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Local Interactions as a Decentralized Mechanism Coordinating Equilibrium Expectations

by

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LOCAL INTERACTIONS AS A DECENTRALIZED MECHANISM COORDINATING EQUILIBRIUM EXPECTATIONS

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ABSTRACT. In the context of standard two-period pure-exchange economies with sequential trade, this paper proposes a decentralized coordination mechanism for equilibrium-expectations, facilitated by local interactions between agents. Interactions are modelled stochastically by specifying a family of individual Markov processes on a two-dimensional integer lattice $\mathbb{Z}^2$ in continuous time. These processes are interdependent, in that the transition rate of each agent's expectation also depends on expectations of neighboring agents. The particular specification of transition rates chosen in the present paper is known as the (two-dimensional) Voter Model. The composite process has two extremal invariant measures and a continuum of non-extremal invariant measures. The economic content of the stochastic expectations process is twofold. First, the convergence of the expectations process itself constitutes a "sunspot-device". While convergence to either one of the extremal invariant measures corresponds to a sunspot-free coordination state, convergence to a convex mixture of invariant measures engenders a sunspot equilibrium. Thus, non-ergodicity of the expectations process is related to the occurrence of sunspot equilibria. Second, it explains how coordination of expectations is actually achieved through direct interactions between agents. Any particular coordination state (defined as a limiting measure of the process) can be traced back to a set of initial configurations or more general initial distributions of expectations.

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1. Introduction

It is well-known that in two-period pure-exchange economies sunspot equilibria can arise if agents come to expect that the equilibrium obtaining in the second period depends on the occurrence of some event which is otherwise irrelevant to the economy and if the occurrence of this event is not insurable. (See Cass and Shell (1983) for the seminal paper on sunspot equilibria and Cass (1989) for the particular model underlying the present paper.) Yet issues such as the nature and the origin of sunspot events and the modelling of mechanisms by which agents coordinate expectations on a sunspot event have received relatively little attention so far, adding to the open problem of equilibrium selection and expectation coordination in GET (see e.g. Allen et al. (2002)).

In this context, the present paper suggests a decentralized mechanism coordinating equilibrium expectations, modelled as a continuous-time random process facilitated by local interactions between agents. The process consists of a family of individual processes, indexed by the two-dimensional integer lattice $\mathbb{Z}^2$. The index set $\mathbb{Z}^2$ corresponds to the set of agents in the economy. Each individual process has a common state space consisting of two distinct equilibrium expectations. The processes are locally interdependent in that the transition rate of the individual expectation held by an agent also depends on the configuration of individual expectations of his nearest neighbors (with respect to $\mathbb{Z}^2$). The particular specification of transition rates is known as the (two-dimensional) Voter Model.

The basic properties of the above process are the following. It is non-ergodic with two extremal invariant measures. It can converge to either one of its extremal invariant measures, in which mass is concentrated on a "full-consensus" configuration (a configuration in which all agents hold the same expectation), or to one element of the continuum-set of non-extremal invariant measures (being convex mixtures of invariant measures), each of which has both full-consensus configurations as its support.

The economic content of the stochastic expectations process is twofold. First, the convergence of the expectations process itself constitutes a "sunspot-device". While convergence to either one of the extremal invariant measures corresponds to a sunspot-free coordination state, convergence to a convex mixture of invariant measures engenders a sunspot equilibrium (in both cases with an underlying economy as the one analysed by Cass (1989), for concreteness). Thus, non-ergodicity of the expectations process is related to the occurrence of sunspot equilibria. Second, it explains how coordination of expectations is actually achieved through direct interactions between agents. Any particular coordination state (defined as a limiting measure of the process) can be traced back to a set of initial configurations or more general initial distributions of expectations.

