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Information Disclosure in Innovation Contests

by

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Abstract

In innovation contests, the progress of the competing firms in the innovation process is usually their private information. We analyze an innovation contest in which research firms have a stochastic technology to develop innovations at a fixed cost, but their progress is publicly announced. We make a comparison with the case of no information revelation: if the progress is disclosed, the expected profit of the firms is higher, but the expected profit of the sponsor is lower. Additionally, we show that firms may voluntarily reveal their information.

JEL: O32, D82, D72
Keywords: contest; innovation; information revelation

1 Introduction

Contests have been used to stimulate research in a variety of contexts: from refrigerators over computer programs to aerospace research. To win the contest, only the best final innovation of all competitors matters. Nevertheless, if the progress of the participating firms is publicly known, intermediate stages of the research process already reveal interim leaders. This knowledge influences future research efforts. It is thus important to identify the impact of intermediate information revelation both from the participants' and from the contest designer's viewpoint. Intuitively, information disclosure has two major opposing effects on research effort. On the one hand, the publication can serve as a kind of positive coordination device for the participants, prohibiting excessive research: a firm will decrease research effort due to the observation of a very valuable or a worthless innovation made by her opponent. On the other hand, the additional information can also expand research effort: if the competitor of a firm turns out to unexpectedly have a slightly better innovation, a firm might discover the need for an improvement.

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From the firms' perspective, the disclosure policy leading to lower research costs is preferable. In contrast, the contest designer cares about the value of the best innovation. In this paper, we use a contest model with multiple stochastic research opportunities to compare two settings: obligatory intermediate information revelation by the firms opposed to keeping their progress secret. Both the firms' and the contest designer's view are analyzed. Furthermore, we study the possibility of endogenous information revelation.

Examples for information disclosure in contests occur in different areas. When a new drug needs to be developed, different pharmaceutical firms conduct research. To test the effectiveness of a new drug – and thus its chances of beating the rivals' developments – firms conduct clinical trials. These trials can be publicly registered in a trial registry like clinicaltrials.gov, giving also the opportunity to post a short result summary. Specifically, for drugs, biologics and medical devices regulated by the US Food and Drug Administration, U.S. Public Law forces sponsors of clinical trials to post results on their effectiveness in such a trial registry[1]. Additionally, some voluntary disclosure of research results takes place in the trial registries and peer-reviewed journals. Similarly, the performance of participants in the Netflix Prize (www.netflixprize.com) can be seen on a public leaderboard. Netflix, a popular video renting company, pays a prize of $1,000,000 for a new algorithm to predict the movie preferences of a user based on the past ratings he submitted. The accuracy of an algorithm is measured by a single number, which can only be ascertained by submitting the algorithm to the website. Interestingly, the website publishes the best result of each contestant automatically.

To capture the influence of intermediate information revelation on the participants' incentives to innovate, we compare two settings in the framework of an innovation contest, which only differ in the treatment of intermediate information. We model an innovation contest in the spirit of Taylor (1995): two firms have the possibility to make stochastic innovations at a fixed cost. Firms can develop up to two independent innovations. They decide sequentially whether they innovate or not. As it is common in contests, only the best of all innovations wins a fixed prize. The main decision problem of a firm appears after the first innovation is made: how good are the chances to beat the other firm with the current innovation? Should a second one be developed? Of course, information on the quality of the opponent's innovation has significant impact on the firm's decision. Hence, we compare two different versions of the model: in the benchmark setting, following Taylor (1995), no information about the first innovations is revealed. In our basic setting, intermediate information disclosure is mandatory. We extend it to include the possibility of voluntary information revelation, the main focus of this paper.

In most of the paper, a key assumption is the independence of innovations. It is motivated by interpreting different innovations as substantially different ideas that have to be explored independently. Specifically, we treat one innovation as fully developed and neglect small improvements due to extended research on an already completed innova-

[1] see e.g. Groves (2008)

tion[2]. Consequently, in case of information revelation, the model does not leave room for spillovers between the firms. In a sense, spillovers are assumed to be smaller than the difference between firms' innovation values and would thus have no effect on the contest winner anyway. This is also in line with the revelation policy in both examples. There, only simple summary statistics of the contestants' performances are publicly available. Hence, competitors know how good their opponents are – but they do not know how they did it, so no direct spillovers are possible. Furthermore, in an extension of the basic model, we use a different interpretation of a multi-round innovation contest and model the innovation process as an improvement of a single idea over several stages.

Surprisingly, only very mild assumptions on the distribution of innovation values are needed for the analysis of the basic model, which has two firms and two periods. In fact, the results essentially turn out to hold true independent of the specific functional form of the distribution of innovation values. Instead, the relative size of the final prize to the cost of developing an innovation is the most important parameter for the firms' incentives. The analysis of the basic model with mandatory information disclosure shows that both firms innovate in the first period in case the prize is not too low compared to the costs of developing an innovation. Then, second-period equilibrium behavior depends on the value of the first-period innovation according to two cutoffs: if one firm has an innovation value in the high range, the leading firm is confident to win, while the probability for the following firm to develop something better is too low compared to the costs. Hence, both stop innovating. Similarly, if the highest innovation is in the intermediate range, only the follower continues to innovate – and if both innovations are below the lower cutoff, both firms continue. We show that the total number of innovations – and thus the research costs – is lower in this equilibrium compared to the equilibrium with secret innovation values. Thus, there is a coordination effect which is favorable for the firms: a contest with information disclosure leads to lower expected research costs and thus a higher expected payoff for the firms. Yet, this does not necessarily mean that the prize sponsor prefers the setting without information disclosure: he cares about the expected value of the highest innovation, which is different from the total number of innovations. As firms stop innovating when they observe a high innovation value, the coordination effect could be strong enough to compensate for the lower total number of innovations. We show that this is not the case if the prize/cost-ratio is sufficiently high. Consequently, the prize sponsor gets a higher expected innovation in the setting without information disclosure. If a prize sponsor is able to enforce this secrecy, he should thus do so. However, if he does not do so, firms might be willing to voluntarily reveal their first-period value. We pursue this question by modeling voluntary disclosure in two different ways: in the first version, the firms decide in an ex ante-game whether they are going to reveal after the first period or not. In the second version, the decision to disclose is delayed until firms

[2] Another way to think about independent innovations is the proof of a theorem: one approach might fail and its a completely different one that will lead to a success.

learn their first-period innovation value. In both cases, it turns out that there is essentially a unique equilibrium in which both firms disclose. Continuing this train of thought, the voluntary revelation has consequences for the contest designer: if he chooses the size of the prize optimally, he should choose it with respect to the setting where information is revealed in case he does not prevent voluntary disclosure. We also prove the existence of an equilibrium with voluntary revelation in case there are n firms or m periods.

In the extension with improving innovations, given mandatory information revelation the decision whether to continue research in the second period does not only depend on the leader's value, but also on the value of the runner-up. The cutoffs identified for the basic model still exist qualitatively but change in their quantitative value. Particularly, the incentives to continue research increase for the firms, as it becomes more likely that the runner-up can produce a better second-period innovation. Nevertheless, the main result of this paper carries over to this setting with improving innovations: there is an equilibrium where firms voluntary reveal their own value.

This paper extends the analysis of research tournaments by Taylor (1995). In his model with a secret innovation process, there is a unique symmetric equilibrium in which firms continue to innovate if their best innovation value does not exceed a certain threshold. Due to the information disclosure, which we introduce in our version of the model, a second cutoff value arises – the contestants are able to coordinate. Of course, the approach followed by Taylor (1995) is not the only one to model research contests. For example, Che and Gale (2003) find the optimal contest to be an auction given a deterministic research technology, Schoettner (2008) builds on the famous model by Lazear and Rosen (1981) to show that given a stochastic innovation technology, fixed-prize contests may in fact be superior to a first-price auction. Also building on Taylor (1995), Fullerton et al. (2002) study auction-style research tournaments. Finally, Baye and Hoppe (2003) show that there is a strategic equivalence between different models of rent-seeking, patent races and innovation contests.

The idea of intermediate revelation of research results is also studied by Gill (2008) in the context of patent contests with exogenously given leader and follower. In his model, research is a two-stage process where both steps are necessary to develop a single innovation. We use value distributions similar to his distributions in our extension with improving innovations. In Gill's model, the leader decides whether or not to disclose his performance after the first stage. Then, the follower may choose to drop out after the first stage. Whether or not the leader discloses depends on the research costs. By contrast, in our model leader and follower are endogenously determined, as multiple innovations can be developed. Furthermore, in Gill (2008) the patent winner is determined randomly, while in our model the best innovation wins for sure. In Aoyagi (2010) all information on intermediate performance is controlled by the contest designer. Related to our model, performance is stochastic. Furthermore, it is additive over the two rounds, while we mostly consider multiple independent innovations. The optimal feedback policy to the participants regarding this information is derived – it depends on the shape of the cost function whether a no-feedback or a full-feedback policy is optimal. In a related paper, Gershkov and Perry (2009) study the design of midterm reviews. Given a fixed prize, it is always optimal to have such a review, if the results of intermediate and final review are optimally aggregated.

This paper also connects to the literature on multiple-round contests. In Konrad and Kovenock (2009), contestants have to win several component contests, modeled as all-pay auctions, to win the overall prize. Contrary to our model, the follower is not fully discouraged from continuing the contest even if he is far behind. Moldovanu and Sela (2006) investigate how to split contestants over sub-contests where only the winners continue to compete. In Yildirim (2005), building on work by Dixit (1987), heterogeneous participants can split their effort over two rounds with observable first-round effort. Similar to one result in this paper, information disclosure can be endogenized by an ex ante game: agents can choose between non-observable effort (which equals one-shot play there) or two-round effort with intermediate revelation. In equilibrium they decide to reveal effort. In our model, we also get voluntary revelation – however, it is revelation of (stochastic) innovation values and not of effort. Furthermore, our model does not boil down to one-shot play in case of secret intermediate results.

Finally, an experimental study of information disclosure is provided by Ludwig and Lunser (2008). They compare two settings with and without intermediate information release, where equilibrium play is not affected by the information structure. Nevertheless, subjects in the experiments behave differently if they observe their opponent's effort.

The paper is organized as follows: the basic model and equilibrium behavior with information disclosure is presented in Section 2. We compare it to the benchmark case without disclosure in Section 3. In Section 4 we endogenize information revelation. Extensions with a second innovation that improves the first one and with n firms and m periods are considered in Section 5. Finally, we conclude in Section 6. Proofs can be found in the Appendix.

2 The Model and Equilibrium Derivation

We consider two risk-neutral research firms, $i = 1, 2$. They compete in an innovation contest to win a fixed prize $p > 0$. Firms are assumed to know the prize sponsor's utility function over research outcomes. Both firms have an innovation technology similar to Taylor (1995): research is modeled as drawing an innovation x out of a probability distribution F with strictly positive density f. F is defined on $[0, b]$ with $b \leq \infty$. Each innovation draw is associated with a cost of $c > 0$ for each firm. Firms are not capital constrained. There are two periods $t = 1, 2$ in which firms may innovate. Innovation values x_i^t are independent across periods and firms. For each firm, only the best draw $(\max\{x_i^1, x_i^2\})$ is relevant for the contest. The firm with the highest draw wins the contest and the prize of p. Ties are randomly broken. We assume that innovations that do not win have a value of zero outside the contest, so that loosing innovations cannot be sold afterwards. In contrast to Taylor (1995), in the basic version of our model we assume in the spirit of Yildirim (2005) that first-period innovations become common knowledge after both firms have made their decision whether to conduct research or not, and have taken their draw.

We first analyze equilibrium behavior of the two firms. We look for subgame perfect Nash equilibria by backward induction and thus start with the second period. First note that for $p < c$ both firms would make a loss from conducting research. Thus, both do not conduct any research (neither in the first nor in the second period). Consequently, we focus on the case $c \leq p$. Additionally, we will narrow the reasonable prize/cost combinations further down later.

2.1 Second Period

Suppose at least one firm has taken a draw in the first period, such that one of the two firms has taken the lead, $x_H^1 > x_L^1 \geq 0$. H stands for the firm with the *higher* first round innovation (the leader) and L for the firm with the *lower* innovation (the follower). We calculate best responses:

If the follower does not continue to innovate, it is a best response for the leading firm to stop innovating as well – she will win in any case.

So suppose now the firm with the higher value does not draw again. Then, the firm with the lower value wants to continue if the following condition holds:

$$P\left(x_L^2 > x_H^1\right) p - c \geq 0 \iff \left(1 - F\left(x_H^1\right)\right) p - c \geq 0 \iff F\left(x_H^1\right) \leq 1 - \frac{c}{p}.$$

This inequality defines a threshold x^* indicating an innovation high enough to make all firms stop research. x^* solves the following equation:

$$F\left(x^*\right) = 1 - \frac{c}{p}. \tag{1}$$

Then, if some $x > x^*$ is drawn by any of the two firms, the contest stops immediately and no new research will be conducted in the second round: the follower has no incentive to draw again if the leader has already drawn such a high innovation. Then, the leader will obviously not draw again as well.

Now consider the case $x_H^1 \leq x^*$, such that the firm with the lower value wants to draw again if the leader does not. What is the best response of the leader against the drawing follower? The firm with the higher value wants to draw again as well if the following condition holds:

$$\begin{aligned} & \left[P\left(x_H^2 > x_L^2 > x_H^1\right) + P\left(x_H^1 > x_L^2\right)\right] p - c \geq P\left(x_H^1 > x_L^2\right) p \\ \iff & \frac{1}{2}\left(1 - F\left(x_H^1\right)\right)^2 p - c \geq 0 \\ \iff & F\left(x_H^1\right) \leq 1 - \sqrt{2\frac{c}{p}}. \end{aligned}$$

This inequality defines a threshold $\bar{x}$ making both firms innovate again if there is no innovation above it. $\bar{x}$ solves

$$F(\bar{x}) = 1 - \sqrt{2\frac{c}{p}} \tag{2}$$

and note that $\bar{x} < x^*$. What is the best response of the follower against a leader drawing again for $x_H^1 \leq \bar{x}$? Drawing again is a best response according to the following condition:

$$\begin{aligned} & P\left(x_L^2 > x_H^1, x_H^2\right) p - c \geq 0 \\ \iff & \left[\frac{1}{2}\left(1 - F\left(x_H^1\right)\right)^2 + \left(1 - F\left(x_H^1\right)\right)\left(F\left(x_H^1\right)\right)\right] p - c \geq 0. \end{aligned} \tag{3}$$

We know that

$$\frac{1}{2}\left(1 - F\left(x_H^1\right)\right)^2 p - c \geq 0$$

because $x_H^1 \leq \bar{x}$. Hence, (3) is fulfilled. Consequently, the follower wants to draw again in the second round as well. This is intuitive: the leader already has an advantage after the first round, so incentives for the follower to draw again are even higher.
We summarize our findings in the following proposition:

Proposition 1 *Given first-period innovations $x_H^1 > x_L^1$, there are the following second-period equilibrium strategies:*

- *If $x_H^1 > x^*$ both firms stop their research effort and the contest ends after the first period.*
- *If $x^* \geq x_H^1 > \bar{x}$ only the follower conducts research in the second period.*
- *If $\bar{x} \geq x_H^1$ both firms conduct research in the second period.*

Note that for small prize values $p < 2c$ we get $\bar{x} < 0$, thus, the leader will never draw again in the second period. Furthermore, the proposition implies that there are no mixed equilibria:

Corollary 2 *Given first-period innovations $x_H^1 > x_L^1$ there is no second-period equilibrium in which players mix at values other than $\bar{x}$ and x^*. Thus, the equilibrium in Proposition 1 is almost everywhere unique.*

It follows immediately from Proposition 1 that a leading firm with $x_H^1 > \bar{x}$ does not do any research irrespective of the following firm's behavior and is thus playing a pure strategy. Similarly, a follower with $x_L^1 < \bar{x}$ will always do research. Thus, neglecting the cutoff values, there is always at least one firm playing a pure strategy, with a pure best reply by the other firm according to Proposition 1.
Let us now consider the case that both firms did not innovate in the first period, which is important for the calculation of first-period equilibrium behavior.

