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Delegation and Strategic Compensation in Tournaments

by

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Delegation and Strategic Compensation in Tournaments*

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Abstract

This paper considers a two-stage game with two owners and two managers. On the first stage, the owners choose a linear combination of profits and sales as incentives for their managers. On the second stage, the two managers compete in a tournament against each other. In a symmetric equilibrium, both owners induce their managers to maximize profits. In asymmetric equilibria, however, one owner puts a positive weight on sales and the other a negative weight.

1 Introduction

The problem of the separation of ownership from management in a publicly owned firm has been widely discussed. While it is well-known that owners should compensate their managers according to profits instead of sales in an isolated (static) context, the same may not hold in a strategic context. For example, when firms compete against each other, owners may wish their managers to act more aggressively by putting a positive weight on sales in the managerial incentive contracts. Some papers discussed the optimal strategic incentives for managers in an oligopoly, assuming that owners can choose a linear combination of profits and sales. The results show that under Cournot competition it is optimal for owners to put a positive weight on sales, whereas under Bertrand competition owners should put a negative weight on sales.[1]

However, the Cournot and the Bertrand game are not the only forms of competition for which the question of strategic incentives arises when owners have to delegate decisions to managers. This paper combines the approaches of Fershtman and Judd (1987), and Lazear and Rosen (1981) to discuss the optimal linear combination of profits and sales when managers compete in a tournament game.[2] Tournaments can be characterized as a rather strong form of competition. In the context of managerial competition, firms may end up as tournament winners and get high sales whereas losing firms only receive low sales. There are a lot of real situations that can be better described by tournament competition than by the Cournot or the Bertrand model: for example, many cases in which firms must spend resources in advance to compete for a highly profitable order from a public institution or from a private enterprise. Such situations can be often found in the professional service sector (e.g., advertising firms compete for a given budget of an industrial enterprise by elaborating proposals for a new publicity campaign). This paper will show that in a symmetric equilibrium there are no strategic interactions between the owners, who induce their managers to maximize profits. This result does not hold in the case of asymmetric equilibria. There, one owner puts a positive and the other a negative weight on sales.

In Section 2, a general two-stage model of delegated competition in tournaments is discussed. Section 3 considers a special case with quadratic costs and uniformly distributed luck. Section 4 concludes.

2 Model and Results

In analogy to Fershtman and Judd (1987), a model with two risk neutral owners and two risk neutral managers is considered where owner i $(i = 1, 2)$ chooses a linear combination

$$O_i = \alpha_i \Pi_i + (1 - \alpha_i) S_i \tag{1}$$

of profits Π_i and sales S_i as incentives for his manager on stage 1 of the game with $\alpha_i > 0$.[3] On the second stage, the two managers compete in a tournament against each other. This tournament subgame follows the basic model by Lazear and Rosen (1981). The managers simultaneously spend resources $\mu_i \geq 0$ to generate a performance or outcome

$$q_i = \mu_i + \varepsilon_i \qquad\qquad (i = 1, 2), \tag{2}$$

where ε_1 and ε_2 denote error terms that are i.i.d.. The usage of resources entails costs to the firm according to $c(\mu_i)$ with $c'(\cdot) > 0$ and $c''(\cdot) > 0$ $(i = 1, 2)$. The winner of the tournament receives high sales S_H, whereas the less successful firm only gets low sales $S_L\,(< S_H)$. Manager i wins when $q_i > q_j$ ($i, j = 1, 2$; $i \neq j$). As in Fershtman and Judd (1987) there is no disutility of effort for managers. Owners want to maximize their expected profits and pay the managers their reservation values in expected terms.[4] Let $g(\cdot)$ denote the density function of the composed random term $\varepsilon_j - \varepsilon_i$. For this model, the following result can be derived:

Proposition 1 *Suppose the existence of a symmetric equilibrium on the tournament stage. Then there will exist a symmetric subgame perfect equilibrium in the two-stage game where owners choose* $\alpha_i^* = 1$ *and managers spend resources* $\mu_i^* = c'^{-1}(\Delta S \cdot g(0))$ $(i = 1, 2)$ *with* $\Delta S = S_H - S_L$.

