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by

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Externalities in Recruiting*

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Abstract

According to the previous literature on hiring, firms face a trade-off when deciding on external recruiting: From an incentive perspective, external recruiting is harmful since admission of external candidates reduces internal workers' career incentives. However, if external workers have high abilities hiring from outside is beneficial to improve job assignment. In our model, external workers do not have superior abilities. We show that external hiring can be profitable from a pure *incentive* perspective. By opening its career system, a firm decreases the incentives of its low-ability workers. The incentives of high-ability workers can increase from a homogenization of the pool of applicants. Whenever this effect dominates, a firm prefers to admit external applicants. If vacancies arise simultaneously, firms face a coordination problem when setting wages. If firms serve the same product market, weaker firms use external recruiting and their wage policy to offset their competitive disadvantage.

Key Words: contest; externalities; recruiting; wage policy.

JEL Classification: C72; J2; J3.

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1 Introduction

When deciding on external recruiting, a firm faces the following well-known trade-off:[1] On the one hand, filling vacancies with external applicants destroys career incentives of internal workers. In particular, allowing external workers to apply for a vacant position already discourages internal candidates, who optimally react by reducing their efforts.[2] On the other hand, expanding the pool of applicants can improve the pool's average quality and, therefore, lead to a better staffing than without external applicants.

In our paper, we show that this traditional trade-off between better job assignment and reduced incentives does not necessarily hold. On the contrary, we show that external recruiting can be beneficial for a firm to *improve incentives*. Expanding the pool of applicants leads to a discouragement of a firm's workforce but possibly also to a more homogeneous field of applicants, which increases incentives. If this advantage dominates discouragement, the firm will optimally decide in favor of external recruiting. In our model, external candidates do not have superior talents. Thus, if a firm admits external candidates, the traditional benefit of improving the pool of applicants cannot play any role.

We consider two firms employing heterogeneous workers. Workers have either a high ability or a low ability. If a firm has to fill a vacant position and thinks about external recruiting, it must keep the following externalities in mind:[3] Since the number of workers competing for the vacant position increases, external recruiting discourages own high-ability and low-ability workers. If the ability difference between the two types of workers is sufficiently large and the number of high-ability workers exceeds a critical value, then the low-ability workers will be completely discouraged and choose zero

[1]See, e.g., Chan (1996), Chen (2005), Tsoulouhas et al. (2007).

[2]Moreover, the firm harms its reputation of honoring good performance of its workers via job-promotion to higher hierarchy levels.

[3]See Konrad (2009), chapter 5, on other externalities in contests.

efforts. Thus, only the high-ability workers remain active in the competition. These workers' incentives are boosted by the homogenization of the set of effective players. If this advantage outweighs the lost incentives of the low-ability workers, the hiring firm will admit external applicants from a pure incentive perspective.

This paper completely focuses on incentives. Including the quality of the recruiting decision (i.e., the ability of the worker that is assigned to the vacant position) would even strengthen our argument for external recruiting: Without external candidates, both internal low-ability and internal high-ability workers have a positive probability of being promoted. If, in the situation described above, external workers are allowed to apply, so that low-ability (internal and external) workers are completely discouraged, the vacant position is filled with a high-ability worker for sure.

In the second part of the paper, we address those externalities in recruiting that arise if firms serve the same product market and/or have simultaneous vacancies. If the two firms A and B compete for the same customers but only firm A has a vacant position, this firm A is less likely to allow for external applications compared to the basic model with separate product markets. Under product market competition, opening of A's career system for external workers generates a positive externality for the other firm B. The workforce of firm B gets incentives for free, which makes B a stronger competitor to A in the product market. Consequently, external recruiting becomes less attractive for firm A.

Firms A and B face a different problem when they have simultaneous vacancies but serve different customers. Now, positive externalities of generating incentives for the other workforce when opening its career system to outsiders leads to a coordination problem. If workers are not too heterogeneous, there will exist two pure equilibria. In either equilibrium, one firm creates incentives for all workers by attaching a positive wage to the vacancy, whereas the other firm free rides on the given incentives and chooses a zero wage.

If firms A and B have vacant positions and operate in the same market, externalities affect both work incentives and the competitive situation of the firms. In case of heterogeneous firms, the stronger one is interested in generating high incentives in order to increase its competitive advantage, whereas the weak firm wants to destroy incentives. As an interior solution, only mixed equilibria exist where the strong (weak) firm puts relatively more probability mass on high (low) wages.

Our paper is related to the contest literature,[4] in particular to those contest papers that also address the problem of external recruiting. Chan (1996) considers a homogeneous internal workforce and finds that opening up the contest to external candidates reduces work incentives for existing employees. To restore incentives, outsiders can be handicapped (disadvantaged). Tsoulouhas et al. (2007) examine the trade-off between incentives and the sorting of high ability workers into the top positions. If external candidates are sufficiently better than insiders, it can be optimal for the firm to handicap current employees. In Waldman (2003), the time-inconsistency problem built into this trade-off is considered: for incentive reasons the firm should favor internal workers ex ante but should not distinguish between internal and external workers ex post. An internal labor market can serve as a commitment device to avoid this problem. Chen (2005) shows that external recruitment can be optimal from a pure incentive perspective if internal workers can choose between productive activities and sabotage. Allowing external competition reduces the effectiveness of sabotage and thus workers will substitute productive effort for sabotage. A similar argument applies for preventing workers' collusion.

All of these contributions consider a homogeneous internal workforce[5].

[4]For an overview see Konrad (2009).

[5]In Waldman (2003), internal workers are heterogeneous. However, during the contest

This situation gives strong incentives to existing workers since success is highly dependent on individual effort. The introduction of external workers thus works clearly in the direction of decreasing overall incentives for internal candidates (when collusion is not a concern). In contrast, we consider heterogeneous internal workers. For this setting, Lazear and Rosen (1981) and Nalebuff and Stiglitz (1983) have argued that handicapping the more able contestants can increase overall incentives. However, this kind of handicap is only possible when the ability of each worker is known to the firm. We show that the firm has another possibility to create a more balanced contest when only the distribution of types in- and outside the firm is known: By allowing external candidates to apply, internal low ability workers will drop out of the competition and incentives for the remaining high ability workers are increased.

The remainder of the paper is organized as follows: In the next two sections, the basic model is described and solved. Section 4 considers product market competition. In Section 5, we examine simultaneous vacancies. Section 6 concludes.

2 The Basic Model

We consider two adjacent hierarchy levels in each of two firms A and B. At the lower hierarchy level, firm F ($F = A, B$) employs n_{FL} workers of type L and n_{FH} workers of type H with $n_{FL} + n_{FH} \geq 2$. Let N_{FL} and N_{FH} denote the corresponding sets of players, that is N_{FL} (N_{FH}) describes the set of L-type (H-type) workers employed by firm F, consisting of $\#N_{FL} = n_{FL}$ ($\#N_{FH} = n_{FH}$) elements. In addition, let $N_F = N_{FL} \cup N_{FH}$ denote the set of all workers employed at the lower hierarchy level of firm F, and $n_F = n_{FL} + n_{FH}$ the respective number of these workers. The four numbers

stage abilities are not known to anyone and therefore all workers have the same expected ability.

n_{AL}, n_{AH}, n_{BL} and n_{BH} are common knowledge of all players, but only the individual worker knows his own type. The type of a worker refers to his ability so that L indicates a low-ability worker, whereas the subscript H corresponds to a worker with high ability. Let $n_L = n_{AL} + n_{BL}$ denote the total number of L-type workers and $n_H = n_{AH} + n_{BH}$ the respective number of H-type workers, and $n = n_L + n_H$ the overall number of workers that are located at the lower tiers of both firms' hierarchies. The corresponding sets are labeled N_L, N_H and N, respectively. The two firms or employers A and B and all n workers are assumed to be risk neutral. Workers are protected by limited liability so that their wages must be non-negative. Furthermore, each worker has a zero reservation value.

It is assumed that nature chooses one of the two firms randomly to have a vacant position at the higher hierarchy level that must be filled. The respective firm F can either promote one of its n_F internal candidates or fill the vacancy with an external hire. In other words, firms A and B have comparable technologies in the sense that working on the lower level of either firm qualifies a worker to fill a vacancy at the higher level of both firms.

The n workers choose non-negative efforts e_i at personal cost e_i/t_i with $t_i \in \{t_L, t_H\}$, $t_H > t_L > 0$, reflecting worker i's talent or ability $(i \in N)$. Hence, firm F has n_{FL} (n_{FH}) workers of talent t_L (t_H). Workers' efforts e_i $(i \in N_F)$ lead to the value $v\left(\sum_{i\in N_F} e_i\right)$ for employer F with $v(\cdot) > 0$, $v'(\cdot) > 0$, $\lim_{x\to\infty} v'(x) = 0$ and $v''(\cdot) < 0$. In words, the value function is monotonically increasing, strictly concave with vanishing increments as well as strictly positive for all feasible arguments. Neither efforts e_i nor the value $v\left(\sum_{i\in N_F} e_i\right)$ are directly observable by the employer. For example, the firm's value of workers' efforts will be realized in the future or it corresponds to a rather complex good or service whose quality cannot be directly determined.[6]

However, an employer can use a coarse signal on relative performance

[6]See MacLeod (2003), p. 219, on this point.

for filling the vacant position. With probability $p_i(e_1, \ldots, e_i, \ldots, e_m)$, this signal tells firm F that worker i has performed best, so that worker i gets the contract offer for the vacant position. Here, m denotes the number of workers that are included in the employer's chosen career system (i.e., either $m = n$ or $m = n_F$). Let M denote the set of these workers. In any case, the firm does not have information on who has performed second-best and so on. This kind of coarse signal particularly holds for those situations where the m workers compete against each other in the same market with only the winner becoming visible. For example, we can think of competition between salesmen for a certain key customer where the only public information is the identity of the salesman who is accepted by the customer. As a second example, we can imagine a situation with different industrial researchers competing in the same innovation race. Competition immediately stops when one of them has made the innovation. In that situation, it is difficult to know who would have succeeded next. Given these examples, the value function $v\left(\sum_{i \in N_F} e_i\right)$ indicates that, from the firm's point of view, finishing the observable task (e.g., acquiring a key customer or making an innovation) is only one valuable aspect of workers' effort choices.

