Bonn Econ Discussion Papers

Discussion Paper 09/2008

Perfect Competition in an Oligopoly (including Bilateral Monopoly)

by

Pradeep Dubey and Dieter sondermann

May 2008

Bonn Graduate School of Economics
Department of Economics
University of Bonn
Adenauerallee 24 - 42
D-53113 Bonn

The Bonn Graduate School of Economics is
sponsored by the

Deutsche Post World Net

MAIL EXPRESS LOGISTICS FINANCE

Perfect Competition in an Oligopoly (including Bilateral Monopoly)

In honor of Martin Shubik*

Pradeep Dubey[†] Dieter Sondermann[‡]

May 16, 2008

Abstract

We show that if limit orders are required to vary smoothly, then strategic (Nash) equilibria of the double auction mechanism yield competitive (Walras) allocations. It is not necessary to have competitors on any side of any market: smooth trading is a substitute for price wars. In particular, Nash equilibria are Walrasian even in a bilateral monopoly.

Keywords: Limit orders, double auction, Nash equilibria, Walras equilibria, mechanism design

JEL Classification: C72, D41, D44, D61

*It is a pleasure for us to dedicate this paper to Martin Shubik who founded and developed (in collaboration with others, particularly Lloyd Shapley) the field of Strategic Market Games in a general equilibrium framework. This research was partly carried out while the authors were visiting IIASA, Laxenburg and The Institute for Advanced Studies, Hebrew University of Jerusalem. Financial support of these institutions is gratefully acknowledged. The authors also thank two anonymous referees for constructive remarks.

[†]Center for Game Theory, Dept. of Economics, SUNY at Stony Brook and Cowles Foundation, Yale University.

[‡]Department of Economics, University of Bonn, D-53113 Bonn.

1 Introduction

As is well-known Walrasian analysis is built upon the Hypothesis of Perfect Competition, which can be taken as in Mas-Colell (1980) to state: "...that prices are publicly quoted and are viewed by the economic agents as exogenously given". Attempts to go beyond Walrasian analysis have in particular involved giving "a theoretical explanation of the Hypothesis itself" (Mas-Colell (1980)). Among these the most remarkable are without doubt the 19th century contributions of Bertrand, Cournot and Edgeworth (for an overview, see Stigler (1965)). The Cournot approach was explored intensively, in a general equilibrium framework, in the symposium issue entitled "Non-cooperative Approaches to the Theory of Perfect Competition" (Journal of Economic Theory, Vol. 22 (1980)).
The features common to most of the symposium articles are:

(a) The strategies employed by the agents are of the Cournot type, i.e., consist in quoting quantities.

(b) The (insignificant) size of any agent relative to the market is the key explanatory variable for the tendency of strategic behavior to approximate perfect competition and, in its wake, to lead to Walrasian outcomes (Mas-Colell (1980), p.122).

The extension of pure quantity strategies from Cournot's partial equilibrium model of oligopoly to a general equilibrium framework, however, does raise questions. Underlying the Cournot model is a demand curve for the particular market under consideration which enables the suppliers to relate quantities, via prices, to expected receipts. If such a close relationship is not provided by the market, then it seems more natural to us that an agent will no longer confine himself to quoting quantities, i.e., to pure buy-or-sell market orders. To protect himself against "market uncertainty - or illiquidity, or manipulation by other agents" (Mertens (2003)), he will also quote prices limiting the execution of those orders, consenting to sell q units of commodity j only if its price is p or more, or buy $\tilde{q}$ units only if its price is $\tilde{p}$ or less. By sending multiple orders of this kind an agent can approximate any monotone demand or supply curve in a market by a step function, as was done in

Dubey (1982, 1994). Here we go further and give each agent *full* manoeuvrability. He places a continuum of infinitesimal limit-price orders, which in effect enables him to send any monotone, continuous demand or supply curve for each commodity[1]. The upshot is a striking result: provided only that all commodity markets are "active" (i.e. there is positive trade in them), and no matter how thin they are, *strategic (Nash) equilibria (SE) coincide – in outcome space – with competitive (Walras) equilibria (CE).* Our result thus provides a rationale, based on strategic competition, for Walrasian outcomes even in the case of a bilateral monopoly. This brings it in sharp contrast to Dubey (1982, 1994), where it was necessary to allow for price wars via competition on both sides of each market (in the sense of there being at least two active buyers and two active sellers for each commodity) in order to conclude that SE are CE [2]. The key point of our paper is that *continuous trading is a substitute for price wars and yields perfect competition.* A monopolist may be in sole command of his own resource, but nevertheless he will be reduced to behaving as if he had cut-throat rivals, once continuous trading sets in. [3]

We present our result in two different models. In Section 2, our focus is first on an oligopoly in which each trader conjectures he can exert monopoly power. Thus a seller may be (or else feel to be) in sole control of commodity j. Given an (inverse) price-quantity demand curve $D_j(q)$ for j, he will try to appropriate the entire consumer surplus under the demand curve D_j by perfect price discrimination; i.e., selling first to the highest-priced buyer of j, then to the next highest, and so on. Similarly, a buyer of commodity j, who feels himself to be in a monopolistic position facing several sellers, will first take the cheapest offer of j, then the next cheapest, etc., in order to appropri-

[1]It must be emphasized that our model is based on *decentralized* markets, and is therefore an order-of-magnitude simpler than that of Mertens (2003), where cross-market limit orders are permitted. SE form a large superset of CE in Mertens' model – for instance, the SE of Shapley's windows model (see Sahi and Yao (1989)) are also SE there.

[2]Indeed, in Dubey's model, the coincidence of SE and CE fails drastically if there is a monopolistic agent in any market. In particular, in a bilateral monopoly, every individually rational and strategically feasible allocation is sustained by SE!

[3]A related phenomenon was analyzed in Coase (1972) . There, too, a monopolist was shown to forfeit his power, but this happened in the setting of durable goods which could be sold sequentially over time to infinitely patient customers. In our model the monopolist loses power even with perishable goods which are traded at one instant of time.

ate the entire producer surplus in market j. Indeed, in the extreme case of a bilateral monopoly, perfect price discrimination of this kind is to be expected. Our equilibrium point (EP) captures what happens when agents make such optimistic conjectures of wielding monopoly power (Section 2.1). In Section 2.2 we then show that, under the additional assumption that strategies are *smooth* (C^1), the same result also holds under "realistic expectations" . The notion of EP is akin to Walrasian equilibrium, with the important difference that prices are not fixed from the outside by an imaginary auctioneer, but are set by the agents themselves, each of whom realizes and exerts his ability to influence prices. Therefore we think that EP is an interesting concept in its own right.

In the second model (Section 3) we turn to a standard market game, as in Dubey (1982) and Dubey (1994). The market functions like a stock exchange, with all higher bid (or, lower ask) prices serviced before a new purchase (or, sale) order is taken up. To accommodate economies in which CE consumptions could occur on the boundary, it becomes needful here to introduce a "market maker" who has infinitesimal inventories of every good, and stands ready to provide them if sellers renege on their promises of delivery. It turns out that, at our SE, the market maker is never active. But it is important for agents to imagine his presence when they think about what they could get were they to unilaterally deviate. [4]

Though the two models are built on quite different behaviorial hypotheses, we find their equilibria (the EP and the SE) lead to the same outcomes, namely Walrasian.

