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A Note on Negative Electoral Advertising

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Abstract

In their seminal paper, Harrington and Hess (1996) discuss a model where candidates differ along two dimensions - ideology which is modeled by the standard Hotelling-Downs formulation and valence factors which encompass traits which all voters agree as desirable. While valence factor is given, the voter perception of a candidate's ideology can be influenced via advertising. In this expository note, we extend the model model to take account of valence as well as ideological advertising but we restrict our attention only to negative advertising. We find that when the available resources are sufficiently small and certain technical conditions are fulfilled, the expected result holds, namely, the candidate with the higher initial valence index will run a relatively personal campaign while the candidate with the lower initial valence index will run an ideological campaign.

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1 Introduction

Negative campaign advertising has been an integral part of American political campaigns over the last fifty years. While it has received a lot of attention among journalists, political scientists and in communication studies and there have been both experimental and empirical studies on the topic (see Ansolabehere et al. (1994), Wattenburg and Brians (1999)), there has been relatively meagre formal modelling on this topic.

In their seminal paper, Harrington and Hess (1996) discuss a model of two-candidate electoral competition where candidates differ along two dimensions - ideology and personal attributes or valence factors where the latter encompass traits which all voters agree as desirable. Kinder (1988) identifies them as leadership, integrity, competence and empathy. Ideology is modelled via the Hotelling-Downs spatial model. At the beginning of the race, candidates inherit a certain valence index and a certain voter perception of their ideologies. They can subsequently influence these by allocating a certain amount of resources across positive and negative ideological advertising. Positive ideological advertising relocates a candidate's ideology towards the swing voter or marginal voter (i.e., the voter indifferent between the two candidates) while negative ideological advertising shifts the opponent's ideology away from the marginal voter. They find that the candidate with the higher initial valence factor will run a relatively positive campaign while the candidate with the lower initial valence factor will run a negative campaign.

It is important to note that in their model, there is a complete absence of valence advertising. This is acknowledged by the authors themselves, namely, "an equally important component of negative campaigning is with respect to the opponent's personal attributes. The history of campaigning is replete with denigration of the character of one's opponent. One should not infer from the absence in our model of negative campaigning with respect to personal attributes that we believe such campaigning is unimportant ... an interesting topic of future research would be to extend our model to incorporate both types of campaigning." (Harrington and Hess, 1996).

In this expository note, we extend the Harrington and Hess (1996) model to incorporate both types of campaigning - namely, campaigning on valence as well as policy issues but we restrict our attention only to negative advertising. Ideological advertising moves the opponent's ideology away from the marginal voter (or the voter indifferent between the two candidates) while valence advertising reduces the valence index of the opponent. Our principal finding is that, under certain restrictions, the expected result will hold, namely the candidate with the higher initial valence index will run a relatively personal campaign while the candidate with the lower initial valence index will run an ideological campaign. The goal of the note is fairly modest, namely to elaborate on some mathematical aspects of a seminal model of political advertising. We are agnostic about the relevance or usefulness of this model. Those issues can only be settled by empirical testing. We will subsequently proceed as follows. In section 2, we discuss the assumptions. In section 3, we discuss the results. In section 4, we conclude.

2 Assumptions

We consider a two candidate model of electoral competition. Candidates are perceived as differing in terms of both their ideology and personal traits. A candidates ideology is represented by a certain location on the ideology space represented by the $[0, 1]$ interval. Personal traits are measured by a valence index which lies in $[0, \infty)$. We use the Enlow and Hinich (1982) formulation where the utility of a voter located at y from candidate i, $i = 1, 2$ is specified to be[1]

$$U(y, i) = \alpha_i - V(|y - x_i|). \tag{1}$$

where $V : \mathbb{R}_+ \to \mathbb{R}_+$. Voters are uniformly distributed with density 1 along the $[0, 1]$ interval.

Candidates inherit a certain ideological position x_i^0 and a certain valence index α_i^0. Without loss of generality we assume $\alpha_1^0 > \alpha_2^0$ and $x_1^0 < x_2^0$, namely candidate 1 is to the left of candidate 2 and has a higher valence index as well.

Candidates can influence voter perception using advertising. In this model, we restrict ourselves to negative advertising and assume each candidate is endowed with an equal amount of advertising resources A. The candidate can allocate his advertising resources in two ways: ideological advertising which influence the perception

[1] Or rather a voter whose most favoured ideological position is located at y.

of the opponent's ideology and valence advertising which influences the perception of the opponent's valence index.

