Bonn Econ Discussion Papers

Discussion Paper 9/2006

A Percolation-Based Model Explaining Delayed Take-Off in New-Product Diffusion

by

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April 2006

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this version: April 2006

Abstract

A model of new-product diffusion is proposed in which a site-percolation dynamics represents socially-driven diffusion of knowledge about the product's characteristics in a population of potential buyers. A consumer buys the new product if her valuation of it is not below the price of the product announced by the firm in a given period. Our model attributes the empirical finding of a delayed "take-off" of a new product to a drift of the percolation dynamics from a non-percolating regime to a percolating regime. This drift is caused by learning-effects lowering the price of the product, or by network-effects increasing its valuation by consumers, with an increasing number of buyers.

JEL classification: C15, L15, 033

Key words: new-product diffusion, innovation adoption, spatial stochastic processes, percolation

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*We are grateful to Paul A. David for many valuable comments on an earlier version of this paper. Any conceptual and technical shortcomings of the paper belong to us.

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1 Introduction

Innovation is a central and crucial aspect of the functioning of capitalistic economies (see Schumpeter (1911, 1942)). In particular, there exists a rich literature analyzing the incentives for industrial innovation, starting with Arrow (1962). In the present paper, however, we take as given that a new product has emerged and concentrate on the time-profile of its spread in a population of consumers.

The analysis of the process of adoption of a new product (in the following termed *new-product diffusion*) constitutes an important research area in both marketing science and economics. From a practitioner's perspective, relevant questions are, for instance, how to forecast whether the new product will "take-off" (see Garber et al. (2004)), or, once it did, the level of its future sales depending on the use of elements of the marketing mix (see e.g. Bass et al. (2000); see also Chandrasekaran and Tellis (2005) for a general overview). From a more theoretical perspective, one is interested, for instance, in why consumers develop preferences for new products (see Witt (2001)), or whether such process tend to be "path-dependent" or "ergodic" (see David (1985)).

Three main approaches to quantitative modeling of the time-profile of new-product diffusion can be distinguished. First, there are phenomenological models of new-product diffusion. This literature starts with Bass (1969). His model has seen numerous refinements over the years (for an overview, see Mahajan et al. (1990, 1995)), and can reproduce the evolution of sales over a wide range of the product life cycle employing appropriate parameter fits. Second, micro-models of new-product diffusion focusing on rational individual decision-making were proposed (see, for instance, David and Olson (1986, 1992)). These models typically ascribe to consumers a high degree of sophistication, in particular they correctly foresee the future evolution of the market. The dynamics of diffusion is driven by the interplay of expectations and maximization. Third, there appeared stochastic micromodels of new-product diffusion which focus on collective effects, often with a myopic model of decision making. These models are variants of the spatial stochastic process called *percolation*[1] (see e.g. Allen (1982), Mort (1991), David and Foray (1994), Solomon et al. (2000), Goldenberg et al. (2000), Silverberg and Verspagen (2002)).

Our present model is percolation-based. It is motivated by the empirical phenomenon that in the early stages of new-product diffusion low levels of sales often persist over a prolonged period of time before a "take-off" occurs (for a detailed discussion of this phe-

[1]For an introduction to percolation and its applications see Stauffer and Aharony (1994). An advanced mathematical treatment of percolation can be found in the monograph by Grimmett (1999).

nomenon see Golder and Tellis (1997) and Geroski (2003)). Serving as a prototypical example of this phenomenon, Figure 1 (top) depicts the cumulative number of adopters of a novel agricultural technique in Iowa in the first half of the last century. The data in Figure 1 (top) is adapted from Ryan and Gross (1943). More examples of long-tailed diffusion curves along with a discussion of the phenomenon of a delayed "take-off" of new products can be found in Mort (1991) and Golder and Tellis (1997).

We find that our model provides a possible analytical explanation for delayed take-off in new-product diffusion. It does so with a myopic individual decision-making model, i.e. avoiding a self-fulfilling-prophecy mechanism relying on rational expectations. Up to our knowledge, it is the first model capable of explaining delayed take-off as a purely collective coordination phenomenon.[2] The structure of the paper is as follows: Section 2 specifies the basic model. Section 3 introduces macroscopic feedbacks and shows by Monte Carlo simulations that the latter can lead to a diffusion-dynamics exhibiting a delayed take-off. The paper concludes with a brief discussion of some additional aspects of our model.

2 The basic model

We model the process of diffusion of a new product[3] (the emergence of which is *assumed* rather than explained in our model) among a large population of consumers. Time is discrete. In any period t, a consumer may buy either one unit of the product or none, with at most one unit bought over the entire time horizon. The individual decision model of a consumer consists of three steps: firstly, learning the product's characteristics, secondly, forming an individual (subjective) valuation of it, and thirdly, comparing one's individual valuation with the price set by the producer.

