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When Randomization in Collective Tournaments is Profitable for the Principal

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Abstract

In the context of principal-agent theory risk is largely seen as a source that causes inefficiencies and lowers incentives and accordingly is not in the principal's interest. In this paper I compare two different designs of a collective tournament where the output of a team is generated through a particular two-stage production process. I show within a theoretical tournament framework that risk in terms of chance is beneficial from the point of view of a profit maximizing principal who organizes the tournament. Selecting an agent randomly that has to work at the final stage after all agents exerted effort at the first stage helps the principal to overcome a trade-off in incentive provision he faces, when selecting the agent who works at the final stage before the tournament starts. This trade-off causes optimal efforts to be lower in a tournament without random selection compared to a tournament with random selection. As the higher efforts overcompensate additional wage costs the principal earns higher expected profits when selecting the agent that has to work at the second stage randomly after the first stage.

JEL Classification: D2, J3, M5

Keywords: collective tournament, incentives, randomization, risk

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1 Introduction

Especially in business contexts collective tournaments between teams are organized to create incentives for the agents to work on a specified problem in the organizer's interest.[1] For example, Starbucks announced in 2008 to shut down 600 of its underperforming U.S. coffee shops (Oregon Business News 2008). This strategy can be interpreted as an (indirect) tournament between the coffee-shops (as teams). In announcing to shut down underperforming shops the organizer created incentives for the employees to focus on his objective of good performance of the shops through a collective tournament. The well performing shops survive and hence "win" the tournament, whereas the losers, meaning the underperforming shops, are shut down. Another example was the strategy of General Motors (GM) in 1992 when they decided to shut down plants, putting them into head-to-head competition to see which would survive (Ward's Auto World 1992).

Assuming that the organizer of such a tournament is a profit maximizing principal adds his objective of profit maximization. Hence, he wants to create high incentives at low costs in order to maximize his expected profits from the tournament. This assumption directly leads to the question whether the principal as the organizer can influence his expected profits through the design of the tournament. The aim of this paper is therefore to analyze the influence of the design of a collective tournament from the point of view of a profit maximizing principal who organizes the tournament on his expected profits.

With regard to tournaments between single agents numerous contributions focus on this question and analyze how the design of a tournament (or contest) can be influenced by the organizer who pursues certain maximization objectives. For example Moldovanu and Sela (2006), Fu and Lu (2008), Clark and Konrad (2007) and Epstein et al. (2008) analyze how tournament design effects efforts exerted by the participants. Demougin and Fluet (2003) focus on the influence of inequity-averse agents on the principal's expected profits in tournaments and Eriksson (1999) is able to show empirically a positive relation between firm performance and tournament design with respect to the pay structures using a rich data set on Danish firms.

In contrast, only a small number of studies focuses on collective tournaments

[1]For analyzes of incentive effects of tournaments in general see for example Nalebuff and Stiglitz (1983) Lazear and Rosen (1981) or O'Keefe, Viscusi, and Zeckhauser (1984).

between teams where individual efforts are not verifiable.[2] Drago et al. (1996) show that first-best effort choices of the agents can be implemented in a collective tournament through an incentive compatible scheme. Gürtler (2006) introduces limited liable agents into a tournament between teams and shows that this aggravates the free-rider problem extremely. He also analyses the influence of sabotage in collective tournaments and finds that sabotage against the weakest team members always decreases a team's performance more significantly than sabotage against the stronger members (Gürtler 2008).

But with respect to collective tournaments the influence of tournament design on its outcome has not yet been studied.

With regard to the business examples mentioned above it is the fundamental interest of a profit maximizing principal to organize a tournament according to profit maximization. One dimension of collective tournaments that can easily be influenced by the organizer and shows to have substantial impact on the profitability of a tournament from the principal's point of view, is the design of the production process among the agents. Hence, I will analyze the influence of the design of a tournament between two teams, on the expected profits of a profit maximizing principal and analyze in addition how the design influences optimal efforts and prizes.

As the output of teamwork is predominantly produced successively the production process within a team is modeled as a two-stage production process: firstly, all members in a team work together and prepare a project jointly (preparation stage). Afterwards, only one of the team members presents or implements the project (implementation stage). The output of a team is therefore generated through efforts exerted at both stages and the teams are finally ranked according to their joint output. Furthermore, I assume that the principal's profits do not only depend on the performance of the winning team, but of all participating teams. Hence, his profits are determined by the sum of output of both teams and the wage costs he has to pay in terms of the tournament prizes.

One example of such a tournament are case study competitions between teams:[3] at the preparation stage a team works together on a given case study while the final presentation of this joint work is predominantly done by just one of the

[2] There exist also various studies on group contests but they assume that individual efforts are verifiable (see for example Lee (1995) or Nitzan (1991)).

[3] See for example case study competitions of Deutsche Post World Net, HP, Ernst & Young or Roden Scholars'.

team members. The winner in this competition is the team that produced the highest output across both stages. Although only one of the teams is chosen as the winning-team based on its output, the organizers are typically not only interested in the output of this winning team. Organizers of case study competitions with an innovative question, for example, use these competitions to generate new ideas and solutions for their business. Therefore, they do not only consider the ideas of the winning team, but the proposals of all teams. Another example are competitions between teams of interior designers or product designers.

Furthermore, this model can be interpreted as a theoretical analysis of the tournament incentive system for teachers in Israel studied by Lavy (2002). It is often argued that the teaching process is characterized by team production (see for example Eberts et al. 2002, Lazear 2003) which means that the output from teaching (for example pupils performance in high-school matriculation exams or dropout-rates, measured on an individual, class or school level) is sequentially generated by a group of teachers. In general, the teaching-process can be roughly divided into two different parts: firstly, a group of teachers exerts effort to teach pupils basic knowledge (preparation stage). Afterwards, only one teacher exerts effort again to prepare pupils in a given class for the final exams. Finally, schools are ranked according to their output (e.g. pupils performance in high-school matriculation exams and dropout-rates) and the tournament prizes are paid out. It is obvious that the organizer does not only care about the output of the winning-team, that is the "best-performing school" but of all schools because the main purpose of this incentive system is to improve pupils outcomes in all participating schools in the pre-determined dimension(s).

Assuming that all members of a team are homogenous and disregarding the possibility to choose the agent for implementation on the basis of performance at the preparation stage leaves basically two possibilities to organize the production across both stages:[4] either each agent knows before the tournament starts whether he has to prepare the project only or whether he is the one that will also have to implement it finally. Hence, each agent knows exactly when to work before the tournament starts. I will refer to this scenario as the *No-Randomization-Scenario*

[4]In the following I refer to these possibilities as scenarios.
An analysis where performance at the first stage is used as an indicator for implementation at the second stage can be found in Kräkel and Schöttner (2008) or Ohlendorf and Schmitz (2008). In this case the principal can save further costs because rents generated at the second stage can be used to generate indirect incentives at the first stage.

(NRS). Another possibility is to introduce some risk through randomization in the tournament and select the agent for the final implementation of the project (randomly) after efforts have been exerted at the preparation stage. In this case selection is also independent from performance at the preparation stage but an agent does not know whether he is the one that has to implement the final project when exerting effort at the preparation stage. Accordingly, I refer to this scenario as the *Equal-Randomization-Scenario* (ERS) because there exists an equal chance for all members of a team to be randomly selected for implementation at the second stage.

In the following both scenarios are analyzed and compared from the point of view of a profit maximizing organizer of such a tournament between teams. Therefore, I assume that the principal is the one who organizes the tournament, sets up its basic structure and decides how to organize the production across both stages. This analysis is conducted within a tournament setting based on the formal model introduced by Lazear and Rosen (1981). In contrast to the firm's zero profit constraint introduced in their model the tournament is evaluated with respect to its profitability for the principal in this paper. It is additionally assumed that the participating agents are strictly protected by limited liability, i.e. the principal cannot extract rents by using negative tournament prizes. Given the applications and examples above, this is a reasonable assumption. Respective outputs of the two teams competing in the tournament are generated through efforts exerted by all members of a team at the production stage and the additional effort of one of these members at the implementation stage.

The main finding of this paper is that when introducing risk into a tournament between teams - through the random selection of the agent for implementation - the principal's expected profits are always larger than without this risk. This result is driven by a trade-off between the incentives for the agents in different roles when they know their roles before the tournament starts, whereas there exists no such trade-off when the agent for implementation is randomly selected after efforts have been exerted for preparation. This trade-off occurs because incentives for the agents in a team are among other things provided through the shares they receive from the prize finally won. As the prize won by a team has to be shared somehow between the team members, it is crucial for incentive provision whether the agents know the division and therefore, how much they can earn before the tournament starts or not. Because the agents have to share a given pie among each other - the prize won - their incentives resulting from the particular shares are not independent from each other. In the *No-Randomization-Scenario* incentives resulting from the shares for the agents in different roles work in opposite directions because the larger the share for one agent, the smaller the remaining for the other one. In the *Equal-Randomization-Scenario*, in contrast, the distribution of the prize between the agents in different roles has no direct incentive effect at the first stage, because the agents do not know their role at this point in time. Consequently, the optimal distribution of prizes as well as the optimal prizes have to balance incentives working in opposite directions for both agents in the *No-Randomization-Scenario* but not in the *Equal-Randomization-Scenario*. While Drago et al. (1996) assume that prizes are divided equally among all members of a team, it is shown in this paper, that this sharing rule is not optimal from the point of view of the profit maximizing principal. Moreover, the optimal division of prizes among the agents varies between both scenarios.