To simplify matters, we adopt the signal structure that is frequently used in the literature on innovation races (e.g., Loury 1979, Dasgupta and Stiglitz 1980, Denicolo 2000, Baye and Hoppe 2003). Given effort e_i, let

$$G(\tau_i|e_i) = 1 - \exp(-h \cdot e_i \cdot \tau_i) \tag{1}$$

denote the probability that worker i succeeds (i.e., acquires a certain key customer or solves a certain problem by making an innovation) before time τ_i. (1) describes an exponential distribution with density $g(\tau_i|e_i) = dG(\tau_i|e_i)/d\tau_i$ and hazard rate $h > 0$. The workers' success times are assumed to be stochastically independent, so that worker i's conditional probability of succeeding first and, hence, winning the recruiting contest is given by

$$P(i \text{ wins}|\tau_i) = \prod_{j\in M\setminus\{i\}} P(\tau_j > \tau_i) = \prod_{j\in M\setminus\{i\}} [1 - G(\tau_i|e_j)]$$
$$= \exp\left(-h\cdot\tau_i \sum\nolimits_{j\in M\setminus\{i\}} e_j\right).$$

Therefore, worker i's unconditional winning probability can be computed as

$$p_i(e_1,\ldots,e_i,\ldots,e_m) = \int_0^\infty \exp\left(-h\tau_i \sum\nolimits_{j\in M\setminus\{i\}} e_j\right) g(\tau_i|e_i)\, d\tau_i$$
$$= \int_0^\infty he_i \exp\left(-h\tau_i \sum\nolimits_{j\in M} e_j\right) d\tau_i = \frac{e_i}{\sum_{j\in M} e_j}. \quad (2)$$

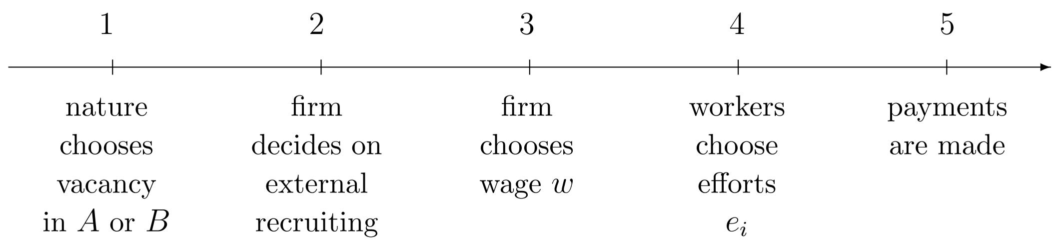
For the special case of $\sum_{j\in M} e_j = 0$ we assume that each worker's winning probability is given by $1/m$.[7]

In order to focus on different firms that compete with their career systems in the same labor market we assume that each firm can credibly commit to assign the best performer to the higher hierarchy level in case of a vacancy.[8] Moreover, we neglect other possible incentive schemes. The only possibility of a firm to generate incentives is to design a recruiting contest for the vacant position at the higher level. Here, firm F can either restrict competition to internal candidates or widen worker competition by accepting external candidates as well. To install a recruiting contest, the firm announces a wage $w \geq 0$ that is attached to the vacant job. The best performing worker gets this job. All other workers get zero wages as optimal contest loser prizes since workers are protected by limited liability and have zero reservation values.[9] We concentrate on incentive issues and, at the end of Section 3,

[7]See already Tullock (1980).
[8]E.g., the signal on the best performer is verifiable.
[9]In other words, since the firm does not have more information on workers' ranking, any positive loser prize would only increase the firm's labor costs and decrease workers'

shortly comment on the consequences of job assignment on firm profits.

We can summarize the time schedule of the basic model as follows:



At the first stage of the game, nature randomly selects one of the firms A and B to have a vacancy on the higher hierarchy level. At stage 2, this firm F has to make the policy decision whether to accept external candidates or not. For the chosen career system – with or without external recruiting – the firm solves

$$\max_{w \geq 0} v \left(\sum\nolimits_{i \in N_F} e_i \right) - w \tag{3}$$

at stage 3. The optimal wage attached to the vacant job also describes the contract offered to each of the internal workers at the lower hierarchy level. Any worker will accept a feasible contract with $w \geq 0$ since workers have zero reservation values but a non-negative payoff when participating in the career game and choosing zero effort. Thus, we do not have to care for the workers' participation constraints when solving the game. In stage 4, all n workers observe the firm's recruiting policy (including w) and simultaneously choose efforts to compete for the vacant position. Finally, the best performing worker that is assigned to the vacant higher-level job gets w, whereas the other workers get zero. The firm F that has filled its vacancy earns profit (3) and the other firm $\hat{F} \in \{A, B\} \backslash \{F\}$ receives $v \left(\sum\nolimits_{i \in N_{\hat{F}}} e_i \right)$. After having solved the game of the basic model we will turn to the case of both firms

incentives.

competing in the same product market.

3 Solution to the Basic Model

We solve the game by backwards induction starting with stage 4, where the m workers simultaneously choose their efforts. Of course, if workers of firm $\hat{F}$ cannot apply for the vacant position since firm F has excluded candidates from outside they will optimally choose zero efforts in order to save effort costs. However, workers of firm F are always included in the recruiting contest. Let m_H denote the number of H-type workers and m_L the number of L-type workers that are allowed to apply for the vacant job with wage $w > 0$. We obtain the following result:

Proposition 1 *There exists a unique and symmetric equilibrium in which workers of the same type choose identical effort levels. If* $t_H (m_H - 1) \geq m_H t_L$*, then L-type workers choose* $e_L^* = 0$ *in equilibrium and H-type workers* $e_H^* = \frac{m_H - 1}{m_H^2} t_H w$*, otherwise*

$$e_L^* = w \frac{t_H t_L (m-1)(m_H t_L - (m_H - 1) t_H)}{(m_H t_L + m_L t_H)^2} \quad \textit{and} \tag{4}$$

$$e_H^* = w \frac{t_H t_L (m-1)(m_L t_H - (m_L - 1) t_L)}{(m_H t_L + m_L t_H)^2}. \tag{5}$$

Proof. See Appendix A. ■

Proposition 1 shows that we have two possible outcomes at the contest stage. Either outcome is symmetric in the sense that H-type workers choose identical efforts and L-type workers choose identical efforts. If the H-type workers are sufficiently more able than the L-type workers, the latter ones will be completely discouraged and drop out of the competition by choosing zero effort. The larger the number of H-type workers the more likely will be this outcome. In particular, for $m_H \to \infty$ the L-type workers will even drop out if the H-type workers have only a marginally higher ability since condition $t_H \geq \frac{m_H}{m_H-1} t_L$ becomes $t_H \geq t_L$. The number of H-type workers also discourages the high-ability workers. They will not drop out, but their equilibrium effort level monotonically decreases in m_H. Recall that either $m_H = n_{AH} + n_{BH}$ or $m_H = n_{FH}$. Hence, if L-type workers drop out under pure internal competition they will drop out as well if firm F opens its career system for external hires, whereas the opposite result does not necessarily hold. Altogether, opening the career system to outsiders can generate strong externalities by discouraging the weak internal workers.

If $t_H (m_H - 1) < m_H t_L$, the recruiting contest will have an equilibrium with both types of workers exerting positive efforts. From (4) and (5) we can see that equilibrium efforts increase in the wage w and that $e_H^* > e_L^*$ since $m_L t_H - (m_L - 1) t_L > m_H t_L - (m_H - 1) t_H$. Moreover, the level of a worker's equilibrium effort crucially depends on two factors – the number of contestants and the degree of heterogeneity between the workers. These two factors can be highlighted by considering them separately. In order to point out the impact of the number of contestants, let $m_H = m_L = \bar{m}$. In that case, we obtain

$$e_L^* + e_H^* = \frac{w t_H t_L (2\bar{m} - 1)(t_H + t_L)}{\bar{m}^2 (t_L + t_H)^2},$$

which is clearly decreasing in $\bar{m}$. Thus, analogously to the case of a corner solution considered in the paragraph before, each worker is discouraged if the number of opponents increases.

To emphasize the role of heterogeneity let, for illustrating purposes, $m_H = m_L = 1$.[10] The sum of equilibrium efforts boils down to

$$e_L^* + e_H^* = w \frac{t_H t_L}{t_L + t_H}.$$

[10] Since by assumption of the basic model, $n_{FL} + n_{FH} \geq 2$, we are in a situation where external workers are excluded.

Hence, for a given amount of collective talent, $t_L + t_H$, workers' efforts are maximized if heterogeneity diminishes (i.e., $t_L = t_H$). This finding is quite intuitive and also in line with results in other contest models: The closer the race between the contestants the more effort each player will choose in equilibrium. Both effects – discouragement by a larger number of contestants and encouragement by a small degree of heterogeneity among the workers – are crucial for firm F's decision whether to allow external recruiting or not.

Anticipating the workers' behavior in the recruiting contest, at stages 2 and 3 firm F solves the design problem for filling the vacancy at its higher hierarchy level. Let V denote the inverse of the marginal value function $v'(\cdot)$. Then we get the following results:

Proposition 2 *Let firm F strictly prefer a positive wage.*[11] *F allows external workers to apply for the vacancy iff*

$$t_H \frac{n_{FH}-1}{n_{FH}} < t_L \leq t_H \frac{n_H - 1}{n_H} \quad \text{and} \tag{6}$$

$$\frac{(n_F - 1)\, n_H^2}{n_{FH}\,(n_H - 1)\, n_{FL}} - \frac{n_{FH}}{n_{FL}} < \frac{t_H}{t_L}. \tag{7}$$

In that case, F optimally chooses

$$w^* = \Phi_1 \cdot V(\Phi_1) \quad \text{with} \quad \Phi_1 = \frac{n_H^2}{n_{FH}\,(n_H - 1)\, t_H}. \tag{8}$$

In all other cases, F does not admit external applications and chooses

$$w^* = \Phi_2 \cdot V(\Phi_2) \quad \text{with} \quad \Phi_2 = \begin{cases} \frac{n_{FH}}{(n_{FH}-1)t_H} & \text{if } t_L \leq t_H \frac{n_{FH}-1}{n_{FH}} \\ \frac{n_{FH}t_L + n_{FL}t_H}{t_H t_L (n_F - 1)} & \text{otherwise.} \end{cases} \tag{9}$$

Proof. See Appendix A. ■

[11]Hence, we must have that $v'(0) \min\left\{ \frac{n_{FH}-1}{n_{FH}}, \frac{t_L(n_{FH}+n_{FL}-1)}{n_{FH}t_L+n_{FL}t_H}, \frac{n_{FH}(n_H-1)}{n_H^2} \right\} t_H > 1$.

Remark *There exist feasible parameter constellations that satisfy (6) and (7) at the same time. Consider, for example,* $n_{\hat{F}H} = n_{FL} = \eta > 0$ *and* $n_{FH} = 1$ *with* $\hat{F}$ *denoting the other firm. For this parameter constellation, conditions (6) and (7) boil down to*

$$0 < t_L \leq t_H \frac{\eta}{1+\eta} \quad \text{and} \quad t_L < t_H \frac{\eta}{\left((\eta+1)^2 - 1\right)}.$$

There are feasible values of t_L *and* t_H *that satisfy both inequalities for any positive integer* η.

From Proposition 1 we know that L-type workers will drop out and choose zero effort, if the number of H-type workers is sufficiently large. Hence, from the perspective of firm F we can differentiate between three cases – (1) the number of internal H-type workers is so large that L-type workers even drop out without external competition, (2) L-type workers only drop out if F opens the career system for external candidates but not under pure internal competition, (3) L-type workers never drop out. Proposition 2 shows that only in case (2) firm F may be interested in allowing external applications. In that case, F strictly benefits from the strong externalities induced by the outsiders. F will prefer an open career system if the increased effort levels of its H-type workers exceed the lost efforts of its L-type workers who become completely discouraged and drop out. In particular, three effects are at work that crucially influence firm F's decision to allow external recruiting: (i) Since the L-type workers drop out, there is pure homogeneous competition among H-type workers. As equilibrium efforts are highest the more homogeneous the players, F strictly profits from an active homogeneous workforce. (ii) Firm F loses the valuable efforts of his L-type workers, who exert zero efforts. (iii) Allowing external candidates changes the number of active contestants. In general, a single worker will be discouraged and, hence, supply less effort the larger the number of his opponents. Whereas F strictly benefits from

(i) and suffers from (ii) the direction of this third effect is not clear. On the one hand, the number of active players decreases as L-type workers drop out, which encourages each remaining H-type worker. On the other hand, additional H-type workers from the other firm enter the competition, which increases the number of active players.