Proposition 3 *Suppose both firms did not innovate in the first period. Then, there are the following second-period equilibrium strategies:*

- *If $p \leq 2c$, there is an equilibrium where both firms do not conduct any research in the second period.*
- *If $p \geq 2c$ there is an equilibrium where both firms conduct research in the second period.*

The proof is given in the Appendix. Note that if at least one firm takes a draw in the first period, a tie appears with zero probability, and thus second-period equilibrium play is almost everywhere unique in the sense of Corollary 2 for almost all possible first-period realizations. For this reason, we can safely skip the calculation of equilibria in case $x_1^1 = x_2^1$: this case will appear with zero probability given any first-period play and we will thus not need it in future calculations.

2.2 First Period

The first-period pure-strategy equilibria can be now derived, taking into account second-period equilibrium play. As the main focus of this paper is on information revelation after the first period, we are especially interested in the conditions under which both firms start innovating in the first period. If they do not innovate in the first period, information revelation is only of minor interest. It turns out that the size of the prize compared to the innovation costs is the crucial parameter for first-period innovation to take place. We make use of the following short notations: $r := \frac{c}{p}$ and $s := \sqrt{2r}$.

Proposition 4 *Let v^* be the solution of the following equation:*

$$\frac{1}{6} - v^* + \frac{2}{3} v^* \sqrt{2v^*} - \frac{1}{2} (v^*)^2 - \frac{1}{2} (v^*)^3 = 0$$

Then, $v^* < \frac{1}{2}$ and in the first period, we get the following pure-strategy equilibrium behavior with firms continuing in the second period as described in Proposition 1:

- *For $r > \frac{1}{2}$ both firms do not conduct any research in the first period.*
- *For $0 < r < v^*$ both firms conduct research in the first period.*
- *For $\frac{1}{2} \geq r > v^*$ equilibrium behavior is asymmetric – one firm conducts research, the other does not.*

Proof See Appendix. □

Numerically, v^* is given by $v^* \approx 0.2428$ and by Proposition 4 both firms conduct research if $\frac{c}{p} = r < 0.2428$. This means that a prize value of $p \approx 4c$ is high enough to ensure the maximum amount of research in the first period.

The proposition shows that if the prize is too low compared to the costs, both firms will invest neither in the first nor in the second period. Additionally, there are two pure-strategy equilibria if r takes intermediate values. Furthermore, there is a more prominent symmetric mixed strategy equilibrium in this case as well, which we do not calculate here because we focus on $r < v^*$ in the following: we are interested in information revelation with firms in fact doing research in the first period. This problem has no meaning if the setting is such that firms do not have full incentives to invest in the first period – and these incentives are already given at a very reasonable prize level. There is thus no need to consider the mixed equilibrium here.

3 Comparison with No Information Release

In this section, we compare the setting with information release after the first period, which we just analyzed, with the setting known from the literature (Taylor 1995) where information is kept secret after the first period. We want to find the preferred setting for both the firms and the contest designer. First, we compare the settings from the perspective of the firms, then we turn to the contest designer.

3.1 Firms' Perspective

To analyze the firms' perspective, we compare the expected number of innovation draws in the setting with information revelation to no information revelation after the first period – firms prefer the setting with lower research costs, which means less innovation draws in this context. The first step is to calculate the expected number of draws $d_R(r)$ in the equilibrium with information release, given that both firms do research in the first period.

Proposition 5 *Given $r < v^*$ the expected number of draws in equilibrium fulfills $d_R(r) = 4 - 2s + r^2$.*

Proof See Appendix. □

We now come back to the setting of Taylor (1995), where no information is released. He shows that there is a unique equilibrium in which firms play a stopping strategy with stop value z: they take draws as long as they do not have an innovation that exceeds z and stop as soon as an innovation exceeds z. However, Taylor does not calculate the z explicitly but characterizes it implicitly. We rewrite his implicit characterization to make it suitable for our purposes. According to Proposition 2 in Taylor (1995), z is the solution of the following equation:

$$0 = p \int_z^b \left[F^2(z) + \left(1 - F^2(z)\right) \frac{F(x) - F(z)}{1 - F(z)} - F^2(z) \right] dF(x) - c.$$

Calculating the integral, this can be rewritten as follows:

$$
\begin{aligned}
0 &= p\left(1+F(z)\right)\left[\int_z^b F(x)f(x)dx - F(z)\int_z^b f(x)dx\right] - c \\
&= p\left(1+F(z)\right)\left[\frac{1}{2}\left(1-F^2(z)\right) - F(z)\left(1-F(z)\right)\right] - c \\
&= p\frac{1}{2}\left(1+F(z)\right)\left(1-F(z)\right)^2 - c \\
\Longleftrightarrow \qquad 0 &= \left(1+F(z)\right)\left(1-F(z)\right)^2 - 2r \qquad (4)
\end{aligned}
$$

The first line follows by factoring out $1+F(z)$ and changing the notation of the integration. The second line uses integration by parts. Unfortunately, the explicit solution of this equation is quite messy. The following lemma gives a feeling of the size of z.

Lemma 6 *For $p > 2c$ the stop value in the setting without information release is between the two thresholds of the setting with information release, $\bar{x} < z < x^*$.*

Proof See Appendix. □

We make a comparison between the setting of Taylor (1995) and our setting. As the expected number of innovations a firm makes represents her cost, we compare the number of draws the firms take in expectation in each setting. For our case with information revelation we already calculated the expected number of draws ($d_R(r)$, Proposition 5). For the setting without information revelation, the expected number of draws can be written as $d_{NR}(r) = 2(1+F(z))$ (a firm is drawing again if and only if the first period value did not exceed z, this happens with probability $F(z)$). z is implicitly defined by (4) for a given r.

Proposition 7 *Considering $0 < r < v^*$, the expected number of draws $d_{NR}(r)$ in case no information is revealed after the first period is larger then the expected number of draws $d_R(r)$ in case information is revealed, $d_{NR}(r) > d_R(r)$.*

Proof See Appendix. □

We immediately get the following corollary, as both players win in expectation $\frac{1}{2}p$ in equilibrium in both settings, but have lower costs in the setting with information disclosure because they take less draws:

Corollary 8 *For $0 < r < v^*$, both research firms prefer the setting with information disclosure over the setting without information disclosure.*

Note that $r < v^*$ is exactly the range of r-values guaranteeing research draws by both firms in the first period. This is the range we focus on as revelation decisions after the first period are only interesting if firms do innovate in the first period.

3.2 Designer's Perspective

From the prize sponsor's perspective, a higher number of innovation draws is in principle favorable, as more draws suggest a higher expected final prize. However, it is not obvious that this relationship really holds in this context: draws are taken conditional on already realized innovations. Thus, if a draw is not taken, a good innovation has already been made. But the equilibrium decision rules whether another draw is taken differ between the two settings. Thus, a higher number of draws is an indicator for a higher expected final innovation, but does not allow a sure conclusion.

The key to the comparison from the designer's perspective is the highest expected innovation generated by the two different settings. The designer prefers the setting yielding the higher one.

To calculate the highest expected innovation for the two settings, we need the respective distribution functions of the highest innovation. In the setting without information release, the two firms are innovating independently. Let Φ be the distribution of the highest innovation for a single firm. Then, the joint distribution is given by Φ^2. Using the result by Taylor (1995) regarding Φ, we get

$$\Phi^2(x) = \begin{cases} F^4(x) & \text{if } x \leq z \\ \left(F(x) - F(z) + F(z)F(x)\right)^2 & \text{if } x > z \end{cases}$$

For the setting with information revelation, the two firms do not innovate independently. The distribution Ψ of the joint highest innovation has the following structure, given the equilibrium play of the two firms – they both draw in the first period as we assume $r < v^*$:

$$\Psi(x) = \begin{cases} F^4(x) & \text{if } x \leq \bar{x} \\ A & \text{if } \bar{x} < x \leq x^* \\ B & \text{if } x^* < x \end{cases}$$

Denote the highest innovation in period j by $x^j_{(1)}$. Then, A and B are given according to

$$\begin{aligned} A &= P\left(x^1_{(1)} < \bar{x}\right) P\left(x^2_{(1)} < x\right) + P\left(\bar{x} < x^1_{(1)} \leq x\right) P\left(x^2_{(1)} < x\right) \\ &= F^2(x)F^2(\bar{x}) + F(x)\left(F(x)^2 - F(\bar{x})^2\right) \\ B &= P\left(x^1_{(1)} < \bar{x}\right) P\left(x^2_{(1)} < x\right) + P\left(\bar{x} < x^1_{(1)} < x^*\right) P\left(x^2_{(1)} < x\right) + P\left(x^* < x^1_{(1)}\right) \\ &= F(x)^2F(\bar{x})^2 - F(x)F(\bar{x})^2 + F(x)F(x^*)^2 + F(x^2) - F(x^*)^2 \end{aligned}$$

Given these distribution functions, we can calculate which setting provides the higher expected innovation – no information revelation is preferred if the following condition holds:

$$\int_0^b 1 - \Phi^2(x)dx \geq \int_0^b 1 - \Psi(x)dx \iff \int_0^b \Phi^2(x) - \Psi(x)dx \leq 0. \tag{5}$$

Note that, different to the results from the firms' perspective, it depends on F whether condition (5) is fulfilled or not. This is because the designer cares about the absolute value of the innovations, while the firms care about their relative ranking. Additionally, the size of r is crucial for the profitability of the settings. We provide a bound on r such that (5) is fulfilled independent of F. This bound is called v':

Theorem 9 *The expected value of the highest innovation is larger in the setting without information revelation if $r < v'$ holds. Then, this setting is preferred by the prize sponsor.*

The derivation of v' can be found in the Appendix. It basically uses a stochastic dominance argument: the integrand of the integral on the left-hand side of (5) is shown to be negative on the whole interval $[0, b]$ when $r < v' = 0.1647$. However, this bound is in general not binding, as the solution to (5) (with equality) differs for each F. For example, for F being the uniform distribution on $[0, 1]$, a calculation of (5) shows that the designer prefers the setting without information revelation for all relecant r-values ($r < v^*$).

4 Endogenous Information Release

We have seen in the previous section that firms prefer the setting with information disclosure after the first draw to the setting without information disclosure. However, the contest designer has opposite preferences, and he is the one to choose the setup. This raises the question whether firms could play the information revelation setting by voluntary revelation of their first-period innovation value.[3] We take two approaches to model this: first, we extend our model by adding a stage zero where firms can ex ante decide whether to disclose the level of their innovation after the first draw or not. This is an extension in the spirit of the analysis in Yildirim (2005). Second, we consider an intermediate decision, where the firms only decide whether they disclose the information after having observed the value of the first-period innovation.

4.1 Ex Ante Decision

We add an initial stage zero in which the firms simultaneously decide whether to reveal their information (action R) or whether they do not reveal (action N). The decision is observable. It is our goal to identify equilibria of this simultaneous-move game to find out whether the analysis in the previous sections can be supported by endogenous information revelation. This would be the case if (R, R) is an equilibrium of this game. In case both firms play R, the contest following afterwards is the same as the one described in the previous sections. Hence, we already know the corresponding equilibrium strategies. The same holds true in case both firms play N. Then, we are back in the setting of Taylor (1995). To derive the best responses in this initial stage, we need to deduce the equilibrium strategies in the case of asymmetric information revelation. In the resulting contest, one

[3] We implicitly assume that the contest designer either does not set rules to prevent voluntary revelation or is not able to enforce such rules.

firm reveals her first draw, the other one does not. We will analyze equilibria by backward induction. To provide incentives for research, we focus on the main case $p > 2c$ in the following, and assume thus $r < 0.5$.

For the second-period equilibrium, we take the first draw as given. One firm has played R in the initial stage, we denote her draw by x_R^1 and call her firm R. The draw of the firm playing N (short: firm N) is denoted by x_N^1.

Proposition 10 *In the setting with asymmetric information release, given first-period innovations x_R^1 and x_N^1, there are the following second-period equilibrium strategies:*

- *Firm R takes a second draw iff $x_R^1 < z$.*
- *Firm N takes a second draw iff $x_N^1 < x_R^1 < x^*$ or $\bar{x} > x_N^1 > x_R^1$.*

In case firm N does not take a draw in the first period, it is the best reply for firm R to take a second draw iff $\bar{x} > x_R^1$. Firm N takes a draw in the second period if $x_R^1 < x^*$.
In case firm R does not take a draw in the first period, it is the best reply for firm N to take a second draw iff $\bar{x} > x_N^1$. Firm R always takes a draw in the second period.

Proof See Appendix. □

Roughly speaking, firm R thus behaves as in the setting with no information release, while firm N plays the same strategy as with full information release. If both firms do not innovate in the first period, they both take a draw in the second period, as we assumed $p > 2c$. Note that Proposition 10 w.l.o.g. ignores the case $x_R^1 = x_N^1$ for values larger than zero, as it appears with zero probability – it is thus not payoff relevant and we can safely omit it here.

