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How to Avoid a Hedging Bias

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Hedging strategies for derivatives which are considered in theory and applied in practice are understood to perform self–financing and to duplicate the final payoff. Of course, this is only valid with respect to an assumed model, called "hedging model", which specifies a set of postulates about the evolution of the underlying stock prices. For example, without lifting the model assumptions of Black and Scholes (1973), an option payoff can be replicated by continuously rebalancing a portfolio consisting of two underlying assets. On the one hand, these portfolio strategies may fail to be effective if the "true" asset price dynamics deviate from the assumed ones. On the other hand, there are further sources of market incompleteness, i.e. trading restrictions, which impede the theoretical concept of *perfect hedging*. In this paper, the effects of so–called model misspecification and the effects of dropping the assumption that continuous rebalancing is possible are examined. In particular, the analysis of the combined effects gives rise to some interesting insights into the topic of hedging.

Due to continuous time trading, the analysis of the implications of model misspecification for pricing and hedging contingent claims has already achieved great acknowledgement in scientific research. By assuming that the hedging strategies are carried out according to a model which differs from the true dynamic of market prices, the effectiveness of such strategies is analysed in El Karoui, Jeanblanc-Picqué and Shreve (1998) and Dudenhausen, Schlögl and Schlögl (1998). The key result states that if the true volatility is locally bounded, then the hedging strategies implied by Black/Scholes-like models[1] corresponding to the upper volatility bound are robust with respect to convex payoff-functions. The strategies which are self-financing and duplicating with respect to the "hedging model" dominate the payoff of the derivative for all stock prices which occur with positive probability, i.e. almost surely under any equivalent measure. Therefore, the use of (theoretically incompatible) lognormal models may be justified by volatility uncertainty in the specification of the "true" model.

Unfortunately, strategies which are robust if applied continuously fail to be robust if applied in discrete time. In particular, they incorporate a hedging bias which originates from the effects of time–discretising strategies meant to be applied continuously. This is in particular transparent if one abstracts from model misspecification and only concentrates on discrete trading, i.e. testing the consistency of the hedge model. It is often stated that hedging discretely does not bias the outcome of the hedge in either direction when all other parameters (volatility, rates and dividends) are known, c.f. Derman and Kamal (1999). However, this is *not* true unless the asset prices are martingales under the *real world measure.* The discrete application of Black/Scholes-like strategies to hedge convex payoffs yields a subhedge on average, i.e. it is not sufficiently hedged in the mean, if the asset price dynamic with respect to the objective probability measure includes a drift component which does not change its sign.