The principle behavioral assumption in the proposed coordination-mechanism is that an agent who must form an expectation about the occurrence of a future event but lacks relevant information to do so tends to align himself with the expectations held by other agents in his "reference group". The reader is referred to empirical results in social psychology which provide evidence that there is a tendency to socially-driven alignment of opinion in humans. Pioneering results in that field of research are those of Asch (1951, 1956) and Festinger (1954). The present paper does not attempt to provide an *explanatory* model for such reference-group influence in expectation formation. (An explanatory model for social effects in consumption was provided by Bernheim (1994).)

Also, it should be stressed that a stochastic modelling of expectations does not imply that expectations are necessarily to be thought of as random in a behavioral sense. One might well propose deterministic interactive behavioral mechanisms on the micro-level, possibly more explanatory in character. However, when applied in modelling large systems, such deterministic models would most likely produce a degree of complexity which is solvable neither analytically nor numerically for long-enough periods of time. Therefore, the stochastic modelling approach to large economies should be considered as a statistical one[1] - a descriptive shortcut providing the possibility of analyzing analytically the aggregate behavior of a large economy with direct interactions between agents.

The organisation of the paper is as follows. Section 2 outlines the basic specifications and properties of the proposed stochastic process of expectations based on well-known mathematical results concerning the Voter Model. Section 3 provides two examples of how the process can be incorporated into concrete GET-models. In Section 3.1, we present the main example, in which "endogenous-sunspot equilibria" are introduced based on Cass's (1989) leading example of sunspot-equilibria in a two-period economy with nominal assets. In Section 3.2, an example of equilibrium-selection facilitated by expectation-coordination in decoupled spot-economies ("trivial sunspot-equilibria" in the terminology of Mas-Colell (1992)) is briefly outlined. Finally, Section 4 contains a few additional comments on the results presented.

2. A stochastic model of interactions-driven dynamics of expectations

This section introduces a family of locally interacting stochastic processes, representing in the present paper the time-evolution of individual equilibrium expectations. Such a family can be viewed as a single process on an appropriate state space. Assume that at any given

[1] Such a modelling approach corresponds to a concept called *Statistical Economics*. The terminus has been suggested by J.M. Grandmont (1992) alluding to Statistical Physics. Seminal work in this direction was done by Hildenbrand (1971) and Föllmer (1974).

time $t \in \mathbb{R}_+$, an agent can expect one of two[2] equilibria to obtain in the following trading period. These individual expectation states are denoted by e_1 and e_2 and the individual state space of the process by $S = \{e_1, e_2\}$.[3] The countably infinite set of agents is denoted by A, and by $X := S^A$ the set of expectation configurations of agents in A. (The space X is compact in the product topology.) Let $\eta_{a,t} \in S$ denote the expectation of agent a at time $t \in \mathbb{R}_+$ and $\eta_t \in X$ a configuration of expectations at time t.

To specify a topology of local interactions, the set of agents A will be endowed with a time-invariant graph structure (implying that each agent a always interacts with the same subset $N(a) \subset A$ of other agents). In the present paper we identify A with the two-dimensional integer lattice $\mathbb{Z}^2$ and set $N(a) = \{b \in \mathbb{Z}^2 : |b - a| = 1\}$ with $|\cdot|$ denoting the Euclidean distance in $\mathbb{R}^2$.

The particular specification of transition rates chosen in this paper is known as the Voter Model.[4] The evolution mechanism is given by the assumption that $\eta_{a,t} \in S$ changes to the other type of expectation at a rate

$$c(a, \eta_t) = \frac{1}{4} \sum_{b \in \mathbb{Z}^2 : |b-a|=1} 1_{\{\eta_{a,t} \neq \eta_{b,t}\}} \tag{1}$$

The transition rate of an agent's equilibrium expectation is thus proportional to the number of neighboring agents holding a different equilibrium expectation. Clearly, if the expectations of agents were independent, the individual transition rate $c(a, \cdot)$ would depend only on $\eta_{a,t}$; in the case specified by the above rate, expectations are interdependent, but direct interaction is restricted to next-neighbors.