2 Walrasian Outcomes via Equilibrium Points

2.1 Optimistic Expectations

In this version of our model no attempt is made to construct a full-fledged game. Since agents are optimistic and *conjecture* that they can exert perfect price discrimination and appropriate the entire surplus in every market, it is

[4] If we restrict attention to interior economies, the market maker can be dispensed with. See Remark in Section 3.5 .

infeasible to assign outcomes, i.e. trades and prices, to arbitrary collections of strategies. Indeed agents' strategies become jointly compatible precisely at an *equilibrium point* (see below), and it is only then that outcomes are defined. But joint compatibility of the conjectures forces a single effective price to form at each market. Furthermore – and this is a more subtle point – the smoothness of the conjectures imply that the ensuing trades will in fact be Walrasian.

Our analysis is in the tradition of a Walrasian tâtonnement process, i.e., no outcomes are defined outside equilibrium (see e.g. Malinvaud (1974)). We are aware that this is an unsatisfactory shortcut from the view point of providing a game-theoretic foundation of the Walras model (a matter to which we shall turn in Section 3). Nevertheless it meets the Walras model on its own terms and goes beyond it in some important ways:

(a) Prices are not quoted from the outside by some "fictitious auctioneer", but set by the agents themselves. In particular, the "law of one price" per market is not postulated: it is derived out of the continuum of prices that agents can strategically set for each commodity, in the course of competing against one another.

(b) It is not assumed that the economic agents face perfectly elastic supply and demand curves. On the contrary, they face the curves that are strategically set by their competitors and these always display elasticity as trades exceed certain threshold.

(c) Strategies of the individuals (i.e. supply and demand curves submitted to the market) need not be based on their true characteristics (preferences and endowments) – indeed, in equilibrium, they never are!

In addition, as shown in Section 2.3, equilibrium points are stable w.r.t. unilateral deviations; i.e., have the same property as a strategic (Nash) equilibrium.

The economic message of our model is that smooth trading leads to perfect competition. From the work of Aumann (1964) we know that a continuum of *agents* is needed to obtain perfect competition in a cooperative game context. Here, in a noncooperative context, what is required is a continuum of *strategies* that enables agents to trade infinitesimal amounts at smoothly varying prices. Continuous trade of this form by just two (or more) agents generates perfect competition in our model. This is a similar effect as in the Black-Scholes (1973) model, where continuous trade in only two commodities spans a continuum of contingent claims.

We now turn to a precise description of our model:

Let $N = \{1, \dots, n\}$ be the set of agents who trade in k commodities. Each agent $i \in N$ has an initial endowment $e^i \in \mathbb{R}^k_+ \setminus \{0\}$ and a preference relation $\succsim_i$ on $\mathbb{R}^k_+$ that is convex, continuous and monotonic (in the sense that $x \geq y$, $x \neq y$ implies $x \succ_i y$). We assume that $\sum_{i \in N} e^i \gg 0$, i.e. every named commodity is present in the aggregate.

An agent may enter a market either as a buyer or a seller, but not both (although he may switch roles), and submit to each of the k commodity markets a marginal (inverse) demand or supply curve. Formally, let

$$M^+ = \{f : \mathbb{R}_+ \to \mathbb{R}_{++} | \ f \text{ is continuous and non-decreasing}\}$$
$$M^- = \{f : \mathbb{R}_+ \to \mathbb{R}_{++} | \ f \text{ is continuous and non-increasing}\}.$$

Then a *strategic choice* σ^i of agent i is given by

$$\sigma^i = (\sigma^i_1, \dots, \sigma^i_k \,|\, \sigma^i_j = d^i_j \in M^- \ \textit{or}\ \sigma^i_j = s^i_j \in M^+, \ \text{for}\, j = 1, \dots, k)$$

In the interpretation $d^i_j(q^i_j)$ is the price at which agent i is willing to buy an infinitesimal, incremental unit of commodity j, once his level of purchases has reached q^i_j. The supply curve has an analogous meaning. Denote $\sigma \equiv (\sigma^1, \dots, \sigma^n)$ and let S^σ_j, D^σ_j be the aggregate[5] supply, demand curves.

We suppose that agent i acts under the optimistic conjecture that he can exert perfect price discrimination, i.e., that he can sell (buy) starting

[5]The aggregation is *horizontal*. In other words, taking *price* to be the independent variable, each s^i_j is a non-decreasing *correspondence*; and so S_j can be viewed to be the sum (over i) of these correspondences, which is also non-decreasing. Reverting to quantity as the independent variable, S_j is a non-decreasing function. D_j is similarly defined as a non-increasing function.

at the highest (lowest) prices quoted by the buyers (sellers). This means that agent i calculates his receipts (or expenditures) on the market j as the integral, starting from 0, under the curve D_j^σ (or S_j^σ). The generally non-convex budget set $B^i(\sigma)$ is then obtained by the requirement that (perceived) expenditures do not exceed (perceived) receipts, i.e.,

$$B^i(\sigma) = \{e^i + t \mid t \in \mathbb{R}^k, e^i + t \in \mathbb{R}_+^k, \sum_{j=1}^k E_j^\sigma(t_j) \leq \sum_{j=1}^k R_j^\sigma(t_j)\}$$

where

$$E_j^\sigma(t_j) = \int_0^{t_j} S_j^\sigma(q)dq \quad \text{if } t_j > 0,\ 0 \text{ otherwise,}$$

$$R_j^\sigma(t_j) = \int_0^{|t_j|} D_j^\sigma(q)dq \quad \text{if } t_j < 0,\ 0 \text{ otherwise.}$$

(Note that $t_j^i > 0$ ($t_j^i < 0$) means that i buys (sells) j . In the sequel we will drop the integration variable dq .)

The collection of strategic choices σ will be called an *equilibrium point* (EP) if there exist trade vectors $t^1, \ldots, t^n$ in $\mathbb{R}^k$ such that

(i) $e^i + t^i$ is $\succsim_i$ -optimal on $B^i(\sigma)$ for $i = 1, \ldots, n$

(ii) $$\sum_{i=1}^n t_j^i = 0 \text{ for } j = 1, \ldots, k$$

(iii) $$\sum_{i:t_j^i > o} t_j^i = sup\{q_j \mid S_j^\sigma(q_j) \leq D_j^\sigma(q_j)\} \text{ for } j = 1, \ldots, k$$

Conditions (i) and (ii) require that agents optimize and that markets clear. Condition (iii) says that no trade can be enforced, i.e., it stops when the (marginal) supply price for the first time exceeds the demand price; and, at the same time, in equilibrium all trades compatible with the submitted strategies are actually carried out.

An EP will be called *active* if there is positive trade in each market.

First let us establish that at an active EP all trade $T_j := \sum_{i\,:\,t_j^i>0} t_j^i$ in any commodity j takes place at *one* price, p_j.

Lemma 1. *The curves* S_j^σ *and* D_j^σ *coincide and are constant on* $[0, T_j]$ *at any EP .*

Proof. For any j, let $G_j := \{i : t_j^i > 0\}$, $H_j := \{i : t_j^i < 0\}$. Then

$$
\begin{aligned}
(1) \qquad \sum_{i \in H_j} R_j^\sigma(t_j^i) &= \sum_{i \in H_j} \int_0^{|t_j^i|} D_j^\sigma \\
&\geq \int_0^{T_j} D_j^\sigma \\
&\geq D_j^\sigma(T_j) \cdot T_j \\
&\geq S_j^\sigma(T_j) \cdot T_j \\
&\geq \int_0^{T_j} S_j^\sigma \\
&\geq \sum_{i \in G_j} \int_0^{t_j^i} S_j^\sigma \\
&= \sum_{i \in G_j} E_j^\sigma(t_j^i).
\end{aligned}
$$

The third inequality follows from (iii); the other four follow from the fact that supply (demand) functions are non-decreasing (non-increasing).