An essential ingredient of our model is a "spatial" dynamics facilitating individual assessment of the product's value by each potential buyer. Underlying this dynamics is a social network – *exogenous* in the present model[4] – which we take to be a two-dimensional

[2]Delayed take-off of new products has been explained in the model of David and Olson (1986, 1992) in the context of rational expectations.

[3]We believe that our diffusion model can be applied to the more general issue of diffusion of innovations. However, that more general context would require a more specific analysis of the question of why and how innovations get adopted (see, for instance, Nelson et al. (2004)). In the present paper in the context of a new product, we confine ourselves to specifying an abstract framework using the notion valuation which is popular in abstract decision models in economics.

[4]An interesting question is how such networks emerge in social systems. This question is beyond the scope of our present investigation, but see, for instance, the paper of Schnegg (2006) for an investigation of this question in a related context.

square lattice. It can be represented by a graph, with $\mathbb{Z}^2$ as the set of nodes and a link between any two $a, b \in \mathbb{Z}^2$ if and only if $||a - b|| = 1$, with $|| \cdot ||$ denoting the Euclidean distance. In our finite model, the set of consumers is represented by a finite square-shaped subset $\Lambda \subset \mathbb{Z}^2$. Two consumers who are directly linked are called *nearest neighbors.* Each consumer – except those at the boundary of Λ – has thus four nearest neighbors.

Our particular choice of the network model is presumably not a realistic one. Alas, we are not aware of empirical studies investigating topologies of interactions in our particular context, while results of studies investigating sociological network topologies related with other types of human interactions do not appear to be a-priori transferable (see Schnegg (2006)). Yet the principle mechanism by which a "take-off dynamics" is generated in our model does not depend on the specific topology of the underlying network (with an exceptional case to be discussed in the last section of this paper).

In each period, the nearest neighbors of those consumers who bought the product in the immediately preceding period acquaint themselves with the product.[5,6] Based on that experience they form their individual valuations of the product[7] reflected in the reservation price θ_a (i.e. the highest price at which consumer a would buy). That assumption implies that it is only via experiencing the product via one's immediate social environment that a consumer forms the valuation of it. In that sense, the innovation is "socially transmitted".

We assume in our model that the transfer of "experience of the product" from one consumer to another is "neutral" in the sense that the valuation formed by consumer a does not depend on the valuation of that buyer who triggered the formation of a's valuation. Thus we specify that θ_a is a realization of the random variable Θ_a with the family $(\Theta_a)_{a\in\Lambda}$ independently identically distributed. To directly relate our basic model to the standard percolation model, each random variable is equi-distributed on $[0, 1]$.

Finally, the consumer's decision to buy the new product is the following: consumer a buys the product if her individual valuation θ_a exceeds or equals the price p.

We employ a simple specification of the supply side as consisting of a "non-maximizing" monopolist using mark-up pricing i.e. the price p is given by the formula

$$p = (1 + m)c \tag{1}$$

[5]In the first period, the dynamics is initialized by the introduction of a fixed number of early buyers located randomly in the population. The origin of such "early birds" is exogenous to our model.

[6]We assume that buyers enable all their nearest neighbors to experience the product corresponding to the case of pure site-percolation, i.e. bonds are always "open".

[7]We assume that the formation of the individual valuation θ_a is made only once thus it is not reassessed if in a later period another nearest neighbor of consumer a buys the product.

with c denoting the unit production costs and m a positive number, the time-constant mark-up. (See Blinder (1991) and Hall et al. (1997) for empirical evidence that firms indeed use mark-up pricing.) In accordance with the specification of the range of individual valuations let us assume that $p \in [0, 1]$.

The dynamics of the model specified so far is well-known from the literature on percolation models. In the following we briefly describe some basic properties of these models. In the simplest case of an (atemporal) site-percolation model with some underlying graph structure, each site of the graph is randomly assigned a value from $\{0, 1\}$, with probability P for a realization of the value 1. The assignment of each value is stochastically independent of the values assigned to other sites. *Percolation* is said to occur if there appears at least one infinite unbounded cluster[8] of sites with value 1. It turns out that there is a threshold-value for the probability P, denoted by P_c, such that such an infinite cluster of "active" sites occurs with probability 1 for $P > P_c$ and with probability 0 for $P < P_c$ (see Stauffer and Aharony (1995)). For the particular graph structure specified in the paper (two-dimensional square lattice) we have approximately $P_c = 0.592743$.

To apply Monte-Carlo techniques for the analysis of percolation models, dynamic processes were proposed enabling to decide whether or not percolation occurs in a given model based on the behavior of the associated process. For such processes the percolation threshold P_c corresponds to that value of the probability P above which diffusion spreads over the entire graph with a significant probability, and below which it "dies out" unless for extremely rare instances. The dynamics of our model specified above corresponds to the Leath-algorithm of percolation (Leath (1976)).

