit. The passage of this Act facilitated banks ability to operate widespread multi-state branching networks and transformed the American banking system from small banking into large banking. As a result there was a large banks' merger wave. Pana, Park and Query (2010) demonstrate empirically that banks mergers led to an increase in banks' liquidity levels. Higher liquidity leads consequently to an increase in banks' lending possibilities.

Independent of the happenings in the financial market, the macroeconomic literature provides also other explanations for the credit boom observed before the 2007 crisis. A very prominent explanation is the rise in global imbalances. Caballero and Krishnamurthy (2009), Caballero, Farhi and Gourinchas (2008), Obstfeld and Rogoff (2009), Blanchard and Milesi-Ferretti (2009) and other contributions put forward the idea that mainly U.S. but also other OECD countries faced a large capital inflow coming from many emerging, recently industrialized countries, like South Korea, Singapore, Hong Kong and especially China. These countries experienced fast economic growth but because of the recent collapse of their domestic asset markets, their local store-of-value instruments were not sufficient. As a result there was an increasing demand for U.S. saving instruments, which consequently led to an increase of risky assets held by the U.S. economy.

From the whole financial system, the housing and the mortgage market attracted special attention in the global 2007 crisis and the reason was the extraordinary size that this market had reached. The constant rise in house prices (Attanasio, Blow, Hamilton and Leicester (2005)) was a key factor in the boom in the housing market. Thus from the mid-1990s until 2007 house prices were following a continuously increasing pattern. This considerably relaxed the credit constraints that the household faced and people started asking for more and more housing. Another contributor to the burst housing bubble was a decline in the monetary interest rate during the the three years prior to the crisis (e.g., Schularick and Taylor (2009) and Taylor (2007)). From the 1980s up to 2003 the Fed was reacting to

changes in inflation and real GDP in a very systematic and predictable way. However, in the period 2003 to 2006, the monetary policy targets deviated substantially by imposing interest rates that were much lower than the ones experienced the last two decades. Low federal funds rate stimulated demand for borrowing. Housing financing in particular were very cheap and this contributed to the extraordinary mortgage market boom that we observed before the crisis. Finally, Campbell and Hercowitz (2006) claim that deregulations that took place in the beginning of the 1980s also contributed as well to the colossal housing demand. The Monetary Control and the Garn-St. Germain Acts of 1980 and 1982 allowed market innovations that dramatically reduced the equity requirements: In addition they offered a greater access to sub-prime mortgages, mortgage refinancing, and home equity loans which reduced effective down payments and increased effective repayment periods.

1.3 Empirical Motivation

In this section we provide some stylized facts on the development of the prime and secondary market for risk, we document the changes in aggregate volatility that took place in the 1980s and we provide graphical evidence on how the prime and secondary markets are related to macroeconomic risk.

As already mentioned, over the last 20 years the U.S. financial market was characterized by big increase of the trading volume of the prime market for risk. Figure 1 displays the time series of total household debt, business sector debt and financial sector debt as percent of total real GDP. The time series span the period from the second quarter of 1952 up to the second quarter of 2010 and the vertical sky-blue line denotes the first quarter of 1984. It is obvious that the financial liabilities in all sectors from the mid 1980s until the crises of 2007 are much higher than from the 1950s to the mid-1980s.

We would like to distinguish before and after the first quarter of 1984 because our benchmark measure for the macroeconomic risk is the standard deviation of quarterly real GDP growth rate and McConnell and Perez-Quiros (2000) identify the first quarter of 1984 at the break date of this time series. Figure 2 plots real GDP growth rates and its volatility from 1952Q2 to 2010Q2.² We compute the volatility of GDP growth as the sample estimate of 10-quarter rolling windows.³ The figure clearly reveals the sharp drop in volatility in the mid 1980's which was quite persistent until the 2008 crisis. More specifically, the decrease

²The right y axes (blue) indicates the measure for the standard deviation and the left y axes (red) indicates the measure for the GDP growth rate.

³However our results are robust to alternative ways of calculating the volatility of the GDP growth rate. For robustness check, we have also calculated the volatility as a sample estimate of an overlapping 5-year windows, of a AR(1) process and of a GARCH (1,1) process. The drop in volatility is present in all the different estimation methods.

in macroeconomic volatility in the subsample (1952Q2:1984Q1) is around 50% compared to the subsample (1984Q2:2010Q2).

In addition to the prime market, the secondary market for risk, thus the credit derivative market and securities markets, started to develop rapidly as well. Figures 3 and 4 show the evolution of the securitization market and of the credit derivative market for the US commercial banks. Figure 3 presents the trading volume for the mortgage-backed securities, asset-backed securities and the collateral debt obligations from 1995 to 2008. The data are yearly and they are taken from the International Monetary Fund. Figure 4 presents aggregate estimates of the credit derivatives sold and bought from 1997 to 2008. The original data are quarterly and their source is the Federal Reserve Bank of Chicago. In order for the results of figure 4 to be comparable with the one of figure 3 we have converted the quarterly data of credit derivatives to yearly by taking yearly averages. As we can clearly see, even though in the mid-1990s these markets had a very small trading volume, thereafter they expanded rapidly. But when did these innovative products first appear at the financial markets?

Figure 5 presents the time line when the different financial innovations took place. In February 1970, the U.S. Department of Housing and Urban Development created the first credit derivative, the mortgage-backed security (MBS), as an application of the securitization technology in the mortgage market. The first institutions to issue them were the government-owned corporations Ginnie Mae and Freddie Mac. This date denoted in some sense the creation date of the modern structured finance because thereafter many other derivatives were introduced in the financial market and securitisation techniques were applied in several different sectors. The most important ones were interest rate swaps, currency swaps and zero coupon bonds introduced in 1981, the collateralized mortgage obligation (CMO) and junk bonds in 1983, asset-backed securities (ABS) in 1985 and finally the collateralized debt obligation (CDO) and credit default swaps (CDS) that were created in 1995 by JPMorgan. Moreover, in the 1970s a number of investment funds, like hedge funds, money market funds, and mutual funds, were established and started to grow rapidly. In addition, the investment banking industry began to change to a more "transactional" form and a large number of boutique investment banks were established. For a more detailed review of the history of financial instruments and institutions, see Allen and Gale (1995).

One would expect that the banks would immediately explore these new opportunities and invest more in risky assets in order to increase profits. However, the data show that until the beginning of the 1990s, the size of these new markets for risk was relatively modest and banks increased their investment in risky loans only by a conservative amount. Instead, after the 1990s both the prime and the secondary market for risk expanded substantially.

To summarize, in the mid-1980s aggregate volatility declined sharply. Shortly after, in the 1990s, we observed an explosion in the size of the prime and secondary markets for risk. Our hypothesis is that the decrease in aggregate volatility caused the credit boom in the prime market for credit and consequently increased demand for credit in the prime market led to a boom also in the secondary market as well.

Before closing this section we conduct a simple exercise in order to see graphically how the macroeconomic risk co-moves with the financial liabilities. To investigate the relationship between the macroeconomic volatility and the prime market for credit we divide our sample in nonoverlapping 5-quarter windows and we compute the standardized standard deviation of quarterly GDP growth and the inverse of the logarithm of the household, business and financial sector debt. The results are plotted in figure 6. Figure 6 show a strong negative correlation between macroeconomic volatility and the volume of each of the prime markets under consideration. During the periods of low aggregate risk, the prime market tends to expand and the economy has high levels of debt. The opposite happens when volatility is high. As a control check we repeat the same exercise for another measure of the macroeconomic risk: the volatility of the aggregate consumption growth rate. Figure 7 presents the results of this exercise. The bottom-right subplot of figure 7 shows that average value of the standard deviation of consumption growth rate is much lower after the mid 1980s. The rest of the subplots reconfirm the result of figure 6. Therefore, the volatility of aggregate consumption is also negatively correlated with the financial debt of the three sections under consideration. Notable in this second exercise is the co-movement of the household sector debt and the volatility of real consumption growth.

1.4 The model

The model presents a two period, one good economy. There is a continuum of regions indexed by i , where $i \in [0, 1]$. In each region is located a bank, which is managed by a risk adverse banker with CARA preferences, $U = -e^{-\Gamma W_0}$, where Γ is constant coefficient of absolute risk aversion (CARA) and W_0 is initial wealth. The interests of the banker are perfectly aligned with the interests of the bank. We assume that the banker is risk averse and this may be, for example, because the bank is facing Value-at-Risk (VaR) constraints. Danielsson and Zigrand (2003) show that the present of VaR constraints impose a risk adverse behaviour on the financial institutions, even in the case when they are fundamentally risk neutral⁴.

⁴The VaR constrains capture the nature of risk sensitive capital requirement regulations like the Basel II. The Basel II agreement is implemented in the banking system of all OECD countries and it is expected to

There are two kinds of assets in the economy: risky and riskless. The risky asset promises a return $R = R^i + R^a$ where R^i is the idiosyncratic (regional) component and R^a is the aggregate one. Both R^i and R^a are normally distributed where $R^i \sim N[E(R_i), Var(R_i)]$ and $R^a \sim N[E(R_a), Var(R_a)]$ at time 1. The idiosyncratic component is identically and independently distributed across regions and uncorrelated to the aggregate component. Thus $R \sim N[E(R), Var(R)] = N[(E(R_i) + E(R_a)), (Var(R_i) + Var(R_a))]$. The riskless asset offers a sure return of R^f . In order for banks to have incentive to take risk, we assume that in expectation the return of the risky asset is higher than the safe asset, $E(R) > R^f$. Each bank possesses some initial wealth W_0 and we assume that the initial wealth is high enough in order to make sure that the banks are never capital constrained. Investment decisions take place in the first period and in the second period uncertainty is realized and consumption occurs. In this model we study two different scenarios of the economy: in autarky and in financial innovation. When the economy is in autarky, the banks choose an optimal portfolio by investing in safe and in risky regional assets. Thus they can invest only in their region. A plausible explanation for banking system segmentation is that each bank has superior information about the production firms in its own region that their risky investment is related to. In this way the monitoring cost for investing in firms in their own regions would be lower than in other regions. In an extreme case the monitoring cost of investing in their own region is zero and the monitoring cost of investing in other regions is infinite. As a consequence banks are forced to invest only in their own region. Therefore by construction the portfolio that they hold is not fully diversified. Under the financial innovation scenario, we introduce in the basic model a new risky asset, the credit derivative that expands the investment choices of the bankers. Below we elaborate further on how the credit derivatives are modelled and their role in the banking system.

1.4.1 Autarky

Each bank invests its initial wealth W_0 in a portfolio comprising of both riskless and risky assets (y^f, y) . The time-zero budget constraint is $W_0 \geq y^f + y$. The bankers maximize their expected utility subject to the budget constraint by choosing the optimal asset holdings. The assumptions of the CARA preferences and the Normal returns permit to obtain a simpler expected utility function, which is an explicit function of the expected portfolio return and

be implemented soon also in a big number of fast-growing, developing countries like China, Thailand, Chile, Mexico and Brazil.

the portfolio variance. Therefore the problem of the bank i is formulated as⁵

$$\begin{aligned} \text{Max}_{y,y^f} E[U(W_1)] &= -e^{-\Gamma E(W_1) + \frac{1}{2}\Gamma^2 \text{Var}(W_1)} \\ \text{s.t. } W_1 &= yR + y^f R^f \end{aligned}$$

where $W_1 = y^f R^f + yR$ is wealth at time-one. From the first order conditions of this optimization problem we can derive the bank's demand for risky asset.

Lemma 1.1 (*Optimal Portfolio*) *The optimal portfolio of risky assets for the bank has the mean-variance form:*

$$y = \frac{E(R) - R^f}{\Gamma(\text{Var}(R_i) + \text{Var}(R_a))} \quad (1)$$

The demand for risk is positively related to the excess return and negatively related to risk aversion and the idiosyncratic and aggregate risk.

Proof. See Appendix. ■

1.4.2 Financial Innovation

The credit derivative promises a return R^d , that is jointly normal with the idiosyncratic part of the risky asset. Thus $(R^d, R^i) \sim N\left[\begin{pmatrix} E(R^d) \\ E(R^i) \end{pmatrix}, \Sigma\right]$.⁶ Σ is the variance-covariance matrix and ρ is the correlation coefficient between the credit derivative and the idiosyncratic component of the risky asset. The credit derivative is negatively correlated with the idiosyncratic part of the risky asset, hence ρ will take values only in the interval $[-1, 0)$. The smaller is ρ , the higher is the degree of correlation between the risky asset and the credit derivative and therefore the bigger will be the hedging opportunities for the bank. As the values of ρ approaches the -1 , the credit derivatives become more efficient hedging instruments. Given that the innovation in this model is defined in terms of the ability of the banks to better hedge risk, ρ represents a measure for the degree of financial innovation of the banking sector. In addition, given that the banks are symmetric in all dimensions except of the idiosyncratic risk that they face, hence lower ρ means that the idiosyncratic risk plays a much smaller role in their portfolio decisions and cross-regionally they become more similar. As a consequence, the use of the credit derivatives decreases the regional differences between the different banks and this contributes to the creation of a more integrated and homogenous banking system.

⁵We suppress the i notation because even though the bankers face different regional risk, the setting of their optimization problem is the same.

⁶ $\Sigma = \begin{bmatrix} \text{Var}(R^d) & \rho\sqrt{\text{Var}(R^i)\text{Var}(R^d)} \\ \rho\sqrt{\text{Var}(R^d)\text{Var}(R^i)} & \text{Var}(R^i) \end{bmatrix}$.

For simplicity we assume that the expected return of the credit derivative is equal to the risk free rate, thus $E(R^d) = R^f$. Therefore the credit derivatives are new instruments that the bank can use in order to hedge its idiosyncratic risk, without gaining any extra return. In reality the products of the secondary market are very complex instruments. We choose to model them in this simplified way in order to capture one of their principle benefits which is risk hedging. However, our modelling strategy is in accordance with other contributions in the literature (see Duffie (2007), Wagner and Marsh (2006) and Wagner (2008)) that stress out the diversification motive for the use of the credit derivatives.

The problem of the bank in this scenario is exactly the same as in autarky, the only difference being that instead of choosing only safe and risky assets, the bank acquires also credit derivatives, d . Therefore the banker's problem consists of choosing the optimal triplet (y^f, y, d) that maximizes expected utility

$$\begin{aligned} \text{Max}_{y,y^f,d} E[u(W_1)] &= E[-e^{-\Gamma W_1}] = -e^{-\Gamma E(W_1) + (\Gamma^2/2) \text{Var}(W_1)} \\ \text{s.t } W_1 &= Ry + dR^d + y^f R^f \end{aligned}$$

Definition 1.2 *An equilibrium allocation in the economy is given by a triplet of risky investment, safe asset and credit derivative $[y^*, (y^f)^*, d^*]$ such that every bank i , for each $i \in [0, 1]$, solves its optimization problem.*

As above, from the first order conditions of the bank's optimization problem we derive the optimal demand for risky asset and also for credit derivatives

Lemma 1.3 *(Optimal Portfolio) The optimal portfolio of risky assets and credit derivatives for the bank has the form;*

$$d = -\rho y \frac{\text{Var}(R^i)}{\text{Var}(R^d)}, \quad (1a)$$

$$y = \frac{E(R) - R^f}{\Gamma[\text{Var}(R^a) + (1 - \rho^2)\text{Var}(R^i)]}. \quad (1b)$$

Equation (1a) presents the demand for credit derivatives as a function of the risky assets and equation (1b) presents the demand for risky asset as a function of the parameters of the model.

Proof. See Appendix. ■

1.4.3 Comparative Statics

Given the assumptions of CARA preferences and normally distributed returns we are able to derive closed form solutions for the optimization problem of the bank. Therefore we obtain closed-form solutions for the demand of the risky asset and the demand of credit derivatives and this enables us to pursue comparative statics on the banks choices and characteristics. We must underline that other preferences or distribution specifications most probably would not allow us to obtain closed-form solutions.

As a first exercise we study how changes in the degree of the financial innovation affect banks' choices i.e. the demand for risky assets and credit derivatives and banks' characteristics, i.e. the portfolio variance and the expected utility on risky and safe investments.

Proposition 1.4 *An increase in the degree of financial innovation of the banking sector (decrease in ρ), increases the demand for risk in both the prime and the secondary market for risk.*

Proof. Directly from equations (1a) and (1b) we have⁷

$$\frac{\partial y}{\partial \rho} = \frac{2(E(R) - R^f) \rho \Gamma \text{Var}(R^i)}{(\Gamma[\text{Var}(R^a) + (1 - \rho^2)\text{Var}(R^i)])^2} < 0 \quad (2a)$$

$$\frac{\partial d}{\partial \rho} = -\frac{\text{Var}(R^i)}{\text{Var}(R^d)} \left(y + \rho \frac{\partial y}{\partial \rho} \right) < 0. \quad (2b)$$

■

An increase in the degree of financial innovation offers the possibility to the banks to better hedge their idiosyncratic risk. As a result banks face less total risk in their investment and therefore they acquire more risky assets than in autarky. The effect of financial innovation on the demand for credit derivatives is both direct and indirect. An increase in the degree of financial innovation directly increases the demand for credit derivatives because they automatically become more efficient instruments for risk hedging. In addition, the demand of credit derivatives increases even more as a result of the extra demand for risky assets in the prime market.

Next, we study the effect of financial innovation on banks stability, where bank stability is expressed as the portfolio variance of the bank.

Proposition 1.5 *An increase in the degree of financial innovation (a decrease in ρ) reduces bank stability.*

⁷Keep in mind that $(-1 < \rho < 0)$.

Proof. Differentiating the portfolio variance with respect to ρ we get

$$\frac{\partial \text{Var}(W_1)}{\partial \rho} = \frac{2\rho (E(R) - R^f)^2 \text{Var}(R^i)}{[\text{Var}(R^a) + (1 - \rho^2)\text{Var}(R^i)]^2} < 0. \quad (3)$$

See the Appendix for the derivation of the portfolio variance. ■

Proposition 5 shows that in more financially integrated banking system, banks face a higher portfolio variance. The economic intuition behind this result is that, even though banks can better hedge their idiosyncratic risk compared to autarky, their portfolio variance increases because their risk appetite increases as well. By acquiring more risky assets, except of idiosyncratic risk, which can be hedged with credit derivatives, banks also get more aggregate risk that cannot be diversified. As a result banks become riskier and less stable than in autarky. Instefjord (2005) concludes in a similar manner, i.e. that financial innovation of credit derivatives may lead to a less stable banking system. However, destabilization effects in Instefjord (2005) depend on the competitiveness of the credit derivative market, instead in our framework they depend on the introduction of more aggregate risk in the final portfolio composition.

Until now we have seen that financial innovation induces banks to invest more in risky assets and derivatives but on the other hand their portfolio volatility goes up as well. Hence is it optimal for the banks to acquire more risky assets or not? Proposition 6 gives the answer to this question.

Proposition 1.6 *An increase in financial innovation (a decrease in ρ) increases the bankers' welfare, $\left(\frac{\partial E(U)}{\partial \rho} < 0\right)$.*

Proof.

$$\frac{\partial E(U)}{\partial \rho} = \left(\frac{-\rho (E(R) - R^f)^2}{[\text{Var}(R^a) + (1 - \rho^2)\text{Var}(R^i)]^2} \right) (-e^{-\Gamma[E(W_1) - \frac{1}{2}\Gamma\text{Var}(W_1)]}) < 0 \quad (4)$$

■

Therefore it is beneficial for banks to engage in more risk taking because their expected utility increases. Thus even though the portfolio volatility increases, the expected portfolio returns from acquiring more risky assets increases as well. In this framework the second effect is bigger than the first one, so the expected utility of the banks increases with a higher degree of financial innovation.

Next we study how the effect of financial innovation in the banking systems varies with different levels of macroeconomic risk.

Proposition 1.7 *The effect of financial innovation on banks risk taking is stronger, the lower is the aggregate risk.*

Proof. We differentiate the results of Proposition 1, (2a) and (2b), with respect to $Var(R^a)$ and we get

$$\frac{\partial \left(\frac{\partial y}{\partial \rho} \right)}{\partial Var(R^a)} = - \frac{4 (E(R) - R^f) \rho \Gamma Var(R^i)}{(\Gamma [Var(R^a) + (1 - \rho^2) Var(R^i)])^3} > 0$$

$$\frac{\partial \left(\frac{\partial d}{\partial \rho} \right)}{\partial Var(R^a)} = - \frac{Var(R^i)}{Var(R^{cd})} \left(\frac{\partial y}{\partial Var(R^a)} + \rho \frac{\partial \left(\frac{\partial y}{\partial \rho} \right)}{\partial Var(R^a)} \right) > 0.$$

■

Proposition 7 highlights the importance of macroeconomic risk, for the effect of financial innovation on banks' risk taking. In order to illustrate better Proposition 7 we present a graphical demonstration of how these nonlinear effects manifest themselves in the context of real changes in macroeconomic risk. Thus, we characterize the behaviour of the equilibrium banks' demand for risky investment and credit derivatives by plotting the model's solution for these quantities and by feeding in the historical values of $Var(R^a)$ estimated in the first section⁸. Figure 8 displays the time series of the demand for risky assets. The data are quarterly and span the period from the first quarter of 1967 to the fourth quarter of 2009. We have computed the demand for risk for different degrees of financial innovation, thus for different values of ρ . As we can see, for all levels of financial innovation, the demand for risk increases substantially after the mid-1980s. This demonstrates that the decrease in aggregate volatility has a large impact on bank risk taking. In addition, it is clear that the fall in aggregate risk adds important nonlinear effects, which become stronger and stronger as the degree of financial innovation increases. Before the 1980s, the presence of credit derivatives does not add significantly in terms of risk acquisition, instead in the period after that, we can see that the degree of financial innovation plays a key role in banks risk taking.

Figure 9 displays the demand for credit derivatives. Again it is obvious that the changes in macroeconomic conditions in the 1980s have significantly affected the expansion of the secondary markets for risk, as the credit derivative market. For this specific calibration of the model, on average the demand for credit derivatives in the case of partial financial innovation ($\rho = -0.7$) is two times larger after the 1980s than the period before it. Instead in the case of total financial innovation is more than three times larger.

⁸The calibrated values for the coefficient of risk aversion, the return of the risky asset and the return of the risk free asset are 1, 20 and 1 respectively.

In addition we study also the portfolio variance in order to gain some insight on how the stability of banking sector has changed through time. Figure 10 displays the portfolio variance of the bank. As Proposition 1 shows, the presence of credit derivatives increases finally the portfolio variance. Even though on one hand banks have the possibility to hedge the idiosyncratic risk by investing in credit derivatives, on the other hand they acquire more risky assets which contain also non diversifiable aggregate risk and thus in the end the portfolio variance increases.

As in the case of the demand for risky assets, the demand for credit derivatives and the portfolio variance, there are nonlinear effects on the expected utility from changes on the degree of financial innovation and macroeconomic risk. Proposition 8 states this result and Figure 11 provides a graphical illustration of it.