Hence

$$
(2) \qquad \sum_{i=1}^{n} R_j^\sigma(t_j^i) \geq \sum_{i=1}^{n} E_j^\sigma(t_j^i) \quad \text{for } j = 1, \ldots, k.
$$

From the monotonicity of preferences, and the fact that each agent has optimized, we have

$$\sum_{j=1}^{k} R_j^\sigma(t_j^i) = \sum_{j=1}^{k} E_j^\sigma(t_j^i) \quad \text{for } i = 1, \dots, n. \tag{3}$$

(2) and (3) together imply:

$$\sum_{i=1}^{n} R_j^\sigma(t_j^i) = \sum_{i=1}^{n} E_j^\sigma(t_j^i) \quad \text{for } j = 1, \dots, k. \tag{4}$$

From (4) it follows that all the inequalities in (1) must, in fact, be equalities. Therefore

$$S_j^\sigma(T_j) = D_j^\sigma(T_j) =: p_j \tag{5}$$

and

$$\int_0^{T_j} D_j^\sigma = p_j T_j = \int_0^{T_j} S_j^\sigma. \tag{6}$$

Since by (iii), $D_j^\sigma \geq S_j^\sigma$ on $[0, T_j]$ we get, from (6), and the monotonicity of D and S

$$D_j^\sigma = S_j^\sigma \text{ on } [0, T_j]. \tag{7}$$

□

In view of Lemma 1 we can talk not only of the allocation but also the prices produced at an active EP . These are the constant values of S_j^σ, D_j^σ on $[0, T_j]$ for $j = 1, \dots, k$. Note that these prices are positive by assumption.

Proposition 1. *The prices and allocation at an active equilibrium point are Walrasian.*

Proof. Let σ be an EP with trades $t^1, \ldots, t^n$ and prices p . We need to show that, for each i , $e^i + t^i$ is $\gtrsim_i$ -optimal on the set

$$B^i(p) := \{e^i + t : t \in \mathbb{R}^k, e^i + t \in \mathbb{R}^k_+,\ p \cdot t = 0\}.$$

W.l.o.g. fix $i = 1$, put

$$\begin{aligned}
J_1 &:= \{j : t^1_j > 0\} \\
J_2 &:= \{j : t^1_j < 0\} \\
J_3 &:= \{j : t^1_j = 0\} \\
T_j &:= \sum_{i : t^i_j > 0} t^i_j \\
\delta_j &:= min[|t^1_j|, T_l : j \in J_1 \cup J_2, l \in J_3] \\
N_j &:= \{\alpha \in \mathbb{R} : |t^1_j - \alpha| < \delta_j\} \\
F_j &:= E_j - R_j
\end{aligned}$$

(Since the EP is active, $\delta_j > 0$). Now we claim, for $j = 1, \ldots, k$:

(8) F_j is continuously differentiable and strictly increasing on N_j and its derivative at t^1_j is p_j.

This follows from the continuity and strict positivity of S_j and D_j, and from Lemma 1 which implies:

(9) $F_j(q)$ coincides with $E_j(q) = p_j q$ if $j \in J_1,\ 0 \leq q \leq t^1_j$

(10) $F_j(q)$ coincides with $-R_j(q) = p_j q$ if $j \in J_2,\ t^1_j \leq q \leq 0$

(11) $F_j(q) = p_j q$ if $j \in J_3,\ q \in N_j$.

W.l.o.g. fix commodity $j = 1$. Since $F_1, \ldots, F_k$ are all strictly increasing and $\sum_{j=1}^{k} F_j(t^1_j) = 0$, and $F'_1(t^1_1) = p_1 > 0$, it follows from the implicit function theorem that there is a neighborhood V of $(t^1_2, \ldots, t^1_k)$ in $N_2 \times \ldots \times N_k$

such that if $(t_2, \ldots, t_k) \in V$ then there is a unique t_1 which satisfies the equation $F_1(t_1) + \ldots + F_k(t_k) = 0$. Thus we have an implicit function $G(t_2, \ldots, t_k) = F_1^{-1}(-F_2(t_2) - \ldots - F_k(t_k))$ defined on V which is clearly continuously differentiable. Finally the point $t^1 = (t_1^1, \ldots, t_k^1)$ belongs by construction to the smooth hypersurface $M = \{(G(t_2, \ldots, t_k), t_2, \ldots, t_k) : (t_2, \ldots, t_k) \in V\} \subset B^1(\sigma)$ and, by (8), the tangent plane H to M at this point has normal p .

Since we are at an EP, $e^1 + t^1$ is $\succsim_1$-optimal on $(e^1 + M) \cap \mathbb{R}_+^k$. Suppose that there is some $x \in H_+ := (e^1 + t^1 + H) \cap \mathbb{R}_+^k$ such that $x \succ_1 e^1 + t^1$. By continuity of $\succsim_1$ we can find a neighborhood Z of x (in $\mathbb{R}_+^k$) with the property: $y \in Z \Rightarrow y \succ_1 e^1 + t^1$. But since M is a smooth surface there exists a point y^* in Z, such that the line segment between y^* and $e^1 + t^1$ pierces $e^1 + M$ at some point $z^* \in (e^1 + M) \cap \mathbb{R}_+^k$ (see Fig.1). By convexity of $\succsim_1$, we have $z^* \succ_1 e^1 + t^1$, contradicting that $e^1 + t^1$ is $\succsim_1$-optimal on $(e^1 + M) \cap \mathbb{R}_+^k$. We conclude that $e^1 + t^1$ is $\succsim_1$-optimal on H_+ . But we have $e^1 \in H_+$ (simply set trades to be zero, i.e., pick $-t^1$ in H). Therefore, in fact, $H_+ = B^1(p)$. Since the choice of $i = 1$ was arbitrary, the proposition follows. □

.... Insert Figure 1 here!................

Proposition 2. *If the trades $t^1, \ldots, t^n$ and prices $p \gg 0$ are Walrasian, then they can be achieved at an EP.*

Proof. For any i let

$$
\begin{aligned}
J_1^i &= \{j : t_j^i > 0\} \\
J_2^i &= \{j : t_j^i < 0\} \\
J_3^i &= \{j : t_j^i = 0\} \\
f_j^i &= \text{any strictly decreasing function with } f_j^i(t_j^i) = p_j \\
g_j^i &= \text{any strictly increasing function with } g_j^i(t_j^i) = p_j
\end{aligned}
$$

and consider, for a suitably small $\delta > 0$, the strategies

$$
s_j^i(x) = \begin{cases} p_j + \delta & \text{if } j \in J_1^i \cup J_3^i \\ max\{p_j, g_j^i(x)\} & \text{if } j \in J_2^i \end{cases}
$$

$$d^i_j(x) = \begin{cases} p_j - \delta & \text{if } j \in J^i_2 \cup J^i_3 \\ min\{p_j, f^i_j(x)\} & \text{if } j \in J^i_1 \end{cases}$$

Then it is readily checked that these strategies constitute an EP and produce the trades $t^1, \ldots, t^n$ at prices p. □

2.2 Realistic Expectations

The expectation of each agent that he can exert perfect price discrimination may appear unduly optimistic, especially when there are several rivals on every market. But our results on equilibrium points in Section 2.1 remain intact also under the assumption of "realistic expectations". In contrast to Section 2.1, each agent here is realistic and realizes that the prices he will get are apropos his own quotations, not the best going; and thus he calculates his expenditure (or, revenue) as the integral under his own demand (or, supply) curve. Furthermore he is aware that he will only be served if his demand (supply) curve lies above (below) the intersection of the market supply and demand curves.

