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The Evolutionary Stability of Optimism, Pessimism and Complete Ignorance

by

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The Evolutionary Stability of Optimism, Pessimism and Complete Ignorance*

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Abstract

We provide an evolutionary foundation to evidence that in some situations humans maintain optimistic or pessimistic attitudes towards uncertainty and are ignorant to relevant aspects of the environment. Players in strategic games face Knightian uncertainty about opponents' actions and maximize individually their Choquet expected utility. Our Choquet expected utility model allows for both an optimistic or pessimistic attitude towards uncertainty as well as ignorance to strategic dependencies. An optimist (resp. pessimist) overweights good (resp. bad) outcomes. A complete ignorant never reacts to opponents' change of actions. With qualifications we show that optimistic (resp. pessimistic) complete ignorance is evolutionary stable / yields a strategic advantage in submodular (resp. supermodular) games with aggregate externalities. Moreover, this evolutionary stable preference leads to Walrasian behavior in those classes of games.

Keywords: ambiguity, Knightian uncertainty, Choquet expected utility, neo-additive capacity, Hurwicz criterion, Maximin, Minimax, Ellsberg paradox, overconfidence, supermodularity, aggregative games, monotone comparative statics, playing the field, evolution of preferences.

JEL-Classifications: C72, C73, D01, D43, D81, L13.

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1 Introduction

The motivation for this work is twofold: First, we want to provide an evolutionary foundation for why humans maintain in some situations an optimistic or pessimistic attitude towards uncertainty and are ignorant to strategic aspects. This is closely related to the question about when do optimism, pessimism and ignorance yield a strategic advantage. Second, on a more theoretical level, we want to study how we can restrict by evolutionary arguments degrees of freedom in models with Knightian uncertainty, ambiguity or imprecise beliefs. In particular, we want to endogeneize by evolutionary arguments a player's attitude towards Knightian uncertainty as well as the amount of Knightian uncertainty over opponents' actions in strategic games.

In the literature on social psychology, there is evidence for biases in information processing and their relation to "success". For example, Seligman and Schulman (1986) found that more optimistic health insurance agents sold more policies during the first year of employment and were less likely to quit. Cooper, Dunkelberg, and Woo (1988), using interviews, found that self-assessed chances of new entrepreneurs' success are uncorrelated with education, prior experience, and start-up capital, and are overly optimistic. Taylor and Brown (1988) found that mentally healthy individuals maintain some unrealistic optimism whereas depressed individuals have more accurate perceptions. Most studies focus on a positive bias in self-evaluations or overconfidence in own skills and abilities (e.g. Svenson, 1981). This is perhaps distinct from a systematic bias in judging the impact of uncertain events on one's life. Studies on individual decision making show that the majority of subjects shy away from uncertain prospects like in the Ellsberg's paradox (Ellsberg, 1961, for a survey see for example Camerer and Weber, 1992). To summarize, there is evidence on optimism in psychology and evidence on pessimism in the literature on individual decision making. This may suggest that both types of belief biases are present in the majority of the population and are not stable across situations. An individual may hold optimistic beliefs in some situations but pessimistic beliefs in some other situations. We do not know of any experimental study that analyzes this hypothesis. In this article we seek an evolutionary explanation and show that the bias may depend on the strategic situation.

We model Knightian uncertainty, ambiguity or imprecise beliefs by Choquet expected utility theory (CEU). This concerns situations where probabilities may be unknown or imperfectly known as opposed to a situations under *risk* where probabilities are known, a distinction made by Knight (1921). In CEU the decision maker's belief is represented by *non-additive probabilities* so called *capacities*, i.e., probabilities that do not necessarily add up to one, see Schmeidler (1989). Decision makers maximize the expected value of a utility function with respect to a capacity, and the expectation is the Choquet integral (Choquet, 1953-54, Schmeidler, 1989). Various axiomatizations of CEU have been presented such as by Schmeidler (1989), Gilboa (1987) and Sarin and Wakker (1992). We make use of a parameterized version of CEU with respect to neo-additive capacities (see Chatenauneuf, Eichberger, and Grant, 2005). This class includes as special cases Subjective expected utility (SEU) as well as various preferences for decision making under complete ignorance such as Minimax (Wald, 1951), Maximax, and Hurwicz preferences (Hurwicz, 1951, Arrow and Hurwicz, 1972).

Since CEU is a generalization of conventional Subjective expected utility theory (SEU), it has more degrees of freedom like the degrees of ignorance and degrees of optimism and pessimism defined in the next section. It is natural to ask how to select among the degrees of optimism/pessimism and ignorance. A possible answer could be provided in an evolutionary framework: If evolution chooses preferences parameterized by those degrees of freedom, which one would it choose? To study such questions, we make use of the literature on Choquet expected utility in strategic games, in which players face Knigthian uncertainty about opponents' actions (see for instance Dow and Werlang, 1994, Eichberger and Kelsey, 2000, 2002, Marinacci, 2000, and Eichberger, Kelsey, and Schipper, 2005). Contrary to the major motivation for Knightian uncertainty, according to which it is difficult to assign probabilities to unfamiliar situations, we show that even after a very large number of repeated interactions implicitly assumed to be behind evolution, players may be still have biased beliefs and are completely ignorant.

Our work is directly related to the growing literature on the evolution of preferences (see the special issue of the Journal of Economic Theory, 2001, and for example more recent work by Heifetz, Shannon and Spiegel, 2005) that started with Güth and Yaari (1992) and Güth (1995). However, we employ Schaffer's (1988, 1989) notion of evolutionary stability for finite populations because we believe that in many situations of economic relevance, players "play the field", i.e., all players in a finite set of players play a game. Schaffer's notion also allows for an interpretation as contest since it is closely related to relative profit maximization (for other applications of Schaffer's notion see Shubik and Levitan, 1980, Hehenkamp, Leininger, and Possajennikov, 2004, Alós-Ferrer and Ania, 2005). There are a few articles that study the evolution of attitudes towards risk (Rubin and Paul, 1979, Karni and Schmeidler, 1986, Cooper, 1987, Sinn and Weichenrieder, 1993, Sinn, 2003, Robson, 1996a, 1996b, To, 1999, Dekel and Scotchmer, 1999, Wärneryd, 2002) although in a very different setting compared to ours. None of those studies concerns strategic uncertainty. Most of those studies lend support for expected utility theory. Robson (1996a) also considers an extension of his model in which non-expected utility can evolve. A study which is probably most closely related to ours is Skaperdas (1991), who studies the advantage of risk attitudes in a specific conflict game. We don't know of any study that considers the evolution of non-expected utility in a strategic context.

The classes of strategic games studied in this work are submodular games as well as supermodular games with aggregation and externalities. These classes include many prominent examples in economics (see the discussion section). We can make use of results on monotone comparative statics of optimal solutions of submodular or supermodular functions (see Topkis, 1998). Moreover, we can apply results on finite population evolutionary stability for submodular and supermodular games with aggregation (Schipper, 2003, 2005, Alós-Ferrer and Ania, 2005).

To illustrate the intuition for our results, consider for example a version of a Nash bargaining game. Let there be two players who simultaneously demand a share of a fixed pie. If the demands sum up to less than 100% of the pie, then the pie is shared according to the demands. Otherwise, players receive nothing. Let each player face Knightian uncertainty about the opponents' demand. Suppose that a player is a pessimist. Then she overweights bad demands by the opponent, i.e., she overweights large demands by the opponent. The more pessimistic the belief, the lower is the best-response demand because the player fears the incompatibility of demands resulting in zero payoffs. Could such pessimistic beliefs be evolutionary stable? Suppose the opponent (the "mutant") is not as pessimistic, then his best-response is a larger demand. If demands add up to less than 100%, this opponent is strictly better off than the pessimist, otherwise both get nothing and he is not worse off. Thus pessimism can't be evolutionary stable.

Is optimism evolution stable? Suppose the opponent is an extreme optimist in the sense that he believes that the opponent will demand zero, then the best-response is to demand 100%. If the opponent does indeed demand zero, the extreme optimist is strictly better off. Otherwise, if the opponent does demand some strict positive share, then both receive nothing and the extreme optimist is not worse off. Thus there is no preference with an attitude towards Knightian uncertainty that would successfully invade a set of extremely optimistic players because an optimist can not be made worse off relative to other players.

The example of the Nash bargaining game is also an example for the evolutionary stability of complete ignorance. A completely ignorant player demands as if the opponent was not there. The evolutionary stability of (to some degree) optimistic preferences with complete ignorance holds with qualifications not only for the Nash bargaining game but for an entire class of games characterized by a general notion of strategic substitutes, so called submodular games. Similarly, we show that preferences reflecting extreme pessimism and complete ignorance are evolutionary stable in games with some general notion of strategic complements, i.e., supermodular games.

Our contributions are as follows: To our knowledge, we present the first study of the evolution of Choquet expected utility including Maximin, Maximax, Hurwicz preferences and Subjective expected utility in strategic contexts. Moreover, it is the first study that tries to endogenize the players' attitudes towards Knightian uncertainty and ignorance towards strategic dependencies in games. Hence we are able to select among equilibria of Knightian uncertainty. We show with qualifications that a preference with optimism (resp. pessimism) and complete ignorance is evolutionary stable in submodular (resp. supermodular) games with aggregate externalities. Moreover, this evolutionary stable preference leads to Walrasian behavior in those classes of games. Our results on the existence and monotone comparative statics of equilibrium under Knightian uncertainty, that are helpful for deriving our results, may be of interest on its own right since they are more general than what is known in the literature.

The article is organized as follows: In the next section we present some concepts that turn out to be useful in the analysis. In section 3 we present strategic games with Knightian uncertainty, define equilibrium and prove existence. This is followed by results on the monotone comparative statics of equilibrium with respect to optimism/pessimism in section 4. Our main results on evolutionary stable preferences are contained in section 5. We finish in section 6 with a discussion, including a discussion on the irrelevance of the observability of preferences for our results, the interpretation of our results as contests and potential applications. Some proofs are collected in the appendix.

2 Preliminaries

2.1 Orders

A *partially ordered set* $\langle X, \trianglerighteq \rangle$ is a set X with a binary relation $\trianglerighteq$ that is reflexive, antisymmetric, and transitive. The *dual* of a set X with a partial order $\trianglerighteq$ is the same set X with a partial order $\trianglerighteq'$ such that for $x', x'' \in X$, $x' \trianglerighteq' x''$ if and only if $x'' \trianglerighteq x'$. A *chain* is a partially ordered set that does not contain an unordered pair of elements, i.e., a totally or completely ordered set. A *lattice* $\langle X, \trianglerighteq \rangle$ is a partially ordered set in which each pair of elements $x, y \in X$ has a least upper bound *(join)* denoted by $x \vee y = \sup_X \{x, y\}$ and a greatest lower bound *(meet)* denoted by $x \wedge y = \inf_X\{x, y\}$ contained in this set. A lattice $\langle X, \trianglerighteq\rangle$ is *complete* if for every nonempty $Y \subseteq X$, $\sup_X Y$ and $\inf_X Y$ exist in X. A *sublattice* Y of a lattice X is a subset $Y \subseteq X$ for which each pair of elements in Y the join and meet is contained in Y. A sublattice Y of a lattice X is *subcomplete* if for each nonempty subset $Y' \subseteq Y$, $\sup_X(Y')$ and $\inf_X(Y')$ exist and is contained in Y. The *interval-topology* on a lattice X is the topology for which each closed set is either X, $\emptyset$, or of type $\{y \in X | x \trianglerighteq y, y \trianglerighteq z\}$. A lattice is complete if and only if it is compact in its interval-topology (Frink, 1942, Birkhoff, 1967, see Topkis, 1998, pp. 29). We assume that all lattices are endowed with a topology finer then the interval-topology, and that all products of topological spaces are endowed with the product topology.[1] For a lattice $\langle X, \trianglerighteq\rangle$ with $A, B \subseteq X$, B is higher than A if $a \in A$, $b \in B$ implies that $a \vee b \in B$ and $a \wedge b \in A$ (strong set order). We then abuse notation and write $B \trianglerighteq A$. A function (correspondence) f from a partially ordered set X to a partially ordered set Y is *increasing (decreasing)* if $x'' \trianglerighteq x'$ in X implies $f(x'') \trianglerighteq (\trianglelefteq) f(x')$. It is *strictly increasing (decreasing)* if we replace "$\trianglerighteq$ ($\trianglelefteq$)" with its non-reflexive part "$\triangleright$($\triangleleft$)" in the previous sentence.