The first-period equilibrium behavior can be summarized as follows (again, we do not calculate possible mixed equilibria, as we later on focus on r-values inducing an equilibrium with research in the first period):

Proposition 11 *Let $\hat{v}$ be the solution to the following equation:*

$$-\frac{1}{24} - \frac{1}{3}\hat{v}^3 + \frac{1}{2}F(z) - \frac{1}{4}F(z)^2 - \frac{1}{6}F(z)^3 + \frac{1}{8}F(z)^4 - F(z)\hat{v} = 0$$

where $F(z)$ is determined by (4) with $r = \hat{v}$.
Furthermore, let $\tilde{v}$ be the solution to

$$\frac{1}{6} - 2\tilde{v} - \frac{1}{2}\tilde{v}^2 - \frac{1}{6}\tilde{v}^3 + 2\sqrt{2\tilde{v}}\tilde{v} - \frac{1}{2}\sqrt{2\tilde{v}}\tilde{v}^2 = 0.$$

Then, in the first period of the contest with asymmetric information disclosure we get the following pure-strategy equilibrium behavior with firms continuing in the second period as described in Proposition 10:

- *For $r < \hat{v}$ there is an equilibrium where both firms draw in the first period.*

- *For* $0.5 > r > \tilde{v}$ *there is an equilibrium where firm* R *draws in the first period and firm* N *does not.*
- *For* $0.5 > r > \hat{v}$ *there is an equilibrium where firm* N *draws in the first period and firm* R *does not.*

The proof is given in the Appendix. Numerically, we can approximate $\hat{v} \approx 0.2623$ and $\tilde{v} \approx 0.1722$. Note that firm R plays different strategies in the two equilibria involving a draw by firm R: as Proposition 10 shows, firm R will continue to innovate in less cases if firm N does not innovate in the first period. Consequently, the best reply of firm N is affected by the change in strategy, yielding two different equilibria involving a draw by firm R in the range $\tilde{v} < r < \hat{v}$. We focus in our analysis on the symmetric equilibrium involving draws by both firms. It is also unique for small r-values.
With this characterization of pure-strategy equilibria we are ready to address the main question of this section: are the two firms willing to ex ante commit to revealing their information after the first draw or not? The answer is given by the following theorem:

Theorem 12 *Let* $\bar{v}$ *solve*

$$\frac{5}{24} - 2\bar{v} + \frac{2\sqrt{2}}{3}\bar{v}^{\frac{3}{2}} - \frac{1}{6}\bar{v}^3 - \frac{1}{2}F(z) + \frac{1}{4}F(z)^2 + \frac{1}{6}F(z)^3 - \frac{1}{8}F(z)^4 = 0, \tag{6}$$

where $F(z)$ *is determined by* (4) *with* $r = \bar{v}$.
For $r < \bar{v}$ *there is a subgame perfect Nash equilibrium in which both firms ex ante commit to revealing their information after the first period. For* $r < \tilde{v}$ *it is unique.*

The proof is given in the Appendix. A numerical approximation gives $\bar{v} \approx 0.2325$. Hence, we have shown that the disclosure of information can be endogenized – the firms are voluntarily agreeing to it ex ante.

4.2 Intermediate Decision

So far, we modeled the revelation decision as taking place before any research is done by the firms. In that setup, firms need to be able to commit to their decision. In the following, we drop the assumption that ex ante commitment is possible – the revelation decision is postponed after the first period, when firms are able to observe their first innovation. As the revelation decision works as a kind of signaling device, a firm holds a belief on the value of the other firm's innovation. We thus refine our equilibrium concept to Perfect Bayesian equilibrium. Nevertheless, firms reveal the information voluntarily, as the following theorem shows:

Theorem 13 *If firms* $i = 1, 2$ *can make their revelation decision simultaneously after learning their first-period innovation value* x_i*, in a Perfect Bayesian equilibrium both firms reveal their value if* $x_i \neq x^*$*. If* $x_i = x^*$ *firm* i *is indifferent between revealing or* *not. The revelation decision in a Perfect Bayesian equilibrium is thus unique up to firms' behavior for value x^*. Off the equilibrium path, in case one firm does not reveal, the other firm believes the deviating firm has value x^* with probability 1 and reacts accordingly.*

The intuition for the proof is as follows: no firm has an incentive to hide her value – then, she would be treated as a firm with value x^*, which is no improvement no matter what the true value of the firm is. Revelation in combination with this punishment thus forms an equilibrium. To show the uniqueness, one has to consider the fact that a firm wants to show that she has a high type (and discourage lower types from continuing to innovate) or a low type (and make intermediate types stop innovating). For intermediate types, one can show that if a firm keeps the information secret, she does so for an interval of values. However, for the lowest of these values a firm has an incentive to reveal – she does not want to pool with higher values against which the other firm would more often like to continue innovating. The details of the proof are given in the Appendix.

5 Extensions

5.1 Second Innovation as Improvement

So far, we modeled the two innovations in the two periods as substantially different ideas: the resulting innovation values do not depend on each other and represent fully developed innovations. A different way of thinking about a multi-period contest is to interpret the second-period innovation not as a new idea, but as an extension of the first-period innovation that improves the innovation value. As a consequence, the distributions of the innovation values in the two periods are not the same (as they have been in our model so far), but the second-period distribution depends on the value of the first-period innovation. In this section, we adapt our model to this interpretation and show that voluntary revelation also appears when the second-period innovation builds on the first-period innovation.

Notation and assumptions stay the same except for the distribution functions: for tractability reasons we assume in this section that F is a uniform distribution on $[0, 1]$ with $F(x) = x$. Furthermore, in the second period, firm i can improve her innovation by taking a draw from the distribution $F_i(x|x_i^1)$ at costs c. We assume that F_i is derived from F and fitted to the interval $[x_i^1, 1]$ according to

$$F_i(x|x_i^1) := F\left(\frac{x - x_i^1}{1 - x_i^1}\right) = \frac{x - x_i^1}{1 - x_i^1}.$$

We start our analysis by identifying the equilibrium behavior of the two firms in the second period in case information is revealed after the first period. Again, we assume that the prize is high enough compared to the costs such that both firms innovate in the first period, which surely guarantees $r < \frac{1}{2}$. The leading firm, with the higher first-period innovation value, is once more denoted by H, the following firm with the lower innovation value by L. Thus, we have $x_H^1 > x_L^1$, again omitting the equality case as it appears with zero probability and is thus not payoff relevant.

Proposition 14 *Given first-period innovations $x_H^1 > x_L^1$, there are the following second-period equilibrium strategies:*

- *If $x_H^1 > 1-(1-x_L^1)r$ both firms stop innovating and the contest ends after the first period.*
- *If $1-(1-x_L^1)2r < x_H^1 \leq 1-(1-x_L^1)r$ only firm L innovates in the second period.*
- *If $x_H^1 \leq 1-(1-x_L^1)2r$ both firms innovate in the second period.*

Proof See Appendix. □

Compared to the equilibrium with independent innovations, the continuation decision does not depend only on the leader's value, but also on the value of the runner-up. This leads to an increased amount of research. Particularly, as in the independent case, the runner-up will always continue to innovate if the leader's value is below $1-r = x^*$ – but additionally, he will also continue to innovate for higher values of firm H if his own first-period value, x_L^1, is not too far behind. Similarly, the leading firm will always innovate if her own value is smaller than $1-2r > 1-\sqrt{2r} = \bar{x}$, which is already a larger set than in the case with independent values (where firm H only continues for $x_H^1 \leq \bar{x}$). Furthermore, the leading firm will also continue if the runner-up is only close behind. This is a major strategic difference to the case with independent values: two innovations of approximately the same size are worth almost the same. It is much less important which firm has the lead.

What is the effect of this strategic difference on voluntary revelation? If the revelation decision is made after the first period, the equilibrium in Theorem 13 uses a maximum punishment idea: if firm i does not reveal its value firm j believes firm i has value x^*, making firm j continue to innovate for the largest possible set of values – which is a bad thing for the hiding firm i. On the contrary, if the second innovation builds on the first one, firm i with a first-period innovation value above x^* can in fact profit from keeping the value secret for some values of firm j in the top range: if firm j believes to face a firm i with value x^*, hiding goes along with an underestimation of i's value by firm j, making j stop innovating for these values. However, at the same time a firm j with a value at the lower end will continue to innovate although she would stop if she knew the true value of i. We thus have two opposing effects. In the following theorem we show that the latter effect is the dominating one and voluntary revelation extends to this model of improving innovations.

Theorem 15 *If firms $i = 1, 2$ can make their revelation decision simultaneously after learning their first-period innovation value x_i^1 and the second innovation always improves* *the first innovation, there is a Perfect Bayesian equilibrium in which both firms reveal their value. Off the equilibrium path, in case one firm does not reveal, the other firm believes the deviating firm has value x^* with probability 1 and reacts accordingly.*

Proof See Appendix. □

Note that there will be no equilibrium in which both firms always hide their value: there is always an interval at the lower end of possible values for which it is beneficial to reveal, showing the opponent that the own value is much lower than he expected. Compared to no revelation, this makes the opponent stop innovating for some medium values and is thus profitable for a firm with a low value realization.

5.2 n Firms and m Periods

Voluntary revelation of intermediate research results is not limited to the case of two firms and two periods we have studied in detail until now. In this section, we extend the main result with independent research draws and an intermediate revelation decision to n firms (and two periods) and m periods (and two firms).

We start with the case of n firms and two periods, otherwise the setting is the same as with two firms. Again, we assume that the prize is large enough compared to the cost to make all firms innovate in the first period. Particularly, all participating firms should make nonnegative profit as they would not innovate at all otherwise. We thus assume that

$$p > nc \iff r < \frac{1}{n}.$$

In second-period equilibrium play with information disclosure, compared to the case with only two firms, incentives to innovate are lower if more competing firms are present. Particularly, if firm i has a first-period innovation better than x^*, no other firm will try to beat firm i in the second period. Furthermore, as long as no other firm is continuing to innovate, the incentives for firm $j \neq i$ to draw in the second period are the same as in the case with only the two firms i and j. Hence, some research is going on in period two if the highest value of the first period, x_H^1, is smaller than x^*. In a pure strategy equilibrium, only one firm will continue to innovate for values slightly below x^*, and there will be additional thresholds at lower values of the leading firm for which more firms continue to innovate. As this type of equilibrium is asymmetric, it comes along with a coordination problem. We will thus focus on a symmetric equilibrium which is in mixed strategies: for values slightly lower than x^*, all firms will continue to innovate with a positive probability depending on x_H^1, $q\left(x_H^1\right)$. This probability is obviously fixed by making all firms that are not in the lead indifferent between drawing or not. The largest value of x_H^1 for which all other firms draw with probability one is denoted by $\hat{x}$:

$$\hat{x} := \max\{x_H^1 | q\left(x_H^1\right) = 1\}$$

By definition, all firms who are not in the lead make zero profit if $x_H^1 = \hat{x}$. Thus, as all these firms draw with probability one in this case, we can conclude that each of these firms wins the contest with probability $r = \frac{c}{p}$, as $p \cdot \frac{c}{p} - c = 0$. Consequently, the remaining winning probability is with the leading firm, who wins with probability $1 - (n-1)r$ and does not draw herself, as due to the current leadership the incentives to draw are strictly lower for this firm. Thus, the leading firm wins exactly if all drawing firms have a second-period value lower than $\hat{x}$, and we can conclude that

$$F\left(\hat{x}\right)^{n-1} = 1 - (n-1)r \iff F\left(\hat{x}\right) = \sqrt[n-1]{1-(n-1)r}.$$

We summarize these results in the following proposition[4].

Proposition 16 *Given the largest first-period innovation x_H^1, in the symmetric second-period equilibrium strategies*

- *no firm draws if $x_H^1 > x^*$,*
- *non-leading firms draw with probability $q\left(x_H^1\right)$ if $\hat{x} \leq x_H^1 \leq x^*$, with $q\left(x_H^1\right) \in (0,1)$ for $\hat{x} < x_H^1 < x^*$,*
- *non-leading firms draw if $\hat{x} > x_H^1$,*
- *the leading firm does not draw if $\hat{x} \leq x_H^1$.*

As in the previous sections, we endogenize the information disclosure by letting firms decide whether they reveal or not after learning their first-period value. Again, the equilibrium we derive builds on maximum punishment: if a firm hides her first-period innovation value, the other firms believe that a hiding firm has value $\hat{x}$, as stated in the following theorem:

Theorem 17 *If firms $i = 1, 2, \ldots, n$ can make their revelation decision simultaneously after learning their first-period innovation value x_i^1, there is a Perfect Bayesian equilibrium in which all firms reveal their value. Off the equilibrium path, in case one firm does not reveal, the other firms believe the deviating firm has value $\hat{x}$ with probability 1 and reacts accordingly.*

Proof See Appendix. □

[4]Note that Proposition 16 is not a full equilibrium characterization but contains only the parts necessary for our purposes.

If all other firms have a value smaller than $\hat{x}$, they will all continue to innovate and the punishment is maximal. However, contrary to the case with two firms, there is potentially some room for benefiting from these beliefs about a hiding firm. Suppose the second highest value is x_L^1, and the values are ordered as follows: $x^* > x_H^1 > x_L^1 > \hat{x}$. If firm H hides her value, the remaining firms will believe that firm L is in fact the leading firm. Particularly, this will make firm L stop innovating in the second period – this is in the interest of firm H. Hence, as this constellation of values only happens with some probability, the main part of proving the effectiveness of the punishment is thus to show that the expected loss from the other value constellations outweighs this potential gain.

Next, we consider m periods and two firms. The prize is assumed to be large enough compared to the costs such that both firms innovate in the first period. Suppose first that revelation is mandatory. Then, if one firm has an innovation with a value above x^*, incentives to continue innovating are similar to the second period of the two period case and it is never beneficial to continue innovating. The following corollary is a direct consequence of the corresponding argument in Proposition 1.

Corollary 18 *Consider an innovation contest with two firms, m periods and mandatory information revelation. Suppose firm i made an innovation in period t with $x_i^t > x^*$. Then, in any following period both firms do not innovate. In case the firm i made the highest innovation in period t and $x_i^t = x^*$, firm $j \neq i$ is indifferent between innovating or not in any following period where $x_i^t = x^*$ is still the highest innovation.*

Now suppose the revelation decision of the firms is voluntary and they can decide after each period whether to reveal or not. As a consequence of Corollary 18, it is immediate to see that the threat of Theorem 13 has bite with m periods as well:

Corollary 19 *Suppose firms $i, j = 1, 2$ can make a revelation decision simultaneously after learning their innovation value of each period $t = 1, \ldots, m-1$. Then, there is a Perfect Bayesian equilibrium in which both firms always reveal their value. Off the equilibrium path, in case firm i does not reveal, firm j believes the deviating firm i has value x^* with probability 1. Then, firm j continues to innovate until she has an innovation better than x^* or firm i reveals such an innovation.*

For firm i, hiding the own value will lead to the maximum punishment, firm j innovates in the next period for all values smaller than x^*. This is always worse for firm i then revealing, as there is no potential future advantage of an additional innovation of firm j for firm i.