In showing that the profit maximizing principal benefits from the introduction of risk this paper highlights a new aspect related to the vast literature focusing on the relationship between risk and incentives. While the existing empirical and theoretical literature focuses on the relationship between (exogenous) risk and incentives in individual incentive schemes and finds mixed results with respect to the direction of the relationship,[5] this paper contributes to this strand of literature by analyzing the influence of (endogenous) risk on team incentives in tournaments. On the one hand it is shown that endogenously imposing risk on the agents leads to higher wage costs for the principal in terms of higher optimal tournament prizes (compared to a situation without this risk). On the other hand the introduction of risk leads to higher optimal efforts exerted by the agents. Thus, the (additional) risk has a positive as well as a negative impact on the principal's profit. This paper shows that the positive effect of higher optimal efforts dominates the negative effect of increased optimal prizes. Consequently, the (additional) risk is in the principal's interest.

The theoretical model is introduced in the following section. Section 3 presents the analysis and results of the model. While optimal efforts for both scenarios are derived and compared in section 3.1 the optimal tournament contracts are derived and analyzed in section 3.2. Further extensions of the model are discussed

[5] An overview about the findings of numerous empirical studies can be found in Prendergast (2002). Different explanations for a positive relationship between risk and incentives can be found in Baker and Jorgensen (2003), Prendergast (2000, 2002) and Wright (2004) for example.

in section 4. The final section of this paper concludes.

2 The model

Consider two teams ($k = A,\ B$) that consist of two risk-neutral and ex-ante homogeneous agents ($r = 1, 2$) each - in the following the indices 1 and 2 denote the possible roles the agents can take, not their identities - and a risk-neutral principal, who offers a tournament contract to the agents. Output of both teams is sequentially generated across two stages due to the following production function: $y^k(e_r^k, \sum h_r^k) = h_1^k + h_2^k + e_r^k + \varepsilon^k$, where $h_r^k \geq 0$ is the effort exerted by an individual in role r in team k at the first stage (preparation stage). $e_r^k \geq 0$ is the effort exerted by an individual in role r in team k at the second stage (implementation stage). ε^k is an exogenous noise term in the tournament in the production of team k. Both ε^A and ε^B are assumed to be stochastically independent and identically distributed. Let $G(\cdot)$ denote the cumulative distribution function of the composed random variable $\varepsilon^B - \varepsilon^A$ and $g(\cdot)$ its density function, where $g(\cdot)$ is unimodal with mode at zero. The final output y^k is equal to the principal's observable but not verifiable return. Single efforts exerted by the agents at the different stages can't be observed by the principal. While all agents who are a member of a team exert effort simultaneously at the first stage only one of the team members has to exert effort afterwards at the second stage.

The costs of providing effort for an individual in role r in team k are given by $\kappa(h_r^k)$ for the first stage, where $\kappa(0) = 0$, $\kappa'(0) = 0$, $\kappa'(h_r^k) > 0$, $\kappa'(h_r^k)$ is invertible, $\kappa''(h_r^k) > 0$ and $\kappa'''(h_r^k) > 0$ for all $h_r^k > 0$ and for the second stage by $c(e_r^k)$, with $c(0) = 0$, $c'(0) = 0$, $c'(e_r^k) > 0$, $c'(e_r^k)$ is invertible, $c''(e_r^k) > 0$ and $c'''(e_r^k) > 0$ for all $e_r^k > 0$. Each agent has a reservation utility equal to $\bar{u}$ which is normalized to 0. The teams compete for the monetary tournament prizes w_H and w_L, where $w_H > w_L \geq 0$, because the participating agents have no (monetary) resources of their own (limited liability).

Output is generated across the two-stage production process and the teams are finally ranked according to their output. Afterwards, the tournament prizes are paid out due to the final ranks of the teams, e.g. the team that produced the highest output gets the winner prize w_H and the other one the loser prize $w_{L.}$

In order to fully pay out the prizes at the end of the tournament they have to be shared somehow between the agents working together in a team. In the following $f_r(w_j) \geq 0$ denotes the share of the prize w_j that gets the agent in role r who exerted effort only at the first stage ($j = H, L$). Assuming that the principal has to pay out the full prize to a team leaves $1 - f_r(w_j) \geq 0$ to the agent who provides effort at both stages.[6] The shares of the prizes are the same for both agents in corresponding roles in both teams and are chosen such that $\frac{\partial f_r(w_j)}{\partial w_j} = f_r$ so $f_r(w_j) = f_r w_j$, and $f_r \epsilon \, [0,1]$. Furthermore $f_r \Delta w = f_r w_H - f_r w_L$ and accordingly $(1 - f_r)\Delta w = (1 - f_r) w_H - (1 - f_r) w_L$. The tournament prizes and also the shares received by the agents are known by all agents and the principal before the tournament starts.

Without loss of generality it is assumed in the following that an agent in role 2 is the one that exerts effort at both stages, while an agent in role 1 exerts effort only at the first stage. At the beginning of the tournament this definition of the roles is known by all agents. Thus, $f_1 w_j$ denotes the share of prize j that gets the agent who exerts effort only at the first stage (role 1) and $(1 - f_1) w_j$ denotes the share of prize j that gets the agent who exerts effort at both stages (role 2).

The following scenarios of team production will be analyzed and compared:

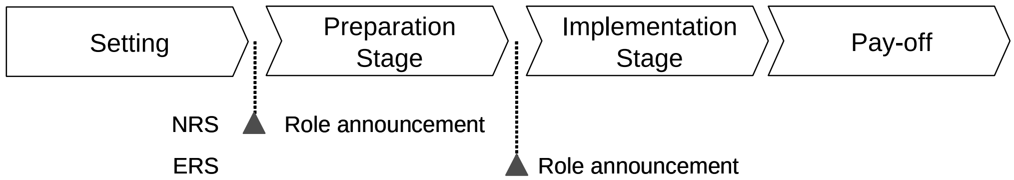
Scenario 1: Both agents know in which role they are before the tournament starts. Therefore, each agent knows if he has to exert effort only at the first stage or at both stages when exerting effort at the first stage. I will refer to this scenario as the *No-Randomization-Scenario* (nr) in the following.

Scenario 2: The agent that has to exert effort at the second stage is randomly chosen after efforts have been exerted by both agents at the first stage, e.g. the agents do not know in which role they are when exerting effort at the first stage. They get to know their roles and hence who has to exert effort at the second stage just before it starts and after they exerted effort at the first stage (there exists an equal chance for all $n = 2$ team members to be chosen. Therefore, the probability to be chosen at the second stage is equal to $q = \frac{1}{n} = \frac{1}{2}$)[7]. I will refer to this scenario as the *Equal-Randomization-Scenario* (er).

[6] Only the sum of payments for both stages together is relevant for incentive provision for the agent that exerts effort at both stages. The distribution between stages does not matter. Therefore, in the following only the sum of payments for this agent is considered and anlyzed.

[7] It can be shown that it is optimal from the principal's point of view and for given prizes that their exists an equal probability for both agents to be chosen at the second stage (see appendix).

The following figure summarizes the timing of the two scenarios:



All in all, both scenarios differ with regard to the knowledge the principal and the agents have about the agents' roles. In the *No-Randomization-Scenario* the principal and both agents know in which role they are before the preparation stage. In the *Equal-Randomization-Scenario,* however, the agents and the principal get to know in which role an agent is **after** all agents exerted effort at the preparation stage but before the implementation stage.

3 Analysis

In the following the agents' effort decisions as well as the optimal tournament prizes and shares are derived by backward induction. Furthermore, they are analyzed and compared for both scenarios from the point of view of a profit maximizing principal who can influence the design of the collective tournament up to different degrees. Firstly, both scenarios are compared for exogenously given shares and prizes. Secondly, the comparison is made for endogenously chosen prizes and exogenously given shares. Lastly, the decision about the division of the shares among the agents is also made endogenously by the principal.

3.1 The first-best solution

In order to interpret the results derived in the subsequent sections it is instructive to consider briefly the first-best solution where efforts are directly contractible as the reference solution. In a first-best world efforts would be chosen such that the expected total surplus of the participating principal and agents is maximized. Hence, the first-best decisions are given by

$$c'\left(e^*_{FB}\right) = 1 \Rightarrow e^*_{FB} = c'^{-1}(1)$$

$$\kappa'(h^*_{1FB}) = \kappa'(h^*_{2FB}) = 1 \Rightarrow h^*_{1FB} = h^*_{2FB} = \kappa'^{-1}\left(1\right)$$

First-best efforts are thus characterized by the equality of marginal costs of effort and marginal returns of production.

3.2 Optimal efforts

Firstly, the optimal effort decisions at the second stage are derived where only one agent has to exert effort after the prizes and shares have been fixed and efforts at the first stage have been exerted. Afterwards optimal efforts for both agents at the first stage are derived. Therefore, optimal efforts at both stages are determined for given prizes and shares.