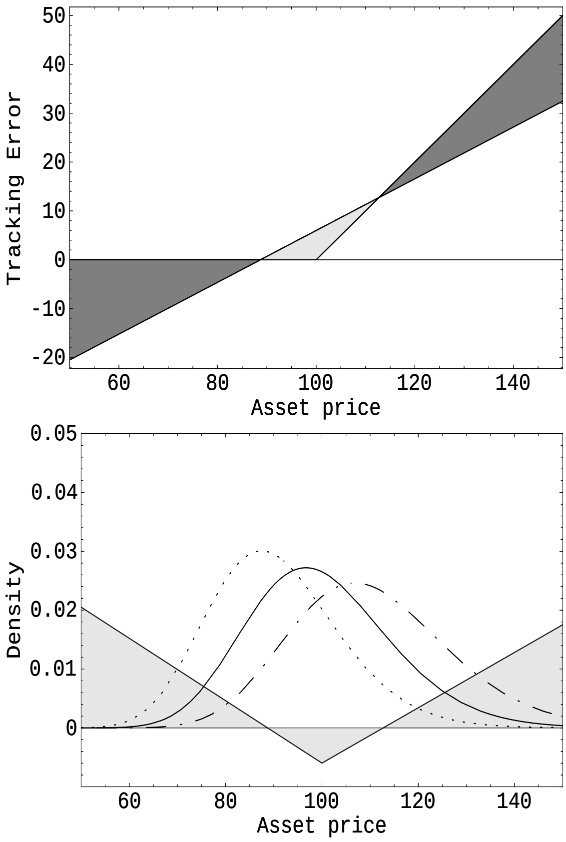
Understanding the bias intuitively

Figure 1 gives an intuitive way to understand the duplication bias. The first graph illustrates the difference of the payoff of a European–call–option with strike 100 and the terminal value of a three months (static) Black/Scholes hedging strategy composed at–the–money and with hedging–volatility $\sigma = 0.3$. Dominating the payoff results in negative costs, subdominating the payoff gives positive costs. Additionally, three densities are given resulting from the assumption that the asset price process follows a geometric Brownian motion with volatility equal to the volatility of the Black/Scholes hedge, i.e. $\sigma = 0.3$, and drift coefficient μ equal to -0.4, 0 and +0.4 respectively. Notice that the tracking error vanishes in expectation if and only if there is no drift in the underlying asset price process, i.e. $\mu = 0$. Otherwise ($\mu = -0.4$ or $\mu = +0.4$) there is a shift of probability mass to the area of positive costs such that the duplication costs are positively biased, i.e. the payoff is not sufficiently hedged in the mean.

Obviously, the bias motivated above can only be

[1] In particular, Black/Scholes-like models or Gaussian models are based on the assumption of a deterministic volatility structure such that the model is complete in the sense of Harrison and Pliska (1983), guaranteeing the existence of a self-financing trading strategy duplicating the payoff of the claim to be hedged.

Figure 1: Tracking error and duplication bias



avoided if a discrete–time hedging model is specified on the basis of the asset price dynamics under the real world measure. However, in contrast to this, the discrete Black/Scholes hedge must be interpreted as the result of specifying a discrete–time hedging model on the basis of the dynamics under the martingale measure which explains the duplication bias. The interesting question can be formulated as follows:
How is it possible to obtain a robust hedge in terms of superhedging and to avoid at the same time the duplication bias which is caused by discrete time trading?

To give an answer to this question, a financial market consisting of two assets X and Y is considered. With respect to this framework, the hedging of an option to exchange X for Y at maturity date T, i.e. a European option with payoff $[X_T - Y_T]^+$ is analysed. For notational convenience, a forward market in terms of the numeraire Y is assumed such that actually the assets $Z = \frac{X}{Y}$ and 1 are traded. The dynamic of Z is described by a strictly positive, continuous semimartingale. Assuming that the underlying probability space $(\Omega, \mathcal{F}, P)$ supports a d–dimensional Brownian motion W and adding some technical requirements, the dynamic can without loss of generality be written as

$$(1) \qquad dZ_t = Z_t \left(\mu_t^{(Z)}\, dt + \sigma_t^{(Z)}\, dW_t \right).$$

At first a short review is given of the robustness result concerning the problem of misspecification without the introduction of trading restrictions. After this, the discretisation bias is derived which arises if the robust continuous–time strategies are applied due to discrete time. Finally, a robust discrete–time hedging model is introduced which gives the solution for robust hedging in discrete time without causing a hedging bias.

Robustness of the Black/Scholes model

In a model where the quotient process $Z := \frac{X}{Y}$ is lognormal, the hedge portfolio $\Phi = (\Phi)_{0 \le t \le T}$ for the exchange option in terms of the assets X and Y is given by

$$(2) \qquad \begin{aligned} \phi_t^X &:= \mathcal{N}(h^{(1)}(t, Z_t)) \quad \text{units of } X \\ \text{and} \quad \phi_t^Y &:= -\mathcal{N}(h^{(2)}(t, Z_t)) \quad \text{units of } Y. \end{aligned}$$