Strategies are defined as before, but now we require that they are *continuously differentiable* (C^1). Define

$$Q^\sigma_j \equiv sup\{q : D^\sigma_j(q) \geq S^\sigma_j(q)\}$$

(with $Q^\sigma_j = 0$ if either $D^\sigma_j = \phi$, or $S^\sigma_j = \phi$, or S^σ_j is strictly above D^σ_j). Let each agent i think that he can buy t_j units of commodity j, where $0 \leq t_j \leq Q^\sigma_j$, for the expected expenditure $\widehat{E}^\sigma_j(t_j) = \int\limits_0^{t_j} d^i_j(q)dq$ so long as $d^i_j(q) \geq D^\sigma_j(Q^\sigma_j)$. To buy more than Q^σ_j, he would need to "enhance" his d^i_j to $\widetilde{d}^i_j$ by some $\Delta > 0$, i.e., set $\widetilde{d}^i_j(q) = d^i_j(q) + \Delta$ for all q. Since $S^\sigma_j \in C^1$, the extra quantity bought varies smoothly with Δ, and so does the integral $\widehat{E}^\sigma_j(t_j)$. Similarly, if he is a seller of commodity j, he can sell $0 \leq |t_j| \leq Q^\sigma_j$ for the expected receipt $\widehat{R}^\sigma_j(t_j) = \int\limits_0^{|t_j|} s^i_j(q)dq$ so long as $s^i_j(q) \leq S^\sigma_j(Q^\sigma_j)$, and to sell more than Q^σ_j lower his s^i_j by some $\Delta > 0$, i.e., set $\widetilde{s}^i_j(q) = s^i_j(q) - \Delta$ for all q. (Remember: $t_j \leq 0$ if it represents a sale).

Now define the (realistic) budget set $\widehat{B}^i(\sigma)$ just like $B^i(\sigma)$ with $E_j^\sigma(t_j)$ and $R_j^\sigma(t_j)$ replaced by $\widehat{E}_j^\sigma(t_j)$ and $\widehat{R}_j^\sigma(t_j)$. After this define EP exactly as before.

We submit that Propositions 1 and 2 still hold. To see this first note that (see footnote 5)

$$\sum_{i\in H_j} \int_0^{|t_j^i|} s_j^i = \int_0^{|T_j|} S_j^\sigma \,, \quad \text{and} \quad \sum_{i\in G_j} \int_0^{t_j^i} d_j^i = \int_0^{T_j} D_j^\sigma,$$

and then verify Lemma 1 by rereading the proof with $\widehat{E}_j^\sigma$, $\widehat{R}_j^\sigma$ in place of R_j^σ, E_j^σ (reversing the chain of inequalities). Then the proof of Proposition 1 remains unchanged, except that condition (8) now follows from the smoothness of S_j and D_j. Clearly, the strategies in the proof of Proposition 2 can also be chosen to be in C^1.

2.3 Stability of Equilibrium Points

We can now show that at an EP of both versions of our model no agent can improve by unilateral deviation of his strategy.

Proposition 3. *Active EPs are stable w.r.t. unilateral deviations.*

Proof. Let $\sigma = (\sigma^i)_{i\in N}$ be an active EP with price $p = (p_j)_{j\in K}$ and net trades $(t_j^i)_{i\in N, j\in K}$. Define Q_j^σ as before. If $0 \le t_j^i < Q_j^\sigma$, agent i can buy more (or, less if $t_j^i > 0$) of commodity j at price p_j . But if $t_j^i = Q_j^\sigma$ he can only buy more of j by raising his demand function d_j^i in order to increase Q_j^σ , thus increasing the average price he has to pay above p_j . But the same is true under optimistic expectations. There he imagines that he can buy more at prices $S_j^\sigma > p_j$, thus also raising the average price above p_j. Similarly, if $-Q_j^\sigma < t_j^i \le 0$, agent i can sell more (or, less if $t_j^i < 0$) of commodity j at price p_j . But in case of $t_j^i = -Q_j^\sigma$ he has to lower his supply function s_j^i below p_j, thus lowering his average price for his sales.

For any strategy choice $\widetilde{\sigma}^i$ of agent i denote by $(\sigma|\widetilde{\sigma}^i)$ the n-tuple $(\sigma^1, \ldots, \sigma^{i-1}, \widetilde{\sigma}^i, \sigma^{i+1}, \ldots, \sigma^n)$ and consider the budget set $B^i(\sigma|\widetilde{\sigma}^i)$ defined as before with σ replaced by $(\sigma|\widetilde{\sigma}^i)$. Since preferences are monotone, for the EP

$\sigma = (\sigma|\sigma^i)$, for each i, total expenditures E^i equal total receipts R^i . Thus, for any deviation $(\sigma|\widetilde{\sigma}^i)$ and net trade $\widetilde{t}^i \in B^i(\sigma|\widetilde{\sigma}^i)$, any increase of $\widetilde{t}^i_j > t^i_j$, i.e., buying more or selling less of commodity j, has to be compensated by some decrease in $\widetilde{t}^i_k < t^i_k$ (selling more or buying less of commodity k) in order to stay within the budget set $B^i(\sigma|\widetilde{\sigma}^i)$. But as we have seen before, such changes in the demand/supply schedule and net trades can only be obtained at prices which are less favorable than the equilibrium prices p_j resp. p_k. Thus the budget set $B^i(\sigma|\widetilde{\sigma}^i)$ of agent i's feasible deviations $(\sigma|\widetilde{\sigma}^i)$ from $(\sigma|\sigma^i)$ is a subset of $B^i(p)$ of Walrasian trades, since in $B^i(p)$ he can trade freely at fixed prices p, which are at least as favorable as those associated with $(\sigma|\widetilde{\sigma}^i)$. But t^i is $\succeq^i$-optimal in the set $B^i(p)$ (since the EP is a CE). Hence t^i is also $\succeq^i$-optimal in the set $B^i(\sigma|\widetilde{\sigma}^i) \subset B^i(p)$. □

3 Strategic Market Game: Implementing Walras Equilibria with an Infinitesimal Market Maker

The foregoing analysis can be recast in terms of a full-fledged strategic market game. We shall adopt the perspective of the mechanism design literature. The aim is to *prescribe* a market mechanism which Nash-implements the Walras correspondence. Of course Maskin's well known results (see Maskin (1999)) imply that this is impossible unless the domain of economies is restricted so as to ensure that the final Walrasian consumption is strictly in the interior of $\mathbb{R}^k_+$ for each agent. But we shall place no such restrictions here. Instead we shall imagine a "market maker" who has inventory of $\varepsilon_j > 0$ units of each commodity $j \in K \equiv \{1, ..., k\}$ and who is ready to bring them to market if any seller reneges on his promise to deliver, thereby giving the buyers *something* to look forward to. We show that *no matter how small* $\varepsilon = (\varepsilon_1, ..., \varepsilon_k)$ is, so long as it is positive, CE are implemented as SE. Our analysis thus sheds some light on Maskin's result. It shows that the *breakdown of the desired mechanism is not drastic, but of the size of an arbitrarily small* ε. Moreover, given the leeway of ε, we do not have to imagine esoteric mechanisms (such as those which entail the strategic announcement of unboundedly large integers – see, again, Maskin (1999)) : the simple double auction, in its smooth incarnation, will do the job.