3.2.1 No-Randomization-Scenario

Given the compensation scheme (w_H, w_L) and an exogenous distribution of these prizes among the participating agents they choose their effort levels such that their expected utilities are maximized. The expected utility of an agent in role 2 in team A at the beginning of the tournament is given by

$$EU_2^A(e_2^A, \sum h_r^A) = (1-f_1)w_l + (1-f_1)\Delta wp - c(e_2^A) - \kappa(h_2^A)$$

where p is the probability that team A wins, e.g. that $y^A > y^B$, so

$$p = prob\left\{y^A > y^B\right\} = prob\left\{\varepsilon^B - \varepsilon^A < \sum_{r=1}^{2} h_r^A - \sum_{r=1}^{2} h_r^B + e_2^A - e_2^B\right\} = G(\sum_{r=1}^{2} h_r^A - \sum_{r=1}^{2} h_r^B + e_r^A - e_r^B)$$

The expected utility of an agent in role 1 in team A is given by

$$EU_1^A(e_2^A, \sum h_r^A) = f_1 w_L + pf_1\Delta w - \kappa(h_1^A)$$

The expected utilities for agents in roles 1 and 2 in the corresponding situations in team B are given accordingly to those for the agents in team A. If a subgame perfect equilibrium in pure strategies exists, the agents in both teams that are in corresponding situations will choose identical efforts. Hence, the symmetric equilibrium will be described by the following first order conditions:[8]

$$c'(e_2^{A^*}) = c'(e_2^{B^*}) = (1-f_1)\Delta wg(0) =: c'(e_2^*)$$
$$\kappa'(h_2^{A^*}) = \kappa'(h_2^{B^*}) = (1-f_1)\Delta wg(0) =: \kappa'(h_2^*)$$
$$\kappa'(h_1^{A^*}) = \kappa'(h_1^{B^*}) = f_1\Delta wg(0) =: \kappa'(h_1^*)$$

[8] The existence of a pure-strategy equilibrium in tournaments is typically not automatically assured. See for example Lazear and Rosen (1981), page 845 fn.2 or Nalebuff and Stiglitz (1983). To guarantee the existence of a solution, $g(\cdot)$ is assumed to be sufficiently flat and $c(e)$ and $\kappa(h)$ have to be "sufficiently convex" for the objective functions to be concave.

Leading to optimal effort decisions according to

$$c'^{-1}\left((1-f_1)\Delta w g(0)\right) = e_2^* \tag{1}$$

$$\kappa'^{-1}\left((1-f_1)\Delta w g(0)\right) = h_2^* \tag{2}$$

$$\kappa'^{-1}\left(f_1 \Delta w g(0)\right) := h_1^* \tag{3}$$

As in standard tournaments (see for example Lazear and Rosen 1982 or Nalebuff and Stiglitz 1983) efforts depend positively on the prize spread between the winner and loser prize and on $g(0)$, which is a measure for the importance of luck in the tournament (see Lazear 1995). The larger $g(0)$ the smaller the influence of luck in the tournament and therefore the larger the amount of effort exerted. Furthermore, incentives at both stages are enhanced by flat marginal cost functions. The flatter these functions the steeper their inverses and the larger the corresponding optimal efforts.

In addition, efforts depend positively on the shares the agents receive from the prize won. The effect of a higher share of the final prize received by an agent goes thus in the same direction as the effect of a higher prize spread. In contrast to the results of standard tournaments the effect of an increased prize spread is not translated one-to-one to individual efforts for $(1 - f_1) < 1$ and $f_1 < 1$.

While higher incentives generated through an increased prize spread cause direct costs for the principal, because this means to enlarge the pie for the agents,[9] incentives generated through increased shares cause no direct costs for the principal, because they are generated through the distribution of a given pie. Nevertheless, it is important to note that incentives generated through the shares cause indirect costs. A change in the shares received by one agent automatically causes a converse effect on the shares awarded to the other agent: increasing the share for the agent in role 2 $(1-f_1)$ automatically lowers the share received by the agent in role 1 (f_1) and vice versa. Hence, higher incentives generated through a larger share for one agent come at the cost of lower incentives for the other agent.

From (1) and (2) it can be seen that it is the total share an agent in role 2 receives in the tournament that matters for the amounts of effort exerted at both stages. It does not matter for incentive provision if particular shares are explicitly attached to one of the two stages. Even if an agent in role 2 knew that he would not get any money for effort exertion at one of the two stages he has an incentive to exert effort also at this stage because this increases his probability of winning the

[9] Assuming that he can not just lower the loser prize but has to increase the winner prize.

tournament. Consequently, it is the total share he receives that creates incentives at both stages simultaneously.

3.2.2 Equal-Randomization-Scenario

This scenario is solved in the same way by backward induction as the *No-Randomization-Scenario*. In contrast to the *No-Randomization-Scenario,* where the agents in different roles have different expected utilities at the beginning of the tournament, they have the same expected utility in this scenario at the beginning of the tournament. This is due to the fact that both agents do not know their role at this point in time and therefore face the same situation at the beginning of the tournament. For agents in team A the expected utility is given by:[10]

$$EU_r^A(e_r^A, \sum h_r^A, q) = q(1-f_1)w_L + (1-q)f_1 w_L + qp(1-f_1)\Delta w + (1-q)pf_1\Delta w - qc(e_r^A) - \kappa(h_r^A)$$

Assuming the existence of an equilibrium in pure strategies and an equal chance for both members to be in role 2 (that is $q = \frac{1}{2}$), it will be described by the following first order conditions:[11]

$$c'(e_r^{A^*}) = c'(e_r^{B^*}) = g(0)(1 - f_1)\Delta w =: c'(e_r^*)$$

$$\kappa'(h_r^{A^*}) = \kappa'\left(h_r^{B^*}\right) = g(0)\frac{1}{2}\Delta w =: \kappa'(h_r^*)$$

which yields the following optimality conditions for effort provision:

$$c'^{-1}\left((1 - f_1)\Delta w g(0)\right) = e_r^* \tag{4}$$

$$\kappa'^{-1}\left(\frac{1}{2}\Delta w g(0)\right) = h_r^* \tag{5}$$

The condition for effort provision at the second stage is exactly the same as in the *No-Randomization-Scenario.* In both scenarios the agents that have to exert effort at the second stage again are in corresponding situations: efforts have already been exerted at the first stage and at the beginning of the second stage they know that they have to exert effort again.

In contrast to the *No-Randomization-Scenario* where the conditions for optimal effort provision at the first stage differ for both agents, they are the same for both agents in this scenario. Up to this point in time both agents are symmetric and

[10] The expected utilities for agents in team B are given accordingly.

[11] The conditions and assumptions for the existence of a pure strategy equilibrium in this scenario are analog to those in the NRS.

do not know in which role they are. Therefore, the distribution of the prize spread between the agents in different roles has no incentive effect on the amount of effort exerted at this stage. Only (half of) the spread between winner and loser prize is decisive for the amounts of effort exerted, as well as $g(0)$.

3.2.3 Comparison of optimal efforts for a given prize spread Δw

Comparing the conditions for optimal effort provision at the two stages in the *No-Randomization-Scenario* (e_2^*, h_2^* and h_1^*) to those in the *Equal-Randomization-Scenario* (e_r^* and h_r^*) reveals the influence of the different scenarios and the corresponding designs of team production on efforts.

$$e_2^* = c'^{-1}\left((1-f_1)\Delta w g(0)\right) = e_r^* \tag{6}$$

$$h_2^* = \kappa'^{-1}\left((1-f_1)\Delta w g(0)\right) \gtreqless \kappa'^{-1}\left(\frac{1}{2}g(0)\Delta w\right) = h_r^* \tag{7}$$

$$h_1^* = \kappa'^{-1}\left(f_1 \Delta w g(0)\right) \gtreqless \kappa'^{-1}\left(\frac{1}{2}g(0)\Delta w\right) = h_r^* \tag{8}$$

It is obvious that the difference in the design affects only the amounts of effort exerted at the first but not at the second stage. Efforts exerted at the second stage are exactly the same in both scenarios because agents exerting effort at the second stage face exactly the same situation at this stage of the tournament. At the first stage individual efforts in the *No-Randomization-Scenario* can be smaller, larger or equal to those in the *Equal-Randomization-Scenario*. $h_2^* \gtreqless h_r^*$ and $h_1^* \lesseqgtr h_r^*$ for $[(1-f_1)\Delta w] \gtreqless f_1\Delta w$, e.g. for $\frac{1}{2} \gtreqless f_1$.