$\mathcal{N}$ denotes the cumulative distribution function of the standard normal distribution and the functions $h^{(1)}$ and $h^{(2)}$ are given by

$$\begin{aligned} h^{(1)}(t, z) &= \frac{\ln(z) + \frac{1}{2}\int_t^T \|\tilde{\sigma}_Z(s)\|^2 ds}{\sqrt{\int_t^T \|\tilde{\sigma}_Z(s)\|^2 ds}} \\ h^{(2)}(t, z) &= h^{(1)}(t, z) - \sqrt{\int_t^T \|\tilde{\sigma}_Z(s)\|^2 ds}. \end{aligned}$$

In particular, the price process of the exchange option which is assumed for hedging purposes is given by

$$C(t, X_t, Y_t) = X_t \mathcal{N}(H_t^{(1)}) - Y_t \mathcal{N}(H_t^{(2)})$$

or

$$C^*(t, Z_t) = Z_t \mathcal{N}(H_t^{(1)}) - \mathcal{N}(H_t^{(2)}),$$

where $H_t = h(t, Z_t)$.
Notice that the assumed volatility $\tilde{\sigma}$ may deviate from the "true" volatility σ. Therefore, with respect to a "Gauss" hedger, we call the expression

$$\tilde{v}_T(t) := \sqrt{\int_t^T \|\tilde{\sigma}_Z(s)\|^2 ds}$$

hedging volatility.

At maturity T of the exchange option, the duplication costs arising from misspecification are given by

$$L_T^*(\phi) = [Z_T - 1]^+ - C^*(0, Z_0) - \int_0^T \phi_s^X \, dZ_s.$$

Assuming that the "true" dynamic of Z is given by equation (1), it is a well known result, cf. El Karoui, Jeanblanc-Picqué and Shreve (1998) or Avellaneda, Levy and Parás (1995), that the *misspecification costs* are given by

$$(3) \qquad L_t^*(\phi) = \int_0^t Z_u \frac{\mathcal{N}'(H_u^{(1)})}{2\tilde{v}_T(u)} \left(\|\sigma_Z(u)\|^2 - \|\tilde{\sigma}_Z(u)\|^2 \right) du.$$

Thus, if the purpose of hedging is the complete elimination of risk, given uncertainty about present and future volatility, one should hedge at the upper volatility bound. In cases where this upper bound is too high for this to be practicable, one should hedge at the upper bound for some confidence interval for the volatility, resulting in a superhedge as long as the realised volatility remains below this upper bound. However, the above hedging decision should not be used if continuous time trading is not possible.

Trading restrictions

Assume now that trading is only possible at a discrete set of dates, i.e.

$$\tau = \{t_0 = 0 \le t_1, \cdots \le t_N = T\}.$$

An application of the Gauss hedge, cf. equation (2), with respect to the discrete set of trading dates τ yields the following duplication costs

$$L_T^*(\phi, \tau) = \sum_{j=1}^{N} C^*(t_j, Z_{t_j}) - \left(\phi_{t_{j-1}}^X Z_{t_j} + \phi_{t_{j-1}}^Y \right).$$

Using

$$C^*(t_j, Z_{t_j}) = C^*(t_{j-1}, Z_{t_{j-1}}) + \int_{t_{j-1}}^{t_j} \phi_s^X \, dZ_s + \int_{t_{j-1}}^{t_j} dL_s^*(\phi)$$

implies

$$(4) \qquad L_T^*(\phi, \tau) = L_T^*(\phi) + \int_0^T \phi_s^X \, dZ_s - \sum_{j=1}^N \phi_{t_{j-1}}^X \left(Z_{t_j} - Z_{t_{j-1}} \right).$$

The duplication costs associated with the discrete time Gauss hedge can thus be interpreted as the sum of *misspecification costs* and the difference of trading gains resulting from continuous–time hedging and discrete–time hedging. The *discretisation error* can thus be defined by

$$D^*(\phi, \tau) := \int_0^T \phi_s^X \, dZ_s - \sum_{j=1}^N \phi_{t_{j-1}}^X \left(Z_{t_j} - Z_{t_{j-1}} \right).$$

A change of measure and an application of Itô's lemma gives the following representation of the *discretisation bias.*