We can identify these three effects when looking at condition (7).[12] This inequality is more likely to be satisfied if t_H is rather large and t_L rather small. The larger t_H the more F will profit from enhanced competition between his H-type workers. The smaller t_L the smaller will be F's losses from his L-type workers, who become completely passive. A similar interpretation can be obtained for n_{FL}: Condition (7) is equivalent to

$$\frac{t_L\left(n_{FH}+n_{FL}-1\right)}{\left(n_{FH}t_L+n_{FL}t_H\right)}<n_{FH}\frac{n_H-1}{n_H^2}.$$

Differentiating the left-hand side with respect to n_{FL} gives

$$\frac{\partial}{\partial n_{FL}}\left(\frac{t_L\left(n_{FH}+n_{FL}-1\right)}{\left(n_{FH}t_L+n_{FL}t_H\right)}\right)=\frac{n_{FH}t_L\left(t_L-\frac{n_{FH}-1}{n_{FH}}t_H\right)}{\left(t_Ln_{FH}+n_{FL}t_H\right)^2},$$

which is strictly positive according to (6). Hence, the smaller n_{FL} the smaller will be F's losses from completely discouraging all of his L-type workers and the more F will tend to open its career system for external workers. Finally, the left-hand side of (7) is non-decreasing (and for $n_H > 2$ strictly increasing) in n_H. This finding is quite intuitive, following effect (iii) above. Recall that n_H also contains the number of H-type workers of the other firm, $n_{\hat{F}H}$. The larger this number, the larger will be the number of active contestants when allowing external candidates to apply. Since the equilibrium effort level of a single H-type worker decreases in the number of opponents when the field of players is completely homogeneous (see Proposition 1), a larger value of $n_{\hat{F}H}$

[12] Condition (6) only states that we are in case (2).

makes opening the career system for firm F less attractive.

The argument given at the end of the last paragraph exactly explains why firm F does not open its career system in case (1) described above. The only effect of such opening would be a discouragement of the internal H-type workers since m_H increases from $m_H = n_{FH}$ to $m_H = n_{FH} + n_{\hat{F}H}$. The remaining case (3) deals with the scenario where L-type workers never give up by choosing zero efforts. At first sight, it is not clear whether opening of the career system may be profitable for F. Of course, allowing external applications unambiguously increases the number of contestants, which discourages each internal worker. However, maybe the additional contestants lead to a better mixture of workers so that the field becomes more homogeneous. Proposition 2 shows that this possible advantage is not strong enough to justify opening of the career system.

In this paper, we do not address the firm's consequences of assigning a worker with certain talent t to the vacant position at the higher hierarchy level. However, since the vacant position is typically accompanied by higher responsibility and influence on firm profits, the firm should prefer $t = t_H$ to $t = t_L$ for the new job holder. Note that given such preference the firm additionally profits in case (2) from ensuring the assignment of an H-type worker to the higher position. Since all L-type workers drop out of the competition and, thus, have a zero probability of winning the contest, opening the career system guarantees optimal selection of workers as a by-product.

Finally, equations (8) and (9) show that the workers' abilities and the numbers of different types of workers play an ambiguous role for firm F's choice of the optimal wage, w^*. This can be exemplarily seen from (8). Note that $V(\cdot)$ is monotonically decreasing since the value function v is increasing and concave. On the one hand, a high talent t_H corresponds to high equilibrium efforts and makes investing in incentives rather attractive for F. On the other hand, marginal returns from effort supply are decreasing due to the concavity of the value function, which makes incentivizing workers less attractive to F.

4 Product Market Competition

We now turn to the case where both firms compete in the same product market. Again, firm F has to fill a vacancy and has to decide whether or not to open its career system for the workers of its competitor $\hat{F}$.

The basic structure of the model remains the same as in Section 2. However, under product market competition, the profit of firm F does not only depend on its own workers' efforts but also on the efforts of its competitor $\hat{F}$'s workers. The higher the total effort of the rival firm's workforce, the lower should be F's profit. This effect seems to be natural if firms directly compete against each other. To model this effect, firm F is assumed to maximize the profit function

$$\max_{w \geq 0} \psi \left(\sum\nolimits_{i \in N_F} e_i - \sum\nolimits_{j \in N_{\hat{F}}} e_j \right) - w \tag{10}$$

where the function $\psi(\cdot)$ has the following properties: ψ is a monotonically increasing, strictly positive, continuously differentiable and bounded function on $\mathbb{R}$ which is strictly concave on $\mathbb{R}^+$ and for which $\psi(x) + \psi(-x)$ is constant in x. The last assumption captures the idea that the two firms are competing for a market of fixed size.

Since, at the contest stage, there are no changes in the situation from the point of view of the workers, equilibrium effort levels for a given wage w are still described by Proposition 1.

As can be seen from the profit function (10), the introduction of competition renders external recruiting less attractive. The reason is that the recruiting contest gives incentives to all participating workers, which includes the workforce of the competing firm in case of external recruiting. Since in our stylized model all incentives are tied to F's recruiting decision, workers who are not admitted to the contest thus have no incentive to spend any effort. If firm F shuts down its contest for external candidates we have $\sum_{j \in N_{\hat{F}}} e_j = 0$. Therefore, the introduction of product market competition can only yield new results in the case where the firm would open its career system to external workers in the absence of competition. This case is described by conditions (6) and (7) of Proposition 2. In the remainder of this section we restrict our attention to this situation. Let Ψ denote the inverse of function ψ'. Then we obtain the following result:

Proposition 3 *Consider the case that conditions (6) and (7) hold, so that firm F would admit external applicants in the absence of product market competition. Furthermore, let firm F strictly prefer a positive wage. Firm F still allows external workers to apply despite product market competition iff $n_{FH} > n_{\hat{F}H}$ and*

$$\frac{(n_F - 1)\, n_H^2}{(n_{FH} - n_{\hat{F}H})\,(n_H - 1)\, n_{FL}} - \frac{n_{FH}}{n_{FL}} < \frac{t_H}{t_L}. \tag{11}$$

In that case, F optimally chooses

$$w^* = \Phi_3 \cdot \Psi(\Phi_3) \quad \textit{with} \quad \Phi_3 = \frac{n_H^2}{(n_{FH} - n_{\hat{F}H})\,(n_H - 1)\, t_H}. \tag{12}$$

Otherwise, F does not admit external applications and chooses a wage w^* corresponding to the second case of (9) with V being replaced by Ψ.

Proof. See Appendix A. ■

Proposition 3 shows that with product market competition two additional conditions – $n_{FH} > n_{\hat{F}H}$ and inequality (11) – need to hold for F to open up its career system. Firm F now has to consider the negative externalities in form of the career incentives for the workers in firm $\hat{F}$. These externalities only arise for H-type workers since the L-type workers in both firms will be completely discouraged and drop out of the job-competition. Firm F

thus has to consider the number of H-type workers $n_{\hat{F}H}$ at the competing firm, which yields the two additional conditions. If $n_{FH} < n_{\hat{F}H}$ firm $\hat{F}$ will gain more from career incentives than firm F since $\hat{F}$ employs more H-type workers. In that case, firm F would unambiguously harm itself by opening its career system for external hires. Thus, $n_{FH} > n_{\hat{F}H}$ describes a necessary condition for firm F to admit external candidates.

Note that, from the viewpoint of job assignment, opening its career system should especially benefit a firm if it has only few high-ability workers. In that case, admitting external candidates can be very useful for a firm to increase the average quality of its workforce. This motive for external recruiting is well-known in the literature (e.g., Chan 1996). However, Proposition 3 shows that from an incentive perspective a relatively small number of high-ability workers may counteract the admission of external hires due to the career system's negative externalities when both firms are located in the same market. The other way round, firm F will rather tend to accept external applications if its rival $\hat{F}$ employs many L-type workers (who completely drop out) and only few H-type workers (who are motivated by the career system).

Moreover, opening the career system requires condition (11) to hold. Again, the number of H-type workers of the other firm $\hat{F}$ turns out to be crucial. There are several effects of a large value of $n_{\hat{F}H}$. First, $n_{\hat{F}H}$ has to be sufficiently large to make firm F's L-type workers drop out by choosing zero efforts. Second, the larger $n_{\hat{F}H}$ the larger will be the number of firm $\hat{F}$'s H-type workers that benefit from the career incentives. Third, the larger $n_{\hat{F}H}$ the more the H-type workers in both firms will be discouraged since the equilibrium effort level of the H-type workers,

$$e_H^* = \frac{(n_{FH} + n_{\hat{F}H}) - 1}{(n_{FH} + n_{\hat{F}H})^2} t_H w,$$

decreases in $n_{\hat{F}H}$. Note that the first effect is covered by (6), which guarantees that $n_{\hat{F}H}$ is sufficiently large so that L-type workers will drop out if all workers compete against each other in a single contest. The second effect is covered by the necessary condition $n_{FH} > n_{\hat{F}H}$. Hence, the third effect – discouragement of H-type workers in both firms – remains. This effect should harm firm F more than firm $\hat{F}$ because of $n_{FH} > n_{\hat{F}H}$. Thus, the larger $n_{\hat{F}H}$ the less condition (11) should be satisfied. The comparison of conditions (7) and (11) shows that this conjecture is correct. The only difference between (7) and (11) is the replacement of n_{FH} by $n_{FH} - n_{\hat{F}H}$ in the denominator of the first expression at the left-hand side. Hence, condition (11) is stricter than condition (7) so that under product market competition firm F will open its recruiting system less often to external applicants than without competition. Since the left-hand side of (11) is monotonically increasing in $n_{\hat{F}H}$, (11) is less likely to be satisfied for large values of $n_{\hat{F}H}$.

5 Simultaneous Vacancies

The findings of the previous sections have shown that a firm can profit from opening its career system to external hires in order to improve incentives of its workforce. This section uses such a situation as the starting point: We assume that the firm lacks appropriate candidates for the vacant position, so that without external hiring there is no worker competition and internal incentives are zero. Hence, a firm must open its career system to external applicants if it wants to generate strictly positive incentives. Up to now only one firm had to fill a vacancy. Now we consider the case where both firms have a vacant position that needs to be staffed. In order to keep the analysis tractable, we restrict our attention to two firms each employing only one worker at the lower hierarchy level.

As before, the four numbers n_{AL}, n_{AH}, n_{BL} and n_{BH} are common knowledge of all players. However, this time, since we have $n_A = n_B = 1$, this assumption implies that firms know the type of each individual worker. Let $t_F \in \{t_L, t_H\}$ be the talent of the worker that is employed by firm F at the lower hierarchy level ($F = A, B$) while the other worker, being employed by firm $\hat{F}$, has talent $t_{\hat{F}} \in \{t_L, t_H\}$ ($\hat{F} \neq F$). The timeline is similar to that of the basic model, with the exception that now there are two firms that move simultaneously in stages 1 and 2: First, both firms have to decide on whether to accept an application from the external candidate or not. At the second stage, firms A and B attach wages w_A and w_B to their vacant positions. At the third stage, workers simultaneously choose efforts. Finally, workers are assigned to jobs and payments are made. We assume that each firm must fill its vacancy with one of the workers.