6 Conclusion

We show that in a basic innovation contest with multiple rounds, firms and contest designer have opposing interests regarding the revelation policy of intermediate research results. Although the contest designer prefers firms to keep intermediate information secret, they are able to establish voluntary revelation of their research progress. For most of our analysis of the basic model – which has two firms, two periods and independent innovations – only mild assumptions on the research technology are needed. Furthermore, our main result of voluntary revelation turns out to be very robust: we consider extensions to n firms, m periods and improving innovations. The possibility of voluntary revelation has an impact on the prize setting by the contest designer. Suppose he wants to set his prize optimally, uses a setting without information disclosure (which he prefers) and does not prevent voluntary revelation. Then, if the firms decide to disclose on their own, using the optimal prize with respect to secret information can lead to a lower payoff for the designer than the optimal prize with respect to mandatory information disclosure. Consequently, the contest designer should then choose his prize as if information disclosure was mandatory.

Considering further extensions of the model, the most prominent one would be a joint examination of n firms, m periods and improving innovations. The existing results suggest that voluntary revelation would extend to this setting as well. Furthermore, we did not fully characterize the equilibrium research behavior for multiple firms and periods in the setting with mandatory information disclosure. Particularly, we simply assumed that the prize is large enough compared to the costs such that all firms start innovating in the first period. From a quantitative perspective, it would be possible to explicitly calculate the respective critical prize/cost ratios, although it has no impact on the qualitative nature of the results. Furthermore, our extension with improving innovations only considers a uniform distribution – it would be interesting to see the impact of a change in distribution. A completely different extension could be made by considering heterogeneous firms with different research costs or different research technologies. As long as the heterogeneity is only mild, we do not expect qualitative effects on the results, although quantitatively heterogeneity will lead to different cutoffs for the firms.

References

Aoyagi, M. (2010). Information feedback in a dynamic tournament. *Games and Economic Behavior*, forthcoming.

Baye, M. R. and H. C. Hoppe (2003). The strategic equivalence of rent-seeking, innovation, and patent-race games. *Games and Economic Behavior 44*(2), 217–226.

Che, Y. and I. Gale (2003). Optimal design of research contests. *The American Economic Review 93*(3), 646–671.

Dixit, A. (1987). Strategic behavior in contests. *The American Economic Review 77*(5), 891–898.

Fullerton, R. L., B. G. Linster, M. McKee, and S. Slate (2002). Using auctions to reward tournament winners: Theory and experimental investigations. *The RAND Journal of Economics 33*(1), 62–84.

Gershkov, A. and M. Perry (2009). Tournaments with midterm reviews. *Games and Economic Behavior 66*(1), 162–190.

Gill, D. (2008). Strategic disclosure of intermediate research results. *Journal of Economics & Management Strategy 17*(3), 733–758.

Groves, T. (2008). Mandatory disclosure of trial results for drugs and devices. *British Medical Journal 336*(7637), 170.

Konrad, K. A. and D. Kovenock (2009). Multi-battle contests. *Games and Economic Behavior 66*(1), 256–274.

Lazear, E. P. and S. Rosen (1981). Rank-Order tournaments as optimum labor contracts. *Journal of Political Economy 89*(5), 841.

Ludwig, S. and G. Luenser (2008). Knowing the gap - intermediate information in tournaments. *working paper*.

Moldovanu, B. and A. Sela (2006). Contest architecture. *Journal of Economic Theory 126*(1), 70–96.

Schoettner, A. (2008). Fixed-prize tournaments versus first-price auctions in innovation contests. *Economic Theory 35*(1), 57–71.

Taylor, C. R. (1995). Digging for golden carrots: An analysis of research tournaments. *The American Economic Review 85*(4), 872–890.

Yildirim, H. (2005). Contests with multiple rounds. *Games and Economic Behavior 51*(1), 213–227.

A Appendix: Proofs

Proof of Proposition 3

Suppose firm i decides not to draw again. Then, not drawing again is a best response for firm $j \neq i$ in case

$$P\left(x_j^2 \geq x^1\right) p - c \leq \frac{1}{2} p \iff \left(1 - F\left(x^1\right)\right) p - c \leq \frac{1}{2} p \iff \frac{1}{2} - \frac{c}{p} \leq F\left(x^1\right) \tag{7}$$

Thus, we get that both firms not drawing again is an equilibrium if (7) holds.
Here, we can directly see that both firms do not want to draw in the second period in case $p < 2c$. Even if both firms did not invest in the first period, and a firm could win for sure by conducting research, expected profit is higher if no research is done.

Let us get to the best response in case firm i decides to draw in the second period. Then, drawing is a best response for firm $j \neq i$ according to the following condition:

$$\frac{1}{2}p - c \geq P\left(x_i^2 \leq x^1\right)\frac{1}{2}p \iff \frac{1}{2}p - c \geq F\left(x^1\right)\frac{1}{2}p \iff 1 - 2\frac{c}{p} \geq F\left(x^1\right) \tag{8}$$

Hence, both firms drawing again is an equilibrium if (8) is fulfilled, which is obviously the case for $p \geq 2c$. Again, we can see that a firm does not want to draw again in case $p < 2c$. □

Proof of Proposition 4

To derive first-period equilibrium play, first consider the case $p < 2c$. As we have seen, both firms will not invest in the second period in case no research is done in the first period. If research is conducted by at least one firm, only the lower firm might invest again, because $\bar{x} < 0$ if $p < 2c$. By backward induction, we can conclude that both firms will not draw in the first period: we have seen in the analysis of the second period that a single draw is too expensive for a firm even when it wins for sure. In the first period, incentives for conducting research are even lower. An investing firm will not win for sure, as the other firm might decide to invest in the second period. Hence, both firms will not invest in the first period if the prize is too low. This is no problem for the firms, as they make a positive expected profit of $\frac{1}{2}p$. It is a problem of the prize sponsor, who will get no research done but has to pay the prize anyway.

So let us consider the case $p \geq 2c$. What is the best response against an opponent not taking a draw in the first period? Note that we know the following:

$$\begin{aligned}
P\left(x_i^2 > x^*\right) &= 1 - F\left(x^*\right) = \frac{c}{p} = r \\
P\left(\bar{x} < x_i^2 \leq x^*\right) &= F\left(x^*\right) - F\left(\bar{x}\right) = \sqrt{2\frac{c}{p}} - \frac{c}{p} = s - r \\
P\left(x_i^2 \leq \bar{x}\right) &= F\left(\bar{x}\right) = 1 - \sqrt{2\frac{c}{p}} = 1 - s
\end{aligned}$$

We can thus write down the condition for player i taking a draw in the first round against a player $j \neq i$ not taking a draw in the first round, bearing in mind second-period equilibrium behavior:

$$\begin{aligned}
&\Bigg[P\left(x_i^1 > x^*\right) + P\left(\bar{x} < x_i^1 \leq x^*\right)\left(P\left(x_j^2 \leq \bar{x}\right) + \frac{1}{2}P\left(\bar{x} < x_j^2 \leq x^*\right)\right) \\
&\quad + P\left(x_i^1 \leq \bar{x}\right)\Bigg(P\left(x_i^2 > \bar{x}\right)\left(P\left(x_j^2 \leq \bar{x}\right) + \frac{1}{2}P\left(x_j^2 > \bar{x}\right)\right) \\
&\quad + \frac{2}{3}P\left(x_i^2 \leq \bar{x}\right)P\left(x_j^2 \leq \bar{x}\right)\Bigg)\Bigg]p - c - P\left(x_i^1 \leq \bar{x}\right)c \geq \frac{1}{2}p - c \\
\iff \quad &\Bigg[r + (s - r)\left((1 - s) + \frac{1}{2}(s - r)\right) + (1 - s)\left(s\left((1 - s) + \frac{1}{2}s\right)\right.
\end{aligned}$$

$$
\begin{aligned}
& \left. \left. +\frac{2}{3}(1-s)(1-s)\right)\right] p-(1-s)c \geq \frac{1}{2}p \\
\Longleftrightarrow \quad & \left[r+(s-r)\left(1-\frac{1}{2}(s+r)\right)+(1-s)\left((s-r)+\frac{2}{3}(1-2s+2r)\right)\right] p \\
& -(1-s)c \geq \frac{1}{2}p \\
\Longleftrightarrow \quad & \left[s-r+\frac{1}{2}r^2+\frac{2}{3}-\frac{1}{3}s+\frac{1}{3}r-\frac{2}{3}s+\frac{2}{3}r-\frac{1}{3}rs\right] p-(1-s)c \geq \frac{1}{2}p \\
\Longleftrightarrow \quad & \left[\frac{1}{6}-\frac{1}{3}rs+\frac{1}{2}r^2\right]-(1-s)r \geq 0 \\
\Longleftrightarrow \quad & \frac{1}{6}-r+\frac{2}{3}rs+\frac{1}{2}r^2 \geq 0 \\
\Longleftrightarrow \quad & \frac{1}{6}-r+\frac{2}{3}\sqrt{2}r^{\frac{3}{2}}+\frac{1}{2}r^2 \geq 0 \qquad (9)
\end{aligned}
$$

We thus have to show now that (9) holds. To check this, we calculate the minimum of the left side in (9) with the help of the substitution $t := \sqrt{r}$. The FOC with respect to r is

$$
\begin{aligned}
& -1+\sqrt{2}r^{\frac{1}{2}}+r=0 \\
\Longleftrightarrow \quad & t^2+\sqrt{2}t-1=0 \\
\Longrightarrow \quad & t=-\sqrt{\frac{1}{2}}+\sqrt{\frac{3}{2}}=\frac{\sqrt{3}-1}{\sqrt{2}}
\end{aligned}
$$

Only the positive solution matters here, as $t = \sqrt{r}$ is restricted to be positive. Hence, we get $r = \frac{\left(\sqrt{3}-1\right)^2}{2} \approx 0.2679$, leading to an expected gain from drawing compared to not drawing of approximately $0.0654 > 0$, which is clearly a minimum on $[0; 0.5]$ ($r \leq 0.5$ holds as $p \geq 2c$). Hence, it is always a best response to draw in the first period if the opponent does not take a draw.

Finally, we get to the best response of firm i in case the other agent $j \neq i$ takes a draw in the first period. We compare the expected profit of drawing as well (and thus playing the same strategy and sharing the prize) with the expected profit of not drawing in the first period. Note that we just calculated above the expected share of the prize a firm gets when taking a draw in the first period against a firm not taking a draw in the first period. We can thus subtract this share from the whole prize to get the share of the firm not drawing against a drawing firm.

$$
\begin{aligned}
& \frac{1}{2}p-c-\left[P\left(x_i^1 \leq \bar{x}\right)P\left(x_j^1 \leq x^*\right)+\frac{1}{2}P\left(\bar{x}<x_i^1 \leq x^*\right)P\left(\bar{x}<x_j^1 \leq x^*\right)\right]c \\
& \geq \left[1-\left(\frac{2}{3}-\frac{1}{3}rs+\frac{1}{2}r^2\right)\right]p-P\left(x_j^1 \leq x^*\right)c \qquad (10)
\end{aligned}
$$

Computing the probabilities yields

$$
\frac{1}{2}p-c-\left[(1-s)(1-r)+\frac{1}{2}(s-r)(s-r)\right]c \geq \left[\frac{1}{3}+\frac{1}{3}rs-\frac{1}{2}r^2\right]p-(1-r)c
$$

$$\begin{aligned}
\iff \quad & \frac{1}{2}p - c - \left[1 - s + \frac{1}{2}r^2\right] c \geq \left[\frac{1}{3} + \frac{1}{3}rs - \frac{1}{2}r^2\right] p - (1-r)\, c \\
\iff \quad & \left[\frac{1}{6} - \frac{1}{3}rs + \frac{1}{2}r^2\right] - \left[1 - s + r + \frac{1}{2}r^2\right] r \geq 0 \\
\iff \quad & \frac{1}{6} - r + \frac{2}{3}rs - \frac{1}{2}r^2 - \frac{1}{2}r^3 \geq 0 \qquad (11)
\end{aligned}$$

We can see that the left side of (11) is decreasing by checking the first derivative, bearing in mind that $r \in [0, 0.5]$:

$$-1 + \sqrt{2r} - r - \frac{3}{2}r^2 \leq -r - \frac{3}{2}r^2 \leq 0$$

Numerically, we get that the left side of (11) equals zero for $r \approx 0.2428$ – we call this boundary value v^*. Hence, drawing as well is a best response for all $r < v^* = 0.2428$. For larger r values, firm i does not want to draw in the first period if firm j takes a draw. □

Proof of Proposition 5

Both firms take a draw in the first period. At least one additional draw is taken in case no innovation has a value above x^*:

$$P(x_i^1 \leq x^*)P(x_j^1 \leq x^*) = (1-r)^2.$$

A second additional draw is taken in case both values are below $\bar{x}$:

$$P(x_i^1 \leq \bar{x})P(x_j^1 \leq \bar{x}) = (1-s)^2.$$

This gives us a total number of

$$d_R(r) = 2 + (1-r)^2 + (1-s)^2 = 4 + r^2 - 2s$$

concluding the proof. □

Proof of Lemma 6 First, we show that the right-hand side of (4) is decreasing in $F(z)$. Using $F(z) < 1$ and the substitution $y = F(z)$ we can write the first derivative as follows:

$$\frac{\mathrm{d}}{\mathrm{d}y}\left((1+y)(1-y)^2 - 2r\right) = 3y^2 - 2y - 1 < 0. \qquad (12)$$

To get $x^* > z$, we plug (1) into the right-hand side of (4), yielding

$$(1 + F(x^*))(1 - F(x^*))^2 - 2r = (2-r)\, r^2 - 2r = 2r\,(r-1) - r^3 < 0,$$

which holds as $r < \frac{1}{2}$. We thus can conclude that

$$(1 + F(x^*))(1 - F(x^*))^2 - 2r < 0 = (1 + F(z))(1 - F(z))^2 - 2r$$

and consequently $F(x^*) > F(z)$ by (12). As F is increasing, this shows $x^* > z$. Similarly, for $\bar{x} < z$ we use (2):

$$(1 + F(\bar{x}))(1 - F(\bar{x}))^2 - 2r = (2-s)\, 2r - 2r = 2r\,(1-s) > 0,$$

which holds as $s = \sqrt{2r} < 1$ by $r < \frac{1}{2}$. Consequently, $\bar{x} < z$ follows as above. □

Proof of Proposition 7

First note that

$$d_{NR}(r) > d_R(r) \iff 2(1+F(z)) > 4 - 2\sqrt{2r} + r^2 \iff F(z) > 1 - \sqrt{2r} + \frac{1}{2}r^2,$$

where $F(z)$ depends on r. To show the proposition, it is thus sufficient to prove $F(z) > 1 - \sqrt{2r} + \frac{1}{2}r^2$. In the proof of Lemma 6 we showed that the right-hand side of (4) is decreasing in $F(z)$. Hence, it is sufficient to plug $1 - \sqrt{2r} + \frac{1}{2}r^2$ into the right-hand side of (4) and show that the resulting expression is greater than 0. As a consequence, $F(z) > 1-\sqrt{2r}+\frac{1}{2}r^2$ directly follows as $F(z)$ solves (4) (and thus yields a lower right-hand side than $1 - \sqrt{2r} + \frac{1}{2}r^2$).