Definition 1 (Supermodular) *A real valued function on a lattice $f : X \longrightarrow \mathbb{R}$ is supermodular in x on X if for all $x, y \in X$,*

$$f(x) + f(y) \leq f(x \vee y) + f(x \wedge y). \tag{1}$$

A real valued function on a lattice is *submodular* if $-f$ is supermodular. A function that is both supermodular and submodular is called a *valuation*.

Definition 2 (Increasing / Decreasing Differences) *A real valued function f on a partially ordered set $X \times T$ has increasing (decreasing) differences in (x, t) on $X \times T$ if for all $x'' \triangleright x'$ and $t'' \triangleright t'$,*

$$f(x'', t') - f(x', t') \leq (\geq) f(x'', t'') - f(x', t''). \tag{2}$$

If inequality (2) holds strictly then f has strictly increasing (decreasing) differences.

Functions defined on a finite product of chains that have increasing differences on this product are also supermodular on this product (Topkis, 1998, Corollary 2.6.1.). A familiar characterization of increasing differences in many economic problems is as follows: If $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ is twice-continuously differentiable then f has (strictly) increasing differences on $\mathbb{R}^n$ if and only if $\frac{\partial^2 f(x)}{\partial x_i \partial x_j} \geq (>) 0$ for all $i \neq j$ and x. An analogous result holds for (strictly) decreasing differences.

A few results will hold for a weaker ordinal version of increasing (decreasing) differences, the *(dual) single crossing property* (Milgrom and Shannon, 1994). A real valued function f on a partially ordered set $X \times T$ satisfies the (dual) single crossing property in (x, t) on $X \times T$ if for all $x'' \triangleright x'$ and $t'' \triangleright t'$,

$$f(x'', t') \geq f(x', t') \quad \Rightarrow (\Leftarrow) \quad f(x'', t'') \geq f(x', t''), \tag{3}$$
$$f(x'', t') > f(x', t') \quad \Rightarrow (\Leftarrow) \quad f(x'', t'') > f(x', t''). \tag{4}$$

[1] This is important later for existence of Equilibrium under Knightian uncertainty.

We say that f satisfies the strict (dual) single crossing property in (x,t) on $X \times T$ if for all $x'' \rhd x'$ and $t'' \rhd t'$,

$$f(x'',t') \geq f(x',t') \quad \Rightarrow (\Leftarrow) \quad f(x'',t'') > f(x',t''). \tag{5}$$

It is straight-forward to verify that increasing (decreasing) differences implies (dual) single crossing property but not vice versa. The same holds for the strict versions.

2.2 Strategic Games with Ordered Actions

Let N be the finite set of players $i = 1,...,n$. Each player's set of actions is a sublattice A_i of a lattice X. We write $A^n = \times_{i \in N} A_i$, with a typical element being $\mathbf{a} \in A^n$. Player i's payoff function is $\pi_i : A^n \longrightarrow \mathbb{R}$. We denote by $A_{-i} = \times_{j \in N \setminus \{i\}} A_j$. A typical element of A_{-i} is a_{-i}. $G = \langle N, (A_i), (\pi_i) \rangle$ denotes a *strategic game* (with lattice action space). A *Nash equilibrium in pure actions* of the strategic game G is an action profile $\mathbf{a}^* \in A^n$ such that for all $i \in N$,

$$\pi_i(a_i^*, a_{-i}^*) \geq \pi_i(a_i, a_{-i}^*), \text{ for all } a_i \in A_i. \tag{6}$$

Let $\mathcal{E}(G) \subseteq A^n$ denote the set of pure Nash equilibria of the game G.

A strategic game G *is (strictly) supermodular* if for each player $i \in N$ the payoff function π_i is supermodular in a_i on A_i for each $a_{-i} \in A_{-i}$ and has (strictly) increasing differences in (a_i, a_{-i}) on A^n. A strategic game G *is (strictly) submodular* if for each player $i \in N$ the payoff function π_i is (strictly) supermodular (!) in a_i on A_i for each $a_{-i} \in A_{-i}$ and has (strictly) decreasing differences in (a_i, a_{-i}) on A^n.

A strategic game G has *positive (negative) externalities* if for each player $i \in N$ the payoff $\pi_i(a_i, a_{-i})$ is increasing (decreasing) in a_{-i} on A_{-i} for each $a_i \in A_i$. A strategic game G has *strictly positive (negative) externalities* if the strict versions hold.

Remark 1 *Assume that A_{-i} is a subcomplete sublattice of A^n for all $i \in N$. If G has positive (negative) externalities then $\arg\max_{a_{-i} \in A_{-i}} \pi_i(a_i, a_{-i}) \supseteq \sup_{A^n} A_{-i}$ $(\inf_{A^n} A_{-i})$ and $\arg\min_{a_{-i} \in A_{-i}} \pi_i(a_i, a_{-i}) \supseteq \inf_{A^n} A_{-i}$ $(\sup_{A^n} A_{-i})$ for all $a_i \in A_i$.*

In the following text we will assume that A_{-i} is a subcomplete sublattice of A^n for all $i \in N$.

Definition 3 (Aggregative Game) *A strategic game G with ordered action space is aggregative if there exists an aggregator $\aleph : X \times X \longrightarrow X$ such that*

(i) Idempotence: $\aleph^1(a_i) := a_i$ for all $a_i \in A_i \subseteq X$ and all $i \in N$;

(ii) Induction: $\aleph^k(a_1,...,a_k) = \aleph(\aleph^{k-1}(a_1,...,a_{k-1}), a_k)$, for $k = 2,...,n$;

(iii) Symmetry: $\aleph^k$ is symmetric for $k = 1,...,n$, i.e., $\aleph^k(a_1,...,a_k) = \aleph^k(a_{f(1)},...,a_{f(k)})$ for all bijections $f : \{1,...,k\} \longrightarrow \{1,...,k\}$, for $k = 1,...,n$;

(iv) Order-preservation: $\aleph^k$ is order-preserving for $k = 1,...,n$, i.e., if $(a_1,...,a_k) \unlhd (a_1',...,a_k')$ implies $\aleph^k(a_1,...,a_k) \unlhd \aleph^k(a_1',...,a_k')$, for $k = 1,...,n$;

and the payoff function π_i is defined on $A_i \times X$ for all players $i \in N$.

This definition is similar to Alós-Ferrer and Ania (2005). Aggregative games are also considered by Corchón (1994), Schipper (2003) and Cornes and Hartley (2005). Many games with ordered action sets in the literature have the aggregation property. See discussion section for examples.

Definition 4 (Aggregate Taking Strategy) *An action a_i° is an optimal Aggregate Taking Strategy (ATS) if*

$$\pi_i(a_i^\circ, \aleph^n(\mathbf{a}^\circ)) \geq \pi_i(a_i, \aleph^n(\mathbf{a}^\circ)), \text{ for all } a_i \in A_i. \tag{7}$$

a_i° is a strict ATS if the inequality holds strictly.

This notion is due to Possanjenikov (2002). Note that an ATS generalizes naturally the competitive outcome or the Walrasian outcome in oligopoly games (see also Vega-Redondo, 1997, Schipper, 2003, Alós-Ferrer and Ania, 2005). Alós-Ferrer and Ania (2005) provide a straight-forward existence result for games satisfying a weaker ordinal property of increasing differences, i.e., the single-crossing property, based on Topkis (1998), Milgrom and Shannon (1994) and Tarski's fixed-point theorem.

We say that an *aggregative game G is (strictly) supermodular* if it is supermodular and for each player $i \in N$ the payoff function π_i has (strictly) increasing differences in $(a_i, \aleph^n(\mathbf{a}))$ on $A_i \times X$. An *aggregative game G is (strictly) submodular* if it is (strictly) submodular and for each player $i \in N$ the payoff function π_i has (strictly) decreasing differences in $(a_i, \aleph^n(\mathbf{a}))$ on $A_i \times X$.

We say that an *aggregative game G has positive (resp. negative) aggregate externalities* if it has positive (negative) externalities and for each player $i \in N$ the payoff $\pi_i(a_i, \aleph^n(\mathbf{a}))$ is increasing (resp. decreasing) in $\aleph^n(\mathbf{a})$ for each $a_i \in A_i$. An *aggregative game G has strict positive (resp. negative) aggregate externalities* if the strict versions hold.

Lemma 1 *Let $G = \langle N, (A_i), (\pi_i) \rangle$ be an aggregative strategic game, and let $\mathbf{a}^\circ$ be a symmetric ATS profile and $\mathbf{a}^*$ be a symmetric Nash equilibrium of G. If the aggregative game G is such that π_i satisfies*

(i) the dual single crossing property in $(a_i, \aleph^n(\mathbf{a}))$ on $A_i \times X$ and G has strict negative (resp. positive) aggregate externalities, or

(ii) the strict dual single crossing property in $(a_i, \aleph^n(\mathbf{a}))$ on $A_i \times X$ and G has negative (resp. positive) aggregate externalities,

then $\mathbf{a}^\circ \trianglerighteq \mathbf{a}^*$ (resp. $\mathbf{a}^\circ \trianglelefteq \mathbf{a}^*$).

The proof is contained in the appendix. As a corollary, the lemma implies the result for aggregative strict submodular games with aggregate externalities or aggregative submodular games with strict aggregate externalities.

2.3 Optimism, Pessimism and Complete Ignorance

We model Knightian uncertainty by Choquet expected utility theory. Let Ω be a space of mutually exclusive states and Σ be a corresponding sigma-algebra of events, then a *capacity* is a function $\nu : \Sigma \longrightarrow \mathbb{R}_+$ satisfying *monotonicity*, if $E \subseteq F$, $E, F \in \Sigma$ then $\nu(F) \geq \nu(E)$, and *normalization*, $\nu(\Omega) = 1$ and $\nu(\emptyset) = 0$. For simplicity and to set the stage for monotone comparative statics, we model a decision maker's ambiguous belief by a neo-additive capacity.

Definition 5 (Neo-additive capacity) *A decision maker's ambiguous belief is represented by a neo-additive capacity* $\nu_i(E) = \alpha_i \delta_i + (1 - \delta_i)\mu_i(E)$, $\emptyset \subsetneqq E \subsetneqq \Omega$, $\nu_i(\emptyset) = 0$, $\nu_i(\Omega) = 1$, μ_i *a probability distribution on* (Ω, Σ), *and* $\alpha_i, \delta_i \in [0, 1]$.

Given that a player's belief is represented by a neo-additive capacity, her (Choquet) expected utility from an action $a_i \in A_i$ is represented by the Choquet integral (for a proof see Chatenauneuf, Eichberger, and Grant, 2005).

Lemma 2 (Choquet Integral) *Let* $\pi_i : A_i \times \Omega \longrightarrow \mathbb{R}$ *be player* i*'s payoff function such that* $\heartsuit_i(a_i) := \max_{\omega \in \Omega} \pi_i(a_i, \omega)$ *and* $\maltese_i(a_i) := \min_{\omega \in \Omega} \pi_i(a_i, \omega)$ *exist. Player* i*'s Choquet expected utility from an action* a_i *with respect to her neo-additive capacity* ν_i *is given by*

$$u_i(a_i, \nu_i) = \delta_i[\alpha_i \heartsuit_i(a_i) + (1 - \alpha_i)\maltese_i(a_i)] + (1 - \delta_i)\mathbb{E}_{\mu_i}[\pi_i(a_i, \omega)], \tag{8}$$

with $\mathbb{E}_{\mu_i}[\pi_i(a_i, \omega)]$ *being the expected payoff with respect to the probability distribution* μ_i *on* (Ω, Σ).

Our particular parametrization of the neo-additive capacities allows us to differentiate between the amount of ambiguity δ_i faced by the decision maker i and her attitude towards this ambiguity α_i.[2] We call the parameter δ_i the *degree of ignorance* whereas α_i is the *degree of optimism*. We often simply say that (α_i, δ_i) is the preference of player i.

Definition 6 (Optimism, Pessimism and (Complete) Ignorance) *Given a Choquet expected utility maximizer* i *with a neo-additive capacity, we interpret* $\alpha_i' \geq \alpha_i$ *as* α_i' *being more optimistic than* α_i *(or* α_i *being more pessimistic than* α_i'*) for a given* δ_i*. We interpret* $\delta_i' \geq \delta_i$ *as* δ_i' *being more ignorant then* δ_i *for a given* α_i*. We say that* i *is completely ignorant if* $\delta_i = 1$*. We say that* i *is a realist if* $\delta_i = 0$*.*[3]

Intuitively, a neo-additive capacity describes a situation in which the decision maker i has an additive probability distribution μ_i over outcomes but also lacks confidence in this belief.