If a firm does not open its career system to the external worker, only the internal worker will compete for the vacant job. Career incentives will not work in such one-person contest since the internal candidate will be promoted with certainty. Consequently, he will exert zero effort. Firm F anticipates this behavior and chooses $w_F = 0$. As, by assumption, each firm must fill its vacancy with one of the workers, it is always optimal for the firms to accept external applications. However, they are free to choose between $w_F = 0$ and $w_F > 0$.

Given both firms' wages $w_F \geq 0$ and $w_{\hat{F}} \geq 0$, the two workers will compete for the higher wage $\max\{w_F, w_{\hat{F}}\}$. Let e_F denote the effort level chosen by the worker in firm F ($F = A, B$) and $e_{\hat{F}}$ the effort of the worker being employed by firm $\hat{F}$. The worker of firm F gets $\max\{w_F, w_{\hat{F}}\}$ with probability $e_F/(e_F + e_{\hat{F}})$ and $\min\{w_F, w_{\hat{F}}\}$ with probability $1 - e_F/(e_F + e_{\hat{F}}) = e_{\hat{F}}/(e_F + e_{\hat{F}})$. He maximizes his expected utility

$$\max\{w_F, w_{\hat{F}}\} \frac{e_F}{e_F + e_{\hat{F}}} + \min\{w_F, w_{\hat{F}}\} \left(1 - \frac{e_F}{e_F + e_{\hat{F}}}\right) - \frac{e_F}{t_F}$$

$$= \min\{w_F, w_{\hat{F}}\} + \frac{e_F}{e_F + e_{\hat{F}}} |w_F - w_{\hat{F}}| - \frac{e_F}{t_F} \qquad (F, \hat{F} = A, B;\ F \neq \hat{F}).$$

Straightforward calculations show that his optimal effort is given by

$$e_F^* = |w_F - w_{\hat{F}}| \cdot T_F \quad \text{with } T_F := \frac{t_F^2 t_{\hat{F}}}{(t_F + t_{\hat{F}})^2}. \tag{13}$$

Obviously, if both firms attach zero wages to their vacant positions or offer identical wages, both workers' optimal efforts will be zero.

5.1 Firms in Separate Product Markets

In this subsection, we assume that both firms operate in different product markets. Hence, we are back in the situation of Section 3, which is now extended by a simultaneous vacancy in the second firm. Firm F solves

$$\max_{w_F \geq 0} v(e_F^*) - w_F = \max_{w_F \geq 0} v(|w_F - w_{\hat{F}}| \cdot T_F) - w_F, \tag{14}$$

while at the same time firm $\hat{F}$ maximizes

$$v(|w_F - w_{\hat{F}}| \cdot T_{\hat{F}}) - w_{\hat{F}} \quad \text{with } T_{\hat{F}} := \frac{t_F t_{\hat{F}}^2}{(t_F + t_{\hat{F}})^2}. \tag{15}$$

Assume for a moment that $w_{\hat{F}} = 0$. Then we have $e_F^* = w_F T_F$ according to (13). Hence, firm F's best response $w_F^*(w_{\hat{F}})$ to $w_{\hat{F}} = 0$ maximizes $v(w_F T_F) - w_F$:

$$w_F^*(0) = \begin{cases} \frac{1}{T_F} V\left(\frac{1}{T_F}\right) =: w_{F\hat{F}}^* & \text{if } T_F v'(0) > 1 \\ 0 & \text{otherwise,} \end{cases} \tag{16}$$

where $w_{F\hat{F}}^*$ follows from F's first-order condition. Let analogously $w_{\hat{F}}^*(w_F)$ denote $\hat{F}$'s best response to w_F. Given $w_F = 0$, the best response $w_{\hat{F}}^*(0) = w_{\hat{F}F}^*$ if $T_{\hat{F}} v'(0) > 1$ can be derived in the same way as (16). Note that any relation $w_{LH}^* \gtreqless w_{HL}^*$ is possible since $\frac{(t_H+t_L)^2}{t_H t_L^2} > \frac{(t_H+t_L)^2}{t_H^2 t_L}$, but $V(\cdot)$ is monotonically decreasing. We obtain the following results for the optimal wage policies of firms F and $\hat{F}$ ($F, \hat{F} \in \{A, B\}; F \neq \hat{F}$):

Proposition 4 *Let* $\frac{t_H t_L^2}{(t_H+t_L)^2} v'(0) > 1$. *For simultaneous vacancies and firms operating in different product markets, there are two scenarios: (1) If workers are homogeneous (i.e.,* $t_F = t_{\hat{F}} =: t \in \{t_H, t_L\}$*), there are two pure equilibria* $(w_F^*, w_{\hat{F}}^*) = (0, w_{\hat{F}F}^*)$ *and* $(w_F^*, w_{\hat{F}}^*) = (w_{F\hat{F}}^*, 0)$ *with* $w_{F\hat{F}}^* = w_{\hat{F}F}^* = \frac{4}{t} V\left(\frac{4}{t}\right)$. *There also exists a symmetric equilibrium in mixed strategies. (2) If workers are heterogeneous, so that* $t_F, t_{\hat{F}} \in \{t_H, t_L\}$, $t_F \neq t_{\hat{F}}$, *with* $w_{F\hat{F}}^* < w_{\hat{F}F}^*$, *then* $(w_F^*, w_{\hat{F}}^*) = (0, w_{\hat{F}F}^*)$ *will be the unique equilibrium iff*

$$v\left(w_{\hat{F}F}^* T_{\hat{F}}\right) - v\left(w_{F\hat{F}}^* T_{\hat{F}}\right) > w_{F\hat{F}}^* + w_{\hat{F}F}^*; \tag{17}$$

otherwise there are two equilibria $(w_F^*, w_{\hat{F}}^*) = (0, w_{\hat{F}F}^*)$ *and* $(w_F^*, w_{\hat{F}}^*) = (w_{F\hat{F}}^*, 0)$.

Proof. See Appendix A. ■

The condition given at the beginning of Proposition 4 excludes corner solutions where both firms choose zero wages. The results show that in the pure equilibria exactly one firm chooses a positive wage. This main finding is due to the fact that, given a zero wage w_F of firm F, the other firm $\hat{F}$ generates a positive externality by choosing a positive wage, which induces incentives to both workers. Firm F now must decide whether to free-ride and keep the zero wage $w_F = 0$, or to deviate to a strictly positive wage $w_F > 0$. However, in the latter case any rational positive wage must be at least twice as high as $w_{\hat{F}}$ because otherwise F destroys existing incentives (see (13)) at positive costs. The proof of Proposition 4 shows that such a deviation by F does not pay out for the firms in case of homogeneous workers or moderate degrees of heterogeneity.

If both workers are homogeneous or not too heterogeneous (i.e., $t_H - t_L$ is sufficiently small), then the two firms will face a coordination problem similar to the battle of the sexes. Both firms strictly favor the outcome that one of them creates incentives and the other one free rides by choosing a zero wage, but each of them prefers to be the free rider. If the firms fail to coordinate, they will end up in a situation with minimal (in the homogeneous case: zero) incentives. As worst possible outcome, both firms choose positive wages to generate incentives, but the two wages just offset each other in (13).

In case of strong heterogeneity, the two abilities t_H and t_L can differ so much that condition (17) is satisfied. Now workers' incentives are strictly more valuable to one of the two firms. This firm always prefers to generate incentives by choosing a positive wage irrespective of whether the other firm offers a positive wage or not. This strong preference solves the coordination problem. In the unique equilibrium, the first firm induces high incentives, whereas the latter firm optimally decides to free ride.

Numerical approximations show that mixed equilibria are also characterized by firms attempting to free-ride on the incentives set by the opponent. Figure 1 displays the equilibrium of a discretized game for a concrete choice of parameters.[13] In equilibrium, both firms set a wage of zero with a substantial probability and mix rather evenly over an interval above zero with the remaining mass.[14]

From a welfare perspective, the positive externality by inducing incentives for the external worker leads to an additional inefficiency. Consider, for example, the case of homogeneous workers ($t_F = t_{\hat{F}} = t$). Efficient or first-best effort e^{FB} maximizes $v(e) - \frac{e}{t}$, thus leading to

$$e^{FB} = V\left(\frac{1}{t}\right).$$

[13]See Appendix B for technical details.

[14]We strongly conjecture that a similar mixed equilibrium exists also for the heterogeneous case. For discretized versions of the game, existence follows from results such as Harsanyi (1973) showing that typical games possess an odd number of Nash equilibria.

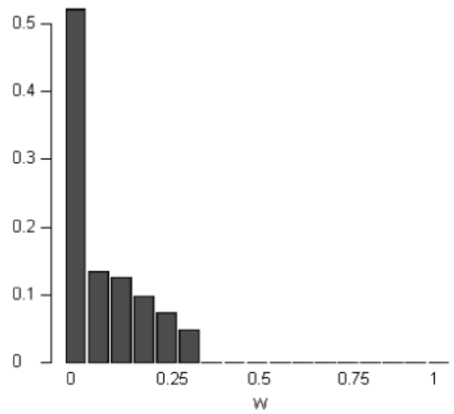


Figure 1: Symmetric mixed equilibrium in a discrete example

Optimal effort of a homogeneous worker in a two-person contest for a wage w is described by (13): $e^{**} = \frac{wt}{4}$. If a multi-plant corporation that consists of locations F and $\hat{F}$ organizes an internal job-promotion contest with wage w as winner prize, it will solve[15]

$$\max_{w \geq 0} 2v\left(e^{**}\right) - w = \max_{w \geq 0} 2v\left(\frac{wt}{4}\right) - w.$$

The solution is $w = \frac{4}{t} V\left(\frac{2}{t}\right)$ implying optimal effort

$$e^{**} = V\left(\frac{2}{t}\right),$$

which is strictly smaller than e^{FB} since V is monotonically decreasing. This inefficiency is well-known in the principal-agent literature: The firm cannot extract the full surplus from his workers, who are protected by limited liability (i.e., the firm is not allowed to choose a negative loser prize). Therefore, it induces less than efficient effort. In our context with a positive externality, the firm that sets a positive wage in equilibrium chooses $w^*_{F\hat{F}} = \frac{4}{t} V\left(\frac{4}{t}\right)$

[15]Recall that the participation constraint is satisfied because of the limited-liability constraint and the worker's zero reservation value. As a direct implication, a firm prefers to choose a zero loser prize.

according to Proposition 4. This wage leads to optimal effort

$$e^* = V\left(\frac{4}{t}\right) < e^{**} < e^{FB}.$$

The ranking of the three effort levels is quite intuitive. Since the value generated by the external worker does not accrue to firm F, optimal incentives are smaller than in the two-person job-promotion contest organized by the multi-plant corporation. Thus, from a welfare perspective both firms A and B should merge to a multi-plant firm in order to internalize the positive externalities in incentive creation.

5.2 Product Market Competition

As in Section 4, the two firms are assumed to serve the same product market. Therefore, a firm's profit function is described by (10). However, as a crucial difference to Section 4, now both firms have a vacant position at the higher hierarchy level and simultaneously compete for the workers at the lower hierarchy levels. We keep the assumption introduced at the beginning of Section 5 that each firm has exactly one worker at the lower hierarchy level. Let, w.l.o.g., $\Delta t := t_F - t_{\hat{F}} \geq 0$ with t_F and $t_{\hat{F}}$ denoting the talents of the two workers at the lower hierarchy level in firm F and firm $\hat{F}$, respectively. Hence, either both firms have equally talented workers in the initial situation or firm F has an H-type worker and firm $\hat{F}$ an L-type worker.