Proof. See the Appendix. ■

The proposition states that in a symmetric equilibrium each owner induces his manager to maximize profits. The intuition behind this result is straightforward: The proof of Proposition 1 shows that on the tournament stage of the game (stage 2) manager i $(i = 1, 2)$ reacts according to $\mu_i^* = c'^{-1}(\Delta S \cdot g(0)/\alpha_i)$. Thus, μ_i^* only depends on α_i and not on α_j, which implies that owner j cannot influence μ_i^* by strategically choosing his incentive parameter α_j. The best each owner can do is to induce his manager to maximize profits. Therefore, both owners choose $\alpha_i^* = \alpha_j^* = 1$ in equilibrium.[5]

The equilibrium values μ_i^* $(i = 1, 2)$ can also be interpreted intuitively: Since $c(\cdot)$ is a convex function, $c'^{-1}(\cdot)$ increases in the sales spread ΔS (i.e., in the spread of the tournament prizes) and decreases, when luck becomes more important for the outcome of the tournament (i.e., when $g(0)$ becomes small).[6]

While there is no strategic interaction between the owners in the case of a symmetric equilibrium, the same does not hold for asymmetric equilibria:

Proposition 2 *If $g(\cdot)$ has a unique mode at zero and there exist asymmetric subgame perfect equilibria $\left(\alpha_i^*, \alpha_j^*, \mu_i^*, \mu_j^*\right)$ with $\alpha_i^* \neq \alpha_j^*$ and $\mu_i^* \neq \mu_j^*$, these equilibria will have the following properties: Either*

(a) $\alpha_j^* < 1 < \alpha_i^*$ and $\mu_i^* < \mu_j^*$ with $\partial\mu_i^*/\partial\alpha_i < 0$, $\partial\mu_j^*/\partial\alpha_i < 0$, $\partial\mu_i^*/\partial\alpha_j > 0$, $\partial\mu_j^*/\partial\alpha_j < 0$, or

(b) $\alpha_i^* < 1 < \alpha_j^*$ and $\mu_i^* > \mu_j^*$ with $\partial\mu_i^*/\partial\alpha_i < 0$, $\partial\mu_j^*/\partial\alpha_i > 0$, $\partial\mu_i^*/\partial\alpha_j < 0$,

$\partial\mu_j^*/\partial\alpha_j < 0$.

Proof. See the Appendix. ■

The results of Proposition 2 show that in the case of asymmetric equilibria one owner puts a positive weight on sales whereas the other owner chooses a negative weight. Since the two subcases (a) and (b) are rather similar, we can consider the scenario (a), for example: Now, there is strategic interaction between the owners (i.e., $\mu_i^*(\alpha_i, \alpha_j)$ and $\mu_j^*(\alpha_i, \alpha_j)$), and owner j puts a positive weight on sales (i.e., $\alpha_j^* < 1$), whereas owner i decides to put a negative weight on sales (i.e., $\alpha_i^* > 1$). Looking at the partial derivatives that describe the managers' reactions to the owners' choices of α_i and α_j, respectively, this result can be explained intuitively. In the scenario (a), it is advantageous for owner j to choose a small α_j, because lowering α_j leads to an increase in μ_j^* and decreases μ_i^* (i.e., $\partial\mu_j^*/\partial\alpha_j < 0$ and $\partial\mu_i^*/\partial\alpha_j > 0$) which both raises manager j's probability of winning the tournament. Owner i is in a quite different situation, because due to the strategic interaction lowering α_i would result in both managers spending more resources in the tournament ($\partial\mu_i^*/\partial\alpha_i < 0$ and $\partial\mu_j^*/\partial\alpha_i < 0$).

The inverse relation between μ_i^* and α_i (or μ_j^* and α_j, respectively) becomes clear from the managers' objective functions

$$EO_i(\mu_i) = S_L + \Delta S \cdot G\left(\mu_i - \mu_j\right) - \alpha_i \cdot c(\mu_i) \tag{3}$$

and

$$EO_j\left(\mu_j\right) = S_L + \Delta S \cdot \left[1 - G\left(\mu_i - \mu_j\right)\right] - \alpha_j \cdot c\left(\mu_j\right) \quad , \tag{4}$$

where $\Delta S = S_H - S_L$, and $G(\cdot)$ denotes the cumulative distribution function of $\varepsilon_j - \varepsilon_i$. Thus, a small α_i (or α_j) leads to low costs and, thereby, to a more aggressive behavior of manager i (or j) in the tournament, i.e. the manager spends more resources, which results in a higher probability of winning. But there are also negative spillover effects in the case of asymmetric equilibria, because increasing one's own expenditures μ_i^* (or μ_j^*) decreases the other manager's marginal and absolute probability of winning. In scenario (a), $\alpha_j^* < 1$ makes manager j discouraging manager i in the tournament by spending large resources μ_j^*, whereas in (b) the opposite holds due to $\alpha_i^* < 1$.