Expected discretisation costs

$$(5) \qquad E_P\left[D_T^*(\phi, \tau^n)\right] = Z_0 \sum_{j=0}^{n-1} \int_{t_j^n}^{t_{j+1}^n} \left(e^{\int_0^{t_{j+1}^n} \mu_u^{(Z)} du} - e^{\int_0^s \mu_u^{(Z)} du} \right) \left[\frac{E_{\hat{P}^Z}\left[\mathcal{N}'(H_s^{(1)})\right]}{\tilde{v}_T(s)} \left(\mu_s + \frac{1}{2} \left(\|\sigma_Z(s)\|^2 - \|\tilde{\sigma}_Z(s)\|^2 \right) \right) - \frac{E_{\hat{P}^Z}\left[H_s^{(1)} \mathcal{N}'(H_s^{(1)})\right]}{2\tilde{v}_T^2(s)} \left(\|\sigma_Z(s)\|^2 - \|\tilde{\sigma}_Z(s)\|^2 \right) \right] ds,$$

where $d\hat{P}_t^Z = D_t^Z \, dP_t$ with

$$D_t^Z := \exp\left\{ \int_0^t \sigma_Z(u) \, dW_u - \frac{1}{2} \int_0^t \|\sigma_Z(u)\|^2 \, du \right\}.$$

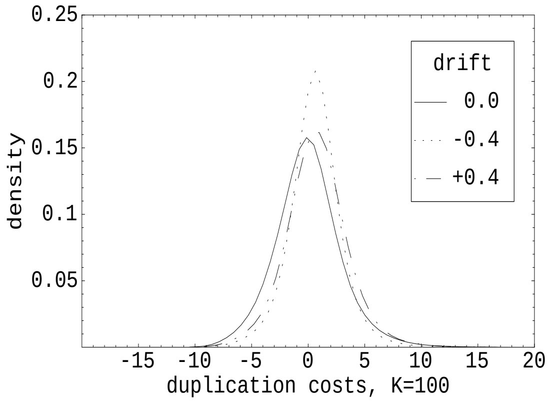
A detailed proof is provided in Dudenhausen (2002). Notice that the strategies under consideration are understood to be self-financing with respect to an assumed model called "hedging model" only. However, while facing the problem of model misspecification combined with trading restrictions, it is worth mentioning that the discrete time Gaussian hedge is in fact, cf. equation (5), neither consistent with the hedging model nor with the true model, i.e.:

If $\sigma_Z(t, \omega) = \tilde{\sigma}_Z(t, \omega)$ and either $\mu_Z(t, \omega) > 0$ or $\mu_Z(t, \omega) < 0$ for $\lambda^1 \otimes P$–almost all $(t, \omega) \in [0, T] \times \Omega$, then the discrete–time Gaussian strategy is positively biased, i.e.

$$E_P\left[D_T^*(\phi, \tau^n)\right] > 0.$$

This result has already been motivated, cf. figure 1, and is further illustrated in figure 2 which shows the distribution of a discrete–time Gaussian hedge under different drift scenarios. Each cost distribution is generated by a Gauss–kernel density estimation from 50000 simulated hedging paths. The parameters for the underlying asset price process Z under the objective probability measure are given

Figure 2: Distributions of hedging costs for a discrete time Gauss hedge under different drift scenarios



by $\sigma_Z = 0.3$ and $\mu_Z = 0.4$ (-0.4 and zero respectively). Time to maturity of the (plain vanilla call) option to be hedged is one year, the initial underlying price is equal to the strike $K = 100$. The duplication portfolio is composed according to $\tilde{\sigma}_Z = \sigma_Z$ but is rebalanced only monthly instead of continuously. In particular, figure 2 illustrates that the average final *profit/loss* is zero if and only if $\mu_Z = 0$, i.e. the real world measure P is already a martingale measure. However, for positive asset price drift, i.e. for $\mu_Z = 0.4$, *and* for negative asset price drift, i.e. for $\mu_Z = -0.4$, a positive duplication bias is observed.