Proposition 1 *The principal's expected profits are always higher in the Equal-Randomization-Scenario than in the No-Randomization-Scenario for given prizes. Only in the case of equally divided prizes among team-members, that is* $(1-f_1) = f_1 = \frac{1}{2}$, *expected profits are the same in both scenarios.*

Proof. The conditions for optimal effort provision at the second stage are exactly the same in both scenarios and because of given prizes the corresponding costs for the principal are the same as well. Consequently, only efforts exerted at the first stage are decisive and have to be compared. For $(1-f_1) = f_1 = \frac{1}{2}$ it follows that $h_1^* + h_2^* = 2\kappa'^{-1}(\frac{1}{2}\Delta w g(0)) = 2h_r^*$. Hence, profits are the same as well. For $1-f_1 \neq f_1$ efforts at the first stage are given by $h_1^* + h_2^* = \kappa'^{-1}\left((1-f_1)\Delta w g(0)\right) + \kappa'^{-1}\left(f_1\Delta w g(0)\right)$ in the *No-Randomization-Scenario* and by $2h_r^* = 2\kappa'^{-1}(\frac{1}{2}\Delta w g(0))$

in the *Equal-Randomization-Scenario.* Δw and $g(0)$ are the same in both scenarios and κ'^{-1} is strictly concave hence $\kappa'^{-1}(\frac{1}{2}(1-f_1)\Delta w g(0)+\frac{1}{2}f_1\Delta w g(0)) > \frac{1}{2}\kappa'^{-1}\left((1-f_1)\Delta w g(0)\right)+\frac{1}{2}\kappa'^{-1}\left(f_1\Delta w g(0)\right)$ for $f_1 \epsilon \left[0,1\right] \setminus \left\{\frac{1}{2}\right\}$. ■

If prizes are exogenously given and the principal is free to choose between both scenarios to organize a tournament, he would always choose the *Equal-Randomization-Scenario* in order to maximize his expected profits. While the costs for the principal are the same in both scenarios, the sum of efforts exerted in this scenario is always greater than or equal to the sum exerted in the *No-Randomization-Scenario.* The decisive factor for this result are the amounts of effort exerted at the first stage in both scenarios, because they vary between both scenarios, while efforts exerted at the second stage are the same.

In the *Equal-Randomization-Scenario* the distribution of the prize spread between the agents has no incentive effect on efforts exerted at the first stage, because the agents do not know in which role they are when exerting effort at this stage. Incentives are therefore only generated through the difference between the given winner and loser prize at this stage. In contrast, in the *No-Randomization-Scenario* incentives are generated through the difference between the prizes and also through the shares the agents in different roles receive. Therefore, the distribution of the prize spread matters. But the shares for both agents are not independent from each other because they have to share a given pie. The larger the share for one agent the smaller the share for the other. Hence, there exists a trade-off in the distribution of shares among the agents in the *No-Randomization-Scenario*: increasing the share for one agent for given prizes automatically leads to a lower share for the other agent and therefore lower incentives for effort provision for this agent. This trade-off taken together with the convex cost functions causes lower optimal efforts in the *No-Randomization-Scenario* for unequal shares than in the *Equal-Randomization-Scenario* for a given Δw. If both agents receive half of the prize won the amount of effort exerted at the first stage is maximized in the *No-Randomization-Scenario* and is just equal to the amount exerted in the *Equal-Randomization-Scenario* for equal shares.

Comparing these results for both scenarios with the first-best solution reveals that the following condition has to be fulfilled in order to implement first-best efforts with and without randomization:

$$(1-f_1)\Delta w = f_1\Delta w = \frac{1}{2}\Delta w = \frac{1}{g(0)}$$

This means the sum of payments to both agents must be the same irrespective of whether they provide effort at both or only at one stage of the production process in the tournament. Hence, the prize won has to be divided equally. This result replicates the findings of Drago et al. (1996). They show that first best efforts can be implemented in a team setting where risk-neutral agents chose multi-dimensional efforts when prizes are divided equally between all members of a team. Efforts exerted at the first stage in this model can be interpreted as helping efforts in the sense of Drago et al. Nevertheless, it should be emphasized that agents in their model are homogenous with respect to the dimensions of effort exerted whereas they are heterogenous in this regard in this model because only one out of two agents exerts effort twice and the other one only once.

Due to the fact that prizes have to be divided equally to induce first-best efforts these efforts are simultaneously exerted by both agents and at both stages once the appropriate share is chosen for one agent.

3.3 The optimal tournament contract

Given the optimal behavior of the agents in equilibrium in both scenarios (formally given by equations 1-5) the principal anticipates their behavior and chooses the optimal tournament prizes to maximize his expected profits P subject to the agents incentive and participation constraints and the wealth constraint $w_L \geq 0$. To be able to derive explicit solutions for the optimal contract and to compare the results also quantitatively in the following the cost functions are assumed to be given by $c(e_r^k) = \frac{1}{3}(e_r^k)^3$ and $\kappa(h_r^k) = \frac{1}{3}(h_r^k)^3$.

3.3.1 No-Randomization-Scenario

The agents' participation constraints in both teams are given by the following conditions, where PC_1 is the participation constraint of an agent in role 1 and PC_2 for an agent in role 2 and $\bar{u} = 0$.[12]

$PC_1 : f_1 w_L + \frac{1}{2} f_1 \Delta w - \kappa(h_1^*) \geq \bar{u}$

$PC_2 : [(1 - f_1) w_L] + \frac{1}{2} [(1 - f_1) \Delta w] - c(e_2^*) - \kappa(h_2^*) \geq \bar{u}$

Due to the assumption that the agents have no (monetary) resources of their own the principal has to take the limited liability constraint $w_L \geq 0$ into account.

[12] Note that the chance of winning/losing is equal to $P = G(0) = \frac{1}{2}$ in equilibrium due to the symmetry of agents and teams.

It is straightforward to see that $w_L = 0$ has to hold in the optimum.[13] Moreover, the participation constraints can be ignored since the agents can always obtain a non-negative expected utility by accepting the contract and choosing zero efforts.

Optimal efforts in the *No-Randomization-Scenario* are therefore given by:

$$e_2^* = \sqrt{(1-f_1)w_H g(0)}$$

$$h_2^* = \sqrt{(1-f_1)w_H g(0)}$$

$$h_1^* = \sqrt{f_1 w_H g(0)}$$

Due to the assumption that the cost functions are the same at both stages the optimal amounts of effort exerted by an agent in role 2 are the same at both stages. Because of the symmetry of both teams the principal's maximization problem is given by $\underset{w_H}{max} \;\; 2E\left[y(e_2^*, h_1^*, h_2^*)\right] - w_H$, that means he chooses the optimal w_H to maximize his expected profit taking the agents incentive constraints into account:

$$\underset{w_H}{max} \quad 2\left[\sqrt{\left[(1-f_1)w_H\right]g(0)} + \sqrt{f_1 w_H g(0)} + \sqrt{\left[(1-f_1)w_H\right]g(0)}\right] - w_H$$

which yields the following optimality conditions:

$$\hat{w}_H = g(0)\left[2\sqrt{(1-f_1)} + \sqrt{f_1}\right]^2 \tag{9}$$

$$\hat{h}_2 = g(0)\left[2\left(1-f_1\right) + \sqrt{\left(1-f_1\right)f_1}\right] \tag{10}$$

$$\hat{h}_1 = g(0)\left[2\sqrt{\left(1-f_1\right)f_1} + f_1\right] \tag{11}$$

$$\hat{e}_2 = g(0)\left[2\left(1-f_1\right) + \sqrt{\left(1-f_1\right)f_1}\right] \tag{12}$$

Obviously, $g(0)$ – as a measure for the importance of luck in the tournament – has a positive impact on optimal efforts as well as the optimal winner prize. The larger $g(0)$ and hence the smaller the influence of luck, the larger the optimal amounts of efforts exerted and the larger the optimal prize. The last effect is thereby driven by the first one: the smaller the influence of luck the larger the amounts of effort exerted, because they are the decisive factor in the determination of the winner and loser in the tournament. To compensate the agents for the higher effort costs caused by higher efforts, the optimal prize has also to increase the smaller the influence of luck.

[13] If the principal chooses $w_L > 0$, then he could reduce w_L and w_H by the same amount, induce still optimal efforts and therefore lower his costs.

However, the influence of f_1 on optimal efforts and the optimal prize is ambiguous. While $\hat{w}_H$, $\hat{h}_2$ and $\hat{e}_2$ increase for small values of f_1 and decrease for values larger than a certain cutoff, $\hat{h}_1$ increases up to a certain certain value of f_1 and decreases only for large values.[14] $\hat{h}_2$ and $\hat{e}_2$ increase for small values of f_1 because this means by implication larger values of $(1 - f_1)$ and hence a larger share of the prize for the agent in role 2. The reverse argumentation holds for the agent in role 1 and accordingly for $\hat{h}_1$. The influence of f_1 on the optimal winner prize is ambiguous, because $\hat{w}_H$ is chosen such that it is optimal for agents in both roles, that means, that it has to balance the positive/negative effects of f_1 and $(1 - f_1)$ on the agents' incentives in the different roles. This ambiguous effect on the optimal prize is carried over to optimal efforts and causes the ambiguous effect of f_1 on $\hat{h}_1$, $\hat{h}_2$ and $\hat{e}_2$ (i.e. that $\hat{h}_2$ and $\hat{e}_2$ increase for small values and decrease for large and the reversed effects on $\hat{h}_1$). Therefore, the ambiguous effect of f_1 on optimal efforts and $\hat{w}_H$ reflects the trade-off the principal faces when providing incentives in this scenario. The incentives for both agents are interrelated because both have to share a pie - the tournament prize - among each other. Furthermore, the incentives for one agent conversely affect those of the other agent. Due to the structure of the production function and cost functions it is optimal for the principal to set incentives for both agents to exert effort. Consequently, he has to trade off the incentives for one agent against that for the other.