The condition that $g(\cdot)$ has a unique mode at zero is not very restrictive. It holds for a wide range of known probability distributions.[7] For example, when ε_i and ε_j are normally distributed with mean zero, the convolution $g(\varepsilon_j - \varepsilon_i)$ is also a normal distribution with mean zero.[8] When ε_i and ε_j are uniformly distributed over $[-\bar{\varepsilon}, \bar{\varepsilon}]$, the composed random variable $\varepsilon_j - \varepsilon_i$ is triangularly distributed over $[-2\bar{\varepsilon}, 2\bar{\varepsilon}]$ with mode at zero.

Unfortunately, the general model considered above does not allow concrete statements whether an owner is better off in the symmetric or in the asymmetric equilibrium. Therefore, a special case will be discussed in the following section.

3 A Special Case: Quadratic Costs and Uniformly Distributed Luck

In this section, it is assumed that each firm i $(i = 1, 2)$ has a quadratic cost function $c(\mu_i) = \frac{k}{2}\mu_i^2$, and that ε_1 and ε_2 are uniformly i.i.d. over $[-\bar{\varepsilon}, \bar{\varepsilon}]$ which implies a triangularly distributed $\varepsilon_j - \varepsilon_i$ over $[-2\bar{\varepsilon}, 2\bar{\varepsilon}]$.[9] Let, for simplicity, $S_L = 0$. In the case of a symmetric equilibrium with $\alpha_i^* = \alpha_j^* = 1$ we obtain

$$\mu_i^* = \mu_j^* = \frac{\Delta S}{2k\bar{\varepsilon}}, \tag{5}$$

which yields expected profits

$$E\Pi_i(\alpha_i^*) = E\Pi_j(\alpha_j^*) = \Delta S \cdot \frac{4k\bar{\varepsilon}^2 - \Delta S}{8k\bar{\varepsilon}^2} \tag{6}$$

for the two owners. Because the two asymmetric scenarios are rather similar, the following considerations are restricted to scenario (a). Here, we have

$$\mu_i^* = \frac{2\Delta S\bar{\varepsilon}\alpha_j}{\Delta S(\alpha_i - \alpha_j) + 4\alpha_i\alpha_j k\bar{\varepsilon}^2}, \quad \mu_j^* = \frac{2\Delta S\bar{\varepsilon}\alpha_i}{\Delta S(\alpha_i - \alpha_j) + 4\alpha_i\alpha_j k\bar{\varepsilon}^2} \tag{7}$$

on the tournament stage and

$$\alpha_i^* = 1 + \frac{\Delta S}{4\alpha_j^* k\bar{\varepsilon}^2}, \quad \alpha_j^* = 1 - \frac{\Delta S}{4\alpha_i^* k\bar{\varepsilon}^2} \tag{8}$$

on the first stage. The expressions for μ_i^* and μ_j^* show that the two owners have to take notice of the strategic interactions on the tournament stage when choosing α_i and α_j on the first stage. Since $\Delta S < 4\alpha_i^* k\bar{\varepsilon}^2$ must hold for the concavity of the managers' objective functions on the tournament stage, we see that $\alpha_j^* \epsilon\,(0,1)$ and $\alpha_i^* > 1$. Thus, owner j (owner i) puts a positive (negative) weight on sales.

Comparing the owners' expected profits for the symmetric and the asymmetric equilibrium yields that owner j prefers the asymmetric to the symmetric equilibrium: His expected profits are higher with an aggressively acting manager that discourages the other manager in the asymmetric case than with a profit maximizing manager in the symmetric case. On the other hand, there exists a critical value $\Delta\hat{S}$ for the sales spread so that for $\Delta S > \Delta\hat{S}$ owner i prefers to have a defensive manager in the asymmetric case to the symmetric case with a profit maximizing manager. For $\Delta S < \Delta\hat{S}$ the opposite holds. The intuition for the last result is indicated by the expressions (5) and (6). Differentiating $E\Pi_i\left(\alpha_i^*\right)$ with respect to ΔS gives

$$\frac{\partial E\Pi_i\left(\alpha_i^*\right)}{\partial \Delta S} = \frac{2k\bar{\varepsilon}^2 - \Delta S}{4k\bar{\varepsilon}^2} \quad . \tag{9}$$

Equation (9) shows that owner i's expected profits decrease in the sales spread for large values of ΔS in the symmetric equilibrium. This seems to be plausible, because μ_i^* raises in ΔS (see (5)) so that $c(\mu_i^*)$ becomes very large for large values of ΔS.