The point of our analysis in Section 3 is not only that Maskin's result on the impossibility of Nash-implementation of non-interior CE can be overcome with an infinitesimal market maker. Nor is it to add to the list of abstract mechanisms which implement the Walras correspondence. Many such have already been presented (see, e.g., Hurwicz (1979), Hurwicz, Maskin, and Postlewaite (1980), Postlewaite (1985), Schmeidler (1980), Giraud and Stahn (2003) – all of which, incidentally, require at least three agents, in addition to interior CE, and bypass the case of a bilateral monopoly). We are instead inspired by the fact that the double auction has a long and rich history, not only in academia, but in real market processes (see Friedman and Rust (1993) for an excellent survey). Indeed, our analysis reveals that a "smoothened" version of the double auction will make for efficiency and help to break monopoly power. It thereby implies that, if the "price-jumps" permitted in bidders' strategies are reduced by mandate of the auction-designer, every such reduction will come with efficiency gains. To that extent, we hope that our analysis below will also be of some interest to applied economists who are concerned with the general properties of double auctions.

Prior to the formalities, it might help to point out an essential feature of anonymity in our market game. Agents submit supply and demand curves as before. In addition, each seller is required to put up collateral to cover his sales. The decision of *how much* of his endowment to reveal, by way of collateral, is left to the seller as a strategic option. Next, the inventories $\varepsilon = (\varepsilon_j)_{j \in K}$ of the infinitesimal market maker are fixed *invariant* of the initial endowments $(e^i)_{i \in N}$ in the economy. The quantity ε_j is made available when the collateral of any seller fails to cover his imputed sales on market j – these sales being determined, in turn, solely from the (supply and demand) curves submitted to the market. Thus the market mechanism *does not know – or need to know* – agents' private characteristics. It operates on only the curves and collaterals that agents send to the market of their own strategic volition.

Let us turn to a precise description of our game.

3.1 The Subeconomy $\mathcal{E}_J$

It will be useful to define subeconomies $\mathcal{E}_J$ of the whole economy $\mathcal{E} = (e^i, \succeq_i)_{i\in N}$ for any subset $J \subset K \equiv \{1, ..., k\}$ of commodities. For a vector $y \in \mathbb{R}^K$, denote $y_J \equiv (y_j)_{j\in J} \in \mathbb{R}^J$. Then the set of agents in $\mathcal{E}_J$ is $\{i \in N : e^i_J \neq 0\}$, with endowments e^i_J and preferences $\succeq_{i,J}$ on $\mathbb{R}^J_+$ given by the rule: $z \succeq_{i,J} y$ iff $(z, e^i_{K\backslash J} \succeq_i (y, e^i_{K\backslash J})$.

3.2 Strategy Sets

There is a market for each commodity, as before. An agent must enter each market either as a buyer or as a seller (and, for simplicity, not both). If i enters as a buyer for commodity j, he must submit a strategic demand function $d^i_j : \mathbb{R}_+ \longrightarrow \mathbb{R}_{++}$ which is non-increasing, *and* smooth (i.e., continuously differentiable)[6]. The interpretation is that i is willing to pay $\int_0^t d^i_j(q)dq$ units of "fiat money" in order to purchase t units of commodity j.

In the same vein, if i enters market j as a seller he must submit a strategic supply function $s^i_j : \mathbb{R}_+ \longrightarrow \mathbb{R}_{++}$ which is non-decreasing, smooth and (only for ease of presentation) satisfies $\lim\limits_{q\to\infty} s^i_j(q) = \infty$. In *addition*, i must put up $\tilde{\theta}^i_j > 0$ (with $\tilde{\theta}^i_j \leq e^i_j$) as "collateral" for his intention to sell j. Finally we stipulate that each agent must enter at least one market as a seller. Thus the strategy set $\sum^i$ of agent i is given by

$$\begin{aligned} \textstyle\sum^i \;=\; & \{(d^i_j, s^i_j, \tilde{\theta}^i_j)_{j\in K} : \text{one, and only one,} \\ & \text{of } d^i_j, s^i_j \text{ is } \phi \text{ for every } j;\ s^i_j \neq \phi \text{ for at} \\ & \text{least one } j;\ 0 < \tilde{\theta}^i_j \leq e^i_j \text{ if } s^i_j \neq \phi; \\ & \tilde{\theta}^i_j = 0 \text{ if } s^i_j = \phi\} \end{aligned}$$

where the functions d^i_j, s^i_j satisfy the conditions mentioned.

[6]If i does not enter market j as a buyer (or, seller), we write $d^i_j = \phi$ (or, $s^i_j = \phi$). The symbol ϕ means that the curve referred to is missing.

3.3 Outcomes

The market does a sequence of computations based on the N-tuple

$$\sigma \equiv (\sigma^i)_{i\in N} \in \mathop{\mathbf{X}}_{i\in N} \sum\nolimits^i$$

of submitted strategies, in order to impute commodity trades and monetary payments to the agents. The idea is simple. Trade is allowed up to the intersection of the aggregate demand and supply curves, with priority accorded to the higher (lower) priced buyer (seller). But two kinds of default can occur. An agent may be unable or unwilling to deliver the goods he is called upon to. Or else he may go into a budget deficit when his purchases exceed the proceeds of his sales. We require that either kind of default be severely punished. One may think here of the obvious stipulation that the entire initial endowment of any defaulter is confiscated (as in, e.g., Peck, Shell, and Spear (1992), or Weyers (1999) or Giraud and Stahn (2003)); or that (as in Shubik and Wilson (1977)) sufficient disutility is inflicted on him by extraneous (unmodeled) means . This does make for a very swift description of the mechanism. But it presupposes that the market has knowledge of agents' private characteristics. In contrast, as already pointed out, the market is blind to them in our scenario: it merely confiscates the entire collateral of any agent who defaults (either on delivery or on budget balance) and prevents him from trading. The confiscated goods are, of course, made available to buyers. The only subtlety is that default on delivery must be dealt with first, since this affects what buyers purchase, and thereby budget balances. We spell the process out precisely:

Step 1 Compute the aggregate demand D_j^σ and aggregate supply S_j^σ for each $j \in K$ as before. (Those curves which are missing are naturally ignored in the aggregation. If $s_j^i = \phi$ for *all* i , then the aggregate supply S_j^σ is also deemed missing and we write $S_j^\sigma = \phi$. Similarly $D_j^\sigma = \phi$ if $d_j^i = \phi$ for *all* i .)

Step 2 Compute the set $J \subset K$ of markets in which D_j^σ and S_j^σ intersect[7] (at, necessarily, a unique price p_j - see Figure 2).

[7]At markets $j \in K \backslash J$, the intersection fails to occur either because S_j lies above D_j, or because one of the curves S_j or D_j is missing.

Step 3 In each market $j \in J$, compute sales by agents until the price p_j, rationing proportionately quantities offered for sale at the margin price p_j in the event that there is excess supply at p_j (see Figure 2). Denote these sales $(\theta_j^i)_{i \in N}$. (Some θ_j^i could be zero, provided $s_j^i = \phi$ or $s_j^i(0) > p_j$.)