Let us now return to our particular model context. The probability for a consumer to buy the product, given she comes to form her valuation (the latter condition is referred to as C), is the probability that her valuation θ_a falls into the interval $[p, 1]$. Thus $\text{Prob}(a \text{ buys}|C) = 1 - p$. According to what was said above, there is in our model a threshold value for the price p such that for $p > p_c$ the diffusion of the product will "die out" but will spread over the population for $p < p_c$. Thus we have $p_c = 1 - P_c$, the numerical value being approximately $1 - P_c = 0.407$. A generic time profile of the adoption dynamics of the basic model is illustrated in Figure 2: percolation occurs for $p = 0.39$, but does not occur for $p = 0.52$. Figure 3 (top) depicts the final share of buyers as a function of the price. A drastic decrease of that share occurs at $p_c = 0.407$.

Note that while the functional form of the time-profile of sales in our model depends on the particular network structure, the occurrence of spread over the entire population of

[8] A cluster is a set of connected "occupied" sites.

consumers depends only on whether the prevailing price p is above or below p_c.

In the next section we will extend our basic model by macroscopic feedbacks which can affect the price or the valuation (or both). It turns out that this feature can produce a "drift" of the percolation dynamics from a "non-percolating regime" to a "percolating regime", thereby facilitating a dynamics corresponding to a delayed "take-off".

3 New-product diffusion with macroscopic feedbacks

In the following we introduce macroscopic feedbacks affecting the supply side or the demand side (or both). In the extended model the price and the individual valuation may be time-dependent such that the general decision rule reads: consumer a buys in t with $t \geq t_a$ if

$$\theta_{a,t} \geq p_t \ \text{ and } \ \theta_{a,t} < p_\tau \ \text{ for all } \tau : t_a \leq \tau < t$$

with t_a denoting the time period in which consumer a learns the product's characteristics and forms an initial valuation.

We first turn to feedback affecting the supply side assuming that unit production costs decrease with the cumulative quantity of units already produced. The decrease of unit production costs is empirically well established and explained by learning within the firm. Decreasing unit production costs are associated with the "learning curve" (see e.g. Yelle (1979)) and with the related notion of "economies of scale" (see e.g. Scherer and Ross (1990)). In our model, the "learning curve" is represented by a functional relationship $c_t = f(\frac{N_{t-1}}{N})$ with N_{t-1} denoting the number of consumers who bought the product up to period $t-1$ and N denoting the total number of consumers. The function f should satisfy $f(x) > 0, f'(x) < 0$ and $f''(x) > 0$ for the non-negative real numbers to comply with empirical data. Thus, from Eq. 1 follows

$$p_t = (1+m) f(\frac{N_{t-1}}{N}). \tag{2}$$

We specify feedback affecting the demand side by assuming that for each consumer a the initial valuation θ_a is increased by an amount proportional to N_{t-1}. This effect reflects the notion of "network externalities" increasing the utility of a product with the number of other adopters (David (1985), Katz and Shapiro (1992)). Taking this effect into account, we have a time-dependent individual valuation

$$\theta_{a,t} = \theta_a + \mu \frac{N_{t-1}}{N}, \tag{3}$$

with some constant μ which we assume to be independent of a. Note that it is not required that $\theta_{a,t} \in [0,1]$, see Eq. 4.

Depending on the nature of the product considered, either one of the feedback effects might vanish. For instance, computer software presumably exhibits only the second kind of feedback effect, while household electronics exhibit only the first.

Note that the two types of macroscopic feedback effects are mathematically equivalent in the sense that with increasing N_{t-1} the existing gaps between the price of the product and individual valuations of consumers who have not yet bought the product tend to vanish. For that reason, many qualitative results to not depend upon which type of feedback is considered.

The probability of buying thus increases over time. Indeed, for a consumer who forms her evaluation in period t (condition C), the probability to buy in period t we get

$$Prob(a \text{ buys in } t|C) = \begin{cases} 1 - p_t + \mu\frac{N_{t-1}}{N} & \text{if } 0 \leq 1 - p_t + \mu\frac{N_{t-1}}{N} \leq 1 \\ 0 & \text{if } 1 - p_t + \mu\frac{N_{t-1}}{N} < 0 \\ 1 & \text{if } 1 - p_t + \mu\frac{N_{t-1}}{N} > 1. \end{cases} \tag{4}$$

Moreover, in each period the decision of a consumer who formed her evaluation in some earlier period and has not yet bought might be revised. As a result, in our model with feedbacks there exists a range of initial prices (in the parameter setting depicted in Figure 3 (bottom) between approximately 0.41 and 0.53) for which the product "takes off" eventually, despite it would not take-off in the basic model of Section 2. For this range of initial prices, the per-period sales curve exhibits two specific phases. First, a very low sales level persists corresponding to the system being in the non-percolating regime. The dynamics may exhibit a temporary decrease of per-period sales resulting from local diffusion seeds which "die out" before reaching the percolating regime. Second, a "take-off" phase occurring when diffusion seeds which "survived" long enough enter the percolating regime of the dynamics.

