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Can price discrimination lead to product differentiation? A vertical differentiation model

by

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Can price discrimination lead to product differentiation? A vertical differentiation model

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Abstract

In this paper, I compare two-part tariff competition to linear pricing in a vertically differentiated duopoly. Consumers have identical tastes for quality but differ in their preferences for quantity. The main finding is that quality differentiation occurs in equilibrium if and only if two-part tariffs are permitted. Furthermore, two-part tariff competition encourages entry, which in turn increases welfare. Nevertheless, two-part tariff competition decreases consumers' surplus compared to linear pricing.

JEL classification: D43, L11, L13

Keywords: Duopoly, Two-part tariff, Vertical differentiation

1 Introduction

Price discrimination can be observed in many markets, for example in telecommunication markets or electricity services. It is also widely used by airlines, movie theaters, and various clubs (health, golf etc.).[1] In particular two-part tariffs are often observed. Typically contracts in the telecommunication market or in the market for electricity contain a fixed fee payment and a per unit charge. Two-part

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[1]For more examples of markets where price discrimination is used see Wilson (1992) or Tirole (1988, ch.3).

tariffs are also widely practiced in vertical contracts. Manufacturers often charge of their retailers a franchise or permission fee. In the literature on nonlinear pricing in oligopolies with complete information, firms are typically assumed to be horizontally differentiated.[2] In contrast, I develop a model of two-part tariff competition in a vertically differentiated duopoly. Thus, consumers have the same ordinal ranking for the goods. Consequently, when all firms use the same tariff, only the commonly preferred firm has a positive market share. Quality can be seen as a "vertical" product feature in the sense that each consumer prefers higher quality. One example for quality differentiation and nonlinear pricing are health clubs: these clubs often levy a membership fee plus a per use charge and they offer a fixed quality (equipment and service).[3] In many cities there is more than one health club in the city center (no spatial differentiation), and in most cases these clubs differ in quality.

The idea that firms can relax price competition via vertical product differentiation is due to Shaked and Sutton (1982). I extend the well-known Shaked and Sutton unit-demand model, or rather the Choi and Chin (1992) version with quasi-linear utility functions, to multi-unit demands. The analysis presented in this paper is based on a non-cooperative three-stage game. In the first stage, the two potential duopolists decide whether to enter the market. After observing the entry decisions, each firm in the market chooses a quality level for its product. Finally, observing entry and quality decisions, firms select a two-part tariff (or a linear price schedule). At each stage, firms act simultaneously and independently.

In the presented model consumers differ only in their preferences for quantity. The main finding of this paper is that product differentiation occurs in equilibrium if and only if price discrimination is permitted. This implies that with linear pricing quality differentiation does not relax price competition. Since consumers have identical tastes for quality, firms' incentives for quality differentiation are based on heterogeneity in consumers' preferences for quantity. This is a new contribution to the theory of product differentiation. By contrast, in earlier work on vertical differentiation, consumers are heterogeneous with regard to their (induced) tastes for quality. In these unit-demand models with linear pricing there are always differentiated products in the market.[4] What is the intuition behind the observation that only quality differentiation in combination with two-part tariffs can relax competition? Consumers are homogeneous with respect to their preferences for quality. Thus, consumers have common preferences for price quality pairs. Consequently, when firms are restricted to use linear pricing at most one firm makes positive profits. On the other hand, firms cannot exploit consumers' heterogeneity to relax price

[2] The subdivision in vertical and horizontal differentiation is due to Lancaster (1979).

[3] Linear pricing means that the tariff is a linear function of the quantity q, hence a linear tariff has the following form: $T(q) = p \cdot q$. Note, a two-part tariff $T(q) = A + p \cdot q$ is an affine function.

[4] More precisely, firms choose differentiated products provided that consumers are sufficiently heterogeneous.

competition via tariff differentiation on its own: one firm serves the high demand consumers by offering a tariff with a high fixed fee and a low marginal price while the other firm chooses a tariff with a low fixed fee and a high marginal price to serve the low demand consumers. This is not a Nash equilibrium of the tariff game for equal qualities (perfect substitutes). The reasoning is similar to the one behind the well-known Bertrand paradox: at least one firm has an incentive to slightly undercut its rival's tariff. But, when firms can choose qualities at a stage before the tariff competition stage, this quality game is like a commitment device to share the market. The high quality firm serves the high demand consumers and the low quality firm serves the low demand consumers. If the products are sufficiently differentiated and thus competition is softened then market sharing via tariff differentiation is incentive compatible.

Moreover, the implications of price discrimination policies on welfare, consumers' surplus, and industry profits are investigated. I show that welfare and industry profits are higher and consumers' surplus is lower if two-part tariffs are feasible. If two-part tariff competition is permitted the firms can relax competition and thus both firms enter the market. The increase in entry supports welfare.

To the best of my knowledge, this paper is the first with vertically differentiated firms and multi-unit demand. Because of the asymmetric structure of vertical differentiation models, the models in the literature are solved for specific utility functions only. Since former research is primarily concerned on unit-demand models, there has been no need to design a tractable utility function for the multi-unit demand framework. One contribution of this paper is to provide a tractable formulation. The assumed utility function can be interpreted not only as a consumer's utility function but also as the profit function of a retailing firm. Nonlinear contracts are widely used and discussed in the context of vertical relations. I present two applications where the retailer is i) a local monopolist, ii) a price taker. This paper also contributes to the theory of vertical restraints. The model of the paper can be interpreted as a model of vertical relations where two manufacturers compete for retailing firms. The effect of banning vertical restraints (franchise fees) on upstream competition is analyzed. The finding is that banning franchise fees reduces upstream competition.

The structure of the paper is as follows: After a brief review of the related literature, Section 2 describes the framework of the model. In Section 3, the model is solved by backwards induction. Section 4 compares the results of two-part tariff competition and linear pricing. The final section summarizes the main findings and concludes.

Related Literature: Price discrimination is often observed in oligopolistic markets. With several notable exceptions, the existing literature on price discrimination focuses on the monopoly problem (cf. Wilson (1992) for monopoly pricing). The following articles analyze competitive third-degree price discrimination.[5] Holmes (1989) studies a duopoly model with differentiated goods in which both firms operate in two distinct markets. The remarkable result of his analysis is that profits can decrease when price discrimination across markets is permitted. Corts (1998) obtains a similar finding for an oligopoly model with vertical differentiation. Borenstein (1985) studies a free-entry circular-city model where firms' possibility to discriminate is based either on the strength of brand preferences or consumers' reservation utilities. Liu and Serfes (2005) analyze a vertically differentiated duopoly where firms can purchase information about consumers' preferences. They find that only the high-quality firm acquires the information and uses a discriminating tariff. In contrast to these articles, I analyze second-degree price discrimination.

There exist only few papers on second-degree price discrimination or nonlinear pricing in oligopoly. A classic paper on this topic is Katz (1984). Katz analyzes an economy with informed high demand consumers and uninformed low demand consumers. Informed consumers purchase from the cheapest store while uninformed consumers choose a store at random. In equilibrium, the firms choose tariffs to separate these two groups. A seminal contribution to the literature on nonlinear pricing in oligopoly is Armstrong and Vickers (2001), who study a general framework with spatially differentiated firms that compete in nonlinear tariffs.[6] They show that under certain conditions firms choose welfare optimal two-part tariffs in equilibrium. Nonlinear pricing in spatial competition models is also analyzed by Stole (1995) and Rochet and Stole (2002).[7] The approaches of Stole and Rochet and Stole are highly related to Armstrong and Vickers if the quality is interpreted as quantity. In these models consumers have unit-demand and firms discriminate via different quality-levels. Since quality and quantity have similar properties, these approaches can be reinterpreted as nonlinear pricing. A logit demand model with two-part tariffs is analyzed by Yin (2004).