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[2] In the literature (e.g. Eichberger, Kelsey and Schipper, 2005), neo-additive capacities are also presented by $\nu_i(E) = \lambda_i + (1 - \lambda_i - \gamma_i)\mu_i(E)$, $\emptyset \subsetneqq E \subsetneqq \Omega$, $\nu_i(\emptyset) = 0$, $\nu_i(\Omega) = 1$, μ_i a probability distribution on a state space Ω, and $\lambda_i, \gamma_i \in [0, 1]$ such that $0 \leq \gamma_i \leq 1$, $0 \leq \lambda_i \leq 1 - \gamma_i$. The Choquet integral is given by

$$u_i(a_i, \nu_i) = \lambda_i \heartsuit_i(a_i) + \gamma_i \maltese_i(a_i) + (1 - \gamma_i - \lambda_i)\mathbb{E}_{\mu_i}[\pi_i(a_i, \omega)] \tag{9}$$

This is isomorph to the parametrization used here. I.e., we can set $\lambda_i = \alpha_i \delta_i$ and $\gamma_i = (1 - \alpha_i)\delta_i$.

[3] Note that if $\delta_i = 0$, then the parameter $\alpha_i \in [0, 1]$ can be arbitrary.

She reacts to this ambiguity with overweighting good or bad outcomes. A decision maker is optimistic (pessimistic) if she overweights good (bad) outcomes. The latter interpretation is based on Wakker (2001) and justified in our context by Chatenauneuf, Eichberger, and Grant (2005), who provide also an axiomatization of Choquet expected utility with neo-additive capacities.

CEU with neo-additive capacities entails several familiar decision theoretic approaches as special cases. Cases 1 to 3 concern decision making under complete ignorance.

1. If $\delta_i = 1$ and $\alpha_i = 0$, preferences have the *Minimax* form and are extremely pessimistic (Wald, 1951);
2. if $\delta_i = 1$ and $\alpha_i = 1$, preferences have the *Maximax* form and exhibit a maximal degree of optimism;
3. if $\delta_i = 1$ and $\alpha_i \in [0, 1]$, these preferences coincide with the *Hurwicz criterion*, (see Hurwicz, 1951, and Arrow and Hurwicz, 1972);
4. if $\delta_i = 0$ and $\alpha_i \in [0, 1]$, the belief coincides with a conventional probability distribution. In particular the capacity is additive, i.e., $A, B \subseteq \Omega$, $A \cap B = \emptyset$, $\nu_i(A \cup B) = \nu_i(A) + \nu_i(B)$. This is the case of *Subjective expected utility (SEU).*

3 Strategic Games with Optimism and Pessimism

A neo-additive capacity ν_i is defined by α_i, δ_i, and the additive probability distribution μ_i. Since players in games face strategic uncertainty about opponents' actions, μ_i represents a probabilistic conjecture over opponents' actions. This probability distribution is to be determined endogenously in equilibrium. In contrast, α_i and δ_i will be treated exogenously for a game. We focus here on equilibria in pure strategies only. Hence a player's probability distribution over opponents' actions is degenerate in the sense that it assigns unit probability to one opponents' action profile. Therefore we write for player i's Choquet expected payoff from an action a_i given i's belief $u_i(a_i, a_{-i}, \alpha_i, \delta_i) = \delta_i[\alpha_i \heartsuit(a_i) + (1 - \alpha_i)\maltese(a_i)] + (1 - \delta_i)\pi_i(a_i, a_{-i})$. Set $(\alpha) = (\alpha_1, ..., \alpha_n)$ and $(\delta) = (\delta_1, ..., \delta_n)$. Let $G((\alpha), (\delta)) = \langle N, (A_i), (u_i(\alpha_i, \delta_i))\rangle$ be a "perturbed" strategic game derived from G by replacing π_i with $u_i(\alpha_i, \delta_i)$ for all $i \in N$. Note that $G(0^n, 0^n) = G$.

Definition 7 (Equilibrium under Knightian Uncertainty) *In a game G an Equilibrium under Knightian uncertainty (EKU)* $\mathbf{a}^*((\alpha), (\delta)) \in A^n$ *with degrees of ignorance (δ) and degrees of optimism (α) is a pure Nash equilibrium of the game $G((\alpha), (\delta))$, i.e., for all $i \in N$,*

$$u_i(a_i^*, a_{-i}^*, \alpha_i, \delta_i) \geq u_i(a_i, a_{-i}^*, \alpha_i, \delta_i), \text{ for all } a_i \in A_i. \quad (10)$$

In the literature on Knightian uncertainty in strategic games, several solution concepts have been suggested. In the discussion-section we will show that our solution implies notions by Dow and Werlang (1994), Eichberger and Kelsey (2000) and Marinacci (2000).

(Strict) Supermodularity or submodularity of G is preserved under "perturbations" with Knightian uncertainty as modelled by Choquet expected utility with neo-additive capacities.

Lemma 3 *If G is (strictly) supermodular then $G((\alpha),(\delta))$ is (strictly) supermodular for any $((\alpha),(\delta)) \in [0,1]^n \times [0,1]^n$. The analogous result holds for G being (strictly) submodular.*

Proof. We need to show that if π_i is supermodular in a_i on A_i and has increasing (decreasing) differences in (a_i, a_{-i}) on A^n then $u_i(\alpha_i, \delta_i)$ is supermodular in a_i on A_i and has increasing (decreasing) differences in (a_i, a_{-i}) on A^n for each $(\alpha_i, \delta_i) \in [0,1]^2$. If π_i is supermodular in a_i on A_i and has increasing (decreasing) differences in (a_i, a_{-i}) on A^n, then for any scalar $\gamma \geq 0$ also $\gamma\pi_i$ is supermodular in a_i on A_i and has increasing (decreasing) differences in (a_i, a_{-i}) on A^n (Topkis, 1998, Lemma 2.6.1. (a)). $\heartsuit_i$ and $\maltese_i$ are both supermodular in a_i on A_i by definition and constant in a_{-i} on A_{-i}. Since u_i is for each $(\alpha_i, \beta_i) \in [0,1]^2$ a sum of supermodular functions in a_i on A_i having increasing (decreasing) differences in (a_i, a_{-i}) on A^n, it is supermodular in a_i on A_i and has increasing (decreasing) differences (a_i, a_{-i}) on A^n (Topkis, 1998, Lemma 2.6.1. (b)). By analogous arguments, the result extends to the strict versions (see Topkis, 1998, p. 49). □

Often the literature makes use of weaker ordinal notions of supermodularity. Those notions may not appropriate for our model as following remark asserts.

Remark 2 *The ordinal version of supermodularity, quasisupermodularity (see Milgrom and Shannon, 1994), may not need to be preserved under Knightian uncertainty since the sum of two quasisupermodular functions does not need to be quasisupermodular unless either is supermodular (Topkis, 1998, pp. 62). An analogous conclusion holds for functions satisfying the ordinal version of increasing (decreasing) differences called (dual) single crossing property (see Milgrom and Shannon, 1994).*

Given the game $G((\alpha),(\beta))$, let player i's best response correspondence be defined by

$$b_i(a_{-i}, \alpha_i, \delta_i) := \{a_i \in A_i : u_i(a_i, a_{-i}, \alpha_i, \delta_i) \geq u_i(a'_i, a_{-i}, \alpha_i, \delta_i), \forall a'_i \in A_i\}. \quad (11)$$

Lemma 4 *If G is supermodular (submodular) then for any $i \in N$ and any $(\alpha_i, \delta_i) \in [0,1]^2$, the best response $b_i(a_{-i}, \alpha_i, \delta_i)$ is a sublattice of A_i and increasing (decreasing) in a_{-i} on $\{a_{-i} \in A_{-i} : b_i(a_{-i}, \alpha_i, \delta_i) \neq \emptyset\}$.*

Proof. By Lemma 3, if G is supermodular (submodular), then $G((\alpha),(\delta))$ is supermodular (submodular) for each $(\alpha, \delta) \in [0,1]^n \times [0,1]^n$. Thus u_i is supermodular in a_i on A_i and has increasing (decreasing) differences in (a_i, a_{-i}) on A^n. Since u_i is supermodular in a_i on A_i, $b_i(a_{-i}, \alpha_i, \delta_i)$ is a sublattice of A_i for each $a_{-i} \in A_{-i}$ and $(\alpha_i, \delta_i) \in [0,1] \times [0,1]$ by Topkis (1998), Theorem 2.7.1. Since u_i has increasing (decreasing) differences in (a_i, a_{-i}) on A^n, $b_i(a_{-i}, \alpha_i, \delta_i)$ is increasing (decreasing) in a_{-i} on $\{a_{-i} \in A_{-i} : b_i(a_{-i}, \alpha_i, \delta_i) \neq \emptyset\}$ by Topkis (1998), Theorem 2.8.1. □

If the Hurwicz criterion is satisfied, i.e., the player is completely ignorant, then her objective function does not depend on the opponents' actions. This leads to the following observation:

Remark 3 *If $\delta_i = 1$, then $b_i(a_{-i}, \alpha_i, 1)$ is trivially constant in a_{-i} on A_{-i} for any $\alpha_i \in [0,1]$.*

We are able state general results on the existence of equilibrium under Knightian uncertainty.

Proposition 1 (Existence in Supermodular Games) *If G is supermodular and for all $i \in N$, A_i is a non-empty complete lattice, and π_i is upper semicontinuous on A_i, then for any $((\alpha),(\delta)) \in [0,1]^n \times [0,1]^n$ the set of equilibria under Knightian uncertainty is a complete lattice and a greatest and least equilibrium exist.*

Proof. Note that if π_i is upper semicontinuous on A_i then u_i is upper semicontinuous on A_i since limits are preserved under algebraic operations. The result follows then from Lemmata 3 and 4 and Zhou's (1994) generalization of Tarski's fixed point theorem. □

Note that in Proposition 1 we do not claim that the set of equilibria under Knightian uncertainty is a sublattice of A^n. Thus if $\mathbf{a} = (a_1, ..., a_n)$ and $\mathbf{a}' = (a'_1, ..., a'_n)$ are both equilibria under Knightian uncertainty for $G((\alpha),(\delta))$ then $(a_1 \vee a'_1, ..., a_n \vee a'_n)$ and $(a_1 \wedge a'_1, ..., a_n \wedge a'_n)$ *may not* be equilibria under Knightian uncertainty for the game $G((\alpha),(\delta))$ (see for an example Zhou (1994), p. 299).

Remark 4 *If G is supermodular with $n = 2$, A_i is a non-empty complete chain, and π_i is upper semicontinuous on A_i for $i = 1, 2$, then for any $((\alpha),(\delta)) \in [0,1]^n \times [0,1]^n$ the set of equilibria under Knightian uncertainty is a subcomplete sublattice and a greatest and least equilibrium exist.*

Proof. This follows from Proposition 1 and a result by Echenique (2003) who observed that the set of Nash equilibria forms a sublattice in two-player games with totally ordered action sets and for which each player's best response correspondence is increasing in the strong set order (e.g. supermodular games). □

Since there is no general fixed-point theorem applicable to decreasing best responses, we use a different approach to existence in submodular games based on Kukushkin (1994) and Novshek (1985).

Proposition 2 (Existence in Submodular Games) *If G is submodular and for all $i \in N$, A_i is a non-empty compact subset of $\mathbb{R}_+$, π_i is defined on A_i and the range of sums of opponents actions, and is continuous in both variables, then for any $((\alpha),(\delta)) \in [0,1]^n \times [0,1]^n$ the set of equilibria under Knightian uncertainty is non-empty.*

Proof. Submodularity of G implies by Lemma 3 submodularity of $G((\alpha),(\delta))$ for all $(\alpha,\delta) \in [0,1]^n \times [0,1]^n$. Thus for all $i \in N$, $u_i(\alpha_i,\delta_i)$ has decreasing differences in $(a_i, \sum_{j \in N \setminus \{i\}} a_j)$. By Lemma 4, $b_i(\alpha_i,\delta_i)$ is decreasing in $\sum_{j \in N \setminus \{i\}} a_j$. Since π_i is continuous in both variables, $u_i(\alpha_i,\delta_i)$ is continuous in a_i and $\sum_{j \in N \setminus \{j\}} a_j$ because limits are preserved under algebraic operations. Thus $b_i(\sum_{j \in N \setminus \{i\}} a_j, \alpha_i, \delta_i)$ is non-empty for any $\sum_{j \in N \setminus \{i\}} a_j$. It also implies that $b_i(\alpha_i,\delta_i)$ is upper-hemicontinuous in $\sum_{j \in N \setminus \{i\}} a_j$. Thus the conditions are sufficient for a theorem by Kukushkin (1994) by which there exists a Nash equilibrium in pure actions of the game $G((\alpha),(\delta))$. □

Kukushkin (1994) requires for each player a real-valued compact action set, an upper hemicontinuous best response correspondence with a single-valued selection that is decreasing in the additive aggregate of opponents' actions. Last assumption is slightly weaker than decreasing best responses in the additive aggregate of opponents' actions that result from decreasing differences of the Choquet expected payoffs in (a_i, a_{-i}) on A^n in our case. The pseudo-potential approach to existence by Dubey, Haimanko and Zapechelnyuk (2004) would be an alternative to Kukushkin (1994). This approach could be used for games both with increasing and decreasing best responses. Note also that Proposition 1 could be applied for existence in two-player submodular games in which action sets are chains by replacing one player's ordered action set with the dual. This transforms the game into a two-player supermodular game.