Proposition 1.8 *The effect of financial innovation on banks expected utility is stronger, the lower is the aggregate risk, $\left(\frac{\partial\left(\frac{\partial E(U)}{\partial \rho}\right)}{\partial \text{Var}(R^a)} > 0\right)$.*

Proof. We differentiate equation (4) with respect to $\text{Var}(R^a)$ and we get

$$\frac{\partial\left(\frac{\partial E(U)}{\partial \rho}\right)}{\partial \text{Var}(R^a)} = \left(\frac{\rho(E(R) - R^f)^4}{2[\text{Var}(R^a) + (1 - \rho^2)\text{Var}(R^i)]^6}\right) (-e^{-\Gamma[E(W_1) - \frac{1}{2}\Gamma\text{Var}(W_1)]}) > 0$$

■

Before we conclude this section, we would like to stress that our concern in this paper is not to match the short to medium term movement in the trade volume of the prime and secondary markets for risk. Instead, we are interested to show in a very simple model how the aggregate risk might have effected bank's decisions regarding risk acquisition, not only explicitly but also implicitly through its effect on the financial innovation. Nevertheless we believe that a precise quantitative assessment would be a very interesting exercise to pursue. However, a much richer model framework is needed for such an exercise and thus we leave this for future research.

1.4.4 Extension: General Equilibrium

Next make a simple extension of the model to a general equilibrium framework in order to assess the effect of what is the effect of financial innovation and macroeconomic risk on equilibrium prices. In order to do so we first have to integrate the individual demand functions of risky assets over the whole banking system. Hence the aggregate demand for

risky assets is;

$$Y = \int_0^1 y_i di = \frac{E(R) - R^f}{\Gamma(\text{Var}(R^a) + (1 - \rho^2)\text{Var}(R^i))}.$$

As one can notice the aggregate demand for risky assets is exactly the same as the individual demands. The reason is that all the banks in our banking system are completely the same except of the fact that they face idiosyncratic investment risk. The idiosyncratic risk persists and it does not disappear in general equilibrium because of market incompleteness. Therefore, the total demand function is defined as a function of exactly the same parameters as the the individual demand functions of regional banks. We assume that there is a fixed supply of asset in this economy, i.e. the supply of risky assets is \bar{Y} . In order to obtain market clearing the aggregate demand and the aggregate supply must be equal, thus $Y = \bar{Y}$. The excess returns from investing in risky assets instead of safe one is

$$E(R) - R^f = \bar{Y}\Gamma(\text{Var}(R^a) + (1 - \rho^2)\text{Var}(R^i)), \quad (5)$$

and the safe interest rate in general equilibrium is

$$R^f = E(R) - \bar{Y}\Gamma(\text{Var}(R^a) + (1 - \rho^2)\text{Var}(R^i)).$$

Given the information above we can also calculate the Sharpe Ratio which is the excess returns per unit of return's volatility,

$$S(R) =: \frac{E(R) - R^f}{\sqrt{\text{Var}(R)}} = \frac{\bar{Y}\Gamma(\text{Var}(R^a) + (1 - \rho^2)\text{Var}(R^i))}{\sqrt{\text{Var}(R^a) + \text{Var}(R^i)}}. \quad (6)$$

Proposition 1.9 *An increase in financial innovation (a decrease in ρ) or a decrease in aggregate volatility, decreases the excess returns,*

$$\left(\frac{\partial(E(R) - R^f)}{\partial \rho} > 0, \frac{\partial(E(R) - R^f)}{\partial \text{Var}(R^a)} > 0 \right).$$

Proof. We differentiate equation (5), with respect to ρ and $\text{Var}(R^a)$ and we get

$$\begin{aligned} \frac{\partial(E(R) - R^f)}{\partial \rho} &= -2\bar{Y}\Gamma\rho\text{Var}(R^i) > 0 \\ \frac{\partial(E(R) - R^f)}{\partial \text{Var}(R^a)} &= \bar{Y}\Gamma > 0 \end{aligned}$$

■

The result of Proposition 9 is very intuitive. Both an increase in the degree of financial innovation and a decrease in aggregate risk contribute to higher demand for risky asset acquisition. However, given that in the aggregate the supply of risky asset is fixed, the

opportunity cost of investing in risky asset should increase and therefore the excess return in each risky investment goes down. This result still holds when we take into account the return's volatility. Proposition 10 states the effect of financial innovation and aggregate risk on the Sharpe Ratio.

Proposition 1.10 *An increase in financial innovation (a decrease in ρ) or a decrease in aggregate volatility, decreases the Sharpe Ratio, $\left(\frac{\partial S(R)}{\partial \rho} > 0, \frac{\partial S(R)}{\partial Var(R^a)} > 0\right)$.*

Proof. We differentiate equation (6), with respect to ρ and $Var(R^a)$ and we get

$$\begin{aligned}\frac{\partial S(R)}{\partial \rho} &= \frac{-2\bar{Y}\Gamma\rho Var(R^i)}{\sqrt{Var(R^a) + Var(R^i)}} > 0 \\ \frac{\partial S(R)}{\partial Var(R^a)} &= \frac{\bar{Y}\Gamma[Var(R^a) + (1 + \rho^2)Var(R^i)]}{2(Var(R^a) + Var(R^i))^{3/2}} > 0\end{aligned}\quad (7)$$

■

In the partial equilibrium analyses the effect of the financial innovation on all the variables under consideration, was directly affected from changes in macroeconomic risk. In the general equilibrium framework equilibrium prices do not exhibit non-linear effects. The reason is that the demand for risky assets in this economy is linear and the supply is constant therefore prices in general equilibrium are a linear combination of the aggregate risk and of the degree of financial innovation, ρ . However, Proposition 11 shows that the non-linear effects are manifested in the case of the Sharpe Ratio.

Proposition 1.11 *The marginal effect on the Sharpe Ratio of a change in the degree of financial innovation is bigger, the lower is aggregate risk $\left(\frac{\partial(\frac{\partial S(R)}{\partial \rho})}{\partial Var(R^a)} > 0\right)$.*

Proof. We differentiate the first equation of (7), with respect to $Var(R^a)$ and we get

$$\frac{\partial\left(\frac{\partial S(R)}{\partial \rho}\right)}{\partial Var(R^a)} = \frac{\bar{Y}\Gamma\rho Var(R^i)(Var(R^a) + Var(R^i))^{-1/2}}{Var(R^a) + Var(R^i)} < 0$$

■

Proposition 9-11 show that more efficient secondary markets and lower macroeconomic risk lead to smaller equity premium. This result is in line with the empirical literature in finance (i.e. Blanchard (1993), Jagannathan, McGrattan, and Scherbina (2000), Fama and French (2002)) that show that the equity premium has substantially decreased after the 1990s.

1.5 Conclusion

From the beginning of the 1990s until 2006, the U.S. financial market was characterized by a substantial expansion of banks investment on risky loans and a rapid development of the secondary market for risk. In this paper we studied how the financial innovation in the banking system in combination with changes in aggregate risk observed in the mid-1980s, have contributed to bank risk taking. The first part of the paper presented how the aggregate risk and the prime and secondary market for credit have changed over time. It also examined empirically the correlation of the macroeconomic risk with the prime market for risk. The empirical results show that changes in macroeconomic conditions are closely related to the evolution of the prime market for risk. In the second part of the paper we analyzed banks' optimal behaviour in a simple portfolio optimization model. Bankers are modelled as risk averse traders who choose their optimal portfolio by investing in risky and safe assets and the returns of the risky assets depends both on idiosyncratic and on aggregate risk. We computed the optimal portfolio choices in two different scenarios; (i) without a secondary market for risk ("autarky") and (ii) with a secondary market for risk ("financial innovation"). In the "financial innovation" scenario banks have access to secondary markets and they can acquire credit derivatives in order to hedge their idiosyncratic risk. The results of the model show that the decline in macro risk and the increase of financial integration increases bank appetite for risky investment, credit derivatives acquisition and the portfolio variance. As a consequence the banking system becomes less stable. The model also highlights the fact that the strength of financial integration effect on the banking sector is stronger in environments with low aggregate macroeconomic risk. Lastly the extension of our model in general equilibrium demonstrates that financial innovation and the decline in macro risk lead to a decrease in the equity premium.

The analysis in this paper is an initial, preliminary attempt to study the effect of financial and macroeconomic changes on bank risk taking. However, much more needs to be done. For example, the assumptions of CARA utility and normally distributed returns are very handy for obtaining closed form solutions and pursue comparative statics but this works against the accuracy of the quantitative results. Therefore, in order to perform a more careful quantitative evaluation, the basic model needs to be extended to a more general framework. Broer and Kero (2011) makes a step further in this direction and analyze the quantitative impact of the Great Moderation on asset prices in a calibrated general equilibrium asset pricing model. Another way to go is the empirical evaluation. Frame and White (2006) stress that there is very little empirical work on financial innovation. It would therefore be interesting to test empirically and measure the effect of financial innovation on

the macroeconomy. Finally, it would be interesting to study the changes in macro risk and financial innovation in a regulated banking sector. The reason is that these changes affect banks' net worth and in a model in which the banks face constraints in their net worth, for example VaR constraints in the spirit of Danielsson and Zigrand (2003) or Danielsson, Shin and Zigrand (2003), interesting dynamics may emerge.

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1.6 Appendix

Proof. Lemma 1

The bankers have CARA preferences with a coefficient of absolute risk aversion Γ . Thus their utility is $U(W_1) = -e^{-\Gamma W_1}$ where $W_1 = y^f R^f + yR = (W_0 - y)R^f + yR$ is wealth at time one. Given that the payoff $R \sim N[E(R), Var(R)]$, the conditional expected utility of the banker can be expressed as

$$E[U(W)] = -e^{-\Gamma E(W_1) + \frac{1}{2}\Gamma^2 Var(W_1)}$$

where $E(W_1)$ is the mean and $Var(W_1)$ is the variance of the portfolio. $E(W_1) = [E(R) - R^f]y + W_0 R^f$ and $Var(W_1) = y^2 Var(R)$. Thus the problem of the banker is

$$\begin{aligned} Max_y E[U(W^1)] &= -e^{-\Gamma E(W_1) + \frac{1}{2}\Gamma^2 Var(W_1)} \\ s.t \ W_1 &= yR + (W_0 - y)R^f \end{aligned}$$

The First Order Conditions

$$\frac{\partial E[U(W^1)]}{\partial y} = \Gamma[E(R) - R^f] - \Gamma^2 y Var(R) = 0$$

$$y = \frac{E(R) - R^f}{\Gamma Var(R)}.$$

■

Proof. Lemma 3

The wealth of the banker at time $t = 1$ is the sum of the payoffs of the safe and risky assets, and the value of the credit derivatives. Thus $W_1 = (R - R^f)y + (R^{cd} - R^f)d + W_0 R^f$. The portfolio mean and variance in this case are $E(W_1) = y[E(R) - R^f] + d[E(R^{cd}) - R^f] + W_0 R^f$ and

$$\begin{aligned} Var(W_1) &= y^2 Var(R) + d^2 Var(R^{cd}) + 2yd Cov(R, R^{cd}) = y^2 Var(R) + d^2 Var(R^{cd}) + \\ &2yd[Cov(R^i, R^{cd}) + Cov(R^a, R^{cd})] = y^2 Var(R) + d^2 Var(R^{cd}) + 2yd\rho\sqrt{Var(R^i)Var(R^{cd})} \end{aligned}$$

Thus the bank's problem is

$$\begin{aligned} Max_{y,y^f,d} E[U(W^1)] &= E[-e^{-\Gamma W_1}] = -e^{-\Gamma E(W_1) + (\Gamma^2/2)Var(W_1)} \\ s.t \ W_1 &= (R - R^f)y + (R^{cd} - R^f)d + W_0 R^f \end{aligned}$$

The First Order Conditions

$$\frac{\partial E[U(W^1)]}{\partial y} = \Gamma[E(R) - R^f] - \Gamma^2[yVar(R) + d\rho\sqrt{Var(R^i)Var(R^{cd})}] = 0$$

$$y = \frac{E(R) - R^f - \Gamma d\rho\sqrt{Var(R^i)Var(R^{cd})}}{\Gamma Var(R)}$$

$$\frac{\partial E[U(W^1)]}{\partial d} = \{\Gamma[E(R^{cd}) - R^f] - \Gamma^2[d\text{Var}(R^{cd}) + y\rho\sqrt{\text{Var}(R^i)\text{Var}(R^{cd})}]\} = 0$$

$$d = \frac{E(R^{cd}) - R^f - \Gamma y \rho \sqrt{\text{Var}(R^i)\text{Var}(R^{cd})}}{\Gamma \text{Var}(R^{cd})}$$

Given the assumption that $E(R^{cd}) - R^f = 0$

$$d(y) = -\rho y \frac{\sqrt{\text{Var}(R^i)}}{\sqrt{\text{Var}(R^{cd})}}$$

The demand for credit derivatives depends positively on the demand for risky assets. By combining the first order conditions for y and d we get the demand for risky assets.

$$y = \frac{E(R) - R^f}{\Gamma[\text{Var}(R^a) + (1 - \rho^2)\text{Var}(R^i)]}.$$

■

Problem 1.12 Derivation of the Portfolio Variance

$$\begin{aligned} \text{Var}(W^1) &= y^2 \text{Var}(R) + d^2 \text{Var}(R^{cd}) + 2yd\rho\sqrt{\text{Var}(R^i)\text{Var}(R^{cd})} \\ &= y^2 \text{Var}(R) + \rho^2 y^2 \frac{\text{Var}(R^i)}{\text{Var}(R^{cd})} \text{Var}(R^{cd}) - 2\rho^2 y^2 \frac{\sqrt{\text{Var}(R^i)}}{\sqrt{\text{Var}(R^{cd})}} \sqrt{\text{Var}(R^i)\text{Var}(R^{cd})} \\ &= y^2 \text{Var}(R) + \rho^2 y^2 \text{Var}(R^i) - 2\rho^2 y^2 \text{Var}(R^i) \\ &= y^2 \text{Var}(R) - \rho^2 y^2 \text{Var}(R^i) \\ &= y^2 [\text{Var}(R^a) + (1 - \rho^2)\text{Var}(R^i)] \\ &= \left(\frac{E(R) - R^f}{\Gamma[\text{Var}(R^a) + (1 - \rho^2)\text{Var}(R^i)]} \right)^2 [\text{Var}(R^a) + (1 - \rho^2)\text{Var}(R^i)] \\ &= \left(\frac{E(R) - R^f}{\Gamma} \right)^2 [\text{Var}(R^a) + (1 - \rho^2)\text{Var}(R^i)]^{-1} \text{Var}(W_1) \end{aligned}$$

$$\text{Var}(W^1) = \frac{(E(R) - R^f)^2}{\Gamma^2[\text{Var}(R^a) + (1 - \rho^2)\text{Var}(R^i)]}.$$

1.7 Data Appendix

The sources and description of each data series that we use is listed below.

Consumption is quantified as the *Total Real Personal Consumption Expenditures* measured in quantity index [index numbers, 2005 = 100]. The data are quarterly, seasonally adjusted and their source is the National Economic Accounts of the Bureau of Economic Analysis (BEA).

Credit derivatives, includes historical data on credit derivatives bought and sold from commercial banks in the US. This series is an aggregation of different off-balance sheet derivatives like Credit Default Swaps, Return Swaps and other credit derivative instruments. The data are quarterly and bank-specific. The source of the data is the Federal Bank of Chicago. The Federal Reserve data are from FR Y-9C reports led by the banks.

GDP is the quarterly gross domestic product, measured in 2005-chained dollars. The source is the National Economic Accounts of the Bureau of Economic Analysis (BEA).

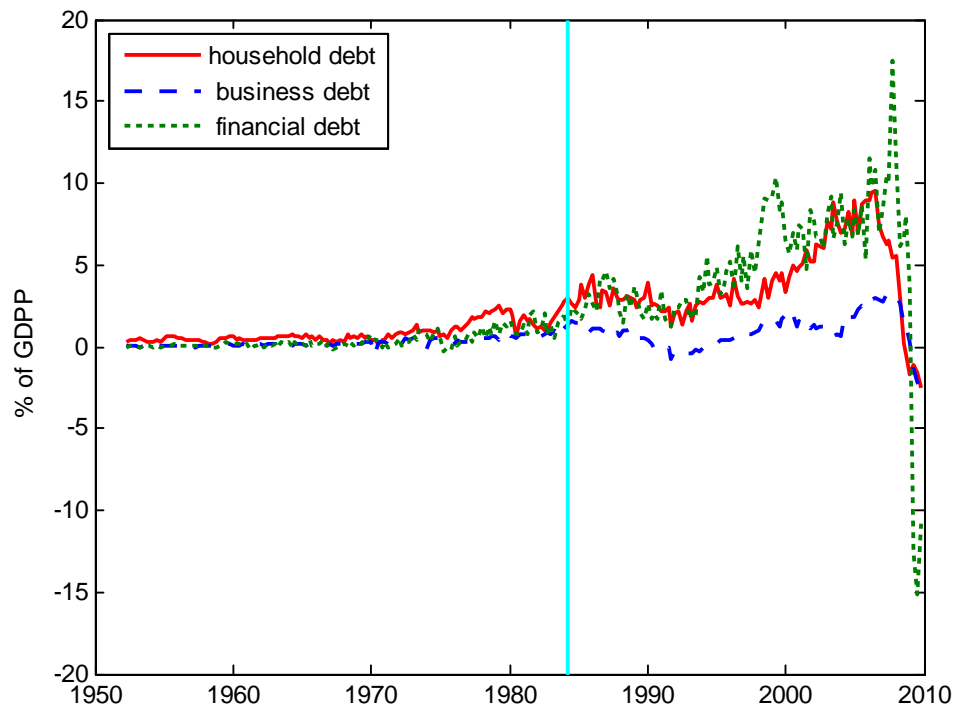
Population is quantified as the *Midperiod Population* of each quarter. The data source is the National Economic Accounts of the Bureau of Economic Analysis (BEA).

Securities (MBS, ABS, CDO), are U.S. Private-Label Securitization Issuances. The data are yearly and the data source is the IMF.

US financial liabilities, includes historical data on total household borrowing, total business borrowing and total borrowing of the domestic financial sector. The data are quarterly and seasonally adjusted annual rates. The source is the Flow of Funds Accounts of the United States of the United States.

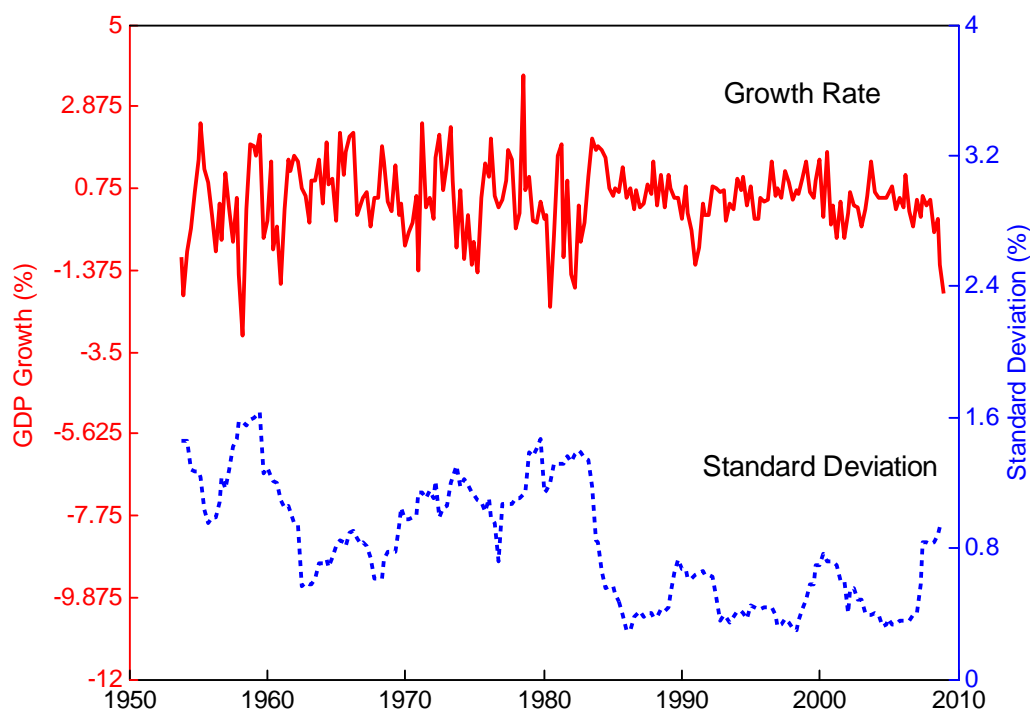
1.8 Figures

Figure 1: U.S. Financial Liabilities per Sector



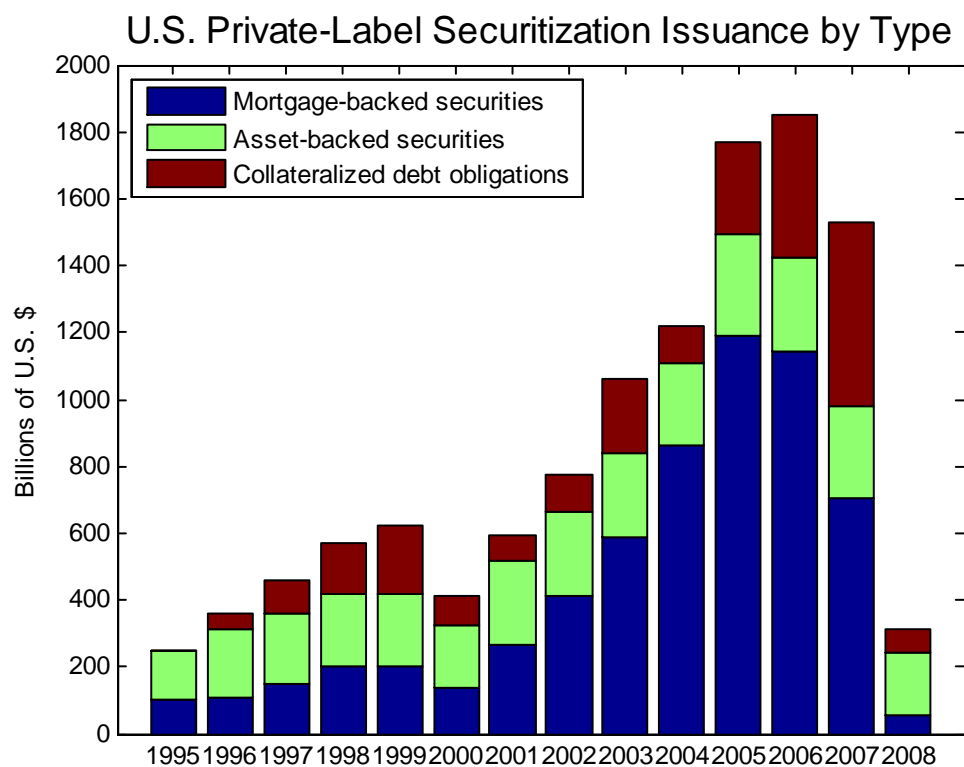
Note: This figure plots the growth rate of the U.S. Financial Liabilities for the household, the business and the financial sector as percent of the Gdp. All variables are measured in 2005-chained dollars. The financial liabilities are defined in debt terms. The data are quarterly and they span the period 1952Q2-2010Q2. The data source for the GDP is the BEA and for the financial data is the Flow of Funds Accounts of the United States.

