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Preemptive Behavior in Sequential-Move Tournaments with Heterogeneous Agents*

Peter-J. Jost** and Matthias Kräkel***

Abstract

Rank-order tournaments are usually modeled simultaneously. However, real tournaments are often sequential. We show that agents' strategic behavior in sequential-move tournaments significantly differ from the one in simultaneous-move tournaments: In a sequential-move tournament with heterogeneous agents, there may be either a first-mover or a second-mover advantage. Under certain conditions the first acting agent chooses a preemptively high effort so that the following agent gives up. The principal is able to prevent preemptive behavior in equilibrium, but he will not implement first-best efforts although the agents are risk neutral.

JEL classification: J3, M12, M5.

Key words: preemption, tournaments.

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1 Introduction

Rank-order tournaments have been extensively discussed in the literature.[1] In the basic model, two agents compete for tournament prizes by choosing their effort levels simultaneously. The agent who produces the highest output receives the winner prize whereas the other agent gets the loser prize. The main result of this literature is that, under certain circumstances, the principal can design tournament prizes such that the agents choose the first-best effort level even in cases where agents' efforts and outputs are unverifiable. In these cases, standard compensation schemes like bonus payments or piece rates are non-contractible. The principal would always be able to save labor costs by claiming that the agents' outputs were low. Tournament incentive schemes, however, consists of contractible prizes that have been fixed in advance so that the principal cannot gain by misrepresenting the agents' performances (Malcomson 1984, 1986). This explains why tournaments are often observed in practice: For example, sales persons compete for bonus payments (Mantrala, Krafft and Weitz 2000). Employees compete in job promotion tournaments to reach a better paid job on a higher rank in the firm's hierarchy (e.g., Baker, Gibbs and Holmström 1994a, 1994b).[2] Managers of the same industry compete against each other in a kind of tournament due to relative performance compensation (Antle and Smith 1986, Gibbons and Murphy 1990, Eriksson 1999). Tournaments can be even observed in connection with broiler production (Knoeber 1989, Knoeber and

[1]See, e.g., Lazear and Rosen (1981), Nalebuff and Stiglitz (1983), Green and Stokey (1983), O'Keefe, Viscusi, and Zeckhauser (1984), Rosen (1986), McLaughlin (1988), Lazear (1989).

[2]The contractibility property of tournaments may explain the major puzzle of Baker, Jensen, and Murphy (1988) why promotion- instead of bonus-based incentive schemes are so often observed in practice.

Thurman 1994). Moreover, further advantages have been attributed to tournaments; especially, low measurement costs, and the filtering of common noise.

The common assumption of the previous tournament models is that the competing agents choose their effort levels simultaneously. This assumption may hold in some contexts. In many other contexts, however, agents do not decide about their efforts at the same time. Real tournaments (e.g., promotion tournaments or tournaments between salesmen) show that agents often act sequentially and may be able to observe their competitors' efforts when deciding on their own effort. Hence, the agents may get some information during the tournament, which will influence their succeeding effort choices. Obviously, these features cannot be discussed within a simultaneous-move tournament. In addition, some tournaments are even organized sequentially in practice, which holds for diverse sport contests (see, for example, Ehrenberg and Bognanno [1990a, 1990b] for an empirical analysis of golf tournaments).

In this paper, we consider tournaments in which the agents are assumed to choose their efforts sequentially. In a two-agent tournament, one of the agents first chooses his effort. After that, the other agent observes this effort and then has to decide about his own effort level. We show that this sequential-move tournament substantially differs from the standard simultaneous-move tournament. In particular, the sequential-move tournament allows for additional strategic behavior by the agents: Either, the first acting agent can use his position as Stackelberg leader to discourage the second agent. He is even able to choose a preemptively high effort level. Or, the second acting agent can use his role as Stackelberg follower to outrival the first agent. The analysis shows that, as a necessary condition for preemption, marginal costs have to be positive at the origin or, alternatively, luck has to be distributed over a finite interval.

There are parallels to the discussion of Dixit (1987) and Baye and Shin (1999) about precommitment in contests. As Dixit (1987, p. 892) states homogeneous players lead to the symmetric simultaneous-move outcome in a game where players choose their efforts sequentially. However, Baye and Shin (1999) show that this result will only hold if the contest success function satisfies a technical third-order condition. Our results highlight that sequential-move tournaments fundamentally differ from simultaneous-move ones: In our model, there will never be a symmetric equilibrium in the sequential-move tournament as it is always the case in the simultaneous-move tournament given homogeneity. Of course, there is at least one crucial difference between our model and the Dixit model: Dixit and Baye and Shin analyze contests with linear costs whereas we consider tournaments with convex cost functions.

There are also some parallels to the literature on preemptive behavior in other setups. Fudenberg et al. (1983) and Fudenberg and Tirole (1985) discuss preemptive behavior in R&D races, whereas Fishman (1988, 1989) considers preemptive takeover bidding. Our paper is most related to the literature on preemptive behavior in (rent-seeking) contests (see Leininger and Yang 1994; Baik 1998; Weimann et al. 2000).

Besides the difference with respect to the players' cost functions there is at least one further difference between contests and tournaments. Often prizes are exogenously given in contests. On the contrary, prizes are endogenous and optimally chosen by the principal in tournaments. More generally, the most important difference to the contest literature is that the principal can choose the design of the tournament. The analysis shows that considering heterogeneous tournaments the underdog will drop out of the competition if the spread between winner and loser prize is sufficiently large and that the principal prevents this outcome by choosing an optimal prize spread. Moreover, the principal does not implement first-best efforts, although both agents are risk neutral.

The paper is organized as follows: In Section 2 we introduce the general model of a sequential-move tournament. In Section 3 we derive necessary conditions for preemptive behavior in the general model. Section 4 considers the special case of uniformly distributed luck and quadratic costs in order to derive explicit solutions.[3] Section 5 concludes.

2 The general model

We follow the model of Lazear and Rosen (1981) and consider a tournament between two risk neutral agents. According to the ranks of their outputs the agents receive a winner prize w_1 or a loser prize w_2, where $w_1 > w_2$. The output q_i of agent i $(i = A, B)$ is given by the linear production function

$$q_i = e_i + \varepsilon_i \tag{1}$$

where $e_i \geq 0$ denotes agent i's effort and ε_i an exogenous error term. The error terms ε_A and ε_B are assumed to be independently and identically distributed (i.i.d. assumption). Let the difference of ε_A and ε_B be denoted by $Y := \varepsilon_B - \varepsilon_A$ with distribution function $F_Y(\cdot)$ and density $f_Y(\cdot)$. Note that the convolution $f_Y(\cdot)$ is symmetric around zero, which implies $F_Y(-y) = 1 - F_Y(y)$. The principal, who is also risk neutral, is assumed

[3] For a discussion of sequential-move tournaments in which the agents' outputs mostly depend on luck rather than effort see Jost (2000).

to observe neither e_i nor ε_i, but he can observe the unverifiable output q_i.[4] Effort e_i entails some costs for agent i. These costs can be described in monetary terms by the function $c_i(e_i)$ with $c_i(0) = 0$, $c_i'(e_i) > 0$ and $c_i''(e_i) > 0$, $\forall e_i > 0$. The subscripts of the cost functions indicate that agents are allowed to be heterogeneous. Hence, we may have a mixed tournament between a less talented agent with a steeper or more convex cost function and a more able agent whose cost function is less steep. We can define that agent i is less talented than agent j if $c_i'(e_i) > c_j'(e_j)$ and $c_i''(e_i) > c_j''(e_j)$. The principal maximizes his expected surplus, i.e. the sum of the expected outputs $(E(q_A) + E(q_B))$ minus the labor costs $w_1 + w_2$. According to this objective function he chooses appropriate tournament prizes to generate optimal incentives for the two agents. Each agent maximizes his expected tournament prize minus his effort costs $c_i(e_i)$. If agent i $(i = A, B)$ decides to participate in the tournament, he will at least receive his reservation utility $\bar{u} \geq 0$.

We consider the following three-stage game (see Figure 1).

[Figure 1]

In stage 1 the principal decides about the tournament prizes w_1 and w_2, and effort level implementation. In stage 2, agent A chooses e_A. In stage 3, agent B observes e_A and then chooses e_B. The realizations of ε_A and ε_B are not known by either agent when exerting effort. After the principal has observed q_A and q_B, the most successful agent gets w_1, whereas the other receives w_2.