In the later analysis, complete ignorance will play a prominent role. For this special case, no matter whether the game is supermodular or submodular, existence and uniqueness is rather straight-forward.

Proposition 3 (Existence under Complete Ignorance) *Let $G = \langle N, (A_i), (\pi_i)\rangle$ be a strategic game with for all $i \in N$, A_i being a non-empty complete lattice, π_i being upper semincontinuous and supermodular on A_i for all $i \in N$, and the Hurwicz criterion is satisfied (i.e., $\delta_i = 1$ for all $i \in N$). Then for all $(\alpha) \in [0,1]^n$, the set of equilibria under Knightian uncertainty is a complete lattice and a greatest and least equilibrium exist.*

Proof. By Remark 3, if for all $i \in N$, $\delta_i = 1$, then $b_i(a_{-i}, \alpha_i, 1)$ is trivially constant in a_{-i} on A_{-i} for any $\alpha_i \in [0,1]$ for all $i \in N$. Hence $b_i(a_{-i}, \alpha_i, 1)$ is trivially increasing in a_{-i} on A_{-i} for any $\alpha_i \in [0,1]$ for all $i \in N$. Note that if π_i is upper semicontinuous on A_i, then u_i is upper semicontinuous on A_i since limits are preserved under algebraic operations. Note further that if π_i supermodular in a_i on A_i, then by the proof of Lemma 3 u_i is supermodular in a_i on A_i. The result follows then from Zhou's (1994) generalization of Tarski's fixed-point theorem. □

While there is a direct alternative elementary proof for above proposition, we choose to present the proof as a special case for Nash equilibrium for games with increasing best-response correspondence (Zhou, 1994) in order to point out the connection to previous results.

Proposition 4 (Uniqueness under Complete Ignorance) *Let $G = \langle N, (A_i), (\pi_i)\rangle$ be a strategic game with for all $i \in N$, π_i being strictly concave on A_i and the Hurwicz criterion being satisfied (i.e., $\delta_i = 1$). If there exists an equilibrium under Knightian uncertainty then it is unique and each player's equilibrium action is her dominant action.*

Proof. If π_i is strictly concave, then u_i is strictly concave since it is a weighted sum of strictly concave functions. Strict concavity of u_i is sufficient for $b_i(a_{-i}, \alpha_i, 1)$ being unique for all $a_{-i} \in A_{-i}$, $\alpha_i \in [0,1]$. By Remark 3, if $\delta_i = 1$ then $b_i(a_{-i}, \alpha_i, 1)$ is constant for all $a_{-i} \in A_{-i}$, $\alpha_i \in [0,1]$. Hence, if there exists an equilibrium under Knightian uncertainty with $\delta_i = 1$ for all $i \in N$, then it must be unique with each player choosing her unique dominant action. □

Finally, we like to remark that Vives (2000, Theorem 2.8, Remark 17) shows the following observation: Consider a symmetric submodular game. If the slope of any selection of the best-response correspondence is strictly greater -1 then if there exists an equilibrium, it must be unique and symmetric.

4 Monotone Comparative Statics

To analyze the effect of mutants in equilibrium under Knightian uncertainty, it will be helpful to study first the monotone comparative statics of equilibrium under Knightian uncertainty. Given the level of generality, these results may be of interest for their own right.

Lemma 5 *If π_i has [increasing differences in (a_i, a_{-i}) on A^n and positive (resp. negative) externalities] or [decreasing differences in (a_i, a_{-i}) on A^n and negative (resp. positive) externalities] then u_i has increasing (resp. decreasing) differences in (a_i, α_i) on $A_i \times [0,1]$ for all $a_{-i} \in A_{-i}$ and $\delta_i \in [0,1]$. The result extends to the strict versions.*

The proof is contained in the appendix.

Lemma 6 (Monotone Optimal Selections) *If $u_i(a_i, a_{-i}, \alpha_i, \delta_i)$ is supermodular in a_i on A_i and has increasing (resp. decreasing) differences in (a_i, α_i) on $A_i \times [0,1]$ for each $a_{-i} \in A_{-i}$ and $\delta_i \in [0,1]$, then $b_i(a_{-i}, \alpha_i, \delta_i)$ is increasing (resp. decreasing) in α_i on $\{\alpha_i \in [0,1] : b_i(a_{-i}, \alpha_i, \delta_i) \neq \emptyset\}$ for $\delta_i \in [0,1]$. If in addition $u_i(a_i, a_{-i}, \alpha_i, \delta_i)$ has strictly increasing (resp. decreasing) differences in (a_i, α_i) on $A_i \times [0,1]$ for each $a_{-i} \in A_{-i}$ and $\delta_i \in [0,1]$, $\alpha_i'' > \alpha_i'$ in $[0,1]$, and for any $a_{-i} \in A_{-i}$, $a_i' \in b_i(a_{-i}, \alpha_i', \delta_i)$ and $a_i'' \in b_i(a_{-i}, \alpha_i'', \delta_i)$, then $a_i' \trianglelefteq (\trianglerighteq) a_i''$. In this case, if one picks any $a_i(\alpha_i)$ in $b_i(a_{-i}, \alpha_i, \delta_i)$ for each α_i with $b_i(a_{-i}, \alpha_i, \delta_i)$ nonempty, then $a_i(\alpha_i)$ is increasing (resp. decreasing) in α_i on $\{\alpha_i \in [0,1] : b_i(a_{-i}, \alpha_i, \delta_i) \neq \emptyset\}$.*

The proof is contained in the appendix.

Corollary 1 *If π_i is supermodular in a_i on A_i, has increasing differences in (a_i, a_{-i}) on A^n and has positive (resp. negative) externalities, then $b_i(a_{-i}, \alpha_i, \delta_i)$ is increasing (resp. decreasing) in α_i on $\{\alpha_i \in [0,1] : b_i(a_{-i}, \alpha_i, \delta_i) \neq \emptyset\}$ for each $\delta_i \in [0,1]$. The same comparative statics obtains if π_i has decreasing differences in (a_i, a_{-i}) on A^n and negative (resp. positive) externalities. If π_i has strictly increasing differences in (a_i, a_{-i}) on A^n and positive (resp. negative) externalities, $\alpha_i'' > \alpha_i'$ in $[0,1]$, given $\delta_i \in [0,1]$, and for any $a_{-i} \in A_{-i}$, $a_i' \in b_i(a_{-i}, \alpha_i', \delta_i)$ and $a_i'' \in b_i(a_{-i}, \alpha_i'', \delta_i)$, then $a_i' \trianglelefteq (\trianglerighteq) a_i''$. In this case, if one picks any $a_i(\alpha_i)$ in $b_i(a_{-i}, \alpha_i, \delta_i)$ for each α_i with $b_i(a_{-i}, \alpha_i, \delta_i)$ nonempty, then $a_i(\alpha_i)$ is increasing (resp. decreasing) in α_i on $\{\alpha_i \in [0,1] : b_i(a_{-i}, \alpha_i) \neq \emptyset\}$. The same comparative statics obtains if π_i has strictly decreasing differences in (a_i, a_{-i}) on A^n and negative (resp. positive) externalities.*

Proposition 5 (Mon. Comp. Statics - Supermodularity) *If G is a supermodular game with positive (resp. negative) externalities, then the greatest and least equilibrium under Knightian uncertainty is increasing (resp. decreasing) in optimism (and decreasing (resp. increasing) in pessimism).*

Proof. This follows from previous results and Topkis (1998, Theorem 4.2.2). □

As usual in the literature, there is no dual result for submodular games. However, if the Hurwicz criterion is satisfied (i.e., under complete ignorance), then we can derive a dual but more special result for submodular games with externalities.

Proposition 6 (Mon. Comp. Statics - Submodularity and Complete Ignorance) *If G is a submodular game with negative (resp. positive) externalities and the Hurwicz criterion ($\delta_i = 1$) is satisfied for all players $i \in N$, then the greatest and least equilibrium under Knightian uncertainty is increasing (resp. decreasing) in optimism (and decreasing (resp. increasing) in pessimism).*

Proof. Let G be a submodular game with negative externalities and $(\delta) = 1^n$, and consider the greatest equilibrium under Knightian uncertainty $\bar{\mathbf{a}}(\alpha)$ when the profile of degrees of optimism is (α). Let $(\alpha') \geq (\alpha)$. Suppose now to the contrary that the greatest equilibrium under Knightian uncertainty $\bar{\mathbf{a}}(\alpha')$ is smaller or unordered to $\bar{\mathbf{a}}(\alpha)$. In both cases there exist a player i whose equilibrium actions satisfy $\bar{a}_i(\alpha')$ is strictly smaller or unordered to $\bar{a}_i(\alpha)$. Note that $\bar{a}_i(\alpha') \in b_i(\bar{a}_{-i}(\alpha'), \alpha'_i, 1)$ and $\bar{a}_i(\alpha) \in b_i(\bar{a}_{-i}(\alpha), \alpha_i, 1)$. Since $\delta_i = 1$, b_i is constant in a_{-i} by Remark 3. Hence, $b_i(\bar{a}_{-i}(\alpha), \alpha_i, 1) \neq b_i(\bar{a}_{-i}(\alpha'), \alpha'_i, 1)$ only if $(\alpha) \neq (\alpha')$. By Corollary 1, $b_i(\bar{a}_{-i}(\alpha), \alpha_i, 1)$ is increasing in α_i. Hence there exists $\tilde{a}_i(\alpha') \in b_i(a_{-i}(\alpha'), \alpha'_i, 1)$ with $\tilde{a}_i(\alpha') \geq \bar{a}_i(\alpha)$, a contradiction to $\bar{a}_i(\alpha')$ being a component of the greatest equilibrium under Knightian uncertainty with (α') and $(\delta) = 1^n$. An analogous arguments holds for positive externalities and for the least equilibrium under Knightian uncertainty. □

5 Evolutionary Stable Preferences

We will restrict the analysis to symmetric games. That is, we assume for all $i \in N$, $\pi_i = \pi$, $A_i = A$, and $\pi(a_i, a_{-i}) = \pi(a_i, a'_{-i})$ if a'_{-i} is a permutation of a_{-i}. Note that symmetry of G implies $\heartsuit_i = \heartsuit$ and $✠_i = ✠$. Furthermore, we assume that each player's set of actions A is a chain. This assumption is done in light of our results on the monotone comparative statics. We do not know how to assess the equilibrium payoff of a mutant playing an action unordered to an non-mutant's action.

Denote by T_G an arbitrary *collection of any player's preferences* that can possibly be defined in a game form $\langle N, A \rangle$ of the strategic game $G = \langle N, A, \pi \rangle$. For instance, we could parameterize player i's Choquet expected utility functions over outcomes in the game G with respect to neo-additive capacities by $t_i = (\alpha_i, \beta_i) \in T_G = [0, 1]^2$ but we want to allow for a much more general set of preferences including preferences over other players' payoffs, beliefs etc. For our results in this section, we only require that T_G includes some specific form of Choquet expected utility functions over outcomes in the strategic game G with respect to neo-additive capacities. Lets denote a profile of all players' preferences by $\mathbf{t} \in T_G^n$. Let $G(\mathbf{t})$ be the strategic game played when $\mathbf{t}$ is the profile of players' preferences in the game form $\langle N, A \rangle$ of the strategic game $G = \langle N, A, \pi \rangle$. Further, let $\mathcal{E}(G(\mathbf{t}))$ be the set of pure strategy equilibria given the game $G(\mathbf{t})$. Since the set of preferences may not be well-defined, the equilibrium notion may not be well-defined either. The only requirement we have is that for all Choquet expected utility players with neo-additive capacities, inequality (10) of Definition 7 holds for any equilibrium. This implies that if all players are Choquet expected utility maximizers with neo-additive capacities, then an equilibrium is defined in Definition 7. It also implies that if all players are Expected utility maximizers, then an equilibrium is defined by Nash equilibrium. We restrict the set of abstract equilibria further by focusing on *intra-group symmetric equilibria* defined by

$$\mathcal{E}^{sym}(G(\mathbf{t})) := \{\mathbf{a}(\mathbf{t}) \in \mathcal{E}(G(\mathbf{t})) | t_i = t_j \text{ implies } a_i(\mathbf{t}) = a_j(\mathbf{t})\}.$$

This is the set of equilibria in which players with the same preference play also the same action. Since for an abstract set of preferences, existence of such equilibria can not be guarantied, we assume $\mathcal{E}^{sym}(G(\mathbf{t})) \neq \emptyset$ for all $\mathbf{t} \in T_G^n$. A different interpretation is that we consider only preferences T_G for which such equilibria can be defined and exist.