Figure 2: U.S. Real GDP growth Rate and its Volatility



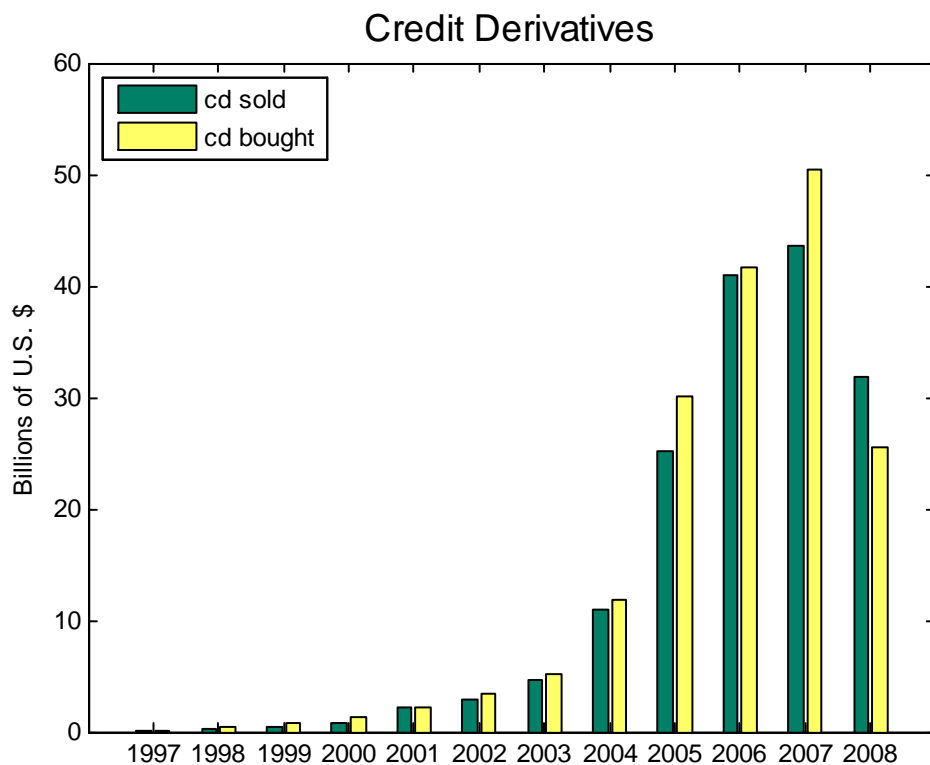
Note: This figure plots the growth rate of the real GDP and its standard deviation estimated in 10-quarter rolling windows. Output is defined in per-capita terms, calculated as ratio of the real gross domestic product, measured in 2005-chained dollars, over total population. The data are quarterly and they span the period 1952Q2 – 2010Q2. The data source is the BEA. The estimates are in percent.

Figure 3: U.S. Private-Label Securitization Issuance by Type



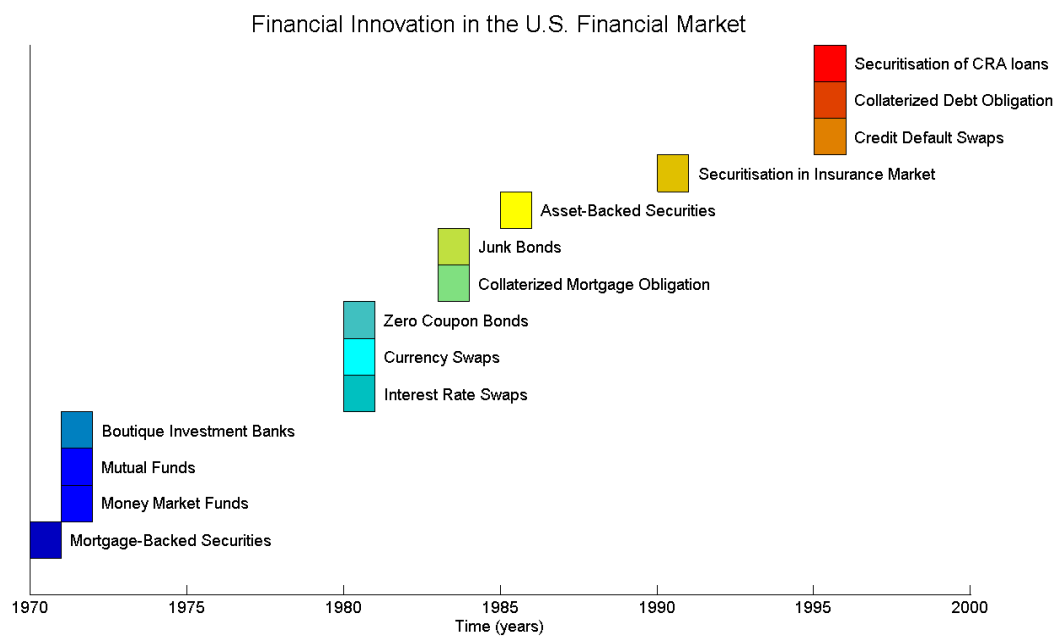
Note: This figure plots the U.S. private-label mortgage-backed securities, asset-backed securities and the collateralized debt obligation. The data are yearly and they span the period 1995-2008. The data source is the IMF. The estimates are in billions of U.S. dollars.

Figure 4: Aggregate Volume of Credit Derivatives Bought and Sold



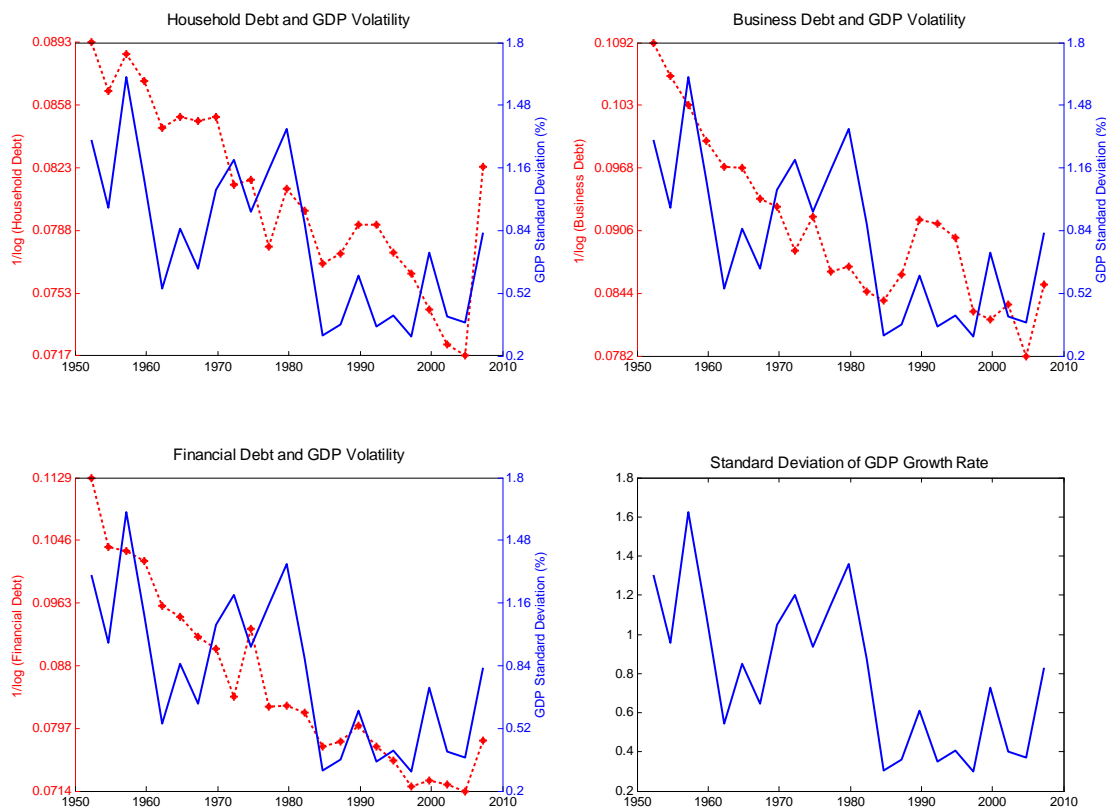
Note: This figure plots the aggregate volume of the credit derivatives both bought and sold in the financial sector. The estimates are yearly averages of quarterly data which span the period 1997Q1 – 2008Q4. The data source is the Federal Bank of Chicago. The estimates are in billions of U.S. dollars.

Figure 5: Time Line



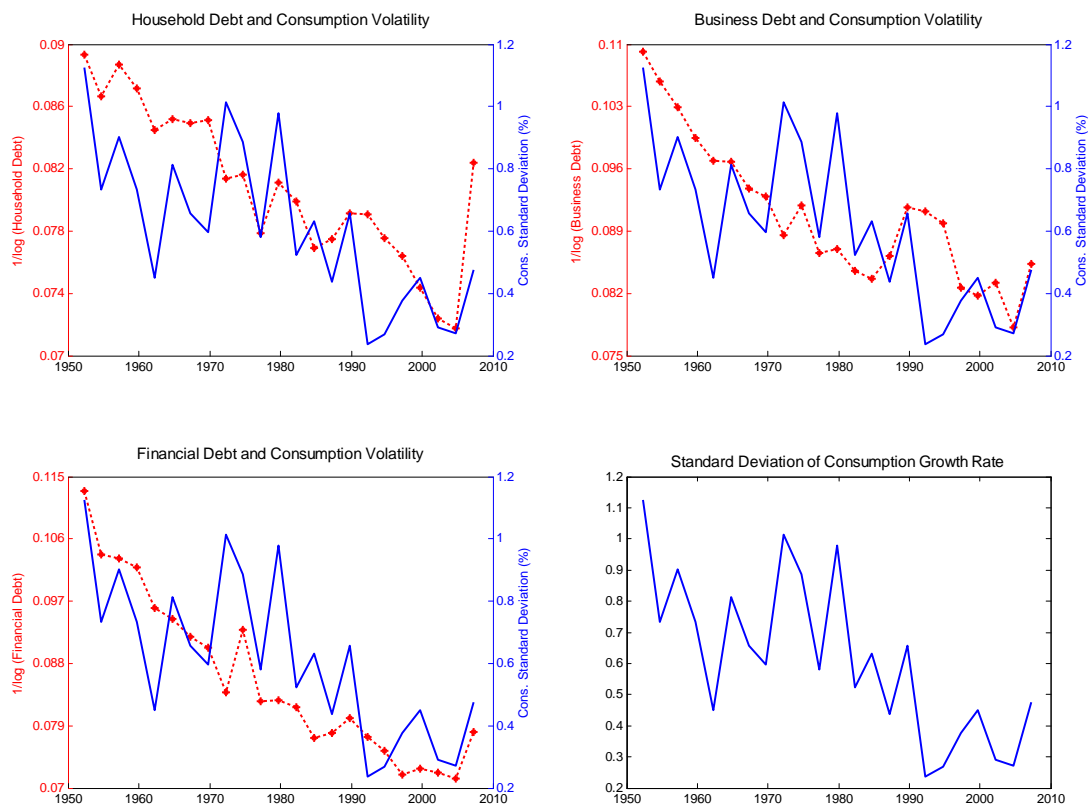
Note: This figure presents a time line that shows when the different secondary market products appeared in the U.S. financial market. The time period covered is from 1970 to 1998.

Figure 6: Comovements of GDP Volatility and the Prime Market



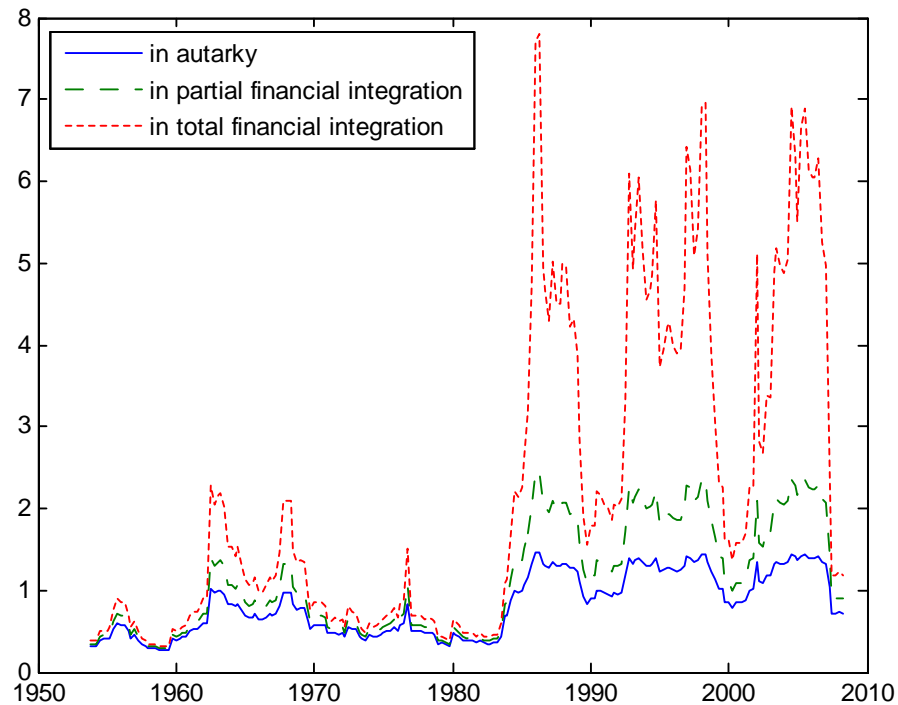
Note: Three of the four subplots of this figure plot jointly the inverse of the logarithm of the financial liabilities for the three different sectors and the standard deviation of the real gdp growth rate. The estimates are in 10-quarter non-overlapping windows. The fourth subplot presents the estimates for the standard deviation of real gdp growth only. Gdp is defined in per-capita terms, calculated as ratio of the real gross domestic product, measured in 2005-chained dollars, over total population. The financial data are measured in 2005-chained dollars as well. All the data are quarterly and they span the period 1952Q2–2010Q2. The data source for the gdp is the BEA and for the financial liabilities is the Flow of Funds Accounts of the United States.

Figure 7: Comovements of Consumption Volatility and the Prime Market



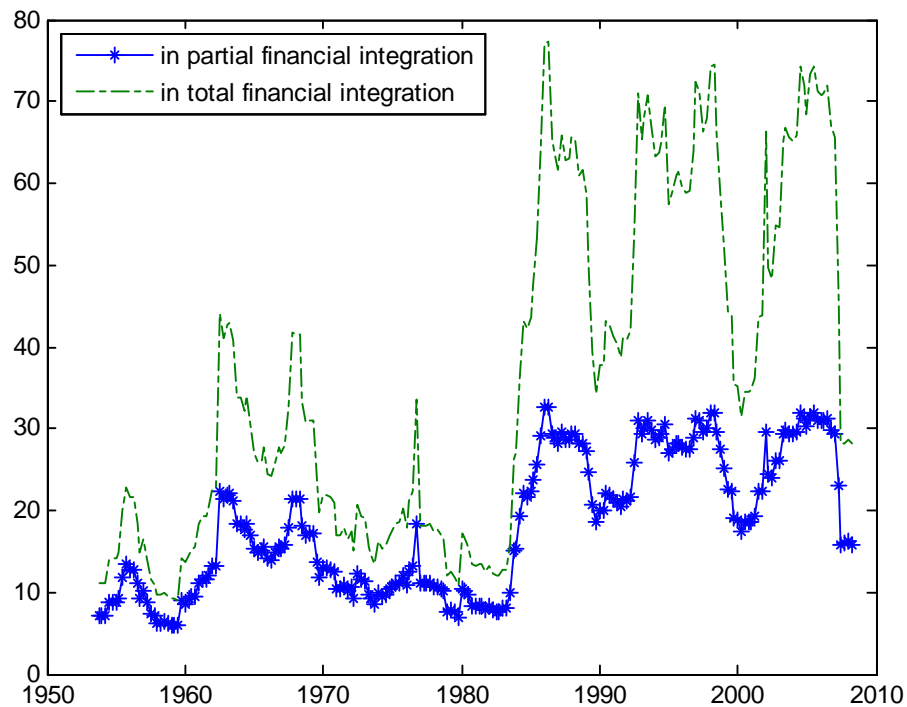
Note: Three of the four subplots of this figure plot jointly the inverse of the logarithm of the financial liabilities for the three different sectors and the standard deviation of the real consumption growth rate. The estimates are in 10-quarter non-overlapping windows. The fourth subplot, present the estimates for the standard deviation of real consumption growth only. Consumption is defined in per-capita terms, calculated as ratio of the total real personal consumption expenditures, measured in 2005-chained dollars, over total population. The financial data are measured in 2005-chained dollars as well. All the data are quarterly and they span the period 1952Q2 – 2010Q2. The data source for consumption is BEA and for the financial liabilities is the Flow of Funds Accounts of the United States.

Figure 8: The Demand for Risky Assets



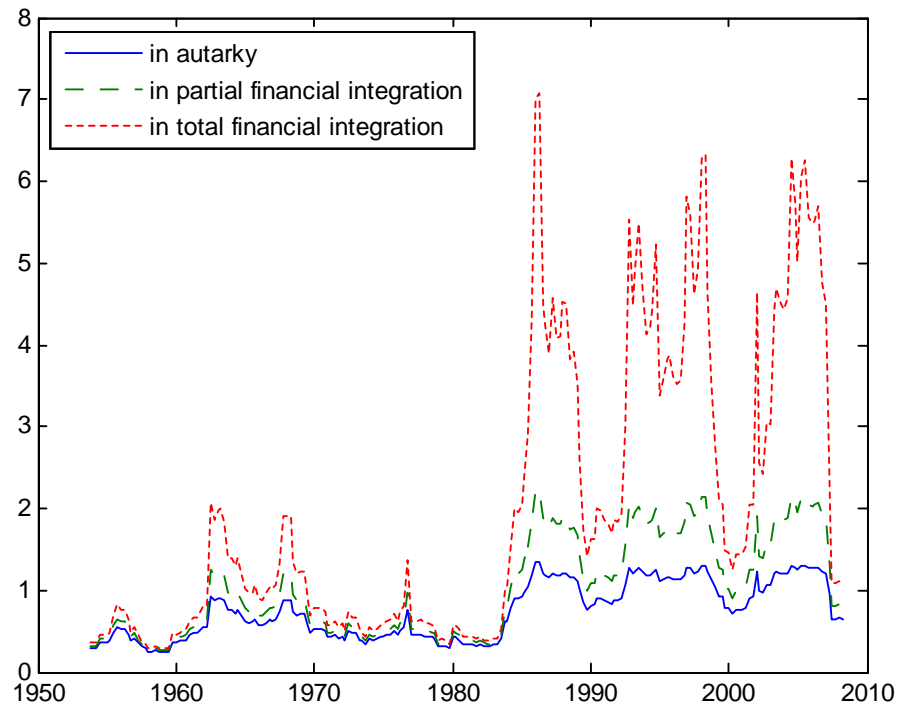
Note: This figure displays the time series of the demand for risky assets as a function of the historical values of $Var(R^a)$ estimated in the third section. The data are quarterly and span the period from the first quarter of 1952 to the fourth quarter of 2009. We have computed the demand for risk for different degrees of financial innovation, thus for different values of ρ : *autarky* $\equiv \rho = 0$, *partial financial integration* $\equiv \rho = -0.7$ and *total financial integration* $\equiv \rho = -1$.

Figure 9: The Demand for Credit Derivatives



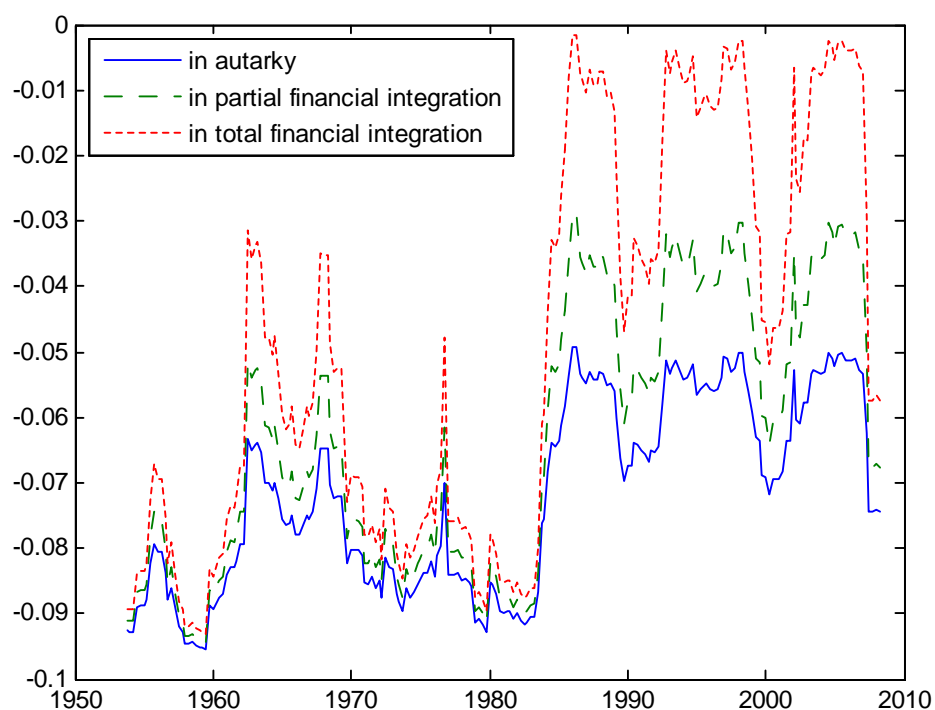
Note: This figure displays the time series of the demand for credit derivatives as a function of the historical values of $Var(R^a)$ estimated in the third section. The data are quarterly and span the period from the first quarter of 1952 to the fourth quarter of 2009. We have computed the demand for risk for different degrees of financial innovation, thus for different values of ρ : *partial financial integration* $\equiv \rho = -0.7$ and *total financial integration* $\equiv \rho = -1$.

Figure 10: The Portfolio Variance



Note: This figure displays the time series of the portfolio variance as a function of the historical values of $Var(R^a)$ estimated in the third section. The data are quarterly and span the period from the first quarter of 1952 to the fourth quarter of 2009. We have computed the demand for risk for different degrees of financial innovation, thus for different values of ρ : *autarky* $\equiv \rho = 0$, *partial financial integration* $\equiv \rho = -0.7$ and *total financial integration* $\equiv \rho = -1$.