Equilibrium effort levels in the recruiting contest are again given by (13). Inserting into (10) shows that firm F solves

$$\max_{w_F \geq 0} \psi\left(|w_F - w_{\hat{F}}| \cdot T \cdot \Delta t\right) - w_F, \quad \text{with } T := \frac{t_F t_{\hat{F}}}{(t_F + t_{\hat{F}})^2}, \tag{18}$$

whereas $\hat{F}$ solves

$$\max_{w_{\hat{F}} \geq 0} \psi \left(- \left| w_F - w_{\hat{F}} \right| \cdot T \cdot \Delta t \right) - w_{\hat{F}}. \tag{19}$$

The solution of the game between firms F and $\hat{F}$ can be characterized as follows:

Proposition 5 *If workers are homogeneous (i.e., $t_F = t_{\hat{F}} =: t \in \{t_H, t_L\}$), there exists the unique equilibrium $(w_F^*, w_{\hat{F}}^*) = (0,0)$. Under heterogeneous workers (i.e., $t_F = t_H$ and $t_{\hat{F}} = t_L$), either a pure equilibrium $(w_F^*, w_{\hat{F}}^*) = (0,0)$ exists or an equilibrium in mixed strategies.*

Proof. See Appendix A. ■

If firms are homogeneous, no one can achieve a competitive advantage by inducing incentives. Consequently, each firm chooses a zero wage to save costs. If firms are heterogeneous but marginal returns are too small, there will be a corner solution with both firms again setting zero wages. In case of heterogeneous firms and an interior solution, only mixed equilibria exist. Figure 2 displays a discrete approximation of such an equilibrium in a numerical example.[16] We see that firms mix over the same support. Firm F puts a substantial probability mass on the highest wage in the support, while firm $\hat{F}$ puts considerable mass on zero. In this example, firm F earns a payoff of about 2 while firm $\hat{F}$ earns about 0.5. If both firms would escape competition by setting a wage of zero, both would earn a bit more than 1.5. Compared to this, due to its stronger position firm F can gain about 0.5 while the sum of payoffs is reduced by 0.5 in equilibrium.

The logic behind this equilibrium is rather intricate: Firm F prefers the two wages to be as far apart as possible, while firm $\hat{F}$ prefers them to be

[16] See Appendix B for technical details.

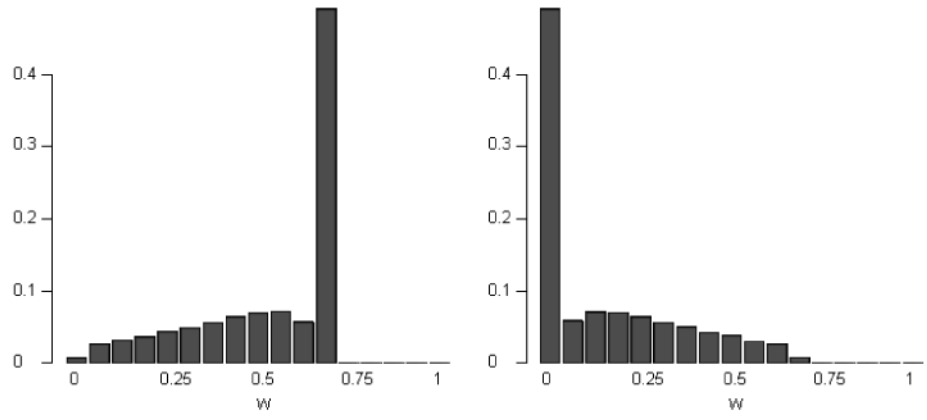


Figure 2: Mixed equilibrium strategies of F and $\hat{F}$ in a discrete example

close together. Moreover, both firms prefer to set small wages. Since firm F often plays high wages, firm $\hat{F}$ sometimes plays high wages as well in order to reduce the wage difference. But firm $\hat{F}$ cannot do this with too high probability because then firm F would have an incentive to free-ride and set a wage of zero – which has a direct negative effect on firm $\hat{F}$'s payoff. Thus, firm $\hat{F}$ sets a wage of zero with a substantial probability and firm F only attempts to free-ride with a comparatively small probability.

It is instructive to note that here firm $\hat{F}$ sets positive wages in order to *reduce* workers' incentives. This is because the firm knows that a strong wage difference between firms enhances competition between the two firms' workers. But since firm F has the more skilled workforce, this increased competition has the consequence that firm $\hat{F}$ loses market shares.

6 Conclusion

We have addressed two kinds of externalities that arise if a firm chooses external recruiting. First, opening the career system can lead to both negative and positive externalities for worker competition. Negative externalities always arise since, for a given vacancy, the enlarged pool of applicants leads to worker discouragement. Positive externalities are generated if external recruiting induces a homogenization of active players which boosts the incentives of a firm's high-ability workers. The firm prefers external recruiting, if the positive externalities dominate the negative ones.

Second, there are externalities between the firms' wage policies in case of simultaneous vacancies since high wages attached to vacant positions offset each other. If we have one strong firm and one weak firm, the latter one uses external recruiting and strategic wage setting to eliminate its competitive disadvantage in the product market. This case shows that – besides employee poaching – a firm may choose its personnel policy to strategically harm a competing firm.

Appendix A

Proof of Proposition 1:

If $e_{L1}, \ldots, e_{Lm_L}$ denote the efforts of the L-type workers and $e_{H1}, \ldots, e_{Hm_H}$ those of the H-type workers, L-type worker α will maximize

$$EU_{L\alpha}(e_{L\alpha}) = \frac{e_{L\alpha}}{e_{L\alpha} + \sum_{i \in \{1,\ldots,m_L\} \setminus \{\alpha\}} e_{Li} + \sum_{j \in \{1,\ldots,m_H\}} e_{Hj}} w - \frac{e_{L\alpha}}{t_L},$$

whereas H-type worker β chooses effort $e_{H\beta}$ to maximize

$$EU_{H\beta}(e_{H\beta}) = \frac{e_{H\beta}}{e_{H\beta} + \sum_{i \in \{1,\ldots,m_L\}} e_{Li} + \sum_{j \in \{1,\ldots,m_H\} \setminus \{\beta\}} e_{Hj}} w - \frac{e_{H\beta}}{t_H}.$$

If $w > 0$, there cannot be an equilibrium with each worker exerting zero effort because then one of the workers can switch to a marginal amount of positive effort and wins w for sure. Since each worker has a strictly concave objective function, worker α either optimally chooses $e_{L\alpha}^* = 0$ if $EU_{L\alpha}'(0) \leq 0$, or

$e^*_{L\alpha} > 0$ with $EU'_{L\alpha}(e^*_{L\alpha}) = 0$ if $EU'_{L\alpha}(0) > 0$. In analogy, we obtain

$$e^*_{H\beta} \begin{cases} = 0 & \text{if} \quad EU'_{H\beta}(0) \leq 0 \\ > 0 \text{ with } EU'_{H\beta}\left(e^*_{H\beta}\right) = 0 & \text{if} \quad EU'_{H\beta}(0) > 0. \end{cases}$$

Hence, a corner solution $e^*_{L\alpha} = 0$ satisfies

$$\frac{\sum_{i\in\{1,\ldots,m_L\}\backslash\{\alpha\}} e_{Li} + \sum_{j\in\{1,\ldots,m_H\}} e_{Hj}}{\left(e^*_{L\alpha} + \sum_{i\in\{1,\ldots,m_L\}\backslash\{\alpha\}} e_{Li} + \sum_{j\in\{1,\ldots,m_H\}} e_{Hj}\right)^2} w \leq \frac{1}{t_L} \Leftrightarrow$$
$$\frac{1}{\sum_{i\in\{1,\ldots,m_L\}\backslash\{\alpha\}} e_{Li} + \sum_{j\in\{1,\ldots,m_H\}} e_{Hj}} w \leq \frac{1}{t_L},$$

and an interior solution $e^*_{L\alpha} > 0$

$$\frac{1}{\sum_{i\in\{1,\ldots,m_L\}\backslash\{\alpha\}} e_{Li} + \sum_{j\in\{1,\ldots,m_H\}} e_{Hj}} w > \frac{1}{t_L}$$

with $e^*_{L\alpha}$ being described by the first-order condition

$$\frac{\sum_{i\in\{1,\ldots,m_L\}\backslash\{\alpha\}} e_{Li} + \sum_{j\in\{1,\ldots,m_H\}} e_{Hj}}{\left(e^*_{L\alpha} + \sum_{i\in\{1,\ldots,m_L\}\backslash\{\alpha\}} e_{Li} + \sum_{j\in\{1,\ldots,m_H\}} e_{Hj}\right)^2} w = \frac{1}{t_L}. \tag{20}$$

Next, we show that there is a unique equilibrium with all workers of the same type choosing identical effort levels. To show uniqueness of the Nash equilibrium we follow an approach put forward by Cornes and Hartley (2005). Let $E \equiv \sum_{i\in\{1,\ldots,m_L\}} e_{Li} + \sum_{j\in\{1,\ldots,m_H\}} e_{Hj}$. From (20) we know that for $e^*_{L\alpha} > 0$ we must have $\frac{E - e^*_{L\alpha}}{E^2} w = \frac{1}{t_L}$ or

$$e^*_{L\alpha} = E\left(1 - \frac{E}{wt_L}\right).$$

Let $e^*_{L\alpha}(E) \equiv \max\left\{E\left(1 - \frac{E}{wt_L}\right), 0\right\}$, which is the unique possible equilibrium value of $e_{L\alpha}$ given that the sum of all effort levels is equal to E.

Similarly, define $e_{H\beta}^{*}(E) \equiv \max\left\{E\left(1-\frac{E}{wt_H}\right),0\right\}$. Then, a necessary condition for $(e_{L1},...,e_{Lm_L},e_{H1},...,e_{m_H})$ being an equilibrium is that the sum of these effort levels E is equal to the sum of the equilibrium effort levels from $e_{L\alpha}^{*}(E)$ and $e_{H\beta}^{*}(E)$. Formally, we must have:

$$\begin{aligned} E &= \sum_{i\in\{1,...,m_L\}} e_{Li}^{*}(E) + \sum_{j\in\{1,...,m_H\}} e_{Hj}^{*}(E) \Leftrightarrow \\ 1 &= \sum_{i\in\{1,...,m_L\}} \max\left\{1-\frac{E}{wt_L},0\right\} + \sum_{j\in\{1,...,m_H\}} \max\left\{1-\frac{E}{wt_H},0\right\}. \end{aligned} \tag{21}$$

The RHS of (21) is decreasing in E, has value $m>1$ for $E=0$, and tends to 0 for $E\to\infty$. Hence, a unique value E^{*} exists satisfying (21). Since $e_{L\alpha}^{*}(E)$ and $e_{H\beta}^{*}(E)$ constitute the unique equilibrium candidate for a given value E, the unique equilibrium is given by $e_{L\alpha}^{*}(E^{*})$ and $e_{H\beta}^{*}(E^{*})$. Thus there exists a unique equilibrium and it has the property that all workers of the same type choose identical effort levels.