As standard in the literature on evolution of preferences, each player i chooses according to her ex-ante objective function indexed by t_i. However, the player's *fitness* is evaluated by her material payoff π_i. This conforms to fitness considerations in business or academia. The success of a manager (resp. assistant professor) is assessed by her realized profit (resp. publications) and not by what the manager (resp. assistant professor) originally expected.

In many economic contexts such as markets with imperfect competition etc., a finite number of players interact repeatedly in an strategic context. For such environments, Schaffer (1988, 1989) introduced a notion of evolutionary stable strategy as an extension of the standard evolutionary stable strategy for large populations to finite populations in which each player plays against all other players ("playing-the-field"). An action $a \in A$ is a *finite population evolutionary stable strategy (ESS)* in a symmetric strategic game $G = \langle N, A, \pi \rangle$ if[4]

$$\pi(a, a', a, ..., a) \geq \pi(a', a, ..., a) \text{ for all } a' \in A.$$

We will apply Schaffer's notion of finite population ESS to the evolution of preferences in aggregative games. As convention we denote by j always the mutant and by $i \neq j$ a non-mutant.

Definition 8 (ESP) *$t \in T_G$ is a Finite Population Evolutionary Stable Preference (ESP) in a symmetric aggregative game $G = \langle N, A, \pi \rangle$ if for all mutants $t' \in T_G$,*

$$\pi(a_i^*(t_j', t_{-j}), \aleph^n(\mathbf{a}^*(t_j', t_{-j}))) \geq \pi(a_j^*(t_j', t_{-j}), \aleph^n(\mathbf{a}^*(t_j', t_{-j}))), \tag{12}$$

for an $\mathbf{a}^*(t_j', t_{-j}) \in \mathcal{E}^{sym}(G(t_j', t_{-j}))$.

A finite population evolutionary stable preference $t \in T_G$ is robust (RESP) in a symmetric aggregative game G if for all mutants $t' \in T_G$ inequality (12) holds for all $\mathbf{a}^*(t_j', t_{-j}) \in \mathcal{E}^{sym}(G(t_j', t_{-j}))$.

A finite population evolutionary stable preference $t \in T_G$ is globally stable (GESP) in a symmetric aggregative game G if for all mutants $t' \in T_G$,

$$\pi(a_i^*(t', \overset{m}{...}, t', t, ..., t), \aleph^n(\mathbf{a}^*(t', \overset{m}{...}, t', t, ..., t))) \geq \pi(a_j^*(t', \overset{m}{...}, t', t, ..., t), \aleph^n(\mathbf{a}^*(t', \overset{m}{...}, t', t, ..., t))), \tag{13}$$

for an $\mathbf{a}^*(t', \overset{m}{...}, t', t, ..., t) \in \mathcal{E}^{sym}(G(t', \overset{m}{...}, t', t, ..., t))$ for all $m \in \{1, ..., n-1\}$.

A finite population evolutionary stable preference $t \in T_G$ that is robust globally stable in a symmetric aggregative game G if for all mutants $t' \in T_G$ inequality (13) holds for all $\mathbf{a}^*(t', \overset{m}{...}, t', t, ..., t) \in \mathcal{E}^{sym}(G(t', \overset{m}{...}, t', t, ..., t))$ for all $m \in \{1, ..., n-1\}$.

We say that *complete ignorance is an evolutionary stable preference* in G if there is an $t_i = u_i(\alpha_i, \delta_i)$ with $\delta_i = 1$ that is an evolutionary stable preference in G. That is, the evolutionary

[4] $\pi(a, a', a, ..., a)$ denotes the payoff of a player playing a when one opponent plays a' and all other opponents play each a.

stable preference satisfies the Hurwicz criterion and involves complete ignorance. Likewise for (robust) globally stable preferences.

We say that a *preference with optimism (pessimism) and complete ignorance is evolutionary stable* in G if $t = u(\alpha, \delta)$ with $\delta = 1$ and $\alpha \geq \max(\min)\{\alpha' \in [0,1] | \mathbf{a}((\alpha'), (1, ..., 1)) \in \mathcal{E}^{sym}(G((\alpha'), (1, ..., 1)) \cap \mathcal{E}^{sym}(G)\}$ is an evolutionary stable preference in G. I.e., if the preference with complete ignorance involves a higher degree of optimism as any symmetric preference profile with complete ignorance that would lead to a symmetric Nash equilibrium in G. This resembles the intuition that a preference leading to Nash equilibrium can not be behaviorally distinguished from a realist. Note that $\max(\min)\{\alpha' \in [0,1] | \mathbf{a}((\alpha'), (1, ..., 1)) \in \mathcal{E}^{sym}(G((\alpha'), (1, ..., 1)) \cap \mathcal{E}^{sym}(G)\}$ is nonempty if for any symmetric Nash equilibrium $\mathbf{a} \in A^n$ of the game G there exist a symmetric equilibrium under Knightian uncertainty $\mathbf{a}((\alpha), (\delta))$ of G with $\delta = 1$ for all players such that $\mathbf{a} = \mathbf{a}((\alpha), (\delta))$. Otherwise, the property might be trivially satisfied.

We say that a *preference with extreme optimism (resp. pessimism) and complete ignorance is evolutionary stable* in G if $t = (\alpha, \delta)$ with $\alpha = 1$ (resp. $\alpha = 0$) and $\delta = 1$ is an evolutionary stable preference in G.

Assumption 1 *For any symmetric ATS profile* $\mathbf{a} \in A^n$ *of the game* G *let there exist a symmetric equilibrium under Knightian uncertainty* $\mathbf{a}((\alpha), (\delta))$ *of* G *with* $\delta_i = 1$ *for all* $i \in N$ *such that* $\mathbf{a} = \mathbf{a}((\alpha), (\delta))$.

For any symmetric optimal aggregate taking strategy, there should exist a capacity that satisfies the Hurwicz criterion such that every player maximizing individually her Choquet expected payoffs with respect to this capacity leads to an equilibrium under Knightian uncertainty that equals to the optimal aggregate taking strategy.

Note that given aggregate (positive or negative) externalities, the assumption is essentially an interior condition. The Hurwicz expectation is a weighted average of the best and worst outcome corresponding to the largest (lowest) aggregate of actions. The assumption says that the ATS lies somewhere between the best and worst outcome, i.e., the largest and lowest outcome. The assumption is violated if for example there exists an action that dominates any other action, and the aggregate taking strategy is different from the Nash equilibrium. Then clearly for any aggregate of opponents' actions, the dominant action would be the best-response to any opponents' actions. This is then also the Nash equlibrium action as well as the equilibrium action in any equilibrium under Knightian uncertainty since Knightian uncertainty respects dominance. If we assume that the (dominant) Nash equilibrium action is different from any ATS, then there can not exist a best-response equivalent to an ATS.

Proposition 7 *Let* $G = \langle N, A, \pi \rangle$ *be a symmetric strict submodular aggregative game with aggregate externalities and* T_G *be a collection of preferences that includes Hurwicz preferences. Suppose that Assumption 1 holds and that a symmetric ATS exists. Then we conclude the following:*

(i) There exists a preference with optimism and complete ignorance (a Hurwicz preference) that is globally evolutionary stable in G.

(ii) A symmetric equilibrium under Knightian uncertainty resulting from play with this evolutionary stable preference profile equals to a symmetric ATS profile.

(iii) If π is strictly concave in the player's own action $a_i \in A$ for all $a_{-i} \in A_{-i}$, then there exists a preference with optimism and complete ignorance (a Hurwicz preference) that is a robust globally evolutionary stable preference.

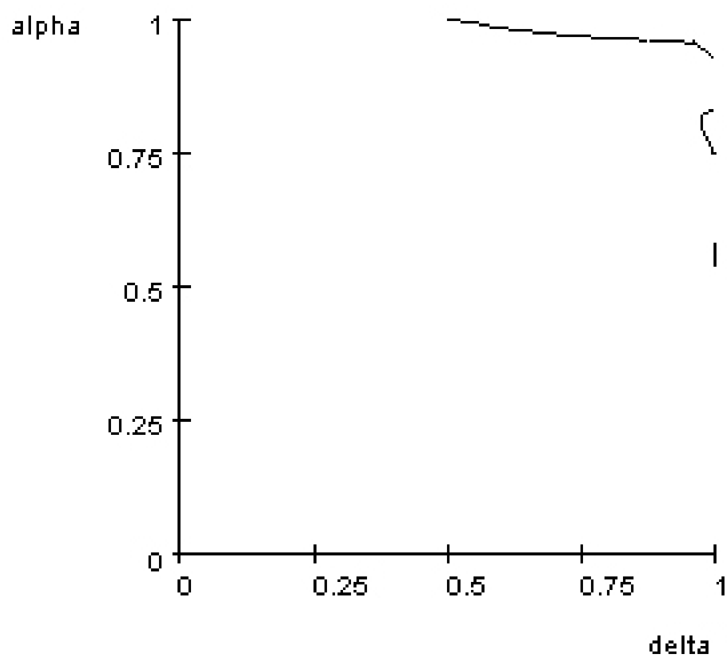
The proof is contained in the appendix.

In Proposition 7, complete ignorance (and thus the Hurwicz criterion) together with optimism is a sufficient condition for the evolutionary stability of a preference. This poses the question whether complete ignorance is also necessary. In Example 1 we present a standard Cournot duopoly with linear demand and convex cost where complete ignorance and extreme optimism is the unique preference with an equilibrium under Knightian uncertainty that equals the ATS profile and hence implies ESS. However, in Example 2 we show in a Cournot duopoly with multiplicative aggregation that complete ignorance and thus the Hurwicz criterion is not necessary for the evolutionary stability of a preference. Together Example 1 and 2 imply that complete ignorance is a minimal sufficient condition in the sense that it applies to all aggregative strict submodular games satisfying the assumptions of Proposition 7. Note that complete ignorance allows us to use arguments of Proposition 6, i.e., a result on the monotone comparative statics of equilibrium under Knightian uncertainty for submodular games with externalities. Otherwise changes in equilibrium under Knightian uncertainty may be ambiguous. Hence, it appears that a full characterization of all evolutionary stable degrees of ignorance and optimism is impossible with this general approach.

Example 1 (Cournot duopoly with linear demand and convex cost) Consider two players, $i = 1, 2$, and symmetric payoff functions $\pi(a_1, a_2) = (100 - a_1 - a_2)a_1 - \frac{1}{2}a_1^2$ with actions a_i being in a suitable positive real-valued interval. This is a standard Cournot duopoly with linear demand and quadratic cost. It has decreasing differences and negative aggregate externalities. There exists a unique Nash equilibrium as well as ATS. In this standard game, the only degree of ignorance and optimism for which the EKU equals the ATS profile is complete ignorance and extreme optimism, i.e., $\delta_i = 1$ and $\alpha_i = 1$. Any lower degree of ignorance would require a larger degree of optimism, which is impossible. Since for class of games under consideration, ATS implies ESS (Schipper, 2003, Alós-Ferrer and Ania, 2005), and an EKU corresponding to the ATS involved complete ignorance, we can conclude that extreme optimism and complete ignorance is an GESP.

Example 2 (Cournot duopoly with multiplicative aggregate) This example is to show that complete ignorance is not necessary in Proposition 7. Consider two players, $i = 1, 2$, and symmetric payoff functions $\pi(a_1, a_2) = (100 - a_1 a_2)a_1 - \frac{1}{2}a_1^2$ with actions a_i being in a suitable positive real-valued interval. This game resembles a Cournot duopoly with a multiplicative aggregate of actions. It has decreasing differences and negative aggregate externalities. There exists a unique Nash equilibrium and ATS. One can compute that $\delta_i = 1$ and $\alpha_i = 0.95244$ leads to a unique EKU that equals the ATS. Thus Assumption 1 holds. Since it involves complete ignorance, this is sufficient to conclude that (δ_i, α_i) is an GESP. Are there other combinations of δ_i and α_i that are ESP? A simple (albeit inefficient) way of searching for it is to compute all preferences for which an EKU equals to an ATS, and check whether those players can not be made worse off by mutants in equilibrium. Following figure presents a plot of all combinations for which the EKU equals an ATS profile if $\delta_i < 1$. It can be verified that indeed there are

Figure 1: Degrees of optimism and ignorance leading to ATS in Example 2



evolutionary stable preference without complete ignorance, for instance $\delta_i = \frac{1}{2}$ and $\alpha_i = 1$. Note that there appears to be a negative relationship between the degree of ignorance and the degree of optimism at least for some range of those parameters.