Figure 11: Banks' Expected Utility



Note: This figure displays the time series of the banks' expected utility as a function of the historical values of $Var(R^a)$ estimated in the first section. The data are quarterly and span the period from the first quarter of 1952 to the fourth quarter of 2009. We have computed the demand for risk for different degrees of financial innovation, thus for different values of ρ : *autarky* $\equiv \rho = 0$, *partial financial integration* $\equiv \rho = -0.7$ and *total financial integration* $\equiv \rho = -1$.

CHAPTER 2

GREAT MODERATION OR GREAT MISTAKE: CAN OVERCONFIDENCE IN LOW MACRO-RISK EXPLAIN THE BOOM IN ASSET PRICES?

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ABSTRACT

The fall in US macroeconomic volatility from the mid-1980s coincided with a strong rise in asset prices. Recently, this rise, and the crash that followed, have been attributed to overconfidence in a benign macroeconomic environment of low volatility. This paper introduces learning about the persistence of volatility regimes in a standard asset pricing model. It shows that the fall in US macroeconomic volatility since the mid-1980s only leads to a relatively small increase in asset prices when investors have full information about the highly persistent, but not permanent, nature of low volatility regimes. When investors infer the persistence of low volatility from empirical evidence, however, the model can deliver a strong rise in asset prices by up to 45%. Moreover, depending on the learning scheme, the end of the low volatility period leads to a strong and sudden crash in prices.

JEL classification: E, G.

Keywords: Macroeconomic Risk, Asset Prices

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“From the Great Moderation to the Great Conflagration: The decline in volatility led the financial institutions to underestimate the amount of risk they faced, thus essentially (though unintentionally) reintroducing a large measure of volatility into the market.”

Thomas F. Cooley, Forbes.com, 11 December 2008

“The stress-tests required by the authorities over the past few years were too heavily influenced by behavior during the Golden Decade. [...] The sample in question was, with hindsight, most unusual from a macroeconomic perspective. The distribution of outcomes for both macroeconomic and financial variables during the Golden Decade differed very materially from historical distributions.”

Andrew Haldane, Bank of England, 13 February 2009

“But what matters is how market participants responded to these benign conditions. They are faced with what is, in essence, a complex signal-extraction problem. But whereas many such problems in economics involve learning about first moments of a distribution, this involves making inferences about higher moments. The longer such a period of low volatility lasts, the more reasonable it is to assume that it is permanent. But as tail events are necessarily rarely observed, there is always going to be a danger of underestimating tail risks.”

Charles Bean, European Economic Association, 25 August 2009

2.1 Introduction

The fall in macroeconomic volatility in the United States and other countries from the mid-1980s, later coined the "Great Moderation", coincided with a strong rise in asset prices. After the economic crisis that started in 2007, both policy-makers and academics attributed part of this rise, and the subsequent fall in prices, to overconfidence in the benign macroeconomic environment of the "golden decade" (Haldane et al 2009). According to this argument, in their attempt to infer the distribution of future shocks on the basis of observed data, investors overestimated the persistence of a low volatility environment, thus bidding up the price of assets beyond their fundamental value. This paper introduces learning about the persistence of volatility regimes into a standard asset pricing model. It shows that the fall in US macroeconomic volatility since the mid-1980s only leads to a relatively small increase in asset prices when investors have full information about the highly persistent, but not permanent, nature of low volatility regimes. When investors optimally infer the persistence of low volatility from empirical evidence using Bayes' rule, however, the model can deliver a much stronger rise in asset prices similar to that observed in the data. Moreover, depending

on the learning scheme, the end of the low volatility period leads to a strong and sudden crash in prices.

Previous studies have found that a fall in macroeconomic volatility of the magnitude observed in the United States between the late 1980s and early 1990s would have to be, essentially, permanent to explain a significant proportion of the subsequent boom in equity prices (Lettau et al 2007). However, while some authors have attributed the great moderation to structural changes in developed economies that are indeed very persistent, or potentially permanent, such as central bank independence, the increase in world trade, or the development of new financial products to diversify risk, others have pointed to its transitory origins, such as an unusually long period of small exogenous shocks ("good luck") that hit western economies during this period (see section 2 for more detail). Moreover, similar uncertainty about the origins and persistence of the Great Moderation can be found in analysis by market participants. In the aftermath of the economic crisis that started in 2007, both policymakers and academics have attributed the boom in asset prices and their subsequent crash to overconfidence of investors in a benign macroeconomic environment of low volatility (Cooley 2008, Haldane 2009, Bean 2009). For example, Haldane (2009) argues that data availability was such that the high volatility period preceding the Great Moderation was often neglected in the estimation of quantitative asset pricing models. Similarly, Bean (2009) attributes part of the boom and bust in asset prices to rising investor confidence that the low volatility environment would be permanent.

This paper looks at the behaviour of asset prices in an environment where investors have to infer the persistence of changes in macro-volatility from the data. Specifically, we interpret the economic experience of the US economy after the Second World War as consisting of realisations of high and low volatility regimes, whose transition probabilities are unknown to investors. This allows us to analyse the behaviour of asset prices in a general equilibrium where investors use optimal bayesian learning rules to infer the persistence of periods of low macro-volatility. Specifically, we study an economy in which investors simply update their priors about transition probabilities in line with observed realisations of high and low volatility regimes according to Bayes' rule (Cogley and Sargent (2008)). The model delivers a boom and bust in asset prices much stronger than in the absence of uncertainty about transition probabilities, and explains about 45 percent of the boom in US asset prices between the early 1980s and their fall that started in 2007. As a robustness exercise, we look at alternative learning schemes. First, we analyse an alternative Bayesian learning rule based on two popular hypotheses that explained the great moderation either by an unusually long sequence of small shocks ("good luck") or by permanent structural change ("good policy").

Under this alternative learning scheme we assume that transition probabilities during normal times are known, but that there is a small ex ante-probability that a low-volatility regime turns out to be permanent. This scheme leads to asset price dynamics that are qualitatively similar, but even stronger in magnitude, compared to our benchmark learning scheme. Finally, we also look at non-optimal, "adaptive" learning schemes, where investors use simple statistical rules to update their inference about volatility on the basis of observed data. This ad hoc learning results in strong overvaluation of assets, relative to the prices implied by full information about data generating process, but does not yield a strong crash after the end of the Great Moderation (which we identify with the beginning of the economic crisis in 2007).

This paper is most related to the literatures on asset pricing with time-varying volatility, and with learning about features of the economic environment. After earlier papers on the effect of changes in economic volatility for asset prices in stationary environments (Bonomo and Garcia (1994, 1996) and Drifil and Sola (1998)), more recently Bansal and Lundblad (2002), Lettau et al (2008) ask whether a persistent change to a low macro-volatility regime can help explain the boom in US asset prices of the 1990s and early 2000s. They find that the low volatility environment would have to be, essentially, permanent to explain the data.³ Most papers look at environments where agents learn about the mean growth rate of output or consumption. For example, Cogley and Sargent (2008) assume that after the Great Depression, investors had pessimistic priors about the probability of transitions from a high to a low-growth state. Using a learning mechanism that is identical to one of those analysed in our study, they show how this may explain a sustained fall over time from an initially high equity premium, as learning leads to rising confidence in high growth. More recently, Adam and Marcet (2010), show how learning about an unknown process for cum-dividend equity returns introduces a self-referential element in equity prices that leads to persistent bubbles and occasional crashes. There has also been a growing number of contributions that study learning about risk. Branch and Evans (2010) employ self-referential adaptive learning about asset prices and return volatility in order to explain high frequency booms and busts in asset prices. Weitzman (2007) adopts a consumption-based asset-pricing model

³Lower macro-volatility is only one item on a long list of potential reasons behind the asset price boom of the 1990s and 2000s. Others are a lower equity premium (Blanchard (1993), Jagannathan, McGrattan, and Scherbina (2000), Fama and French (2002)), higher long-run growth (Jagannathan, McGrattan, and Scherbina (2000), Fama and French (2002), Campbell and Shiller (2003), although Siegel (1999) finds no evidence for this), stronger intangible investment in the 1990s (Hall (2000)) saving during the 1990s by the baby boom generation (Abel (2003)), redistribution of rents towards owners of capital (Jovanovic and Rousseau (2003)) or reduced costs of stock market participation and diversification (Heaton and Lucas (1999), Siegel (1999), Calvet, Gonzalez-Eiras, and Sodini (2003)).

and replaces rational expectations with Bayesian learning about consumption growth rate volatility, which allows him to solve a number of asset pricing puzzles.

Most relevant for this paper are two studies that link the asset price boom and bust of 1990s and 2000s to learning about regime changes in key parameters of the economic environment. Boz and Mendoza (2010) study a partial equilibrium model in which investors face an exogenous leverage constraint that follows a two-state markov process with unknown transition probabilities. Assuming Bayesian learning as in Cogley and Sargent (2008), the authors show that with little prior information, the observation of a string of high leverage periods can lead to overoptimism about their persistence, and thus a boom in asset prices, leverage and consumption, which crashes abruptly once the economy switches back to a tighter constraint. While one of our learning mechanisms also follows Cogley and Sargent (2008), we analyse, in general equilibrium, exogenous changes in macro-volatility, rather than in regimes of financial regulation. This focus is similar to Lettau et al. (2008), who also study the asset price effect of changes in macro volatility-regimes under limited information about the environment. Particularly, while knowing all parameters of the environment, including the persistence of volatility regimes, agents in their model ignore whether the economy is currently in a high or low volatility regime.⁴ Rather than incorporating learning explicitly, they then calculate asset prices given the sequence of posterior state probabilities implied by a regime-switching model estimated on post-war consumption data for the US. Our work differs to theirs in several ways: first, based on our reading of the academic literature and the business press (see section 2), we assume agents were sure that the US economy had experienced a change in aggregate volatility with the Great Moderation, but were uncertain about its persistence. Second, we explicitly look at different optimal and ad hoc learning mechanisms to ask how their implied asset price behaviour relates to that of an environment of full information about transition probabilities, and US data. Importantly, in Lettau et al (2008) asset prices are, essentially, weighted averages of full information prices. The model-implied prices are therefore always lower than those that would prevail in the most benign low-volatility regime with full information. In our model, on the other hand, agents may overestimate the persistence of the Great Moderation, leading to significant overvaluation of asset prices relative to full information.

The rest of the paper is organised as follows. To motivate our approach in more detail, section II reviews the main empirical facts on the Great Moderation as well as the debate

⁴Lettau et al (2008) also have two states of different mean growth, leaving four states of the economy in total.

2.2. MOTIVATION: THE GREAT MODERATION, ITS UNCERTAIN CAUSE AND PERSISTENCE

about its causes among academics and market participants. Section III presents the model. Section IV gives the main results and section V shows how robust these are to changes in the underlying assumptions.

2.2 Motivation: The Great Moderation, its uncertain cause and persistence, and the boom in asset prices

2.2.1 Asset Prices and the Great Moderation: Stylized Facts

Figure 1 and figure 2 present the time series of real GDP and consumption growth rates and their corresponding volatilities (computed as the standard deviation over 10-quarter rolling windows). Both series exhibit a significant and abrupt fall in volatility, which persisted until the beginning of the current crisis. The timing of the drop, however, differs: while GDP volatility declined around the middle of the 1980s, the fall occurred somewhat later, at the beginning of the 1990s, for consumption growth.

Enter Figure 1 about here

Using quarterly data from 1952Q2 to 2010Q2, table 1 and 2 quantify this decline in volatility for different subperiods.⁵ The end dates of the first subperiod are 1984Q1 for GDP and 1992Q1 for consumption, while the second ends with the start of the financial crisis in 2007⁶. Whereas there is almost no change in mean growth across the first two subperiods, there is a significant fall in volatility of more than 50 percent for both aggregate output and consumption growth. In the third sub-sample that covers the recent crisis, we observe a sharp decrease in mean growth for both GDP and consumption and a strong rise in volatility.

Figure 2 shows how the decline in macroeconomic volatility coincided with a strong rise in asset prices and a fall in the US price-dividend ratio for the *S&P* 500. Importantly, this fall was much less abrupt than the decline in volatility itself. Again, table 3 quantifies this effect for 3 subperiods, choosing 1995Q1 as the start of the second subperiod⁷. The price-dividend ratio more than doubled across the first two periods, but fell back to levels seen in the 1960s and 1970s with the start of the recent crisis.

⁵See the Data Appendix for a more detailed description of the data series.

⁶McConnell and Perez-Quiros (2000) provide evidence that 1984Q1 was the break date for the GDP growth series and Lettau et al. (2008) provide evidence that 1992Q1 was the break date for the aggregate consumption growth series.

⁷Lettau et al. (2008) provide evidence that 1995Q1 was the break date for the aggregate consumption growth series.

2.2. MOTIVATION: THE GREAT MODERATION, ITS UNCERTAIN CAUSE AND PERSISTENCE

Moments of GDP growth

<i>Date</i>	<i>Mean</i>	<i>StDev</i>
1952Q2 : 1983Q4	0.53	1.1
1984Q1 : 2006Q4	0.51	0.51
2007Q1 : 2010Q2	-0.16	0.90

Table 2.1: The table reports sample estimates for the mean and the standard deviation of real GDP growth rate. Output is defined in real per-capita terms. The GDP and the population data are taken from the Bureau of Economic Analysis. The data are quarterly and span the period 1952Q2 – 2010Q2. The estimates are in percent.

Moments of Consumption Growth

<i>Date</i>	<i>Mean</i>	<i>StDev</i>
1952Q2 : 1991Q4	0.57	0.82
1992Q1 : 2006Q4	0.61	0.36
2007Q1 : 2010Q2	-0.19	0.50

Table 2.2: The table reports sample estimates for the mean and the standard deviation of real consumption growth rate. Consumption is defined in real per-capita terms. The consumption and the population data are taken from the BEA. The data are quarterly and span the period 1952Q2 – 2010Q2. The estimates are in percent.

US Equity Prices

<i>Date</i>	<i>Mean</i> $\frac{p}{d}$	<i>Mean</i> $\frac{p}{e}$
1952Q2 : 1994Q4	27.49	15.54
1995Q1 : 2006Q4	62.25	30.03
2007Q1 : 2010Q2	46.48	21.18

Table 2.3: The table reports sample estimates for the mean of the price-dividend and the price-earning ratio based on the *S&P* 500. Consumption is defined in real per-capita terms. The data are taken from the Robert Shiller's homepage. The data on prices are monthly instead the data on dividends and on the price-earning ratio are quarterly. We calculate quarterly estimates for the prices by taking quarterly averages over the monthly data. The data span the period 1952Q2 – 2010Q2. The estimates are in percent.

Enter Figure 2 about here.

2.2.2 Uncertainty about Origin and Persistence of the Great Moderation

By the second half of the 1990s, both the academic literature (Kim and Nelson (1999), McConnell and Perez-Quiros (1997, 2000)) and the business press had noticed a break in the volatility properties of US output growth around the middle of the preceding decade. Somewhat later, a similar decline in volatility was documented for a broader set of US macroeconomic variables (Blanchard and Simon (2001), Stock and Watson (2002)), as well as for other industrial countries (Stock and Watson 2003). However, although the Great Moderation itself had become a stylised fact, there was no consensus about its causes. While some authors explained the phenomenon with reference to changes in the structure of industrial economies, such as financial innovation (Dynan et al 2006), improved inventory management, or financial and trade liberalisation (see Wachter (2006) for a brief summary), the two perhaps most prominent hypotheses competed under the heading of "Good Policy or Good Luck?". Specifically, following the seminal article by Stock et al (2003), several studies⁸ used time-varying VAR models to find that a string of unusually small shocks, rather than changes in their transmission to main macroeconomic variables or in the conduct of monetary policy, were at the root of the decline in macro-volatility. Against this, both academics (Benati et al 2008) and policymakers (Tucker 2005, Bernanke 2004) argued that reduced-form models were likely to mistake effects of improved monetary policy, such as more stable but unobserved inflation expectations, for changes in the variance-covariance-properties of exogenous economic shocks. For example, Bernanke (2004) argued that "some of the benefits of improved monetary policy may easily be confused with changes in the underlying environment". Importantly, the lack of consensus about the causes of the observed fall in macro-volatility left it unclear whether the phenomenon was likely to be permanent, as suggested by structural change or possibly improved policy environments, or transitory, in line with the "good luck" hypothesis.

How did market participants perceive the Great Moderation and its effect on prices? Investment analysts explicitly attributed part of the observed fall in the equity risk premium since the late 1980s to the decline in macro-volatility. For example, Goldman Sachs research (2002) noted that an estimated 8 percentage point fall in the risk premium since the 1970s was "underpinned by dramatic improvements in the economic environment. Inflation fell sharply, and the volatility of GDP growth, inflation and interest rates all declined significantly." (p.

⁸Giorgio E. Primiceri (2005), Christopher Sims and Tao Zha (2006), and Luca Gambetti, Evi Pappa, and Fabio Canova (2008)

2). But while investors acknowledged the effect of the Great Moderation on asset prices, they were also aware of the uncertain persistence of this low-volatility environment, and thus of the decline in equity premia. For example, regarding risk premia in fixed income securities, Unicredit analysts (2006) argued that "the ongoing deterioration in surprise risk should be seen as one of the arguments behind the declining risk premium. Whether this is due to a more effective central bank policy, a major improvement in the forecast ability of economic observers around the globe, sheer luck or maybe a mix of all three factors can't finally be answered." (p. 10). Researchers at JP Morgan (2005), on the other hand, attribute most of the fall in volatility to a changed orientation of policymakers towards a "Stability Culture", which, however, is not certain to persist.

We draw three conclusions from this evidence: first, the fall of macro-volatility since the mid-1980s was accepted as a stylised fact, and widely seen as a contributing factor to higher asset prices during the 1990s and 2000s. Second, as Lettau et al (2008) have shown, standard asset pricing models predict significantly higher asset prices only when a fall in volatility is permanent, or extremely persistent. Finally, during the Great Moderation it was exactly this persistence that investors were uncertain about. This paper therefore puts learning about the persistence of volatility changes at the center of its analysis. In particular, it asks two questions: first, can rising confidence in the persistence of low volatility explain the strong and gradual rise in asset prices during the Great Moderation; and second, how much of this rise is due to overconfidence in the great moderation, equivalent to an overvaluation of assets relative to the prices implied by the true value of persistence.

2.3 The model

This section presents a standard general equilibrium asset pricing model and adds learning about the persistence of volatility regimes.

2.3.1 Preferences

We consider an endowment economy with an infinitely-lived representative agent who solves the following problem

$$\max_{C_t, S_t} U_t \quad (2.1)$$

$$s.t. S_t P_t + C_t = S_{t-1} P_t + D_t \quad (2.2)$$

$$S_{-1} \text{ given} \quad (2.3)$$

where U_t denotes an expected utility index at time t , C_t denotes consumption, S_t are the agent's stockholdings, P_t is the stock price and D_t are dividends. Preferences U_t are as in Epstein and Zin (1989, 1991) or Weil (1989)

$$U(C_t) = [(1 - \beta)C_t^{\frac{1-\gamma}{\alpha}} + \beta(E_t U_{t+1}^{1-\gamma})^{\frac{1}{\alpha}}]^{\frac{\alpha}{1-\gamma}}$$

where E_t is the mathematical expectation with respect to the agents subjective probability distribution conditional on period t information, $\alpha = \frac{1-\gamma}{1-\frac{1}{\psi}}$, γ is the coefficient of relative risk aversion, and ψ the intertemporal elasticity of substitution.

The first order condition associated to this problem is

$$P_t = E_t^s[M_{t+1}(P_{t+1} + D_{t+1})] \quad (2.4)$$

where M_{t+1} is the stochastic discount factor, which, with Epstein-Zin preferences, equals

$$M_{t+1} = (\beta(\frac{C_{t+1}}{C_t})^{-\frac{1}{\psi}})^{\alpha} R_{w,t+1}^{\alpha-1}$$

Here, $R_{w,t+1}^{\alpha-1}$ is the return on the aggregate wealth portfolio of the representative agent, whose returns equal aggregate consumption.

2.3.2 The Processes for Consumption and Dividend Growth

We choose a simple and transparent way to model an economy that goes through periods of low and high macro-volatility by assuming that the log consumption follows an exogenous random walk with drift

$$g_t = \Delta \ln C_t = \bar{g} + \varepsilon_t$$

where \bar{g} is constant mean consumption growth.⁹ Shocks ε_t are independently normally distributed, and their variance follows a two-state Markov process

$$\varepsilon_t \sim N(0, \sigma_t^2) \quad \sigma_t^2 \in \{\sigma_l^2, \sigma_h^2\}$$

The transition probabilities for the Markov process are

$$\begin{aligned} \Pr(\sigma_{t+1}^2 = \sigma_l^2 \mid \sigma_t^2 = \sigma_l^2) &= F_{ll} \\ \Pr(\sigma_{t+1}^2 = \sigma_h^2 \mid \sigma_t^2 = \sigma_h^2) &= F_{hh} \end{aligned}$$

⁹Previous studies have looked at time-variation in \bar{g} . Here, we assume \bar{g} to be constant over time, and concentrate instead on changes over time in the variance of shocks ε_t .

which yields the transition probability matrix as

$$\mathbf{F} = \begin{bmatrix} F_{ll} & 1 - F_{ll} \\ 1 - F_{hh} & F_{hh} \end{bmatrix}$$

Following Mehra and Prescott (1985), and in line with the endowment nature of the economy, it is common to assume that dividend flows equal consumption flows. To capture the higher empirical volatility of dividends, we follow Campbell (1986), Abel (1999), or Lettau et al (2008), and use a generalised version of the standard model where shocks to dividend growth are a multiple of those to consumption

$$\Delta \ln D_t = \bar{g} + \lambda \varepsilon_t \quad \lambda \geq 1$$

Dividends thus follow the same volatility pattern as consumption, but are on average more volatile.