Let I_i be the amount of ideological advertising and V_i be the amount of valence advertising of candidate i. Then a candidate's ideological and valence advertising are subject to the following constraints:

$$\begin{aligned} I_i + V_i &\leqslant A; \\ I_i &\geqslant 0; \\ V_i &\geqslant 0. \end{aligned}$$

Then the impact of candidates' campaigns on voter perception of their ideology and valence index is modelled through the post-advertising ideology and valence index as follows;

$$\begin{aligned} x_1 &= x_1^0 - g(I_2); \\ x_2 &= x_2^0 + g(I_1); \\ \alpha_1 &= \alpha_1^0 - h(V_2); \\ \alpha_2 &= \alpha_2^0 - h(V_1); \end{aligned}$$

where $h : \mathbb{R}_+ \to \mathbb{R}_+$ and $g : \mathbb{R}_+ \to \mathbb{R}_+$ model the impact of advertising on valence index and ideology. A marginal voter is a voter who is indifferent between the two candidates. Negative ideological advertising moves the opponent away from the marginal voter. Negative valence advertising simply reduces the valence index of the opponent. We make two key assumptions:

$$\begin{aligned} \textit{Assumption1}: \quad & V' > 0,\ V'' > 0,\ h' > 0,\ V(0) = 0, \\ h'' &< 0,\ g' > 0,\ g'' < 0,\ h(0) = 0,\ g(0) = 0. \\ \textit{Assumption 2}: \quad & V(x_2^0 - x_1^0) - h(A) > \alpha_1^0 - \alpha_2^0 > h(A) > 0. \end{aligned}$$

Assumption 1 implies that the advertising technology is concave and the distance function is convex. There are many examples of advertising technology that fit this specification. One of them is given in Appendix 1. Assumption 2 ensures the existence of a marginal voter (see lemma 1) and hence an interior solution by assuming that the amount available for advertising is *sufficiently small.* If advertising expenditures are large enough that the post-advertising valence profiles are reversed, the results may not hold.

Each candidate is interested in maximizing his or her vote share. Hence, all resources available for advertising will be fully expended. We will solve for the Nash equilibrium (V_1^*, V_2^*), where V_i^* is the amount of valence advertising that maximizes the vote share of candidate i, and $I_i^* = A - V_i^*$.

3 Results

In this section, we will show our main result, namely that the candidate with the higher valence factor conducts a campaign based on valence issues while a candidate with a lower valence factor conducts a largely ideological campaign, namely in the Nash equilibrium (V_1^*, V_2^*), $V_1^* \geqslant V_2^*$ which automatically implies $I_1^* \leqslant I_2^*$. The proofs have technical similarities with Harrington and Hess (1996). If the marginal voter exists, his or her position is given by $z(x_1, x_2, \alpha_1, \alpha_2)$ obtained by solving:

$$\alpha_1 - V(z - x_1) = \alpha_2 - V(x_2 - z). \tag{2}$$

It is intuitively obvious from the standard Hotelling-Downs model that if a marginal voter exists, then all voters $y \in [0, z)$ vote for candidate 1 and all voters $y \in (z, 1]$ vote for candidate 2. In Lemma 1, we show this rigorously as well as prove the existence of the marginal voter.

Lemma 1 *Under assumption 2, for all advertising levels, an unique marginal voter z exists where $z \in (x_1, x_2)$ and voter $y \in [0, z)$ votes for candidate 1 and voter $y \in (z, 1]$ votes for candidate 2.*

Proof. Let z be defined implicitly by $\psi(z) = 0$ where $\psi(y)$ is the difference in utility derived from candidate 1 and candidate 2 for voter y, namely,

$$\psi(y) = [\alpha_1 - V(|y - x_1|)] - [\alpha_2 - V(|y - x_2|)].$$

Let us assume (we will show this later on) that

$$\psi(x_1) > 0 > \psi(x_2).$$

Then by the intermediate value theorem, there exists $z \in (x_1, x_2)$ such that $\psi(z) = 0$. Furthermore, since

$$\psi'(y) = -V'(|y - x_1|) - V'(|y - x_2|) < 0$$

for all $y \in (x_1, x_2)$, z is unique. Also since $\psi(z) = 0$ and for $y \in (x_1, z)$, $\psi'(y) < 0$, therefore $\psi(y) > 0$ which implies $U(y, 1) > U(y, 2)$.