[4] By the assumption of unverifiable outcomes we rule out the possibility that the principal can induce proper incentives by using individual incentive schemes like piece rates.

3 Preemptive behavior as equilibrium outcome in the general model

To focus on preemptive behavior as equilibrium outcome consider more closely the behavior of the two agents. Given agent A chooses e_A in stage 2, agent B then chooses e_B in stage 3 to maximize

$$\begin{aligned} EU_B(e_B) &= w_2 + \Delta w \cdot \text{prob}\{q_A < q_B\} - c_B(e_B) \\ &= w_2 + \Delta w \cdot [1 - F_Y(e_A - e_B)] - c_B(e_B). \end{aligned} \tag{2}$$

with $\Delta w := w_1 - w_2$ as prize spread. Let $e_B^* = e_B^*(e_A)$ describe B's best response. Agent A's objective function then is given by

$$EU_A(e_A) = w_2 + \Delta w \cdot F_Y(e_A - e_B^*) - c_A(e_A) \tag{3}$$

In order to use his first-mover position to gain a strategic advantage, A may have the opportunity to preempt the second mover B, i.e. to choose a sufficiently high effort so that B prefers to drop out of the tournament by choosing zero effort instead of competing against A. Such preemption will be an equilibrium outcome, if agent B prefers to drop out at the third stage for a given effort of agent A, and if A prefers to choose a preemptive effort at the second stage given B's reaction function. Let $e_{A,pre}$ denote agent A's preemptive effort in this case. Then, at the third stage, agent B will drop out, if

$$w_2 + \Delta w \cdot [1 - F_Y(e_{A,pre})] \geq w_2 + \Delta w \cdot [1 - F_Y(e_{A,pre} - e_B)] - c_B(e_B) \qquad , \forall e_B.$$

At the second stage, agent A will choose preemption if

$$w_2 + \Delta w \cdot F_Y(e_{A,pre}) - c_A(e_{A,pre}) \geq w_2 + \Delta w \cdot F_Y(e_A - e_B^*) - c_A(e_A) \qquad , \forall e_A.$$

Altogether, we obtain the following result:

Proposition 1 *A preemptive equilibrium* $(e_{A,pre}, 0)$ *exists if and only if*

$$\Delta w \left[F_Y\left(e_{A,pre}\right) - F_Y\left(e_{A,pre} - e_B\right)\right] \leq c_B\left(e_B\right), \forall e_B, \qquad \text{and} \qquad (4)$$

$$c_A\left(e_{A,pre}\right) - c_A\left(e_A\right) \leq \Delta w \left[F_Y\left(e_{A,pre}\right) - F_Y\left(e_A - e_B^*\right)\right], \forall e_A. \qquad (5)$$

According to condition (4), agent B will choose to give up, if the additional expected return from competing is lower or equal than the associated costs that can be saved from non-competing. Condition (5) shows that agent A prefers to choose his preemptive effort if additional costs from preemption are at most as high as the additional expected return. Note that a preemptive outcome of a tournament with heterogeneous agents will be most likely if the first mover is the favorite and the second mover the underdog: The more convex $c_B(\cdot)$ the larger the right-hand side of (4), and the less convex $c_A(\cdot)$ the smaller the left-hand side of (5). In this situation, the drop-out gains for B, $c_B(e_B)$, are quite large, whereas A's preemption costs, $c_A(e_{A,pre}) - c_A(e_A)$, are not prohibitively high.

Typically, if agent B drops out for a given value of agent A's effort, he will also give up for even larger values of A's effort so that the right-hand side of (5) is positive and we can combine conditions (4) and (5) to

$$\frac{c_A\left(e_{A,pre}\right) - c_A\left(e_A\right)}{F_Y\left(e_{A,pre}\right) - F_Y\left(e_A - e_B^*\right)} \leq \Delta w \leq \frac{c_B\left(e_B\right)}{F_Y\left(e_{A,pre}\right) - F_Y\left(e_{A,pre} - e_B\right)}, \forall e_A, \forall e_B. \qquad (6)$$

Condition (6) emphasizes that a preemptive equilibrium exists for intermediate values of the prize spread Δw. The intuition for this result can be explained by the ambiguity of Δw in case of preemption: On the one hand, a large prize spread leads to high expected returns for B from competing with A, which works against preemption. In addition, effort incentives and, therefore, effort costs will be quite high for agent A if the prize spread is large. On the other hand, a small value of Δw implies restricted gains from preemption for A, which also makes a preemptive outcome unlikely.

By inspection of (4) and (5) a necessary condition for a preemptive equilibrium can be given:

Corollary *For the existence of a preemptive equilibrium at least one of the following two conditions must hold: (1)* $c_B'(0) > 0$*, (2)* $\varepsilon_B - \varepsilon_A$ *is distributed over a finite interval.*

If $c_B'(0) = 0$ and $f_Y(e_A - e_B)$ (i.e., the marginal probability of winning) is always positive,[5] a preemptive equilibrium cannot exist. Instead of dropping out, agent B would always prefer to exert at least marginal positive effort, since for $e_B = 0$ marginal costs $c_B'(e_B)$ are zero but marginal gains $\Delta w f_Y(e_A - e_B)$ are strictly positive. In other words, condition (4) can never be met. Hence, we have two possible types of preemptive equilibria: (1) Given $e_{A,pre}$ agent B drops out, since competition would be too costly (type-I preemption with $c_B'(0) > 0$); (2) Given $e_{A,pre}$ agent B gives up, since luck is restricted and $e_{A,pre}$ shifts $f_Y(\cdot)$ out of the finite interval so that marginal gains from competition are zero for agent B (type-II preemption with $\varepsilon_B - \varepsilon_A$ being distributed over a finite interval). In either case, preemptive effort $e_{A,pre}$ has to be sufficiently high to make B give up.

Of course, the existence of preemptive equilibria in the general model introduced in Section 2 depends on the specific shape of the agents' cost functions and on the specific shape of the distribution function $F_Y(\cdot)$. Since Leininger and Yang (1994), who use Tullock's (1980) contest success function[6] and a linear cost function as specific assumptions, already considered the

[5]This holds, e.g., for the case in which ε_A and ε_B are independently and normally distributed so that $\varepsilon_B - \varepsilon_A$ also follows a normal distribution.

[6]Note that Tullock's contest success function corresponds to a probit or tournament model with exponentionally distributed noise; see, e.g., Loury (1979).

case of type-I preemption we will focus for the rest of this paper on type-II preemption.

4 A special case: quadratic costs and uniformly distributed noise

In order to analyze type-II preemption and to derive explicit solutions for the agents' equilibrium behavior, we now use concrete specifications for the probability distribution and the agents' cost functions. In particular, we assume that luck is distributed uniformly and the agents have quadratic costs $c_A(e_A) = 0.5\tau k e_A^2$ and $c_B(e_B) = 0.5 k e_B^2$ with $\tau > 0$ and $k > 0$. Here the parameter τ characterizes the types of the two agents. If $\tau = 1$, we will have a tournament with homogeneous agents. If $\tau < 1$ ($\tau > 1$), we will consider heterogeneous competition with agent A being the favorite (underdog) and B the underdog (favorite). When luck ε_i is uniformly distributed over $[-\bar{\varepsilon}, \bar{\varepsilon}]$ with $\bar{\varepsilon} > 0$, the convolution $f_Y(y)$ with $Y = \varepsilon_B - \varepsilon_A$ and $y = e_A - e_B$ is triangular with[7]

$$f_Y(y) = \begin{cases} \frac{1}{2\bar{\varepsilon}} + \frac{y}{4\bar{\varepsilon}^2} & \text{if } -2\bar{\varepsilon} \leq y \leq 0 \\ \frac{1}{2\bar{\varepsilon}} - \frac{y}{4\bar{\varepsilon}^2} & \text{if } 0 < y \leq 2\bar{\varepsilon} \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

and

$$F_Y(y) = \begin{cases} 0 & \text{if } y < -2\bar{\varepsilon} \\ \frac{y}{2\bar{\varepsilon}} + \frac{y^2}{8\bar{\varepsilon}^2} + \frac{1}{2} & \text{if } -2\bar{\varepsilon} \leq y \leq 0 \\ \frac{y}{2\bar{\varepsilon}} - \frac{y^2}{8\bar{\varepsilon}^2} + \frac{1}{2} & \text{if } 0 < y \leq 2\bar{\varepsilon} \\ 1 & \text{if } y > 2\bar{\varepsilon}. \end{cases} \tag{8}$$

[7] For construction of this convolution see analogously Kräkel (2000).