In contrast to the articles mentioned so far, I analyze nonlinear pricing in a model of vertical rather than horizontal differentiation.[8] In a classic contribution on vertical differentiation, Gabszewicz and Thisse (1979) analyze a price equilibrium of an oligopoly game. Consumers differ in income m and obtain utility $U = s \cdot (m - p)$ when they buy quality s at price p. The qualities of the firms are fixed exogenously in this model. Shaked and Sutton (1982) extend the Gabszewicz-Thisse

[5]Pigou (1920) considers three kinds of price discrimination: first-degree price discrimination is perfect price discrimination, second degree price discrimination is discrimination across quantities and for third degree price discrimination the prices differ for distinguishable consumers.

[6]Armstrong and Vickers (2001) also analyze third-degree price discrimination.

[7]A similar framework is studied by Desai (2001) but with a different focus.

[8]There is no commonly accepted definition of nonlinear pricing. Following Wilson (1992 p.5), I denote a tariff as nonlinear if the average charge is a function of the purchased quantity. In unit-demand models where firms offer various pairs of quality and price, however, these offers are often denoted as nonlinear pricing function. Based on the second definition, nonlinear pricing in a vertically differentiated duopoly is also analyzed by Champsaur and Rochet (1989) and Johnson and Myatt (2003).

model by endogenizing quality levels. The main result is that in equilibrium firms produce distinct qualities and thereby relax price competition. Tirole (1988) shows robustness of these earlier results for the case of the Mussa-Rosen utility function, i.e. for the case in which $U = \theta \cdot s - p$, where θ denotes the consumer's type.[9] While Tirole focuses on parameter values such that the market is fully covered in equilibrium, Choi and Shin (1992) analyze the model when the market is not covered. A complete characterization of quality choices in a duopoly model in which consumers have a Mussa-Rosen utility function is given by Wauthy (1996). All these models assume that consumers have unit-demand. In contrast, I analyze the effects of quantity discounts in a vertically differentiated duopoly, which cannot be captured by one of the utility functions mentioned above. Consequently, I introduce a novel tractable utility function for the framework with vertically differentiated firms and multi-unit demand.

2 Description of the model

There are two potential firms ($i = 1, 2$) producing (distinct) substitute goods. The two firms play a non-cooperative three-stage game.[10] At the first stage, they decide independently and simultaneously whether or not to enter the market. In case of entry, a firm incurs fixed cost $K > 0$. At stage two, each firm observes whether its rival has entered the market. Thereafter, both firms independently and simultaneously choose their respective quality level $s_i \in \{0, \frac{1}{3}, \frac{2}{3}, 1\}$.[11] This stage will be referred to as the quality game. At the third stage, both firms independently and simultaneously choose a tariff, having observed the rival's quality level. Each firm i chooses a two-part tariff: $T_i(q) = A_i + p_i \cdot q$.[12] Here, q denotes quantity, $p_i \geq 0$ is the price per unit, and $A_i \geq 0$ a fixed fee. The third stage will be called tariff game. If price discrimination is banned or infeasible, then $A_i = 0$. The focus of this paper is on firms' strategic incentives for product differentiation. To separate this effect, I assume that costs are zero for all quality levels.

There exists a continuum of consumers with measure one. I assume that consumers practice "one-stop shopping". That is, consumers make all their purchases from one firm. Thus, each consumer takes one of the following three actions: purchase from firm 1, purchase from firm 2, or do not purchase at all. Consumers differ in a taste parameter θ that is uniformly distributed on the unit-interval. I assume a simple linear demand function. When a consumer of type θ buys from firm i, his

[9] Mussa and Rosen (1978) characterize the optimal price-quality schedule for a monopolist.

[10] This three stage game is similar to the one considered by Shaked and Sutton (1982) for their unit-demand approach.

[11] The presented analysis can be generalized to quality-levels $s_i \in [0, 1]$. The existence of tariff game equilibria, however, then is intricate to show.

[12] It is assumed that general nonlinear tariffs are infeasible.

demand is given by:

$$q_i(p_i, \theta) = \begin{cases} \theta(s_i - p_i) , & \text{for } p_i \leq s_i \\ 0 \quad , & \text{for } p_i > s_i \end{cases} . \tag{1}$$

It is assumed that consumers have quasi-linear utility functions and the reservation utility is normalized to zero. Let $v_i(p_i, \theta)$ be the surplus function of a consumer of type θ. By Roy's identity, $q_i(p_i, \theta) \equiv -\frac{\partial v_i(p_i,\theta)}{\partial p_i}$. Thus, the surplus function corresponding to the demand function above has the following form

$$v_i(p_i, \theta) = \begin{cases} \frac{1}{2}\theta(s_i - p_i)^2 , & \text{for } p_i \leq s_i \\ 0 \quad , & \text{for } p_i > s_i \end{cases} . \tag{2}$$

The surplus is the maximum net utility a consumer of type θ can receive excluding a potential fixed fee payment.[13] The type parameter θ is a simple multiplier in the consumer's surplus function. Consumer i's surplus is increasing in the quality he consumes and decreasing in the marginal price he has to pay. The consumer's surplus is a weighted quadratic function of the product's "net value". If firms practice marginal cost pricing ($p_i = 0$), then the utility of a consumer with type θ is given by $U = \frac{1}{2}\theta \cdot s_i^2 - A_i$.[14]

Furthermore, it is assumed that consumers have full information about the tariffs and the quality levels in the market. Consumers' tastes are private information, only the distribution is known by the firms. If a consumer is indifferent between firm 1 and firm 2, he purchases the higher-quality product. If quality levels are equal, the consumer chooses a store at random.[15]

The equilibrium concept employed is subgame perfect Nash equilibrium in pure strategies. Thus, the game is solved by backwards induction.

In the next section I discuss two applications of the model described so far to vertical relations. Readers not interested in this topic may skip the next section.

2.1 Applications to vertical relations

The above surplus function can also be interpreted as a profit function of a retailer. Two-part tariffs are widely used and discussed in the context of vertical relations.[16] Assume that firms, called manufacturers in this section, do not sell their products

[13] Let the utility of consuming the good under consideration with quality s_i be $u(q, s_i, \theta)$. Then $v_i(p_i, \theta) \equiv \max_q \{u(q, s_i, \theta) - p_i q\}$. It is assumed that the utility maximization problem has an interior solution. A utility function corresponding to the assumed surplus function is discussed in Section 4.

[14] In this case the consumer's utility function is similar to the well known Mussa and Rosen (1978) utility function for unit demand models with distinct qualities, where $U = \theta \cdot s - p$. The Mussa and Rosen utility function is used in various models of vertically differentiated markets, for instance Choi and Shin (1992), or in an augmented version in Rochet and Stole (2002).

[15] This assumption is not crucial, however, it simplifies some proofs.

[16] Classic contributions to the theory of vertical restraints, in particular on upstream competition and two-part tariffs, are Rey and Stiglitz (1988) and Bonnano and Vickers (1988).

to consumers directly but to a retail firm. A retail firm can only sell the product of a single manufacturer. As far as vertical relations are concerned, when the manufacturer sets a fixed fee, this fixed fee can be interpreted as franchise fee. For instance, the decision of a potential retailer may be to open either a McDonald's, a Pizza Hut, or neither. A model of vertical differentiation and market power on the upstream market is also analyzed in Avenel and Caprice (2006). In their model a monopolist produces a high quality product and the low quality product is produced by a competitive fringe. Product differentiation on the upstream market is exogenous. The focus of Avenel and Caprice is on retailers' product lines. In the reinterpretation of the model introduced so far, both upstream firms have market power and the degree of quality differentiation is endogenous. But the retailers are exclusive dealers, who cannot sell the products of both manufacturers.