Plugging $1 - \sqrt{2r} + \frac{1}{2}r^2$ into the right-hand side of (4) we get

$$\begin{aligned}
&\left(1+\left(1-\sqrt{2r}+\frac{1}{2}r^2\right)\right)\left(1-\left(1-\sqrt{2r}+\frac{1}{2}r^2\right)\right)^2 - 2r\\
=&2r - 2\sqrt{2}r^{\frac{3}{2}} - 2\sqrt{2}r^{\frac{5}{2}} + 3r^3 + \frac{1}{2}r^4 - \frac{3}{4}\sqrt{2}r^{\frac{9}{2}} + \frac{1}{8}r^6\\
>&2r - 2\cdot\frac{3}{4}r - 2\cdot\frac{3}{4}r^2 + 3r^3 + \frac{1}{2}r^4 - \left(\frac{3}{4}\right)^2 r^4 + \frac{1}{8}r^6\\
=&\frac{1}{2}r - \frac{3}{2}r^2 + 3r^3 - \frac{1}{16}r^4 + \frac{1}{8}r^6\\
>&\frac{1}{2}r - \frac{3}{8}r + 3r^3 - \frac{1}{64}r^3 + \frac{1}{8}r^6\\
>&0.
\end{aligned}$$

The third line follows by $r < v^* < 0.25$ and thus $-\sqrt{2r} > -\frac{3}{4}$. Similarly, the fifth line follows by $-r > -\frac{1}{4}$ and the last line by $r > 0$. $\square$

Proof of Theorem 9

We derive a condition on r making $\int_0^b \Phi^2(x) - \Psi(x)dx < 0$ in a rather coarse way by looking for a non-positive integrand on the whole interval $[0, b]$. We proceed in several steps, cutting the interval into different parts:

i) $[0, \bar{x}]$

In this case, it is easy to see that $\int_0^{\bar{x}} \Phi^2(x) - \Psi(x)dx = \int_0^{\bar{x}} 0dx = 0$ holds.

ii) $(\bar{x}, z]$

Here, we get

$$\int_{\bar{x}}^{z} \Phi^2(x) - \Psi(x)dx = \int_{\bar{x}}^{z} \underbrace{\left(F(x)^2 - F(x)\right)}_{<0}\underbrace{\left(F(x)^2 - F(\bar{x})^2\right)}_{>0} dx < 0.$$

iii) $(z, x^*]$

First, we rewrite

$$\int_z^{x^*} \Phi^2(x) - \Psi(x)dx = \int_z^{x^*} \underbrace{\begin{aligned} &F(z)^2 + F(x)\left(F(\bar{x})^2 - 2F(z) - 2F(z)^2\right) \\ &+F(x)^2\left(1 + 2F(z) + F(z)^2 - F(\bar{x})^2\right) - F(x)^3 \end{aligned}}_{=:h(x)} dx$$

We now show that the integrand $h(x)$ is negative by analyzing its first derivative, which is given as follows:

$$h'(x) = F(\bar{x})^2 - 2F(z) - 2F(z)^2 + 2F(x)\left(1 + 2F(z) + F(z)^2 - F(\bar{x})^2\right) - 3F(x)^2$$

At z, h' is positive:

$$h'(z) = \underbrace{\left(F(z)^2 - F(\bar{x})^2\right)}_{>0} \underbrace{(2F(z) - 1)}_{>0 \text{ for } F(z)>\frac{1}{2}}$$

As $F(z)$ is implicitly given by (4) we get

$$F(z) > \frac{1}{2} \iff r < \frac{1}{2} \cdot \frac{3}{2} \cdot \left(\frac{1}{2}\right)^2 = 0.1875$$

and consequently $h'(z)$ is positive in this case. Additionally, a numerical check shows that $h'(x^*)$ is positive as well (for all $z \in [0, b]$). Furthermore, h' is a quadratic function which has a maximum (this follows from $h'''(x) = -6$). Taking these facts together, we get that h' is positive on $[z, x^*]$ given $r < 0.1875$. Hence, h is increasing on $[z, x^*]$. A numerical check shows that $h(x^*) < 0$ for $r < 0.1647$ – thus, for these r-values h is negative on the whole interval (as it is largest at x^*).

iv) $(x^*, b]$

In this case, we get the following:

$$\begin{aligned} &\int_{x^*}^{b} \Phi^2(x) - \Psi(x)dx \\ &\quad = \int_{x^*}^{b} \underbrace{\begin{aligned} &F(z)^2 + F(x^*)^2 + F(x)\left(F(\bar{x})^2 - 2F(z) - 2F(z)^2 - F(x^*)^2\right) \\ &+F(x)^2\left(2F(z) + F(z)^2 - F(\bar{x})^2\right) \end{aligned}}_{=:l(x)} dx \end{aligned}$$

As $l(x^*) = h(x^*)$, we know that $l(x^*)$ is negative for $r < 0.1647$. Furthermore, l is a quadratic function having a minimum (as $l''(x) = 2\left(2F(z) + F(z)^2 - F(\bar{x})^2\right) > 0$). Hence, as $l(b) = 0$, l is negative on $(x^*, b]$.

Thus, we can conclude that the integrand (and thus the whole integral) is negative if $r < 0.1647 = v'$ holds. □

Proof of Proposition 10

First, we know from Proposition 1 that no firm will draw again in case she knows that an innovation larger than x^* has been drawn. The conclusion of this proposition applies to asymmetric information release as well: in the situation of Proposition 1 a firm does not want to draw again even if she knows that she is behind. If a firm with such a high draw does not know the opponent's draw, incentives for drawing again are even lower.

Additionally, Proposition 1 implies that firm N will not draw again if $x_N^1 > \bar{x}$ and $x_N^1 > x_R^1$. We first consider the following case: both firms have taken a draw in the first round. Firm R has a draw $\bar{x} < x_R^1 < x^*$ and faces the decision whether to draw again or not. For the moment we assume that firm N behaves according to Proposition 1 and thus draws again if she is behind (the case of equality of draws can be ignored from firm R's perspective as it is a zero probability event). It is beneficial for firm R to draw again if the following condition holds:

$$\begin{aligned}\Bigg[P\left(x_N^1 < x_R^1\right)\left(P\left(x_N^2 < x_R^1\right) + \frac{1}{2}P\left(x_N^2 > x_R^1\right)P\left(x_R^2 > x_R^1\right)\right)&\\ + \frac{1}{2}P\left(x_N^1 > x_R^1\right)P\left(x_R^2 > x_R^1\right)\Bigg]p - c \geq P\left(x_N^1 < x_R^1\right)P\left(x_N^2 < x_R^1\right)p&\end{aligned}$$

This yields the following probabilities:

$$\begin{aligned}&\left[F\left(x_R^1\right)\left(F\left(x_R^1\right) + \frac{1}{2}\left(1 - F\left(x_R^1\right)\right)^2\right) + \frac{1}{2}\left(1 - F\left(x_R^1\right)\right)^2\right]p - c \geq F\left(x_R^1\right)^2 p\\ \Longleftrightarrow\quad &\left(1 + F\left(x_R^1\right)\right)\left(1 - F\left(x_R^1\right)\right)^2 - 2\frac{c}{p} \geq 0 \qquad (13)\end{aligned}$$

Note that (13) has the same structure as (4). Hence, firm R will draw again exactly in case her first draw is smaller than z, which solves both (4) and (13). We denote $F(z) =: w$ in the following.

For the calculation above, we assumed that firm N follows the strategy described in Proposition 1, but it is not clear that this strategy is a best reply. Obviously, it is a best reply in case firm N is leading, as $x_R^1 > \bar{x}$. Not drawing is then profitable even against an opponent who draws. However, it could be profitable for firm N to stop drawing in case she is behind and firm R has a draw $\bar{x} < x_R^1 < z$ with $x_R^1 > x_N^1$. In this case, firm R will draw again as well – she would not do so if she knew that she is in front, as it is the case in the situation of Proposition 1. We check whether it is anyway profitable to draw again for firm N:

$$\begin{aligned}&P\left(x_N^2 > x_R^1\right)\left(P\left(x_R^2 < x_R^1\right) + \frac{1}{2}P\left(x_R^2 > x_R^1\right)\right)p - c\\ =&\left(1 - F\left(x_R^1\right)\right)\left(F\left(x_R^1\right) + \frac{1}{2}\left(1 - F\left(x_R^1\right)\right)\right)p - c\\ >&\frac{1}{2}\left(1 + F\left(x_R^1\right)\right)\left(1 - F\left(x_R^1\right)\right)^2 p - c\\ \geq&0\end{aligned}$$

The strict inequality holds by direct comparison (and $0 < F\left(x_R^1\right) < 1$). The last inequality holds as $x_R^1 < z$ in this case and (13) applies. Hence, it is in fact a best reply for firm N to follow the strategy derived in Proposition 1.

If the draw of firm R fulfills $x_R^1 < \bar{x}$, the incentives to draw again are the same for firm N as in Proposition 1. Hence, firm N behaves similarly here. For firm R, we consider an estimate of her profit from drawing again, looking only at the largest terms:

$$\begin{aligned}
&\left[P\left(x_N^1 < x_R^1\right)\left(P\left(x_N^2 < x_R^1\right) + \frac{1}{2}P\left(x_N^2 > x_R^1\right)P\left(x_R^2 > x_R^1\right)\right)\right.\\
&\qquad\qquad\left. + \frac{1}{2}P\left(x_N^1 > \bar{x} > x_R^1\right)P\left(x_R^2 > \bar{x} > x_R^1\right)\right]p - c\\
&= \left[F\left(x_R^1\right)\left(F\left(x_R^1\right) + \frac{1}{2}\left(1 - F\left(x_R^1\right)\right)^2\right) + \frac{1}{2}\left(1 - F\left(\bar{x}\right)\right)^2\right]p - c\\
&> \left[F\left(x_R^1\right)F\left(x_R^1\right) + \frac{1}{2}\left(1 - \left(1 - \sqrt{2\frac{c}{p}}\right)\right)^2\right]p - c\\
&= F\left(x_R^1\right)F\left(x_R^1\right)p
\end{aligned}$$

The latter is the expected profit of firm R without a second draw. Hence, drawing again is beneficial for firm R.

What happens if one of the firms plays a strategy where she does not take a draw in the first period? If firm N faces a firm R taking no draw, the second period behavior is similar to playing against a firm with a draw of zero. For firm R, things change: if she faces a firm not drawing in the first period, her best reply is similar as in the situation of full information release. Thus, if she believes with probability one that she faces a not-drawing firm, she plays the same strategy as firm N in that case: she will only draw again if $x_R^1 < \bar{x}$. □

Proof of Proposition 11

In the first period, both firms have to compare the expected profits of taking a draw with the expected profits of waiting one period. Consider first the case of firm R not drawing in the first round. What is the best reply of firm N? This is basically the same exercise as deriving inequality (9), with one slight difference: firm N is not able to discourage firm R from taking a draw in case $x_N^1 > x^*$. This slightly reduces the probability of winning the prize for firm N compared to the setting of full revelation: it is now possible that firm R beats firm N with a draw $x_R^2 > x_N^1 > x^*$. This is the case with probability $\frac{1}{2}\left(1 - F\left(x^*\right)\right)^2 = \frac{1}{2}r^2$. We can include this probability change into (9) by subtracting $\frac{1}{2}r^2$, which gives us the following condition for a profitable draw in the first round:

$$\frac{1}{6} - r + \frac{2}{3}\sqrt{2}r^{\frac{3}{2}} \geq 0 \tag{14}$$

The analysis of the first order condition shows that the left side of (14) has a minimum at $r = \frac{1}{2}$. For $r = \frac{1}{2}$, equality holds in (14). Hence, taking a draw is profitable for firm N in the first period in this case.