As a benchmark we can derive first-best efforts which are defined as those effort levels that maximize $E[q_i] - c_i(e_i)$. In this case, we obtain

$$e_A^{FB} = \frac{1}{\tau k} \text{ and } e_B^{FB} = \frac{1}{k} \tag{9}$$

as first-best efforts. To ensure that these efforts lie in the interval $[0, 2\bar{\varepsilon}]$, we assume throughout the paper that agents' marginal effort costs are sufficiently high at the boundary of the joint error distribution, that is $c_i'\left(e_i^{FB}\right) = 1 < c_i'(2\bar{\varepsilon})$ $(i = A, B)$, i.e.

$$1 < 2\tau k\bar{\varepsilon} \text{ and } 1 < 2k\bar{\varepsilon}. \tag{10}$$

We will now consider the outcome of the sequential-move tournament, in which agent A moves first whereas agent B follows at the next stage:[8]

Proposition 2 *Given uniformly distributed noise and quadratic costs, there exists a critical-value function $\hat{\tau}(\Delta w)$ such that the following results hold:*
(a) Let $\tau > \hat{\tau}(\Delta w)$. Then

$$e_A^* = \frac{8\Delta w k\bar{\varepsilon}^3}{\tau\Delta w^2 + (2\tau - 1)\, 4\Delta w k\bar{\varepsilon}^2 + 16\tau k^2\bar{\varepsilon}^4} \text{ and} \tag{11}$$

$$e_B^* = \frac{2\tau\bar{\varepsilon}\Delta w\left(4k\bar{\varepsilon}^2 + \Delta w\right)}{\tau\Delta w^2 + (2\tau - 1)\, 4\Delta w k\bar{\varepsilon}^2 + 16\tau k^2\bar{\varepsilon}^4}. \tag{12}$$

(b) Let $\tau \leq \hat{\tau}(\Delta w)$. If $\Delta w < 4k\bar{\varepsilon}^2$, then

$$e_A^* = \frac{8\Delta w k\bar{\varepsilon}^3}{\tau\Delta w^2 - (2\tau - 1)\, 4\Delta w k\bar{\varepsilon}^2 + 16\tau k^2\bar{\varepsilon}^4} \text{ and} \tag{13}$$

$$e_B^* = \frac{2\tau\bar{\varepsilon}\Delta w\left(4k\bar{\varepsilon}^2 - \Delta w\right)}{\tau\Delta w^2 - (2\tau - 1)\, 4\Delta w k\bar{\varepsilon}^2 + 16\tau k^2\bar{\varepsilon}^4}. \tag{14}$$

If, otherwise, $\Delta w \geq 4k\bar{\varepsilon}^2$ then

$$e_A^* = \frac{\Delta w}{2k\bar{\varepsilon}} \text{ and } e_B^* = 0. \tag{15}$$

[8] For the concrete shape of the critical-value function $\hat{\tau}(\Delta w)$ see the appendix.

Proof. See the appendix. ■

Note first that, the principal will not be able to implement first-best efforts for both agents, if he wants to do this, which becomes obvious by inspection of (9) and (11)–(15). Figure 2 illustrates the optimal behavior of the two agents according to Proposition 2.

[Figure 2]

For sufficiently large values of τ, the second mover B is more aggressive – in the sense of choosing higher efforts – than the first mover A. Interestingly, as Figure 2 shows, this may even happen for $\tau < 1$, i.e. for situations in which agent B is the underdog and A the favorite. However, for sufficiently small values of τ, the first mover A exerts more effort than the second mover B. According to Figure 2, there exist parameter values $\tau > 1$, where the more aggressive player A is the underdog and the less aggressive one, B, is the favorite. Note that there will never be a symmetric equilibrium if the agents are homogeneous (i.e., if $\tau = 1$).

Most interestingly, if the prize spread Δw is sufficiently large, the aggressive player A chooses a preemptive effort so that the second mover B drops out. Such preemptive behavior can only happen if τ is sufficiently small which is just in line with the findings of Proposition 1. Note that the result concerning the sufficiently large prize spread does not contradict the intuition given in the discussion of condition (6), which describes a closed interval for preemptive values of Δw. In the parametric case of Proposition 2, there is the same trade-off as in the discussion above: A high prize spread Δw implies large gains from preemption, but also high incentives for both players and, therefore, high effort costs for A when preempting agent B. Technically, as Figure 2 and the functional form of $\hat{\tau}(\Delta w)$ show (see the appendix), the critical-value function $\hat{\tau}(\Delta w)$ is monotonically decreasing for

$\Delta w \geq 4k\bar{\varepsilon}^2$. Hence, if Δw becomes arbitrarily large, τ has to be very small – i.e. heterogeneity has to be very large with agent A being a clear favorite – for preemption to be an equilibrium outcome in the tournament.

To summarize, preemptive behavior will be optimal for agent A, if he is sufficiently more talented than agent B and if the additional gains from preemption, Δw, are quite large. Moreover, note that there also exist parameter values for Δw slightly above $4k\bar{\varepsilon}^2$ for which agent A preempts the second mover B, although A is the underdog (i.e., $\tau > 1$). In addition, the preemption condition $\Delta w \geq 4k\bar{\varepsilon}^2$ also indicates that preemptive behavior will only be optimal, if the cost parameter k and the impact of luck $\bar{\varepsilon}$ are not too large. Of course, if the cost function is too convex, preemptive behavior will be prohibitively costly for agent A. Similarly, if $\bar{\varepsilon}$ is large, agent A has to exert a very high effort to shift $f_Y(\cdot)$ out of the interval $[-2\bar{\varepsilon}, 2\bar{\varepsilon}]$ which again would be prohibitively expensive for A.

The results of Proposition 2 only highlight which agent is the more aggressive one. Now we compare the agents' expected utilities for the different situations to check whether the agents can realize first-mover or second-mover advantages. The preemption case is obvious. Here agent A shifts $f_Y(\cdot)$ out of the interval $[-2\bar{\varepsilon}, 2\bar{\varepsilon}]$ so that B's winning probability is zero and he ends up with the loser prize w_2 whereas A gets an expected utility strictly greater than w_2. Let $\hat{\tau}(\Delta w)$ denote the critical-value function of Proposition 2. Then we obtain the following result:

Proposition 3 *Given uniformly distributed noise and quadratic costs, if* $\tau \leq \hat{\tau}(\Delta w)$ *we will have* $EU_A(e_A^*, e_B^*) > EU_B(e_A^*, e_B^*)$*,but for* $\tau > \hat{\tau}(\Delta w)$ *the opposite holds.*

Proof. See the appendix. ■

Proposition 3 shows that agent A is better off than agent B as long as τ is sufficiently small, but B's expected utility exceeds the one of agent A for relative large values of τ. More interestingly, as Figure 2 shows there are parameter constellations $(\Delta w, \tau)$ with Δw not too large in which agent A has a higher expected utility than agent B although $\tau > 1$, i.e. agent A is the underdog and B the favorite.[9] In these cases we can speak of a first-mover advantage. On the other hand, for sufficiently large values of Δw there are also constellations in which B's expected utility is larger than A's despite $\tau < 1$ so that the underdog B realizes a second-mover advantage. The intuition for these results comes from the fact that the impact of τ – i.e., the impact of heterogeneity – diminishes as Δw becomes large so that incentives are mainly determined by the prize spread. Figure 2 shows that the number of parameter constellations which correspond to a first-mover or a second-mover advantage increases with increasing Δw.

At the first stage of the three-stage game, the principal chooses his optimal tournament prizes. The following result can be derived:

Proposition 4 *Given uniformly distributed noise and quadratic costs, the principal optimally chooses* $\Delta w < 4k\bar{\varepsilon}^2$ *which implements positive efforts for both agents.*

Proof. See the appendix. ■

According to Proposition 4 the principal prefers a relatively small prize spread, which serves two purposes: On the one hand, labor costs are fixed on a moderate level. On the other hand, the principal prevents agent A from preempting agent B. Hence, concerning the complete three-stage game with optimally chosen prizes, preemptive behavior is never an equilibrium outcome. As the proof of Proposition 4 shows, preemption would be completely

[9] Note that the increasing part of $\hat{\tau}(\Delta w)$ is always larger than 1.

detrimental for the principal, because it leads to strictly negative profits. When studying tournaments with destructive behavior, Lazear (1989) shows that from the principal's viewpoint it may be beneficial to choose a low prize spread Δw to decrease the agents' incentives for sabotaging each other. Our analysis gives another argument for choosing a low Δw in practice: By choosing a low prize spread the principal can prevent agent A from exerting preemptive effort.

