BONN ECON DISCUSSION PAPERS

Discussion Paper 6/2008

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March 2008

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Limited Liability and the Risk-Incentive Relationship*

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Abstract

Several empirical findings have challenged the traditional view on the trade-off between risk and incentives. By combining risk aversion and limited liability in a standard principal-agent model the empirical puzzle on the positive relationship between risk and incentives can be explained. Increasing risk leads to a less informative performance signal. Under limited liability, the principal may optimally react by increasing the weight on the signal and, hence, choosing higher-powered incentives.

Key Words: moral hazard, limited liability, risk-incentive relationship

JEL Classification: D82, D86

*Financial support by the Deutsche Forschungsgemeinschaft (DFG), grant SFB/TR 15 ("Governance and the Efficiency of Economic Systems"), is gratefully acknowledged.

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1 Introduction

In the well-known principal-agent model with a risk neutral principal and a risk averse agent we face the typical negative relationship between exogenous risk and incentives: if risk is large and, hence, the risk premium (or risk costs) from high-powered incentives would be high the principal will optimally choose low incentives for the agent. Contrary, if risk is small so that risk costs from induced incentives are rather low, the principal will prefer high-powered incentives.

Prendergast (2002) refers to several empirical studies which point to a *positive* relation between risk and incentives and therefore challenge the traditional view. Consequently, he offers alternative explanations for this empirical puzzle. In his modeling, he assumes that the agent is risk neutral in order to abstract from the classical trade-off. This assumption may be crucial. For example, if in Prendergast (2002) the agent were risk averse and high risk makes the principal prefer output-based to input-based contracts, the well-known traditional trade-off would apply again. Another explanation for the empirical puzzle is introduced by Raith (2003). There, the positive relationship between risk and incentives comes as a by-product. Raith assumes that agents are risk averse so that incentives indeed imply risk costs. However, since agents face a binding participation constraint and principals always realize zero profits due to market competition, neither the agent nor the principal (but society) has to bear the risk costs from inducing incentives. Hence, less extreme competition may lead to different findings concerning the trade-off. Finally, Wright (2004) and Serfes (2005) independently develop a matching approach that can explain the puzzle. If competition makes less risk averse agents match with more risky principals, the outcome may be a positive relation between risks and incentives.

In this note, we offer an alternative explanation for the empirical puzzle which does neither need risk neutral agents nor market competition as a driving force. We only combine two standard contracting problems – risk aversion and limited liability – which should be present in many circumstances so that this approach seems to be the most natural explanation for a positive relationship between risk and incentives. In our model, the agent earns a non-negative rent. If risk increases and, hence, the performance signal becomes less informative, it can pay for the principal to increase the weight on this signal by choosing higher-powered incentives. Since the agent's individual rationality constraint is non-binding at the optimum, the principal's additional incentive costs will not increase too steeply: they only increase in terms of expected wage payments whereas the progressively increasing effort costs simply reduce the agent's rent.

Our note is organized as follows. The next section introduces the model. Section 3 analyzes the possibility of a positive risk-incentive relationship under the optimal contract. Sections 4 contains an illustrating example. Section 5 will conclude.

2 The Model

We consider a typical moral-hazard problem between a risk averse agent and a risk neutral principal based on the binary-signal model used by Demougin and Garvie (1991) and Demougin and Fluet (2001), for example. The agent chooses a non-negative effort a that is unobservable to the principal. The non-contractible value of this effort to the principal is described by the function $v(a)$ with $v'(a) > 0$ and $v''(a) < 0$. In choosing a, the agent incurs a private cost $c(a)$, which, together with his utility $u(w)$ from wealth w, describes his preferences by the utility function $U(w, a) = u(w) - c(a)$. We assume that $u(0) = 0$, $u' > 0$, $u'' \leq 0$, and $c(a)' > 0$, $c(a)'' > 0, \forall a > 0$. To ensure an interior solution we assume $c(0) = 0$, $c'(0) = 0$ and $\lim_{a \to \infty} c'(a) = \infty$. The agent's reservation utility is given by $\bar{U} = 0$.