Therefore, we have symmetric solutions in the sense of $e_{L\alpha}^{*}=e_{L}^{*}$ ($\alpha=1,\ldots,m_L$) and $e_{H\beta}^{*}=e_{H}^{*}$ ($\beta=1,\ldots,m_H$). The condition for the corner solution $e_{L\alpha}^{*}=e_{L}^{*}=0$ boils down to

$$\frac{1}{m_H e_H^{*}} w \leq \frac{1}{t_L}, \tag{22}$$

and the conditions for an interior solution $e_{L\alpha}^{*}=e_{L}^{*}>0$ can be simplified to

$$\frac{1}{m_H e_H^{*}} w > \frac{1}{t_L} \quad \text{and} \tag{23}$$

$$\frac{(m_L-1)\,e_L^{*}+m_H e_H^{*}}{(m_L e_L^{*}+m_H e_H^{*})^2} w = \frac{1}{t_L}. \tag{24}$$

Analogously, we obtain

$$\frac{1}{m_L e_L^{*}} w \leq \frac{1}{t_H} \tag{25}$$

for $e^*_{H\beta} = e^*_H = 0$, and

$$\frac{1}{m_L e^*_L} w > \frac{1}{t_H} \quad \text{and} \tag{26}$$

$$\frac{m_L e^*_L + (m_H - 1)\, e^*_H}{(m_L e^*_L + m_H e^*_H)^2} w = \frac{1}{t_H} \tag{27}$$

for $e^*_{H\beta} = e^*_H > 0$.

First, we can show by contradiction that a solution $e^*_L > 0$ and $e^*_H = 0$ is not possible. For this solution (24) and (25) must hold at the same time. Inserting $e^*_H = 0$ into (24) yields $e^*_L = [t_L (m_L - 1)\, w] / m_L^2$. Plugging into (25) and rewriting gives $t_H m_L \leq t_L (m_L - 1)$, a contradiction.

However, a corner solution with $e^*_L = 0$ and $e^*_H > 0$ is possible. Combining (22) with (27) and $e^*_L = 0$ leads to

$$e^*_H = \frac{(m_H - 1)\, t_H}{m_H^2} w \quad \text{and} \quad t_H \geq \frac{m_H}{m_H - 1} t_L \quad (m_H > 1),$$

where the last inequality is clearly satisfied for $m_H \to \infty$.

Finally, an interior solution with $e^*_L > 0$ and $e^*_H > 0$ is described by the two first-order conditions (24) and (27). Straightforward computations yield (4) and (5).

Proof of Proposition 2:

If $n_L = 0$ or $n_H = 0$, competing workers are homogeneous irrespective of whether firm F allows external applicants or not. In this situation, F strictly benefits from excluding external hires since a worker's individual equilibrium effort decreases in the number of contestants.

The other possible situations can be divided into three cases. Case (1) deals with $t_L \leq t_H \frac{n_{FH}-1}{n_{FH}}$. Then L-type workers drop out with and without external recruiting (see Proposition 1). F solves

$$\max_w v\left(n_{FH}\frac{n_{FH}-1}{n_{FH}^2}t_H w\right)-w$$

when excluding external workers, and

$$\max_w v\left(n_{FH}\frac{n_H-1}{n_H^2}t_H w\right)-w$$

if it allows external workers to apply. Since $(n_{FH}-1)/n_{FH}^2 \geq (n_H-1)/n_H^2$, firm F prefers to exclude external candidates. Because its objective function is strictly concave, the optimal wage is described by the first-order condition

$$v'\left(\frac{n_{FH}-1}{n_{FH}}t_H w^*\right)\frac{n_{FH}-1}{n_{FH}}t_H=1,$$

given that $v'(0)\frac{n_{FH}-1}{n_{FH}}t_H>1$ guarantees an interior solution. The first-order condition can be rewritten to the expression given in the first line of (9).

Case (2) is characterized by $t_H\frac{n_{FH}-1}{n_{FH}}<t_L\leq t_H\frac{n_H-1}{n_H}$. Now, L-type workers drop out with external recruiting but do not drop out without external hires. Using (4) and (5), under pure internal career competition firm F maximizes

$$v\left(n_{FH}\cdot e_H^*+n_{FL}\cdot e_L^*\right)-w=v\left(\frac{t_Ht_L\left(n_F-1\right)}{\left(n_{FH}t_L+n_{FL}t_H\right)}w\right)-w. \tag{28}$$

If F additionally includes external candidates, his L-type workers will drop out and F maximizes

$$v\left(n_{FH}\frac{n_H-1}{n_H^2}t_H w\right)-w. \tag{29}$$

Firm F will prefer external recruiting, if

$$n_{FH}\frac{n_H-1}{n_H^2} > \frac{t_L\left(n_F-1\right)}{\left(n_{FH}t_L+n_{FL}t_H\right)},$$

which can be rewritten to (7). If F prefers to allow external job candidates it will maximize (29), leading to (8). Otherwise, F maximizes (28), yielding the expression in the second line of (9).

Case (3) deals with $t_H\frac{n_H-1}{n_H} < t_L$. Now, L-type workers will not drop out irrespective of whether firm F allows external applicants or not. Thus, the only effect of opening the career system is an increase in the number of L-type and H-type contestants without influencing the number of effort spending internal workers. We can show that such opening does not pay for the firm since the negative incentive effect of an increased number of contestants always dominates a possibly positive incentive effect by a less heterogeneous field of contestants (see the additional pages for the referees).

Proof of Proposition 3:

Let conditions (6) and (7) be fulfilled. As before, L-type workers drop out with external recruiting but do not drop out without external hires. Using (4) and (5), under pure internal career competition firm F maximizes in analogy to (28):

$$\psi\left(n_{FH}\cdot e_H^*+n_{FL}\cdot e_L^*\right)-w=\psi\left(\frac{t_Ht_L\left(n_F-1\right)}{\left(n_{FH}t_L+n_{FL}t_H\right)}w\right)-w.$$

If F additionally invites external job applicants, all L-type workers will drop out and F maximizes

$$\psi\left(n_{FH}\cdot e_H^*-n_{\hat{F}H}\cdot e_H^*\right)-w=\psi\left(\left(n_{FH}-n_{\hat{F}H}\right)\frac{n_H-1}{n_H^2}t_Hw\right)-w. \quad (30)$$

Thus, for any positive wage w firm F will prefer external recruiting iff

$$\left(n_{FH}-n_{\hat{F}H}\right)\frac{n_H-1}{n_H^2}>\frac{t_L\left(n_F-1\right)}{\left(n_{FH}t_L+n_{FL}t_H\right)}.$$

This condition can only be satisfied for $n_{FH}>n_{\hat{F}H}$. In that case, it can be rewritten to (11), and F maximizes (30) leading to (12). Otherwise, we are in the analogous situation as without product market competition where F maximizes (28), yielding the expression in the second line of (9) with function V being replaced by Ψ.

Proof of Proposition 4:

To prove the proposition, we can make use of the following two lemmas:

Lemma 1 *If $w_F>0$, then $w_{\hat{F}}^*\left(w_F\right)\notin\left(0,2w_F\right]$, $F,\hat{F}\in\{A,B\}$, $F\neq\hat{F}$.*

Proof. Given $w_F>0$, firm $\hat{F}$'s objective function (15) is strictly larger for $w_{\hat{F}}=0$ than for $w_{\hat{F}}\in\left(0,2w_F\right]$. ■

Hence, investing in incentives can only be profitable to a firm if the existing incentives induced by the other firm are at least doubled. Otherwise, such investment would deteriorate existing incentives at positive costs.

Lemma 2 *If $w_F\geq w_{\hat{F}F}^*$, then $w_{\hat{F}}^*\left(w_F\right)=0$.*

Proof. $w_{\hat{F}}^*\left(w_F\right)\notin\left(0,2w_F\right]$ due to Lemma 1. $w_{\hat{F}}>2w_F$ cannot be a best reply to $w_F\geq w_{\hat{F}F}^*$ either: Problem

$$\max_{w_{\hat{F}}} v\left(\left(w_{\hat{F}}-w_F\right)T_{\hat{F}}\right)-w_{\hat{F}} \tag{31}$$

is solved by $w_{\hat{F}}=w_{\hat{F}F}^*+w_F\leq 2w_F$. Since (31) is strictly concave, $\hat{F}$ prefers $w_{\hat{F}}=2w_F$ when choosing $w_{\hat{F}}\in\left[2w_F,\infty\right)$. However, $w_{\hat{F}}=0$ would implement the same effort level at zero costs. ■

Lemma 2 states that a firm should completely save costs by choosing a zero wage if the other firm already induces sufficient incentives.

Now we can prove Proposition 4. We start with the case of *homogeneous workers*: $t_F = t_{\hat{F}} =: t$, so that

$$w_F^*(0) = w_{\hat{F}}^*(0) = \frac{4}{t} V\left(\frac{4}{t}\right) = \begin{cases} w_{HH}^* & \text{if} \quad t = t_H \\ w_{LL}^* & \text{if} \quad t = t_L \end{cases} \tag{32}$$

according to (16).

(1) If $w_F = 0$, then $w_{\hat{F}}^*(0) = w_{\hat{F}F}^* = w_{F\hat{F}}^* = \frac{4}{t} V\left(\frac{4}{t}\right)$ according to (32). Given this behavior of $\hat{F}$, firm F has no incentive to deviate (Lemma 2).

(2) If $w_F \geq w_{F\hat{F}}^* = w_{\hat{F}F}^*$, then $w_{\hat{F}}^*(w_F) = 0$ by Lemma 2. Given $w_{\hat{F}} = 0$, firm F will choose $w_F^*(0) = w_{F\hat{F}}^*$ (see (32)) and no firm has an incentive to deviate.

(3) If $w_F \in (0, w_{F\hat{F}}^*] = (0, w_{\hat{F}F}^*]$, then $w_{\hat{F}}^*(w_F) \notin (0, 2w_F]$ (Lemma 1). There are three possibilities: (i) If $\hat{F}$ reacts by choosing $w_{\hat{F}} \geq w_{\hat{F}F}^* = w_{F\hat{F}}^*$, then $w_F^*(w_{\hat{F}}) = 0$ (Lemma 2) and $w_{\hat{F}}^*(0) = w_{\hat{F}F}^* = w_{F\hat{F}}^*$ according to (32). (ii) If $\hat{F}$ reacts by choosing $w_{\hat{F}} = 0$, then $w_F^*(0) = w_{F\hat{F}}^* = w_{\hat{F}F}^*$ (see (32)) and no one deviates. (iii) If $\hat{F}$ reacts by choosing $w_{\hat{F}} \in (2w_F, w_{\hat{F}F}^*] = (2w_F, w_{F\hat{F}}^*]$, then his best reply will solve

$$\max_{w_{\hat{F}} \in \left(2w_F, w_{\hat{F}F}^*\right]} v\left(\frac{(w_{\hat{F}} - w_F)\, t}{4}\right) - w_{\hat{F}}.$$

Since in case (iii), by assumption, $\hat{F}$ does not react by choosing zero effort (i.e., there is not a corner solution at zero as in case (ii))), the first-order condition can be applied, which leads to $w_{\hat{F}} - w_F = \frac{4}{t} V\left(\frac{4}{t}\right) = w_{\hat{F}F}^* \Leftrightarrow w_{\hat{F}} = w_{\hat{F}F}^* + w_F$. Because in case (iii) $\hat{F}$ is restricted to $w_{\hat{F}} \in (2w_F, w_{\hat{F}F}^*]$ and since the firm's objective function is strictly concave, $\hat{F}$ will choose the corner solution $w_{\hat{F}} = w_{\hat{F}F}^*$. Then $w_F^*(w_{\hat{F}}) = 0$ (Lemma 2) and no one deviates.