5 Conclusion

In this paper, we analyzed sequential-move tournaments between two heterogeneous agents in which preemptive behavior by the first acting agent is possible. As a necessary condition for preemption, either marginal costs have to be positive at the origin or luck has to be distributed over a finite interval. Using a parameterized model we then showed that preemption will be only possible if the spread between the winner the loser prize is sufficiently large. However, the principal who anticipates possible preemptive behavior optimally chooses a small prize spread that prevents preemption.

An interesting question remains with respect to the agents' strategic behavior in more general dynamic tournaments: Even if in practice the principal can separate the agents so that their decisions are independent, real tournaments are of repeated nature. That is, the entire tournament consists of several stages and at each stage the agents play a simultaneous-move tournament. Before the next stage, the agents observe their competitors' efforts in the last stage. From the analysis of this paper, one would expect that if the prize spread is sufficiently large, strategic behavior by the agents will be possible. This strategic behavior might include a preemptive effort by the first acting agent. It is, however, also possible that leapfrogging might occur: Although one agent is behind in total output, he might choose with some small probability an effort level such that he leaves his competitor behind.

Appendix

Proof of Proposition 2:

We analyze the sequential-move tournament using backward induction: (1) Given the effort e_A of agent A, we first consider agent B's optimal response $e_B^*(e_A)$ at stage 3. (2) Given this reaction function, we solve for the optimal effort level e_A^* of agent A at stage 2.

(1) Agent B's optimal reaction $e_B^*(e_A)$:

Given e_A agent B chooses $e_B^* = e_B^*(e_A)$ to maximize his expected utility. Using the first-order condition agent B will react according to

$$\Delta w f_Y (e_A - e_B^*) = k e_B^* \tag{A1}$$

To check the second-order condition for a maximum suppose that $e_B^* < e_A$. Then $y = e_A - e_B^* > 0$ and the second-order condition $\Delta w \left(\frac{1}{4\bar{\varepsilon}^2}\right) - k < 0$ is satisfied as long as

$$\Delta w < 4k\bar{\varepsilon}^2 \tag{A2}$$

holds, i.e. the marginal cost function is steeper than the left-hand tail of the triangular density (times Δw). Alternatively, suppose that $e_B^* > e_A$. Then $y = e_A - e_B^* < 0$ and the second-order condition $\Delta w \left(-\frac{1}{4\bar{\varepsilon}^2}\right) - k < 0$ is always satisfied. Note that condition (A2) is equivalent to $e^* < 2\bar{\varepsilon}$, where e^* denotes the symmetric equilibrium effort in case of a simultaneous-move tournament between two homogeneous agents as defined by

$$\Delta w f_Y(0) = k e^* \Leftrightarrow e^* = \frac{\Delta w}{2k\bar{\varepsilon}}.$$

Case 1: Let $\Delta w < 4k\bar{\varepsilon}^2$. Then we can distinguish three subcases.

- Suppose that $e_A \epsilon [0, e^*]$. Since $e_A \leq e^*$ implies $e_B^* \geq e_A$ ($c_B'(e_B^*) = k e_B^*$ intersects with the left-hand tail of the triangular density function (times Δw)) and we have an interior solution described by (A1) and the left-hand tail of $f_Y(y)$:

$$e_B^* = \Delta w \frac{e_A + 2\bar{\varepsilon}}{\Delta w + 4k\bar{\varepsilon}^2}. \tag{A3}$$

Note that e_B^* is linearly increasing in e_A with $e_B^*(0) = \frac{2\bar{\varepsilon}\Delta w}{\Delta w + 4k\bar{\varepsilon}^2}$ and $e_B^*(e^*) = e^*$.[10]

- Suppose that $e_A \epsilon [e^*, 2\bar{\varepsilon}]$. Since $e_A \geq e^*$ implies $e_B^* \leq e_A$ ($c_B'(e_B^*) = ke_B^*$ intersects with the right-hand tail of the triangular density function (times Δw)) and an interior solution is given by (A1) together with the right-hand tail of the density function:

$$e_B^* = \Delta w \frac{2\bar{\varepsilon} - e_A}{4k\bar{\varepsilon}^2 - \Delta w}. \tag{A4}$$

Note that e_B^* is linearly decreasing in e_A with $e_B^*(e^*) = e^*$ and $e_B^*(2\bar{\varepsilon}) = 0$.

- Suppose that $e_A \geq 2\bar{\varepsilon}$. Then we do not have an interior solution and agent B optimally chooses

$$e_B^* = 0. \tag{A5}$$

Case 2: Let $\Delta w \geq 4k\bar{\varepsilon}^2$. Then $e^* \geq 2\bar{\varepsilon}$ and we can distinguish two subcases.

- Suppose that $e_A \epsilon [0, e^*]$. Since $e_A \leq e^*$ implies $e_B^* \geq e_A$, the optimal effort e_B^* is given by (A1) and the left-hand tail of $f_Y(y)$:

$$e_B^*(e_A) = \Delta w \frac{e_A + 2\bar{\varepsilon}}{\Delta w + 4k\bar{\varepsilon}^2}.$$

[10] Note that $EU_B(e_A, e_B^*(e_A)) > EU_B(e_A, 0)$ iff $\Delta w (e_A + 2\bar{\varepsilon})^2 > 0$ which is always satisfied.

Note that this interior solution requires $EU_B(e_A, e_B^*(e_A)) \geq EU_B(e_A, 0)$ to hold. Using agent B's best response function, this condition is satisfied as long as

$$e_A \leq \bar{e}_A := \frac{2\bar{\varepsilon}}{\Delta w + 8k\bar{\varepsilon}^2}\left(\Delta w + \sqrt{2\Delta w^2 + 8\Delta w k\bar{\varepsilon}^2}\right)$$

with $\bar{e}_A \in [2\bar{\varepsilon}, e^*]$. Again, e_B^* is linearly increasing in e_A with $e_B^*(0) = \frac{2\bar{\varepsilon}\Delta w}{\Delta w + 4k\bar{\varepsilon}^2}$.

- Suppose that $e_A > e^*$. Since the left-hand tail of the triangular density is steeper than the marginal cost function, we do not have an interior solution and agent B optimally chooses

$$e_B^* = 0.$$

(2) Agent A's optimal effort e_A^*:

Given tournament prizes w_1 and w_2 agent A chooses e_A^* to maximize his expected utility, taking into account agent B's optimal response $e_B^*(e_A)$. Using the first-order condition agent A will act according to

$$\Delta w f_Y(e_A - e_B^*(e_A))\left(1 - \frac{\partial e_B^*}{\partial e_A}\right) = \tau k e_A. \quad \text{(A6)}$$

Note that the second-order condition for an interior solution is always satisfied for $e_A > e_B^*$ since $-\frac{\Delta w}{4\bar{\varepsilon}^2}\left(1 - \frac{\partial e_B^*}{\partial e_A}\right)^2 - \tau k < 0$, and is satisfied for $e_A < e_B^*$ iff $\frac{\Delta w}{4\bar{\varepsilon}^2}\left(1 - \frac{\partial e_B^*}{\partial e_A}\right)^2 - \tau k < 0$. Using the argumentation above, we can distinguish two possible situations:

Case 1: Let $\Delta w < 4k\bar{\varepsilon}^2$. Then we have three possibilities:

- Suppose that $e_A^* \epsilon [0, e^*]$. Then $e_A < e_B^*$ and $\frac{\partial e_B^*}{\partial e_A} = \frac{\Delta w}{\Delta w + 4k\bar{\varepsilon}^2}$, and the second-order condition reads as

$$4\Delta w k\bar{\varepsilon}^2 < \tau\left(\Delta w + 4k\bar{\varepsilon}^2\right)^2.$$

This condition is satisfied as long as $\tau \geq 1/4$, or if $\tau < 1/4$ and $\Delta w \leq \frac{k\bar{\varepsilon}^2}{2\tau}\left(4 - 8\tau - 4\sqrt{1-4\tau}\right) \in (0, 4k\bar{\varepsilon}^2)$ which can be rewritten as

$$\tau \geq \frac{4\Delta w k\bar{\varepsilon}^2}{\left(\Delta w + 4k\bar{\varepsilon}^2\right)^2} := \hat{\tau}_1(\Delta w).$$