It is to be shown that there exists a process with path-space distribution P^{η_0} (with the superscript indicating that the process starts at the initial configuration η_0 in time 0) such that for each $a \in A$ and each initial configuration η_0

$$P^{\eta_0}[\eta_{a,t} \neq \eta_{a,0}] = c(a, \eta_0)t + o(t) \tag{2}$$

for $t \to 0$. Moreover, in continuous time it is natural to require that the process P^{η_0} is such that at most one coordinate changes in a point in time, i.e. for each $a, b \in A$ and each η_0

$$P^{\eta_0}[\eta_{a,t} \neq \eta_{a,0}, \eta_{b,t} \neq \eta_{b,0}] = o(t) \tag{3}$$

[2]The assumption that there are exactly two possible expectations will enable us to directly refer to the existence results of Cass (1989); see Section 3.1.

[3]The equilibrium expectations constituting the set S might be time-dependent, because individuals only gradually learn to what invariant measure - implying a coordination state - the process converges, and thus which equilibria emerge in the economy; see final paragraph in Sect. 3.1 for an elaboration of this point.

[4]The Voter Model was introduced independently by Clifford and Sudburry (1973) and Holley and Liggett (1975). For an extensive discussion, see Chapter V of Liggett (1986) and Part II of Liggett (1999).

for $t \to 0$. The existence and uniqueness of such a process has been shown, among others, by means of the Markov semigroup approach and the Hille-Yosida theorem. The reader is referred to Liggett (1985) for thorough treatment of these issues (see also the Appendix to this paper for a brief outline).

Let $\mathcal{P}(X)$ denote the set of all probability measures on the set X of expectation-configurations, equipped with the topology of weak convergence, i.e.

$$\mu_n \Rightarrow \mu \in \mathcal{P}(X) \Leftrightarrow \int f d\mu_n \to \int f d\mu \quad \forall f \in C(X)$$

with $C(X)$ denoting the space of continuous functions on X. Suppose $\mu \in \mathcal{P}(X)$ as an initial distribution of the process. The probability measure of the process at time t, denoted by $\mu S(t) \in \mathcal{P}(X)$ is implicitly defined via

$$\int f d[\mu S(t)] = \int S(t) f d\mu \quad \forall f \in C(X).$$

An important set of measures are those which are invariant under shift in time.

Definition 1. A measure $\mu \in \mathcal{P}(X)$ is called invariant for the Markov semigroup $(S_t)_{t\in\mathbb{R}_+}$ if $\mu S_t = \mu$ for all $t \geq 0$. The set of all invariant measures will be denoted by $\mathcal{I}$.

Invariant measures represent equilibrium-states for the underlying stochastic process. The next proposition confirms that only these measures can obtain as limiting distributions of the process (see Liggett, 1985, p.10).

Proposition 1. *If $\nu = \lim_{t\to\infty} \mu S_t$ exists for some initial measure $\mu \in \mathcal{P}(X)$, then $\nu \in \mathcal{I}$.*

The situation is simple if the process converges to a single limit distribution from any initial measure (implying that $\mathcal{I}$ is a singleton), a situation which corresponds to what is called *probabilistic ergodicity* of a process. However, multiplicity of invariant measures is at the core of the present paper, since it aims at modelling a situation with multiple possible economic coordination-states.

For the Voter Model, non-ergodicity is obvious. Because transition rate is zero for an agent in agreement with all his next-neighbors, the Voter Model has as invariant measures at least the point-mass measures concentrated on the full-consensus configurations η_{e_i} with $(\eta_{e_i})_a = e_i \ \forall a$, with $i = 1, 2$. These measures, which are extremal, will be denoted by δ_{e_i}. Every convex combination $\delta_\alpha = \alpha\delta_{e_1} + (1-\alpha)\delta_{e_2}$ with $\alpha \in (0,1)$ is also invariant, though non-extremal. For the latter measures, the empirical distribution is random, with probability α for occurrence of a full-consensus on e_1, and $1-\alpha$ for e_2.

The question of whether there is stable coexistence of opinions, i.e. whether there exist extremal invariant measures other than δ_{e_1} and δ_{e_2}, is of both mathematical and, as will become clear in Section 3, of economic interest. For the present model, the answer is negative[5] (see Liggett (1985, Sect. V 1)).