For $y < x_1$,

$$\psi'(y) = V'(|y - x_1|) - V'(|y - x_2|).$$

Since $|y - x_1| < x_1 - y < x_2 - y = |y - x_2|$ and $V'' > 0$,

$$V'(|y - x_1|) < V'(|y - x_2|)$$

which implies $\psi'(y) < 0$. Now, $\psi(x_1) > 0$ and $\psi'(y) < 0$ for $y < x_1$ implies $\psi(y) > 0$ for $y \in [0, x_1)$. Therefore, for $y \in [0, x_1)$, $U(y, 1) > U(y, 2)$. Hence, for $y \in [0, z)$, $U(y, 1) > U(y, 2)$. Analogously, one can show that $U(y, 2) > U(y, 1)$ for all $y \in (z, 1]$.

The final step is to show that $\psi(x_1) > 0 > \psi(x_2)$ is implied by assumption 2. Now, $\psi(x_1) > 0 > \psi(x_2)$ is equivalent to (given V_1, V_2),

$$\alpha_1^0 - \alpha_2^0 > h(V_2) - h(V_1) + V(0) - V(x_2^0 - x_1^0 + g(I_2) + g(I_1)); \quad (3)$$

$$\alpha_1^0 - \alpha_2^0 < h(V_2) - h(V_1) - V(0) + V(x_2^0 - x_1^0 + g(I_2) + g(I_1)). \quad (4)$$

Consider (4). Since $V(0) = 0$, (4) can be written as

$$h(V_2) - h(V_1) + V(x_2^0 - x_1^0 + g(I_2) + g(I_1)) > \alpha_1^0 - \alpha_2^0. \quad (5)$$

The left hand is increasing in V_2, I_1 and I_2 and decreasing in V_1. Hence, (5) holds for all advertising levels if it holds for $(V_1, V_2, I_1, I_2) = (A, 0, 0, 0)$, namely if

$$-h(A) + V(x_2^0 - x_1^0) > \alpha_1^0 - \alpha_2^0$$

which is precisely assumption 2.

Next consider (3). Since $V(0) = 0$, (3) summarizes to

$$\alpha_1^0 - \alpha_2^0 > h(V_2) - h(V_1) - V(x_2^0 - x_1^0 + g(I_2) + g(I_1)).$$

The right hand side is increasing in V_2 and decreasing in V_1, I_1 and I_2. Hence it holds for all advertising levels if it holds for $(V_1, V_2, I_1, I_2) = (0, A, 0, 0)$, namely if,

$$\alpha_1^0 - \alpha_2^0 > h(A) - V(x_2^0 - x_1^0).$$

From assumption 2, $h(A) - V(x_2^0 - x_1^0) < 0$ and $\alpha_1^0 - \alpha_2^0 > 0$. Hence, the condition is automatically satisfied. That completes the proof. ■

Next, we have the following lemma.

Lemma 2 *Assuming that the marginal voter exists and is located at $z = z(x_1, x_2, \alpha_1, \alpha_2)$. Then*

$$0 < \frac{\left(\frac{\partial z}{\partial \alpha_1}\right)}{\left(\frac{\partial z}{\partial x_1}\right)} < -\frac{\left(\frac{\partial z}{\partial \alpha_2}\right)}{\left(\frac{\partial z}{\partial x_2}\right)} \tag{6}$$

Proof. First note that $\alpha_1 > \alpha_2$ since $\alpha_1^0 - \alpha_2^0 > h(A)$ from assumption 2. This is because the lower bound of α_1 is $\alpha_1^0 - h(A)$ and the upper bound of α_2 is α_2^0. Hence from equation (2), $V(z - x_1) > V(x_2 - z)$. This implies $z - x_1 > x_2 - z$ since $V' > 0$. Now, $V'' > 0$ which implies $V'(z - x_1) > V'(x_2 - z)$ which implies

$$\frac{1}{V'(z - x_1)} < \frac{1}{V'(x_2 - z)}. \tag{7}$$

Partially differentiating (2) with respect to α and x_1, we get

$$\begin{aligned} \frac{\partial z}{\partial \alpha_1} &= \frac{1}{V'(z - x_1) + V'(x_2 - z)}; \\ \frac{\partial z}{\partial x_1} &= \frac{V'(z - x_1)}{V'(z - x_1) + V'(x_2 - z)}. \end{aligned}$$

Hence,

$$\frac{\left(\frac{\partial z}{\partial \alpha_1}\right)}{\left(\frac{\partial z}{\partial x_1}\right)} = \frac{1}{V'(z - x_1)}. \tag{8}$$

Similarly,

$$-\frac{\left(\frac{\partial z}{\partial \alpha_2}\right)}{\left(\frac{\partial z}{\partial x_2}\right)} = \frac{1}{V'(x_2 - z)}. \tag{9}$$