Suppose we have an interior solution. Then simple calculations show that

$$e_A^* = \frac{8\Delta w k\bar{\varepsilon}^3}{\tau\Delta w^2 + (2\tau - 1)\,4\Delta w k\bar{\varepsilon}^2 + 16\tau k^2\bar{\varepsilon}^4} \tag{A7}$$

and that this effort is positive iff $\tau > \hat{\tau}_1(\Delta w)$. Note that $e_A^* \leq e^*$ requires

$$\tau \geq \frac{4k\bar{\varepsilon}^2}{\Delta w + 4k\bar{\varepsilon}^2} =: \hat{\tau}_2(\Delta w)$$

to hold. Furthermore, we have $\hat{\tau}_2(\Delta w) > \hat{\tau}_1(\Delta w)$ for all parameter constellations, and $\hat{\tau}_2(\Delta w) > \frac{1}{4}$ for $\Delta w < 4k\bar{\varepsilon}^2$. Altogether, a feasible interior solution described by (A7) will hold iff $\tau \geq \hat{\tau}_2(\Delta w)$. The corresponding expected utility is given by

$$EU_{A,(I)} = w_2 + \frac{8\tau\Delta w k^2\bar{\varepsilon}^4}{\tau\Delta w^2 + (2\tau - 1)\,4\Delta w k\bar{\varepsilon}^2 + 16\tau k^2\bar{\varepsilon}^4}. \tag{A8}$$

Agent B's optimal effort to e_A^* then is

$$e_B^*\left(e_A^*\right) = \frac{2\tau\bar{\varepsilon}\Delta w\left(4k\bar{\varepsilon}^2 + \Delta w\right)}{\tau\Delta w^2 + (2\tau - 1)\,4\Delta w k\bar{\varepsilon}^2 + 16\tau k^2\bar{\varepsilon}^4}$$

with an expected utility

$$EU_{B,(I)} = w_2 + \Delta w\frac{\tau^2\left(\Delta w + 2k\bar{\varepsilon}^2\right)\left(4k\bar{\varepsilon}^2 + \Delta w\right)^3 - 8\Delta w\tau k\bar{\varepsilon}^2\left(4k\bar{\varepsilon}^2 + \Delta w\right)^2 + 16\Delta w^2 k^2\bar{\varepsilon}^4}{\left(\tau\Delta w^2 + (2\tau - 1)\,4\Delta w k\bar{\varepsilon}^2 + 16\tau k^2\bar{\varepsilon}^4\right)^2}.$$

If, on the other hand, the second-order condition is not satisfied or the interior solution is not feasible, we have a corner solution. Comparing agent A's expected utilities for $e_A = 0$ and $e_A = e^*$ yields

$$\begin{aligned} EU_A\left(0, e_B^*\left(0\right)\right) &= w_2 + \Delta w \frac{8k^2\bar{\varepsilon}^4}{\left(\Delta w + 4k\bar{\varepsilon}^2\right)^2} \\ &< EU_A\left(e^*, e_B^*\left(e^*\right)\right) = w_2 + \Delta w \frac{4k\bar{\varepsilon}^2 - \tau\Delta w}{8k\bar{\varepsilon}^2} \text{ iff} \\ \Delta w &< \frac{\left(4 - 8\tau + 4\sqrt{1+4\tau}\right)}{2\tau} k\bar{\varepsilon}^2 \\ &\Leftrightarrow \Delta w\tau + 4\tau k\bar{\varepsilon}^2 - 2k\bar{\varepsilon}^2 < 2k\bar{\varepsilon}^2\sqrt{1+4\tau}. \qquad \text{(A9)} \end{aligned}$$

If the left-hand side of inequality (A9) is negative, the inequality will always hold. Otherwise we obtain

$$\tau < \frac{4k\bar{\varepsilon}^2\left(\Delta w + 8k\bar{\varepsilon}^2\right)}{\left(\Delta w + 4k\bar{\varepsilon}^2\right)^2}.$$

Since the right-hand side of this inequality is greater than $\hat{\tau}_2(\Delta w)$, (A9) always holds for $\tau < \hat{\tau}_2(\Delta w)$. Hence, agent A optimally chooses the corner solution $e_A^* = e^*$. Agent B then chooses $e_B^*\left(e^*\right) = e^*$ and receives

$$EU_B\left(e^*, e_B^*\left(e^*\right)\right) = w_2 + \Delta w \frac{4k\bar{\varepsilon}^2 - \Delta w}{8k\bar{\varepsilon}^2}$$

- Suppose that $e_A^* \epsilon \left[e^*, 2\bar{\varepsilon}\right]$. Then $e_A > e_B^*$ and we have an interior solution e_A^* which maximizes

$$\begin{aligned} &EU_A\left(e_A, e_B^*\left(e_A\right)\right) = w_2 \\ &+\Delta w \left[\frac{e_A - \Delta w \frac{2\bar{\varepsilon} - e_A}{4k\bar{\varepsilon}^2 - \Delta w}}{2\bar{\varepsilon}} - \frac{\left(e_A - \Delta w \frac{2\bar{\varepsilon} - e_A}{4k\bar{\varepsilon}^2 - \Delta w}\right)^2}{8\bar{\varepsilon}^2} + \frac{1}{2}\right] \\ &-\frac{k}{2}\tau e_A^2. \end{aligned}$$

This yields

$$e_A^* = \frac{8\Delta w k\bar{\varepsilon}^3}{\tau\Delta w^2 - (2\tau - 1)\,4\Delta w k\bar{\varepsilon}^2 + 16\tau k^2\bar{\varepsilon}^4} \qquad \text{(A10)}$$

and

$$EU_{A,(II)} = w_2 + \Delta w \frac{\tau \Delta w^2 - (2\tau - 1)\, 4\Delta w k \bar{\varepsilon}^2 + 8\tau k^2 \bar{\varepsilon}^4}{\tau \Delta w^2 - (2\tau - 1)\, 4\Delta w k \bar{\varepsilon}^2 + 16\tau k^2 \bar{\varepsilon}^4}. \tag{A11}$$

Note that the denominator of (A10) and (A11) is always positive. The numerator of (A11) will be positive, if $\Delta w < \left(4 - \sqrt{8}\right) k\bar{\varepsilon}^2$, or if $\Delta w > \left(4 - \sqrt{8}\right) k\bar{\varepsilon}^2$ and

$$\tau < \frac{4\Delta w k \bar{\varepsilon}^2}{8\Delta w k \bar{\varepsilon}^2 - \Delta w^2 - 8k^2 \bar{\varepsilon}^4} =: \hat{\tau}_3(\Delta w).$$

Moreover, note that e_A^* described by (A10) is always smaller than $2\bar{\varepsilon}$, but $e_A^* > e^*$ iff

$$\tau < \frac{4k\bar{\varepsilon}^2}{4k\bar{\varepsilon}^2 - \Delta w} =: \hat{\tau}_4(\Delta w).$$

Agent B's optimal reaction to (A10) is

$$e_B^*\left(e_A^*\right) = \frac{2\tau \bar{\varepsilon} \Delta w \left(4k\bar{\varepsilon}^2 - \Delta w\right)}{\tau \Delta w^2 - (2\tau - 1)\, 4\Delta w k \bar{\varepsilon}^2 + 16\tau k^2 \bar{\varepsilon}^4}.$$

and

$$EU_{B,(II)} = w_2 + \frac{2\tau^2 \Delta w k \bar{\varepsilon}^2 \left(4k\bar{\varepsilon}^2 - \Delta w\right)^3}{\left(\tau \Delta w^2 - 8\Delta w k \bar{\varepsilon}^2 \tau + 4\Delta w k \bar{\varepsilon}^2 + 16\tau k^2 \bar{\varepsilon}^4\right)^2}.$$

- Suppose that $e_A^* > 2\bar{\varepsilon}$. To induce $e_B^* = 0$, A will choose

$$e_A^* = 2\bar{\varepsilon} \tag{A12}$$

to maximize expected utility which gives

$$EU_{A,(III)} = w_2 + \Delta w \frac{\Delta w - 2\tau k \bar{\varepsilon}^2}{\Delta w}. \tag{A13}$$

B's expected utility then of course is $EU_{B,(III)} = w_2$.