The general principle underlying our model is that the diffusion dynamics may "drift" from the non-percolating regime to the percolating regime. This drift occurs because the probability of buying increases over time with the cumulative number of buyers. In the remainder of this section, we present a few instances of such "drift" which were obtained by Monte Carlo simulations. For simplicity, we maintain the assumption that the initial individual valuation θ_a is equi-distributed on $[0, 1]$.[9] Figure 1 (bottom) depicts a diffusion curve resulting from our model with macroscopic feedback affecting only the demand side for a setting with one initial buyer in period $t = 1$, a 400×400 lattice, a time-independent

[9]With this distribution being, for instance, a truncated normal distribution on $[0, 1]$ all qualitative results were reestablished.

price $p = 0.433$ and the parameter μ equal to 0.4. The data is averaged over 500 simulation runs. The reader may think of this averaged curve as modeling new-product diffusion in a population located in many towns with network externalities affecting the population within a single town only. A comparison of Figure 1 (top) and (bottom) illustrates that our model can explain long flat tails empirically observed in the early stages of new-product diffusion.

Figure 4 depicts the evolution of per-period sales (left-hand side) and cumulative sales (right-hand side) resulting from a specification with one initial buyer in period $t = 1$ on a 400×400 lattice. Macroscopic feedback affects the demand side only; the time-independent price is set to $p = 0.435$ (top) and $p = 0.421$ (bottom) and the constant μ describing the influence of network externalities equals 0.4. For both prices, the curves are obtained by averaging over 500 simulation runs. Note that the threshold product price being approximately 0.407, for both the new product would not spread over the population in the basic model without macroscopic feedbacks. However, because individual valuations increase with the number of buyers N_t, some simulation runs persist up to the point where the additional term in Eq. 3 closes the gap between the average evaluation and the price, so that spread of the product occurs. The length of the long left tail increases with p and decreases with μ ceteris paribus. A comparison of Figure 4 (top) with Figure 4 (bottom) exemplifies the first part of this statement. Furthermore, the decrease of per period sales in the first phase as visible in Figure 4 (top, left-hand side) increases with increasing price.

Figure 5 depicts three curves corresponding to per-period sales, cumulative sales and the evolution of the product price resulting from a single simulation run in a setting with macroscopic feedbacks affecting the supply side only. We specify the time-dependent price p_t (see Eq. 2) as

$$p(n_{t-1}) = p_0 - qn_{t-1} + \alpha n_{t-1}^2, \tag{5}$$

with the fraction of buyers $n_{t-1} = \frac{N_{t-1}}{N}$, the initial price $p_0 \in [0, 1]$ and $q > 0$ and $\alpha > 0$ constant parameters. Figure 5 corresponds to the parameter values $q = 0.5$, and $p_0 = 0.52$ and $\alpha = 0.295$. The initial number of buyers equals 3000 and lattice size is 1501×1501. Initial price p_0 is set to 0.52.

As Figure 5 demonstrates, the characteristic take-off dynamics displayed by the averaged curves of Figure 4 can be obtained from a single simulation run. This fact is significant, because in the case of macroscopic feedbacks affecting the price, sales numbers averaged over multiple simulation runs are difficult to justify as they would involve different price sequences.

4 Discussion

We conclude with two comments. First, the paper does not propose that the square lattice is a realistic representation of real-world interaction topologies involved in new-product diffusion. But while the functional form of the time-profile of sales in our model might depend on the particular underlying topology, the effect of delayed "take-off" itself does not: it relies solely on the existence of a percolation threshold p_c separating a percolating regime from a non-percolating regime of the dynamics. It is the passage of the dynamics from the former to the latter that facilitates the "take-off" phenomenon.

However, for a certain type of graph structures – called *scale-free* networks – the percolation threshold tends to zero with a growing number of sites (see Cohen et al. (2000)). Thus, delayed "take-off" would not occur in our model with a scale-free network representing the topology of local interactions. It is tempting to empirically test this implication of our model, once comparative studies on interaction/communication topologies related with different product categories or technologies are available.