Given the optimality conditions for effort provision and the prize the principal's expected profit in the optimum is given by

$$P_{nr} = 2E\left[y(\hat{e}_2, \hat{h}_1, \hat{h}_2)\right] - \hat{w}_H = g(0)(4 - 3f_1 + 4\sqrt{(1 - f_1)\, f_1}) > 0 \text{ for } f_1 \leq 1 \quad (13)$$

He always makes positive profits in the optimum, no matter how the shares are chosen. If the principal is free to choose the shares, he will do this in order to maximize his expected profits:

$$\max_{f_1} \quad P_{nr} = 2E\left[y(\hat{e}_2, \hat{h}_1, \hat{h}_2)\right] - \hat{w}_H$$

Because $\frac{\partial P_{nr}}{\partial f_1} \lesseqgtr 0$ there exists an interior solution for the maximization problem of the principal and his expected profits are maximized for $\hat{f}_1 = \frac{1}{5}$. This value is equal to the exact cutoff-value of f_1 for $\hat{w}_H$. Up to this point $\hat{w}_H$ increases in f_1 and afterwards decreases. Hence, it is exactly the point at which the incentives for an agent in role 1 are optimally traded off against that of an agent in role 2.

[14] The exact cutoff-value of $f_1$for $\hat{h}_2$ and $\hat{e}_2$ is $f_1^{\hat{e}_2} = f_1^{\hat{h}_2} = \frac{1}{2} - \frac{1}{5}\sqrt{5}$, for $\hat{h}_1$ it is given by $f_1^{\hat{h}_1} = \frac{1}{10}\sqrt{5} + \frac{1}{2}$ and for $\hat{w}_H$ it is give by $f_1^{\hat{w}_H} = \frac{1}{5}$.

In order to further interpret this result it is important to note that this exact value of $\hat{f}_1$ does not only depend on the general characteristics of this scenario, but also on the assumptions on the cost and production functions. Therefore, it is instructive to analyze the range of the optimal f_1, namely, that $0 < \hat{f}_1 < \frac{1}{2} \Leftrightarrow \hat{f}_1 < \left(1 - \hat{f}_1\right)$. Hence, the prize is unevenly divided in the optimum and the agent that exerts effort at both stages receives a larger share. This result is intuitively plausible: while f_1 generates incentives directly only at one stage, $(1 - f_1)$ generates incentives directly at both stages. Furthermore, it is optimal to choose f_1 larger than zero because it has a direct incentive effect on the effort exerted by the agent in role 1 at the first stage. As the cost functions for both agents are assumed to be convex and the same at the first stage, it is optimal to set incentives for both agents to exert effort at this stage. But the incentives for an agent in role 1 come at the cost of lower incentives for an agent in role 2 at the first **and** second stage, because the larger f_1 the smaller $(1 - f_1)$. Thus, $f_1 < (1 - f_1)$.

3.3.2 Equal-Randomization-Scenario

Due to the symmetry of the agents at the beginning of the tournament the participation constraints are the same for both agents in both teams at this point in time and are given by PC_r.

$PC_r : \frac{1}{2}(1 - f_1) w_L + \frac{1}{2} f_1 w_L + \frac{1}{4}(1 - f_1) \Delta w + \frac{1}{4} f_1 \Delta w - \frac{1}{2} c(e_r^*) - \kappa(h_r^*) \geq \bar{u}$

As argued above $w_L = 0$ in the optimum and the participation constraints can be ignored. The principal's maximization problem is given by: $\underset{w_H}{max}\ 2E\left[y(e_r^*, h_r^*)\right] - w_H$.

This gives the following conditions for optimal effort provision:

$$e_r^* = \sqrt{g(0)(1 - f_1) w_H}$$

$$h_r^* = \sqrt{g(0)\frac{1}{2} w_H}$$

Therefore, the principal solves

$$\underset{w_H}{max}\, 2\left[2\sqrt{g(0)\frac{1}{2} w_H} + \sqrt{g(0)(1 - f_1) w_H}\right] - w_H$$

which yields the following solutions:

$$\tilde{w}_H = 2g(0)\left(1 + \frac{1}{2}\sqrt{2(1 - f_1)}\right)^2 \tag{14}$$

$$\tilde{h}_r = g(0)\left(1 + \frac{1}{2}\sqrt{2\,(1-f_1)}\right) \tag{15}$$

$$\tilde{e}_r = g(0)\left(\sqrt{2\,(1-f_1)} + (1-f_1)\right) \tag{16}$$

The influence of $g(0)$ on optimal efforts and the prize is positive as it is in the *No-Randomization-Scenario* for the same reason. In contrast to the ambiguous effect of f_1 on optimal efforts and the optimal prize in the *No-Randomization-Scenario*, its effect on these variables is uniformly negative in this scenario. All these variables decrease in f_1 because f_1 has no direct incentive effect for effort provision in this scenario but a higher value of f_1 means a lower value of $(1-f_1)$. This share, in contrast, has a direct and positive incentive effect on efforts. Consequently, $\tilde{w}_H$ decreases in f_1 that means increases in $(1-f_1)$ to compensate the agents for higher effort costs. The negative effect of f_1 on the optimal prize is carried over to $\tilde{h}_r$. As shown above (equation 6) optimal efforts at the first stage are independent from the distribution of shares for given prizes. As the prize is now set optimally from the principal's point of view $\tilde{h}_r$ also depends negatively on f_1.

The expected profit of the principal in the optimum is given by

$$P_{er} = 2E\left[y(\tilde{e}_r, \tilde{h}_r)\right] - \tilde{w}_H = g(0)(3 + 2\sqrt{2\,(1-f_1)} - f_1) > 0 \text{ for } f_1 \leq 1 \tag{17}$$

Because $\frac{\partial P_{er}}{\partial f_1} < 0$ there exists no interior solution for the maximization problem of the principal and his profits are maximized for $\tilde{f}_1 \rightarrow 0$.[15] This result is very intuitive and is driven by the fact that f_1 has a negative incentive effect on efforts exerted at both stages. At the first stage the distribution of the prize between both agents doesn't have a direct incentive effect, because the agents do not know their roles and there exists an equal probability to be in each role. Hence, they do not care about the division of the prize at this point in time. The division influences optimal efforts only indirectly at the first stage through the optimal prize which decreases in f_1. Therefore, also optimal efforts at the first stage decrease in f_1. At the second stage incentives result from $(1-f_1)$. Therefore, the higher f_1 the lower the incentives at the second stage and therefore the smaller the amounts of effort exerted at this stage. This means that f_1 is like a "cost" for the principal. But the principal does not benefit from "paying this cost", which means setting f_1 larger than zero. This does not create any additional incentives but lowers equilibrium efforts by lowering $(1-f_1)$. Hence the principal would always set f_1 as close as possible to zero if he is free to decide about the shares for the agents.

[15] As corner solutions are not excluded the principal would optimally choose $\tilde{f}_1 = 0$.

3.3.3 Comparison of results

In the following the results of both scenarios are compared on the one hand for a exogenously given shares and on the other hand for optimally chosen shares from the point of view of the profit maximizing principal.

For exogenously given shares

The principal may not have the possibility to decide about the division of the prize among the agents but he may be free to choose between the *Equal-Randomization-Scenario* and the *No-Randomization-Scenario* when organizing a tournament. Therefore, it is instructive to compare both scenarios with respect to optimal efforts and prizes for an exogenously given distribution of the prize among the agents which is the same in both scenarios. This analysis leads to the following proposition:

Proposition 2 *The amounts of effort exerted at both stages together and separately are always greater in the Equal-Randomization-Scenario than or equal to the amounts of effort exerted in the No-Randomization-Scenario. Individual efforts at the first stage can be larger, equal or smaller in the Equal-Randomization-Scenario compared to the No-Randomization-Scenario.*

Proof. In the *Equal-Randomization-Scenario* optimal efforts at the second stage are always larger than or equal to efforts in the *No-Randomization-Scenario.*

1. Check equality of $\tilde{e}_2$ and $\hat{e}_2$:
   $g(0)\left(\sqrt{2\left(1-f_1\right)}+\left(1-f_1\right)\right)=g(0)\left[2\left(1-f_1\right)+\sqrt{\left(1-f_1\right)f_1}\right]$
   $\Leftrightarrow 0=\sqrt{1-f_1}(\sqrt{1-f_1}-\sqrt{2}+\sqrt{f_1}) \Leftrightarrow f_1=\frac{1}{2} \vee f_1=1$
2. Check inequality of $\tilde{e}_2$ and $\hat{e}_2$: $0 \gtrless \sqrt{1-f_1}(\sqrt{1-f_1}-\sqrt{2}+\sqrt{f_1})=\hat{e}_2-\tilde{e}_2$

A: $0>\sqrt{1-f_1}(\sqrt{1-f_1}-\sqrt{2}+\sqrt{f_1})$ which means $\tilde{e}_2>\hat{e}_2$, hence we need:

   (a) $\sqrt{1-f_1}>0 \wedge(\sqrt{1-f_1}-\sqrt{2}+\sqrt{f_1})<0$ which means $\sqrt{1-f_1}>0 \Leftrightarrow f_1<1$ and $(\sqrt{1-f_1}-\sqrt{2}+\sqrt{f_1})<0 \Leftrightarrow 0<(1-\sqrt{2 f_1})^2 \Leftrightarrow f_1 \lesseqgtr \frac{1}{2}$ or

   (b) $(\sqrt{1-f_1}-\sqrt{2}+\sqrt{f_1})>0 \wedge \sqrt{1-f_1}<0 \Leftrightarrow 1<f_1$ not feasible because $f_1 \epsilon[0,1]$

B: $0<\sqrt{1-f_1}(\sqrt{1-f_1}-\sqrt{2}+\sqrt{f_1})$ which means $\tilde{e}_2<\hat{e}_2$, hence we need

(a) $\sqrt{1-f_1} > 0 \wedge (\sqrt{1-f_1} - \sqrt{2} + \sqrt{f_1}) > 0$ which has no solution or

(b) $\sqrt{1-f_1} < 0$ and $(\sqrt{1-f_1} - \sqrt{2} + \sqrt{f_1}) < 0$ not feasible.