In the following, I will give two examples of how the surplus function (2) can be interpreted as a profit function of a retailer who sells manufacturer i's products.

Example 1: Retailer is local monopolist Consider a retailer of type $\theta \in (0, 1]$ who sells the products of manufacturer i. Assume that the retailer is a local monopolist and operates without costs. The inverse demand function he faces is

$$P(q, \theta) = s_i - \frac{1}{2}\frac{q}{\theta} \,.$$

Here, θ measures the "market size" in the downstream market. A higher θ corresponds to a retailer with higher demand. Given a tariff T_i offered by manufacturer i to each potential retailer, the profit of a retailer of type θ who sells firm i's products is

$$\pi_R^{LM}(q, i, \theta) = \left(s_i - \frac{1}{2}\frac{q}{\theta}\right) q - p_i q - A_i \,. \tag{3}$$

The retailer maximizes his profit with respect to the quantity he sells to consumers (which equals the quantity he purchases from manufacturer i). From the first-order condition of this profit maximization problem, the supply function of this retailer is immediately obtained:

$$q_i^*(p_i, \theta) = \theta(s_i - p_i) \,. \tag{4}$$

Substituting (4) into (3) yields,

$$\pi_R^{LM}(q^*, i, \theta) = \frac{1}{2}\theta(s_i - p_i)^2 - A_i \,. \tag{5}$$

Hence, ignoring the fixed fee, the retailer's profit function is equivalent to the surplus function (2).

Example 2: Retailer is price taker Consider a retail firm that operates as a price taker. Suppose there exists a competitive fringe selling the same qualities as produced by the manufacturers. The market price depends on the quality the retailer sells and is given by

$$P(s_i) = s_i \; . \tag{6}$$

For selling q units of a product purchased by manufacturer i, the retailer has cost

$$c(q, \theta) = \frac{1}{2} \frac{q^2}{\theta} \; , \tag{7}$$

where θ measures how efficient the retailer is. If θ is high, the retailer has relatively low costs for serving consumers. The profit of the retailer when selling the products of firm i is then given by

$$\pi_R^{PT}(q, i, \theta) = s_i q - \frac{1}{2} \frac{q^2}{\theta} - p_i q - A_i \; , \tag{8}$$

where A_i is a franchise fee the retailer has to pay for the permission to sell firm i's products. Maximizing $\pi_R^{PT}(q, i, \theta)$ with respect to q allows to obtain the following supply function of a retailer with cost parameter θ:

$$q_i^*(q, \theta) = \theta(s_i - p_i) \; . \tag{9}$$

Inserting the optimal supply of the retailer in his profit function yields,

$$\pi_R^{PT}(q^*, i, \theta) = \frac{1}{2}\theta(s_i - p_i)^2 - A_i \; . \tag{10}$$

Again, the indirect profit function is equivalent to the surplus function (2) minus the fixed fee.

In what follows, I focus on the case where each producer ($i = 1, 2$) offers his products to consumers directly. Nevertheless, the above applications allow one to interpret consumers as retail outlets. The implications of the model for the context of vertical relations are discussed in the conclusions.

2.2 Preliminary remarks on the quality game

In this section, I establish that when both firms are active in the market, they produce distinct quality levels.

Lemma 1 *Suppose that both firms produce the same quality and two-part tariffs are feasible, then in the unique tariff game equilibrium both firms use the cost-based linear tariff $T^* = 0 \cdot q$ and earn zero profits.*

Proof: All proofs are given in the appendix.

The intuition is similar to the reasoning behind the well-known Bertrand paradox. Assume, for a sake of contradiction, that both firms produce the same quality and at least one firm makes positive profits. Then the firm with lower profits can increase its profits by slightly undercutting the rival's tariff.[17] Firm i undercutting firm j's tariff means that in an expenditure-quantity diagram the tariff of firm i is completely below the tariff of firm j. This logic is still true when the firms have equal positive profits. When firm i slightly undercuts firm j's tariff, firm i obtains all customers of firm j and almost always keeps some of its former customers. Consequently, slightly undercutting the rival's tariff increases profits. Hence, for equal qualities I obtain that the well-known Bertrand result also holds when two-part tariffs are feasible.

Without quality differentiation there is perfect competition and price discrimination is infeasible. Therefore, I obtain the following result for the subgame perfect equilibrium.

Proposition 1 *In any subgame perfect equilibrium in which both firms enter, the firms produce distinct quality levels.*

This result extends Proposition 1 in Shaked and Sutton (1982) to the multi-unit approach with two-part tariffs.

3 Formal analysis

3.1 The tariff game

Suppose that $s_2 > s_1$, so that there is a high-quality supplier (firm 2) and a low-quality supplier (firm 1). The net surplus of a consumer of type θ, given the tariffs of the two firms, depending on the consumer's quality decision is:

$$V(\theta) = \begin{cases} \frac{1}{2}\theta(s_2 - p_2)^2 - A_2, & \text{if he buys from firm 2 (high-quality)} \\ \frac{1}{2}\theta(s_1 - p_1)^2 - A_1, & \text{if he buys from firm 1 (low-quality)} \\ 0 & , \text{ otherwise} \end{cases} . \qquad (11)$$

In equilibrium, consumers with relatively strong tastes for the product buy from the high-quality firm. For "middle-type" consumers, the high-quality firm is too expensive, hence they purchase the low-quality product at a (very) cheap tariff. Consumers with relatively low tastes do not purchase at all. I will call consumers who are indifferent between two options "marginal consumers". Therefore, in the economy there exist two kinds of marginal consumers: one is indifferent between buying from firm 1 and firm 2, whereas the other marginal consumer is indifferent between buying from firm 1 or not buying at all (see Figure 1). The first type of

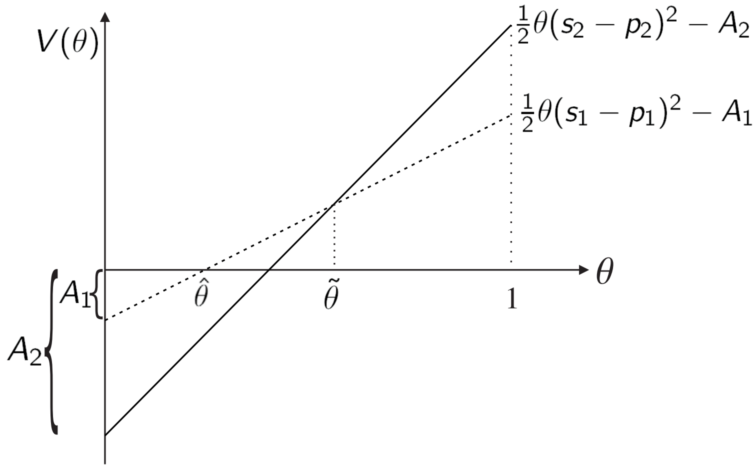

Figure 1: Optimal choices for different consumer types

marginal consumer will be denoted by $\tilde{\theta}$, the latter type by $\hat{\theta}$.