What is the best reply for firm R against this strategy of firm N? We first calculate the probability for firm R to win the prize if she is taking a draw in the first period (and following the equilibrium strategy of the second period afterwards).

$$\begin{aligned}
&P\left(x_R^1 > x^*\right)\left[P\left(x_N^1 < x^*\right) + \frac{1}{2}P\left(x_N^1 > x^*\right)\right] \\
&+P\left(z < x_R^1 < x^*\right)\left[\frac{1}{2}P\left(z < x_N^1 < x^*\right)\left(\frac{2}{3}P\left(z < x_N^2 < x^*\right) + P\left(x_N^2 < z\right)\right)\right. \\
&\left. + P\left(x_N^1 < z\right)\left(P\left(x_N^2 < z\right) + \frac{1}{2}P\left(z < x_N^2 < x^*\right)\right)\right] \\
&+P\left(\bar{x} < x_R^1 < z\right)\left[\frac{1}{2}P\left(x_N^1 > x^*\right)P\left(x_R^2 > x^*\right)\right. \\
&+ P\left(z < x_N^1 < x^*\right)\left(P\left(x_R^2 > x^*\right) + \frac{1}{2}P\left(z < x_R^2 < x^*\right)\right) \\
&+ P\left(\bar{x} < x_N^1 < z\right)\left(\frac{1}{2}\left(P\left(x_R^2 > z\right) + \frac{1}{3}P\left(\bar{x} < x_R^2 < z\right)\right)\right. \\
&+ \frac{1}{2}\left[P\left(x_R^2 > z\right)\left(\frac{1}{2}P\left(x_N^2 > z\right) + P\left(x_N^2 < z\right)\right)\right. \\
&+ P\left(\bar{x} < x_R^2 < z\right)\left(P\left(x_N^2 < \bar{x}\right) + \frac{3}{4}P\left(\bar{x} < x_N^2 < z\right)\right) \\
&\left.\left. + P\left(x_R^2 < \bar{x}\right)\left(P\left(x_N^2 < \bar{x}\right) + \frac{2}{3}P\left(\bar{x} < x_N^2 < z\right)\right)\right]\right) \\
&+ P\left(x_N^1 < \bar{x}\right)\left(P\left(x_R^2 > z\right)\left(P\left(x_N^2 < z\right) + \frac{1}{2}P\left(x_N^2 > z\right)\right)\right. \\
&+ P\left(\bar{x} < x_R^2 < z\right)\left(P\left(x_N^2 < \bar{x}\right) + \frac{2}{3}P\left(\bar{x} < x_N^2 < z\right)\right) \\
&\left.\left. + P\left(x_R^2 < \bar{x}\right)\left(P\left(x_N^2 < \bar{x}\right) + \frac{1}{2}P\left(\bar{x} < x_N^2 < z\right)\right)\right)\right] \\
&+P\left(x_R^1 < \bar{x}\right)\left[\frac{1}{2}P\left(x_N^1 > \bar{x}\right)P\left(x_R^2 > \bar{x}\right)\right. \\
&+ P\left(x_N^1 < \bar{x}\right)\left(P\left(x_R^2 > \bar{x}\right)\left(P\left(x_N^2 < \bar{x}\right) + \frac{1}{2}P\left(x_N^2 > \bar{x}\right)\right)\right. \\
&\left.\left. + \frac{1}{2}P\left(x_R^2 < \bar{x}\right)P\left(x_N^2 < \bar{x}\right)\right)\right]
\end{aligned}$$

Using the short notations $P(x > \bar{x}) = s$, $P(x < z) = w$ and $P(x > x^*) = r$, simplifying and subtracting the costs, this reduces to the following expected profit:

$$\left(\frac{7}{24} - \frac{1}{3}r^3 + \frac{1}{6}s^3 + \frac{1}{2}w - \frac{1}{4}w^2 - \frac{1}{6}w^3 + \frac{1}{8}w^4\right)p - (1+w)\,c \tag{15}$$

Furthermore, we have to calculate the expected profit of firm R when she is waiting for the second period without taking a draw (and faces a drawing firm N):

$$\begin{aligned}
&\left[\frac{1}{2}P\left(x_N^1>\bar{x}\right)P\left(x_R^2>\bar{x}\right)+P\left(x_N^1<\bar{x}\right)\left(\frac{1}{3}P\left(x_R^2<\bar{x}\right)P\left(x_N^2<\bar{x}\right)\right.\right.\\
&\left.\left.+P\left(x_R^2>\bar{x}\right)\left(P\left(x_N^2<\bar{x}\right)+\frac{1}{2}P\left(x_N^2>\bar{x}\right)\right)\right)\right]p-c\\
=&\left[\frac{1}{3}+\frac{1}{6}s^3\right]p-c. \qquad (16)
\end{aligned}$$

Drawing in the first period is thus profitable if the value of (15) is larger than the value of (16). Comparing these two terms, we get

$$\begin{aligned}
&\left(\frac{7}{24}-\frac{1}{3}r^3+\frac{1}{6}s^3+\frac{1}{2}w-\frac{1}{4}w^2-\frac{1}{6}w^3+\frac{1}{8}w^4\right)p-(1+w)\,c\geq\left[\frac{1}{3}+\frac{1}{6}s^3\right]p-c\\
\Longleftrightarrow\quad&\left(-\frac{1}{24}-\frac{1}{3}r^3+\frac{1}{2}w-\frac{1}{4}w^2-\frac{1}{6}w^3+\frac{1}{8}w^4\right)p-wc\geq 0\\
\Longleftrightarrow\quad&-\frac{1}{24}-\frac{1}{3}r^3+\frac{1}{2}w-\frac{1}{4}w^2-\frac{1}{6}w^3+\frac{1}{8}w^4-wr\geq 0.
\end{aligned}$$

A numerical analysis shows that the left-hand side equals zero for $r\approx 0.2623$ – we call this critical value $\hat{v}$. For larger r values firm R prefers to wait for the second period to take her draw. In this case, we showed that there is an asymmetric equilibrium with firm N drawing in the first and firm R drawing in the second period. Firm N then follows her second-period equilibrium strategy.

For smaller r values, firm R takes a draw in the first period as well. To confirm that this constellation is consistent with an equilibrium behavior, we have to check the incentives of firm N to take a draw in this case. If she does not take a draw, her expected profit is

$$\begin{aligned}
&\left[P\left(z<x_R^1<x^*\right)\left(P\left(x_N^2>x^*\right)+\frac{1}{2}P\left(z<x_N^2<x^*\right)\right)\right.\\
&+P\left(x_R^1<z\right)\left(\frac{1}{3}P\left(x_N^2<z\right)P\left(x_R^2<z\right)\right.\\
&\left.\left.+\;P\left(x_N^2>z\right)\left(P\left(x_R^2<z\right)+\frac{1}{2}P\left(x_R^2>z\right)\right)\right)\right]p-P\left(x_R^1<x^*\right)c\\
=&\left[\frac{1}{2}-\frac{1}{2}w+\frac{1}{2}w^2-\frac{1}{6}w^3-\frac{1}{2}r^2\right]p-(1-r)c. \qquad (17)
\end{aligned}$$

We compare this with the expected profit of taking a draw. As part of (15), we already calculated the probability that firm R wins the contest in case both firms take a draw in the first round. Consequently, this number and the probability that firm N wins this contest add up to one. Hence, firm N makes an expected profit according to the following expression:

$$\left[\frac{17}{24}+\frac{1}{3}r^3-\frac{1}{6}s^3-\frac{1}{2}w+\frac{1}{4}w^2+\frac{1}{6}w^3-\frac{1}{8}w^4\right]p-\left(2-s+\frac{1}{2}r^2\right)c. \qquad (18)$$

Comparing (18) with (17), we get the following condition for a profitable draw in the first period:

$$\begin{aligned}
&\left[\frac{17}{24}+\frac{1}{3}r^3-\frac{1}{6}s^3-\frac{1}{2}w+\frac{1}{4}w^2+\frac{1}{6}w^3-\frac{1}{8}w^4\right]p-(2-s+\frac{1}{2}r^2)c\\
&\geq\left[\frac{1}{2}-\frac{1}{2}w+\frac{1}{2}w^2-\frac{1}{6}w^3-\frac{1}{2}r^2\right]p-(1-r)c\\
\Longleftrightarrow\ &\frac{5}{24}-\frac{1}{6}r^3+\frac{1}{3}w^3-\frac{1}{4}w^2-\frac{1}{8}w^4-\frac{1}{6}s^3-\frac{1}{2}r^2-r+rs\geq 0.
\end{aligned}$$

Again, a numerical analysis shows that the left hand side equals zero for $r \approx 0.2939$. For smaller r values, the inequality is fulfilled and drawing in the first period is profitable for firm N – we found an equilibrium in that case. For larger r values, firm N's best reply is not to draw in the first round. We thus have to check how firm R's best reply against a waiting firm N looks like (with respect to correct beliefs). Note that firm R will only draw again in the second-period equilibrium if $x_R^1 < \bar{x}$. Hence, incentives to draw are similar to the case of full information release and result in condition (9). The analysis of that condition showed that it is thus profitable for firm R to draw against a waiting firm N. Finally, we have to analyze the incentives of the waiting firm N – is it profitable to draw against a drawing firm R who believes to face a firm N that does not draw? The expected profit of drawing can be calculated as follows:

$$\begin{aligned}
&\Bigg[\frac{1}{2}P\left(x_R^1>x^*\right)P\left(x_N^1>x^*\right)+P\left(\bar{x}<x_R^1<x^*\right)\Big(P\left(x_N^1>x^*\right)\\
&+P\left(\bar{x}<x_N^1<x^*\right)\left(\frac{1}{2}+\frac{1}{2}\left(P\left(x_N^2>x^*\right)+\frac{1}{3}P\left(\bar{x}<x_N^2<x^*\right)\right)\right)\Big)\\
&P\left(x_R^1<\bar{x}\right)\left(\frac{1}{2}P\left(x_N^1<\bar{x}\right)+P\left(x_N^1>\bar{x}\right)\left(\frac{1}{2}P\left(x_R^2>\bar{x}\right)+P\left(x_R^2<\bar{x}\right)\right)\right)\Bigg]p\\
&-\left(2-s+\frac{1}{2}r^2\right)c\\
=&\left[\frac{1}{2}-\frac{1}{2}s^2+\frac{2}{3}s^3+\frac{1}{3}r^3-\frac{1}{2}r^2s\right]p-\left(2-s+\frac{1}{2}r^2\right)c.
\end{aligned}$$

If firm N does not draw in the first period, she is in the same situation as in the right hand side of (10). We compare the expected profits of drawing and not drawing:

$$\left[\frac{1}{2}-\frac{1}{2}s^2+\frac{2}{3}s^3+\frac{1}{3}r^3-\frac{1}{2}r^2s\right]p-\left(2-s+\frac{1}{2}r^2\right)c\geq\left[\frac{1}{3}+\frac{1}{3}rs-\frac{1}{2}r^2\right]p-(1-r)\,c.$$

Simplifying and using $s=\sqrt{2r}$, we get that drawing is profitable in case

$$\frac{1}{6}-2r-\frac{1}{2}r^2-\frac{1}{6}r^3+2sr-\frac{1}{2}sr^2\geq 0.$$

This condition holds for $r < 0.1722$, as a numerical analysis shows. We call this critical value $\tilde{v}$. Given this condition, we are back in the situation where both want to draw (and our previous analysis showed that this is an equilibrium for this range of r-values). For $r > 0.1722$, firm N does not want to draw and we are hence in an equilibrium as well – the best reply for firm R against a firm N that does not draw is to draw. □

Proof of Theorem 12 We focus our analysis on the first equilibrium identified in Proposition 11. In this equilibrium, both firms take a draw in the first period and it is unique for $r < \tilde{v}$. In the initial stage zero, where firms choose whether to reveal or not, we now have to identify the best responses of the two firms. What is the best response of a firm, if the other firm chooses to play R? If she plays R as well, they share the prize in expectation and $2 - s + \frac{1}{2}r^2$ research draws are taken by each of the firms. If a firm deviates to play N, the expected costs of drawing do not change (as she still gets the same information and plays the same strategy). However, it may happen that she receives in expectation less than half of the prize after the deviation, as given by the following condition ($w = F(z)$):

$$\left[\frac{17}{24} + \frac{1}{3}r^3 - \frac{1}{6}s^3 - \frac{1}{2}w + \frac{1}{4}w^2 + \frac{1}{6}w^3 - \frac{1}{8}w^4\right] p - \left(2 - s + \frac{1}{2}r^2\right) c = \frac{1}{2}p, \tag{19}$$

The left-hand side of (19) states the profit for the firm deviating to N, as derived in (18). A r-value of $\bar{v} \approx 0.2325$ solves (19) (which is equivalent to (6)), and for $r < \bar{v}$ a firm playing N against R receives in expectation less than half the share of the total prize. Combined with the fact that research costs do not change, it is the best response against a firm playing R to play R as well for these values.

What is the best response against a firm playing N? Playing N as well gives in expectation half of the prize while taking $1 + w$ draws. As we have just seen, a firm playing R receives in expectation more than half the prize against a firm playing N for $r < \bar{v}$. Additionally, she has to take the same number of draws in expectation. Hence, it is profitable to play R against a firm playing N.

A firm will thus always play reveal in the initial stage, no matter whether the other firm plays reveal as well or not. □

Proof of Theorem 13 We first show that it is in fact an equilibrium. Note that the point of revealing (or not revealing) is to make the other firm stop researching in as many cases as possible. Suppose firm i deviates and does not reveal her value. This deviation cannot be beneficial: if firm j has a value $x_j > x^*$, the reaction of this firm does not change – she always stops researching in this case. Additionally, if $x_j < x^*$, firm j will continue to do research, and thus goes on in the maximum number of cases. Revealing a value $x_i < x^*$ would have made a firm with value $x_j \in (x_i, x^*)$ stop researching, increasing the chances of firm i to win.

To show the uniqueness, suppose there is another equilibrium in which at least one firm hides a value different from x^*. Consider the strategy of firm i, and first assume that this firm always keeps the information secret in case $x_i \in X_1 \subset (x^*, b]$ (and reveals her value for $x_i \notin X_1$). Thus, in equilibrium, if firm j observes that firm i does not reveal any information, she correctly believes that $x_i > x^*$. Consequently, firm j stops innovating, no matter what value her first-period innovation has. This provides firm i with an incentive to always keep her information secret, as this will make firm j stop. Hence, in any equilibrium where information is kept secret for values in X_1, this has to be done also for some values

$x_i \in X_2 \subset [0, x^*]$. Furthermore, X_2 has to be large enough such that firm j continues to innovate for some values x_j when receiving no information by firm i (and believing correctly that $x_i \in X_1 \cup X_2$). However, if $x_i \in X_1$, firm i has a profitable deviation by simply revealing her value and making firm j stop innovating in any case. Thus, there cannot be an equilibrium in which firm i with value $x_i \in (x^*, b]$ keeps this value secret.

A similar reasoning applies in case we assume that information is kept secret only for values $x_i \in X_3 \subset [0, \bar{x}]$ – firm j with any $x_j \in (\bar{x}, x^*)$ would stop innovating, and firm i with $x_i \in (\bar{x}, x^*)$ had an incentive to keep her information secret and make firm j stop for these x_j. Additionally, consider the case of a set $X_4 \subset (\bar{x}, x^*)$ for which values are kept secret on top of X_3 (making some $x_j \in (\bar{x}, x^*)$ continue to innovate): then, it is profitable for $x_i \in X_3$ to reveal and make firm j stop innovating for all $x_j \in (\bar{x}, x^*)$. Thus, there cannot be an equilibrium in which firm i with value $x_i \in [0, \bar{x}]$ keeps this value secret.

Finally, consider the case where information is kept secret by firm i for values $x_i \in X_5 \subset (\bar{x}, x^*)$. Then, firm j will continue to innovate for all $x_j < \inf X_5$ if she does not observe any information by firm i. As a firm i with a value $x_i \in X_5$ decides to keep her information secret, firm i in equilibrium cannot be better off by revealing (and making firm j continue for all $x_j < x_i$). Thus, there can be no set X_6 with $x_j \in X_6$ continuing to innovate in equilibrium and $X_6 \cap (\inf X_5, x^*)$ having a positive mass. Otherwise, there would be some $x'_j \in X_6 \cap (\inf X_5, x^*)$ dividing this set in two parts with a positive mass. Consequently, some $x_i \in X_5 \cap (\inf X_5, x'_j)$ would exist for which firm i had a profitable deviation by revealing her type (and making firm j stop innovating in the part above x'_j). This shows that in equilibrium firm j does not continue to innovate for all $x_j \in (\inf X_5, x^*)$, if she does not observe information by firm i. Keeping this in mind, we can conclude that X_5 is in fact an interval of the form $(\inf X_5, x^*)$ (possibly including the end points, which we ignore for notational purpose). Suppose this were not the case. Then, there is some $x_i \in (\inf X_5, x^*)$ for which firm i would reveal her value. However, she could do strictly better for that value by keeping the information secret and making firm j stop innovating for all $x_j \in (\inf X_5, x_i)$.