4 Conclusions

This paper discusses a two-stage game with the owners choosing a linear combination of sales and profits as incentives for the managers on the first stage, and the managers competing in a tournament on the second stage. The results show that, contrary to the usual case of oligopolistic competition, there may exist a symmetric equilibrium as well as asymmetric equilibria.[10] In the symmetric equilibrium, there is no direct strategic interaction between the owners, and each owner induces his manager to maximize profits. In the asymmetric case, however, one owner puts a positive weight on sales whereas the other chooses a negative weight. Although only simultaneously acting players have been considered on each stage, the special case of Section 3 indicates that there may be a first-mover advantage when owners have to choose their incentive schemes sequentially: Then it can be profitable to decide first and put a positive weight on sales so that one's own manager becomes the aggressively acting one in an asymmetric equilibrium.

Appendix

Proof of Proposition 1:

In the tournament subgame (stage 2) manager i $(i = 1, 2)$ wants to maximize $EO_i(\mu_i) = \alpha_i E\Pi_i + (1 - \alpha_i) ES_i$, where $S_i = S_H$ $(S_i = S_L)$ if manager i is the winner (loser) of the tournament. Since $E\Pi_i = ES_i - c(\mu_i)$, the manager's objective function can be rewritten as $EO_i(\mu_i) = ES_i - \alpha_i c(\mu_i)$. Expected sales are $ES_i = S_H \cdot$prob$\{i$ wins$\}$ $+ S_L \cdot [1-$prob$\{i$ wins$\}]$ with prob$\{i$ wins$\}$ $=$ prob$\{q_i > q_j\}$ $=$ prob$\{\mu_i + \varepsilon_i > \mu_j + \varepsilon_j\}$ $=$ prob$\{\varepsilon_j - \varepsilon_i < \mu_i - \mu_j\}$ $= G(\mu_i - \mu_j)$ where $G(\cdot)$ denotes the cumulative distribution function of $\varepsilon_j - \varepsilon_i$ and $g(\cdot)$ its density. Thus, $EO_i(\mu_i) = S_H \cdot G(\mu_i - \mu_j) + S_L \cdot [1 - G(\mu_i - \mu_j)] - \alpha_i \cdot c(\mu_i) = S_L + \Delta S \cdot G(\mu_i - \mu_j) - \alpha_i \cdot c(\mu_i)$ with $\Delta S = S_H - S_L$, and $EO_j(\mu_j) = S_L + \Delta S \cdot [1 - G(\mu_i - \mu_j)] - \alpha_j \cdot c(\mu_j)$. The first order conditions for the managers' choices of μ_i and μ_j are

$$\Delta S \cdot g\left(\mu_i^* - \mu_j^*\right) - \alpha_i \cdot c'\left(\mu_i^*\right) = 0 \tag{A1}$$

$$\Delta S \cdot g\left(\mu_i^* - \mu_j^*\right) - \alpha_j \cdot c'\left(\mu_j^*\right) = 0 \;, \tag{A2}$$

which imply

$$\alpha_i \cdot c'\left(\mu_i^*\right) = \alpha_j \cdot c'\left(\mu_j^*\right), \tag{A3}$$

and the second order conditions yield

$$\Delta S \cdot g'\left(\mu_i^* - \mu_j^*\right) - \alpha_i \cdot c''\left(\mu_i^*\right) < 0 \quad \text{(A4)}$$

$$-\Delta S \cdot g'\left(\mu_i^* - \mu_j^*\right) - \alpha_j \cdot c''\left(\mu_j^*\right) < 0. \quad \text{(A5)}$$

Let $\mu_i^*(\alpha_i, \alpha_j)$ and $\mu_j^*(\alpha_i, \alpha_j)$ denote the managers' Nash equilibrium strategies in the tournament subgame. On the first stage, owner i and owner j $(i, j = 1, 2;\ i \neq j)$ maximize their expected profits $E\Pi_i(\alpha_i) = S_L + \Delta S \cdot G(\mu_i^*(\alpha_i, \alpha_j) - \mu_j^*(\alpha_i, \alpha_j)) - c(\mu_i^*(\alpha_i, \alpha_j))$ and $E\Pi_j(\alpha_j) = S_L + \Delta S \cdot [1 - G(\mu_i^*(\alpha_i, \alpha_j) - \mu_j^*(\alpha_i, \alpha_j))] - c\left(\mu_j^*(\alpha_i, \alpha_j)\right)$ by optimally choosing α_i and α_j, respectively.[11] The first order conditions are[12]