How to avoid the hedging bias

Of course, the incompleteness associated with postulating continuous–time asset price dynamics given by equation (1), while hedging in discrete time is non hedgeable in the sense of superhedging. However, one may be tempted to discretise the hedging model instead of time–discretising an originally continuous time trading strategy. This gives the advantage that the strategies under consideration are compatible with their underlying hedging model. Besides, the hedging model *and* its corresponding strategies can be specified according to the asset price dynamics under the real world probability measure, i.e. including μ and σ. What is true concerning the concept of perfect hedging is true for robust hedging, too. Continuous–time trading is independent of μ but discrete–time trading is not. In particular, a discrete–time hedging model is necessary to avoid the positive hedging bias due to convex payoff–profiles.

Discrete–time hedging model

Concerning the above setup a binomial hedging model is needed. It is shown that binomial strategies incorporate similar robustness features as the Gaussian hedge and can be specified such that the duplication bias is avoided. Besides, the distribution of the cost process associated with the binomial hedge coincides, in the limit, with the distribution of the Gaussian hedge.

Without loss of generality, an equidistant set of trading dates τ^n with $t_k^n = \frac{kT}{n}$ is assumed. Furthermore, for ease of notation the parameters defining the up– and down–movements of the binomial hedging model which was first motivated by Cox, Ross and Rubinstein (1979), are given only in dependence of the degree of refinement n. Let g_1 and g_2 be defined by

$$(6)\quad g_1^n(t_k^n, z) := \frac{C_{\text{CRR}}^n(t_{k+1}^n, u_n z) - C_{\text{CRR}}^n(t_{k+1}^n, d_n z)}{(u_n - d_n)z},$$

$$(7)\quad g_2^n(t_k^n, z) := C_{\text{CRR}}^n(t_k^n, d_n z) - g_1^n(t_k^n z) d_n z,$$

where $C_{\text{CRR}}^n(t_n^n, z) = [z-1]^+$ and

$$C_{\text{CRR}}^n(t_k^n, z) = \sum_{j=0}^{n-k} \binom{n-k}{j} \left(\frac{1-d_n}{u_n - d_n}\right)^j \left(\frac{u_n - 1}{u_n - d_n}\right)^{n-k-j} \left[u_n^j d_n^{n-k-j} z - 1\right]^+,$$

for $k = 0, \ldots, n-1$.

Notice that any arbitrage free binomial model can exclusively be specified through d_n and u_n satisfying the condition $d_n < 1 < u_n$. The uniquely defined martingal–measure (concerning the hedging model) is then determined by the transition probabilities $p_n^* = \frac{1-d_n}{u_n - d_n}$ for the up–movement respectively $1 - p_n^*$ for the down–movement of the risky asset. Consequently, C_{CRR}^n denotes the arbitrage free price according to the hedging–model. C_{CRR}^n is relevant for hedging purposes only and does not need to match the true market price. The hedging strategy of a binomial–hedger can thus be specified in terms of g_1^n and g_2^n.

The duplication costs L_T^C for $C_T = [Z_T - 1]^+$ associated with the trading strategy $\Phi^n = \left(\phi_n^X, \phi_n^Y\right)$ in the assets $\left(\frac{X}{Y}, 1\right)$, where

$$\Phi_t^n = \left(g_1^n(t_k^n, Z_{t_k^n}), g_2^n(t_k^n, Z_{t_k^n})\right) \text{ for } t \in]t_k^n, t_{k+1}^n]$$

and $Z_t = \frac{X_t}{Y_t}$, are then given by

$$L_T^C(\Phi^n) = \sum_{j=1}^{n} C_{\text{CRR}}^n\left(t_j^n, Z_{t_j^n}\right) - V_{t_j^n}(\Phi^n).$$