The assumed market structure implies that $A_2 > A_1$ and $(s_2 - p_2) > (s_1 - p_1)$ in equilibrium.[18] From the definitions of the marginal consumers, one immediately obtains the following equations that characterize the fixed fees charged by the two firms:

$$A_1 = \frac{1}{2}\hat{\theta}(s_1 - p_1)^2 \tag{12}$$

$$A_1 = \frac{1}{2}\tilde{\theta}\left[(s_1 - p_1)^2 - (s_2 - p_2)^2\right] + A_2 \tag{13}$$

$$A_2 = \frac{1}{2}\tilde{\theta}\left[(s_2 - p_2)^2 - (s_1 - p_1)^2\right] + A_1 \tag{14}$$

Note that given p_i and the rival's tariff, the fixed fee A_i is uniquely determined by $\tilde{\theta}$. Put differently, the choice of the fixed fee is equivalent to choosing the marginal consumer $\tilde{\theta}$.[19] Each firm i chooses $(p_i, \tilde{\theta})$ to maximize profits. The firms' profit functions, for a given tariff of the competitor, are[20]

$$\pi_2(p_2, \tilde{\theta}) = (1 - \tilde{\theta}) \cdot A_2(p_2, \tilde{\theta}) + p_2 \cdot \int_{\tilde{\theta}}^{1} q_2(p_2, \theta)\, d\theta \tag{15}$$

$$\pi_1(p_1, \tilde{\theta}) = \left(\tilde{\theta} - \hat{\theta}(\tilde{\theta}, p_1)\right) \cdot A_1(p_1, \tilde{\theta}) + p_1 \cdot \int_{\hat{\theta}(\tilde{\theta}, p_1)}^{\tilde{\theta}} q_1(p_1, \theta)\, d\theta \tag{16}$$

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[17]The tariff T_i undercuts tariff T_j if $\forall q \;\; p_i q + A_i < p_j q + A_j$.

[18]Ex ante, there may exist an equilibrium where $A_2 < A_1$ and $(s_2 - p_2) < (s_1 - p_1)$, such that the consumers with high tastes purchase the low quality. Fortunately, this case can be ruled out as an equilibrium candidate for the relevant quality levels. The appendix gives a formal proof of this claim, however, readers should be aware that the claim requires some results presented in Section 3 later on.

[19]Note that in equilibrium the marginal consumers $\tilde{\theta}$ chosen by the two firms are the same.

[20]More precisely, $A_i = A_i(p_i, p_j, A_j, \tilde{\theta})$ for $i \neq j$, but I suppress in the following the rival's tariff parameters.

For now, assume that firms' maximization problems have interior solutions. In the appendix it is shown that this is indeed the case.

3.1.1 Profit maximization problem of firm 2 (high-quality supplier)

More specifically, the profit function of the high quality firm is given by

$$\pi_2(p_2, \tilde{\theta}) = (1-\tilde{\theta}) \cdot A_2 + \frac{1}{2} p_2 (s_2 - p_2)(1-\tilde{\theta}^2) \tag{17}$$

where

$$A_2 = \frac{1}{2}\tilde{\theta}\left[(s_2-p_2)^2 - (s_1-p_1)^2\right] + A_1 \ .$$

First, setting the partial derivative of π_2 with respect to p_2 equal to zero allows to solve for the optimal marginal price of firm 2,

$$p_2^* = \frac{1}{2}(1-\tilde{\theta})s_2 \ . \tag{18}$$

Hence, p_2^* depends only indirectly on rival's tariff via $\tilde{\theta}$. The optimal marginal price p_2^* is determined by the marginal consumer $\tilde{\theta}$. A greater market share of firm 2 leads to a higher marginal price. A greater market share is accompanied by a lower fixed fee. Thus, less of the surplus of the served consumers can be extracted by the fixed fee. This leads to a raise in the optimal marginal price. Note that the optimal marginal price exceeds marginal cost. This result is in contrast to several models of horizontal differentiation, where in equilibrium marginal prices equal marginal costs. For instance, Armstrong and Vickers (2001) show for spatially differentiated markets that the firms offer cost-based two-part tariffs in equilibrium. On the other hand, Yin (2004) points out that in the context of a Hotelling model, marginal prices are higher than marginal costs if the transportation costs are shipping costs.[21] In the model of Armstrong and Vickers consumers' types do not interact with quantity, whereas in Yin's model and the one presented here there is an interaction between consumers' types and quantity. I conclude: regardless of the differentiation framework, marginal prices exceed marginal costs if for a given marginal price consumers with different types prefer different quantities.

The first-order condition for profit maximization of firm 2 with respect to $\tilde{\theta}$ is given by

$$\frac{\partial \pi_2}{\partial \tilde{\theta}} = -A_2 + (1-\tilde{\theta})\frac{1}{2}\left[(s_2-p_2)^2 - (s_1-p_1)^2\right] - p_2(s_2-p_2)\tilde{\theta} \overset{!}{=} 0 \tag{19}$$

Inserting the optimal marginal price, p_2^*, into (19) and rearranging yields

$$A_2^* = \frac{1}{2} \cdot (1-\tilde{\theta}) \left[\frac{1}{4}s_2^2(1-\tilde{\theta}^2) - (s_1-p_1)^2\right] \ . \tag{20}$$

The sign of the derivative of A_2^* with respect to the market share of firm 2 is undetermined.

[21] Anderson and Engers (1994) describe two types of transportation costs. A shipping cost depends on the quantity which is "shipped" and a shopping cost is independent of the amount purchased.

3.1.2 Profit maximization problem of firm 1 (low-quality supplier)

The profit function and the optimization constraints of firm 1 are given by

$$\pi_1(p_1, \tilde{\theta}) = (\tilde{\theta} - \hat{\theta}) \cdot A_1 + \frac{1}{2} p_1 (s_1 - p_1)(\tilde{\theta}^2 - \hat{\theta}^2) \ , \tag{21}$$

where

$$\begin{aligned} \hat{\theta} &= \tilde{\theta}\left[1 - \left(\frac{s_2 - p_2}{s_1 - p_1}\right)^2\right] + \frac{2A_2}{(s_1 - p_1)^2} \\ A_1 &= \frac{1}{2}\tilde{\theta}\left[(s_1 - p_1)^2 - (s_2 - p_2)^2\right] + A_2 \ . \end{aligned}$$

Setting the partial derivative of π_1 with respect to p_1 equal to zero yields an implicit condition for the optimal marginal price p_1^*:

$$\frac{\partial \pi_1}{\partial p_1} \stackrel{!}{=} 0 \quad \Longleftrightarrow \quad 3\hat{\theta} = \tilde{\theta} \ . \tag{22}$$

In equilibrium the low-quality firm serves $\frac{2}{3}$ of the residual demand. Equation (22) can be rewritten as

$$p_1^* = s_1 - \sqrt{\frac{3}{\tilde{\theta}}\left(\frac{1}{2}\tilde{\theta}(s_2 - p_2)^2 - A_2\right)} \ .$$

The optimal marginal price of firm 1 is higher than marginal costs if the net surplus of the marginal consumer $\tilde{\theta}$ is sufficiently small. The net surplus of marginal consumer $\tilde{\theta}$ is quite small if competition between the two firms is not very intense. Hence, when the products of the two firms are sufficiently differentiated the marginal price of firm 1 is positive. The other first-order condition of firm 1 is obtained by setting the partial derivative of the profit function with respect to $\tilde{\theta}$ equal to zero. Rewriting this equation and inserting p_1^* yields to the following formulation for the optimal fixed fee:

$$\begin{aligned} A_1^* = \left(\frac{1}{2}\tilde{\theta}(s_2 - p_2)^2 - A_2\right)&\left[\frac{-3A_2}{\tilde{\theta}(s_2 - p_2)^2} + \frac{7}{2} - \frac{2s_1}{(s_2 - p_2)^2}\sqrt{\frac{3}{2}(s_2 - p_2)^2 - \frac{3A_2}{\tilde{\theta}}}\right] \\ &- \frac{1}{3}\tilde{\theta} s_1 \sqrt{\frac{3}{2}(s_2 - p_2)^2 - \frac{3A_2}{\tilde{\theta}}} \ . \end{aligned} \tag{23}$$