Second, in our model individual valuations of the new product made by consumers are not subject to *local* social influence (which is considered, for instance, in the papers by Goldenberg at al. (2000)) and Solomon et al. (2000)). Rather, we consider only macroscopic feedbacks (externalities). Again, this is not because we believe that local interaction effects are not present - interesting phenomena do appear from such local interdependencies in valuation (see, for instance, Erez et al. (2006)). However in the present model we aim at explaining the occurrence of delayed take-off as simply as possible.

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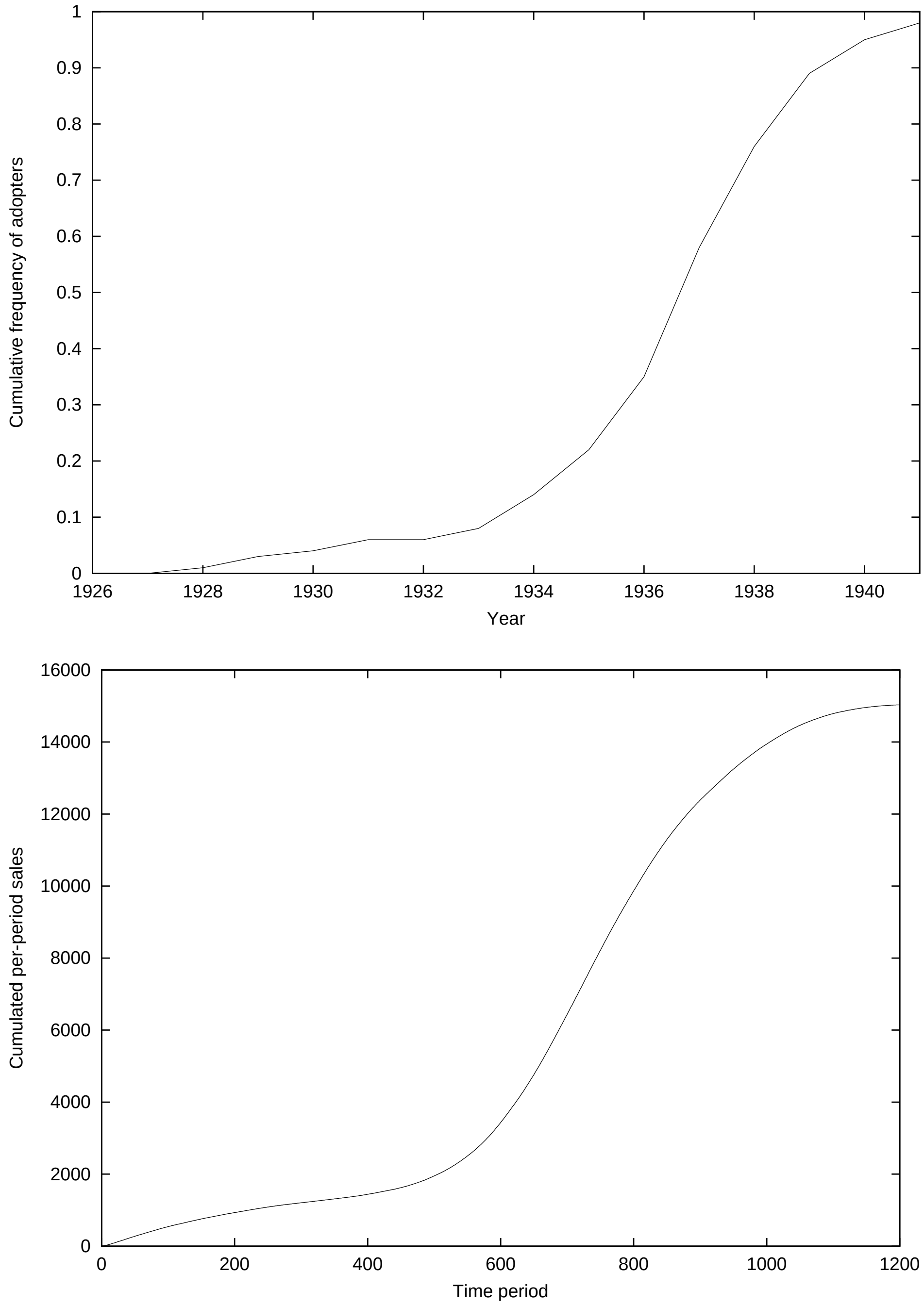

Figure 1: Cumulative frequency of adopters for the diffusion of hybrid corn seed in two Iowa farming communities adapted from [26] (top); cumulative number of buyers in our model with the parameter values $p = 0.433$ and $\mu = 0.4$ (bottom).