So suppose firm i keeps the information secret for such an interval, $(x'_i, x^*) \neq \emptyset$. Then, as we just showed, firm j does not continue to innovate for values $x_j \in (x'_i, x^*)$ if she does not observe any information. Consider some $x''_i \in (x'_i, x^*)$. From the equilibrium derivation in case of full information revelation we know that any $x_j \in (x'_i, x''_i)$ makes a positive profit against x''_i by continuing to innovate. We denote the average expected profit of drawing for firm j against values in (x''_i, x^*) by δ (it is independent of the size of x_j, as long as $x_j < x''_i$). Against all values in (x_j, x''_i), the expected profit is even larger than δ. Now consider some fixed $x_j < x''_i$ for which the probability that firm j is in the lead if she does not receive any information is less or equal to ε. If firm j would deviate for x_j and continue to innovate, this would have two effects: on the one hand, she would make an expected profit of at least δ against firm i having a higher valuation (up to x^*). On the other hand, she could maximally waste the cost of drawing c if she faces a firm i with a value in (x_i', x_j), as the one-sided deviation of an additional draw cannot make firm j loose more often. This only happens with probability ε. Thus, innovating is profitable for firm j with value $x_j \in (x_i', x_i'')$, if the following condition holds:

$$(1-\varepsilon)\delta - \varepsilon c > 0$$

As this condition is fulfilled for ε small enough, firm j has the profitable deviation to continue innovating. Thus our initial assumption is not true and we cannot have an equilibrium where any firm keeps the information secret for values other than x^*. In case their first-period value is x^*, firms are indifferent between revealing or not – but this event has zero probability. □

Proof of Proposition 14

Suppose first the leading firm H does not innovate in the second period. Then, it is (weakly) beneficial for firm L to innovate again iff

$$P\left(x_L^2 > x_H^1\right)p - c \geq 0 \iff \left(1 - F_L\left(x_H^1 | x_L^1\right)\right)p - c \geq 0 \iff F_L\left(x_H^1 | x_L^1\right) \leq 1 - \frac{c}{p}$$
$$\iff x_H^1 \leq 1 - (1 - x_L^1)r$$

For firm H, there is only a possible need of continuing to innovate if the other firm is also innovating (otherwise, firm H would win for sure anyway). Thus, firm H will do so iff

$$\begin{aligned} & \left[P\left(x_H^2 > x_L^2 > x_H^1\right) + P\left(x_H^1 > x_L^2\right)\right]p - c \geq P\left(x_H^1 > x_L^2\right)p \\ \iff & \frac{1}{2}\left(1 - F_L\left(x_H^1 | x_L^1\right)\right)p - c \geq 0 \\ \iff & F_L\left(x_H^1 | x_L^1\right) \leq 1 - 2\frac{c}{p} \\ \iff & x_H^1 \leq 1 - (1 - x_L^1)2r \end{aligned}$$

The second line follows as firm H will always improve her first period innovation and beats a firm L that also improves upon x_H^1 in exactly half of the cases because we assumed a uniform distribution.

Finally, we have to check that firm L has no incentives to refrain from innovating in the second period in the range of values where firm H innovates as well:

$$P\left(x_L^2 > x_H^2\right)p - c \geq 0 \iff \frac{1}{2}\left(1 - F_L\left(x_H^1 | x_L^1\right)\right)p - c \geq 0$$

which is the same condition as for firm H – both firms continuing to innovate is thus an equilibrium if this condition is fulfilled. □

Proof of Theorem 15

We check whether firm i has an incentive to deviate for a value x_i^1. Suppose first that $x_i^1 < x^* = 1 - r$. Then, hiding the value makes firm j continue to innovate for a strictly larger set of first-period values: Proposition 14 shows that firm j makes a second innovation for $x_j^1 \in [0, 1-(1-x^*)2r]$, which is a superset of the set of values for which firm j would draw if she knew x_i^1, $[0, 1-(1-x_i^1)2r]$. Firm i has thus no incentive to hide the value. The more interesting case is given by $x_i^1 > x^*$. We first pin down the expected profit of firm i with given value x_i^1 when both firms reveal their true value. Applying Proposition 14 to determine the ranges for which the two firms continue to innovate and the resulting winning probabilities, the expected profit amounts to

$$\begin{aligned}
&\Bigg[P\left(x_j^1 < 1-\frac{1-x_i^1}{r}\right) + P\left(1-\frac{1-x_i^1}{r} < x_j^1 < 1-\frac{1-x_i^1}{2r} \wedge x_j^2 < x_i^1\right) \\
&\quad + P\left(1-\frac{1-x_i^1}{2r} < x_j^1 < x_i^1 \wedge \left(x_j^2 < x_i^1 \vee x_i^1 < x_j^2 < x_i^2\right)\right) \\
&\quad + P\left(x_i^1 < x_j^1 < 1-(1-x_i^1)2r \wedge x_i^2 > x_j^2\right) \\
&\quad + P\left(1-(1-x_i^1)2r < x_j^1 < 1-(1-x_i^1)r \wedge x_i^2 > x_j^1\right)\Bigg]\, p \\
&\quad - P\left(1-\frac{1-x_i^1}{2r} < x_j^1 < 1-(1-x_i^1)r\right) c \\
=& \Bigg[F\left(1-\frac{1-x_i^1}{r}\right) + \int_{1-\frac{1-x_i^1}{r}}^{1-\frac{1-x_i^1}{2r}} F_j\left(x_i^1|x_j^1\right) f\left(x_j^1\right)\, \mathrm{d}x_j^1 \\
&\quad + \int_{1-\frac{1-x_i^1}{2r}}^{x_i^1} \left(F_j\left(x_i^1|x_j^1\right) + \frac{1}{2}\left(1-F_j\left(x_i^1|x_j^1\right)\right)\right) f\left(x_j^1\right)\, \mathrm{d}x_j^1 \\
&\quad + \int_{x_i^1}^{1-(1-x_i^1)2r} \frac{1}{2}\left(1-F_i\left(x_j^1|x_i^1\right)\right) f\left(x_j^1\right)\, \mathrm{d}x_j^1 \\
&\quad + \int_{1-(1-x_i^1)2r}^{1-(1-x_i^1)r} \left(1-F_i\left(x_j^1|x_i^1\right)\right) f\left(x_j^1\right)\, \mathrm{d}x_j^1\Bigg]\, p \\
&\quad - \left[F\left(1-(1-x_i^1)r\right) - F\left(1-\frac{1-x_i^1}{2r}\right)\right] c
\end{aligned}$$

If firm i hides her own value, firm j believes that firm i has value x^* and acts accordingly. However, as we want to look at the one-sided deviation of firm i, firm j still reveals her value. Depending on the size of x_i^1, the decision whether to innovate in the second period or not changes. To write down the expected profit of firm i when hiding her value, we thus have to make a case distinction.

First case. We start with the case $x_i^1 < 1-(1-x^*)2r = 1-2r^2$, such that firm i will make the following expected profit:

$$\begin{aligned}
&\Bigg[P\left(0 < x_j^1 < 1-\frac{1-x_i^1}{2r} \wedge x_j^2 < x_i^1\right) \\
&\quad + P\left(1-\frac{1-x_i^1}{2r} < x_j^1 < x_i^1 \wedge \left(x_j^2 < x_i^1 \vee x_i^1 < x_j^2 < x_i^2\right)\right) \\
&\quad + P\left(x_i^1 < x_j^1 < 1-(1-x^*)2r \wedge x_i^2 > x_j^2\right) \\
&\quad + P\left(1-(1-x^*)2r < x_j^1 < 1-(1-x_i^1)r \wedge x_i^2 > x_j^1\right)\Bigg]\, p \\
&\quad - P\left(1-\frac{1-x_i^1}{2r} < x_j^1 < 1-(1-x_i^1)r\right) c
\end{aligned}$$

$$= \left[\int_0^{1-\frac{1-x_i^1}{2r}} F_j\left(x_i^1|x_j^1\right) f\left(x_j^1\right) \mathrm{d}x_j^1 + \int_{1-\frac{1-x_i^1}{2r}}^{x_i^1} \left(F_j\left(x_i^1|x_j^1\right) + \frac{1}{2}\left(1 - F_j\left(x_i^1|x_j^1\right)\right) \right) f\left(x_j^1\right) \mathrm{d}x_j^1 \right.$$

$$\left. + \int_{x_i^1}^{1-2r^2} \frac{1}{2}\left(1 - F_i\left(x_j^1|x_i^1\right)\right) f\left(x_j^1\right) \mathrm{d}x_j^1 + \int_{1-2r^2}^{1-(1-x_i^1)r} \left(1 - F_i\left(x_j^1|x_i^1\right)\right) f\left(x_j^1\right) \mathrm{d}x_j^1 \right] p$$

$$- \left[F\left(1-(1-x_i^1)r\right) - F\left(1 - \frac{1-x_i^1}{2r}\right) \right] c$$

It is thus not profitable to hide the own value, iff

$$\left[F\left(1 - \frac{1-x_i^1}{r}\right) + \int_{1-2r^2}^{1-(1-x_i^1)2r} \frac{1}{2}\left(1 - F_i\left(x_j^1|x_i^1\right)\right) f\left(x_j^1\right) \mathrm{d}x_j^1 \right.$$

$$\left. - \int_0^{1-\frac{1-x_i^1}{r}} F_j\left(x_i^1|x_j^1\right) f\left(x_j^1\right) \mathrm{d}x_j^1 - \int_{1-2r^2}^{1-(1-x_i^1)2r} \left(1 - F_i\left(x_j^1|x_i^1\right)\right) f\left(x_j^1\right) \mathrm{d}x_j^1 \right] p \geq 0$$

$$\iff 1 - \frac{1-x_i^1}{r} - \int_{1-2r^2}^{1-(1-x_i^1)2r} \frac{1}{2}\left(1 - \frac{x_j^1 - x_i^1}{1-x_i^1}\right) \mathrm{d}x_j^1 - \int_0^{1-\frac{1-x_i^1}{r}} \frac{x_i^1 - x_j^1}{1-x_j^1} \mathrm{d}x_j^1 \geq 0 \tag{20}$$

First note that $x_j^1 \geq x_i^1$ on $[1-2r^2, 1-(1-x_i^1)2r]$. Hence, we can estimate

$$\int_{1-2r^2}^{1-(1-x_i^1)2r} \frac{1}{2}\left(1 - \frac{x_j^1 - x_i^1}{1-x_i^1}\right) \mathrm{d}x_j^1 < \int_{1-2r^2}^{1-(1-x_i^1)2r} \frac{1}{2} \mathrm{d}x_j^1 = r(r-(1-x_i^1))$$

Furthermore, $\frac{x_i^1 - x_j^1}{1-x_j^1}$ is decreasing in x_j^1, thus

$$\int_0^{1-\frac{1-x_i^1}{r}} \frac{x_i^1 - x_j^1}{1-x_j^1} \mathrm{d}x_j^1 < \int_0^{1-\frac{1-x_i^1}{r}} x_i^1 \,\mathrm{d}x_j^1 = \frac{x_i^1}{r}(r-(1-x_i^1))$$

As $x_i^1 > 1-r$, the condition (20) is thus fulfilled if

$$\left(\frac{1}{r} - r - \frac{x_i^1}{r}\right)(r-(1-x_i^1)) > 0 \iff x_i^1 < 1-r^2.$$

This is true, as by assumption $x_i^1 < 1-2r^2 < 1-r^2$. We can conclude that firm i does not want to deviate and hide her value for $x_i^1 < 1-2r^2$.

Second case. The next case is $1-4r^3 > x_i^1 > 1-2r^2$. The condition stems from requiring $1 - \frac{1-x_i^1}{2r} < 1-2r^2$. Furthermore, the relationship $1-4r^3 > 1-2r^2$ is always fulfilled as

$r < \frac{1}{2}$. If firm i hides her value, she makes the following expected profit:

$$
\begin{aligned}
&\Bigg[P\left(0 < x_j^1 < 1 - \frac{1-x_i^1}{2r} \wedge x_j^2 < x_i^1\right) \\
&\quad + P\left(1 - \frac{1-x_i^1}{2r} < x_j^1 < 1 - 2r^2 \wedge \left(x_j^2 < x_i^1 \vee x_i^1 < x_j^2 < x_i^2\right)\right) \\
&\quad + P\left(1 - 2r^2 < x_j^1 < x_i^1\right) + P\left(x_i^1 < x_j^1 < 1 - (1-x_i^1)r \wedge x_i^2 > x_j^1\right)\Bigg] p \\
&\quad - \left[P\left(1 - \frac{1-x_i^1}{2r} < x_j^1 < 1 - 2r^2\right) + P\left(x_i^1 < x_j^1 < 1 - (1-x_i^1)r\right)\right] c \\
= &\Bigg[\int_0^{1-\frac{1-x_i^1}{2r}} F_j\left(x_i^1 | x_j^1\right) f\left(x_j^1\right) \, \mathrm{d}x_j^1 + \int_{1-\frac{1-x_i^1}{2r}}^{1-2r^2} \left(F_j\left(x_i^1|x_j^1\right) + \frac{1}{2}\left(1 - F_j\left(x_i^1|x_j^1\right)\right)\right) f\left(x_j^1\right) \, \mathrm{d}x_j^1 \\
&\quad + F(x_i^1) - F(1-2r^2) + \int_{x_i^1}^{1-(1-x_i^1)r} \left(1 - F_i\left(x_j^1|x_i^1\right)\right) f\left(x_j^1\right) \, \mathrm{d}x_j^1\Bigg] p \\
&\quad - \left[F\left(1 - (1-x_i^1)r\right) - F(x_i^1) + F(1-2r^2) - F\left(1 - \frac{1-x_i^1}{2r}\right)\right] c
\end{aligned}
$$

Thus, hiding is not profitable iff

$$
\begin{aligned}
&\Bigg[1 - \frac{1-x_i^1}{r} - \int_0^{1-\frac{1-x_i^1}{r}} \frac{x_i^1 - x_j^1}{1-x_j^1} \, \mathrm{d}x_j^1 + \int_{1-2r^2}^{x_i^1} \left(\frac{x_i^1 - x_j^1}{1-x_j^1} + \frac{1}{2} \cdot \frac{1-x_i^1}{1-x_j^1}\right) \mathrm{d}x_j^1 \\
&\qquad - x_i^1 + 1 - 2r^2 - \int_{x_i^1}^{1-(1-x_i^1)2r} \frac{1}{2} \cdot \frac{1-x_j^1}{1-x_i^1} \, \mathrm{d}x_j^1\Bigg] p - \left[x_i^1 - 1 + 2r^2\right] c \\
&\geq 0
\end{aligned}
$$

By calculating the integrals and simplifying, the condition boils down to

$$
2r - \frac{1}{2} - \frac{1}{2}\ln(2(1-x_i^1)) + \frac{1}{4}(1-2r)^2 - \frac{2r^3}{1-x_i^1} \geq 0
$$

Thus, by using $1 - 2r^2 < x_i^1 < 1 - 4r^3$ we can get a lower bound of the left hand side and formulate the following sufficient condition for hiding to be non-profitable:

$$
\begin{aligned}
& 2r - \frac{1}{2} - \frac{1}{2}\ln(2 \cdot 2r^2) + \frac{1}{4}(1-2r)^2 - \frac{2r^3}{4r^3} \geq 0 \\
\iff \quad & 2r - \ln(2r) + \frac{1}{4}(1-2r)^2 \geq 1
\end{aligned}
$$

It remains to show that this condition is fulfilled. For $r = \frac{1}{2}$, it is obviously fulfilled with equality. We show that the left hand side is decreasing on $(0, \frac{1}{2})$ by looking at its first derivative with respect to r:

$$
2 - \frac{1}{r} - (1-2r) \leq 0 \iff 1 - r - 2r^2 \geq 0,
$$

which is true for $0 < r < \frac{1}{2}$.