Comparing (A13) and (A11) yields

$$EU_{A,(III)} < EU_{A,(II)} \Leftrightarrow -2\tau^2 k \bar{\varepsilon}^2 \left(\Delta w - 4k\bar{\varepsilon}^2\right)^2 < 0$$

which is always true. Hence, agent A never prefers preemption under $\Delta w < 4k\bar{\varepsilon}^2$. We know that for $\tau < \hat{\tau}_2(\Delta w)$ there is no feasible interior solution $e_A^* \epsilon [0, e^*]$. Therefore, we have to compare $EU_A(e^*, e_B^*(e^*)) = w_2 + \Delta w \frac{4k\bar{\varepsilon}^2 - \tau\Delta w}{8k\bar{\varepsilon}^2}$ and $EU_{A,(II)}$.[11] We obtain

$$\begin{aligned} w_2 + \Delta w \frac{4k\bar{\varepsilon}^2 - \tau\Delta w}{8k\bar{\varepsilon}^2} &< w_2 + \Delta w \frac{\tau\Delta w^2 - (2\tau - 1)\,4\Delta w k\bar{\varepsilon}^2 + 8\tau k^2\bar{\varepsilon}^4}{\tau\Delta w^2 - (2\tau - 1)\,4\Delta w k\bar{\varepsilon}^2 + 16\tau k^2\bar{\varepsilon}^4} \\ &\Leftrightarrow -\Delta w \left(\Delta w\tau - 4\tau k\bar{\varepsilon}^2 + 4k\bar{\varepsilon}^2\right)^2 < 0, \end{aligned}$$

which is always satisfied. For $\tau > \hat{\tau}_2(\Delta w)$, we have to compare (A8) and (A11):

$$EU_{A,(I)} < EU_{A,(II)} \Leftrightarrow -\Delta w^2 \left(\tau^2\Delta w^2 + \left(4\tau - 3\tau^2 - 1\right)16k^2\bar{\varepsilon}^4\right) < 0.$$

Note that for $\tau \in \left[\frac{1}{3}, 1\right]$ this inequality holds for all values of Δw, k and $\bar{\varepsilon}$. For all other values of τ we have $4\tau - 3\tau^2 - 1 < 0$, and the inequality can be solved for Δw as:

$$\Delta w > 4k\bar{\varepsilon}^2 \frac{\sqrt{3\tau^2 - 4\tau + 1}}{\tau}.$$

Rearranging this inequality gives

$$\begin{aligned} \tau &\in (\hat{\tau}_5(\Delta w), \hat{\tau}_6(\Delta w)) \\ \text{with } \hat{\tau}_5(\Delta w) &= \frac{4k\bar{\varepsilon}^2}{48k^2\bar{\varepsilon}^4 - \Delta w^2}\left(8k\bar{\varepsilon}^2 - \sqrt{16k^2\bar{\varepsilon}^4 + \Delta w^2}\right) \\ \text{and } \hat{\tau}_6(\Delta w) &= \frac{4k\bar{\varepsilon}^2}{48k^2\bar{\varepsilon}^4 - \Delta w^2}\left(8k\bar{\varepsilon}^2 + \sqrt{16k^2\bar{\varepsilon}^4 + \Delta w^2}\right). \end{aligned}$$

We have $\hat{\tau}_5(\Delta w) < \hat{\tau}_2(\Delta w) < \hat{\tau}_6(\Delta w)$ ($\forall\ \Delta w < 4k\bar{\varepsilon}^2$) and $\hat{\tau}_2(\Delta w) \in \left[\frac{1}{3}, 1\right]$. Moreover, $\hat{\tau}_6(0) = 1$ and $\frac{\partial\hat{\tau}_1(\Delta w)}{\partial\Delta w} > 0$, $\forall\Delta w$. Furthermore, $\hat{\tau}_3(\Delta w) > \hat{\tau}_6(\Delta w)$ and $\hat{\tau}_4(\Delta w) > \hat{\tau}_6(\Delta w)$ ($\forall\ \Delta w < 4k\bar{\varepsilon}^2$). To summarize, for $\tau \in [\hat{\tau}_2(\Delta w), \hat{\tau}_6(\Delta w)]$ agent A again optimally chooses (A10), but for $\tau > \hat{\tau}_6(\Delta w)$ he switches to (A7).

Case 2: Let $\Delta w \geq 4k\bar{\varepsilon}^2$. Then we have to consider two possibilities:

[11] Note that $\hat{\tau}_2(\Delta w) < \hat{\tau}_4(\Delta w)$ and $\hat{\tau}_2(\Delta w) < \hat{\tau}_3(\Delta w)$ for all parameter constellations $\Delta w < 4k\bar{\varepsilon}^2$.

- Suppose that $e_A^* \epsilon \left[0, e^*\right]$. Then $e_B^* \geq e_A$ and the second-order condition

$$4\Delta w k\bar{\varepsilon}^2 < \tau\left(\Delta w + 4k\bar{\varepsilon}^2\right)^2 \tag{A14}$$

is satisfied as long as $\tau \geq 1/4$ or if $\tau < 1/4$ and $\Delta w \geq \frac{k\bar{\varepsilon}^2}{2\tau}(4 - 8\tau + 4\sqrt{1-4\tau}) \Leftrightarrow \tau \geq \hat{\tau}_1(\Delta w)$. Recall that $\hat{\tau}_2(\Delta w) > \hat{\tau}_1(\Delta w)$ for all parameter constellations. Hence, again $\tau \geq \hat{\tau}_2(\Delta w)$ ensures a feasible interior solution where agent A chooses e_A^* according to (A7) and has an expected utility given by $EU_{A,(I)}$ (see (A8)). However, if $\tau < \hat{\tau}_2(\Delta w)$ we will have a corner solution with $e_A^* = e_B^*\left(e_A^*\right) = e^*$ and $EU_A\left(e^*, e_B^*\left(e^*\right)\right) = w_2 + \Delta w\frac{4k\bar{\varepsilon}^2 - \tau\Delta w}{8k\bar{\varepsilon}^2}$.

- Suppose that $e_A > e^*$. Then agent B optimally chooses $e_B^* = 0$ and agent A will choose $e_A^* = e^*$ resulting in an expected utility

$$EU_A\left(e^*, 0\right) = w_2 + \Delta w\frac{8k\bar{\varepsilon}^2 - \tau\Delta w}{8k\bar{\varepsilon}^2}. \tag{A15}$$

For $\tau < \hat{\tau}_2(\Delta w)$, we have to compare $EU_A\left(e^*, e_B^*\left(e^*\right)\right) = w_2 + \Delta w\frac{4k\bar{\varepsilon}^2 - \tau\Delta w}{8k\bar{\varepsilon}^2}$ and $EU_A\left(e^*, 0\right)$ (according (A15)). The comparison immediately shows that $EU_A\left(e^*, 0\right) > EU_A\left(e^*, e_B^*\left(e^*\right)\right)$, $\forall \Delta w, \tau, k, \bar{\varepsilon}$. Hence, agent A chooses preemption for sufficiently small values of τ. For $\tau \geq \hat{\tau}_2(\Delta w)$, comparing (A8) and (A15) shows that there exist two critical values

$$\hat{\tau}_7\left(\Delta w\right) = \frac{\left(8k\bar{\varepsilon}^2 + 6\Delta w - 2\sqrt{\left(16k^2\bar{\varepsilon}^4 + 24\Delta w k\bar{\varepsilon}^2 + \Delta w^2\right)}\right)k\bar{\varepsilon}^2}{\Delta w\left(\Delta w + 4k\bar{\varepsilon}^2\right)}$$

and

$$\hat{\tau}_8\left(\Delta w\right) = \frac{\left(8k\bar{\varepsilon}^2 + 6\Delta w + 2\sqrt{16k^2\bar{\varepsilon}^4 + 24\Delta w k\bar{\varepsilon}^2 + \Delta w^2}\right)k\bar{\varepsilon}^2}{\Delta w\left(\Delta w + 4k\bar{\varepsilon}^2\right)}$$

with $\frac{\partial\hat{\tau}_i(\Delta w)}{\partial\Delta w} < 0, i = 7, 8$ for all $\Delta w \geq 4k\bar{\varepsilon}^2$ such that for all $\tau \in \left[\hat{\tau}_7\left(\Delta w\right), \hat{\tau}_8\left(\Delta w\right)\right]$

$$EU_{A,(I)} < EU_A\left(e^*, 0\right).$$

Note that $\hat{\tau}_7(\Delta w) < \hat{\tau}_2(\Delta w) < \hat{\tau}_8(\Delta w)$,$\forall \Delta w, \tau, k, \bar{\varepsilon}$, and that e_A^* according to (A7) satisfies $e_A^* \leq \bar{e}_A$ for $\Delta w \geq 4k\bar{\varepsilon}^2$. Therefore, if $\tau \in [\hat{\tau}_2(\Delta w), \hat{\tau}_8(\Delta w)]$ agent A again prefers preemption, but for $\tau > \hat{\tau}_8(\Delta w)$ he optimally chooses e_A^* according to (A7).