If $\theta_j^i > \tilde{\theta}_j^i$ for *some* $j \in J$ (i.e., i's collateral fails to cover his imputed sale θ_j^i at some market), then i is declared a "*defaulter*" and forbidden to trade across *all* markets, and his collateral is confiscated at *every* market that he submitted them to.

Step 4 At each $j \in J$, define

$$Q_j^\sigma = \begin{cases} \sum_{i \in N} \theta_j^i, \text{ if there is no seller-default at } j \\ \varepsilon_j + \sum_{i \in N} \min \left\{ \theta_j^i, \tilde{\theta}_j^i \right\} \text{ otherwise} \end{cases}$$

(Note that the infinitesimal inventory ε_j is made available precisely when collaterals fail to cover sales of commodity j). The market maker now allocates Q_j^σ to buyers on D_j^σ, starting at the highest price $D_j^\sigma(0)$ in D_j^σ and rationing proportionately the demand at the margin price $D_j^\sigma(Q_j)$ if necessary (i.e., if there is excess demand at this price). Denote these purchases $(\varphi_j^i)_{i \in N}$. If i is already a defaulter in Step 3, he is ignored; otherwise his net debt is computed:

$$\Delta^i = \sum_{j \in J} \int_0^{\varphi_j^i} d_j^i(q) dq - \sum_{j \in J} \int_0^{\theta_j^i} s_j^i(q) dq$$

(For $d_j^i = \phi$ or $s_j^i = \phi$, the integral is taken to be zero.) If $\Delta^i > 0$, then again i is declared a "defaulter" and dealt with as before, i.e., his collateral is confiscated at every market in which he put them up and he is forbidden from trading.

Remark : Our confiscation scheme can equivalently be described as follows. Each seller of good j is required to put up any strictly positive quantity of j by way of collateral and, if the market calls upon him to deliver more, he must do so out of his remaining endowment of j or else forfeit his collateral. On the other hand, if his collateral exceeds the delivery he has to make, the excess collateral is returned to him. We rolled the two steps into one for brevity of exposition.

.... Insert Figure 2 here!................

3.4 Payoffs

Agents $i \in N$ who are not defaulters (as in Step 3 or in Step 4) buy φ_j^i and sell θ_j^i in markets $j \in J$. They obtain payoff $u^i(x^i)$ where

$$x_j^i = \begin{cases} e_j^i + \varphi_j^i - \theta_j^i & \text{if } j \in J \\ e_j^i & \text{if } j \in K \backslash J \end{cases}$$

Defaulting agents i obtain payoff $u^i(y^i)$ where

$$y_j^i = e_j^i - \tilde{\theta}_j^i \text{ for } j \in K.$$

This well defines a game Γ in strategic form on the player set N. By SE we shall mean a strategic (Nash) equilibrium in pure strategies of the game Γ.

3.5 Active SE are Walrasian

Define a market to be *active* in an SE if there is positive trade at that market.

Proposition 4. *At any SE with active markets J, all trade in $j \in J$ takes place at one price p_j. Moreover these prices and the final allocation constitute a CE of the economy $\mathcal{E}_J$.*

Proof. We will prove the proposition for the case $J = K$. (The same argument holds for any $J \subset K$ and the corresponding economy $\mathcal{E}_J$.)

First observe that by lowering d_j^i to $\tilde{d}_j^i$ so that $\tilde{d}_j^i(0) < S_j(0)$ and by raising s_j^i to $\tilde{s}_j^i$ so that $\tilde{s}_j^i(0) > D_j(0)$, any agent i can ensure that he does not trade and so end up consuming his initial endowment e^i. But if i defaults, his utility is less than that of e^i, since he loses his collateral in at least one market and purchases nowhere. We conclude that there is no default in an SE.

Next we assert that (at an SE) in each market j all trade must be taking place at the intersection price p_j. The proof of this is similar to that of Lemma 1. Indeed, no more than the money paid out by agent-buyers goes to agent-sellers, implying $\sum_{i \in N} \Delta^i \geq 0$. But no default also implies $\Delta^i \leq 0$ for all $i \in N$. We conclude that $\Delta^i = 0$ for all $i \in N$. Now if any purchase took place above p_j or any sale below p_j in *some* market j, then (since purchases [or, sales] occur at prices $\geq$ [$or, \leq$] the intersection price at *every* market), we would have: total money paid out by agents across all markets > total money received by agents across all markets. This would imply $\Delta^i < 0$ for some i, a contradiction, proving our assertion.

Consider the bundles that an agent i can obtain via unilateral deviation from his own strategy at the SE. First suppose i is a buyer of commodity j at the SE.

Case 1 There exists at least one other active buyer of j at the SE, or else there is excess supply of j at the SE price $p_{j.}$

In this case, i can buy slightly more of j at the price p_j by simply demanding a slightly higher quantity at p_j. The maneuver works for i even if he is the sole buyer of j. This is because there is no default on seller deliveries in SE - as we just saw - and, consequently, the total collateral put up by sellers covers the total purchase of j prior to i's deviation. But i's deviation is *unilateral* and sellers hold their collaterals fixed while i deviates. If there was excess collateral in SE to begin with, i can buy more on account of the excess; otherwise the inventory of ε_j comes into play (see the first display of Step 4), still enabling i to buy more.

Case 2 Case 1 fails, i.e., i is the sole buyer of j and there is no excess supply of j at the SE price p_j.

In this case, i can demand a little more at a slightly higher price (i.e., raise the flat part of his demand curve, keeping it flat till it intersects S_j). Since S_j is continuously differentiable, the extra quantity purchased by i as well as his expenditure, will vary smoothly with the rise in the intersection price. (The fact that i can indeed buy a little more is once again assured by the infinitesimal inventory of the market maker.)

By a similar argument, i can sell a little more of any commodity j' that he was selling at the SE, either at the same price or at a price that is slightly lower and varies smoothly with the extra quantity sold.

Clearly i can *reduce* his sale and purchase and get the same price as at the SE.

Thus it is feasible for i to enhance trade a *little* beyond his SE trade in a smooth manner. More precisely, he can get consumption bundles on a smooth ε-extension $M(\varepsilon)$ of the flat part of his achievable set of bundles (where the extension is computed using prices smoothly increasing/decreasing away from p_j in accordance with the D_j/S_j curves). The situation is depicted in Figure 1, with the curved bold line extended only slightly beyond the flat part, and representing $M(\varepsilon)$. But the argument in the proof of Proposition 1 applies, *no matter how small the smooth extension* $M(\varepsilon)$ *may be*: if x is not optimal on i's Walrasian budget set, then there exists a point z^* on $M(\varepsilon)$ which yields more utility to i than x, contradicting that i has optimized. $\square$

Define an SE to be *active* if all markets are active in it. Then Proposition 3 implies

Proposition 5. *The prices and allocations at an active SE are Walrasian .*

Remark (Robustness of SE) : At our SE, an agent cannot profit by a unilateral deviation to arbitrary piecewise-continuous monotonic demand and supply curves (e.g., kinked curves, or, worse, step-functions as in Dubey (1982)). This is so because, via such deviations, all he can accomplish is to buy more (or sell more) at prices at least as high (or at least as low) as the SE prices. But since the SE is a CE, he does not even have incentive to trade more at the SE prices. Of course, the SE must first emerge from a game with smooth strategies.