2.3.3 Full Information Price-Dividend Ratios

We can use the first-order condition for share holdings to express the price-dividend ratio $p_t = \frac{P_t^D}{D_t}$ as

$$p_t = \left(\frac{P_t^C}{C_t}\right)^{1-a} E_t \left[\beta^\alpha \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\alpha}{\psi} + \alpha - 1} \left(\frac{P_{t+1}^C}{C_{t+1}} + 1\right)^{\alpha-1} (p_{t+1} + 1) \frac{D_{t+1}}{D_t} \right] \quad (2.5)$$

where $\frac{P_t^C}{C_t}$ is the price-consumption ratio, with P_t^C the price of a claim to aggregate consumption and $\frac{P_t^C}{C_t}$ equals $\frac{P_t^D}{D_t}$ whenever $\lambda = 1$. When the agent knows the true structure of uncertainty, given the random walk nature of consumption and dividends, the price-dividend and price-consumption ratios are functions only of the volatility state, $p_t = p(\sigma_t^2)$, and thus non-random conditional on σ_t^2 . We can thus simplify (2.5) by taking expectations across realisations of log-normal consumption and dividend growth conditional on σ_{t+1}^2 , which gives a recursive expression for the price-dividend and ratios.¹⁰

Note that in the special case when $\lambda = 1$ both consumption and dividend growth follow the same the log-normal distribution. With $\psi = \frac{1}{\gamma}$ (CRRA preferences), this yields an

¹⁰Specifically, $p(\sigma_t^2)$ is defined by

$$p(\sigma_t^2) = \sigma_t^2 = \rho_i^{1-\alpha} \beta^\alpha e^{(-\frac{\alpha}{\psi} + \alpha)\bar{g}} \quad (2.6)$$

$$\left(F_{ii} e^{\frac{(-\frac{\alpha}{\psi} + \alpha - 1 + \lambda)^2}{2} \sigma_i^2} (1 + \rho_i)^{\alpha-1} (1 + p_i) + F_{ij} e^{\frac{(-\frac{\alpha}{\psi} + \alpha - 1 + \lambda)^2}{2} \sigma_j^2} (1 + \rho_j)^\alpha (1 + p_j) \right) \quad (2.7)$$

analytical solution to the vector of price-dividend ratios p as

$$p = \beta F(1+p)e^{(-\gamma+1)\bar{g}}(e^{(-\gamma+1)^2 2\sigma_i^2}) \quad (2.10)$$

$$= \mathbb{F}\beta F e^{(-\gamma+1)\bar{g}}(e^{(-\gamma+1)^2 2\sigma_i^2}) \quad (2.11)$$

where $\mathbb{F} = [I - \beta F \begin{bmatrix} \sigma_l^2 & 0 \\ 0 & \sigma_h^2 \end{bmatrix}]^{-1}$.

2.3.4 Learning and Subjective Beliefs

To study whether a long spell of σ_l can lead to a boom in asset prices via overconfidence in the persistence of a low-volatility environment, we assume that the representative agent does not know the full probabilistic structure of the economy. Specifically, the agent knows that dividend growth is log-normal with mean \bar{g} but ignores some information about its variance. Particular, she uses observations on realised dividends to infer the transition probabilities between high and low volatility states. A later sensitivity analysis studies how the results change when assuming other Bayesian or non-Bayesian learning schemes.

In this section, we look at an environment in which the agent learns about the transition probabilities between volatility states F_{hh} and F_{ll} . The model is thus very similar to Cogley and Sargent (2008), with the difference that the agent learns about transitions between volatility states, rather than mean-growth states. More specifically, the agent knows the structure of the model and all parameter values except the true transition probabilities F_{hh} and F_{ll} . In every period she observes a dividend realization and the distribution that this specific realization is drawn from, parameterised by σ_t^2 . The agent thus forms a best guess about F_{hh} and F_{ll} on the basis of the history of volatility-states $\Sigma^t = \{\sigma_t^2, \sigma_{t-1}^2, \dots, \sigma_2^2, \sigma_1^2\}$.

We assume that the agent has independent beta-binomial prior distributions about F_{hh} and F_{ll}

$$f_0(F_{hh}, F_{ll}) \propto f_0(F_{hh})f_0(F_{ll})$$

with

$$\begin{aligned} f(F_{hh}) &= f(F_{hh} | \Sigma^0) = \text{beta}(n_0^{hh}, n_0^{hl}) \propto F_{hh}^{n_0^{hh}-1} (1 - F_{hh})^{n_0^{hl}} \\ f(F_{ll}) &= f(F_{ll} | \Sigma^0) = \text{beta}(n_0^{ll}, n_0^{lh}) \propto F_{ll}^{n_0^{ll}-1} (1 - F_{ll})^{n_0^{lh}}. \end{aligned}$$

where $\rho = \frac{F_t^C}{C_t}$ follows

$$\rho^\alpha(\sigma_t^2 = \sigma_i^2) = \rho_i^{1-\alpha} \beta^\alpha e^{(-\frac{\alpha}{\psi} + \alpha)\bar{g}} \quad (2.8)$$

$$\left(F_{ii} e^{\frac{(-\frac{\alpha}{\psi} + \alpha)^2}{2} \sigma_i^2} (1 + \rho_i)^{\alpha-1} + F_{ij} e^{\frac{(-\frac{\alpha}{\psi} + \alpha)^2}{2} \sigma_j^2} (1 + \rho_j)^\alpha \right) \quad (2.9)$$

where Σ^0 denotes a prior belief about frequencies n_0^{ij} of transitions from state i to state j . The agent updates this prior on the basis of the likelihood function for the history of volatility states Σ^t conditional on F_{hh} and F_{ll} , which is the product of two independent binomial density functions, thus

$$L(\Sigma^t | F_{hh}, F_{ll}) \propto L(\Sigma^t | F_{hh})L(\Sigma^t | F_{ll})$$

where

$$\begin{aligned} L(\Sigma^t | F_{hh}) &= \text{binomial}(F_{hh}, F_{hh}) \propto F_{hh}^{n_t^{hh} - n_0^{hh}} (1 - F_{hh})^{n_t^{hl} - n_0^{hl}} \\ L(\Sigma^t | F_{ll}) &= \text{binomial}(F_{ll}, F_{ll}) \propto F_{ll}^{n_t^{ll} - n_0^{ll}} (1 - F_{ll})^{n_t^{lh} - n_0^{lh}} \end{aligned}$$

Here, n_t^{ij} is a "counter", that equals the number of transitions from state i to state j up to time t plus the prior frequencies n_0^{ij} . The posterior kernel is the product of the beta prior and the binomial likelihood function,

$$f(F_{hh}, F_{ll} | \Sigma^t) \propto \underbrace{L(\sigma_t^2 | F_{hh}, F_{ll})}_{\text{Likelihood}} \cdot \underbrace{f(F_{hh}, F_{ll} | \Sigma^{t-1})}_{\text{Prior}}$$

which after normalizing by $M(\Sigma^t) = \int \int F_{hh}^{n_t^{hh}-1} (1 - F_{hh})^{n_t^{hl}-1} F_{ll}^{n_t^{ll}-1} (1 - F_{ll})^{n_t^{lh}-1} dF_{hh} dF_{ll}$ yields the posterior density function as the product of independent *Beta* distributions

$$\begin{aligned} f(F_{hh} | \Sigma^t) &= \text{beta}(n_t^{hh}, n_t^{hl}) \propto F_{hh}^{n_t^{hh}-1} (1 - F_{hh})^{n_t^{hl}-1} \\ f(F_{ll} | \Sigma^t) &= \text{beta}(n_t^{ll}, n_t^{lh}) \propto F_{ll}^{n_t^{ll}-1} (1 - F_{ll})^{n_t^{lh}-1} \end{aligned}$$

Note that in this context the counters n_t^{ij} are sufficient statistics for the posterior.

Let $p(\sigma_t^2, F)$ denote the price-dividend ratio when the transition probability matrix is F . Following Cogley and Sargent (2008), p_t^{BL} , the vector of price-dividend ratios under Bayesian learning about transition probabilities can then be written as

$$p_t^{BL} = \int p(\sigma_t^2, F) f(F, \Sigma^t) dF \quad (2.12)$$

where $f(F, \Sigma^t)$ is the posterior distribution of F^{11} . Note that for given F_{hh}, F_{ll} , $p(\sigma_t^2, F)$ is described by the same pair of equations as under full information ((2.9), (2.7)). And the law of iterated expectations implies that we can compute $p(\sigma_t^2, F)$ as a fixed point of these two equations. p_t^{BL} can then easily be calculated by numerical integration across the independent beta posteriors for F_{hh}, F_{ll} .

¹¹For a derivation of equation (1) see Appendix B.

2.4 Quantitative Results for the Benchmark Economy

2.4.1 The exercise

This section presents the results of numerical simulations to answer the two main questions of this paper: can learning about the persistence of the great moderation explain the observed boom and bust in US asset prices? And can overconfidence in this persistence lead to an overvaluation of assets, and a larger fall in prices at the end of the low-volatility period, relative to the case of full information. To answer these questions we analyse a scenario that is similar to the economic experience of the US after World-War II. Particularly, we interpret this experience as a long realisation of high volatility followed by the Great Moderation that ends with the recent crisis. Our data generation process thus consists of three sequences of shocks corresponding to three subperiods of different consumption growth volatility σ_t^2 . Specifically, our analysis starts with a high volatility regime in 1952Q2. Since in our highly stylised model there is no distinction between consumption and GDP, we use a start date for the Great Moderation at the beginning of 1984, as suggested by the fall in GDP volatility, but also look at later dates as suggested by the consumption growth series. In line with the observed rise in volatility in figure 1, we locate the end of the Great Moderation at the beginning of 2007, the start year of the crisis. To compute the fall in asset prices around this end of the Great Moderation we also make the stronger assumption that the economy returned to the high volatility environment observed before the Great Moderation. This assumption is largely used for heuristic purposes. It allows us to isolate the crash in asset prices implied by the disappearing overconfidence in the Great Moderation from other factors that this paper abstracts from.

2.4.2 Parameter choice

2.4.2.1 Preferences

As Bansal and Yaron (2004) have shown, for a rise in consumption volatility to increase asset prices with Epstein-Zin preferences, the intertemporal elasticity of substitution ψ has to be greater than unity. We thus follow Lettau et al (2008) and set $\psi = 1.5$. For our statements about the size of boom and bust to be interesting, the model has to deliver a level of asset prices that is approximately equal to the data in the period before the Great Moderation. Rather than changing parameters across different learning rules to target asset prices exactly, however, we choose $\beta = 0.99325$ to target an interest rate of 2 percent p.a. (which varies very little across models), and set $\gamma = 30$ which yields equity prices that are,

on average across models, close to US data, but not exactly equal to it for any particular model.

2.4.2.2 The Process for Consumption and Dividends

The consumption process in this model is characterised by three parameters: constant mean growth \bar{g} , and the variances in the two subperiods σ_h^2, σ_l^2 , which we estimate directly from quarterly data on US personal real per capita consumption expenditure, using the subperiods from table 1. This yields mean growth 0.6 percent per quarter and variances of 0.82 percent and 0.37 percent respectively.

To capture the underlying uncertainty about the persistence of the Great Moderation, we choose a particularly simple and transparent calibration of the transition matrix \mathbf{F} , by choosing F_{ll}, F_{hh} such that the expected durations of high and low volatility regimes equal the subperiods identified from US data. Specifically, we set $F_{ii} = 1 - \frac{1}{T_i}$, where T_l, T_h are the durations of the Great Moderation and the high-volatility period preceding it, which yields transition matrices equal to

$$\mathbf{F}^{cons} = \begin{bmatrix} 0.989 & 1 - 0.989 \\ 1 - 0.992 & 0.992 \end{bmatrix}$$

It is interesting to note that these transition probabilities are extremely close to those in Lettau et al (2008), based on an estimated markov process on the same data.¹²

Unless otherwise mentioned, we set $\lambda = 4.5$ as suggested by Lettau et al (2008) on the basis of the relative volatility of US consumption and dividends. Table 4 summarises the parameters of preferences and the endowment process for the baseline model.

2.4.2.3 Learning Parameters

In our benchmark Bayesian learning scheme, the only additional free parameters are the prior probabilities. We use relative frequencies that imply high but not extreme persistence of both regimes equal to $f_{ii} = 0.9$. In line with the relative lack of prior knowledge about the Great Moderation, however, in our benchmark case we choose weak priors by giving agents relatively little information about the transitions in a low-volatility regime (equivalent

¹²Their point estimates are

$$\mathbf{F} = \begin{bmatrix} 0.991 & 1 - 0.991 \\ 1 - 0.994 & 0.994 \end{bmatrix}$$

Their process, however, is more complex, as they also include uncertainty about mean growth.

Parameter Values for the Baseline Model		
Preferences		
β	0.99325	Discount Factor
γ	30	Relative Risk Aversion
ψ	1.5	Intertemporal Elasticity of Substitution
Endowment Process		
\bar{g}	0.0059	Mean of Consumption Growth
σ_l	0.0037	Low Volatility of Consumption Growth
σ_h	0.0082	High Volatility of Consumption Growth
F_{ll}	0.989	$P(\sigma_{t+1}^2 = \sigma_l^2 \mid \sigma_t^2 = \sigma_l^2)$
F_{hh}	0.992	$P(\sigma_{t+1}^2 = \sigma_h^2 \mid \sigma_t^2 = \sigma_h^2)$
λ	4.5	Leverage

Table 2.4: Parameter values in the benchmark model.

Prior for the benchmark model with Learning		
n_0^{ll}	3.6	Prior frequency for low-to-low transitions
n_0^{lh}	0.4	Prior frequency for low-to-high transitions
n_0^{hh}	36	Prior frequency for high-to-high transitions
n_0^{hl}	4	Prior frequency for high-to-low transitions

Table 2.5: Learning parameters in the benchmark model

to 1 year of data). In the high volatility regime, with its longer history, we choose 10 years. In another scenario, we look at stronger priors (equivalent to 5 and 20 years of data, respectively).

Table 4 summarises the parameter values.

2.4.3 Asset Price Dynamics: Mean and Variance Effects of Learning and Uncertainty about Transition Probabilities

Figure 4 presents the time path of the PD ratio in US data in the upper panel. The bottom panel depicts both the PD ratio with learning (solid lines) and those under full information about the the data generating process (dashed lines). The first thing to note

Asset prices - Benchmark Model			
	Boom	Overvaluation	Bust
Full Info	15%	0	15%
Learning	44%	28%	34%

Table 2.6: "Boom" denotes the increase in prices until the end of the Great Moderation relative to the high-volatility regime preceding it (computed over windows of 5 years). "Overvaluation" is the overvaluation at the end of the Great Moderation relative to the prices under full information. And "Bust" is the fall in prices in the first period after the Great Moderation.

is that both with and without learning, the model delivers realistic levels of asset prices before the 1980s. Together with a model-implied interest rate of 2%, which is independent of regime and the information structure, the benchmark calibration thus delivers a realistic equity premium. Without learning, however, there is no sustained asset price boom during the Great Moderation. Rather, the model with full information delivers a small jump in prices of around 15% at the beginning of the Great Moderation. With learning, on the other hand, asset prices during the Great Moderation continue to rise after an initial jump. While the model is not able to replicate the hump-shape in PD ratios during the Great Moderation, nor the more than two-fold rise in prices between the early 1980s and 2007, it can explain a strong sustained boom in prices until the early 2000s of around 45%, and a subsequent bust of similar magnitude. The results are summarised in table 6.

The model with learning thus differs from that with full information both in terms of the average level of asset prices and in terms of its dynamics. What is behind this? The slight fall in prices before the great moderation, and the concave shape after the jump that follows the fall in volatility in the 1980s, are very intuitive: as agents observe how the economy remains in the same regime of low or high volatility, they increase their persistence estimates from their relatively loose and moderate prior centered around 0.9. This leads to a fall in prices in the high volatility regime, as agents infer a falling likelihood of moving to a more benign macroeconomic climate of lower volatility. Similar reasoning explains the rise in prices during low volatility. Finally, as information accumulates, the marginal contribution of an additional observation to persistence estimates falls, explaining the concave shape of prices during the Great Moderation. However, with a moderate prior of $F_{ll}^0 = 0.9$, prices at the beginning of the great moderation should, in principle be lower, as agents anticipate a

move back to the high volatility environment of the 1960s and 1970s with higher probability than under full information. Instead, the model with learning predicts asset prices to be around slightly higher than with full information at the beginning of the Great Moderation. Why is this? The answer follows from the fact that, in addition to the mean effect of a less persistent average prior, there is an additional effect of uncertainty about transition probabilities on price levels, which results from the variance of posterior persistence estimates around their expected value. Particularly, PD ratios turn out to be a strongly convex function of persistence at high values of f_u . This implies a Jensen's inequality effect, whereby PD ratios with uncertainty about persistence parameters are higher than without. And the difference is larger the looser are priors, and the higher is therefore the variance of the posterior estimates.

Figure 5 looks at the simplified case of symmetric transition probabilities ($f_u = f_{hh}$), to illustrate both effects. The solid lines depict PD ratios in the absence of persistence. As persistence rises, high-volatility prices fall, as agents are less willing to pay for assets whose payoffs they anticipate to remain volatile with a larger probability. Interestingly, low-volatility prices initially fall, but rise strongly for high values of persistence above 0.99. This non-linearity of the certainty price leads to higher level of prices as priors about persistence become looser. Figure 5 demonstrates this by showing the PD ratios when decreasing the size of the parameters in the beta distribution n^{ii}, n^{ij} while keeping their ratio constant.

Figure 5, however, does not explain why PD ratios are a convex function of persistence in the first place. Figure 6 gives a partial answer by plotting the diagonal and off-diagonal elements of the present discount value matrix $V = \sum_{i=0}^{\infty} \beta^i F^i = [I - \beta * P]^{-1}$. As the figure shows, for other than very high persistence, the geometrically declining probability of remaining in the same state for 1, 2, ..., n periods leads to mixing coefficients in V that are close to $\frac{1}{2}$, and thus asset prices that differ little between regimes. It is thus the geometric nature of present discounted probabilities that leads to the highly non-linear relationship between asset prices and persistence in figure 5.

Note how uncertainty about the value of regime persistence has a fundamentally different effect from that of uncertainty about dividend realisations in this model. The latter reduces prices, as agents are risk averse. The former, however, increases prices, as agents anticipate with positive probability persistence to be at levels above its expected value, where prices rise much more strongly than with no uncertainty.

2.4.3.1 Sensitivity

In the benchmark parameterisation, we assumed an ad hoc prior of moderately persistent regimes ($f_{hh} = f_{ll} = 0.9$). Also, priors were relatively weak, corresponding to 1 and 5 year(s) of data in the low and high volatility regime respectively. In this section, we look at stronger priors and priors equal to the true transition probabilities.

Panel 1 and 2 of figure 7 show how a stronger ad hoc prior reduces the initial jump in the price-dividend ratio at the beginning of the Great Moderation, which is exactly the Jensen's inequality effect described in the previous section. Also, with a stronger prior, the marginal contribution of observed transitions to the posterior is reduced, which lowers the increase in prices during the course of the Great Moderation. The convexity effect is further illustrated by comparing panels 3,4 and 5 of figure 7, which show results for a correct prior about transition probabilities and strengthening information (corresponding to 1 and 5 years, 5 and 10 years, and 50 and 100 years of prior observations on low and high volatility regimes respectively). The results show how, with a true prior, the initial jump in prices is higher than with the less persistent ad hoc prior. More importantly, the jump is much stronger than under full information when the prior is weak, but close to the full information case when it is strong.

The assumption of high risk-aversion was made to target price-dividend ratios that are close to those observed in the period before the Great Moderation. The relative volatility of log-dividend growth $\lambda = 4.5$ is equal to the benchmark value in Lettau et al (2008), whose choice is based on estimates of the relative volatility in post-war US data. Table 9 shows how the results for Bayesian learning change with lower risk aversion of $\gamma = 20$ and dividend volatility of $\lambda = 2.5$. Lower risk aversion reduces the impact of the fall in volatility during the Great Moderation on full information prices by about a third. Similar reductions in the effect can be observed for the Bayesian learning schemes. With lower dividend volatility, the reduction is even stronger, to about half the benchmark value.

2.5 Asset Prices under Alternative Learning Schemes

2.5.1 "Good policy or good luck": Learning when low-volatility is suspected to be permanent

In this section, we propose an alternative learning scheme that tries to explicitly capture the uncertainty about whether the Great Moderation was permanent in nature or not. For this, we assume, as before, that the agent observes the current variances of shocks σ_t^2 . Also, and contrary to the previous section, we assume she knows the transition probabilities

between high and low volatility regimes during normal times. However, whenever the agent observes a move to low-volatility, she attaches a small prior probability \hat{p} to this change being permanent. She then updates this prior probability according to the likelihood of the observed sequence of regimes in normal times relative to that after a permanent change to low volatility. More specifically, the likelihood of observing a sequence of Σ_l^N of N low-variance periods when transitions probabilities are given by \mathbf{F} is simply

$$L(\Sigma_l^N | \sigma_t^2 = \sigma_l^2, \mathbf{F}) = P(\sigma_{t+1}^2 = \sigma_l, \sigma_{t+2}^2 = \sigma_l, \dots, \sigma_{t+N}^2 = \sigma_l | \sigma_t^2 = \sigma_l^2, \mathbb{F}) = F_{ll}^N$$

where $P(A|B, F)$ denotes the probability of event A conditional on event B and transition matrix F . The posterior probability of a permanent shift having occurred in period t , denoted $P(F = \mathbf{1} | \Sigma_l^N)$, for $\mathbf{1}$ the identity matrix, is thus

$$\begin{aligned} P(F = \mathbf{1} | \Sigma_l^N) &= \frac{P(F = \mathbf{1} \wedge \Sigma_l^N)}{P(F = \mathbf{1} \wedge \Sigma_l^N) + P(F = \mathbf{F} \wedge \Sigma_l^N)} \\ &= \frac{\hat{p}}{\hat{p} + F_{ll}^N(1 - \hat{p})} \end{aligned} \quad (2.13)$$

In our analysis we focus on a scenario similar to the experience of the US economy after World War II, which we interpret as a long realisation of high volatility followed by the Great Moderation and a move back to higher volatility with the recent crisis. Accordingly, we assume that a move to permanently low volatility can only happen once. It is thus immediate that equity prices in any high volatility period after the start of the Great Moderation are equal to the full information price p_h . Similar to the full information case, the vector of price dividend ratios under Bayesian learning about a 'permanent vs. transitory' Great Moderation, denoted p_t^{PT} is then described by equations similar to (2.9), (2.7). With $\lambda = 1$, this yields

$$p_{it}^{PT} = \beta e^{(-\frac{\alpha}{\psi} + a)\bar{g}} \left(\mathbf{P}_{ii,t} e^{\frac{(-\frac{\alpha}{\psi} + \alpha)^2}{2} \sigma_i^2} (1 + p_{i,t+1}^{PT})^a + \mathbf{P}_{ij,t} e^{\frac{(-\frac{\alpha}{\psi} + \alpha)^2}{2} \sigma_j^2} (1 + p_{j,t+1}^{PT})^a \right)^{1/a}$$

where again $i, j \in \{h, l\}$ and $\mathbf{P}_{ij,t}$ is the probability of moving from regime i to regime j given the period t posterior probability of the change to low volatility being permanent in equation (2.13). Note, however, that price-dividend ratios under this learning schemes are not simply fixed points to equation (2.5). Rather, the representative agent anticipates that, should low volatility persist next period, the probability of a permanent change increases, and so does the price-dividend ratio. We thus have to compute the whole path of price-dividend ratios jointly.¹³

¹³This is simplified by the fact that price-dividend ratios are easily calculated for permanent regimes with $\mathbb{F} = \mathbf{1}$. Also, it is simple to calculate the path of posterior probabilities $P(F = \mathbf{1} | \sigma_l^N)$ as N rises, and thus the

Asset prices - Learning about transitory vs. Permanent GM			
	Boom	Overvaluation	Bust
No Learning	2%	0	1%
Learning	37%	41%	43%

Table 2.7: "Boom" denotes the increase in prices until the end of the Great Moderation relative to the high-volatility regime preceding it (computed over windows of 5 years). "Overvaluation" is the overvaluation at the end of the Great Moderation relative to the prices without learning. And "Bust" is the fall in prices in the first period after the Great Moderation.