$$\Delta S \cdot g\left(\mu_i^* - \mu_j^*\right)\left(\frac{\partial \mu_i^*}{\partial \alpha_i^*} - \frac{\partial \mu_j^*}{\partial \alpha_i^*}\right) - c'\left(\mu_i^*\right)\frac{\partial \mu_i^*}{\partial \alpha_i^*} = 0 \quad \text{(A6)}$$

$$-\Delta S \cdot g\left(\mu_i^* - \mu_j^*\right)\left(\frac{\partial \mu_i^*}{\partial \alpha_j^*} - \frac{\partial \mu_j^*}{\partial \alpha_j^*}\right) - c'\left(\mu_j^*\right)\frac{\partial \mu_j^*}{\partial \alpha_j^*} = 0. \quad \text{(A7)}$$

As second order conditions we obtain

$$\Delta S g'\left(\mu_i^* - \mu_j^*\right)\left(\frac{\partial \mu_i^*}{\partial \alpha_i^*} - \frac{\partial \mu_j^*}{\partial \alpha_i^*}\right)^2 + \Delta S g\left(\mu_i^* - \mu_j^*\right)\left(\frac{\partial^2 \mu_i^*}{\partial \alpha_i^{*2}} - \frac{\partial^2 \mu_j^*}{\partial \alpha_i^{*2}}\right)$$

$$-c''\left(\mu_i^*\right)\left[\frac{\partial \mu_i^*}{\partial \alpha_i^*}\right]^2 - c'\left(\mu_i^*\right)\frac{\partial^2 \mu_i^*}{\partial \alpha_i^{*2}} < 0 \quad \text{(A8)}$$

$$-\Delta S g'\left(\mu_i^* - \mu_j^*\right)\left(\frac{\partial \mu_i^*}{\partial \alpha_j^*} - \frac{\partial \mu_j^*}{\partial \alpha_j^*}\right)^2 - \Delta S g\left(\mu_i^* - \mu_j^*\right)\left(\frac{\partial^2 \mu_i^*}{\partial \alpha_j^{*2}} - \frac{\partial^2 \mu_j^*}{\partial \alpha_j^{*2}}\right)$$

$$-c''\left(\mu_j^*\right)\left[\frac{\partial \mu_j^*}{\partial \alpha_j^*}\right]^2 - c'\left(\mu_j^*\right)\frac{\partial^2 \mu_j^*}{\partial \alpha_j^{*2}} < 0. \quad \text{(A9)}$$

Proposition 1 assumes the existence of a symmetric equilibrium $\mu_i^* = \mu_j^*$ on the tournament stage. From (A3) we know that then $\alpha_i = \alpha_j$. (A1) and (A2) give

$$\mu_i^* = c'^{-1}\left(\frac{\Delta S \cdot g(0)}{\alpha_i}\right) \quad \text{and} \quad \mu_j^* = c'^{-1}\left(\frac{\Delta S \cdot g(0)}{\alpha_j}\right). \tag{A10}$$

Thus, $\partial \mu_j^* / \partial \alpha_i^* = 0$ in (A6) and $\partial \mu_i^* / \partial \alpha_j^* = 0$ in (A7), and the owners' first order conditions yield

$$\mu_i^* = c'^{-1}\left(\Delta S \cdot g(0)\right) \quad \text{and} \quad \mu_j^* = c'^{-1}\left(\Delta S \cdot g(0)\right). \tag{A11}$$

Comparing (A10) and (A11) we see that the owners optimally choose $\alpha_i^* = \alpha_j^* = 1$. At last, we have to check the second order conditions for the first stage. Because of $\mu_i^* = \mu_j^*$, $\partial \mu_j^* / \partial \alpha_i^* = 0$, and $\partial \mu_i^* / \partial \alpha_j^* = 0$ the owners' second order conditions can be simplified to

$$\left[\frac{\partial \mu_i^*}{\partial \alpha_i^*}\right]^2 \left(\Delta S g'(0) - c''(\mu_i^*)\right) + \frac{\partial^2 \mu_i^*}{\partial \alpha_i^{*2}} \left(\Delta S g(0) - c'(\mu_i^*)\right) < 0 \tag{A12}$$

$$\left[\frac{\partial \mu_j^*}{\partial \alpha_j^*}\right]^2 \left(-\Delta S g'(0) - c''\left(\mu_j^*\right)\right) + \frac{\partial^2 \mu_j^*}{\partial \alpha_j^{*2}} \left(\Delta S g(0) - c'\left(\mu_j^*\right)\right) < 0. \tag{A13}$$

Both equations, (A12) and (A13), consist of four terms. In each equation, the first term is quadratic and therefore positive; the second term is negative, because it describes the managers' second order conditions (A4) and (A5) for $\alpha_i^* = \alpha_j^* = 1$; the fourth term is zero because of the managers' first order conditions evaluated at $\alpha_i^* = \alpha_j^* = 1$.