Principal and agent observe a contractible signal $s \in \left\{s^L, s^H\right\}$ on the agent's performance with $s^H > s^L$. The outcome $s = s^H$ is favorable information about the agent's effort choice in the sense of Milgrom (1981). Let the probability of this favorable outcome be $p(a)$ with $p'(a) > 0$ (strict monotone likelihood ratio property) and $p''(a) < 0$ (convexity of the distribution function condition). Since the signal s is the only observable and verifiable information on the agent's performance, the principal offers a payment scheme $w(s)$ with $w(s^L) = w_L$ and $w(s^H) = w_H$. We assume that the agent is protected by limited liability in the sense of $w_L, w_H \geq 0$.

To make the analysis of the risk-incentive trade-off more precise, we first provide definitions of *more risky* (less informative) outcomes and *higher-powered* or stronger incentives. To that purpose, we compare different binary signals $s, \hat{s} \in \{s^L, s^H\}$ with probabilities $p(a)$ and $\hat{p}(a)$ for the favorable outcome s^H. Thus, the support of the outcome distribution does not change, but probabilities do. We define a performance signal $\hat{s}$ to be *more risky* (less informative with respect to the agent's action) than a signal s, if $\hat{s}$ is a garbling from s, i.e. if there exists a number $b \in (0, 1/2]$ such that

$$\hat{p}(a) = (1-b) \cdot p(a) + b \cdot (1 - p(a)) = b + (1-2b)p(a).$$

This is a special case of Blackwell informativeness, where the garbling is symmetric among realizations.[1] We will use this garbling in the following. $b \in (0, 1/2]$ is without loss of generality. It only makes sure that the favorable outcome in s is also the favorable one in $\hat{s}$. Garbling can easily be interpreted in terms of risk in the binary model since the variance of signal s is $p(a)(1 - p(a))(s^H - s^L)^2$, which is maximized for $p(a) = 1/2$. Obviously, $(1-b) \cdot p(a) + b \cdot (1-p(a))$ is closer to $1/2$ than $p(a)$ for all $p(a) \in [0,1]$ and $b \in (0, 1/2]$.

Finally, we specify the meaning of higher-powered incentives and a positive risk-incentive relationship. The payment scheme

$$\hat{w} = \begin{cases} \hat{w}_H & \text{if } \hat{s} = s^H \\ \hat{w}_L & \text{if } \hat{s} = s^L \end{cases}$$

based on the signal $\hat{s}$ is *higher-powered* than the scheme

$$w = \begin{cases} w_H & \text{if } s = s^H \\ w_L & \text{if } s = s^L \end{cases}$$

based on s if $\hat{w}_H - \hat{w}_L > w_H - w_L$. We will speak of a *positive risk-incentive*

[1] Note that our findings will qualitatively hold for asymmetric garbling. Extending the analysis to asymmetric garbling can only increase the set of possible cases for which a positive risk-incentive relation holds.

relationship if

$$\left.\frac{\partial\left(\hat{w}_H-\hat{w}_L\right)}{\partial b}\right|_{b=0}>0,$$

that is if marginal garbling yields higher-powered incentives.

3 Risk and Incentives

In setting w_L and w_H, the principal's aim is to maximize his profit net of expected wage payments, provided the agent accepts the contract and chooses the desired action, and the wages fulfill the limited-liability constraint:

$$\begin{aligned}
&\max_{w_L,w_H,a} v(a)-p(a)w_H-(1-p(a))\,w_L \\
&\text{subject to} \quad p(a)u(w_H)+(1-p(a))u(w_L)-c(a)\geq 0, && \text{(IR)}\\
&\qquad a\in\arg\max_{\alpha}\left\{p(\alpha)u(w_H)+(1-p(\alpha))u(w_L)-c(\alpha)\right\}, && \text{(IC)}\\
&\qquad w_H,w_L\geq 0. && \text{(LL)}
\end{aligned}$$