To implement this model quantitatively, we choose the same moderately persistent prior about transition probabilities in normal times as in the previous section ($F_{ll} = F_{hh} = 0.9$). We set the conditional probability of the Great Moderation being permanent conditional on an observed change from high to low volatility to 2 percent, which yields an unconditional probability 0.2%. Figure 8 shows the time path that results from this learning scheme, compared to US data. The less persistent prior probabilities lead to a PD ratio that is higher until the early 1980s, but also a smaller initial rise when the agent observes a move to low volatility in 1985. As the economy persists in low volatility, however, the rising posterior probability of a permanent moderation in macro-volatility leads to an S-shaped increase in prices. The magnitude of the boom is with 37% similar to our benchmark learning scheme. The observed end of the Great Moderation comes with a strong bust in asset prices of 49%, as agents update the probability of being in a permanently more benign macroeconomic environment to zero.

2.5.2 Ad hoc learning

It has been argued, by Haldane (2009) example, that overconfidence in a low volatility environment may arise when agents base their inferences about the future predominantly on recent observations of small shocks. This over-reliance on the recent past is opposed to the optimal nature of learning schemes so far, but in line with a large number of studies in which agents follow ad hoc learning rules that map observations into estimates of parameters

transition probabilities $\mathbf{P}_{ij,t}$ increases. Once $P(F = \mathbf{1}|\sigma_t^N)$ is close enough to 1, say after \bar{N} low volatility periods, we know that $p_{l,t+\bar{N}}^{PT}$ equals the price-dividend ratio under permanently low volatility $p(F = \mathbf{1})$, which we can easily calculate as a fixed point to (??) for $F = \mathbf{1}$. This allows us to calculate the sequence of price-dividend ratios at for periods $s = \bar{N} - 1, \bar{N} - 2, \dots, t$ by backward induction.

of interest (see for example Evans and Honkapohja (1999)). To see whether non-optimal learning rules can deliver a boom and bust in asset prices similar to those observed in US data, we assume that the representative agent knows the mean dividend growth \bar{g} and observes the history of shocks $\Omega_t = \{\varepsilon_s\}_{s=0}^t$. But in her estimate about future macro-volatility, she ignores, or chooses to ignore, the two-stage nature of the data generating Markov process. Rather she uses simple ad hoc rules that map observed histories into estimates $\widehat{\sigma}_{t+1}^2$ of the variance of future shocks σ^2

$$\widehat{\sigma}_{t+1}^2 = G(\Omega_t)$$

where $G : R^t \rightarrow R^+$. Specifically, we consider 3 simple mappings G

$$G^{OLS} = \frac{1}{N} \sum_{s=0}^t (\varepsilon_s)^2 \quad (2.14)$$

$$G^{CG} = \xi(\varepsilon_t)^2 + (1 - \xi)G_{t-1}^{CG} = \sum_{s=0}^t \xi(1 - \xi)^{t-s} \varepsilon_s^2, \quad 0 < \xi < 1 \quad (2.15)$$

$$G^{CW} = \frac{1}{n} \sum_{s=t-n}^t (\varepsilon_s)^2 \quad (2.16)$$

Thus, under G^{OLS} agents simply compute their best guess of the future variance as an average over the entire history of shocks. G^{CG} describes a simple "constant-gain" learning rule: the agent computes the variance as a weighted average of his best guess in the previous period and the squared shock today. Relative to G^{OLS} , this overweighs more recent observations, as the weight on more distant observations decays geometrically at rate $1 - \xi$. Finally, G^{CW} uses windows of n most recent observations to compute the variance.

Figure 9 presents the time path of asset prices under the three ad-hoc learning rules, together with full-information prices in the case of high persistence. With OLS learning, the fall in the variance estimate for consumption growth is relatively slow. Moreover, since each estimate weighs all past periods equally, the variance estimate remains an average across high and low volatility periods, resulting in a relatively small rise in prices. The price nevertheless rises above that under full information, which is, again, due to a Jensen's inequality effect. With constant gain learning, the contribution of past periods to the variance estimate falls geometrically over time. This implies that the estimate of consumption variability during the great moderation falls more quickly, and further, than with OLS learning. The boom in prices is thus steeper and stronger, amounting to around 70% at the end of the Great Moderation, far above that implied by full information. When agents compute their estimate of the consumption growth variance as an average across a window of constant length, asset

Asset prices - Ad hoc learning			
	Boom	Overvaluation	Bust
Full Info	17%	0	17%
OLS	22%	3%	0%
Constant Gain	72%	46%	2%
Constant Window	83%	54%	1%

Table 2.8: "Boom" denotes the relative increase in prices until the end of the Great Moderation relative to the high-volatility regime preceding it (computed over windows of 5 years). "Overvaluation" is the overvaluation at the end of the Great Moderation relative to the prices under full information. And "Bust" is the fall in prices in the first period after the Great Moderation.

prices reach an even higher plateau as under constant gain learning, although their path is slightly more convex, as at the beginning of a new regime estimates adjust more slowly. Under all three ad-hoc learning rules, the fall in prices at the end of the Great Moderation is relatively slow: only as information about a change in volatility accumulates, adjust agents their estimates. Contrary to their Bayesian counterparts, recursive, ad hoc learning rules are thus not able to deliver sudden crashes in prices.¹⁴ Table 8 summarises the results for the benchmark case.

2.5.3 Sensitivity

2.5.3.1 Alternative prior probabilities of a permanent Great Moderation

When learning about the permanent vs. transitory character of the Great Moderation, in the benchmark version of the model agents believed the observed move to low volatility to be permanent with a conditional prior probability of $P(F_{ll} = 1) = 2\%$. Here we look at how the results change when reducing this prior probability to 1 and 0.1 percent. As suggested by intuition, the rise in price-dividend ratios during the Great Moderation is slower when agents attach a lower prior probability to it being permanent. As figure 10 shows, with $P(F_{ll} = 1) = 1\%$, the difference is small. But when $P(F_{ll} = 1) = 0.1\%$, the Great Moderation comes to an end before the posterior converges to 1, such that the rise in prices is "cut off", and both the overall boom and the fall in asset prices are smaller.

¹⁴Since the true persistence of regimes has no effect under ad hoc learning, we omit the results for low persistence.

Asset prices - Lower Risk Aversion and Dividend Volatility			
	Boom	Overvaluation	Bust
Full Info (benchmark)	17%	0	17%
Full Info ($\gamma = 20$)	14%	0	14%
Full Info ($\lambda = 2.5$)	8%	0	8%
Trans Probabilities (benchmark)	46%	24%	39%
Trans Probabilities ($\gamma = 20$)	40%	17%	28%
Trans Probabilities ($\lambda = 2.5$)	24%	11%	18%
Perm vs Trans (benchmark)	48%	47%	50%
Perm vs Trans ($\gamma = 20$)	33%	32%	34%
Perm vs Trans ($\lambda = 2.5$)	25%	26%	27%

Table 2.9: "Boom" denotes the relative increase in prices until the end of the Great Moderation relative to the high-volatility regime preceding it (computed over windows of 5 years). "Overvaluation" is the overvaluation at the end of the Great Moderation relative to the prices under full information. And "Bust" is the fall in prices in the first period after the Great Moderation.

2.5.4 Different assumptions for ad hoc learning rules

Figure 11 shows the time path of price-dividend ratios under our ad hoc learning rules with a smaller gain parameter $\xi = 0.01$ and a longer window for the estimation of volatilities of $w^n = 30$ years. As anticipated, the boom is now smaller, as individuals update their volatility estimates less quickly as information about a new low-volatility environment accumulates. The size of the boom in the case of constant gain is reduced from 70 to around 35 percent and with constant windows estimation from 83 to 47 percent.

2.5.5 Lower Risk Aversion and Dividend Volatility

Lower risk aversion has a similar effect on asset price dynamic as under the benchmark learning scheme. For example, with learning about a permanent vs. transitory Great Moderation, the predicted boom falls from around 50 percent in the benchmark case to 33 percent with lower risk aversion, and to 25 percent with lower dividend volatility. The changes with ad-hoc learning are similar, and are therefore omitted.

2.5.6 Alternative Timing of the Great Moderation

All results so far were based on a start of the Great Moderation in 1984, in line with the estimates for GDP. A later start date, for example in the early 1990s, as suggested by the fall in consumption volatility, has very little impact on the sizes of boom and bust in cases where learning is relatively quick, and price-dividend ratios therefore converge to a constant value during the early or middle years of the Great Moderation. When learning is slow, however, for example in the ad-hoc cases with low gain or long windows, or when the prior probability of a permanent move to low-volatility is low, both the size of the boom and the crash are smaller, as the increase in price-dividend ratios in the figures becomes "interrupted" by the end of the Great Moderation.

2.6 Conclusion

From a review of both academic and investment research we conclude that, first, the "Great Moderation" in macro-volatility was perceived as an important factor behind the asset price boom of the 1990s and 2000s, and, second, that academics and investors alike were uncertain about the origins and persistence of the new low-volatility environment. Using different learning mechanisms, we modelled this uncertainty explicitly in a general equilibrium asset pricing model with time-varying volatility. The results confirmed the intuition of policymakers (Bean 2009, Haldane 2009) that overconfidence in a benign macroeconomic environment may have led to an overvaluation of assets beyond their fundamental value. Particularly, we find that Bayesian learning can lead to an asset price boom of around 50 percent at the end of the Great Moderation, as agents had become increasingly confident in its persistence. The end of the low-volatility period, which we identified with the beginning of the recent crisis, in turn leads to a strong crash in prices. Moreover, both boom and crash are an order of magnitude larger than in a model with full information about the data generating process. More ad hoc, or statistical, learning rules predict an even stronger boom in prices, but cannot replicate the fast crash at the end of the Great Moderation period. Future research could extend this study in several directions, by, for example, including time variation in the mean growth of the economy, or by looking at an alternative scenario where agents form expectations about future prices directly, rather than the distribution of dividends as in the model analysed here. Adam and Marcet (2010) show how this can lead to self-fulfilling bubbles and crashes in asset prices, as a rise in prices is sustained by generating expectations of rises in the future. When learning about volatility, this self-referential mechanism is less clear, as higher expected volatility primarily feeds into the level of prices,

and not into their second moment. An in-depth analysis of this issue should be conducted in future work.

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2.7 Appendix

2.7.1 Data Appendix

Consumption is quantified as the *Total Real Personal Consumption Expenditures* measured in quantity index [index numbers, 2005 = 100]. The data are quarterly, seasonally adjusted and their source is the National Economic Accounts of the Bureau of Economic Analysis (BEA).

GDP is quantified as the *Real Gross Domestic Product*, measured in 2005-chained dollars. The data are quarterly, seasonally adjusted and their source is the National Economic Accounts of the Bureau of Economic Analysis (BEA).

Population is quantified as the *Midperiod Population* of each quarter. The data source is the National Economic Accounts of the Bureau of Economic Analysis (BEA).

Asset Price is quantified as the average *S&P 500 Stock Price Index* of each quarter. The data source is the Robert Shiller's homepage. The original data are monthly averages of daily closing prices.

Dividend is quantified as the original quarterly *Dividend Payment* reported in the Robert Shiller's homepage.

Price-Earning Ratio is quantified as the *Cyclically Adjusted Price Earnings Ratio* (P/E10), known also as the *CAPE*. The data source is the Robert Shiller's homepage. The price-earning-ratio series used by us contains only the original quarterly earning data.

2.7.2 Appendix B

2.7.2.1 Numerical solving for the price when agents use Bayesian leaning

The Bayesian agent enters each period with a prior. He observes the realization of the exogenous process and he updates the counters

$$\begin{aligned} n_{t+1}^{ij} &= n_t^{ij} + 1 \text{ if } s_{t+1} = j \text{ and } s_t = i \\ n_{t+1}^{ij} &= n_t^{ij} \quad \text{if } \textit{otherwise}. \end{aligned}$$

The posterior density function is

$$f(F_{hh}, F_{ll} | \Sigma^t) = \textit{beta}(n_t^{hh}, n_t^{hl}) * \textit{beta}(n_t^{ll}, n_t^{lh}). \quad (2.17)$$

We would like to calculate

$$p_t = \int p(S_t, F) f(F | \Sigma^t) dF$$

which can be also expressed as

$$\int p(S_t, F) f(F | \Sigma^t) dF = E_{\Sigma^t}[p(F)]. \quad (2.18)$$

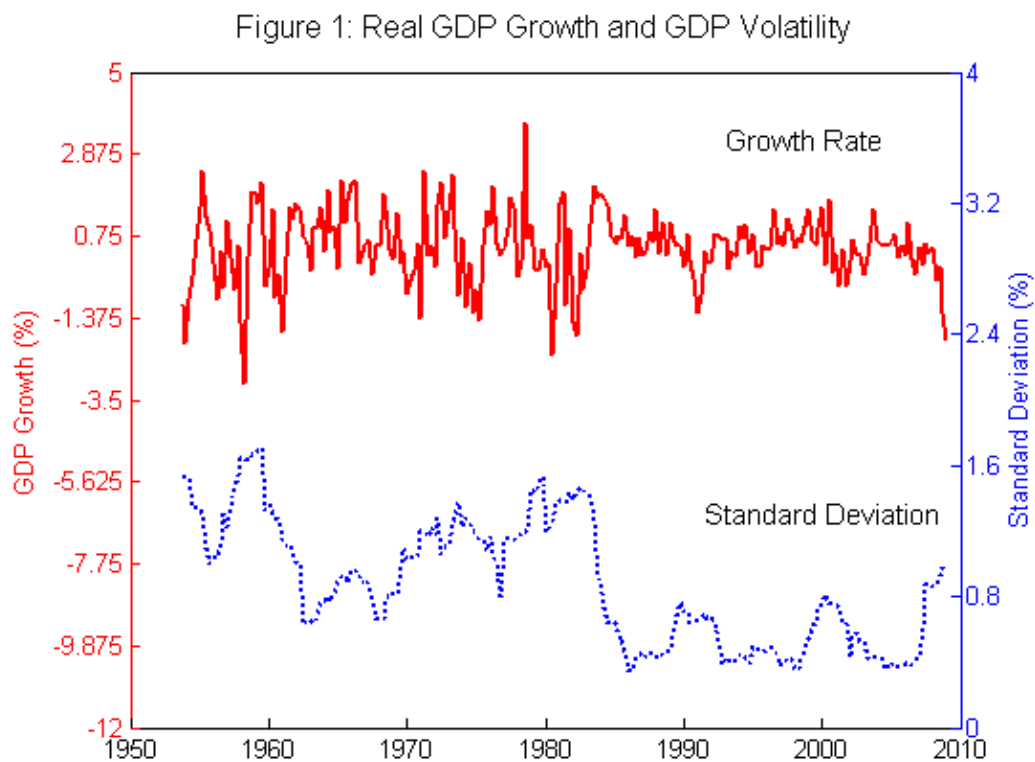
Therefore equation (2) can be approximated as

$$E_{\Sigma^t}[p(F)] \approx \frac{\sum_{i=1}^n p(S_t, F_i)}{n} \quad (2.19)$$

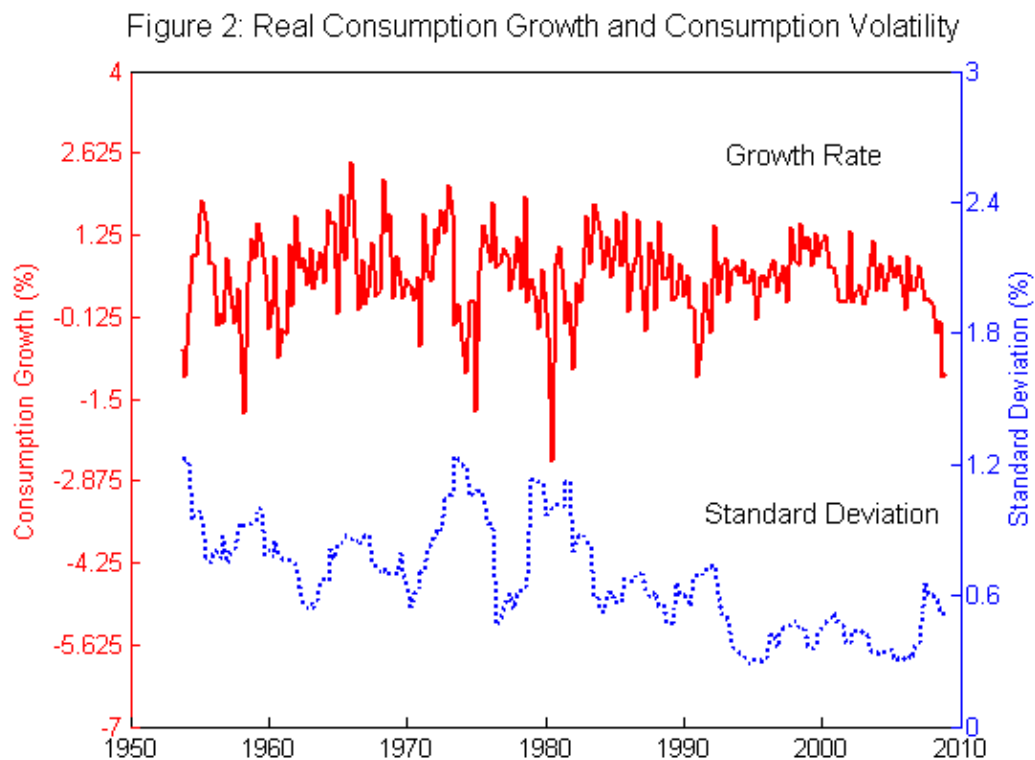
In order to compute equation (3) at each time t we generate a sample of n transition probability matrixes, F , as random observations from equation (1). Therefore in each period the price function can be numerically approximated by the sample average, so

$$p_t \approx \frac{\sum_{i=1}^n p(S_t, F_i)}{n}$$

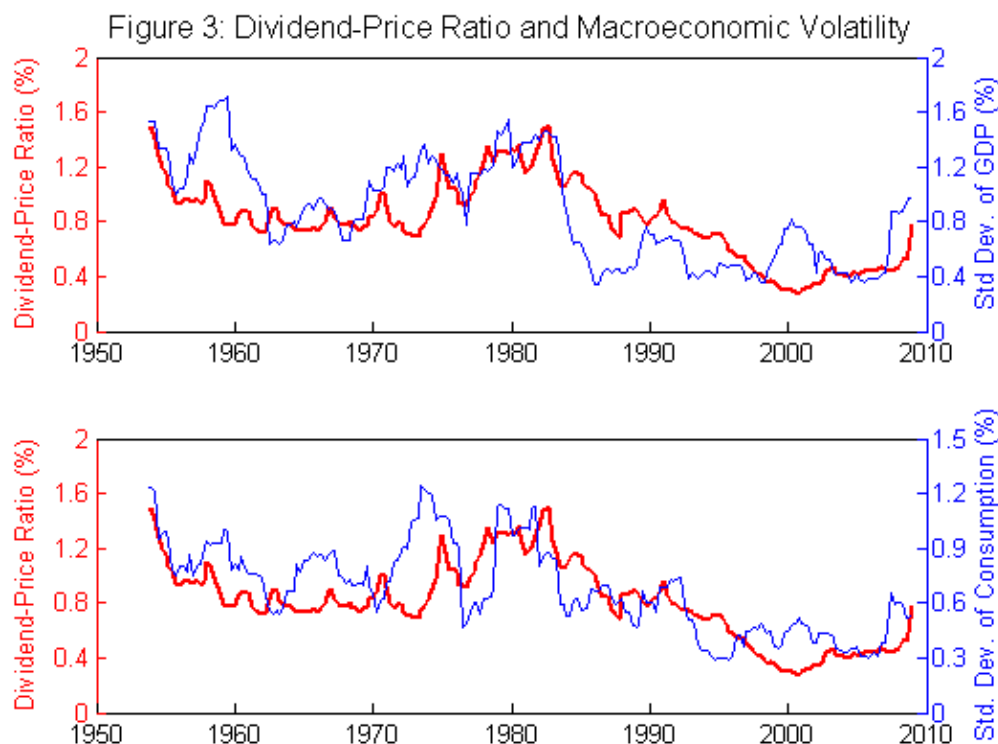
2.8 Figures



Note: The figure plots the growth rate of the real GDP and its standard deviation estimated in 10-quarter rolling windows. Output is defined in per-capita terms, calculated as ratio of the real gross domestic product, measured in 2005-chained dollars, over total population. The data are quarterly and span the period 1952Q2 - 2010Q2. The data source is the BEA. The estimates are in percent.