Notice that

$$\begin{aligned} & C_{\text{CRR}}^n\left(t_j^n, Z_{t_j^n}\right) - V_{t_j^n}(\Phi^n) \\ = \; & C_{\text{CRR}}^n\left(t_j^n, Z_{t_j^n}\right) \\ & - \left(g_1^n(t_{j-1}^n, Z_{t_{j-1}^n}) Z_{t_j^n} + g_2^n(t_{j-1}^n, Z_{t_{j-1}^n})\right). \end{aligned}$$

Inserting g_1^n and g_2^n according to equation (6) and equation (7) and defining $x_{t_j^n} := \frac{Z_{t_j^n}}{Z_{t_{j-1}^n}}$ yields

$$\begin{aligned} & C_{\text{CRR}}^n\left(t_j^n, x_{t_j^n} Z_{t_{j-1}^n}\right) - \\ & \left[\frac{x_{t_j^n} - d_n}{u_n - d_n} C_{\text{CRR}}^n\left(t_j^n, u_n Z_{t_{j-1}^n}\right)\right. \\ & \left. + \frac{u_n - x_{t_j^n}}{u_n - d_n} C_{\text{CRR}}^n\left(t_j^n, d_n Z_{t_{j-1}^n}\right)\right]. \end{aligned}$$

It is well known that $C_{\text{CRR}}^n\left(t_j^n, z\right)$ is convex in z which allows the formulation of a robustness result.

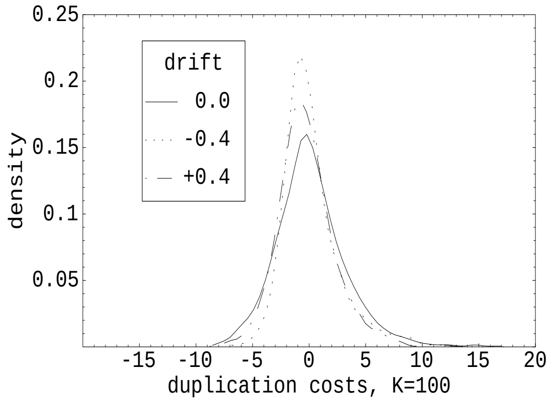
Robustness of the CRR model

A superhedge is achieved if and only if the returns of the underlying asset are within the range of the up– and down– parameters of the assumed binomial process for each trading period, i.e. if and only if

$$\frac{Z_{t_j^n}}{Z_{t_{j-1}^n}} \in [d_n, u_n] \quad P\text{–}a.s. \text{ for all } j = 1, \ldots, n.$$

Obviously, a binomial hedging strategy is not able to dominate the payoff of the exchange option almost surely if the true asset price can move outside the interval. The requirement that the asset price stays in the interval defined by the assumed up– and down–movements of the binomial model outweighs the requirement that the "true" volatility is dominated by the volatility of the continuous–time Gauss hedge. In particular, superhedging according to a binomial strategy requires the incorporation of the upper volatility bound as well as the "true" asset price drift under the real world measure P into the placement of the nodes d_n and u_n. Suitably specifying u_n and d_n in dependence of volatility and drift allows to avoid the discretisation bias associated with a Gaussian hedge which is exclusively defined via the assumed volatility. Again, it is worth mentioning that it is only possible to avoid the discretisation error if the robust discrete–time hedging model is specified on the basis of the asset price dynamics under the real world measure P. Discretising the continuous–time hedging model according to its risk neutral dynamic yields the same duplication bias as the application of the discrete time Gauss hedge.

Figure 3: Distributions of hedging costs for JR–like hedging strategies under known volatility referring to different drift scenarios