Hence: $\tilde{e}_2 = \hat{e}_2$ if $f_1 = 1$ or $f_1 = \frac{1}{2}$ and $\tilde{e}_2 > \hat{e}_2$ for $\forall f_1 \epsilon [0,1) \setminus \left\{\frac{1}{2}\right\}$.

All remaining proofs follow accordingly. ■

Optimal efforts exerted at the first stage evolve in opposite directions in the *No-Randomization-Scenario* for $(1-f_1) \neq f_1$: the larger f_1 the larger the amounts of effort exerted by an agent in role 1, but the smaller the amounts exerted by an agent in role 2 and vice versa. In contrast there is no such effect in the *Equal-Randomization-Scenario*. As it was already shown these converse effects in the *No-Randomization-Scenario* result from the trade-off in incentive provision at the first stage that is caused by the division of a given pie among both agents. But this trade-off is not translated one-to-one from the shares received to the amounts of effort exerted. This effect is due to the fact that a change in the share for an agent in role 2 effects efforts at both stages, while a change in the share for an agent in role 1 effects only efforts exerted at the first stage.

Proposition 3 *The optimal prize is higher in the Equal-Randomization-Scenario than in the No-Randomization-Scenario for unequally shared prizes. If prizes are shared equally the optimal prizes are the same in both scenarios.*

Proof. If $f_1 = \frac{1}{2}$ optimal prizes are exactly the same in both scenarios: $g(0)\left[2\sqrt{(1-f_1)} + \sqrt{f_1}\right]^2 = 2g(0)\left(1 + \frac{1}{2}\sqrt{2(1-f_1)}\right)^2 = g(0)(\sqrt{2} + \frac{1}{2}\sqrt{2})^2$. To compare the optimal prizes for $\forall f_1 \epsilon [0,1] \setminus \left\{\frac{1}{2}\right\}$ it is useful to analyze the difference-function of optimal prizes:

$w(f_1) = \tilde{w}_H - \hat{w}_H = \left(2f_1 + 2\sqrt{2}\sqrt{1-f_1} - 4\sqrt{f_1}\sqrt{1-f_1} - 1\right) g(0)$ with $\frac{\partial w(f_1)}{\partial f_1} = -\frac{1}{\sqrt{f_1}\sqrt{1-f_1}}\left(\sqrt{2}\sqrt{f_1} - 4f_1 - 2\sqrt{f_1}\sqrt{1-f_1} + 2\right) g(0)$. $w(f_1)$ takes only positive values for $\forall f_1 \epsilon [0,1]$ (see figure 1 in the appendix) and $\frac{\partial w(f_1)}{\partial f_1} \mid_{f_1 = \frac{1}{2}} = 0$. Hence, the difference between the optimal prizes in both scenarios is minimized for $f_1 = \frac{1}{2}$ and otherwise larger than zero. Hence $\tilde{w}_H > \hat{w}_H$, $\forall f_1 \epsilon [0,1] \setminus \left\{\frac{1}{2}\right\}$. ■

As shown above optimal efforts are higher in the *Equal-Randomization-Scenario* for unequally shared prizes and hence effort costs are also higher in this scenario. Consequently, the optimal prize is higher in order to compensate the agents for higher effort costs.

Taking propositions 2 and 3 together makes it clear that on the one hand higher efforts are exerted in the *Equal-Randomization-Scenario* in the optimum but on the other hand the optimal prize needed to compensate the agents and to create incentives is also larger in this scenario compared to the *No-Randomization-Scenario*. Accordingly, the costs for the principal are also higher.

If the principal is free to choose between both scenarios for an exogenously given division of prizes among the agents when organizing a tournament he will choose the scenario that gives him the highest expected profits.

Proposition 4 *There exists a certain value $\bar{f}_1$ where expected profits are the same for the principal in both scenarios. For $f_1 \neq \bar{f}_1$ expected profits in the Equal-Randomization-Scenario always exceed those in the No-Randomization-Scenario.*

Proof. If $f_1 = \frac{1}{2}$ expected profits are exactly the same in both scenarios $g(0)(3 + 2\sqrt{2(1-f_1)} - f_1) = g(0)(4 - 3f_1 + 4\sqrt{(1-f_1)f_1} = \frac{9}{2}g(0)$.

To show that expected profits are larger with equal randomization for $\forall f_1 \epsilon [0,1] \setminus \{\frac{1}{2}\}$ the difference-function $d(f_1)$ of expected profits is analyzed:

$d(f_1) = P_{er} - P_{nr} = g(0)(3 + 2\sqrt{2(1-f_1)} - f_1) - 4 + 3f_1 - 4\sqrt{(1-f_1)f_1})$ with

$\frac{\partial d(f_1)}{\partial f_1} = \frac{-\sqrt{f_1(1-f_1)}}{f_1(1-f_1)^{\frac{3}{2}}} \left(2\sqrt{1-f_1} + \sqrt{2f_1(1-f_1)} - 2(1-f_1)\sqrt{f_1} - 4f_1\sqrt{1-f_1}\right) g(0)$.

$d(f_1)$ takes only positive values for $\forall f_1 \epsilon [0,1]$ (see figure 2 in the appendix) and $\frac{\partial d(f_1)}{\partial f_1} \mid_{f_1 = \frac{1}{2}} = 0$. Hence the difference between expected profits in both scenarios is minimized for $f_1 = \frac{1}{2}$ and otherwise larger than zero. Hence $P_{er} > P_{nr}, \forall f_1 \epsilon [0,1] \setminus \{\frac{1}{2}\}$. ■

If $f_1 = (1 - f_1) = \frac{1}{2}$, so the prize is divided equally between both team members, it makes no difference for the principal whether the agents know their roles before the tournament starts or whether one agent is randomly selected for implementation after the preparation stage with equal probability for both agents to be chosen. In this case efforts exerted at both stages are exactly the same in both scenarios and so output is the same as well as the optimal prizes. This result is due to the fact that incentives at the first stage in the *No-Randomization-Scenario* are in this case equal to those in the *Equal-Randomization-Scenario*. In dividing the prize equally the incentive structure of the *Equal-Randomization-Scenario* is replicated in the *No-Randomization-Scenario*.

As shown above for $f_1 \neq (1 - f_1)$ the principals optimal expected profits are always larger in the *Equal-Randomization-Scenario* than in the *No-Randomization-Scenario* because the sum of efforts exerted in the first scenario is always greater than in the second one and overcompensates the higher costs the principal faces in the first scenario. Therefore, it is always more profitable for the principal in such a tournament situation not to tell the agents in advance about their roles in the tournament, but to select the agent who finally presents/implements the project randomly after both agents exerted effort for preparation. It is important to keep in mind that these results are derived for risk-neutral agents and are not driven by any other kind of preferences.

For optimally chosen shares

If the principal is free to chose the division of the prizes between the agents in both scenarios he will set the shares such that his expected profits are maximized. As shown above, he will set $\hat{f}_1 = \frac{1}{5}$ in the *No-Randomization-Scenario* and $\tilde{f}_1$ as small as possible, that is $\tilde{f}_1 = 0$, in the *Equal-Randomization-Scenario.*

Comparing these values to those needed to implement first-best, it becomes obvious that a profit maximizing principal would not want to implement first-best efforts because he would have to divide the prize equally between the participating agents to reach first-best. But an equal division is not optimal from his point of view.

Choosing optimal shares yields the following expected profits for the principal in the two scenarios:

$$\hat{P}_{nr} = 5g(0)$$
$$\tilde{P}_{er} = g(0)\left(3 + 2\sqrt{2}\right)$$

The principal's optimal expected profits are in both scenarios linear in $g(0)$, which is - as noted above - a measure for the importance of luck in the tournament. The larger $g(0)$ the smaller the influence of luck in the tournament and the larger the principal's optimal expected profits. This effect is driven by the positive impact of $g(0)$ on optimal efforts. The smaller the influence of luck the larger the efforts exerted by the agents because efforts are the decisive factor in the production and hence to determine the winner and loser of the tournament.