Defining the critical-value function

$$\hat{\tau}(\Delta w) = \begin{cases} \frac{4k\bar{\varepsilon}^2\left(8k\bar{\varepsilon}^2+\sqrt{16k^2\bar{\varepsilon}^4+\Delta w^2}\right)}{48k^2\bar{\varepsilon}^4-\Delta w^2} \equiv \hat{\tau}_6(\Delta w) & \text{if} \quad \Delta w < 4k\bar{\varepsilon}^2 \\ \frac{2k\bar{\varepsilon}^2\left(4k\bar{\varepsilon}^2+3\Delta w+\sqrt{16k^2\bar{\varepsilon}^4+24\Delta w k\bar{\varepsilon}^2+\Delta w^2}\right)}{\Delta w(\Delta w+4k\bar{\varepsilon}^2)} \equiv \hat{\tau}_8(\Delta w) & \text{if} \quad \Delta w \geq 4k\bar{\varepsilon}^2 \end{cases} \tag{A16}$$

completes the proof.

Proof of Proposition 3:

First, consider the case of $\Delta w < 4k\bar{\varepsilon}^2$ and $\tau \leq \hat{\tau}(\Delta w)$ where $\hat{\tau}(\Delta w)$ is given by (A16). Hence, using the expected utilities that have been computed in the proof of Proposition 2, we must show that $EU_{A,(II)} > EU_{B,(II)}$ in the relevant range. We get

$$\begin{aligned} EU_{A,(II)} > EU_{B,(II)} &\Leftrightarrow \\ \left(4k\bar{\varepsilon}^2-\Delta w\right)^2\left(6k\bar{\varepsilon}^2-\Delta w\right)\tau^2 & \\ -8k\bar{\varepsilon}^2\left(6k\bar{\varepsilon}^2-\Delta w\right)\left(2k\bar{\varepsilon}^2-\Delta w\right)\tau-16k^2\bar{\varepsilon}^4\Delta w &< 0. \end{aligned}$$

The left-hand side of the inequality describes a parabola open to the top with the two roots

$$\begin{aligned} \rho_1(\Delta w) &= \frac{4k\bar{\varepsilon}^2\left(12k^2\bar{\varepsilon}^4-8\Delta w k\bar{\varepsilon}^2+\Delta w^2-\sqrt{2k\bar{\varepsilon}^2(6k\bar{\varepsilon}^2-\Delta w)(12k^2\bar{\varepsilon}^4-6\Delta w k\bar{\varepsilon}^2+\Delta w^2)}\right)}{(6k\bar{\varepsilon}^2-\Delta w)(4k\bar{\varepsilon}^2-\Delta w)^2} < 0 \\ \rho_2(\Delta w) &= \frac{4k\bar{\varepsilon}^2\left(12k^2\bar{\varepsilon}^4-8\Delta w k\bar{\varepsilon}^2+\Delta w^2+\sqrt{2k\bar{\varepsilon}^2(6k\bar{\varepsilon}^2-\Delta w)(12k^2\bar{\varepsilon}^4-6\Delta w k\bar{\varepsilon}^2+\Delta w^2)}\right)}{(6k\bar{\varepsilon}^2-\Delta w)(4k\bar{\varepsilon}^2-\Delta w)^2} > 0. \end{aligned}$$

Hence, the condition $\tau < \rho_2(\Delta w)$ must hold. As we have $\rho_2(\Delta w) > \hat{\tau}_6(\Delta w)$ and $\hat{\tau}_6(\Delta w) \geq \tau$ for $\Delta w < 4k\bar{\varepsilon}^2$, this condition is always satisfied.

Next, we have to consider the preemption case with $\Delta w \geq 4k\bar{\varepsilon}^2$ and $\tau < \hat{\tau}(\Delta w)$. Since agent A receives $EU_A(e^*, 0) > w_2$ according to (A15) and agent B $EU_B = w_2$, we immediately obtain that A's expected utility is greater than B's one.

Finally, consider the case of $\tau > \hat{\tau}(\Delta w)$ for the full range of Δw. Here, we must have

$$\begin{aligned} EU_{A,(I)} &< EU_{B,(I)} \\ \Leftrightarrow \left(\Delta w + 6k\bar{\varepsilon}^2\right)\left(4k\bar{\varepsilon}^2 + \Delta w\right)^2 \tau^2 & \\ -8\bar{\varepsilon}^2 k\left(\Delta w + 2k\bar{\varepsilon}^2\right)\left(\Delta w + 6k\bar{\varepsilon}^2\right)\tau + 16\Delta w k^2\bar{\varepsilon}^4 &> 0. \end{aligned}$$

The right-hand side of this inequality again characterizes a parabola open to the top. Its roots are given by

$$\begin{aligned} \rho_3(\Delta w) &= \frac{4k\bar{\varepsilon}^2\left(8\Delta w k\bar{\varepsilon}^2 + 12k^2\bar{\varepsilon}^4 + \Delta w^2 - \sqrt{2k\bar{\varepsilon}^2(\Delta w + 6k\bar{\varepsilon}^2)(\Delta w^2 + 6\Delta w k\bar{\varepsilon}^2 + 12k^2\bar{\varepsilon}^4)}\right)}{(\Delta w + 6k\bar{\varepsilon}^2)(4k\bar{\varepsilon}^2 + \Delta w)^2} > 0 \\ \rho_4(\Delta w) &= \frac{4k\bar{\varepsilon}^2\left(8\Delta w k\bar{\varepsilon}^2 + 12k^2\bar{\varepsilon}^4 + \Delta w^2 + \sqrt{2k\bar{\varepsilon}^2(\Delta w + 6k\bar{\varepsilon}^2)(\Delta w^2 + 6\Delta w k\bar{\varepsilon}^2 + 12k^2\bar{\varepsilon}^4)}\right)}{(\Delta w + 6k\bar{\varepsilon}^2)(4k\bar{\varepsilon}^2 + \Delta w)^2} > 0. \end{aligned}$$

It suffices to show that the condition $\tau > \rho_4(\Delta w)$ holds $\forall \Delta w$. As $\rho_4(\Delta w) < \hat{\tau}(\Delta w)$, $\forall \Delta w$, and we have $\tau > \hat{\tau}(\Delta w)$ this condition always holds.

Proof of Proposition 4:

The principal's objective function is given by

$$\begin{aligned} \pi &= e_A^* + e_B^* - w_1 - w_2 = e_A^* + e_B^* - \Delta w - 2w_2 \qquad \text{(A17)} \\ &= \begin{cases} \frac{8\Delta w k\bar{\varepsilon}^3 + 2\tau\bar{\varepsilon}\Delta w\left(4k\bar{\varepsilon}^2 + \Delta w\right)}{\tau\Delta w^2 + (2\tau - 1)4\Delta w k\bar{\varepsilon}^2 + 16\tau k^2\bar{\varepsilon}^4} - \Delta w - 2w_2 & \text{if } \tau > \hat{\tau}(\Delta w) \\ \frac{8\Delta w k\bar{\varepsilon}^3 + 2\tau\bar{\varepsilon}\Delta w\left(4k\bar{\varepsilon}^2 - \Delta w\right)}{\tau\Delta w^2 - (2\tau - 1)4\Delta w k\bar{\varepsilon}^2 + 16\tau k^2\bar{\varepsilon}^4} - \Delta w - 2w_2 & \text{if } \tau \leq \hat{\tau}(\Delta w) \text{ and } \Delta w < 4k\bar{\varepsilon}^2 \\ \frac{\Delta w}{2\bar{\varepsilon}k} - \Delta w - 2w_2 & \text{if } \tau \leq \hat{\tau}(\Delta w) \text{ and } \Delta w \geq 4k\bar{\varepsilon}^2 \end{cases} \end{aligned}$$