Nash equilibrium of the tariff game: Given that the firms' profit maximization problems have interior solutions, any pure strategy Nash equilibrium of the tariff game is characterized by equations (12), (13), (18), (20), (22), and (23). In a Nash equilibrium both firms choose the best response given the rival's tariff. Hence, for each firm the two first order conditions must hold.[22] Equation (13) ensures that

[22] The second-order necessary conditions (SOCs) are checked in the appendix, see A.2. I suggest to postpone reading A.2 until the end of Section 3.

both firms choose the same marginal consumer $\tilde{\theta}$, which is necessary for an equilibrium. Condition (12) determines the optimal $\hat{\theta}$ for a given tariff of firm 1. The Nash equilibrium of the tariff game cannot be solved analytically. The tariff game equilibrium is characterized by a polynomial of sixth order in $\tilde{\theta}$. Fortunately, for the relevant quality pairs, it can be shown that this polynomial has exactly one root in $[0, 1]$, which is the solution for $\tilde{\theta}$.

3.2 The quality game

If firm i chooses the lowest possible quality level, $s_i = 0$, it makes non-positive profits. Consequently, quality levels $s_i = 0$ cannot be part of a subgame perfect equilibrium in which both firms enter. In addition, quality levels where the firms are not sufficiently differentiated, following the entry of both firms, can be excluded as candidates for a subgame perfect equilibrium.

Proposition 2 *Assume that $2s_1 \geq s_2 \geq s_1$. Then, there is a pure strategy equilibrium of the tariff game where both firms offer linear tariffs. In equilibrium, the low quality firm has zero market share and consequently earns zero profits.*

I assume that the equilibrium with linear tariffs is played whenever possible.[23] On the other hand, if $0 < s_1 < \frac{1}{2}s_2$, then firm 1 will have a positive market share and earns positive profits in equilibrium. To see this, consider firm 2's incentives not to serve the whole market, $\frac{\partial \pi_2}{\partial \tilde{\theta}}|_{\tilde{\theta}=0} > 0$ provided that

$$\left[(s_2 - p_2)^2 - (s_1 - p_1)^2\right] > 0 \,. \tag{24}$$

Put verbally, the high-quality firm leaves an unsatisfied residual demand if condition (24) is satisfied.[24] In this case, the low-quality firm has a positive market share and consequently positive revenues. Condition (24) always holds for $s_2 = 1$ and $s_1 \leq \frac{1}{2}$, because $(s_2 - p_2) \geq \frac{1}{2}s_2 > s_1 \geq (s_1 - p_1)$. Therefore, in the subgame perfect equilibrium $1 > \tilde{\theta}^* > \hat{\theta}^* > 0$. Hence, both fixed fees are positive and both firms realize strictly positive profits. As mentioned above, the tariff game equilibrium is characterized by a polynomial of sixth order in $\tilde{\theta}$. For $s_2 = 1$ and $s_1 = \frac{1}{3}$, this polynomial has exactly one root for all relevant values of $\tilde{\theta}$. With the above analysis and applying numerical methods the next result follows immediately.

Proposition 3 *Assume that the entry cost, K, is sufficiently small. Then there exists a subgame perfect equilibrium in pure strategies. Both firms have strictly positive market shares and make strictly positive profits in equilibrium. The equilibrium*

[23]For the quality pairs $(1, 1), (\frac{2}{3}, \frac{2}{3}), (\frac{1}{3}, \frac{1}{3}), (0, 0), (1, \frac{2}{3}), (\frac{2}{3}, \frac{1}{3})$, the firms use linear tariffs and the low-quality firm makes a zero profit.

[24]If $\tilde{\theta} = 0$ in equilibrium then $\hat{\theta}$ also equals zero and consequently $A_1 = A_2 = 0$.

values are:

$$s_1 = 1/3 \qquad s_2 = 1$$
$$\pi_1 = .00138222 \qquad \pi_2 = .142833$$
$$p_1 = .0657433 \qquad p_2 = .351995$$
$$A_1 = .00353261 \qquad A_2 = .0550837$$
$$\hat{\theta} = .0986701 \qquad \tilde{\theta} = .29601$$

4 Two-part tariff competition versus linear pricing

In this section, the results of the game with two-part tariffs is compared to the outcome when price discrimination is banned. An interesting result can be obtained from the surplus function (2). Suppose that only linear prices are allowed. In this case, each consumer chooses a firm to maximize the difference $(s_i - p_i),\ i = 1, 2$.

Proposition 4 *Suppose there is a ban on price discrimination and that both firms have entered the market. Then it is impossible that both firms realize strictly positive profits in a subgame perfect equilibrium of the subgame containing the quality game and the tariff game.*

The outcome of the two-stage game (quality game and tariff game) when only linear pricing is feasible, depends on the behavior of firm 1. When firm 1 chooses a quality $s_1 < s_2 \leq 1$, then in equilibrium firm 1 has no positive market share and consequently zero profits. On the other hand, if firm 1 chooses $s_1 = s_2$, there is perfect competition and $\pi_1 = \pi_2 = 0$. In either case firm 1 earns zero profits.

In contrast, if two-part tariffs are allowed the two firms produce distinct qualities and realize strictly positive profits. Relaxing price competition via quality differentiation is possible if and only if two-part tariff competition is permitted. With the above analysis the characterization of firms' optimal entry decisions follows immediately. A firm decides to enter the industry if its expected profits exceed the entry cost.

Proposition 5 *For sufficiently small entry cost, K,*

- *two firms enter the market if price discrimination is permitted.*
- *a ban on price discrimination leads to a monopoly.*

Vertical differentiation is optimal if and only if price discrimination is feasible. In contrast, if price discrimination is banned, the products in the market are not differentiated. What are the driving forces of this result? An explanation can be given by analyzing the utility function corresponding to surplus function (2),

$$u(q, s_i, \theta) = s_i \cdot q - \frac{1}{2}\frac{q^2}{\theta} \qquad \theta \in (0, 1] \; .$$

Note that all consumers have the same tastes for quality, but differ in their preferences for quantity. The type parameter θ determines a consumer's satiation point. Here, firms' incentives for quality differentiation are based on heterogeneity in consumers' preferences for quantity, which is novel in the literature. Former research studied vertical differentiation only with linear tariffs, for instance Shaked and Sutton (1982). In these unit-demand frameworks, consumers are heterogeneous with regard to their tastes for quality.[25]

Now I address the question how a ban on price discrimination affects welfare, consumers' surplus and industry profits. Social welfare is defined as the sum of consumers' surplus and industry profits. From Proposition 5 the next result follows immediately.