Since both the monotone likelihood ratio property and the convexity of the distribution function condition hold, the incentive compatibility constraint can be replaced by the first-order condition

$$p'(a)\left(u(w_H)-u(w_L)\right)=c'(a).$$

In the optimal solution, the individual rationality constraint (IC) will be non-binding and the limited-liability condition (LL) will be binding for w_L. To see this, note that the agent can always obtain a non-negative expected utility by accepting the contract and choosing zero effort. Hence, the agent will always earn a non-negative rent. Since w_L decreases incentives but increases labor costs, the principal will optimally choose $w_L^*=0$. The optimization problem then reduces to

$$\begin{aligned}
&\max_{w_H,a}\pi\left(w_H\right)=v(a)-p(a)w_H\\
&\text{subject to} \quad p'(a)u(w_H)=c'(a).
\end{aligned}$$

Now we introduce our symmetric garbling from Section 2 by replacing $p(a)$ with $\hat{p}(a) = (1-b)p(a) + b(1-p(a))$ ($b \in (0, 1/2]$) so that the corresponding optimization problem can be written as

$$\max_{w_H, a} \pi(w_H) = v(a) - [b + (1-2b)\, p(a)]\, w_H$$
$$\text{subject to} \qquad (1-2b)\, p'(a) u(w_H) = c'(a). \qquad \text{(IC')}$$

The incentive constraint (IC') shows that a more risky performance signal requires higher-powered incentives for implementing the same effort level a.

Proposition 1 *To implement a given action a at minimal cost, higher-powered incentives are necessary under a more risky performance measure.*

Proof. Obvious from (IC'). ■

The intuition is straightforward: if the outcome becomes more risky and, hence, the performance signal is less informative about the agent's effort choice, incentives will decline. To restore former incentives, the principal has to choose a higher weight for the performance signal, i.e. a higher value of w_H.[2] However, it is not clear whether the principal should optimally react to increased risk by increasing incentives as well: a higher value of w_H also increases the principal's labor cost in case of a favorable performance signal and, therefore, the agent's rent. The principal is therefore likely to reduce the implemented action in order to reduce the required wage payment. Which of the two countervailing effects dominates is not clear from the outset.

In case of a finite action space $A = \{a_1, \ldots, a_n\}$, it is easy to construct situations in which a higher wage spread will result: if under s the principal is not indifferent between the implemented action and another, an incremental increase in risk will not result in a change of the desired action, and a positive risk-incentive relationship will apply:

Corollary 2 *If the principal strictly prefers the implemented action in a model with a finite action space, there is a positive risk-incentive relationship.*

[2] Note that this general finding on increased risk and higher powered incentives also holds under unlimited liability.

Proof. Let A be ordered such that $c(a_j) > c(a_{j-1})$ for $j = 2, \ldots, n$. If both the monotone likelihood ratio (MLRP) and the convexity of the distribution function condition (CDFC) as defined by Grossman and Hart (1983) hold,[3] the incentive constraint (IC) can be written as

$$(1-2b)\,[p(a_j) - p(a_{j-1})]u(w_H) \geq c(a_j) - c(a_{j-1}), \tag{IC"}$$

which is fulfilled with equality in the second-best contract. Let a_s be the optimal action to be implemented by performance measure s, with a net profit of Π_s accruing to the principal. Moreover, let $\Pi_{-s} < \Pi_s$ denote the maximum net profit under the actions $a \in A \backslash a_s$. Now consider a garbling of s. From (IC"), the cost of inducing a_s is

$$\begin{aligned} \hat{W}(a_s, b) &= \hat{p}(a_s)u^{-1}\left(\frac{c(a_s) - c(a_{s-1})}{\hat{p}(a_s) - \hat{p}(a_{s-1})}\right) \\ &= [b + (1-2b)p(a_s)]u^{-1}\left(\frac{c(a_s) - c(a_{s-1})}{(1-2b)(p(a_s) - p(a_{s-1}))}\right). \end{aligned}$$