The principal maximizes π according to (A17) subject to the relevant restriction on Δw and the two agents' participation constraints $EU_i(e_i^*) \geq \bar{u}$

$(i = A, B)$.[12] Obviously, the principal chooses w_2 so that the agent with the lower expected utility is just indifferent between accepting the contract or not, that is, his participation constraint is binding. Using our comparison of expected utilities above and solving for w_2 then yields the following three cases:

- If $\tau > \hat{\tau}(\Delta w)$, agent A's reservation constraint becomes binding and we have

$$\begin{aligned} \pi_I(\Delta w) &= \frac{8\Delta w k\bar{\varepsilon}^3 + 2\tau\bar{\varepsilon}\Delta w\left(4k\bar{\varepsilon}^2 + \Delta w\right)}{\tau\Delta w^2 + (2\tau - 1)\,4\Delta w k\bar{\varepsilon}^2 + 16\tau k^2\bar{\varepsilon}^4} - \Delta w \\ &\quad -2\left(\bar{u} - \frac{8\tau\Delta w k^2\bar{\varepsilon}^4}{\tau\Delta w^2 + (2\tau - 1)\,4\Delta w k\bar{\varepsilon}^2 + 16\tau k^2\bar{\varepsilon}^4}\right). \end{aligned}$$

  Solving the principal's profit for the optimal wage spread $\Delta w^* = f(\tau)$ shows that $\tau = f^{-1}(\Delta w^*)$ is strictly decreasing, i.e. the higher the costs for agent A, the smaller the optimal wage spread Δw^*.[13] In particular, $\Delta w^* < 4k\bar{\varepsilon}^2$.

- If $\tau \leq \hat{\tau}(\Delta w)$ and $\Delta w < 4k\bar{\varepsilon}^2$, agent B's reservation constraint is

[12] Note that here and in the following the agents' incentive constraints e_A^* and e_B^* are directly inserted into the principal's objective function and the two agents' participation constraints.

[13] We obtain $f^{-1}(\Delta w^*) = \frac{8k\bar{\varepsilon}^2\left(\Theta + 2\sqrt{\Psi}\right)}{2(4k\bar{\varepsilon}^2 + \Delta w)(\Delta w^3 + 12\Delta w^2 k\bar{\varepsilon}^2 + 64k^2\bar{\varepsilon}^4\Delta w - 8\Delta w k\bar{\varepsilon}^3 - 32k^2\bar{\varepsilon}^5)}$ with

$$\begin{aligned} \Theta &= \Delta w^3 - 2\Delta w^2\bar{\varepsilon} + 8k\bar{\varepsilon}^2\left(2k\bar{\varepsilon}^3 + 2\Delta w k\bar{\varepsilon}^2 + \Delta w^2\right) \\ \Psi &= (\bar{\varepsilon} - \Delta w)\,\Delta w^4\bar{\varepsilon} \\ &\quad -2k\bar{\varepsilon}^3\left((3 + 2\bar{\varepsilon}k)\,\Delta w^4 - 4\Delta w k\bar{\varepsilon}^2\left(16k^2\bar{\varepsilon}^4 + \Delta w^2\right)\right) \\ &\quad +2k\bar{\varepsilon}^3\left(8\Delta w^2 k\bar{\varepsilon}^3\left(6k\bar{\varepsilon} + 4k^2\bar{\varepsilon}^2 - 1\right) + 32k^3\bar{\varepsilon}^7\right). \end{aligned}$$

binding, resulting in

$$\pi_{II}(\Delta w) = \frac{8\Delta w k\bar{\varepsilon}^3 + 2\tau\bar{\varepsilon}\Delta w\left(4k\bar{\varepsilon}^2 - \Delta w\right)}{\tau\Delta w^2 - (2\tau - 1)\,4\Delta w k\bar{\varepsilon}^2 + 16\tau k^2\bar{\varepsilon}^4} - \Delta w$$
$$-2\left(\bar{u} - \frac{2\tau^2\Delta w k\bar{\varepsilon}^2\left(4k\bar{\varepsilon}^2 - \Delta w\right)^3}{\left(\tau\Delta w^2 - 8\Delta w k\bar{\varepsilon}^2\tau + 4\Delta w k\bar{\varepsilon}^2 + 16\tau k^2\bar{\varepsilon}^4\right)^2}\right).$$

- If $\tau \leq \hat{\tau}(\Delta w)$ and $\Delta w \geq 4k\bar{\varepsilon}^2$, the principal maximizes

$$\pi_{III}(\Delta w) = \frac{\Delta w}{2k\bar{\varepsilon}} - \Delta w - 2\bar{u}.$$

  Since $\pi_{III}(\Delta w)$ is monotonically decreasing because of $2k\bar{\varepsilon} > 1$ (see condition (10)), the optimal wage spread is given by $\Delta w^* = 4k\bar{\varepsilon}^2$ and the principal's profit is negative: $\pi_{III}(4k\bar{\varepsilon}^2) = -2\bar{\varepsilon}(2k\bar{\varepsilon} - 1) - 2\bar{u} < 0$.

To summarize, the principal always chooses $\Delta w^* < 4k\bar{\varepsilon}^2$ for all $\tau \geq 0$.

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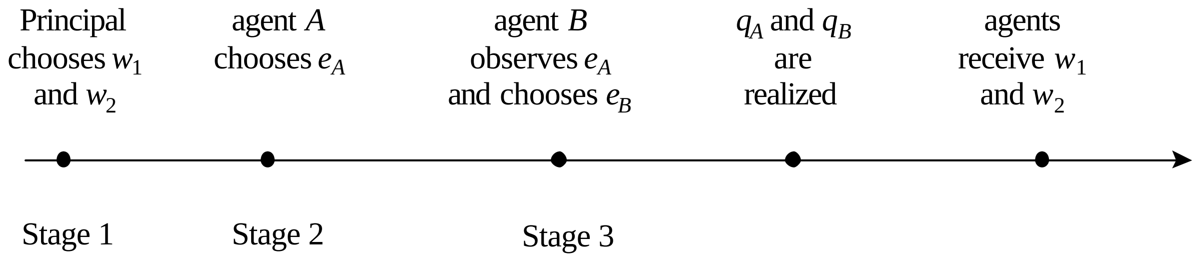

Figure 1: Timing of events

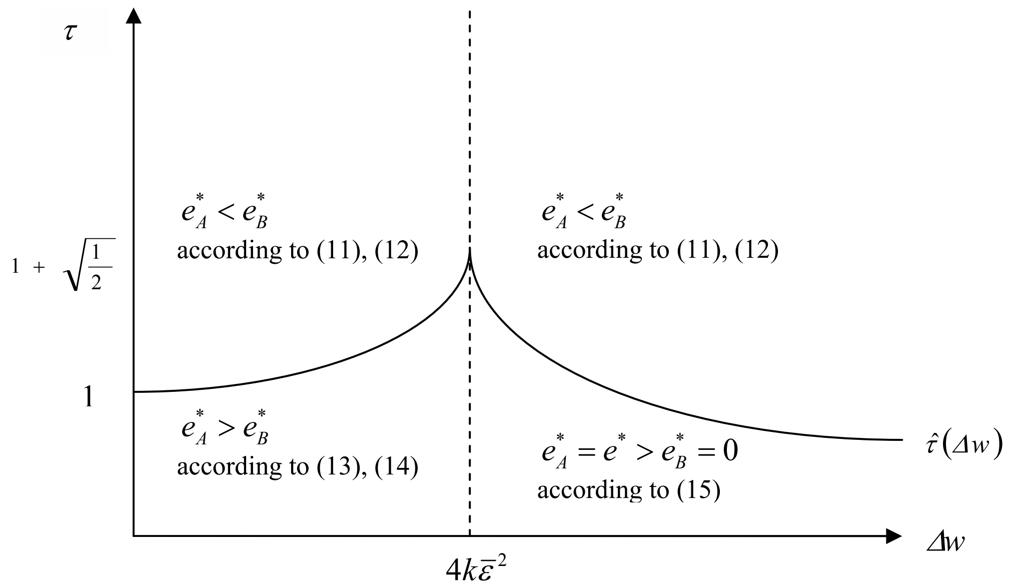

Figure 2: The agents' optimal behavior