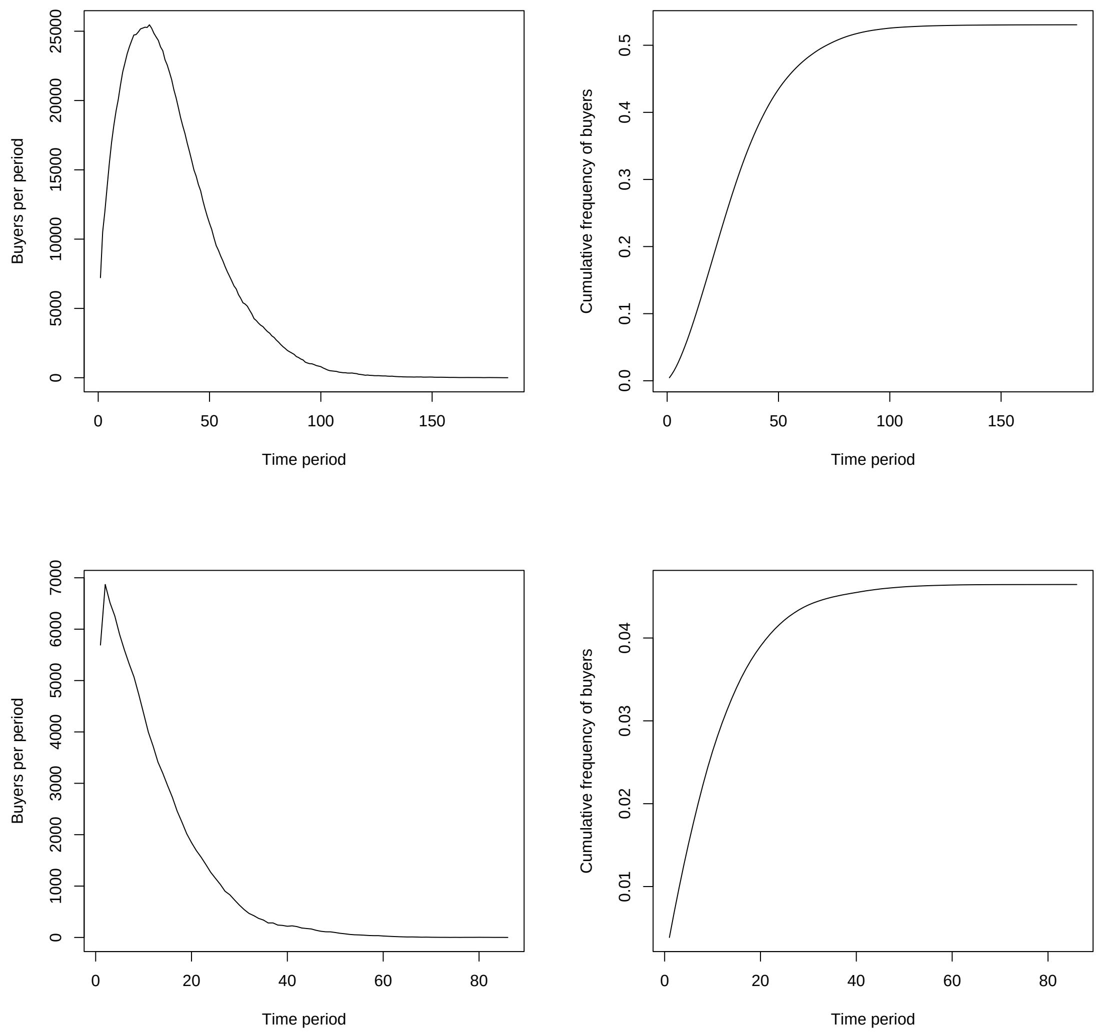

Figure 2: Per-period number of buyers (left-hand side) and cumulative frequencies of buyers over time (right-hand side) in the basic model of Section 2; percolation occurs for a price p=0.39 (top) but does not occur for p=0.52 (bottom); initial number of buyers equals 3000.