Having characterized the set of invariant measures, the next issue relevant for the economic models of Section 3 is convergence to these measures. Theorem 1.9 with Corollary 1.13 from Liggett (1985, p.231) provide necessary and sufficient conditions for convergence for a wide class of initial measures. An example is the following

Proposition 2. *Let $\mu \in \mathcal{P}(X)$ denote a translation-invariant measure with marginals $\mu\{\eta : \eta(a) = e_1\} = \alpha$. Then*

$$\lim_{n\to\infty} \mu S(t) = \alpha\delta_{e_1} + (1-\alpha)\delta_{e_2}.$$

Remark 1. There are initial conditions for which convergence does not occur.

A question crucial for a rational-expectations economic model is whether an agent can infer the weights α and $1-\alpha$ of the extremal components in the limiting distribution from observing the realizations of expectations in a finite subset of agents. (As will be explained below, these weights are equal to the probabilities of what will be defined as sunspot events in the present model.)

The following property, called *clustering*, holds for the two-dimensional Voter Model (see Liggett (1999), Th. 1.3, p.141)

$$\lim_{t\to\infty} P^{\eta_0}(\eta_t(a) \neq \eta_t(b)) = 0 \tag{4}$$

for all $a, b \in \mathbb{Z}^2$ and all initial conditions η_0. Thus, for any arbitrary large finite volume, after a long enough period of time one observes (almost) all agents having assumed expectations of the same type.

With Eq. 4 in mind, there arise two scenarios for the convergence of the two-dimensional Voter Model to a non-extremal measure. First, in any large but finite volume the process settles randomly on one "extremal invariant measure"[6] after some random time T (with probabilities α and $1-\alpha$ respectively), not returning to the other one thereafter. Second, the process oscillates between the two invariant measures infinitely often, with no such T existing. Cox and Griffeath (1986) and, in a more general setting, Cox and Klenke (2000) have shown that the second scenario actually obtains.

Moreover, the weights α and $1-\alpha$ determine the proportion of time spent "close" to either one of the extremal measures (again, restricted to a large finite volume). Because

[5]The answer is positive for lattice-dimension greater than or equal three (for details, see Liggett (1985, Sect. V 1)).

[6]More precisely, what is meant is the convergence in a finite volume to the projection of an infinite-volume invariant measure to that finite-volume.

of these properties, agents can infer the weights of the extremal components by sampling expectations in a finite subset of agents over a long enough period of time.

3. Application to Sunspot-Phenomena and Equilibrium-Coordination

The present section places the model of decentralized coordination of equilibrium expectations, the mathematical properties of which were outlined in the previous section, in the context of two simple economies, each a two-period ($t = 0, 1$) pure-exchange economy. In Section 3.1, the underlying market structure corresponds to the leading example of the existence of sunspot-equilibria in a finite-horizon economy with unrestricted participation but incomplete asset-structure presented by Cass (1989). In this context, the model introduced in the present paper both explains how coordination of expectations comes about, and makes the actual sunspot-mechanism underlying the coordination endogenous. In Section 3.2, the underlying market structure is a pair of decoupled spot-economies (with no financial assets to transfer wealth) with multiple equilibria in the second period. In that context, the model can explain coordination of expectations between the a-priori given equilibria (but the coordination process has no influence on the economy, unless one assumes a causal link from the expectations of agents to the equilibrium actually obtaining).

Throughout Section 3, the coordination process is assumed to evolve, and its limiting distribution to emerge, prior to trading decisions being made in period $t = 0$. The limiting distribution of the process corresponds to a *coordination-state* (see Section 3.1). The realization of a configuration of expectations from the limiting distribution of the coordination process - called the *coordination-outcome* and corresponding to the sunspot-event - is assumed to become known between $t = 0$ and $t = 1$. In the present model the coordination-outcome can be either one of the full-consensus configurations η_{e_1} and η_{e_2}.