Remark (The Infinitesimal Market Maker) : It should be noted that our infinitesimal market maker is not called upon to take any action at the SE of our strategic game. He only lurks in the background. It is enough for every agent i to *believe* that the market maker would make available the infinitesimal inventory ε, were i to unilaterally deviate from SE and thereby trigger a situation in which some sellers of commodity j are unable to deliver on their promises. The belief in the market maker ensures that he is never called upon to prove his existence (see also Proposition 6 in Section 3.7). We feel that this role of the market maker is not pure mathematical gimmickry, but has counterparts in the real world. One need only think of a broker who has a small inventory of company stocks from the past, and who is willing to make them available to his buyer clients to mitigate seller default on deliveries.

Remark (Interior Economies): Define the economy $\mathcal{E} = (e^i, \succeq_i)_{i \in N}$ to be *interior* if, for all $i \in N$,

$$\left\{x \in \mathbb{R}^k_+ : x \succeq_i e^i\right\} \subset \mathbb{R}^k_{++}.$$

Such economies form a standard domain in the mechanism design literature (see, e.g., Maskin (1999)). Consider our game *without* the market maker, take $\varepsilon_j = 0$ in Step 4; and with the stipulation that collaterals must *strictly* cover sales (i.e. $\theta^i_j < \tilde{\theta}^i_j$), otherwise agent i is deemed to have defaulted on his delivery. Then Proposition 3 and 4 still hold by the same proofs, showing standard Nash-implementation of the Walras correspondence on the domain of interior economies. To check this, one need only note that in the unilateral deviation, considered in the proof of Proposition 3, there will always be excess collateral prior to the deviation, obviating the need for the infinitesimal market maker.

3.6 Walrasian outcomes are achieved at SE

Proposition 6. *The prices and allocations at any CE can be achieved at an SE.*

Proof. Let z_i denote the CE consumption of i and $t^i = x^i - e^i$ his net trade. For j such that $t^i_j \geq 0$, let i offer to buy from 0 till t^i_j at the CE price p_j (and to buy more at smoothly and strictly declining prices); and, for j such that $t_j^i < 0$, let him offer to sell from 0 till $|t_j^i|$ at the CE price p_j (and to sell more at smoothly and strictly increasing prices). Finally let $\tilde{\theta}_j^i \in [|t_j^i|, e_j]$ when he is selling j. It is clear that these strategies reproduce the CE prices and consumptions. They also constitute an SE : having optimized on his Walrasian budget set, no agent wants to buy or sell more at the CE prices; and, via unilateral deviation in strategy, he can only buy (sell) at prices higher (lower) than CE prices - which is even worse for him. □

3.7 Refined Nash Equilibria

The gap in our analysis pertains to SE which are not active. These fall outside the purview of Proposition 4 and may well fail to be Walrasian. Indeed suppose all agents take it into their heads to send crazy orders, with sellers asking for exorbitantly high prices and buyers offering absurdly low prices. This will clearly constitute an SE at which all markets are inactive.

To eliminate such spurious inactivity, we introduce an equilibrium refinement (Proposition 7 below). In the process, we also embed the infinitesimal market maker into the trembles of the refinement (Proposition 6 below).

Fix the economy $(e^i, \succeq_i)_{i \in N}$ and let Γ_ε denote the strategic market game when the market maker has inventories $\varepsilon = (\varepsilon_1, ...\varepsilon_k) \in \mathbb{R}^k_{++}$ of the various commodities. Thus Γ_0 is the game without the market maker.

We shall say that an SE σ of Γ_0 is *refined* if there exist SE $\sigma(\varepsilon)$ of Γ_ε such that $\sigma(\varepsilon) \longrightarrow \sigma$ as $\varepsilon \longrightarrow 0$.

It is immediate that the market maker can be removed from the foreground and put into the ε-trembles of the refinement process, so that Proposition 4 may be reworded :

Proposition 7. *Active, refined SE of* Γ_0 *coincide in prices and allocations with the CE of the underlying economy* $(e^i, \succeq_i)_{i \in N}$.

It may be useful to compare our refinement with that of other models, which also invoke an ε-market maker (see, e.g., Dubey and Shubik (1978) and its variants). In those models, agents must think of ε-perturbed games G_ε in which the market maker is trading up to ε in all markets, not just off the SE play but *in* SE. Typically G_ε has multiple SE, all of which are different from the candidate SE of the original game G_0 . The agents must coordinate beliefs on the same SE of G_ε as an approximation of the candidate SE. In contrast, in our model here, the supply and demand curves, and the collaterals, are throughout given objectively by the candidate strategies. They constitute an SE not only of the original game Γ_0 without the market maker, but also of all the perturbed games Γ_ε. The ε-market maker need only be conjectured in Γ_ε, off the fixed SE play, by an agent when he unilaterally deviates. Furthermore the conjecture is rudimentary: each agent i thinks – *without* varying the candidate strategies of the others – that an extra supply ε_j will be available at market j if collaterals fail to cover the sales. Thus, both at a conceptual and interpretational level, and as a mathematical device, our refinement is simplicity itself compared to that of Dubey and Shubik (1978)

We now strengthen the notion of refinement to allow for inactive markets. Once again our notion is simpler in that no wholly new SE of the ε-perturbed game Γ_ε need be coordinated upon by the agents.

Imagine that, in our game Γ_ε, the market maker further endeavors to bolster trade by offering to buy (and, sell) up to $\tilde{\varepsilon}_j > 0$ units of commodity j at some common price $\tilde{p}_j$ and to buy (and, sell) more at smoothly decreasing (and, increasing) prices. Treating the market maker as a strategic dummy, and postulating that he creates the commodity and the money that the mechanism calls upon him to deliver, the game is well-defined even after some subset $J \subset K$ of markets are $\tilde{\varepsilon}_j - \tilde{p}_j$ – perturbed as described. We shall say that an SE $\sigma(\varepsilon)$ of Γ_ε is "*-refined" if there exist $\tilde{\varepsilon}_j - \tilde{p}_j$ – perturbations of the inactive markets in $\sigma(\varepsilon)$ that do not disturb the SE $\sigma(\varepsilon)$. An SE σ of Γ_0 is *strongly refined* if there exist *-refined SE $\sigma(\varepsilon)$ of Γ_ε such that $\lim \sigma(\varepsilon) \longrightarrow \sigma$ as $\varepsilon \longrightarrow 0$. Then we obtain (by an obvious proof):

Proposition 8. *Strongly refined SE of* Γ_0 *coincide in prices and allocations with the CE of* $(e^i, \succeq_i)_{i \in N}$

References

Aumann, R. (1964): "Markets with a Continuum of Traders," *Econometrica*, 32, 39–50.

Black-Scholes (1973): "The pricing of options and corporate liabilities," *Journal of Political Economy*, 81, 637–654.

Coase, R. (1972): "Durability and Monopoly," *The Journal of Law and Economics*, XV(1), 143–149.

Dubey, P. (1982): "Price-Quantity Strategic Market Games," *Econometrica*, 50(1), 111–126.

——— (1994): "Strategic Market Games: A Survey of Recent Results," *Mertens, J.F. and S.Sorin (eds), Game-Theoretic Methods in General Equilibrium Theory.*

Dubey, P., and M. Shubik (1978): "The Noncooperative Equilibria of a Closed Trading Economy with Market Supply and Bidding Strategies," *Journal of Economic Theory*, 17(1), 1–20.