This function is continuous in b and identical to the cost of inducing a_s without garbling for $b = 0$. Therefore, there exists a critical value $\hat{b}$ up to which $\hat{\Pi}_s = v(a_s) - W(a_s, b)$ is greater than Π_{-s}. For such values of b, action a_s will still be optimal under the garbled performance measure. The positive risk-incentive relationship then follows from proposition 1 and (IC"), respectively. ■

In case of a continuous action, let $a^* = a^*(w_H)$ be the agent's incentive compatible effort choice implicitly described by (IC') with

$$\frac{\partial a^*}{\partial w_H} = -\frac{(1-2b)\,p'(a^*)u'(w_H)}{(1-2b)\,p''(a^*)u(w_H) - c''(a^*)} > 0 \tag{1}$$

$$\text{and} \quad \frac{\partial a^*}{\partial b} = \frac{2p'(a^*)u(w_H)}{(1-2b)\,p''(a^*)u(w_H) - c''(a^*)} < 0 \tag{2}$$

so that the first-order condition to the principal's optimization problem can

[3] MLRP here is simply $p(a_j) \geq p(a_{j-1})$ for $j = 2, \ldots, n$, CDFC means that if $c(a_j) = \lambda c(a_i) + (1-\lambda)c(a_k)$ for some $\lambda \in [0,1]$, then $p(a_j) \geq \lambda p(a_i) + (1-\lambda)p(a_k)$.

be written as

$$\frac{d\pi\left(w_H, a^*\left(w_H\right)\right)}{dw_H} = v'(a^*)\frac{\partial a^*}{\partial w_H} - \left[b + \left(1-2b\right)p(a^*)\right] - \left(\left(1-2b\right)p'(a^*)\frac{\partial a^*}{\partial w_H}\right)w_H = 0. \tag{3}$$

Suppose that the principal's problem is well-behaved, that is the second-order condition

$$\begin{aligned}\frac{d^2\pi\left(w_H, a^*\left(w_H\right)\right)}{dw_H^2} &= \left(v''(a^*) - \left(1-2b\right)p''(a^*)w_H\right)\left(\frac{\partial a^*}{\partial w_H}\right)^2 \\ &+ \left(v'(a^*) - \left(1-2b\right)p'(a^*)w_H\right)\frac{\partial^2 a^*}{\partial w_H^2} \\ &- 2\left(1-2b\right)p'(a^*)\frac{\partial a^*}{\partial w_H} < 0\end{aligned}$$

is satisfied.[4]

Now we can analyze the possibility of a positive risk-incentive relationship. Implicit differentiation of (3) yields

$$\frac{\partial w_H}{\partial b} = -\frac{d^2\pi\left(w_H, a^*\left(w_H\right)\right)/dw_H db}{d^2\pi\left(w_H, a^*\left(w_H\right)\right)/dw_H^2}$$

so that

$$sign\left(\frac{\partial w_H}{\partial b}\right) = sign\left(\frac{d^2\pi\left(w_H, a^*\left(w_H\right)\right)}{dw_H db}\right).$$

The sign of this derivative depends on whether the marginal returns or the marginal costs from increasing incentives react stronger to increased risk.

[4] Note that without further specifying the underlying functions, the sign of $\frac{d^2\pi(w_H, a^*(w_H))}{dw_H^2}$ is unclear. For example, the expression $\frac{\partial^2 a^*}{\partial w_H^2}$ depends on the third derivatives of the functions $p\left(a\right)$ and $c\left(a\right)$.