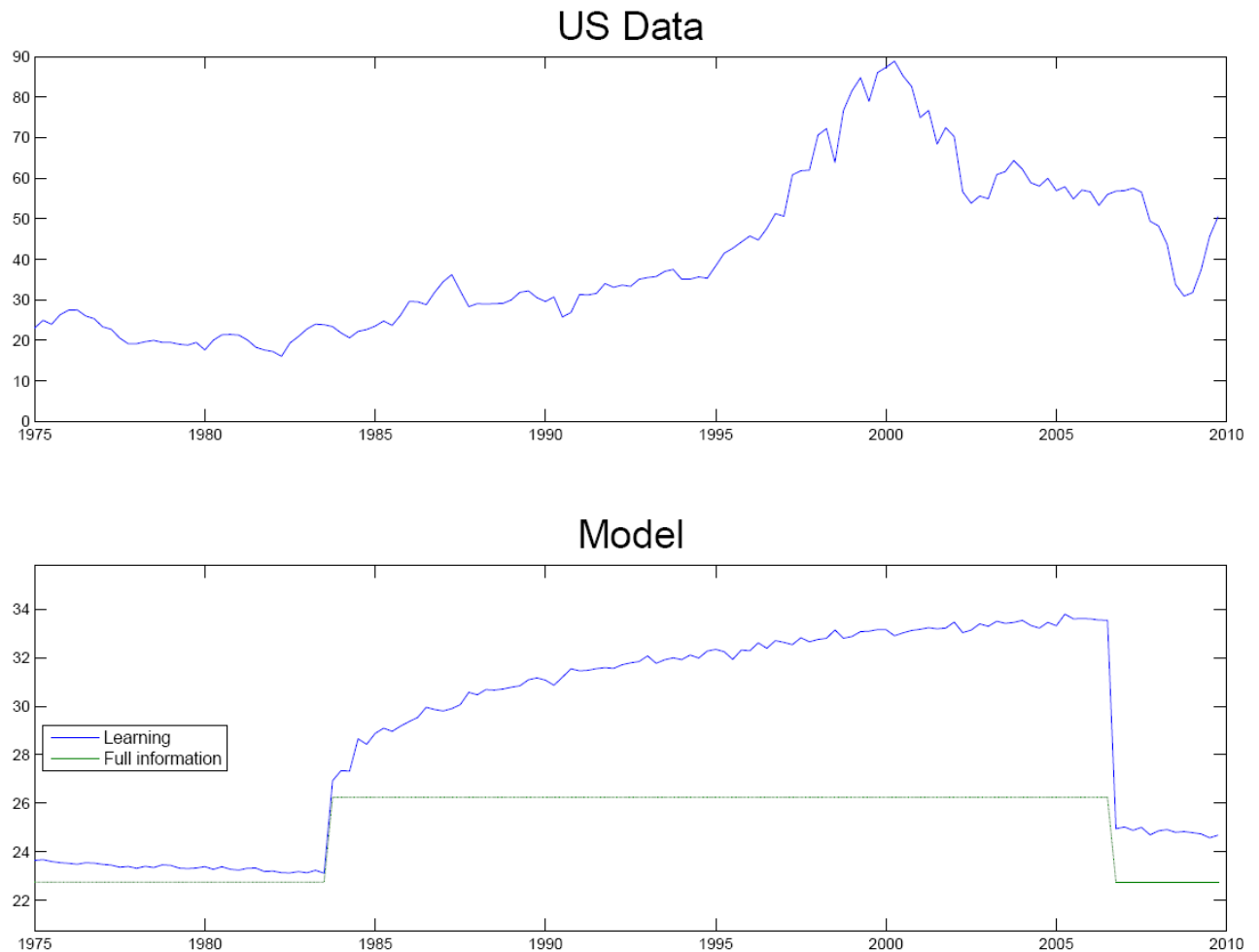


Note: The figure plots the growth rate of the real consumption and its standard deviation estimated in 10-quarter rolling windows. Consumption is defined in per-capita terms, calculated as ratio of the total real personal consumption expenditures, measured in quantity index [index numbers, 2005 = 100], over total population. The data are quarterly and span the period 1952Q2 - 2010Q2. The data source is the BEA. The estimates are in percent.



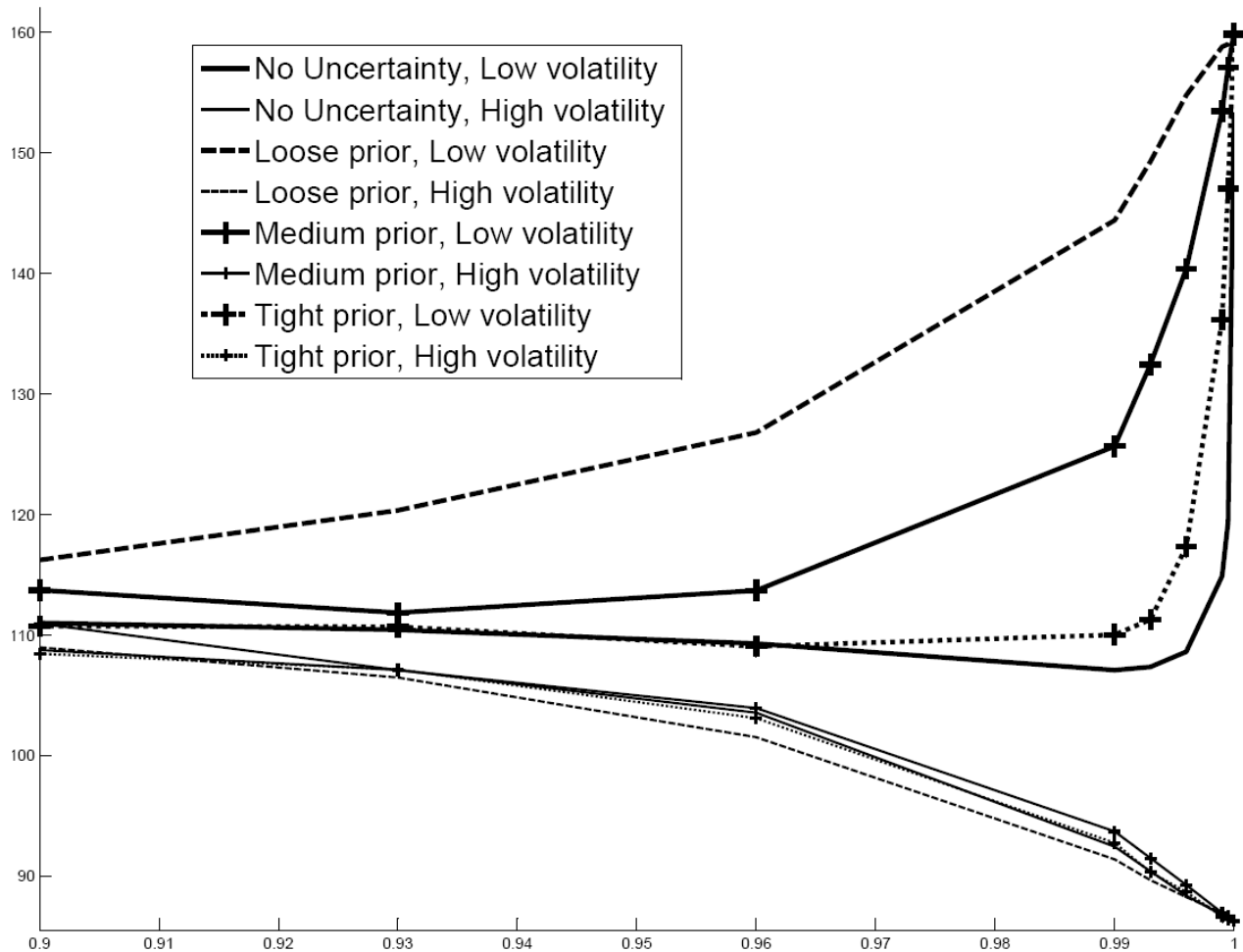
Note: The figure plots the dividend price ratio against the standard deviation of real GDP growth rate (first subplot) and the standard deviation of the real consumption growth rate (second subplot), estimated in 10-quarter rolling windows. Output is defined in per-capita terms, calculated as ratio of the real gross domestic product, measured in 2005-chained dollars, over total population. Consumption is defined in per-capita terms, calculated as ratio of the total real personal consumption expenditures, measured in quantity index [index numbers, 2005 =100], over total population. The financial data are taken from the Robert Shiller's homepage and the rest of the data are taken from the BEA. The estimates are in percent.

Figure 4: Dividend Ratios: Benchmark Model



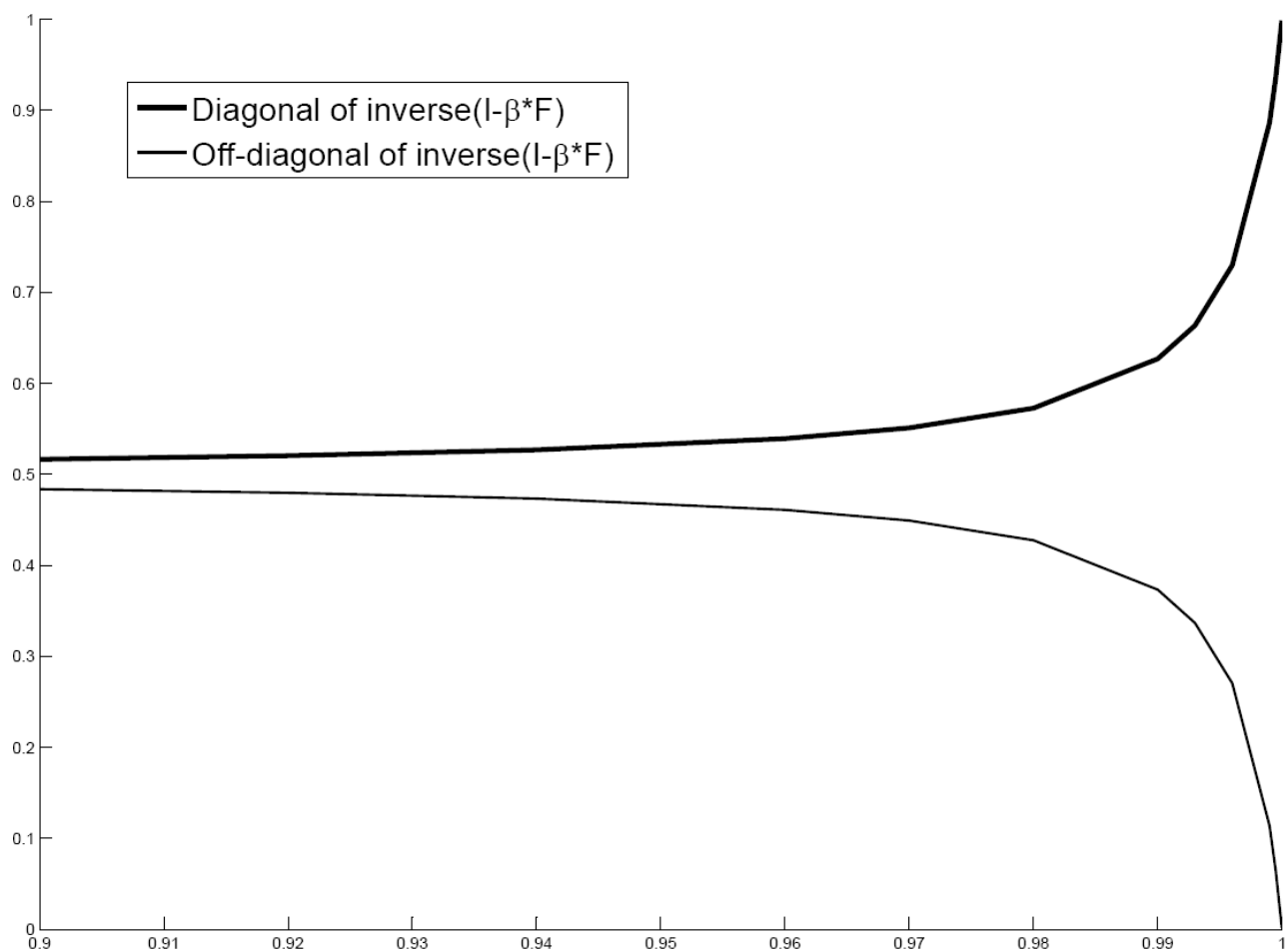
Note: The figure plots the price dividend ratio in US data (upper Panel), and under learning about transition probabilities (lower panel), for the benchmark calibration of the model.

Figure 5: Dividend Ratios as a Function of Persistence and Prior Tightness



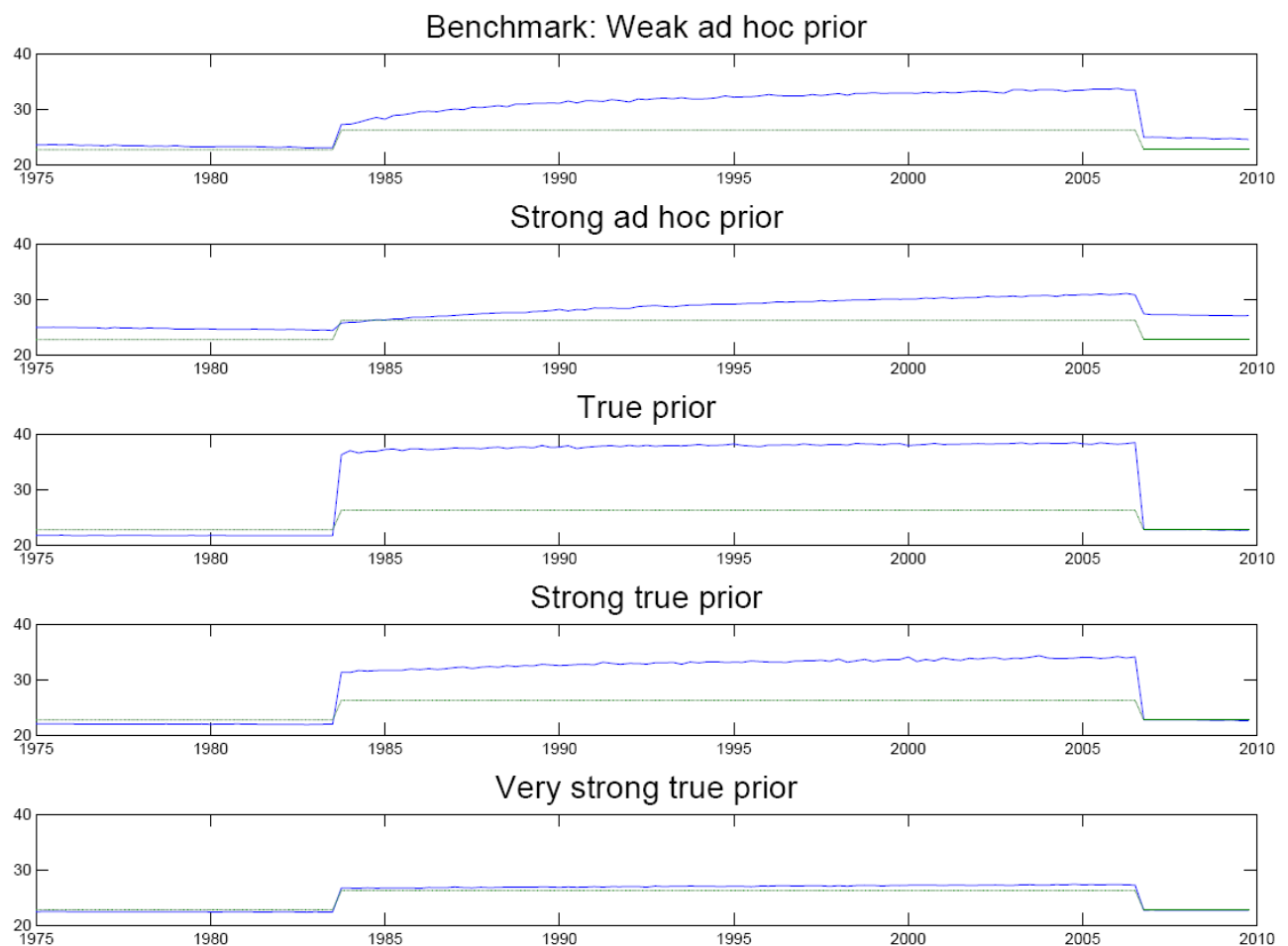
Note: Using the simplified case of symmetric transitions ($f_{jj} = f_{hh}$), the figure plots the price dividend ratio as a function of persistence for different values of the tightness of priors for the benchmark calibration of the model.

Figure 6: Non-Linear Asset Price-Persistence Relation



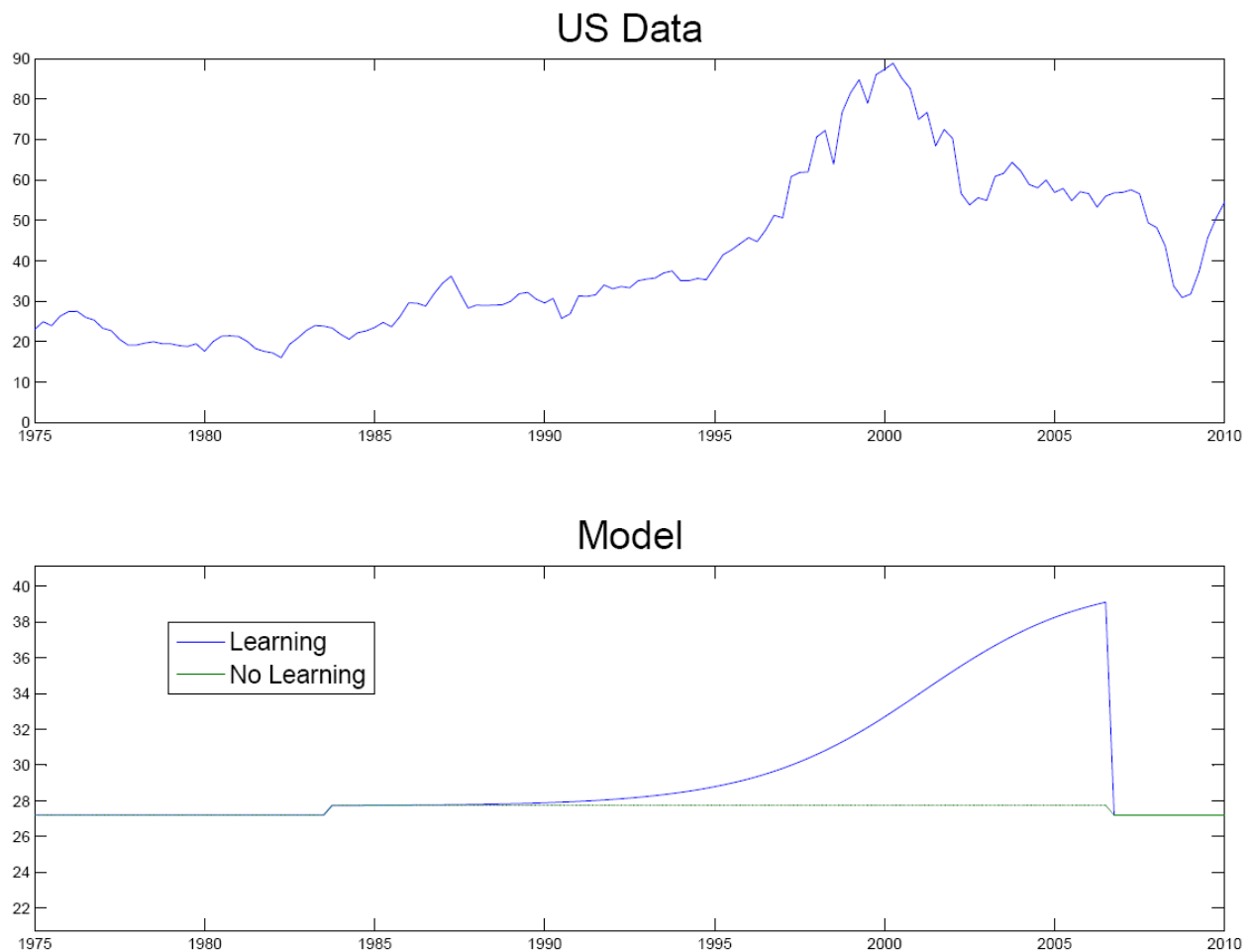
Note: The figure depicts the diagonal and non-diagonal elements of the present discounted value matrix $\text{inv}(I - \beta * P)$.

Figure 7: Learning about Transition Probabilities with Different Priors,
 $F = \mathbf{F}^{hp}$



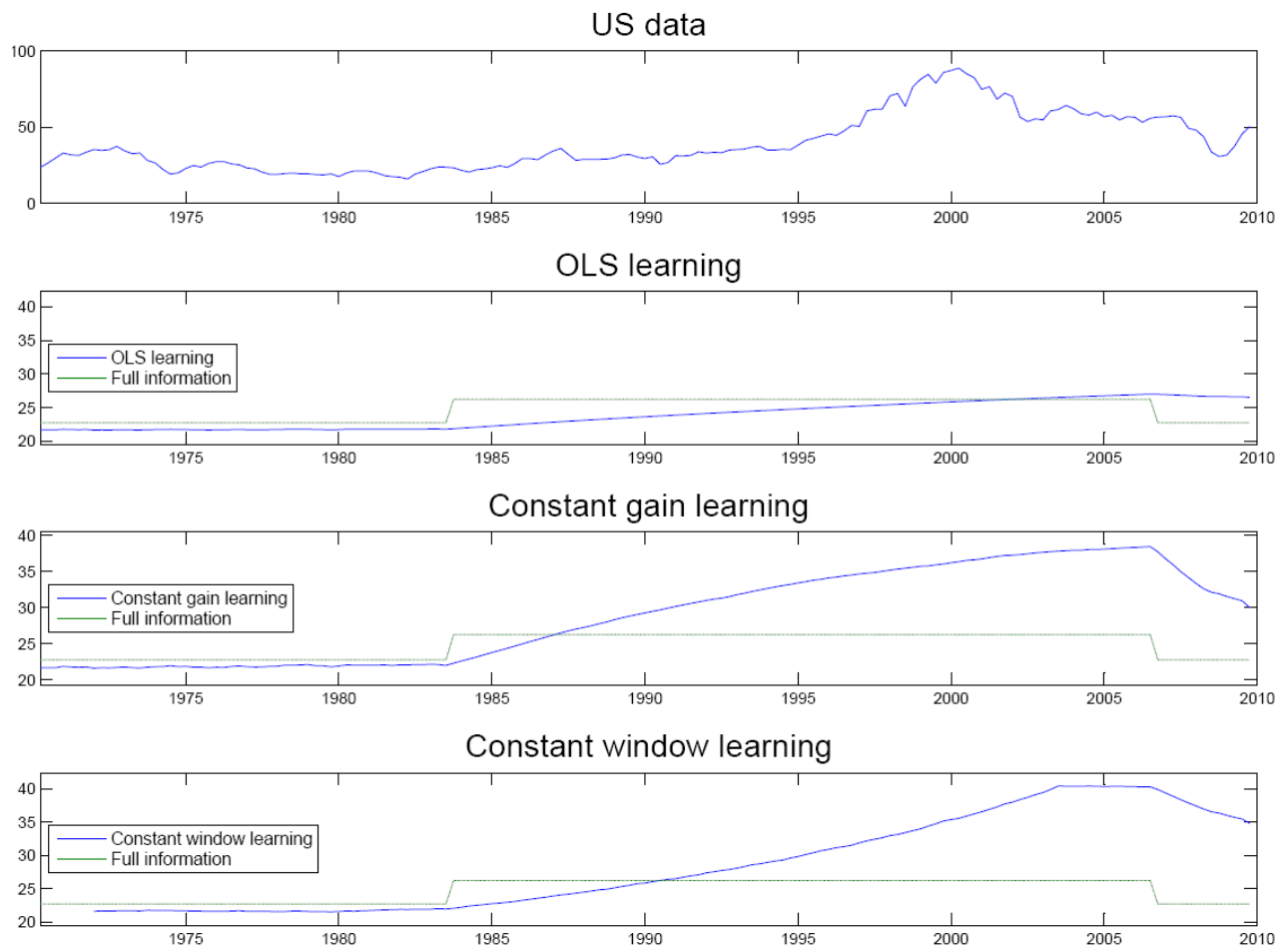
Note: The figure shows the time-path of dividends for the benchmark model with weak ad hoc priors (panel 1), for stronger priors (corresponding to 5 and 10 years of prior observations, panel 2), and for a correct prior about $F = \mathbf{F}^{hp}$ with strengthening information (panel 3 to 5, corresponding to 1 and 5 years, 5 and 10 years, and 5 and 10 years of prior observations respectively).