Friedman, D., and J. Rust (1993): "The Double Auction Market," *Addison-Wesley.*

Giraud, G., and H. Stahn (2003): "Nash-implementation of Competitive Equilibria via a Bounded Mechanism," *CERNSEM Disc. Paper.*

Hurwicz, L. (1979): "Outcome Functions yielding Walrasian and Lindahl Allocations at Nash Equilibrium Points," *Review of Economic Studies*, 46, 217–225.

Hurwicz, L., E. Maskin, and A. Postlewaite (1980): "Feasible Implementation of Social Choice Correspondences by Nash Equilibria," *Mimeo.*

Malinvaud, E. (1974): *Lectures on Microeconomic Theory.* North-Holland.

Mas-Colell, A. (1980): "Noncooperative Approaches to the Theory of Perfect Competition: Presentation," *Journal of Economic Theory*, 22(2), 121–135.

Maskin, E. (1999): "Nash Equilibrium and Welfare Optimality," *Review of Economic Studies*, 66(1), 23–38.

Mertens, J. (2003): "The Limit-Price Mechanism," *Journal of Mathematical Economics*, 39(5-6), 433–528.

Peck, J., K. Shell, and S. Spear (1992): "The Market Game: Existence and Structure of Equilibrium," *Journal of Mathematical Economics*, 21, 271–299.

Postlewaite, A. (1985): "Implementation via Nash Equilibria in Economic Environments," *Hurwics, L. and D. Schmeidler and H. Sonnenschein (eds), Social Goals and Social Organization: A Volume in Memory of Elisha Pazner*, Cambridge Univ. Press, 205–228.

Sahi, S., and S. Yao (1989): "The Non-cooperative Equilibria of a Trading Economy with Complete Markets and Consistent Prices," *Journal of Mathematical Economics*, 18, 315–346.

Schmeidler, D. (1980): "Walrasian Analysis via Strategic Outcome Functions," *Econometrica*, 48, 1585–1594.

Shubik, M., and C. Wilson (1977): "The Optimal Bankruptcy Rule in a Trading Economy Using Fiat Money," *Zeitschrift f. Nationaloekonomie*, 37(3-4), 337–354.

Stigler, G. (1965): "Perfect Competition, historically contemplated," *Essays in the History of Economics, Quart. 8, University of Chicago Press.*

Weyers, S. (1999): "Uncertainty and Insurance in Strategic Market Games," *Economic Theory*, 14, 181–201.

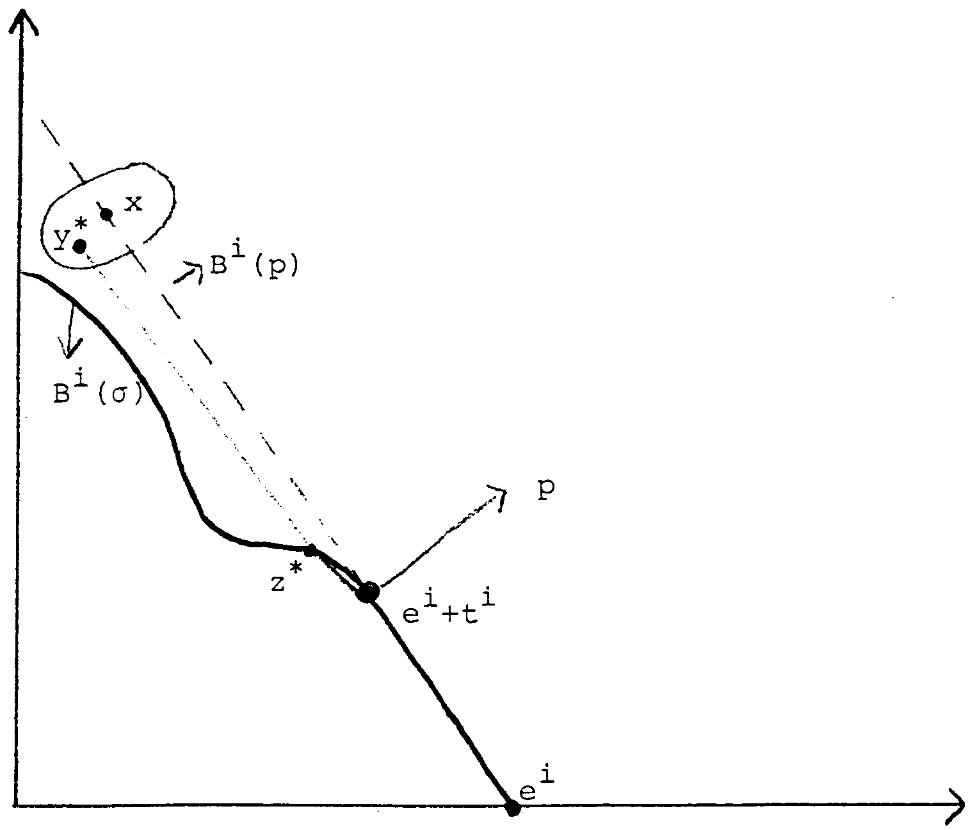
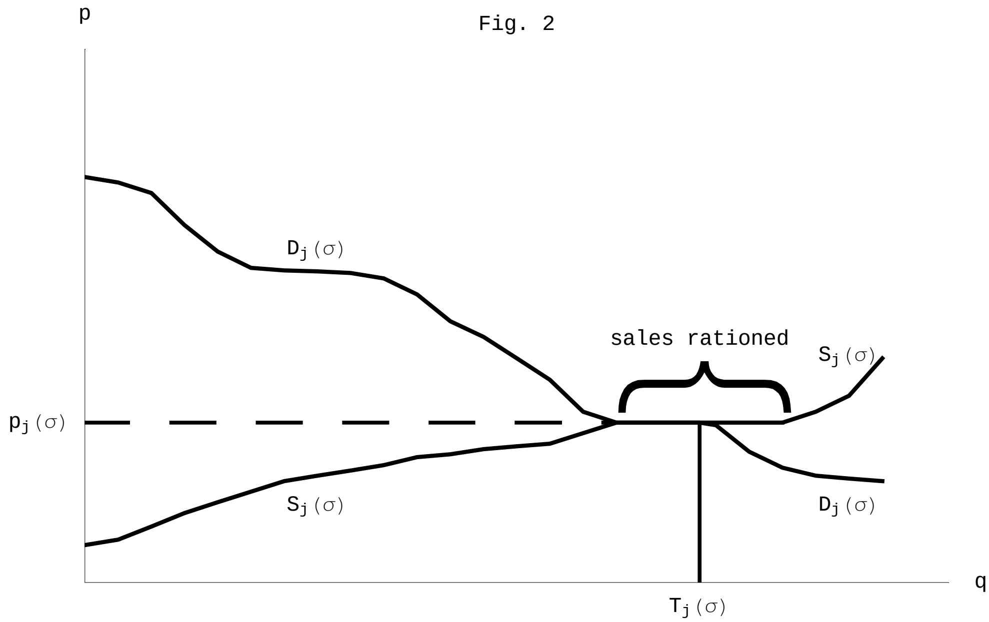


Fig. 1

Fig. 2


p
$D_j(\sigma)$
sales rationed
$S_j(\sigma)$
$p_j(\sigma)$
$S_j(\sigma)$
$D_j(\sigma)$
q
$T_j(\sigma)$