Proposition 6 *Assume that the entry cost, K, is sufficiently small. Then social welfare and industry profits are lower and aggregate consumers' surplus is higher if two-part tariff competition is banned.*

Permitting price discrimination encourages entry, which in turn increases welfare.[26] This finding is in line with the result obtained by Armstrong and Vickers (2001), who analyze non-linear pricing in a free entry circular city model. In their model, as in the one presented here, permitting price discrimination increases industry profits, this encourages entry, which in turn increases welfare. On the other hand, the above proposition is in contrast to the result of Armstrong and Vickers (1993). They study the effect of permitting third-degree price discrimination on entry.[27] Armstrong and Vickers show that for reasonable values of the entry cost, "[...] entry will occur if and only if price discrimination is banned" (Armstrong and Vickers 1993, p.337). Their result is driven by the fact that competitive third-degree price discrimination can lower market profits.

It is worthwhile to point out, that imposing a minimum quality-standard can have the same entry effect as banning price discrimination. If the quality standard is too high, so that the possible profits of firm 1 are lower than the entry cost, then in the subgame perfect equilibrium only one firm will enter the market.

[25] See, for example, Choi and Shin (1992). In Shaked and Sutton (1982), consumers have the same utility function: $u(s, m) = s \cdot m$, and different incomes (m). Note that the income is like a taste parameter for quality.

[26] Market entry in a vertical product differentiation model is also analyzed by Donnenfeld and Weber (1992, 1995) and in particular by Johnson and Myatt (2003), however, these are unit-demand models.

[27] In Armstrong and Vickers (1993) an incumbent sells its products on two markets. In one of these markets the incumbent faces a potential entrant.

5 Summary and conclusions

In this paper, a vertically differentiated duopoly with endogenous degree of product differentiation and two-part tariffs was analyzed. The main finding is that product differentiation occurs in equilibrium if and only if price discrimination is permitted. In the presented model, firms' incentives for quality differentiation are based on consumers' heterogeneity in their preferences for quantity. In earlier models of vertically differentiated industries, consumers differ in their tastes for quality. Here, consumers only differ in their preferences for quantity. Nevertheless, firms choose distinct qualities in equilibrium if two-part tariffs are feasible. The contribution of this paper to the theory of product differentiation is that price discrimination can lead to quality differentiation.

As shown, the model can also be interpreted as a model of vertical relations with upstream competition. As far as vertical restraints are concerned this paper shows that franchise fees can improve upstream competition. This result is in contrast to several classic contributions where the number of upstream producers is exogenously given. For instance, Rey and Stiglitz (1988) point out that "vertical restraints may be used only to decrease competition between producers". When a government authority regulates vertical contracts, as done with the Robinson-Patman Act, the authority should take into account the effect of such regulations on the number of producers in the market.

In addition, for the vertical differentiation model that I introduced the effects of banning price discrimination on entry, welfare, consumers' surplus, and industry profits were analyzed. It was shown that price discrimination encourages entry, which in turn increases welfare. Consumers, however, are worse off when price discrimination is permitted. Moreover, the presented model shows an interesting point for R&D-models. Consider a model of product innovation with surplus function $v = \frac{1}{2}\theta(s-p)^2$, where an incumbent faces a potential entrant.[28] The entrant can invest in R&D. When the entrant invests in R&D it enters with a sufficiently superior product compared to incumbent's one. If price discrimination is banned and the entrant innovates, it thereafter serves the whole market. On the other hand, if two-part tariff competition is permitted both firms share the market when the entrant innovates.

For real-world applications, this model represents just an extreme case, similar as pure spatial competition or logit demand models. In reality, firms are differentiated in more than one characteristic. It is hard to say which differentiating framework is more suitable to describe a real market. Markets for wired telephone services are usually fully covered in developed countries and hence comparable to spatial competition frameworks. If a telephone service provider changes its tariff, this has

[28] Product innovation with a Mussa-Rosen type utility function is analyzed for instance by Hoppe and Lehmann-Grube (2005).

no effect on the total number of fixed-line network subscribers. On the other hand, clubs (health, tennis etc.) offer a fixed-quality, for instance a golf club has only one golf course. Hence, for such markets the vertical differentiation approach chosen here seems more suitable. Other product markets are more like the logit-demand model: when a supplier raises its tariff, some of its former consumers will buy from a rival, others will not participate in the market anymore.

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A Appendix

A.1 Proofs to propositions and lemmas

Proof of Lemma 1: When both firms choose the same quality level, their products are perfect substitutes. For the proof, I distinguish two cases.

Case 1: Suppose $\pi_i > \pi_j \geq 0$ and that the corresponding tariffs are (T_i, T_j). Firm j can increase its profit when it offers the tariff

$$T_j^* = \begin{cases} T_i - \varepsilon & , \text{ if } A_i > 0 \\ (p_i - \varepsilon)q , & \text{ if } A_i = 0 \end{cases} ,$$

where $\varepsilon > 0$ is sufficiently small. The profit of firm j is then arbitrarily close to $\pi_i > \pi_j$.

Case 2:[29] Suppose that $\pi_i(T_i, T_j) = \pi_j(T_j, T_i) > 0$, where T_i is the tariff of firm i and T_j the tariff of firm j. Again, firm j can increase its profit by slightly undercutting its rival's tariff. That is, firm j uses the tariff T_j^* defined in Case 1. The rise in firm j's profit is then

$$\lim_{\varepsilon \to 0} \Big[\pi_j(T_j^*, T_i) - \pi_j(T_j, T_i) \Big] = \lim_{\varepsilon \to 0} \Big[\pi_j(T_j^*, T_i) - \pi_i(T_i, T_j) \Big] \geq$$
$$\Big[1 - \hat{\theta}\Big] A_i + p_i \int_{\hat{\theta}}^{1} q_i(p_i, \theta) \, d\theta - \Big[\lambda \cdot A_i + p_i \int_{1-\lambda}^{1} q_i(p_i, \theta) \, d\theta \Big] > 0 , \quad (A.1)$$

where λ is the fraction of customers that purchase from firm i under tariffs (T_i, T_j) and $\hat{\theta}$ is the marginal consumer who is indifferent between buying or not when firm j would not be present.[30] Note that when firm j slightly undercuts firm i's tariff,

[29] I am grateful to Heidrun C. Hoppe for suggesting the analytical proof for case 2 of Lemma 1.

[30] If the marginal consumer who is indifferent between purchasing or not when only firm i would be present does not exist, then $\hat{\theta}$ is equal to zero.

firm j obtains all customers of firm i, λ, and additionally keeps some of its former customers. Consequently, $\lambda < 1 - \hat{\theta}$.

Hence, when both firms produce the same quality level, there exists no equilibrium where at least one firm earns strictly positive profits.

q.e.d.

Proof of Proposition 1: Follows immediately from Lemma 1.

Proof of Proposition 2: First, it is shown that $p_1 = A_1 = 0$ is a best response for firm 1 given $A_2 = 0$ and $p_2 = s_2 - s_1$.

- If $p_1 > 0$ then firm 1 has no market share for all $A_1 \geq 0$, since $s_2 - p_2 = s_1 > s_1 - p_1$.
- If $A_1 > 0$ and firm 1 has a positive market share then it has to hold that $s_1 - p_1 > s_2 - p_2 = s_1$. This condition is violated for all $p_1 \geq 0$.

Now suppose firm 1 offers the tariff with $p_1 = A_1 = 0$. It is shown that offering $T_2 = 0 + (s_2 - s_1)q$ is a best response for firm 2.

If firm 2's profit maximization problem has an interior solution then T_2 is characterized by FOCs. According to (20), the optimal fixed fee is given by

$$A_2^* = \frac{1}{2}(1-\tilde{\theta})\left[\frac{1}{4}s_2^2(1-\tilde{\theta}^2) - s_1^2\right] . \tag{A.2}$$

The fixed fee has to be non negative, hence the following condition has to hold

$$\frac{1}{4}s_2^2(1-\tilde{\theta}^2) - s_1^2 \geq 0 . \tag{A.3}$$

For $2s_1 \geq s_2$ condition (A.3) is violated and thus the optimal tariff is a corner solution. That is T_2 is either a linear tariff ($A_2 = 0$) or a flat tariff ($p_2 = 0$).