Third case. The remaining case is $x_i^1 > 1 - 4r^3$. The expected profit of firm i from hiding the value amounts to

$$\begin{aligned}
&\left[P\left(0 < x_j^1 < 1 - 2r^2 \wedge x_j^2 < x_i^1\right) + P\left(1 - 2r^2 < x_j^1 < x_i^1\right)\right.\\
&\qquad \left. + P\left(x_i^1 < x_j^1 < 1 - (1 - x_i^1)r \wedge x_i^2 > x_j^1\right)\right] p - P\left(x_i^1 < x_j^1 < 1 - (1 - x_i^1)r\right) c\\
=&\left[\int_0^{1-2r^2} F_j\left(x_i^1 | x_j^1\right) f\left(x_j^1\right) \, \mathrm{d}x_j^1 + F\left(x_i^1\right) - F(1 - 2r^2)\right.\\
&\qquad \left. + \int_{x_i^1}^{1-(1-x_i^1)r} \left(1 - F_i\left(x_j^1 | x_i^1\right)\right) f\left(x_j^1\right) \, \mathrm{d}x_j^1\right] p - \left[F\left(1 - (1 - x_i^1)r\right) - F\left(x_i^1\right)\right] c
\end{aligned}$$

It is not profitable to hide the value iff

$$\begin{aligned}
&\left[1 - \frac{1 - x_i^1}{r} + \int_{1-\frac{1-x_i^1}{r}}^{x_i^1} \frac{x_i^1 - x_j^1}{1 - x_j^1} \, \mathrm{d}x_j^1 + \int_{1-\frac{1-x_i^1}{2r}}^{x_i^1} \frac{1}{2} \cdot \frac{1 - x_i^1}{1 - x_j^1} \, \mathrm{d}x_j^1 - \int_0^{1-2r^2} \frac{x_i^1 - x_j^1}{1 - x_j^1} \, \mathrm{d}x_j^1\right.\\
&\left. - \int_{x_i^1}^{1-(1-x_i^1)2r} \frac{1}{2} \cdot \frac{1 - x_j^1}{1 - x_i^1} \, \mathrm{d}x_j^1 - F\left(x_i^1\right) + F(1 - 2r^2)\right] p - \left[F\left(x_i^1\right) - F\left(1 - \frac{1 - x_i^1}{2r}\right)\right] c\\
\geq& 0
\end{aligned}$$

Again, we calculate the integrals and simplify, to finally get the condition

$$\frac{3}{2} \ln(2r) + (1 - 2r) \left(\frac{3}{4} + \frac{1}{2} r\right) \leq 0.$$

It is fulfilled for $r = \frac{1}{2}$ and increasing on $(0, \frac{1}{2})$, as we can confirm by looking at the first derivative:

$$\frac{3}{2r} - 1 - 2r \geq 0 \iff \frac{3}{2} - r - 2r^2 \geq 0,$$

which is true for $0 < r < \frac{1}{2}$. The proof is thus complete – it is never profitable for one of the firms to deviate and hide the own value. □

Proof of Theorem 17

We show that firm i with value x_i^1 has no incentive to hide her value if all other firms reveal. This is easy to see in case $x_i^1 > x^*$: revealing the value will make all opponents stop innovating. In case $x_i^1 < \hat{x}$, there is no point in hiding – no other firm will be discouraged from drawing if her beliefs of firm i increase to $\hat{x}$ compared to x_i^1. Quite the contrary, for some values it could make a leading firm j with $\hat{x} > x_j^1 > x_i^1$ continue to innovate although she would have stopped if she knew the true value of firm i.

Let us now suppose $\hat{x} \leq x_i^1 \leq x^*$, the remaining case to show. If firm i is not leading, it does not make a difference whether she reveals or not as her value has no influence on the innovation behavior of the other firms. We thus have to check what happens if firm i is in the lead. First note that by Proposition 16 she will not continue to innovate then. If she does not hide her value, she thus wins the contest in case no other firm draws or all drawing firms have a lower value. Hence, her winning probability π can be written as

$$\pi = P\left(\max_{j\neq i} x_j^2 < x_i^1\right) = \sum_{l=0}^{n-1} \binom{n-1}{l} \left(1 - q\left(x_i^1\right)\right)^{n-1-l} \cdot q\left(x_i^1\right)^l \cdot F\left(x_i^1\right)^l .$$

If the leading firm i hides her value, the other firms believe she has value $\hat{x}$. If the second highest first-period innovation value is lower than $\hat{x}$, and firm i is thus still believed to be the leading firm, all other firms continue to innovate for sure. If it is higher, drawing behavior depends on its exact value. The winning probability of a hiding firm i, π_h, is given by

$$\begin{aligned}\pi_h &= P\left(\max_{j\neq i} x_j^2 < x_i^1\right) \\ &= F\left(\hat{x}\right)^{n-1} F\left(x_i^1\right)^{n-1} \\ &\quad + P\left(\hat{x} < \max_{j\neq i} x_j^1 < x_i^1\right) \sum_{l=0}^{n-2} \binom{n-2}{l} \left(1 - q\left(\max_{j\neq i} x_j^1\right)\right)^{n-2-l} q\left(\max_{j\neq i} x_j^1\right)^l F\left(x_i^1\right)^l .\end{aligned}$$

We need to show that $\pi > \pi_h$. To do this, first note that

$$\begin{aligned}P\left(\hat{x} < \max_{j\neq i} x_j^1 < x_i^1\right) &< P\left(\hat{x} < \max_{j\neq i} x_j^1 < x^*\right) \\ &= F(x^*)^{n-1} - F(\hat{x})^{n-1} \\ &= (1-r)^{n-1} - (1-(n-1)r).\end{aligned}$$

Furthermore, we have $q\left(\max_{j\neq i} x_j^1\right) \geq q\left(x_i^1\right)$. If the same number of firms draws more often (with higher probability), this reduces the winning probability of the leading firm. Hence,

$$\begin{aligned}&\sum_{l=0}^{n-2} \binom{n-2}{l} \left(1 - q\left(\max_{j\neq i} x_j^1\right)\right)^{n-2-l} q\left(\max_{j\neq i} x_j^1\right)^l F\left(x_i^1\right)^l \\ \leq &\sum_{l=0}^{n-2} \binom{n-2}{l} \left(1 - q\left(x_i^1\right)\right)^{n-2-l} q\left(x_i^1\right)^l F\left(x_i^1\right)^l\end{aligned}$$

and we can conclude that

$$\begin{aligned}\pi_h &\leq (1-(n-1)r) F\left(x_i^1\right)^{n-1} \\ &\quad + \left((1-r)^{n-1} - (1-(n-1)r)\right) \sum_{l=0}^{n-2} \binom{n-2}{l} \left(1 - q\left(x_i^1\right)\right)^{n-2-l} q\left(x_i^1\right)^l F\left(x_i^1\right)^l .\end{aligned}$$

Hence, to get $\pi > \pi_h$ it is sufficient to show

$$\begin{aligned}&\sum_{l=0}^{n-1} \binom{n-1}{l} \left(1 - q\left(x_i^1\right)\right)^{n-1-l} \cdot q\left(x_i^1\right)^l \cdot F\left(x_i^1\right)^l - \Bigg[(1-(n-1)r) F\left(x_i^1\right)^{n-1} \\ &\quad + \left((1-r)^{n-1} - (1-(n-1)r)\right) \sum_{l=0}^{n-2} \binom{n-2}{l} \left(1 - q\left(x_i^1\right)\right)^{n-2-l} q\left(x_i^1\right)^l F\left(x_i^1\right)^l \Bigg] > 0.\end{aligned} \tag{21}$$

We prove this statement by an induction argument, where we keep $F(x_i^1)$ fixed and take $q = q(x_i^1) \in [0,1]$ as variable. This approach does not use all available information, as it ignores the dependence of $q(x_i^1)$ and $F(x_i^1)$, but it is sufficient for our purposes.
We start with the basis, $n = 3$. For $q = 1$ the left-hand side of (21) boils down to

$$2rF\left(x_i^1\right)^2 - r^2 F\left(x_i^1\right) > 0 \iff 2F\left(x_i^1\right) > r. \tag{22}$$

Note that the range of possible x_i^1 values in $[\hat{x}, x^*]$ depends on n and r. We thus need to make sure that the basis holds for all these combinations: (22) is true for $r < \frac{1}{n}$ and

$$F\left(x_i^1\right) > \sqrt[n-1]{1-(n-1)\frac{1}{n}} = \sqrt[n-1]{\frac{1}{n}} > \frac{1}{n} > r.$$

For $q \in [0,1)$, we show that the left-hand side of (21) is monotone in q by looking at its first derivative with respect to q, which is given by

$$\begin{aligned}
&-2(1-q) + r^2 + F\left(x_i^1\right)\left(2 - 4q - r^2\right) + 2qF\left(x_i^1\right)^2 \\
&\quad = \left(2 - r^2 - 2q\left(1 - F\left(x_i^1\right)\right)\right)\left(F\left(x_i^1\right) - 1\right) \\
&\quad < 0.
\end{aligned}$$

The last step holds as $2 - r^2 - 2q\left(1 - F\left(x_i^1\right)\right) > 2F\left(x_i^1\right) - r^2 > 0$, which we already showed above. Hence, the left-hand side of (21) is decreasing in q, and as it is positive for $q = 1$, it is positive on the whole range.
We now get to the inductive step. Suppose we know that (21) is true for n firms. By multiplying with $1 - q + qF\left(x_i^1\right)$ (this is the change in π when adding another firm), (21) is equivalent to

$$\begin{aligned}
&\Bigg\{\sum_{l=0}^{n-1}\binom{n-1}{l}\left(1-q\left(x_i^1\right)\right)^{n-1-l}\cdot q\left(x_i^1\right)^l\cdot F\left(x_i^1\right)^l - \Big[(1-(n-1)r)F\left(x_i^1\right)^{n-1} \\
&\qquad + \left((1-r)^{n-1} - (1-(n-1)r)\right)\sum_{l=0}^{n-2}\binom{n-2}{l}\left(1-q\left(x_i^1\right)\right)^{n-2-l} q\left(x_i^1\right)^l F\left(x_i^1\right)^l\Big]\Bigg\} \\
&\qquad \cdot\left(1-q+qF\left(x_i^1\right)\right) \\
&> 0 \\
\iff &\Bigg\{\sum_{l=0}^{n}\binom{n}{l}\left(1-q\left(x_i^1\right)\right)^{n-l}\cdot q\left(x_i^1\right)^l\cdot F\left(x_i^1\right)^l \\
&\qquad - \Big[(1-(n-1)r)F\left(x_i^1\right)^{n-1}\cdot\left(1-q+qF\left(x_i^1\right)\right) \\
&\qquad + \left((1-r)^{n-1} - (1-(n-1)r)\right)\sum_{l=0}^{n-1}\binom{n-1}{l}\left(1-q\left(x_i^1\right)\right)^{n-1-l} q\left(x_i^1\right)^l F\left(x_i^1\right)^l\Big]\Bigg\} \\
&> 0
\end{aligned}$$

Next, we subtract the left-hand side of this equation from the left-hand side of (21) with $n+1$ firms, which amounts to

$$
\begin{aligned}
&(1-(n-1)r)F\left(x_i^1\right)^{n-1}\cdot\left(1-q+qF\left(x_i^1\right)\right)-(1-nr)F\left(x_i^1\right)^{n}\\
&\quad-\left((1-r)^n-(1-nr)-(1-r)^{n-1}+(1-(n-1)r)\right)\\
&\quad\cdot\sum_{l=0}^{n-1}\binom{n-1}{l}\left(1-q\left(x_i^1\right)\right)^{n-1-l}q\left(x_i^1\right)^{l}F\left(x_i^1\right)^{l}\\
&=(1-(n-1)r)F\left(x_i^1\right)^{n-1}\left(1-q+qF\left(x_i^1\right)-F\left(x_i^1\right)\right)+rF\left(x_i^1\right)^{n}\\
&\quad-\left((1-r)^{n-1}(1-r-1)-r\right)\cdot\sum_{l=0}^{n-1}\binom{n-1}{l}\left(1-q\left(x_i^1\right)\right)^{n-1-l}q\left(x_i^1\right)^{l}F\left(x_i^1\right)^{l}\\
&=(1-(n-1)r)F\left(x_i^1\right)^{n-1}\left((1-q)\left(1-F\left(x_i^1\right)\right)\right)+rF\left(x_i^1\right)^{n}\\
&\quad+\left((1-r)^{n-1}r+r\right)\cdot\sum_{l=0}^{n-1}\binom{n-1}{l}\left(1-q\left(x_i^1\right)\right)^{n-1-l}q\left(x_i^1\right)^{l}F\left(x_i^1\right)^{l}\\
&>0
\end{aligned}
$$

As the difference is positive, we showed that (21) is fulfilled for $n+1$ as well. This completes the inductive step and the proof. □