Supermodular games have often multiple equilibria. The following assumption guarantees that there are at least two equilibria in pure strategies if $\max A \neq \min A$, i.e., if the set of actions is nontrivial. In particular, the largest and the lowest profile of actions are Nash equilibria.

Assumption 2 *Let $\pi_i(a_i, \inf A_{-i})$ be decreasing in $a_i \in A$ and $\pi_i(a_i, \sup A_{-i})$ be increasing in $a_i \in A$.*

Note that by Milgrom and Roberts (1990, Theorem 5), for any supermodular game there exist both a largest and smallest profile of actions that survive iterated deletion of strictly dominated strategies, and those combinations of actions are pure Nash equilibria. It is known that only iteratively undominated actions are rationalizable. Assumption 2 can be interpreted as focusing on the subset of actions that can not be eliminated by iterated elimination of strictly dominated actions. It is immediate that if an action is strictly dominated then it can not be an equilibrium action under Knightian uncertainty for any degree of optimism and ignorance. However, the assumption is not without loss of generality because in general a player may still take opponents' strictly dominated actions into account for computing the worst or the best case payoff. In Example 4 we illustrate what may happen if Assumption 2 is violated.

Proposition 8 *Let $G = \langle N, A, \pi \rangle$ be a symmetric strict supermodular aggregative game with aggregate externalities and T_G a collection of preferences that includes the Minimax preference. Suppose that Assumptions 1 and 2 hold. Then we conclude the following:*

(i) The preference with extreme pessimism and complete ignorance (the Minimax preference) is evolutionary stable in G.

(ii) A symmetric equilibrium under Knightian uncertainty resulting from play with this evolutionary stable preference profile equals to a symmetric ATS.

(iii) If π is strictly quasi-concave in the player's own action $a_i \in A$ for all $a_{-i} \in A_{-i}$, then the preference with extreme pessimism and complete ignorance (the Minimax preference) is a robust evolutionary stable preference.

The proof is contained in the appendix.

Contrary to Proposition 7, we can not show the global stability for supermodular games. If our proof of the proposition should go through also for global stability, we would need to show that the lower (largest) Nash equilibrium is coalition-proof if the supermodular game has positive (negative) externalities. This is not necessarily case. In contrast, one can show that the largest (lowest) Nash equilibrium is coalition-proof if the game with ordered action space has positive (negative) externalities (Lemma 7 in the appendix).

In order to show that complete ignorance and extreme pessimism is a robust evolutionary stable preference, we require that π is just strictly quasi-concave in the player's own action a_i instead of being strictly concave as in Proposition 7. This is due to extreme pessimism and complete ignorance, such that the Choquet expected payoff is equivalent to the worst-case payoff only. Contrary, in Proposition 7 the Choquet expected payoff is a weighted average of the worst and the best-case payoff. It is well known that a weighted average of two quasi-concave functions does not need to be quasi-concave.

The following example demonstrates that complete ignorance is not necessary for an evolutionary stable preference in the class of games considered in Proposition 8.

Example 3 (Public goods game with multiple Nash equilibria) Consider two players, $i = 1, 2$, and symmetric payoff functions $\pi(a_1, a_2) = \frac{1}{4}(a_1 + a_2)^2 - \frac{1}{2}a_1$ with actions $a_i \in [0, 1]$. This game resembles a public goods game with increasing returns to contribution. Clearly this game has increasing differences and positive aggregate externalities. Since the benefit function $\frac{1}{4}(a_1 + a_2)^2$ is convex in contributions, the only symmetric combinations of actions corresponding to a pure Nash equilibrium are $(0, 0)$ and $(1, 1)$. Thus Assumption 2 holds. It can be verified that $a_i = 0$ is the unique finite population ESS and by the arguments in the proof of Proposition 8 an ATS (see also Alós-Ferrer and Ania, 2005). The corresponding ATS profile equals to an EKU with complete ignorance and extreme pessimism. Since this equilibrium involves complete ignorance, a non-mutant can not be made worse of by any mutant because $a_i = 0$ is an ESS and she does not react to a mutant. However, there is a whole range of parameters (δ_i, α_i) for which the best response is $a_i = 0$ no matter which action a mutant may choose. These parameters are characterized by $\alpha_i \in [0, \frac{1}{2}]$ and $\delta_i \in [\frac{1}{2(1-\alpha_i)}, 1]$. Hence complete ignorance is not necessary for an evolutionary stable preference.

The next example demonstrates what happens if Assumptions 1 and 2 do not hold. It also demonstrates that an ESP may not exist within the class of CEU preferences. In particular, if the game is supermodular and has a unique equilibrium, then there may not exist an evolutionary stable CEU preference.

Example 4 (Public goods game with a dominant action) Consider two players, $i = 1, 2$, and symmetric payoff functions $\pi(a_1, a_2) = (a_1 + a_2)^2 - \epsilon a_1$ with actions $a_i \in [\epsilon, 1]$ and $0 < \epsilon < 1$. This game is a variant of the previous example. However, it possesses a unique strict dominant action $a_i = 1$. Note that Knightian uncertainty respects dominance, i.e., if an action is strictly dominated, then it is never an equilibrium action under Knightian uncertainty. Thus any symmetric equilibrium under Knightian uncertainty must be $(1, 1)$ no matter which degree of optimism and degree of ignorance. However, $(1, 1)$ is never a finite population ESS since an opponent can make a player relatively worse off then herself by deviating to action ϵ. Hence, there is no evolutionary stable preference within the class of CEU preferences. It is straight forward to establish that $a_i = \epsilon$ is the unique finite population ESS, and thus by arguments in the proof of Proposition 8, ϵ is an ATS (see also Alós-Ferrer and Ania, 2005). This example violates Assumption 1 since there exists no profile of preferences within the class of CEU with neo-additive capacities such that the EKU resulting from this preference equals the ATS profile. It also violates Assumption 2 since $\pi_i(a_i, \inf A_{-i})$ is strictly increasing in $a_i \in A$ and not decreasing. From a more general point of view, the example also demonstrates that there is no apparent connection between the unique and strict dominant Nash equilibrium and finite population ESS (as well as no apparent connection between efficiency and finite population ESS).

6 Concluding Discussion

a. *Evolutionary Stable Preferences and Evolutionary Drift:* It is immediate that given the abstract set of preferences, the inequalities defining (robust globally) evolutionary stable preferences may not hold strictly. There may be many other preferences that achieve the same fitness. Hence, our notion of (robust globally) evolutionary stable preference does not preclude evolutionary drift among preferences achieving the same fitness. In this sense, our notion of evolutionary stable preferences is a much closer analogy to the notion of neutrally stable strategies than to the standard notion of evolutionary stable strategies. However, any preference taking over the population by evolutionary drift would be behaviorally indistinguishable from the behavior of a homogenous population with the evolutionary stable preference in our results. To avoid evolutionary drift, specific assumptions on the collection of preferences would be required.

b. *Strategic Advantage - An Interpretation as Contest:* Instead on focusing on the evolutionary interpretation of the results, the present study may be interpreted as analyzing contests or tournaments among players in which one may have a different preference than others. A player with an evolutionary stable preference maximizes relative payoffs, i.e., t is an ESP of the game $G = \langle N, A, \pi \rangle$ if and only if (recall that we denote by j the mutant and by i a non-mutant)

$$t \in \arg\max_{t' \in T} \{\pi(a_j^*(t'_j, t_{-j}), \aleph^n(\mathbf{a}^*(t'_j, t_{-j}))) - \pi(a_i^*(t'_j, t_{-j}), \aleph^n(\mathbf{a}^*(t'_j, t_{-j})))\}.$$

An ESP maximizes relative payoffs, a player deviating to an ESP may decrease her own equilibrium payoff but decreases the equilibrium payoff of others even more.

Consider now two preferences t and t'. Partition the set of players N into two sets N_t and $N_{t'}$. All players in N_t have preference t and analogously for $N_{t'}$. Preference t yields a *strategic advantage* over preference t' if at each equilibrium $\mathbf{a}^* \in A^n$ of the game $G = \langle N, A, (t_i)_{i \in N_t}, (t'_j)_{j \in N_{t'}} \rangle$ we have

$$\pi_i(\mathbf{a}^*) \geq \pi_j(\mathbf{a}^*) \text{ for all } (i,j) \in N_t \times N_{t'}$$

with strict inequality for some $(i,j) \in N_t \times N_{t'}$ (see Koçkesen, Ok and Sethi, 2000a, b). In our case, a preference yielding a strategic advantage is a strict robust globally evolutionary stable preference (that would exclude evolutionary drift as discussed above). Suppose we restrict T_G suitably in a non-trivial way such to exclude the possibility of evolutionary drift. Then our result on symmetric aggregative submodular games with aggregate externalities implies that optimism and complete ignorance yields a strategic advantage over other preferences in the set T_G. Similarly, if $N_{t'}$ is a singleton, then our result on symmetric aggregative supermodular games with aggregate externalities implies that extreme pessimism and complete ignorance yields and strategic advantage over other preferences in the set T_G.

c. *Observability of Preferences:* It has been noted in the literature on the evolution of preferences that the commitment effect of preferences vanishes if preferences are not perfectly observable (e.g., Samuelson, 2001, Ok and Vega-Redondo, 2001, Ely and Yilankaya, 2001, Dekel, Ely and Yilankaya, 2004). Does this critique apply to our setting as well? No, it does not. First, we show that a preference with complete ignorance is evolutionary stable. Thus, an action by player with this evolutionary stable preference does not depend on the observability of the opponents' actions or preferences. Second, in the proofs of Proposition 7 and 8 we consider any actions by any mutants without the need of specifying whether their preferences entail a technology to observe opponents' preferences or not. So the assumption of observability of preferences does not play any role for our results. With a different approach, Heifetz, Shannon and Spiegel (2005) also show that payoff maximizing behavior may not need to prevail even if preferences are imperfectly observable.

d. *The Relationship between Evolutionary Stable Strategy and optimal Aggregate Taking Strategy:* The proof of Proposition 7 relies heavily on the observation that in aggregate (quasi-)submodular games ATS implies globally stable ESS (see Schipper, 2003, Alós-Ferrer and Ania, 2005), and that ESS implies ATS for aggregate (quasi-)supermodular games (Alós-Ferrer and Ania, 2005).

e. *Evolutionary Dynamics:* So far, we just considered a static concept of evolutionary stability and were silent on any dynamic process of preference evolution. Only few authors considered explicitly the dynamics of preference evolution (Huck, Kirchsteiger and Oechssler, 2005, Sandholm, 2001, Heifetz, Shannon and Spiegel, 2004, and Possajennikov, 2005). A dynamic analysis in our setting should consider beside the preference dynamics also a faster learning process for equilibrium under Knightian uncertainty. Unfortunately, no learning process for equilibrium under Knightian uncertainty has been proposed. Leaving this conceptional issue aside, we can propose a process of preference adaption based on imitate-the-best. In reality, testimonies of (un-)successful people educate us in some situations on "think positive" or "be careful" even though this education may not be conscious. Hence it may not be unreasonable to assume that attitudes towards uncertainty may be imitated. Consider following model of imitation a là Vega-Redondo (1997): In each period every player has a strict positive probability bounded away from one to adjust her preference. If a player adjusts, then she mimics the preference of the most successful player in the previous round (note that in contrast to above discussion this imitation process requires that success and preferences are observable). For simplicity consider a finite set of preferences that at least entails complete ignorance and "relevant" degrees of optimism. The imitation dynamics induces a discrete time finite Markov chain on the space of preference profiles. Focusing on intra-group symmetric equilibria it can be shown that the set of absorbing sets includes each identical preference profile. If we assume that each player may make mistakes in imitating preferences (noise), i.e., with a small probability she selects any preference profile when adjusting her preference, then the resulting perturbed Markov chain is ergodic and irreducible. We can now focus on the unique limiting invariant distribution of preferences when the noise goes to zero. This is the long run distribution interpreted as the average proportion of time a player selects each preference (for an exposition of this methodology, see Samuelson, 1997). For aggregative strict submodular games with aggregate externalities we conjecture based on results by Schipper (2003) and arguments in the proof of Proposition 7, that the evolutionary stable preference with complete ignorance and optimism is in the support of long run distribution, i.e., "stochastically stable". Similarly, for aggregative strict supermodular games with aggregate externalities, we conjecture that the evolutionary stable preference with complete ignorance and extreme pessimism is in the support of the long run distribution.