I) Flat tariff ($A_2 > 0, p_2 = 0$): By the definition of the marginal consumer $\tilde{\theta}$, for $p_1 = A_1 = 0$ I obtain

$$A_2 = \frac{1}{2}\tilde{\theta}(s_2 - s_1)^2 . \tag{A.4}$$

Firm 2's profit is given by

$$\pi_2 = (1-\tilde{\theta})\frac{1}{2}(s_2 - p_2)^2 .$$

The profit maximizing marginal consumer is $\tilde{\theta} = 1/2$ and the profit π^{flat} is then

$$\pi^{flat} = \frac{1}{8}(s_2^2 - s_1^2) . \tag{A.5}$$

II) Linear tariff ($A_2 = 0, p_2 > 0$): In the case of linear tariffs it is clear that the optimal marginal price is $p_2^* = s_2 - s_1$. Then each consumer purchases from firm

2. If firm 2 sets a higher price it has no market share, a lower price is not optimal because $p_2^* = s_2 - s_1 \leq \frac{1}{2} = p_M$, where p_M is the price of a monopolist with linear tariff. The profit of firm 2 with linear pricing, π^{lin}, is

$$\pi^{lin} = \frac{1}{2} s_1 (s_2 - s_1) \; . \tag{A.6}$$

A comparison of (A.5) and (A.6) reveals that a linear tariff is optimal for firm 2.

q.e.d.

Proof of Proposition 3: Solving the equations system given by the equations (12), (13), (18), (20), (22), and (23) yields to the following polynomial in $\tilde{\theta}$

$$\begin{aligned} P(\tilde{\theta}) = & \quad 27 s_2^4 (9 - 30\tilde{\theta} - 29\tilde{\theta}^2 + 108\tilde{\theta}^3 + 63\tilde{\theta}^4 - 110\tilde{\theta}^5 - 75\tilde{\theta}^6) \\ & + \; 108 s_2^2 (3 - 17\tilde{\theta} + 14\tilde{\theta}^2 + 46\tilde{\theta}^3 - 33\tilde{\theta}^4 - 45\tilde{\theta}^5) \\ & + \; 108 (1 - 8\tilde{\theta} + 18\tilde{\theta}^2 - 27\tilde{\theta}^4) \\ & - \; 576 s_1^2 (3 - 23\tilde{\theta} + 57\tilde{\theta}^2 - 45\tilde{\theta}^3) \\ & - \; 192 s_1^2 s_2^2 (9 - 48\tilde{\theta} + 58\tilde{\theta}^2 + 40\tilde{\theta}^3 - 75\tilde{\theta}^4) \\ & - \; 16 s_1^2 s_2^4 (27 - 81\tilde{\theta} - 18\tilde{\theta}^2 + 190\tilde{\theta}^3 - 25\tilde{\theta}^4 - 125\tilde{\theta}^5) \\ & \stackrel{!}{=} \; 0 \end{aligned}$$

A plot of the polynomial for the relevant quality levels ($s_1 = 1/3$ and $s_2 = 1$) is given below. The plot (Figure 2) shows that the polynomial has exactly one root for $\tilde{\theta} \in [0, 1]$.

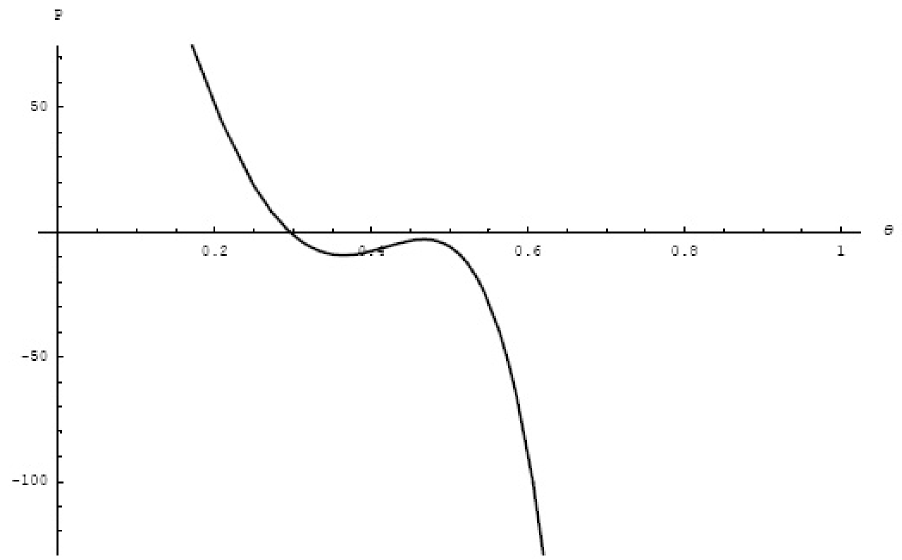

Figure 2: Solution of the equilibrium marginal consumer

Using numerical methods for solving the equations system yields to the equilibrium values for marginal prices, fixed fees, and profits.

q.e.d.

Proof of Proposition 4: Suppose $s_1 \leq s_2 \leq 1$, then consumer θ's net utility is: $\frac{1}{2}\theta(s_i - p_i)^2$ for $i = 1, 2$, if he buys from firm i. Hence, firm 1's profit is,

$$\pi_1 = \begin{cases} p_1(s_1 - p_1)\int_0^1 \theta \, d\theta & , \text{if } s_1 - p_1 > s_2 - p_2 \\ \frac{1}{2} \cdot p_1(s_1 - p_1)\int_0^1 \theta \, d\theta & , \text{if } s_1 - p_1 = s_2 - p_2 \text{ and } s_1 = s_2 \\ 0 & , \text{otherwise} \end{cases} . \tag{A.7}$$

Consequently, firm 1 has an incentive to choose $p_1 \geq 0$ as high as possible such that $s_1 - p_1 > s_2 - p_2$ and it can serve the whole market. Clearly firm 1 will not set p_1 higher than the monopoly price, that is $p_1 \leq \frac{1}{2}s_1$. Note that the problem for firm 2 is similar. Thus for the price game equilibrium one obtains:

$$\begin{array}{llll} \text{if } s_1 = s_2 & \Longrightarrow \quad p_1^* = 0, \ p_2^* = 0 & \text{and} & \pi_1^* = 0, \ \pi_2^* = 0 \\ \text{if } s_1 < s_2 & \Longrightarrow \quad p_1^* = 0, \ p_2^* = s_2 - p_2 & \text{and} & \pi_1^* = 0, \ \pi_2^* > 0 \ , \end{array}$$

If firm 1 is aware that $s_2 = 1$, then the profit of firm 1 is always zero, independent of the quality level s_1. Hence, all quality-levels $s_1 \in \{0, \frac{1}{3}, \frac{2}{3}, 1\}$ are possible in a subgame perfect equilibrium.

q.e.d.

Proof of Proposition 5: Follows directly from Proposition 3 in combination with Proposition 4.