From (7), (8) and (9), (6) follows. ■

Before we present the next lemma, let us introduce some notation. For the strategy tuple (V_1, V_2), we get post-advertising locations and valence indices as functions of

V_1, V_2, namely if,

$$\begin{aligned} x_1(V_2) &= x_1^0 - g(A - V_2); \\ x_2(V_1) &= x_2^0 + g(A - V_1); \\ \alpha_1(V_2) &= \alpha_1^0 - h(V_2); \\ \alpha_2(V_1) &= \alpha_2^0 - h(V_1). \end{aligned}$$

then the equilibrium location is given by $z(x_1, x_2)$ can be expressed as an indirect function of V_1 and V_2. We denote this by

$$\begin{aligned} \widehat{z}(V_1, V_2) &= z(x_1(V_2), x_2(V_1), \alpha_1(V_2), \alpha_2(V_1)) \\ &= z(x_1^0 - g(A - V_2), x_2^0 + g(A - V_1), \alpha_1^0 - h(V_2), \alpha_2^0 - h(V_1)). \end{aligned}$$

Now, we have our main result.

Lemma 3 *Assuming that the marginal voter exists, $V_1^* > V_2^*$.*

Proof. For the Nash equilibrium (V_1^*, V_2^*), the equilibrium location is represented by $\widehat{z}(V_1^*, V_2^*)$. Now,

$$\begin{aligned} &\frac{\partial \widehat{z}}{\partial V_1} \\ &= \frac{\partial z}{\partial \alpha_2} \cdot \frac{\partial \alpha_2}{\partial V_1} - \frac{\partial z}{\partial x_2} \cdot \frac{\partial x_2}{\partial (A - V_1)} \\ &= -\left[h'(V_1) \cdot \frac{\partial z}{\partial \alpha_2} + g'(A - V_1) \cdot \frac{\partial z}{\partial x_2} \right]. \end{aligned} \tag{10}$$

Similarly,

$$\begin{aligned} &\frac{\partial \widehat{z}}{\partial V_2} \\ &= -\left[h'(V_2) \cdot \frac{\partial z}{\partial \alpha_1} - g'(A - V_2) \cdot \frac{\partial z}{\partial x_1} \right]. \end{aligned} \tag{11}$$

We can consider three alternative cases:

Case 1: Let $\dfrac{\partial \widehat{z}(V_1^*, V_2^*)}{\partial V_1} > 0$. Then $V_1^* = A$ which implies $V_1^* \geqslant V_2^*$.

Case 2: Let $\dfrac{\partial \widehat{z}(V_1^*, V_2^*)}{\partial V_2} > 0$. Then $V_2^* = 0$ which implies $V_1^* \geqslant V_2^*$.

Case 3: The final case is $\dfrac{\partial\widehat{z}(V_1^*,V_2^*)}{\partial V_1}\leqslant 0$ and $\dfrac{\partial\widehat{z}(V_1^*,V_2^*)}{\partial V_2}\leqslant 0$. Then from (10) and (11),

$$-\frac{\left(\dfrac{\partial z}{\partial\alpha_2}\right)}{\left(\dfrac{\partial z}{\partial x_2}\right)}\leqslant\frac{g'(A-V_1^*)}{h'(V_1^*)}; \tag{12}$$

$$\frac{\left(\dfrac{\partial z}{\partial\alpha_1}\right)}{\left(\dfrac{\partial z}{\partial x_1}\right)}\geqslant\frac{g'(A-V_2^*)}{h'(V_2^*)}. \tag{13}$$

From (6), (12) and (13),

$$\frac{g'(A-V_1^*)}{h'(V_1^*)}>\frac{g'(A-V_2^*)}{h'(V_2^*)}. \tag{14}$$

Now, suppose $V_2^*\geqslant V_1^*$. By concavity of $g(.)$ and $h(.)$,

$$h'(V_2^*)\leqslant h'(V_1^*)$$

and

$$g'(A-V_2^*)\geqslant g'(A-V_1^*)$$

which implies

$$\frac{g'(A-V_1^*)}{h'(V_1^*)}\leqslant\frac{g'(A-V_2^*)}{h'(V_2^*)}$$

which contradicts (14). Hence $V_1^*>V_2^*$. That completes the proof. ■

Hence, we can show than under certain assumptions, candidates in a two-party electoral contest tend to bet on their opponent's weaknesses. Candidates with higher valence factor tend to emphasize that their opponent is lacking on valence issues, while the less likeable candidate tend to potray the other candidate as holding extreme views. One pertinent issue is of course whether the assumptions are too restrictive. Concavity with regard to the impact of advertising expenditure is a fairly reasonable assumption. After a certain amount of negative advertising, voters become less sensitive to advertising. Assumption 2 simply implies that advertising does not too large an effect on the valence factor. Candidates can influence but not radically alter voter perceptions with regard to valence issues. The other assumption here is the convexity of the distance function. But convexity of the distance function simply implies that as candidates are closer to each other, namely, their ideologies are less distinguishable, valence factor becomes more important in determining utility obtained from a certain candidate. This in itself is a quite reasonable assumption.