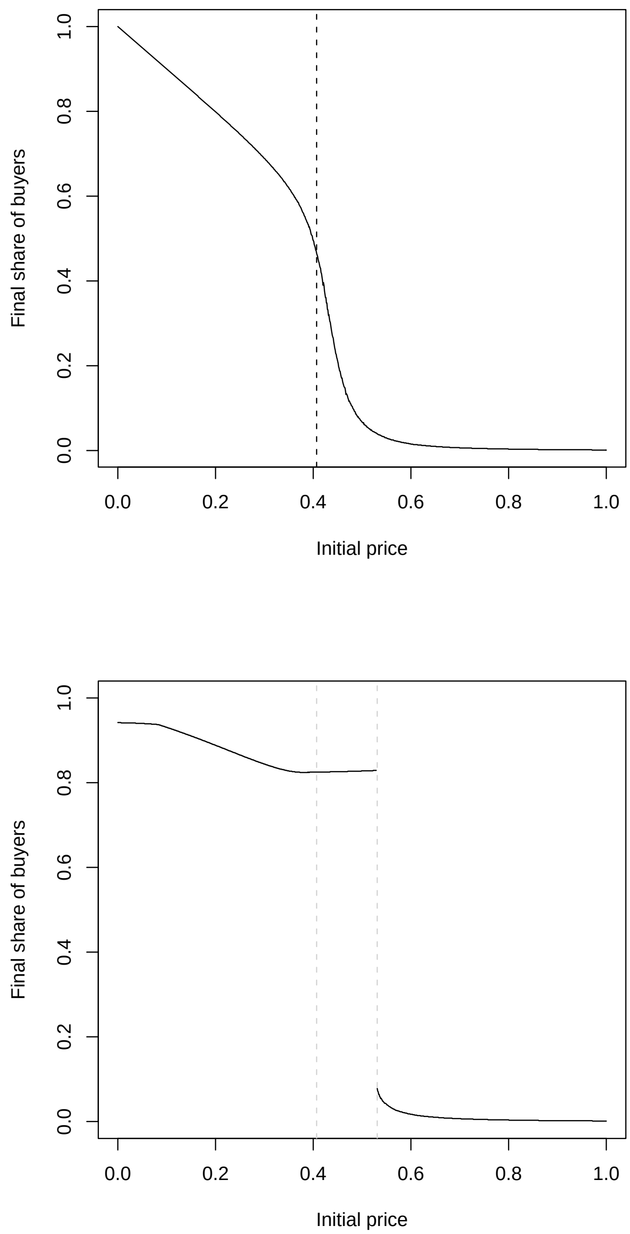

Figure 3: Total (final) share of buyers as a function of price in the basic model of Section 2 (top); total (final) share of buyers as a function of initial price in the model with supply-side-feedbacks (bottom), see Figure 5 for the corresponding time-profiles. Delayed take-off occurs for initial prices in the range enclosed by the two dashed lines, that is for initial prices between 0.407 and 0.531.