Existence of a symmetric mixed equilibrium can be shown as follows:

Denote again by $w^*_{F\hat{F}}$ the wage one firm sets given that the other firm $\hat{F}$ sets a wage of zero. By the concavity of v, firm F will not respond with a wage strictly greater than $w^*_{F\hat{F}}$ to any strategy of firm $\hat{F}$ and vice-versa. Thus, any equilibrium of the restricted game where firms can set wages only in the interval $[0, w^*_{F\hat{F}}]$ must be an equilibrium of the original game as well. Payoffs in the restricted game are continuous and bounded and the action space is compact. Thus, by the main result of Becker and Damianov (2006) the restricted game possesses a symmetric equilibrium which is then also an equilibrium of the unrestricted game. Since there are no symmetric pure equilibria, this equilibrium must be in mixed strategies.

Second, we examine the *heterogeneous* case with $t_F, t_{\hat{F}} \in \{t_H, t_L\}$; $t_F \neq t_{\hat{F}}$. As any relation $w^*_{LH} \gtreqless w^*_{HL}$ is possible and the special case $w^*_{LH} = w^*_{HL}$ has already been discussed in the previous paragraph on homogeneity, without loss of generality it is sufficient to consider the remaining general case $w^*_{F\hat{F}} < w^*_{\hat{F}F}$ with $F, \hat{F} \in \{A, B\}$, $F \neq \hat{F}$.

(1a) If $w_F = 0$, then $w^*_{\hat{F}}(0) = w^*_{\hat{F}F}$. Given this behavior of $\hat{F}$, firm F has no incentive to deviate (Lemma 2).

(1b) If $w_{\hat{F}} = 0$, then $w^*_F(0) = w^*_{F\hat{F}}$. Given $w_F = w^*_{F\hat{F}}$, note that $w^*_{\hat{F}}(w^*_{F\hat{F}}) \notin (0, 2w^*_{F\hat{F}}]$. However, deviation to $w_{\hat{F}} \geq 2w^*_{F\hat{F}}$ can be optimal: Firm $\hat{F}$ solves

$$\max_{w_{\hat{F}} \geq 2w^*_{F\hat{F}}} v\left(\left(w_{\hat{F}} - w^*_{F\hat{F}}\right) T_{\hat{F}}\right) - w_{\hat{F}},$$

which leads to the solution $w_{\hat{F}} = w^*_{F\hat{F}} + w^*_{\hat{F}F} \geq 2w^*_{F\hat{F}}$. $\hat{F}$ will only deviate if this gives a higher expected profit compared to the initial situation $(w_F, w_{\hat{F}}) = (w^*_{F\hat{F}}, 0)$. This is not fulfilled if

$$v\left(w^*_{\hat{F}F} T_{\hat{F}}\right) - v\left(w^*_{F\hat{F}} T_{\hat{F}}\right) \leq w^*_{F\hat{F}} + w^*_{\hat{F}F}.$$

If this condition holds, $(w^*_F, w^*_{\hat{F}}) = (w^*_{F\hat{F}}, 0)$ is an equilibrium. Otherwise, $\hat{F}$ will deviate to $w_{\hat{F}} = w^*_{F\hat{F}} + w^*_{\hat{F}F}$ and we will end up in $(w^*_F, w^*_{\hat{F}}) = (0, w^*_{\hat{F}F})$

due to Lemma 2.

(2a) If $w_F \geq w^*_{\hat{F}F}$, then $w^*_{\hat{F}}(w_F) = 0$ by Lemma 2. Given $w_{\hat{F}} = 0$, firm F will choose $w^*_F(0) = w^*_{F\hat{F}}$ and we are back in the reasoning of (1b), resulting either in $(w^*_F, w^*_{\hat{F}}) = (0, w^*_{\hat{F}F})$ or in $(w^*_F, w^*_{\hat{F}}) = (w^*_{F\hat{F}}, 0)$.

(2b) If $w_{\hat{F}} \geq w^*_{F\hat{F}}$, then $w^*_F(w_{\hat{F}}) = 0$ by Lemma 2. Given $w_F = 0$, firm $\hat{F}$ will choose $w^*_{\hat{F}}(0) = w^*_{\hat{F}F}$ and no firm has an incentive to deviate.

(3a) If $w_F \in (0, w^*_{\hat{F}F}]$, then $w^*_{\hat{F}}(w_F) \notin (0, 2w_F]$ (Lemma 1). There are three possibilities: (i) If $\hat{F}$ reacts by choosing $w_{\hat{F}} \geq w^*_{F\hat{F}}$, then $w^*_F(w_{\hat{F}}) = 0$ (Lemma 2) and $w^*_{\hat{F}}(0) = w^*_{\hat{F}F}$. (ii) If $\hat{F}$ reacts by choosing $w_{\hat{F}} = 0$, then $w^*_F(0) = w^*_{F\hat{F}}$ and we are back in the reasoning of (1b), resulting either in $(w^*_F, w^*_{\hat{F}}) = (0, w^*_{\hat{F}F})$ or in $(w^*_F, w^*_{\hat{F}}) = (w^*_{F\hat{F}}, 0)$. (iii) If $\hat{F}$ reacts by choosing $w_{\hat{F}} \in (2w_F, w^*_{F\hat{F}}]$, then his best reply will solve

$$\max_{w_{\hat{F}} \in \left(2w_F, w^*_{F\hat{F}}\right]} v\left(\left(w_{\hat{F}} - w_F\right) T_{\hat{F}}\right) - w_{\hat{F}}.$$

The first-order condition leads to $w_{\hat{F}} = w^*_{\hat{F}F} + w_F$. Because in case (iii) $\hat{F}$ is restricted to $w_{\hat{F}} \in (2w_F, w^*_{F\hat{F}}]$ and since the firm's objective function is strictly concave, $\hat{F}$ will choose the corner solution $w_{\hat{F}} = w^*_{F\hat{F}}$. Then $w^*_F(w_{\hat{F}}) = 0$ (Lemma 2) followed by $w^*_{\hat{F}}(0) = w^*_{\hat{F}F}$ and no one further deviates.

(3b) If $w_{\hat{F}} \in (0, w^*_{F\hat{F}}]$, we also have to consider three possibilities: (i) If F reacts by choosing $w_F \geq w^*_{\hat{F}F}$, then $w^*_{\hat{F}}(w_F) = 0$ followed by $w^*_F(0) = w^*_{F\hat{F}}$ and we are back in the reasoning of (1b), resulting either in $(w^*_F, w^*_{\hat{F}}) = (0, w^*_{\hat{F}F})$ or in $(w^*_F, w^*_{\hat{F}}) = (w^*_{F\hat{F}}, 0)$. (ii) If F reacts by choosing $w_F = 0$, then $w^*_{\hat{F}}(0) = w^*_{\hat{F}F}$ and no one has an incentive to deviate. (iii) If F reacts by choosing $w_F \in (0, w^*_{\hat{F}F})$ we are back in the reasoning of (3a) resulting into $(w^*_F, w^*_{\hat{F}}) = (0, w^*_{\hat{F}F})$ or $(w^*_F, w^*_{\hat{F}}) = (w^*_{F\hat{F}}, 0)$.

Proof of Proposition 5:
If workers are homogeneous, then $\Delta t = 0$ in (18) and (19), so that each firm chooses a zero wage as dominant strategy. The result on heterogeneous workers follows from the firms' best-response functions.

Lemma 3 *The best response of firm $\hat{F}$ satisfies $w^*_{\hat{F}}(w_F) \leq w_F$.*

Proof. The claim can be proved by contradiction. Suppose that $w_{\hat{F}} > w_F$ in (19). Then $\hat{F}$ strictly gains from switching to $w'_{\hat{F}}$ with $w'_{\hat{F}} < w_{\hat{F}}$ and $|w_F - w_{\hat{F}}| = |w_F - w'_{\hat{F}}|$. ■

If $\frac{t_H t_L \Delta t}{(t_H+t_L)^2}\psi'(0) < 1$, then F's best response to $w_{\hat{F}} = 0$ is given by $w^*_F(0) = 0$ and, applying Lemma 3, both firms end up in equilibrium $(w^*_F, w^*_{\hat{F}}) = (0,0)$. If

$$\frac{t_H t_L \Delta t}{(t_H + t_L)^2}\psi'(0) > 1, \tag{33}$$

then $(0,0)$ is not a Nash equilibrium and, as we argue next, no pure equilibrium exists. Note first that there cannot be a pure equilibrium where both firms set the same wage $w > 0$: Assume (w,w) is a Nash equilibrium. Since costs are linear, if neither firm wants to deviate to $\tilde{w} = w + z$ for some $z > 0$, then due to the linearity of costs both firms setting a wage of 0 must be a Nash equilibrium as well which contradicts our assumption that $(0,0)$ is not an equilibrium.[17]

Moreover, there cannot be a pure equilibrium where firm F sets w and firm $\hat{F}$ sets $\hat{w} > w$ by Lemma 3. Finally, we have to show that there cannot be an equilibrium where firms play wages w and $\hat{w}$ with $w > \hat{w}$. To simplify notation, we define $\phi(x) := \psi(x \cdot T \cdot \Delta t)$. Since this is merely a rescaling, ϕ inherits all properties we assumed for ψ. In order to show that we do not have a Nash equilibrium, it suffices to consider small deviations which

[17] Clearly, the opposite implication does not hold: From (w,w) firms can deviate to both, higher and lower wages while from $(0,0)$ they can only deviate to higher wages. Thus, $(0,0)$ being an equilibrium does not imply that (w,w) is an equilibrium as well.

leave $w - \hat{w}$ positive. Therefore, we can leave away the absolute value and assume that firm F earns a payoff of $\phi(w - \hat{w}) - w$ and firm $\hat{F}$ earns a payoff of $\phi(\hat{w} - w) - \hat{w}$. By our assumption that $\phi(x) + \phi(-x)$ is constant, we have that $\phi'(x) = \phi'(-x)$ and $\phi''(x) = -\phi''(-x)$. Thus, if the first-order condition $\phi'(w - \hat{w}) = 1$ of firm F is satisfied, then $\hat{F}$'s first-order condition $\phi'(\hat{w} - w) = 1$ is satisfied as well. Now, consider second-order conditions: Since we assumed ϕ to be concave for positive arguments, we have $\phi''(w - \hat{w}) < 0$, so that firm F is indeed in a local maximum. The second derivative of firm $\hat{F}$'s payoff is $\phi''(\hat{w} - w) = -\phi''(w - \hat{w}) > 0$. Thus, firm $\hat{F}$ is in a local minimum and prefers to deviate to a marginally smaller or larger wage. Therefore, no pure strategy equilibrium exists.

It remains to be shown that a Nash equilibrium exists. Whenever no pure equilibrium exists this must be a mixed equilibrium. We first argue that firms do not play wages greater than $\overline{w} = \lim_{x\to\infty} \psi(x)$ in any equilibrium: Firm F can guarantee itself a non-negative payoff through setting a wage of 0 regardless of its opponent's strategy. Since setting a wage greater than $\overline{w}$ leads to a negative payoff for firm F regardless of the opponent's strategy, firm F does not play wages outside $[0, \overline{w}]$ in any equilibrium. Now, consider firm $\hat{F}$. By Lemma 3, if firm F plays a pure strategy w, firm $\hat{F}$ is better off playing w than setting a wage strictly higher than w. Likewise, if firm F plays a mixed strategy, firm $\hat{F}$ does not play wages above the support of F's strategy in equilibrium. Therefore, neither firm plays wages outside $[0, \overline{w}]$ in any equilibrium.