Proof of Proposition 2:

Proposition 2 assumes the existence of asymmetric subgame perfect equilibria. First, we have to prove the signs of the partial derivatives for the two subcases (a) and (b). By differentiating the system of implicit functions (A1) and (A2)[13] we obtain

$$\frac{\partial \mu_i^*}{\partial \alpha_i} = \frac{c'\left(\mu_i^*\right)\left(-\Delta S g'\left(\mu_i^*-\mu_j^*\right)-\alpha_j c''\left(\mu_j^*\right)\right)}{D} \tag{A14}$$

$$\frac{\partial \mu_j^*}{\partial \alpha_i} = \frac{-\Delta S g'\left(\mu_i^*-\mu_j^*\right) c'\left(\mu_i^*\right)}{D} \tag{A15}$$

$$\frac{\partial \mu_i^*}{\partial \alpha_j} = \frac{\Delta S g'\left(\mu_i^*-\mu_j^*\right) c'\left(\mu_j^*\right)}{D} \tag{A16}$$

$$\frac{\partial \mu_j^*}{\partial \alpha_j} = \frac{c'\left(\mu_j^*\right)\left(\Delta S g'\left(\mu_i^*-\mu_j^*\right)-\alpha_i c''\left(\mu_i^*\right)\right)}{D} \tag{A17}$$

where the denominator is given by

$$\begin{aligned} D &= -\alpha_j c''\left(\mu_j^*\right)\left[\Delta S g'\left(\mu_i^*-\mu_j^*\right)-\alpha_i c''\left(\mu_i^*\right)\right]+\alpha_i c''\left(\mu_i^*\right)\Delta S g'\left(\mu_i^*-\mu_j^*\right) \\ &= -\alpha_i c''\left(\mu_i^*\right)\left[-\Delta S g'\left(\mu_i^*-\mu_j^*\right)-\alpha_j c''\left(\mu_j^*\right)\right]-\alpha_j c''\left(\mu_j^*\right)\Delta S g'\left(\mu_i^*-\mu_j^*\right). \end{aligned}$$

The sum D consists of two terms. The first term is positive, because $\alpha_i, \alpha_j > 0$, $c''(\cdot) > 0$, and the expression in brackets denotes manager i's or manager j's second order condition, respectively. The second term is also positive. $g'\left(\mu_i^*-\mu_j^*\right)$ may be positive or negative, but the two alternative formulations of D show that this does not matter. Then, from the managers' second order conditions we obtain $\partial\mu_i^*/\partial\alpha_i < 0$ (see (A14)) and $\partial\mu_j^*/\partial\alpha_j < 0$ (see (A17)) for both subcases (a) and (b). The signs of (A15) and (A16) are different for the two subcases. In subcase (a), we have $\mu_i^* < \mu_j^*$, so that $\mu_i^* - \mu_j^*$ is located at the left-hand tail of $g(\cdot)$. Since, by assumption, $g(\cdot)$ has a unique mode at zero, we have $g'\left(\mu_i^* - \mu_j^*\right) > 0$ in subcase (a). This implies $\partial\mu_j^*/\partial\alpha_i < 0$ (see (A15)) and $\partial\mu_i^*/\partial\alpha_j > 0$ (see (A16)) for subcase (a). In subcase (b), $\mu_i^* - \mu_j^* > 0$ being part of the right-hand tail of $g(\cdot)$ with $g'\left(\mu_i^* - \mu_j^*\right) < 0$ due to the unique mode of $g(\cdot)$ at zero. Therefore, in subcase (b) we obtain $\partial\mu_j^*/\partial\alpha_i > 0$ (see (A15)) and $\partial\mu_i^*/\partial\alpha_j < 0$ (see (A16)).