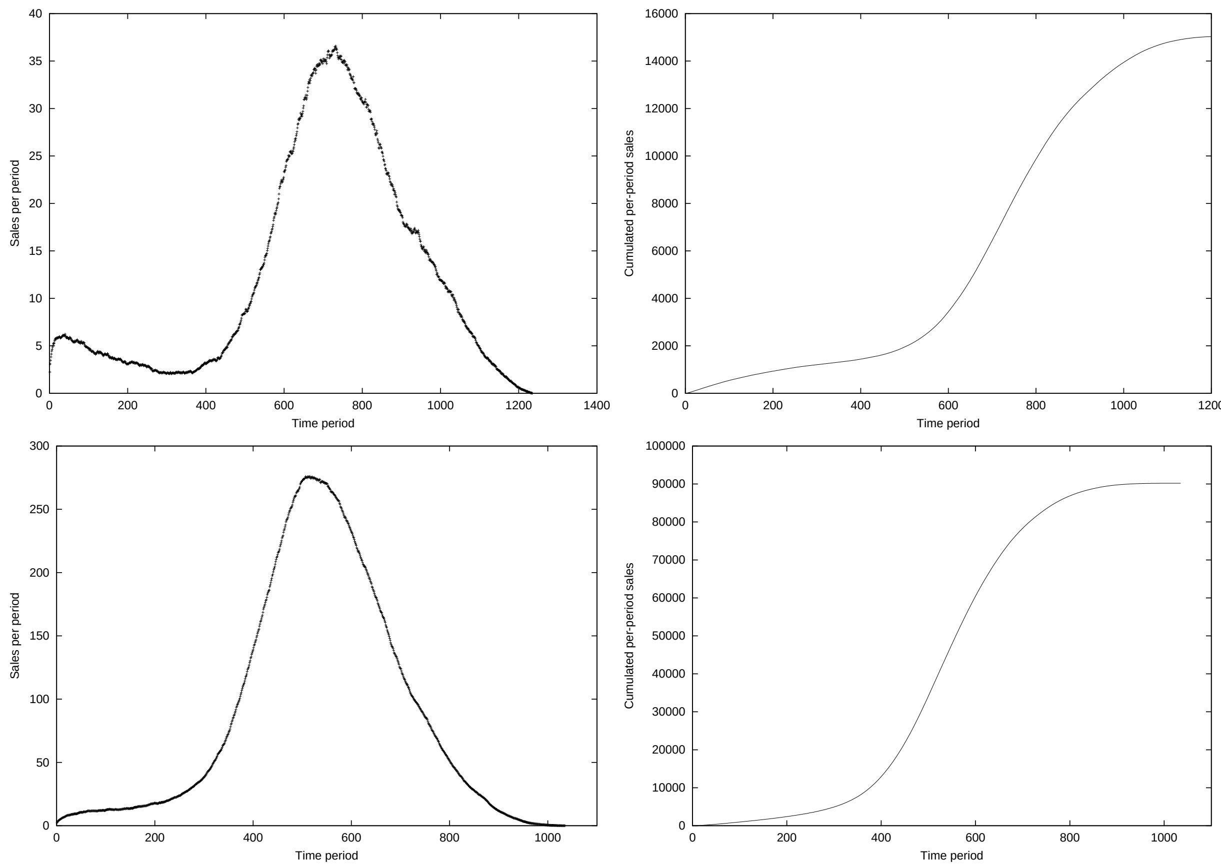

Figure 4: Per-period number of buyers (left-hand side) and cumulative number of buyers (right-hand side) in our model with parameter values $p = 0.435$ and $\mu = 0.4$ (top) and $p = 0.421$ and $\mu = 0.4$ (bottom).

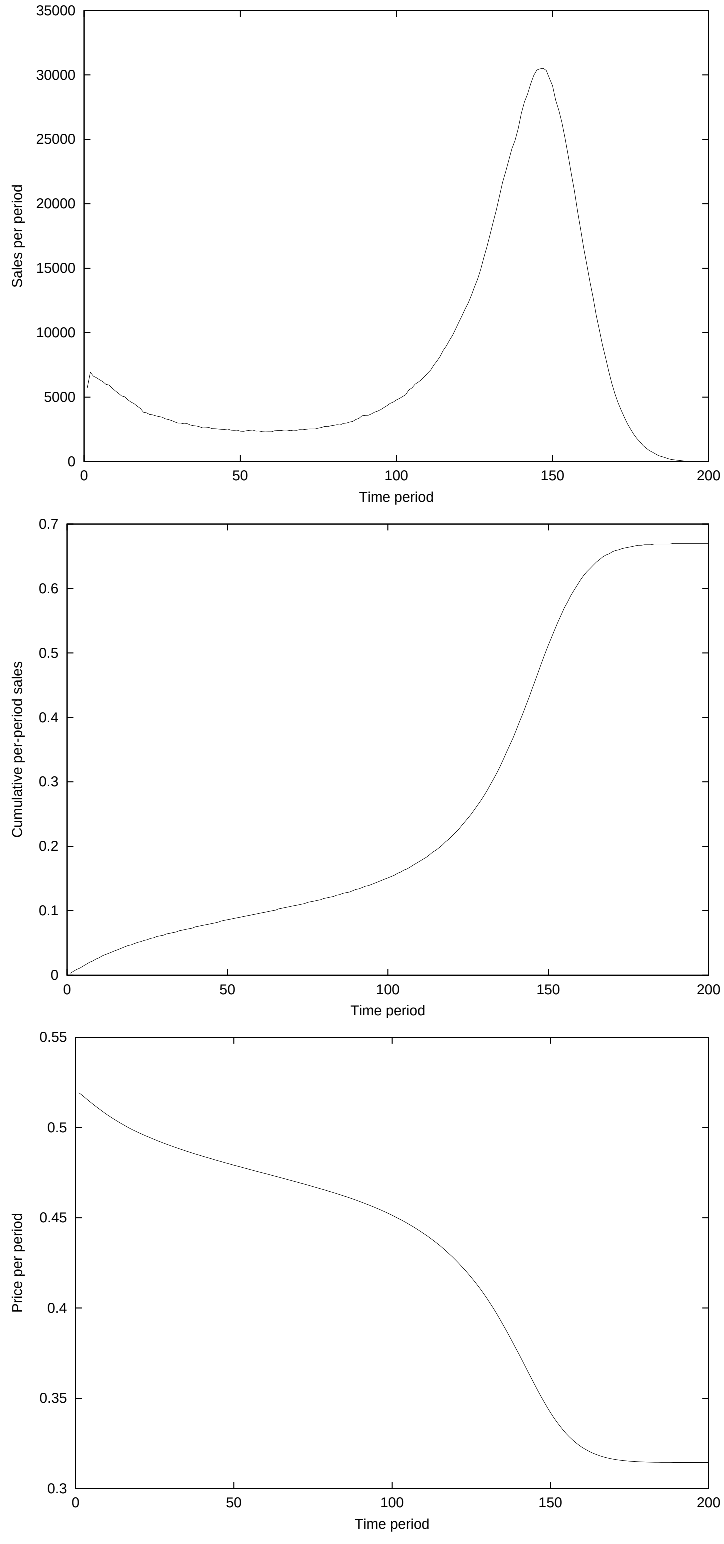

Figure 5: Per-period number of buyers (top), cumulative number of buyers (middle) and the evolution of price (bottom) in our model with macroscopic feedback affecting supply side only; initial price is $p_0 = 0.52$.