f. *Short Run Industry Dynamics:* While evolutionary stability focuses on long run outcomes, in the short run interesting dynamics of profits can arise. Consider a standard Cournot oligopoly and assume that all firms choose individually optimal according to the evolutionary stable preference which involves complete ignorance and optimism. Suppose now that there is a mutant who is a realist. A realist improves his profit compared to his pre-mutant profit before because she plays a best-response to the opponents' quantities. However, she raises the profits of the opponents' even more. Before the realistic mutant is driven out in the medium run, all earn higher profits, which may attract entry by additional firms.

g. *Strategic Delegation and Optimism:* In Eichberger, Kelsey and Schipper (2005), we show among others that a principal prefers to delegate to an optimistic manager in both Cournot and Bertrand oligopoly. This is surprising since results are usually reversed when one goes from standard Cournot to standard Bertrand oligopoly. The reason for our results is that an optimistic manager in Cournot oligopoly is more aggressive and less expensive, while she is less aggressive and less expensive in Bertrand oligopoly. From our analysis in this article is clear that a principal delegating to an optimistic manager in a supermodular Bertrand oligopoly may not survive. Since the manager is less aggressive, the firm may make less material payoffs then a competitor with more pessimistic manager. This is in contrast to Cournot oligopoly, in which the optimistic manager survives.

h. *Applications:* The results in the previous sections concern just a special class of games which are however of substantial interest to economics and social science in general. Many games in economics involve ordered action sets like prices, quantities, qualities, contribution levels, appropriation levels etc. Often there is a natural aggregate of all players actions like total market quantity, total contribution or appropriation etc. or an aggregate can be found that may not have an interpretation in the context (for games with aggregation see see Corchón, 1994, Schipper, 2003, Alós-Ferrer and Ania, 2005, Cornes and Hartley, 2005). Moreover, many games with ordered action space have either some version of strategic substitutes or strategic complements (see Bulow, Geanakoplos and Klemperer, 1985) and can be brought into a framework of supermodular or submodular games. Examples include Cournot oligopoly (Amir, 1996, Vives, 2000), some Bertrand oligopoly (Vives, 2000), common pool resource dilemma (Walker, Gardner, and Ostrom, 1990), some rent seeking games (Hehenkamp, Leininger and Possajennikov, 2004), some bargaining games, some public goods games, some co-ordination games (e.g. Van Hyuck, Battalio and Beil, 1991), arms race and search problems (Milgrom and Roberts, 1990).

i. *Equilibrium under Knightian Uncertainty:* We claimed in section 3 that our notion of equilibrium under Knightian uncertainty implies some equilibrium notions proposed in the literature on ambiguity in strategic games. Consider a general capacity ν_i for player i on the opponents' action space A_i and let suppν_i denote the Dow-Werlang support of the capacity, i.e., a set $supp\nu_i \subseteq A_{-i}$ with $\nu_i(A_{-i} \setminus supp\nu_i) = 0$ and $\nu_i(F) > 0$ for all F such that $A_{-i} \setminus supp\nu_i \subsetneqq F$. An *equilibrium under ambiguity* of a finite strategic game G is a profile of capacities $(\nu_i^*)_{i=N}$ such that for all $i \in N$ there exists a non-empty support $supp\nu_i^*$ for which

$$supp\nu_i^* \subseteq \times_{j \in N \setminus \{i\}} \arg\max_{a_j \in A_j} \int \pi_j(a_j, a_{-j}) d\nu_j^*,$$

where the integral is the Choquet integral. In the two-player case, this is the definition of Dow and Werlang (1994) and in the n-player case the one by Eichberger and Kelsey (2000). Marinacci (2000) introduced a similar definition of equilibrium under ambiguity for two-player strategic games in which he defines the support of a capacity ν_i by the set of all $a_{-i} \in A_{-i}$ with $\nu_i(a_{-i}) > 0$. For neo-additive capacities (Definition 5) used in our study, the Dow-Werlang support and the Marinacci support coincide with the support of the probability distribution μ_i. In Eichberger, Kelsey and Schipper (2005), Proposition 3.1, we show for neo-additive capacities that any equilibrium under Knightian uncertainty implies an equilibrium a là Eichberger and Kelsey (2000), Dow and Werlang (1994) or Marinacci (2000).

A Proofs

A.1 Proof of Lemma 1

Since $\mathbf{a}^\circ$ is an ATS and $\mathbf{a}^*$ is a Nash equilibrium of G, we have by definition

$$\pi_i(a_i^\circ, \aleph(a_i^\circ, a_{-i}^\circ)) \geq \pi_i(a_i^*, \aleph(a_i^\circ, a_{-i}^\circ)), \tag{14}$$
$$\pi_i(a_i^*, \aleph(a_i^*, a_{-i}^*)) \geq \pi_i(a_i^\circ, \aleph(a_i^\circ, a_{-i}^*)), \tag{15}$$

for all $i \in N$. Consider case (i), and let G have strict negative aggregate externalities. Suppose to the contrary that for $i \in N$ we have $a_i^\circ \lhd a_i^*$. By the dual single crossing property of π_i in $(a_i, \aleph(\mathbf{a}))$ on $A_i \times X$,

$$\pi_i(a_i^*, \aleph(a_i^*, a_{-i}^*)) \geq \pi_i(a_i^\circ, \aleph(a_i^\circ, a_{-i}^*)) \Rightarrow \pi_i(a_i^*, \aleph(a_i^*, a_{-i}^\circ)) \geq \pi_i(a_i^\circ, \aleph(a_i^\circ, a_{-i}^\circ)). \tag{16}$$

Since G has strict negative aggregate externalities,

$$\pi_i(a_i^*, \aleph(a_i^*, a_{-i}^\circ)) \geq \pi_i(a_i^\circ, \aleph(a_i^\circ, a_{-i}^\circ)) \Rightarrow \pi_i(a_i^*, \aleph(a_i^\circ, a_{-i}^\circ)) > \pi_i(a_i^\circ, \aleph(a_i^\circ, a_{-i}^\circ)), \tag{17}$$

which is a contradiction to inequality (14). The case for strict positive externalities follows analogously.

Now consider case (ii). Again, suppose to the contrary that for $i \in N$ we have $a_i^\circ \lhd a_i^*$. By the dual single crossing property of π_i in $(a_i, \aleph(\mathbf{a}))$ on $A_i \times X$,

$$\pi_i(a_i^*, \aleph(a_i^*, a_{-i}^*)) \geq \pi_i(a_i^\circ, \aleph(a_i^\circ, a_{-i}^*)) \Rightarrow \pi_i(a_i^*, \aleph(a_i^*, a_{-i}^\circ)) > \pi_i(a_i^\circ, \aleph(a_i^\circ, a_{-i}^\circ)). \tag{18}$$

Since G has negative aggregate externalities,

$$\pi_i(a_i^*, \aleph(a_i^*, a_{-i}^\circ)) > \pi_i(a_i^\circ, \aleph(a_i^\circ, a_{-i}^\circ)) \Rightarrow \pi_i(a_i^*, \aleph(a_i^\circ, a_{-i}^\circ)) > \pi_i(a_i^\circ, \aleph(a_i^\circ, a_{-i}^\circ)), \tag{19}$$

which is a contradiction to inequality (14). The case for positive externalities follows analogously. □

A.2 Proof of Lemma 5

If π_i has decreasing (increasing) differences in (a_i, a_{-i}) on A^n then for all $a_i'' \trianglerighteq a_i'$,

$$\pi_i(a_i'', \sup_{A^n} A_{-i}) - \pi_i(a_i', \sup_{A^n} A_{-i}) \leq (\geq) \pi_i(a_i'', \inf_{A^n} A_{-i}) - \pi_i(a_i', \inf_{A^n} A_{-i}).$$

By Remark 1 it follows that if π_i has [decreasing differences in (a_i, a_{-i}) on A^n and negative (resp. positive) externalities] or [increasing differences in (a_i, a_{-i}) on A^n and positive (resp. negative) externalities], then for all $a_i'' \trianglerighteq a_i'$,

$$\maltese_i(a_i'') - \maltese_i(a_i') \leq (\geq) \heartsuit_i(a_i'') - \heartsuit_i(a_i'). \tag{20}$$

Let $\alpha_i', \alpha_i'' \in [0,1]$ with $\alpha_i'' \geq \alpha_i'$. It follows from last inequality that

$$\begin{aligned} &\alpha_i''[(\heartsuit_i(a_i'') - \heartsuit_i(a_i')) - (\maltese_i(a_i'') - \maltese_i(a_i'))] \geq (\leq) \\ &\quad \alpha_i'[(\heartsuit_i(a_i'') - \heartsuit_i(a_i')) - (\maltese_i(a_i'') - \maltese_i(a_i'))]. \end{aligned} \tag{21}$$

This is equivalent to

$$\begin{aligned} &\alpha_i''[\heartsuit_i(a_i'') - \heartsuit_i(a_i')] - \alpha_i''[\maltese_i(a_i'') - \maltese_i(a_i')] \geq (\leq) \\ &\quad \alpha_i'[\heartsuit_i(a_i'') - \heartsuit_i(a_i')] - \alpha_i'[\maltese_i(a_i'') - \maltese_i(a_i')] \end{aligned} \tag{22}$$

$$\begin{aligned} &\alpha_i''[\heartsuit_i(a_i'') - \heartsuit_i(a_i')] + [\maltese_i(a_i'') - \maltese_i(a_i')] - \alpha_i''[\maltese_i(a_i'') - \maltese_i(a_i')] \geq (\leq) \\ &\quad \alpha_i'[\heartsuit_i(a_i'') - \heartsuit_i(a_i')] + [\maltese_i(a_i'') - \maltese_i(a_i')] - \alpha_i'[\maltese_i(a_i'') - \maltese_i(a_i')] \end{aligned} \tag{23}$$

$$\begin{aligned} &\alpha_i''[\heartsuit_i(a_i'') - \heartsuit_i(a_i')] + (1 - \alpha_i'')[\maltese_i(a_i'') - \maltese_i(a_i')] \geq (\leq) \\ &\quad \alpha_i'[\heartsuit_i(a_i'') - \heartsuit_i(a_i')] + (1 - \alpha_i')[\maltese_i(a_i'') - \maltese_i(a_i')]. \end{aligned} \tag{24}$$

Consider any $\delta_i \in [0,1]$. Then previous inequality implies

$$\begin{aligned} &\alpha_i''\delta_i[\heartsuit_i(a_i'') - \heartsuit_i(a_i')] + (1 - \alpha_i'')\delta_i[\maltese_i(a_i'') - \maltese_i(a_i')] \geq (\leq) \\ &\quad \alpha_i'\delta_i[\heartsuit_i(a_i'') - \heartsuit_i(a_i')] + (1 - \alpha_i')\delta_i[\maltese_i(a_i'') - \maltese_i(a_i')], \end{aligned} \tag{25}$$

which in turn implies

$$\begin{aligned} &\alpha_i''\delta_i[\heartsuit_i(a_i'') - \heartsuit_i(a_i')] + (1 - \alpha_i'')\delta_i[\maltese_i(a_i'') - \maltese_i(a_i')] + \\ &\quad (1 - \delta_i)[\pi_i(a_i'', a_{-i}) - \pi_i(a_i', a_{-i})] \geq (\leq) \\ &\quad \alpha_i'\delta_i[\heartsuit_i(a_i'') - \heartsuit_i(a_i')] + (1 - \alpha_i')\delta_i[\maltese_i(a_i'') - \maltese_i(a_i')] + \\ &\quad (1 - \delta_i)[\pi_i(a_i'', a_{-i}) - \pi_i(a_i', a_{-i})] \end{aligned} \tag{26}$$

$$\begin{aligned} &\delta_i[\alpha_i''\heartsuit_i(a_i'') + (1 - \alpha_i'')\maltese_i(a_i'')] + (1 - \delta_i)\pi_i(a_i'', a_{-i}) \\ &\quad -\delta_i[\alpha_i''\heartsuit_i(a_i') + (1 - \alpha_i'')\maltese_i(a_i')] - (1 - \delta_i)\pi_i(a_i', a_{-i}) \geq (\leq) \\ &\quad \delta_i[\alpha_i'\heartsuit_i(a_i'') + (1 - \alpha_i')\maltese_i(a_i'')] + (1 - \delta_i)\pi_i(a_i'', a_{-i}) \\ &\quad -\delta_i[\alpha_i'\heartsuit_i(a_i') + (1 - \alpha_i')\maltese_i(a_i')] - (1 - \delta_i)\pi_i(a_i', a_{-i}). \end{aligned} \tag{27}$$

Hence we have

$$u_i(a_i'', a_{-i}, \alpha_i'', \delta_i) - u_i(a_i', a_{-i}, \alpha_i'', \delta_i) \quad \geq (\leq) \quad u_i(a_i'', a_{-i}, \alpha_i', \delta_i) - u_i(a_i', a_{-i}, \alpha_i', \delta_i). \tag{28}$$

The proof holds analogously for strict versions. □

A.3 Proof of Lemma 6

Pick any α_i'' and α_i' in $[0,1]$ with $\alpha_i'' > \alpha_i'$, and for any $(a_{-i}, \delta_i) \in A_{-i} \times [0,1]$, $a_i' \in b_i(a_{-i}, \alpha_i', \delta_i)$ and $a_i'' \in b_i(a_{-i}, \alpha_i'', \delta_i)$.