It is important to remark that the time scales of the market exchange process in discrete time (the two-period context can be arbitrarily extended) and the expectation-coordination process in continuous time, are not comparable. It is up to the modeller to specify a relation between them. Therefore, the convergence of the process (the limit $t \to \infty$) need not take "longer" in real time than the time interval between two consecutive trading periods.

3.1. **An example of an endogenous-sunspot equilibrium.** Following Cass (1989), assume a two-period ($t = 0, 1$) pure-exchange economy with a single good available in each period, denoted by y^t, traded on a spot market in period t. In period 0, there is also a financial instrument (bond), denoted by b, with exogenous nominal returns r_1 and r_2, possibly depending on an "endogenous sunspot-event". The price of the good in period t is denoted by p^t, with the price of the bond being normalized to unity in each period. The spot-price of the good in the second period may also depend on the "endogenous sunspot-event". The price vector is denoted by $p = (p^0, p^{1,1}, p^{1,2})$, with the second superscript, referring to the coordination-outcome.

Let us now turn to equilibrium expectations. There are two principle types of limiting distributions of the stochastic process of individual expectations. These types can be interpreted in terms of a coordination state of agents' expectations they imply. First, the process converges to one of its extremal invariant measures, say δ_{e_1}. Then there is no uncertainty as to the coordination-outcome, and agents correctly assign the probability $\pi_1 = 1$ to the occurrence of η_{e_1} and the probability $\pi_2 = 0$ to the occurrence of η_{e_2}. As a result, sunspot-equilibria cannot be induced by the process itself. Second, the limiting distribution of the process is a convex combination $\delta_\alpha = \alpha\delta_{e_1} + (1-\alpha)\delta_{e_2}$ of its extremal invariant measures. When a realization is drawn from the measure δ_α, it is either the full-consensus configuration η_{e_1} (with probability α), or the full-consensus configuration η_{e_2} (with probability $1-\alpha$). Importantly, it follows from the properties of the convergence of the process to a mixed invariant measure, as described in the final part of Section 2, that each agent can learn - by sampling the realizations of the process in any large finite volume - the correct probabilities of the full-consensus outcomes. Thus, each agent will correctly expect an occurrence of η_{e_1} with probability $\pi_1 = \alpha$ and of η_{e_2} with probability $\pi_2 = 1-\alpha$.

With the above motivation, we introduce the following notions.

Definition 2. We define a *coordination-state* as the limiting distribution of the stochastic process on X. We define a *coordination-outcome* as any realization from the limiting distribution of the stochastic process - i.e. any realization from the coordination-state.

Since in each mixed coordination-state of the process, the possible coordination-outcomes are η_{e_1} and η_{e_2}, in which agents are in agreement about the associated equilibrium, these outcomes can play the role of the sunspot-events.

Remark 2. With a more general specification of the coordination process (see comments in Sect. 4), one would be interested only in the empirical distribution of the coordination-state, which is a, in general random, measure on S.

To complete the model, let us now return to the specification of the underlying economy. The present paper will relate to the Cass (1989) example, only with extra emphasis on a countable set of agents. Each agent $a \in A$ is characterized by a differentiable, strictly increasing and strictly concave Neumann-Morgenstern utility function $u_a : \mathbb{R}^2_+ \to \bar{\mathbb{R}}$ with $u_a(0) = -\infty$, and an endowment vector $w_a = (w_a^0, w_a^1, w_a^1) \in \mathbb{R}^3_{++}$. Neither the utility function nor the endowment vector depend on the realization of the expectation coordination process. The individual demand of agent $a \in A$ solves the maximization problem

$$\max_{(y_a^0, b_a, y_a^{1,1}, y_a^{1,2})} \pi_1 u_a(y_a^0, y_a^{1,1}) + \pi_2 u_a(y_a^0, y_a^{1,2}) \tag{5}$$