Figure 8: Dividend Ratios – Learning about a Permanent vs. Transitory Great Moderation



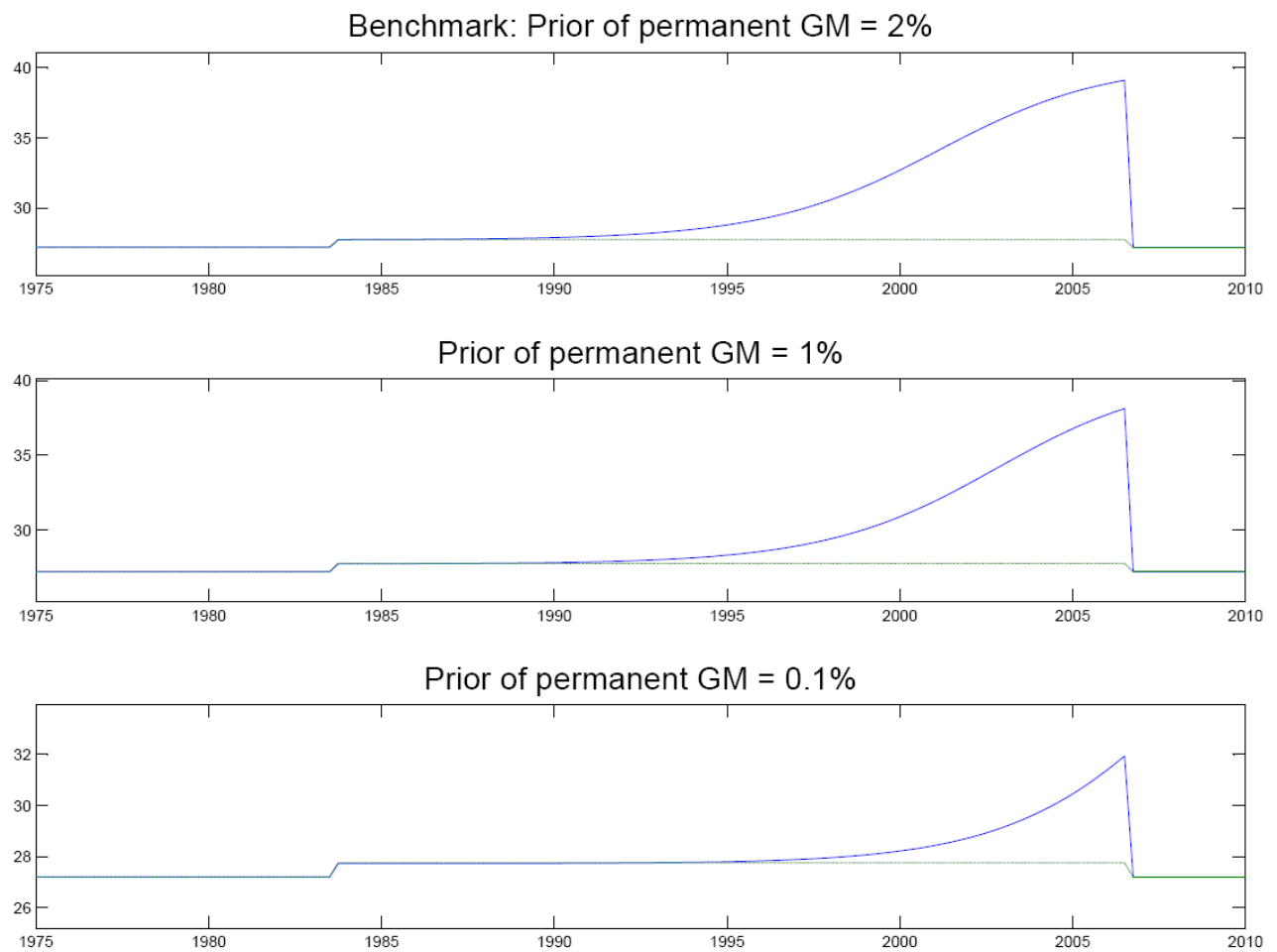
Note: The figure plots the price dividend ratio in US data (upper Panel), and under learning about a permanent vs. transitory Great Moderation (lower panel), for the benchmark calibration of the model.

Figure 9: Dividend Ratios - Ad Hoc Learning



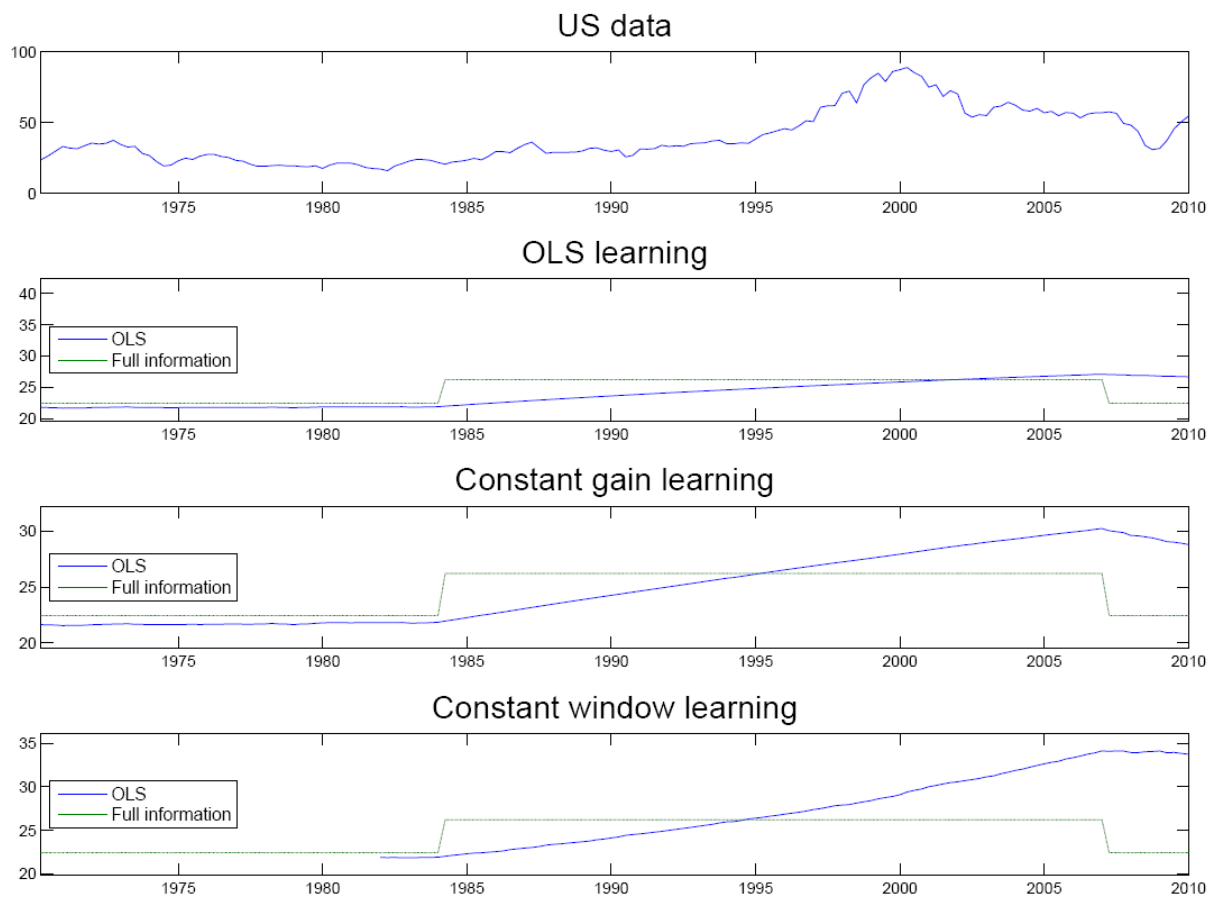
Note: The figure plots the price dividend ratio in US data (upper Panel), and under three ad hoc learning rules: OLS (second panel), constant gain (third panel), and constant window (bottom panel), for the benchmark calibration of the model. The full information prices correspond to the case of high persistence.

Figure 10: Learning about permanent vs. Transitory GM with Different Priors



Note: The figure shows the time-path of dividends with learning about a permanent vs. Transitory Great Moderation with different prior probabilities.

Figure 11: Ad-hoc learning with lower gain and longer windows



Note: The figure shows the time-path of dividends in the data, and in the model with ad-hoc learning for $\xi = 0.01$ and $w^n = 30$ years.

CHAPTER 3

HOUSING AND THE MACROECONOMY: THE IMPACT OF MORTGAGE MARKET INNOVATIONS

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ABSTRACT

We employ the model of Jeske and Krueger (2007) to study the impact of mortgage market innovations on the macroeconomy and their distributional effects. The results show that the elimination of the foreclosure costs have a big impact on the total amount of mortgages and no influence in the aggregate amount of real estate in the economy. On the other hand, the cancelation of the mortgage administration fees does not have any quantitative effect on the equilibrium.

JEL classification: E.

Keywords: Financial Innovation, Housing Market, Mortgage Market.

¹Contact: afroditi.kero@eui.eu. Kero would like to thank for helpful comments and suggestions Georg Duernecker.

3.1 Introduction

Changes in financial regulations that took place in the early 1980s facilitated mortgage lending and the use of durables as collateral. The Monetary Control Act of 1980 and the Garn St Germain Act of 1982 allowed marked innovation that dramatically reduced equity requirements. The development of sub-prime mortgages, the greater access to mortgage refinancing and to home equity loans reduced the effective down payments. The literature has documented these changes and has studied their effect on different fields of the economy (for reference see Chamber et al. (2007), Campbell and Hercowitz (2006) and Scoccianti (2008)). The objective of this work is to explore the macroeconomic and distributional effects of the financial innovations in the U.S mortgage market. In order to do so, we employ a general equilibrium model with real estate and a mortgage market, developed by Jeske and Krueger (2007). The model is in the spirit of Aiyagari (1994), with incomplete markets and heterogeneous agents. In addition to the traditional features, in this model the households can borrow using their real estate property as collateral and besides this they can also default in their debt obligations. In each period households face idiosyncratic endowment and depreciation shocks to their real estate property and they make decisions with respect to consumption (goods and housing services) and investment (bonds, risky real estate and mortgage). In the model, we express mortgage market innovations as a decrease in foreclosure costs and in the mortgage administration fees. The relaxing of these parameters should decrease the mortgage price and bust the demand for mortgages. The results of this simple thought experiment show that the aggregate amount of mortgages is 6 times bigger when there are no foreclosure costs compared to the situation when the bank loses 22% of the value of the collateral in case of default. On the contrary, the impact of this change in the aggregate amount of real estate is negligible. The cancellation of the mortgage administration fees has no quantitative effect neither on the aggregate variables, nor on the invariant distributions. Our guess is that the reason than we don't observe any change is because the calibrated value of this parameter in the baseline economy is very close to zero, thus when we set it to zero, it does not alternate the equilibrium results. The rest of the paper is organized follows: in the second section, we describe the model economy and define the equilibrium. The third section discusses the parametrization, while the fourth section examines the quantitative importance of the mortgage market innovations. Finally the fifth section summarizes and concludes.

3.2 The model

The model is a heterogeneous agent model with incomplete markets, similar to the Aiyagari (1994). In addition to the standard models of this type, this model has a real estate and a mortgage sector, households can borrow using their real estate property as collateral and in addition they can default in their debt obligations.

3.2.1 Household's problem

In this economy there is a continuum of measure one of infinitely lived households. Each period they face idiosyncratic endowment shocks $y \in Y$. The income process is modeled as a five state Markov chain with transition probabilities $\pi(y'/y)$: Except for the endowment shocks, the households face also idiosyncratic depreciation shocks on their housing property $\delta \in D$. The depreciation shocks are iid shocks and $p(\delta')$ denotes their mass probability function. All the households have the same utility function and they derive utility from consuming consumption goods c and housing services h . Thus their one period utility function is $U(c, h)$.

The households have access to two kinds of assets; one period bonds b and real estate g' . The price of a bond is P_b and a unit of real estate costs P_h . Except of deriving housing services from the real estate property, households can also use it as a collateral and issue mortgage debt. The cost of each unit of mortgage is $P_m(m', g')$ and is a function of the mortgage size m' and the size of the collateral g' .

Households have the right to default on their debt obligations at the cost of losing their housing collateral. They make use of this right whenever

$$m' > P_h(1 - \delta')g'$$

The dynamic programming problem of the household is the following

$$v(s) = \max U(c, h) + \beta \sum_{y'} \pi(y'/y) \sum_{\delta'} p(\delta') v(s') \quad (1)$$

subject to the budget constraint:

$$\begin{aligned} c + b'P_b + hP_l + g'P_h - m'P_m(g', m') &= a + g'P \\ a'(\delta', y', m', g', b') &= b' + \max \{0, P_h(1 - \delta')g' - m'\} + y' \end{aligned}$$

where a denote the cash on hand, which is endowment plus assets brought into the current period from the previous one.

3.2.2 The Real Estate Construction Sector

The representative firm in real estate construction sector optimally chooses the output of houses (I) and the input of the consumption good (C_h) so as to maximize:

$$\begin{aligned} & \max_{I, C_h} P_h I - C_h \\ \text{s.t. } & I = A_h C_h \end{aligned} \quad (2)$$

where A_h is a technological constant that measures the amount of consumption good required to build one house. The model assumes that the real estate construction technology is linear and reversible. Thus in the equilibrium

$$P_h = \frac{1}{A_h}.$$

The banking sector is perfectly competitive and it takes prices as given. Thus in the equilibrium they make zero profits. The price of each mortgage issued P_m ; is a function of the size of the loan m' and of the collateral g' that backs the mortgage. The households have the option to default in their debt obligations. In case of default the bank can recover a fraction $\gamma \leq 1$ of the value of the real estate. Because of perfect competition in the market, banks break-even on each single mortgage they finance.

What determines the optimal default behavior of the household is the level of depreciation. Given a mortgage m' and a collateral g' the cut-off level of depreciation $\delta^*(m', g')$ above which the household defaults on his debt obligation is defined as

$$m' = (1 - \delta^*(m', g'))P_h g'$$

where $\delta^*(m', g') \in [\delta_{\min}, \delta_{\max}]$. More specifically the $\delta^*(m', g')$ is determined as

$$\delta^*(m', g') = \begin{cases} \delta_{\min} & \text{if } 1 - \frac{m'}{g'P_h} < \delta_{\min} \\ 1 - \frac{m'}{g'P_h} & \text{if } 1 - \frac{m'}{g'P_h} \in [\delta_{\min}, \delta_{\max}] \\ \delta_{\max} & \text{if } 1 - \frac{m'}{g'P_h} > \delta_{\max} \end{cases}$$

The risk free interest rate in this economy is denoted by r_b ; and thus the refinancing cost is $P_b = \frac{1}{1+r_b}$. In order to issues a mortgage, the bank has to pay some administrative costs. r_w stands for the percentage real resource cost, per unit of mortgage financed by the bank. Thus the effective net cost of the bank for issuing a one dollar mortgage is $r_b + r_w$. Therefore for all the type of mortgages (m', g') where m' and $g' > 0$, the bank chooses the optimal mortgage price $P_m(m', g')$ in order to maximize its profits. Thus the problem of the bank is,

$$\max[-m'P_m(g', m') + \frac{1}{1+r_b+r_w}\{m'P(\delta' \leq \delta^*(m', g')) + \gamma P_h g' \sum_{\delta' > \delta^*(m', g')} (1-\delta) * f(\delta)\}]. \quad (3)$$

3.2.3 Equilibrium

Definition 3.1 *A Stationary Recursive Competitive Equilibrium is characterized by value functions and the policy functions c ; h ; b_0 ; m_0 and g_0 for the households, the policies for the real estate construction sector I and C_h , an invariant distribution and a set of prices P_l , P_h , P_b , and a mortgage pricing function P_m , $\hat{P}_m : R^+ \times R^+ \rightarrow R$, such that*

1. *Given prices P_l, P_h, P_b, P_m , the value function v , solves the household's problem (1), and c, h', b', m', g' are the associated policy functions.*
2. *Given P_h , the optimal policies I, C_h solve the real estate construction company maximization problem (2).*
3. *Given P_h and P_b , the function P_m solves the maximization problem of the banks (3).*
4. *The Rental Market clears*

$$\sum_{b.g.m.y.\delta} g'(b.g.m.y.\delta)\mu(b.g.m.y.\delta) = \sum_{b.g.m.y.\delta} h'(b.g.m.y.\delta)\mu(b.g.m.y.\delta)$$

5. *The Bond Market clears*

$$\sum_{b.g.m.y.\delta} b'(b.g.m.y.\delta)\mu(b.g.m.y.\delta) = \sum_{b.g.m.y.\delta} m'(b.g.m.y.\delta)\mu(b.g.m.y.\delta)$$

6. *(Invariant Distribution, μ) The distribution μ is invariant with respect to the Markov process induced by the exogenous Markov process π and the policy functions b', m' and g' .*

3.3 Calibration

Table 1 presents the calibration for the baseline economy. Given that we build our exercise on Jeske and Krueger (2007) we calibrate most of the parameters as they do. More specifically we calibrate all the preference and mortgage market parameters as in Jeske and Krueger (2007), except of the foreclosure technology which we take it from Pennington and Cross (2004). The technology parameters are taken from Tauchen (1986).

Table 1: Parameter Values for the Baseline Model		
Preferences Parameters		
σ	2.000	Risk Aversion
β	0.8870	Time Discount Factor
θ	0.8590	Share Parameter on Housing
Mortgage Parameters		
r_w	0.002	Mortgage Administration Fee
γ	0.78	Foreclosure Technology
Technology Parameters		
A_h	1	Housing Technology
Π	see below	Transition Matrix for Income
y	see below	Income States

We model that the endowment process as a continuous state AR(1) process

$$\begin{aligned}\log y' &= \rho \log y + \varepsilon \\ E(\varepsilon) &= 0 \text{ and } E(\varepsilon^2) = \sigma_\varepsilon^2\end{aligned}$$

where the persistence $\rho = 0.9$ and $\sigma_\varepsilon = 0.2$. The estimates for the one-period autocorrelation of ρ and the unconditional standard deviation σ_ε vary in the literature. However, most of the estimations show that the estimate for ρ is close to but less than one and the estimate for σ_ε is between 0.2 and 0.4. We then discretize the income process with the approximation procedure developed by Tauchen (1986). The result of the approximation is a five state Markov chain where the labor productivity shock

$$y \in \{0.2525, 0.5025, 1.000, 1.9902, 3.9610\}$$

and the transition matrix

$$\Pi = \begin{bmatrix} 0.8491 & 0.1509 & 0.0000 & 0.0000 & 0.0000 \\ 0.0195 & 0.8962 & 0.0843 & 0.0000 & 0.0000 \\ 0.0000 & 0.0427 & 0.9146 & 0.0427 & 0.0000 \\ 0.0000 & 0.0000 & 0.0843 & 0.8962 & 0.0195 \\ 0.0000 & 0.0000 & 0.0000 & 0.1509 & 0.8491 \end{bmatrix}$$

The depreciation process: We model the depreciation process similar to Jeske and Krueger (2007) and we assume that $\log(1 - \delta) \sim N(-\mu_\delta, \sigma_\delta^2)$ with mean $\mu_\delta = 0.0152$ and

$\sigma_\delta^2 = 0.08$. Therefore the depreciation of real estate in the model follows

$$f(\delta) = \frac{1}{\sigma_\delta(1-\delta)\sqrt{2\pi}} \exp\left\{-\frac{(\ln(1-\delta) + \mu_\delta)^2}{2\sigma_\delta^2}\right\}$$

In order the solution of the problem to be more convenient we discretize the above distribution. We approximate the distribution of the depreciation shock $f(\delta)$ with $\{\delta_i\}_{i=1}^5$ and $\{p_i\}_{i=1}^5$ where p_i is the probability that δ_i realizes. Explicitly what we get is $\delta \in \{-0.1240, 0.1070, 0.3380, 0.5690, 0.8\}$ and their corresponding probabilities are $P = [0.0489, 0.8347, 0.1111, 0.0009, 0.0004]$.

The period utility function is a function of durables and non-durable goods

$$u(c, h) = \frac{c^{\theta(1-\sigma)} h^{(1-\theta)(1-\sigma)} - 1}{1-\sigma}.$$

3.4 Results

Table 2: Numerical Results

	Low γ	High γ	Difference
γ	0.7800	1.0000	0.282
P_l	0.0500	0.0495	-0.010
r_b	0.0200	0.0300	0.500
G	0.0197	0.0198	0.005
M	0.0165	0.1235	6.484
$\mu(m' > 0)$	1.78%	35%	18.67
$\mu(g' > 0)$	2%	2%	-

In Table 2 we document the quantitative results of our experiment. Thus we show the implications of easing the mortgage lending which is represented by a decrease in foreclosure costs. We compare two steady state economies: one with foreclosure costs ($\gamma = 0.78$) and the other without foreclosure costs ($\gamma = 1$). Thus we account for the effect of a 28.2% decrease in the foreclosure costs. As we can see from the results, the main impact on the economy from this change, is to make mortgages much more attractive by decreasing the effective mortgage price. In consequence, the aggregate amount of mortgages in the economy increases more than 6 times. On the other hand, the overall impact on real estate investment is very modest. We observe an increase of only 0.05%. The effect of easing mortgage issuing is reflected also in the invariant distributions of mortgage and collateral. Hence, we can observe a very large change in the number of households that hold mortgage loans and no change in the percentage of households who holds collateral. In addition because of general equilibrium effects, the

equilibrium interest rate increases, thus the price of the bonds decreases. Conversely, we observe a slight decrease in the equilibrium rental price.

Another parameter that influences the mortgage price is the mortgage administration fee r_w . We re-did the same exercise as with γ and tried to see what were the results when we decreased r_w . However in this case the quantitative results were not so enlighting because actually there was no change neither in the aggregate variables, nor in the invariant distributions. Our guess is that given that the value of the mortgage administration fee r_w is already very small in the baseline economy (only 0.002), we don't observe any change when we set it to zero.

3.5 Conclusions

We use a general equilibrium model developed by Jeske and Krueger (2007) to study the quantitative implications of the mortgage market innovations on the macroeconomy and their distributional effects. The model is a typical model with incomplete markets and heterogeneous agents but in addition it has a mortgage and a real estate market. We express the mortgage market innovations as a decrease in foreclosure costs and in the mortgage administration fees. The results of this quantitative experiment show that the elimination of the foreclosure costs has a big impact on the total amount of mortgages and no influence on the aggregate amount of real estate in the economy. On the other hand, the cancelation of the mortgage administration fees does not have any quantitative effect on the equilibrium.

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