Next, we have to prove that either

$$\text{(a)}\ \alpha_j^* < 1 < \alpha_i^* \quad \text{and} \quad \mu_i^* < \mu_j^* \quad \text{or} \tag{A18}$$

$$\text{(b)}\ \alpha_i^* < 1 < \alpha_j^* \quad \text{and} \quad \mu_i^* > \mu_j^* \tag{A19}$$

in asymmetric equilibria. Equation (A3) shows that either $\alpha_j^* < \alpha_i^*$ and $\mu_i^* < \mu_j^*$, or $\alpha_i^* < \alpha_j^*$ and $\mu_i^* > \mu_j^*$ because of $c'(\cdot) > 0$. It remains to show that $\alpha_j^* < 1 < \alpha_i^*$ in subcase (a), and $\alpha_i^* < 1 < \alpha_j^*$ in subcase (b). Equations (A1) and (A2) yield the following characterizations of the managers' equilibrium strategies:

$$\mu_i^* = c'^{-1}\left(\frac{\Delta S \cdot g(\Delta\mu^*)}{\alpha_i}\right) \quad \text{and} \quad \mu_j^* = c'^{-1}\left(\frac{\Delta S \cdot g(\Delta\mu^*)}{\alpha_j}\right) \tag{A20}$$

with $\Delta\mu^* = \mu_i^* - \mu_j^*$. On the first stage, using the partial derivatives (A14)–(A17) the owners' first order conditions (A6) and (A7) can be written as

$$\begin{aligned} 0 &= \Delta S g(\Delta\mu^*)\left(-\frac{\alpha_j^* c'(\mu_i^*) c''(\mu_j^*)}{D}\right) \\ &\qquad -c'(\mu_i^*)\frac{c'(\mu_i^*)\left(-\Delta S g'(\Delta\mu^*) - \alpha_j^* c''(\mu_j^*)\right)}{D} \end{aligned} \tag{A21}$$

$$\begin{aligned} 0 &= -\Delta S g(\Delta\mu^*)\frac{\alpha_i^* c'(\mu_j^*) c''(\mu_i^*)}{D} \\ &\qquad -c'\left(\mu_j^*\right)\frac{c'\left(\mu_j^*\right)\left(\Delta S g'(\Delta\mu^*) - \alpha_i^* c''(\mu_i^*)\right)}{D}. \end{aligned} \tag{A22}$$

After some calculations we obtain

$$\mu_i^* = c'^{-1}\left(\frac{\Omega_j\left(1-\frac{1}{\alpha_i^*}\right)}{\Delta S g'\left(\Delta\mu^*\right)}\right) \quad \text{and} \quad \mu_j^* = c'^{-1}\left(\frac{-\Omega_i\left(1-\frac{1}{\alpha_j^*}\right)}{\Delta S g'\left(\Delta\mu^*\right)}\right) \qquad \text{(A23)}$$

with $\Omega_l = \Delta S g\left(\Delta\mu^*\right)\alpha_l^* c''(\mu_l^*)$ $(l = i, j)$. Comparing (A20) and (A23) we see that the owners choose α_i^* and α_j^*, respectively, so that

$$\frac{\Omega_j\left(1-\frac{1}{\alpha_i^*}\right)}{\Delta S g'\left(\Delta\mu^*\right)} = \frac{\Delta S g\left(\Delta\mu^*\right)}{\alpha_i^*} \qquad \text{(A24)}$$

$$\frac{-\Omega_i\left(1-\frac{1}{\alpha_j^*}\right)}{\Delta S g'\left(\Delta\mu^*\right)} = \frac{\Delta S g\left(\Delta\mu^*\right)}{\alpha_j^*}. \qquad \text{(A25)}$$

which gives

$$-\Delta S g'\left(\Delta\mu^*\right) - \alpha_j^* c''(\mu_j^*)(1-\alpha_i^*) = 0 \qquad \text{(A26)}$$

$$\Delta S g'\left(\Delta\mu^*\right) - \alpha_i^* c''(\mu_i^*)(1-\alpha_j^*) = 0. \qquad \text{(A27)}$$

Equation (A26) ((A27)) describes owner i's (j's) optimal choice of α_i (α_j). In subcase (a), we have $\Delta\mu^* < 0$ which yields $g'\left(\Delta\mu^*\right) > 0$ due to the assumption of a unique mode of $g\left(\cdot\right)$ at zero. Thus, the first term in (A26) is negative so that the second term has to be positive which requires $1-\alpha_i^* < 0 \Leftrightarrow \alpha_i^* > 1$. In (A27) the first term is positive which requires the second term to be negative and, therefore, $1-\alpha_j^* > 0 \Leftrightarrow \alpha_j^* < 1$. Altogether, in subcase (a) we obtain $\alpha_j^* < 1 < \alpha_i^*$. In analogy, subcase (b) yields the inverse relation $\alpha_i^* < 1 < \alpha_j^*$ because here $\Delta\mu^* > 0$, which implies $g'\left(\Delta\mu^*\right) < 0$ due to the unique mode of $g\left(\cdot\right)$ at zero.