4 Conclusion

In this note, we extend the model of Harrington and Hess (1996) in a direction suggested by the authors by taking into account valence advertising. We find that the expected result holds under certain conditions - namely, the candidate with the lower valence factor will divert more resources to ideological advertising and the candidate with the higher valence factor will divert more resources to valence advertising. The validity of this result depends on four central assumptions. First, advertising expenditures are small enough so that they do not radically alter the voter perceptions of the relative valence dimensions of the candidates. Second, the distance function is convex. Third, the advertising technology is concave. Fourthly, near-complete homogeneity is assumed in the sense that both candidates are identical in all respects other than their initial positions and initial valence indices.

If the above assumptions are relaxed then these results will no longer hold. In fact, the very reverse result might hold, namely, the candidate with the lower valence factor actually invests more in valence advertising. One may even end up in solutions where both candidates invest all resources in valence advertising. With different explicit functional forms, different solutions will result. There can, in fact, exist multiple Nash equilibria. Some counter-examples are provided in Appendix 2.

Appendix 1:

This example has been modified from Butters (1977). Consider a situation in which candidates have one ad each for valence advertising and ideological advertising but they can send out this ad several times in various media to N voters located equidistant on a $[0, 1]$ space. The probability of a voter getting the ad each time it is sent out is $1/N$. If N is large, we have a reasonable approximation of the Hotelling-Downs model. The probability of a voter getting an ad each time it is sent out is $1/N$ and the cost of sending the ad out each time is θ.

If the consumer receives the ad in question, it shifts the ideological position or changes the valence index of the opponent by σ. However, if the same ad is received more than once, there is no additional impact. This extreme case is meant to capture the fact that the same ad if viewed more than once has a smaller and smaller additional impact on voter perception.

We will show that $h' > 0$ and $h'' < 0$ for valence advertising. Analogously, one can show that the same holds for ideological advertising.

Given that V_i amount of resources are devoted to valence advertising by candidate i, the number of valence ads sent out are equal to $\frac{V_i}{\theta} = \vartheta_i$ (say).

The probability that x of these ads are received by a given consumer is equal to

$$\begin{pmatrix} \vartheta_i \\ x \end{pmatrix} \left(\frac{1}{N}\right)^x \left(1 - \frac{1}{N}\right)^{\vartheta_i - x}.$$

The probability that a consumer receives zero ads is equal to $\left(1 - \frac{1}{N}\right)^{\vartheta_i}$. In our model, N tends to infinity in which case the above expression can be approximated by $\exp\left(-\frac{\vartheta_i}{N}\right)$. Hence, the probability that a voter gets one or more ads is equal to

$$1 - \exp\left(-\frac{\vartheta_i}{N}\right).$$

Hence, expected change in the valence index of the opponent is equal to

$$\begin{aligned} & \sigma\left(1 - \exp\left(-\frac{\vartheta_i}{N}\right)\right); \\ = \; & \sigma\left(1 - \exp\left(-\frac{V_i}{\theta \cdot N}\right)\right); \\ = \; & h(V_i). \end{aligned}$$

Hence,

$$\begin{aligned} h'(V_i) &= \frac{\sigma}{\theta \cdot N} \exp\left(-\frac{V_i}{\theta \cdot N}\right) > 0; \\ h''(V_i) &= -\frac{\sigma}{\theta^2 \cdot N^2} \exp\left(-\frac{V_i}{\theta \cdot N}\right) < 0. \end{aligned}$$

Appendix 2:

Let $V(k) = k$, $h(k) = k$, $g(k) = k$ for all $k \in \mathbb{R}_+$ namely, the system is fully linear. Then the vote share is completely independent of how advertising expenses are allocated between ideological and valence advertising. All possible allocations are Nash equilibria subject to the constraint that all resources are spent and there are an infinite number of them.

If on the other hand, $V(k) = k$, $h(k) = k^2$, $g(k) = k^2$ for all $k \in \mathbb{R}_+$, there are four Nash equilibria given by $(V_1^*, V_2^*) \in \{(A, 0), (0, A), (0, 0), (A, A)\}$.

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