We obtain

$$\frac{d^2\pi\left(w_H, a^*\left(w_H\right)\right)}{dw_H db} = \underbrace{v''(a^*)\frac{\partial a^*}{\partial b}\frac{\partial a^*}{\partial w_H}}_{A} + \underbrace{\left(v'(a^*) - (1-2b)\, p'(a^*)w_H\right)\frac{\partial^2 a^*}{\partial w_H \partial b}}_{B}$$

$$\underbrace{-\left[(1-2p(a^*)) + (1-2b)\, p'(a^*)\frac{\partial a^*}{\partial b}\right]}_{C}$$

$$\underbrace{+\, 2p'(a^*)\frac{\partial a^*}{\partial w_H}w_H}_{D} \underbrace{-\, (1-2b)\, p''(a^*)w_H\frac{\partial a^*}{\partial b}\frac{\partial a^*}{\partial w_H}}_{E}.$$

Altogether, there are five effects that determine the possibility of a positive risk-incentive relationship. Since $v\left(a\right)$ is concave, reducing a by increased risk (i.e. $\frac{\partial a^*}{\partial b}$) is favorable for creating incentives since now incentives are induced at a higher productivity level. Hence, expression A is positive.

The sign of expression B is not clear. We know that the marginal net profits from increased effort are positive since the first-order condition (3) can be rewritten as

$$\left(v'(a^*) - (1-2b)\, p'(a^*)w_H\right) = \frac{\left[b + (1-2b)\, p(a^*)\right]}{\frac{\partial a^*}{\partial w_H}}.$$

However, the mixed derivative $\frac{\partial^2 a^*}{\partial w_H \partial b}$ may be either positive or negative. It measures how marginal incentives $\frac{\partial a^*}{\partial w_H}$ react to increased risk. On the one hand, the agent may be discouraged if the performance signal becomes less informative. On the other hand, marginal incentives become more effective since productivity (in terms of the probability function $p\left(\cdot\right)$) increases with decreased a due to concavity.

Expression C describes whether the probability of the wage payment w_H is reduced or enhanced by increased risk. First, garbling makes the original probability $p(a^*)$ tend to $1/2$. Depending on whether initially $p(a^*)$ was smaller or larger than $1/2$, garbling increases or decreases the probability of the high wage payment. Second, increased risk reduces optimal effort a^* which is favorable to the principal in the sense of a decreased expected wage payment.

Expression C directly corresponds to the reaction of the agent's rent to increased incentives and risk. The rent is defined as

$$R(w_H) = [b + (1 - 2b)p(a^*)]u(w_H) - c(a^*)$$

with a^* depending on w_H and b according to equations (1) and (2). Applying the envelope theorem yields

$$\frac{dR}{dw_H} = \frac{\partial R}{\partial w_H} = [b + (1 - 2b)p(a^*)]u'(w_H) > 0.$$

Hence, increasing incentives for a given risk will increase the agent's rent. The derivative with respect to b,

$$\left[(1 - 2p(a^*)) + (1 - 2b)\, p'(a^*)\frac{\partial a^*}{\partial b}\right] u'(w_H),$$

shows that if expression C is positive, then the increase of the rent due to higher-powered incentives will decrease in risk, what will favor a positive risk-incentive relationship.

According to expression D, garbling directly reduces marginal expected costs for creating incentives.

Finally, expression E can be interpreted analogously to A. Since $p\,(a)$ is also strictly concave, a reduction of a via increased risk (i.e. $\frac{\partial a^*}{\partial b}$) leads to a higher productivity in terms of probability. If now incentives are increased, additional expected wage costs will be rather large.

To sum up, a higher risk has favorable effects on increasing incentives which may become more effective and less costly. If these positive effects dominate the negative ones in form of a higher wage payment w_H and a less informative performance signal, there will be a positive relationship between risk and incentives under limited liability. Note that the agent's limited liability is crucial for our findings as it makes creating incentives relatively cheap for the principal: the individual rationality constraint is non-binding at the optimum. Hence, increasing incentives via w_H only increases the principal's incentive costs in terms of expected money payment. However, the principal does not have to care about the steeply increasing effort costs $c(a)$, which only decrease the agent's rent.