First, consider strictly increasing differences of u_i in (a_i, α_i), and suppose to the contrary that $a_i' \rhd a_i''$. Then $a_i'' \lhd a_i' \vee a_i''$ and so using the hypothesis that $u_i(a_i, a_{-i}, \alpha_i, \delta_i)$ is for any $(a_{-i}, \delta_i) \in A_{-i} \times [0,1]$ supermodular in a_i and has strictly increasing differences in (a_i, α_i),

$$\begin{aligned} 0 &\leq u_i(a_i', a_{-i}, \alpha_i', \delta_i) - u_i(a_i' \wedge a_i'', a_{-i}, \alpha_i', \delta_i) \\ &\leq u_i(a_i' \vee a_i'', a_{-i}, \alpha_i', \delta_i) - u_i(a_i'', a_{-i}, \alpha_i', \delta_i) \\ &< u_i(a_i' \vee a_i'', a_{-i}, \alpha_i'', \delta_i) - u_i(a_i'', a_{-i}, \alpha_i'', \delta_i) \leq 0, \end{aligned} \tag{29}$$

which yields a contradiction.

Second, consider strictly decreasing differences of u_i in (a_i, α_i), and suppose to the contrary that $a_i' \lhd a_i''$. Then $a_i' \rhd a_i' \vee a_i''$ and so using the hypothesis that $u_i(a_i, a_{-i}, \alpha_i, \delta_i)$ is for any $(a_{-i}, \delta_i) \in A_{-i} \times [0,1]$ supermodular in a_i and has strictly decreasing differences in (a_i, α_i),

$$\begin{aligned} 0 &\leq u_i(a_i'', a_{-i}, \alpha_i'', \delta_i) - u_i(a_i' \wedge a_i'', a_{-i}, \alpha_i'', \delta_i) \\ &\leq u_i(a_i' \vee a_i'', a_{-i}, \alpha_i'', \delta_i) - u_i(a_i'', a_{-i}, \alpha_i'', \delta_i) \\ &< u_i(a_i' \vee a_i'', a_{-i}, \alpha_i', \delta_i) - u_i(a_i', a_{-i}, \alpha_i', \delta_i) \leq 0, \end{aligned} \tag{30}$$

which yields a contradiction. □

The second part of the proof above is analog to Topkis (1978) (see also Topkis, 1998, Theorem 2.8.4.).

A.4 Proof of Proposition 7

(i) First, we show that there exists a preference t° with complete ignorance leading to an EKU that equals an ATS. By assumption, a symmetric ATS exists for G. We denote it by $\mathbf{a}^\circ$. It follows from Assumption 1 that there exists a preference $t^\circ = (\delta^\circ, \alpha^\circ)$ with $\delta^\circ = 1$ (and some α°) such that there is a symmetric Equilibrium under Knightian with $\mathbf{a}^*(\mathbf{t}^\circ) = \mathbf{a}^\circ$.

Second, we show that t° is a globally stable ESP. In particular, we show that if the EKU $\mathbf{a}^*(\mathbf{t}^\circ)$ is a symmetric ATS then t° is a globally stable ESP in G. Consider the case of negative aggregate externalities. Denote by $\mathbf{t}' := (t', \overset{m}{\dots}, t', t^\circ, ..., t^\circ)$ for some m s.t. $1 \leq m \leq n-1$ and $t' \in T$. Recall that we denote by j a mutant (playing t') by i a non-mutant (playing t°). By the definition of ATS,

$$\pi(a_i^*(\mathbf{t}^\circ), \aleph(\mathbf{a}^*(\mathbf{t}^\circ))) \geq \pi(a_j^*(\mathbf{t}'), \aleph(\mathbf{a}^*(\mathbf{t}^\circ))) \tag{31}$$

for all $t' \in T$. By Remark 3 all non-mutants have constant best-response selections. Therefore we can select $\mathbf{a}^*(\mathbf{t}') \in \mathcal{E}^{sym}(G(\mathbf{t}'))$ such that $a_i^*(\mathbf{t}^\circ) = a_i^*(\mathbf{t}')$ for all non-mutants i and $a_i^*(\mathbf{t}^\circ) \geq a_j^*(\mathbf{t}')$ for all mutants j. This implies that $\mathbf{a}^*(\mathbf{t}^\circ) \geq \mathbf{a}^*(\mathbf{t}')$. By decreasing differences

$$\pi(a_i^*(\mathbf{t}^\circ), \aleph(\mathbf{a}^*(\mathbf{t}'))) \geq \pi(a_j^*(\mathbf{t}'), \aleph(\mathbf{a}^*(\mathbf{t}'))). \tag{32}$$

Since we fixed $\mathbf{a}^*(\mathbf{t}') \in \mathcal{E}^{sym}(G(\mathbf{t}'))$ such that $a_i^*(\mathbf{t}^\circ) = a_i^*(\mathbf{t}')$ for all non-mutants i, we have

$$\pi(a_i^*(\mathbf{t}'), \aleph(\mathbf{a}^*(\mathbf{t}'))) \geq \pi(a_j^*(\mathbf{t}'), \aleph(\mathbf{a}^*(\mathbf{t}'))). \tag{33}$$

Since this holds for all m with $1 \leq m \leq n-1$ and all $t' \in T$, we have that t° is a globally evolutionary stable preference in G. The case of positive externalities follows analogously.

Third, we show that t° is optimistic. Since G has strict negative (positive) externalities we have by Lemma 1, $\mathbf{a}^\circ \geq (\leq)\mathbf{a}^*$. Assume for the time being that $\mathbf{a}^\circ \neq \mathbf{a}^*$. We claim that $\alpha^\circ \geq \alpha^*$ where $\alpha^* = \max\{\alpha' \in [0,1] | \mathbf{a}^*((\alpha'), (1, ..., 1)) \in \mathcal{E}^{sym}(G((\alpha'), (1, ..., 1)) \cap \mathcal{E}^{sym}(G)\}$. Suppose to the contrary that $\alpha^\circ < \alpha^*$. Then by Proposition 6 we must have $\mathbf{a}^*(\alpha^\circ) \leq (\geq)\mathbf{a}^*(\alpha^*)$. Since by assumption $\mathbf{a}^*(\alpha^\circ) \neq \mathbf{a}^*(\alpha^*)$ we have a contradiction. Hence $\alpha^\circ \geq \alpha^*$. If $\mathbf{a}^\circ \neq \mathbf{a}^*$ then trivially there exists $\alpha^\circ = \alpha^*$ by Assumption 1. Hence t° is optimistic.

(ii) Follows immediately from previous arguments.

(iii) We note that if for all players π is strictly concave in the player's own action a_i on A for all a_{-i} on A_{-i}, then so is u_i since it is a sum of strictly concave functions, each term multiplied by positive scalar, and because of aggregate externalities, the worst and best-case actions of the opponents do not depend on the player's own action. Hence $b(a_{-i}, t_i)$ is a singleton for all $i \in N$, each a_{-i} and each t_i. Thus, if $\delta_i = 1$ we have by Remark 3 that $b(a_{-i}, \alpha_i)$ is a constant function on A_{-i} for each α_i. Therefore for all non-mutants $a_i^*(\mathbf{t}^\circ) = a_i^*(\mathbf{t}')$ for all $\mathbf{a}^*(\mathbf{t}') \in \mathcal{E}^{sym}(G(\mathbf{t}'))$. Hence t° is a robust globally evolutionary stable preference. This completes the proof of the proposition. □

A.5 Proof of Proposition 8

Lemma 7 *Suppose that the strategic game $G = \langle N, A, \pi \rangle$ has positive (resp. negative) externalities, and let $\bar{\mathbf{a}}$ and $\underline{\mathbf{a}}$ be the greatest and least combination of actions in A^n. If $\underline{\mathbf{a}}$ (resp. $\bar{\mathbf{a}}$) is a Nash equilibrium of G then $\underline{a}$ (resp. $\bar{a}$) is a finite population evolutionary stable strategy in G.*

Proof of Lemma. If G has positive (resp. negative) externalities, let $a := \underline{a}$ (resp. $a := \bar{a}$). Since $\bar{\mathbf{a}}$ is a Nash equilibrium of G,

$$\pi(a, ..., a) \geq \pi(a', a, ..., a) \text{ for all } a' \in A. \tag{34}$$

We need to show that

$$\pi(a, a', a, ..., a) \geq \pi(a', a, ..., a) \text{ for all } a' \in A. \tag{35}$$

Given both inequalities, it is sufficient to show

$$\pi(a, a', a, ..., a) \geq \pi(a, ..., a) \text{ for all } a' \in A. \tag{36}$$

But last inequality follows immediately from positive (resp. negative) externalities. □

Proof of Proposition 8. (i) and (ii): Suppose that the game has positive (negative) aggregate externalities and consider the lowest (highest) symmetric profile of actions $\underline{\mathbf{a}}$ ($\bar{\mathbf{a}}$). By Assumption 2 this profile of actions is a Nash equilibrium of the game G.

If G has positive (resp. negative) externalities, $a := \underline{a}$ (resp. $a := \bar{a}$) is by Lemma 7 a finite population evolutionary stable strategy in G.

We claim that a is an ATS. Since G is an aggregative game, we have by definition of finite population ESS,

$$\pi(a, \aleph^n(a', a, ..., a)) \geq \pi(a', \aleph^n(a', a, ..., a)) \text{ for all } a' \in A. \tag{37}$$

By increasing differences, finite population ESS implies ATS,

$$\pi(a, \aleph^n(a, ..., a)) \geq \pi(a', \aleph^n(a, ..., a)) \text{ for all } a' \in A. \tag{38}$$

By Assumption 1 there exists a preference t with $\delta = 1$ such that a symmetric EKU satisfies $\mathbf{a}^*(\mathbf{t}) = \mathbf{a}$. Consider the preference $t = (\delta, \alpha)$ with $\delta = 1$ and $\alpha = 0$. From Proposition 5 follows that a symmetric EKU with the symmetric profile of preferences $\mathbf{t} = (t, ..., t)$ satisfies $\mathbf{a}^*(\mathbf{t}) = \mathbf{a}$.

Inequality (37) implies

$$\pi(a_i^*(\mathbf{t}), \aleph^n(\mathbf{a}^*(t'_j, t_{-j}))) \geq \pi(a_j^*(t'_j, t_{-j}), \aleph^n(\mathbf{a}^*(t'_j, t_{-j}))) \text{ for all } t' \in T. \tag{39}$$

By Remark 3 all non-mutants have constant best-response selections. Therefore we can select $\mathbf{a}^*(t'_j, t_{-j}) \in \mathcal{E}^{sym}(G(t'_j, t_{-j}))$ such that $a_i^*(\mathbf{t}) = a_i^*(t'_j, t_{-j})$ for all non-mutants with t and any mutant with any $t' \in T$. Hence

$$\pi(a_i^*(t'_j, t_{-j}), \aleph^n(\mathbf{a}^*(t'_j, t_{-j}))) \geq \pi(a_j^*(t'_j, t_{-j}), \aleph^n(\mathbf{a}^*(t'_j, t_{-j}))) \text{ for all } t' \in T, \tag{40}$$

i.e., t is an ESP in the game G.

(iii) We note that if π is strictly quasi-concave in the player's own action a_i on A for all a_{-i} on A_{-i}, then $u_i(1, 0) = \maltese$ is strictly quasi-concave in the player's own action a_i on A. Hence $b(a_{-i}, (1, 0))$ is a singleton for each a_{-i}. By Remark 3 we have that $b(a_{-i}, (1, 0))$ is a constant function on A_{-i}. Hence $a_i^*(\mathbf{t}) = a_i^*(t'_j, t_{-j})$ for all equilibria $\mathbf{a}^*(t'_j, t_{-j}) \in \mathcal{E}^{sym}(G(t'_j, t_{-j}))$. This completes the proof of the proposition. □

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