Due to this finding, a principal would prefer to have only little luck in the production in both scenarios. Nevertheless, it is worthwhile to note that at least some noise is needed in order to guarantee the existence of a pure-strategy equilibrium.

Comparing the values for optimal expected profits in both scenarios leads to the following conclusion:

Conclusion 1 *With optimally chosen shares and prizes the principal's expected profits in the Equal-Randomization-Scenario exceed those in the No-Randomization-Scenario for g(0) > 0*

Hence, only if luck is the dominant factor in the determination of the final rank of the team, both scenarios leave no expected profits for the principal. Otherwise the principal's expected profits are always larger in the *Equal-Randomization-Scenario* than in the *No-Randomization-Scenario* for optimally chosen shares. The larger $g(0)$ that is to say the smaller the influence of luck in the tournament, the larger the expected profits in the *Equal-Randomization-Scenario* compared to the *No-Randomization-Scenario.*

4 Discussion

4.1 The agents perspective

Having shown that a profit maximizing principal will generally favor the *Equal-Randomization-Scenario* compared to the *No-Randomization-Scenario* leads to the question whether the same is true for the agents. Analyzing the participation constraints for the agents in different roles in the two scenarios already reveals that both scenarios are not only different from the perspective of the principal but also for the agents. In the *No-Randomization-Scenario* expected utilities for the agents in different roles are different at the beginning of the tournament. Nevertheless, they are the same for an agent in a given role during the whole tournament until the final ranks of the teams are known. In contrast expected utilities are the same for both agents in the *Equal-Randomization-Scenario* at the beginning of the tournament. But their situation changes at the second stage when both agents get to know in which role they are and efforts at the first stage have already been exerted. From this point in time on the situation is the same for the agents that are in corresponding roles in both scenarios and hence their expected utilities are also the same, i.e. different for both agents in a team.

But the optimal tournament contract is determined at the beginning of the tournament and at this point in time the participation constraints for both agents are the same in the *Equal-Randomization-Scenario.* As shown above, the principal will choose $\tilde{f}_1 = 0$ in order to maximize his expected profits. Hence, the expected utility for the agent in role 1 – after getting to know that he is in this role – is just equal to his cost of effort exerted at the first stage and hence negative for $\tilde{h}_1 > 0$. Moreover, it is equal to the rent the agent in this role receives, irrespective of the final rank of his team in the tournament. As optimal efforts of an agent in role 1 are higher in the *Equal-Randomization-Scenario* than in the *No-*

Randomization-Scenario, efforts costs are also higher in the *Equal-Randomization-Scenario.* It follows immediately that the rent for an agent in role 1 is larger in the *No-Randomization-Scenario*, regardless of the final rank of his team. Because of the change in expected utilities – conditional on getting to know ones role – an agent in role 1 ultimately incurs a loss even if his team has won the tournament. Hence, he would never have taken part in the tournament if he knew this before. Nevertheless, from his point of view it is optimal to exert effort at the first stage because he does not know his role at this point in time. Thus, the chance to be chosen at the second stage and win the (whole) final prize creates incentives for both agents to work at the first stage. The principal does not have to set extra incentives – as in the *No-Randomization-Scenario* – through the shares the agents receive. His higher expected profits in the *Equal-Randomization-Scenario* come thus at the cost of the agent that is randomly chosen to be in role 1. As it is only the agent in role 2 that ultimately wins a prize in the tournament, the optimal prize has to be sufficiently large to create incentives for both agents to exert effort at the preparation stage.

For the agent in role 2 the rents are larger in the *Equal-Randomization-Scenario* than in the *No-Randomization-Scenario* if the team wins the tournament. In contrast, they are less negative for an agent in role 2 in the *No-Randomization-Scenario* if the team loses the tournament.

Summing up these findings reveal that only an agent in role 2 in the winning team gets a higher rent in the *Equal-Randomization-Scenario* than in the *No-Randomization-Scenario.* For all the remaining cases the agents receive higher/less negative rents in the *No-Randomization-Scenario.*

4.2 Heterogenous agents

Up to now it has been assumed that the agents in a team are completely homogenous. Relaxing this assumption and allowing for some heterogeneity among the agents with regard to their effort costs (for efforts exerted at both or just at one of the stages) fundamentally changes the previous results that the principal's expected profits are always larger or equal for given prizes in the *Equal-Randomization-Scenario.*[16] The randomization in choosing the agent for imple-

[16]Examplary the cost functions of one agent can be given as before by $c(e_i^k) = \frac{1}{3}(e_i^k)^3$ and $\kappa(e_i^k) = \frac{1}{3}(e_i^k)^3$ and that of the other agent by $c_\delta(h_i^k) = \frac{1}{3}\delta(h_i^k)^3$ and $\kappa_\delta(h_i^k) = \frac{1}{3}\delta(h_i^k)^3$ where $0 < \delta < 1$ or $\delta > 1$.

mentation has an ambiguous effect if the agents' effort costs differ. On the one hand the positive incentive effect discussed in the previous sections remains. On the other hand the random selection of the agent for implementation implies the risk that by chance the "wrong" agent, e.g. the one with higher effort costs, is chosen to exert effort at both stages. It is very intuitive and can also be easily shown that an agent with higher/lower effort costs will optimally exert less/more effort compared to the other one. In order to maximize his expected profits the principal will optimally choose the agent with lower effort costs to exert effort at both stages in the *No-Randomization-Scenario.* But as this is not possible in the *Equal-Randomization-Scenario* the principal has to take the risk that by chance the agent with higher effort costs is chosen for implementation. Consequently his expected (and finally realized) profits can be smaller in the *Equal-Randomization-Scenario* than in the *No-Randomization-Scenario.*

It can be shown that – for equally divided prizes – the principal's expected profits are always larger in the *No-Randomization-Scenario* than in the *Equal-Randomization-Scenario* if it is assumed that the agents' cost functions are different – for one agent given by: $\kappa(h_r^k) = \frac{1}{3}(h_r^k)^3$and $c(e_r^k) = \frac{1}{3}(e_r^k)^3$ and for the other by: $\kappa_\delta(h_i^k) = \frac{1}{3}\delta(h_i^k)^3$ and $c_\delta(h_r^k) = \frac{1}{3}\delta(h_r^k)^3$ where $\delta > 1$ – and the principal chooses the agent with lower effort costs for implementation in the *No-Randomization-Scenario.*[17]

5 Conclusion

The introduction of randomization into a tournament between teams with a two-stage production process generates higher expected profits for the principal than a tournament without randomization. The key for this findings is a trade-off in incentive provision in the *No-Randomization-Scenario* which is absent in the *Equal-Randomization-Scenario.* This trade-off occurs because the incentives for the agents in a team are among other things, provided through the shares they receive from the prize finally won. This prize is zero-sum and the share one agent receives isn't independent from the share the other agent receives. Therefore, incentives are interrelated as well. In the *No-Randomization-Scenario* - where agents know their roles before the tournament starts - incentives generated by

[17]This result holds also if the agents differ only with respect to their cost functions for implementation and the principal choses the agent with lower effort costs for implementation in the *No-Randomization-Scenario.*

the shares work in opposite directions for both agents. It is intuitively clear that the higher the share for one agent the higher his incentives to exert effort but at the same time the lower the remaining share for the other agent, and accordingly his incentives. When selecting the agent for implementation randomly after the preparation stage the principal can overcome these reverse effects. As none of the agents knows his role at the first stage the shares given to the agents do not directly influence incentives at the preparation stage.

An interesting and economically relevant extension of this model would be the introduction of risk-averse instead of risk-neutral agents. Interpreting risk aversion as a dislike to come in a situation where one has to present (implementation stage) a poorly prepared project (preparation stage) should even strengthen the results derived in this model. The effort decision of an agent who knows from the beginning of the tournament that he will not have to present the project finally isn't affected by this risk. In contrast, both agents face this risk when working on the preparation if the agent for presentation is randomly selected. Consequently, in this case one would expect that these agents will work even harder at the first stage than risk-neutral agents in order to minimize the risk of presenting an insufficiently prepared project.

Appendix

Proof of the optimality of equal-randomization for given prizes:

Assume that the probability for one of the agents in a team to be chosen to exert effort at both stages is given by q. Hence, his probability of being in role 1 is $1-q$ ($q \neq 1-q$). Correspondingly, the probability of the other agent in the team to be in role 2 is $1-q$ and that of being in role 1 is q. Furthermore, it is assumed that q is known by the principal and the agents before the tournament starts. Expected utilities of both agents are therefore given by:

$$EU_{2q} = q(1-f_1)w_L + (1-q)f_1 w_L + qp(1-f_1)\Delta w + (1-q)\, pf_1 \Delta w - qc(e^A_{2q}) - \kappa(h^A_{2q})$$

and

$$EU_{2(1-q)} = (1-q)(1-f_1)w_L + qf_1 w_L + (1-q)p(1-f_1)\Delta w + qpf_1\Delta w - (1-q)c(e^A_{2(1-q)}) - \kappa(h^A_{2(1-q)})$$

Where EU_{2q} denotes the expected utility of the agent who is chosen with probability q to be in role 2 and $EU_{2(1-q)}$ denotes the expected utility of the agent that is chosen to be in role 2 with probability $(1-q)$. Accordingly, h^A_{2q} (e^A_{2q}) denotes the effort at the first (second) stage of an agent in team A who is chosen with probability q to be in role 2 and $h^A_{2(1-q)}$ ($e^A_{2(1-q)}$) that of an agent in team A who is chosen with probability $1-q$ to be in role 2.