Notes

1. See Fershtman and Judd (1987), Sklivas (1987). In a recent paper, Ishibashi (2000) adds quality competition to the discussion of strategic incentives in oligopoly. His results show that now choosing a positive weight on sales may be beneficial for owners under Bertrand competition. For a discussion of relative performance evaluation of managers in an oligopoly see Fumas (1992). For a survey of strategic delegation see Spulber (1992, pp. 566-568).

2. Baik and Kim (1997) also discuss strategic aspects of delegation in contests, which are not identical with tournaments considered here. Moreover, the paper does not discuss the question, whether the owner should put a positive weight on sales or not.

3. Owners will never choose $\alpha_i \leq 0$, since this would induce their managers to spend countless resources. This becomes clear when looking at the managers' objective functions.

4. See Fershtman and Judd (1987), fn. 3 and 6. Thus, the compensation for manager i can be described by $A_i + B_i O_i$ $(B_i > 0)$, where the parameters A_i and B_i are chosen so that expected compensation just equals manager i's reservation value. However, we have to assume that managers dislike the

spending of resources, because otherwise there would be no real conflict of interests between owners and managers.

5. It is well-known in the tournament literature that the existence of equilibria must be assumed in general; see, e.g., Lazear and Rosen (1981, p. 845, fn. 2); Nalebuff and Stiglitz (1983, p. 29); Lazear (1989, p. 565, fn. 3).

6. For the interpretation of $g(0)$ see Lazear (1995, p. 29).

7. In addition, this assumption is not unusual in the tournament literature; see, e.g., Drago, Garvey, and Turnbull (1996, p. 225).

8. See Wolfstetter (1999, pp. 343-344).

9. The derivations of the following results are relegated to an extended version of the paper, which can be requested from the author.

10. Note that in the standard tournament model with two homogeneous and risk neutral players there exists at most one equilibrium in pure strategies which then must be symmetric; see Proposition 1.

11. The two managers receive their reservation values; see fn. 4.

12. For brevity, the managers' equilibrium strategies are written as μ_i^* and μ_j^*.

13. See Chiang (1984, pp. 210-212).

References

Baik, K.H. and I.-G. Kim, 1997, ”Delegation in Contests”, *European Journal of Political Economy*, 13, 281−298.

Chiang, A.C., 1984, *Fundamental Methods of Mathematical Economics*, 3rd edition, Auckland et al.: McGraw-Hill Book Company.

Drago, R., Garvey, G.T., and G.K. Turnbull, 1996, ”A Collective Tournament”, *Economics Letters*, 50, 223−227.

Fershtman, C. and K.L. Judd, 1987, ”Equilibrium Incentives in Oligopoly”, *American Economic Review*, 77, 927−940.

Fumas, V.S., 1992, ”Relative Performance Evaluation of Management”, *International Journal of Industrial Organization*, 10, 473−489.

Ishibashi, K., 2000, ”Strategic Delegation under Quality Competition”, *Journal of Economics*, (forthcoming).

Lazear, E.P., 1989, ”Pay Equality and Industrial Politics”, *Journal of Political Economy*, 97, 561−580.

Lazear, E.P., 1995, *Personnel Economics*, Cambridge, MA: The MIT Press.

Lazear, E.P. and S. Rosen, 1981, ”Rank-Order Tournaments as Optimum Labor Contracts”, *Journal of Political Economy*, 89, 841−864.

Nalebuff, B.J. and J.E. Stiglitz, 1983, ”Prizes and Incentives: Towards a General Theory of Compensation and Competition”, *Bell Journal of Economics*, 14, 21−43.

Sklivas, S.D., 1987, ”The Strategic Choice of Managerial Incentives”, *Rand Journal of Economics*, 18, 452−458.

Spulber, D.F., 1992, ”Economic Analysis and Management Strategy: A Survey”, *Journal of Economics* & *Management Strategy*, 1, 535−574.

Wolfstetter, E., 1999, *Topics in Microeconomics*, Cambridge et al.: Cambridge University Press.