Applying our definition from Section 2 (with $b = 0$) leads to the following result:

Proposition 3 *Let* $\frac{d^2\pi(w_H, a^*(w_H))}{dw_H^2} < 0$. *If*

$$\left(v''(a^*) - p''(a^*)w_H\right)\frac{\partial a^*}{\partial w_H}\left.\frac{\partial a^*}{\partial b}\right|_{b=0} + \left(v'(a^*) - p'(a^*)w_H\right)\left.\frac{\partial^2 a^*}{\partial w_H \partial b}\right|_{b=0}$$
$$-(1 - 2p(a^*)) + \left(2\frac{\partial a^*}{\partial w_H}w_H - \left.\frac{\partial a^*}{\partial b}\right|_{b=0}\right)p'(a^*) > 0,$$

the optimal contract will exhibit a positive risk-incentive relationship.

4 An Example

In this section, we illustrate that a positive risk-incentive relationship can easily be constructed with standard concave and convex polynomial functions. Let $p(a) = a^{\frac{1}{2}}$, $v(a) = 2a^{\frac{1}{2}}$, $u(w_H) = 2(w_H)^{\frac{1}{2}}$, and $c(a) = \frac{1}{2}a^2$.

The incentive constraint for this parameterized version of the model is given by

$$(1 - 2b)^{\frac{1}{3}}(w_H)^{\frac{1}{6}} = a^{\frac{1}{2}}.$$

Inserting into the principal's objective function $v(a) - [b + (1 - 2b)p(a)]w_H$ then leads to expected net profits

$$\pi(w_H) = 2(1 - 2b)^{\frac{1}{3}}(w_H)^{\frac{1}{6}} - \left(b + (1 - 2b)^{\frac{4}{3}}(w_H)^{\frac{1}{6}}\right)w_H.$$

The first-order condition yields

$$\pi'(w_H) = -\left(b + \frac{(7w_H(1 - 2b) - 2)(1 - 2b)^{\frac{1}{3}}}{6w_H^{\frac{5}{6}}}\right) = 0,$$

and the second-order condition

$$-\frac{1}{36w_H^{\frac{11}{6}}}\left((7w_H(1-2b)+10)(1-2b)^{\frac{1}{3}}\right)<0,$$

which is always satisfied. Using $b=0$ in the first-order condition gives the optimal wage

$$w_H=\frac{2}{7}.$$

Finally, we will have a positive risk-incentive relationship if

$$\left.\frac{\partial\pi'(w_H)}{\partial b}\right|_{b=0,w_H=\frac{2}{7}}>0.$$

We obtain

$$\begin{aligned}\left.\frac{\partial\pi'(w_H)}{\partial b}\right|_{b=0,w_H=\frac{2}{7}} &= -1-\left.\frac{(4-56w_H(1-2b))(1-2b)^{\frac{1}{3}}}{18w_H^{\frac{5}{6}}(1-2b)}\right|_{b=0,w_H=\frac{2}{7}}\\ &= 0.89365>0.\end{aligned}$$

Note that (at $b=0$) we have $p(a^*)=(a^*)^{\frac{1}{2}}=(w_H)^{\frac{1}{6}}=0.81156<1$.

5 Conclusion

Given a risk averse agent, providing incentives is costly for the principal since incentive-compatible payment leads to a positive risk premium which usually increases in the magnitude of the exogenous risk. For this reason, the standard principal-agent hidden-action model claims a negative relationship between risk and optimal incentives. Several empirical findings have challenged this traditional view. By combining risk aversion with limited liability – the two standard contracting problems given a verifiable performance signal – we obtain an explanation for a *positive* risk-incentive relationship without relying on additional assumptions from outside the textbook model.

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