subject to the constraint that the budget be balanced in both periods and states

$$(i) \quad p^0 y_a^0 + b_a^0 = p^0 w_a^0 \tag{6}$$

$$(ii) \quad p^{1s} y_a^{1s} = p^{1s} w_a^1 + r^s b_a \qquad s = 1, 2. \tag{7}$$

To define aggregate variables for a countably infinite set of agents, the distribution-economy approach due to Hildenbrand (1970, 1974) is taken. Let $f_y(\cdot, \phi, w) : \mathbb{R}^3_{++} \to \mathbb{R}^3_{++}$ denote an individual good-demand function of an agent with preference $\phi \in \mathcal{P}$ and endowment $w \in T$ derived from the above maximization problem. Let $f_b(\cdot, \phi, w) : \mathbb{R}^3_{++} \to \mathbb{R}$ denote the demand of agent a for the asset. The mean good-demand function $\bar{f}_y$ of the economy obtains as

$$\bar{f}_y(p) = \int_{\mathcal{P}\times T} f_y(p, \phi, w) d\nu(\phi, w)$$

where $\nu(\phi, w)$ denotes a probability distribution on $\mathcal{P} \times T$, while, analogously, the mean asset-demand obtains as

$$\bar{f}_b(p) = \int_{\mathcal{P}\times T} f_b(p, \phi, w) d\nu(\phi, w).$$

In order to directly refer to Cass' (1989) results, we assume that ν is concentrated on a common preference ϕ_c and on just two different endowment vectors w_1 and w_2 (with equal weights for (ϕ_c, w_1) and (ϕ_c, w_2)) so that mean good-demand obtains as

$$\bar{f}_y(p) = \int_{\mathcal{P}\times T} f_y(p, \phi, w) d\nu(\phi, w) = \frac{1}{2} f_y(p, \phi_c, w_1) + \frac{1}{2} f_y(p, \phi_c, w_2) \tag{8}$$

while mean asset-demand obtains as

$$\bar{f}_b(p) = \int_{\mathcal{P}\times T} f_b(p, \phi, w) d\nu(\phi, w) = \frac{1}{2} f_b(p, \phi_c, w_1) + \frac{1}{2} f_b(p, \phi_c, w_2). \tag{9}$$

The conditions for equilibrium read, for the good-markets

$$f_y(p^*, \phi_c, w_1) + f_y(p^*, \phi_c, w_2) = w_1 + w_2 \tag{10}$$

and, for the asset market

$$f_b(p^*, \phi_c, w_1) + f_b(p^*, \phi_c, w_2) = 0. \tag{11}$$

Intuitively, the economy is a countably infinite product of identical two-agent economies from the Cass' (1989) example, such that equilibrium conditions reduce to those in the single two-agent economy.

The results derived by Cass (1989) can thus imply the existence of endogenous sunspot-equilibria, in the sense defined above, for any mixed coordination-state of the expectations process.

Finally, a remark needs to be made about the individual state space S. Since the set of equilibria emerges only upon convergence of the process, it seems unreasonable to assume that agents know the final equilibrium-(price)-expectations e_1 and e_2 before convergence of the coordination process has occurred (in fact, the two-state coordination model would collapse in the case when there is convergence to an extremal invariant measure, and if agents anticipated this). For the present model it suffices to assume that some disequilibrium-expectations $e_1(t)$ and $e_2(t)$ of the equilibrium-price (with disequilibrium referring to the distribution of the process at time t) are obtained by the agents in some sampling process, without specifying explicitly the time-path of their values. The stringent assumption to be made is that agents agree on common expectations at each point of time (because of the specific formulation of transition rates in Eq.(1)). Given the latter assumption, the specific time-path of the common equilibrium expectations $e_1(t)$ and $e_2(t)$ has no consequences and only the final - correct - equilibrium expectations matter.