However, using the specification of Jarrow and Rudd (1983) of the binomial parameters u and d, i.e. let

$$\begin{aligned} u_n &:= \exp\left\{\frac{\mu_Z - \frac{1}{2}\|\tilde{\sigma}_Z\|^2}{n} + \frac{\tilde{\sigma}_Z}{\sqrt{n}}\right\}, \\ d_n &:= \exp\left\{\frac{\mu_Z - \frac{1}{2}\|\tilde{\sigma}_Z\|^2}{n} - \frac{\tilde{\sigma}_Z}{\sqrt{n}}\right\}, \end{aligned}$$

gives no duplication bias caused by the asset price drift. In particular, the stepwise shortfall–probability is independent of μ. In addition, it turns out that the expected duplication costs,

$$E_P\left[L_T^C(\Phi^n)\right] = \sum_{j=1}^{n} E_P\left[C_{\text{CRR}}^n\left(t_j^n, Z_{t_j^n}\right) - V_{t_j^n}(\Phi^n)\right],$$

tend to be negative, i.e. the binomial hedge is over–financing on average. Furthermore, the above specification of u_n and d_n additionally guarantees the convergence of the cost process associated with the binomial hedge to the one of a Gaussian hedge in distribution if the incompleteness arising from trading restriction vanishes, i.e. if $n \to \infty$.

Once again, each cost distribution illustrated in figure 3 is generated by a Gauss–kernel density estimation from 50000 simulated hedging paths. The parameters for the underlying asset price process Z

under the objective probability measure are given by $\sigma_Z = 0.3$ and $\mu_Z = 0.4$ (-0.4 and zero respectively). Time to maturity of the (plain vanilla call) option to be hedged is again one year, the initial underlying price is 100, the strike K of the option is chosen to be 100 and the duplication portfolio is rebalanced monthly. This time, the strategies are given by a binomial model with a Jarrow–Rudd specification as described above. A comparison of these simulation results with the ones based on the time discretised Gauss hedge, cf. figure 2, shows that the model discretisation in form of the binomial hedge is clearly favoured when suitably adjusted to the asset price drift. While, on average, the discrete–time Gauss hedge yields a subhedge only, the binomial hedge even manages to achieve a superhedge.[2]

Conclusion

The results of this paper present a strong argument to discretise the hedging model instead of discretising the hedging strategies if the rebalancing of the portfolio is restricted to a set of discrete–time trading dates. Black/Scholes–type strategies and binomial strategies to hedge derivatives with convex payoff-profiles can be understood to incorporate comparable robustness features in the sense of superhedging. Due to continuous time, dominating the payoff of a contingent claim almost surely with respect to all equivalent measures is obviously independent of the drift of the underlying under the objective measure. However, if market incompleteness is not only due to sources of model and parameter misspecification, but also to trading restrictions in the sense of discrete trading, a non–trivial superhedge cannot be obtained even if volatility is bounded. A discrete–time superhedge requires that asset prices do not move outside an interval. Therefore, it is adequate to allow the strategy to depend on more parameters than only on the hedge volatility with vanishing influence if the distance of trading dates converges to zero. This can easily be done with binomial strategies, but not with a discrete–time version of a Gaussian hedge. In comparison to a simple Black/Scholes strategy, the advantage of the CRR-like hedging strategy is particularly transparent if the market is complete without the introduction of trading restrictions. On the one hand, a Gaussian hedging strategy which is applied in discrete time subdominates the convex payoff to be hedged on average for positive as well as negative asset price trends under the objective probability measure. On the other hand, the binomial hedge which is suitably adjusted to the "real world" drift component is (almost) self–financing in the mean, tending to favour the outcome of the hedge. Since the cost processes coincide in the limit, there is nothing lost by using the binomial hedge instead of the Gaussian hedge if the trading frequency is increased. While the use of (theoretically incompatible) lognormal models may be justified by volatility uncertainty in the specification of the "true" model, the use of binomial models may be justified by trading restrictions even if continuous time price processes are assumed to describe the true dynamics. With respect to discrete–time trading, a discretisation bias for convex payoff–profiles is only avoided if a discrete–time hedging model is specified on the basis of the asset price dynamics under the real world measure.

[2] It is worth mentioning that, in order to keep consistent with the limit, the initial value of the binomial hedge under consideration was not fixed to match the initial value of the corresponding Gauss hedge. In fact, due to the above examples, the t_0–prices of the "drift–adjusted" binomial hedging models were even less than the initial Gauss price.

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