Assuming the existence of an equilibrium in pure strategies, it will be described by the following first order conditions:[18]

$$c'^{-1}\left[(1-f_1)\Delta w g(0)\right] = e^*_{2q} = e^*_{2(1-q)}$$

$$\kappa'^{-1}\left[(q(1-f_1) + (1-q)f_1)\,\Delta w g(0)\right] := h^*_{2q}$$

$$\kappa'^{-1}\left[((1-q)(1-f_1) + qf_1)\,\Delta w g(0)\right] := h^*_{2(1-q)}$$

The condition for optimal effort provision at the second stage is independent from q and hence the same as in the other scenarios as soon as an agent knows that he is in role 2.[19] In contrast efforts at the first stage depend on the probabilities to be chosen for the different roles. So the amounts of effort exerted

[18] The conditions and assumptions for the existence of a pure strategy equilibrium in this case are analog to those quoted above. The first order conditions for effort provision at the second stage are given on the supposition that an agent know that he is in role 2.

[19] Before knowing in which role an agent is optimal efforts at the second stage are given by $[q(1-f_1)\Delta w g(0) + f_1(1-q)\Delta w g(0)] = c'(e^*_{2q})$ and $c'(e^*_{2(1-q)}) = [(1-q)(1-f_1)\Delta w g(0) + f_1 q \Delta w g(0)]$.

at the first stage differ for $q \neq 1-q$. Assuming again that the cost functions are given by $c(e_2^k) = \frac{1}{3}(e_2^k)^3$ and $\kappa(h_i^k) = \frac{1}{3}(h_i^k)^3$ leads to the following sum of efforts at the first stage: $h_{2q}^* + h_{2(1-q)}^* = \sqrt{(q(1-f_1)+(1-q)f_1)\,\Delta w g(0)} + \sqrt{((1-q)(1-f_1)+qf_1)\,\Delta w g(0)}$ where Δw and $g(0)$ are given. As their values do not influence the optimal value of q the following function has to be analyzed: $H(q,f_1) = \sqrt{(q(1-f_1)+(1-q)f_1)} + \sqrt{((1-q)(1-f_1)+qf_1)}$.

$\frac{\partial}{\partial q}H(q,f_1) = -\frac{1}{2}(2f-1)\frac{\sqrt{2fq-q-f+1}-\sqrt{f+q-2fq}}{\sqrt{f+q-2fq}\sqrt{2fq-q-f+1}}$ where $\sqrt{f+q-2fq} > 0$ and $\sqrt{2fq-q-f+1} > 0$ for $q, f_1 \in (0,1)$. Hence, $H(q,f_1)$ is maximized if $q = \frac{1}{2}$ and $f_1 \in [0,1]$ or if $f_1 = \frac{1}{2}$ and $q \in [0,1]$. That means, for given prizes the sum of efforts exerted at the first stage is maximized if there exists an equal chance for both agents to be chosen for effort provision at the second stage. If prizes are divided equally between both agents, that is $f_1 = 1 - f_1 = \frac{1}{2}$, efforts are independent from q.

Figures

The value of $g(\cdot)$ influences only the shape of the difference-functions but neither their minimum, nor their curvature. Therefore they are displayed for a value of 1.

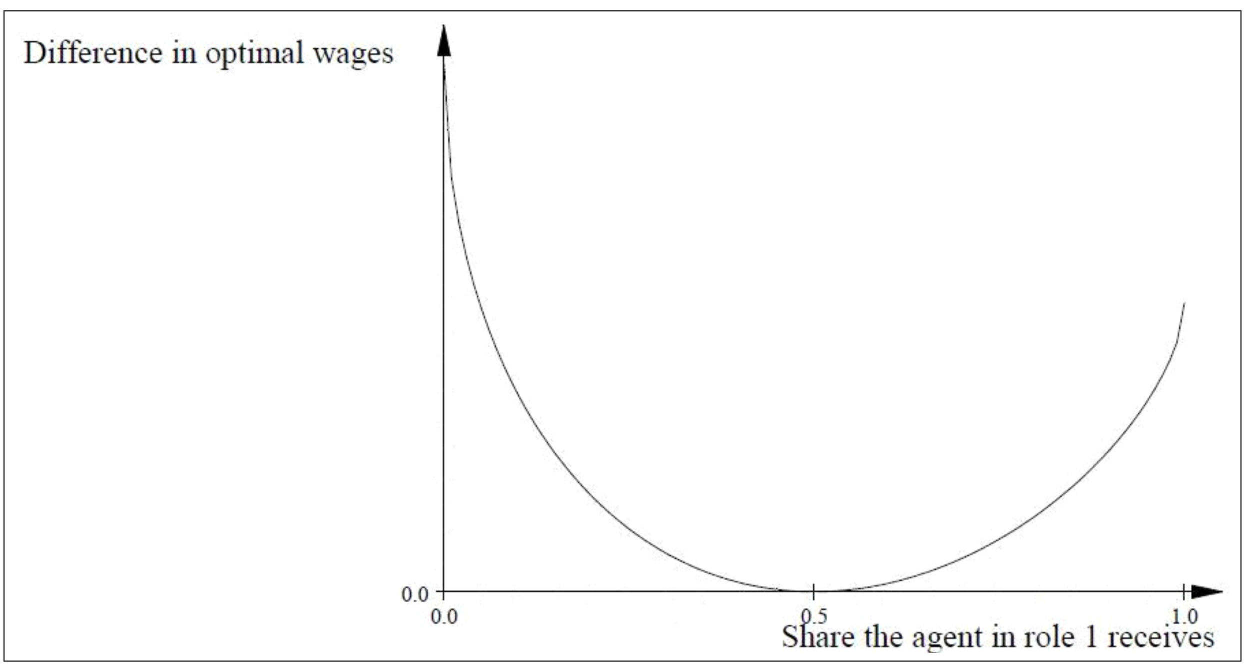


Figure 1: Difference in optimal prizes between both scenarios ($\tilde{w}_H - \hat{w}_H$)

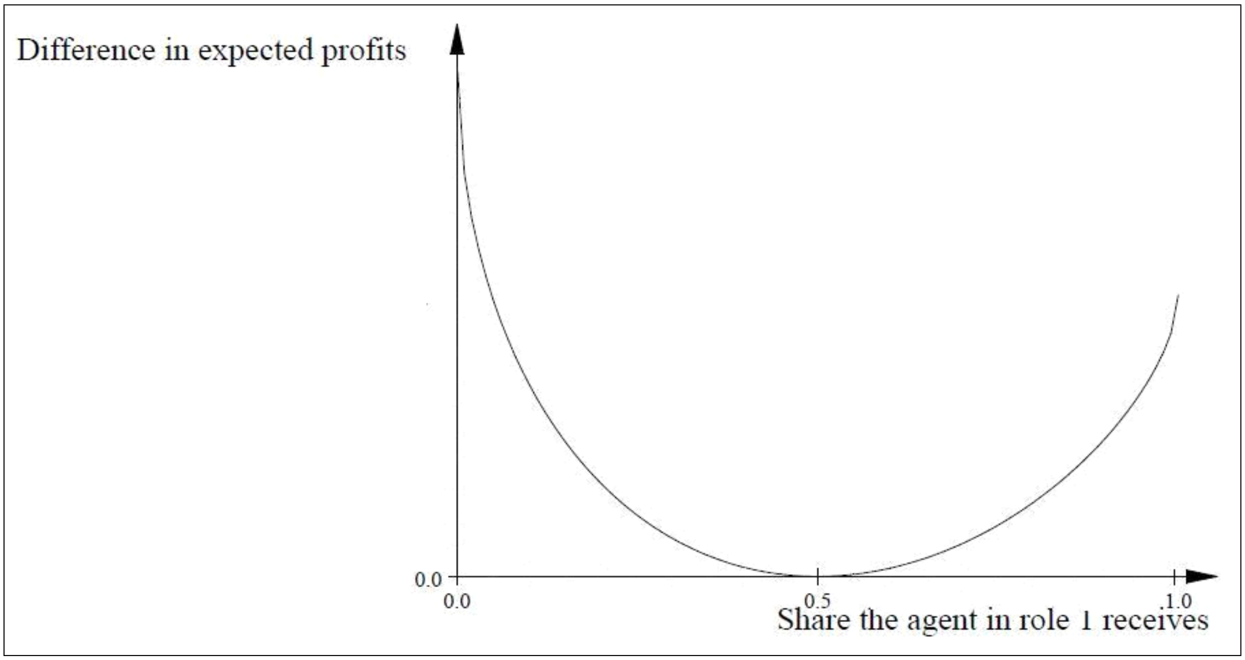


Figure 2: Difference in expected profits between both scenarios ($\mathrm{P}_{er} - P_{nr}$)

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