3.2. **Coordination between multiple equilibria and trivial sunspots.** Let us again consider an economy with a countably infinite set of agents. Assume that the spot market of the economy in $t = 1$ allows for multiple, say two, equilibria (while no actions relevant for period 1 can be taken in period $t = 0$). Let the expectation set S, being the individual state-space of the stochastic coordination model, comprise the expectation that either one of the spot-equilibria will occur. Say the process is started at some configuration η_0 or at some initial distribution μ_0 for which convergence to an invariant measure $\delta \in \mathcal{I}$ is known to occur. In a similar argument as was presented in Section 3.1, two cases can be distinguished. If the limiting measure is extremal, then there is no macroscopic uncertainty, and the equilibrium which is expected to occur indeed occurs. If the limiting measure is non-extremal, with probability weights α and $1 - \alpha$ assigned to the full-consensus states δ_{e_1} and δ_{e_2}, the expectations associated with the full-consensus configurations η_{e_1} and η_{e_2} are $\pi_1 = \alpha$ and $\pi_2 = 1 - \alpha$ respectively. This case corresponds to what Mas-Colell (1992) calls a trivial sunspot-equilibrium.

4. Discussion

It was claimed that in the economic model presented in this paper a "sunspot-event" arises *endogenously* - as the realization of a certain stochastic process representing the coordination process of equilibrium expectations. This claim deserves further elaboration, since it can be argued that, after all, the coordination process of individual expectations is decoupled from the agents' decisions, and thus as much exogenous to the economy as the vaguely specified sunspot-event in the traditional sense.

The reply to this critique is that in the above model, possibly extended to a multi-period setting, a linkage could be established between the coordination process of expectations and the macroscopic variables of the economy. Conceptually, such feedback from macroscopic to microscopic variables was recently elaborated by Hahn (2003). Technically, it can be introduced in the present model by making the a-priori distribution of individual expectations, which is the Bernoulli distribution in the above analysis, related to the realization of one or more macroscopic variables, or trends, patterns etc. in their time-path. In this way, a bi-directional linkage between the expectations process and other economic variables can be established.

Another issue deserving comment is the specific role of the two-dimensional Voter Model as the underlying stochastic process. Indeed, the property of having precisely the two full-consensus point-mass configurations in the support of the invariant measures is specific to the Voter Model for lattice-dimension less or equal two. For lattice-dimension three or more, there appear additional extremal invariant measures not concentrated on full-consensus configurations. The same is true if the transition probabilities are slightly changed. For instance, if the transition rate for an agent is slightly positive despite all his neighbors being in agreement with him, the resulting invariant measures (there are multiple such measures for lattice-dimension two or more) no longer have an empirical distribution concentrated on full-consensus outcomes. Then, though there is no macroscopic uncertainty for translation-invariant extremal measures, a fixed percentage of agents deviates from the majority expectation. There are economic implications of such residual heterogeneity, but they call for a separate treatment and discussion (see Hohnisch (2005)).

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Appendix

It is known from the general theory of Markov processes (see e.g. Gihman and Skorohod (1975)) that a Markov process is uniquely determined by a Markov semigroup.

Definition 3. A Markov semigroup of operators on $C(X)$ is a family $(S_t)_{t\in\mathbb{R}_+}$ of linear operators on $C(X)$ satisfying the following conditions

(1) $S_0 = I$ (identity operator)
(2) The mapping $t \to S(t)f$ from $[0,\infty)$ to $C(X)$ is rightcontinuous for every $f \in C(X)$
(3) $S(t_1+t_2) = S(t_1)S(t_2)$
(4) $S(t)1 = 1 \forall t \geq 0$
(5) $S(t)f \geq 0$ whenever $f \in C(X)$ and $f \geq 0$.

In turn, by the Hille-Yosida Theorem there is a one-to-one correspondence between Markov semigroups on $C(X)$ and Markov generators on $C(X)$. The generator G of a Markov process on X with locally interacting variables $\eta_{a,t}$ is the closure in $C(X)$ of the following pregenerator

$$Gf(\eta) = \sum_a c(a,\xi)[f(\eta^a) - f(\eta)] \tag{12}$$

defined on an appropriate set of functions, with η^a denoting a configuration in X obtaining from η by changing the a-th coordinate. The process thus determined has by construction the required local dynamics.