Thus, an equilibrium of the restricted game where firms can only set wages from $[0, \overline{w}]$ must be an equilibrium of the unrestricted game as well. In the restricted game, payoffs are bounded and continuous and the action space is compact. Therefore, we can apply the result of Glicksberg (1952) to show existence of equilibrium in the restricted game. This implies existence of equilibrium in the unrestricted game.

Appendix B

Details of Numerical Results: The numerical examples of Sections 5.1 and 5.2 are based on the specification

$$v(x) = \psi(x) = \frac{\pi}{2} + \arcsin(12x)$$

which obviously fulfills the requirements we made on v and ψ. In Section 5.1 we consider $t_F = t_{\hat{F}} = \frac{3}{2}$ while in Section 5.2 we choose $t_F = 3$ and $t_{\hat{F}} = 1$.[18] We discretized the game allowing only wages which are multiples of $\frac{1}{16}$. The discretized game was solved using the software package Gambit.[19]

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[18] The structure of the equilibria appeared to be robust to variations of these choices.

[19] Due to limitations of the software, finer discretizations were unavailable and payoffs had to be rounded to four valid digits. An even rougher discretization and rounding leads to results which are hardly distinguishable. Therefore, more accurate approximations can only be expected to lead to small quantitative corrections.

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Additional pages for the referees on Proposition 2, case (3):

Let $e_L^*(m_L, m_H)$ and $e_H^*(m_L, m_H)$ denote the equilibrium efforts being described by (4) and (5). We can first show that increased heterogeneous competition via opening the career system leads to a decrease of internal workers' efforts for almost all feasible parameter constellations. For $e_L^*(n_{FL}, n_{FH})$ we obtain[20]

$$\frac{\partial e_L^*(n_{FL}, n_{FH})}{\partial n_{FH}} = \Omega_1 \cdot [n_{FH}t_L - (2n_{FH} + n_{FL} - 2)\, t_H]$$
$$\text{with } \Omega_1 = \Theta \cdot (n_{FL}t_H - (n_{FL} - 1)\, t_L) > 0 \text{ and } \Theta = \frac{wt_Ht_L}{(n_{FH}t_L + n_{FL}t_H)^3}$$
$$\frac{\partial e_L^*(n_{FL}, n_{FH})}{\partial n_{FL}} = \Omega_2 \cdot [n_{FH}t_L - (2n_{FH} + n_{FL} - 2)\, t_H]$$
$$\text{with } \Omega_2 = \Theta \cdot (n_{FH}t_L - (n_{FH} - 1)\, t_H) > 0.$$

Only the term in square brackets of each derivative can be negative. For the derivatives to be positive we must have that $n_{FH} > 2n_{FH} + n_{FL} - 2 \Leftrightarrow n_{FH} + n_{FL} < 2$, which is impossible because each firm has at least two workers at the lower tier of the hierarchy. For $e_H^*(n_{FL}, n_{FH})$ the comparative statics read as

$$\frac{\partial e_H^*(n_{FL}, n_{FH})}{\partial n_{FH}} = \Omega_1 \cdot [n_{FL}t_H - (2n_{FL} + n_{FH} - 2)\, t_L]$$
$$\frac{\partial e_H^*(n_{FL}, n_{FH})}{\partial n_{FL}} = \Omega_2 \cdot [n_{FL}t_H - (2n_{FL} + n_{FH} - 2)\, t_L].$$

Similar to the derivatives before, only the term in square brackets can be negative. It is positive iff

$$n_{FL}t_H > (2n_{FL} + n_{FH} - 2)\, t_L.$$

[20] Of course, n_{FL} and n_{FH} are integers. However, for n_{FL} and n_{FH} being not too small $e_L^*(n_{FL}, n_{FH})$ and $e_H^*(n_{FL}, n_{FH})$ are monotonically decreasing in the number of workers of both types so that the results on marginal changes of these numbers carry over to discrete changes.

Since the talent of H-type workers is restricted to $t_H \frac{n_{FH}-1}{n_{FH}} < t_L$, to be true the inequality must at least be satisfied for $t_H = \frac{n_{FH}}{n_{FH}-1} t_L$. Inserting into the inequality yields

$$(2 - n_{FH})(n_F - 1) > 0,$$

which only holds for $n_{FH} = 1$ and $n_{FL} \geq 1$, or for $n_{FH} = 0$ and $n_{FL} \geq 2$.

Altogether, the comparative-static results point out that for $n_{FH} \geq 2$ it does not pay off for F to enlarge worker competition by allowing external applications: Internal workers become discouraged, irrespective of the mixture of the two firms' workers at the lower hierarchy level. However, we still have to check out whether increasing $e_H^*(n_{FL}, n_{FH})$ by external recruiting under $n_{FH} = 1$ or $n_{FH} = 0$ outweighs lower values of $e_L^*(n_{FL}, n_{FH})$.

We start with the case of $n_{FH} = 1$. Under pure internal recruiting, firm F maximizes

$$v\left(\frac{t_H t_L n_{FL}}{t_L + n_{FL} t_H} w\right) - w.$$

Allowing external applicants would lead to objective function

$$v\left(\frac{t_H t_L (n_H + n_L - 1)(t_L + t_H n_{FL} + (t_H - t_L)(n_L - n_H n_{FL}))}{(n_H t_L + n_L t_H)^2} w\right) - w.$$

Thus, F will open its career system for external workers if and only if

$$\frac{n_{FL}}{t_L + n_{FL} t_H} < \frac{(n_H + n_L - 1)(t_L + t_H n_{FL} + (t_H - t_L)(n_L - n_H n_{FL}))}{(n_H t_L + n_L t_H)^2}. \tag{34}$$

Note that this inequality does not hold for $n_H = 1$. Hence, we must have $n_H \geq 2$. Differentiating RHS(34) with respect to n_H yields

$$\frac{(2 - n_H) t_L + n_L (t_H - 2t_L)}{(n_H t_L + n_L t_H)^3} (t_L + t_H n_{FL} + (t_H - t_L)(n_L - n_H n_{FL})) \\ - (t_H - t_L) n_{FL} \frac{(n_H + n_L - 1)}{(n_H t_L + n_L t_H)^2},$$

which is negative because $n_H \geq 2$ and $t_H < 2t_L$ (since $t_H < \frac{n_H}{n_H - 1} t_L, \forall n_H \geq 2$). Therefore, if (34) can be satisfied, it must at least hold for the lower limit $n_H = 2$. Inserting into (34) and substituting for $n_L = n_{FL} + n_{\hat{F}L}$ (again, $\hat{F}$ indicates the other firm) gives

$$\frac{n_{FL}}{t_L + n_{FL} t_H} < \frac{\left(n_{FL} + n_{\hat{F}L} + 1\right)\left(t_L + t_H n_{FL} + \left(t_H - t_L\right)\left(n_{\hat{F}L} - n_{FL}\right)\right)}{\left(2t_L + \left(n_{FL} + n_{\hat{F}L}\right) t_H\right)^2}. \tag{35}$$

Differentiating RHS(35) with respect to $n_{\hat{F}L}$ leads to

$$\frac{\left(t_H - 2t_L\right)\left[\left(n_{FL} + 1\right) t_H n_{FL} + n_{\hat{F}L}\left(2t_L + t_H\left(n_{FL} - 1\right)\right)\right]}{\left(2t_L + \left(n_{FL} + n_{\hat{F}L}\right) t_H\right)^3},$$

which is negative due to $t_H < 2t_L$. Thus, if (35) holds, it must at least be true for $n_{\hat{F}L} = 1$. Inserting into (35) and rearranging gives

$$\left[\left(t_H - t_L\right) n_{FL}^3 + \left(3n_{FL} - 2\right) t_L\right] t_H + \left(2t_H - t_L\right) t_L n_{FL}^2 + \left[\left(n_{FL} - 1\right) t_H^2 + 2t_L^2\right] n_{FL} < 0,$$

which cannot be true. To sum up, F will prefer to exclude external workers from competing with internal ones if $n_{FH} = 1$.

Finally, we have to consider the case of $n_{FH} = 0$. If firm F excludes applicants from the other firm, it will maximize

$$v\left(\frac{t_L\left(n_{FL} - 1\right)}{n_{FL}} w\right) - w.$$

Under the external-recruiting policy, F maximizes

$$v\left(n_{FL} \frac{t_H t_L\left(n_H + n_L - 1\right)\left(n_H t_L - \left(n_H - 1\right) t_H\right)}{\left(n_H t_L + n_L t_H\right)^2} w\right) - w.$$

F will prefer the latter policy if and only if

$$\frac{n_{FL} - 1}{n_{FL}} < n_{FL} \frac{t_H\left(n_H + n_L - 1\right)\left(n_H t_L - \left(n_H - 1\right) t_H\right)}{\left(n_H t_L + n_L t_H\right)^2}. \tag{36}$$

Since

$$\frac{\partial RHS\,(36)}{\partial t_H} = -\frac{n_H t_L\,(n_H + n_L - 1)\,(n_H\,(2t_H - t_L) + (n_L - 2)\,t_H)}{(n_H t_L + n_L t_H)^3}$$

is negative,[21] for inequality (36) to be true it must at least hold for $t_H = t_L$. Inserting $t_H = t_L$ into (36) yields

$$\frac{n_{FL} - 1}{n_{FL}} < \frac{n_{FL}\,(n_H + n_L - 1)}{(n_H + n_L)^2}.$$

Note that the RHS is decreasing in n_H: $\frac{\partial}{\partial n_H}\left(\frac{n_H+n_L-1}{(n_H+n_L)^2}\right) = -\frac{(n_H+n_L-2)}{(n_H+n_L)^3} < 0$ as $n_H + n_L \geq 4$. Inserting the best possible case[22] $n_H = 1$ into the last inequality gives

$$\frac{n_{FL} - 1}{n_{FL}} < \frac{n_{FL}\,(n_{FL} + n_{\hat{F}L})}{(1 + n_{FL} + n_{\hat{F}L})^2}.$$

Further, note that

$$\frac{\partial}{\partial n_{\hat{F}L}}\left(\frac{n_{FL} + n_{\hat{F}L}}{(1 + n_{FL} + n_{\hat{F}L})^2}\right) = -\frac{n_{\hat{F}L} + n_{FL} - 1}{(n_{\hat{F}L} + n_{FL} + 1)^3} < 0.$$

Therefore, plugging $n_{\hat{F}L} = 1$ into the last inequality leads to

$$\frac{n_{FL} - 1}{n_{FL}} < \frac{n_{FL}\,(n_{FL} + 1)}{(2 + n_{FL})^2} \Leftrightarrow n_{FL}^2 < 2,$$

which contradicts $n_{FL} \geq 2$. Thus, F will not prefer to open its career system for external hires if $n_{FH} = 0$.

[21] Note that we must have $n_L \geq 2$ since each firm consists of at least two workers at the lower hierarchy level and since $n_{FH} = 0$, which implies $n_{FL} \geq 2$ and, hence, $n_L \geq 2$.

[22] Recall from the beginning of the proof that we can exclude $n_H = 0$.