Proof of Proposition 6: If price discrimination is permitted, the equilibrium tariffs, profits, and quality-levels are given in Proposition 3. The social welfare is then:

$$W_D := \pi_1 + \pi_2 + \int_{\tilde{\theta}}^1 v_2(p_2, \theta) \, d\theta - (1 - \tilde{\theta})A_2 + \int_{\hat{\theta}}^{\tilde{\theta}} v_1(p_1, \theta) \, d\theta - (\tilde{\theta} - \hat{\theta})A_1 \ . \tag{A.8}$$

Substituting the equilibrium values into (A.8), one obtains

$$W_D^* = 0.201913 \ . \tag{A.9}$$

Furthermore, standard calculations show that industry profits and consumers' surplus are

$$\Pi_D^* := \pi_1^* + \pi_2^* = 0.144215 \tag{A.10}$$

$$CS_D^* = 0.0576981 \ . \tag{A.11}$$

On the other hand, if price discrimination is banned, only one firm (M) enters the market. The monopolist chooses $s_M = 1$. The profit maximization problem for the monopolist is:

$$\pi_M = p_M \int_0^1 \theta(1 - p_M) \, d\theta \ \rightarrow \ \max_{p_M} \ . \tag{A.12}$$

The optimal price is $p_M^* = \frac{1}{2}$ and the corresponding equilibrium values for profit, welfare and consumers' surplus are:

$$\begin{aligned}\pi_M^* &= 0.125 &\text{(A.13)}\\ W_M^* &= 0.1875 &\text{(A.14)}\\ CS_M^* &= 0.0625\ . &\text{(A.15)}\end{aligned}$$

q.e.d.

A.2 Examination of second-order conditions

First I check the second-order condition (SOC) for firm 2 (high-quality). The partial derivative of π_2 with respect to p_2 is:

$$\frac{\partial \pi_2}{\partial p_2} = \frac{1}{2} s_2 (1-\tilde{\theta})^2 - (1-\tilde{\theta}) p_2\ . \tag{A.16}$$

From (A.16) it is easy to take the second derivative of π_2 with respect to p_2 and the cross-partial:

$$\begin{aligned}\frac{\partial^2 \pi_2}{\partial p_2^2} &= -(1-\tilde{\theta}) < 0 &\text{(A.17)}\\ \frac{\partial^2 \pi_2}{\partial \tilde{\theta} \partial p_2} &= p_2 - s_2(1-\tilde{\theta}) &\text{(A.18)}\end{aligned}$$

Taking the second-order partial derivative of π_2 with respect to $\tilde{\theta}$ yields

$$\frac{\partial^2 \pi_2}{\partial \tilde{\theta}^2} = -\left[(s_2 - p_2)^2 - (s_1 - p_1)^2\right] - p_2(s_2 - p_2) < 0\ . \tag{A.19}$$

Since the zero points of the FOCs are unique, the FOCs describe a global maximum point if the profit function is concave in the neighborhood of the stationary point. Consequently, it is sufficient to check the sign of the determinant of the Hessian matrix. The determinant of the Hessian is:

$$\det(H) \equiv \frac{\partial^2 \pi_2}{\partial \tilde{\theta}^2} \cdot \frac{\partial^2 \pi_2}{\partial p_2^2} - \left(\frac{\partial^2 \pi_2}{\partial \tilde{\theta} \partial p_2}\right)^2$$

$$\Rightarrow \det(H) = (1-\tilde{\theta})\left[(s_2 - p_2)^2 - (s_1 - p_1)^2 + p_2(s_2 - p_2)\right] - \left(p_2^2 - 2p_2 s_2(1-\tilde{\theta}) + s_2^2(1-\tilde{\theta})^2\right)\ . \tag{A.20}$$

Since a corner solution ($p_2 = 0$ or $p_2 = s_2$) cannot be optimal, the optimal marginal price is characterized by the FOC. Inserting the optimal marginal price $p_2^* = \frac{1}{2}(1 - \tilde{\theta})s_2$ into (A.20) yields

$\det(H) =$

$$(1-\tilde{\theta})\Bigg[\underbrace{(s_2-p_2)^2-(s_1-p_1)^2}_{\geq 0,\ \text{for } A_2\geq 0}+\underbrace{\frac{1}{4}s_2^2(1-\tilde{\theta})(1+\tilde{\theta})-\frac{1}{4}s_2^2(1-\tilde{\theta})}_{>0}\Bigg]>0\ . \quad \text{(A.21)}$$

q.e.d.

Next, I check the SOC for the low-quality firm. To prove that the SOC holds for the low quality firm using the determinant of the Hessian matrix is very tedious. It is easier to show that the profit maximization problem of firm 1 is not solved by a corner solution. Suppose firm 1 sets the marginal price as high as possible, that is, $p_1 = s_1$. Then consumers obtain a non positive surplus if they purchase from firm 1 and consequently $\pi_1 = 0$. Similarly, if firm 1 sets $\tilde{\theta} = 0$ then it has no market share and thus zero profits. On the other hand, if $\tilde{\theta} = 1$ the high-quality firm has no market share and realizes zero profits. This cannot happen in equilibrium: if only one firm is active in the market it is clearly the high-quality firm. To summarize, all corner solutions can be ruled out except that firm 1 sets $p_1 = 0$ and offers a flat tariff.

Suppose $p_1 = 0$ is optimal. The profit maximization problem of firm 1 then is given by

$$\pi_1^{flat}(\tilde{\theta}) := \left(\tilde{\theta}-\hat{\theta}(\tilde{\theta})\right)A_1(\tilde{\theta}) \ \rightarrow \max_{\tilde{\theta}}$$

with

$$A_1 = \frac{1}{2}\tilde{\theta}[s_1^2-(s_2-p_2)^2]+A_2 \ \text{ and } \ \hat{\theta} = \frac{2A_1}{s_1^2}\ .$$

Taking the derivative of π_1^{flat} with respect to $\tilde{\theta}$ yields

$$\begin{aligned}\frac{d\pi_1^{flat}}{d\tilde{\theta}} = \frac{(s_2-p_2)^2}{s_1^2}&\left(\tilde{\theta}\frac{1}{2}\left[(s_1^2-(s_2-p_2)^2\right]+A_2\right)\\ &+\left(\frac{\tilde{\theta}(s_2-p_2)^2-2A_2}{s_1^2}\right)\frac{1}{2}\left[s_1^2-(s_2-p_2)^2\right] \overset{!}{=} 0\ . \quad \text{(A.22)}\end{aligned}$$

Solving the above equation for $\tilde{\theta}$ yields

$$\tilde{\theta} = \frac{2(s_2-p_2)^2-s_1^2}{(s_2-p_2)^2\left[(s_2-p_2)^2-s_1^2\right]}A_2 \quad \text{(A.23)}$$

Evaluating the profit function, π_1^{flat}, at the equilibrium tariff of firm 2 and for $s_1 = \frac{1}{3}$ and $s_2 = 1$ yields

$$\pi_1^{flat} = 0.00129999 < 0.0138222 = \pi_1^*\ .$$

Hence a flat tariff is not optimal for firm 1. All potential corner solutions have been ruled out as profit maximizing solutions, thus the profit maximization problem of firm 1 has an interior solution.

q.e.d.

A.3 Proof of the claim in Footnote 18

Note that for the complete analysis of the tariff game it was not used that $s_2 > s_1$. It was used that firm 2 serves the consumers with high valuation and that firm 1 serves the consumers with low valuations. Now suppose that $s_2 < s_1$ and that firm 2 still serves the consumers with high valuations while firm 1 serves the consumers with low valuations. The assumed market structure implies that

$$s_1 - p_1 < s_2 - p_2 \tag{A.24}$$

and $A_2 > A_1 \geq 0$. The marginal price of the firm that serves the market segment $(1 - \tilde{\theta})$, firm 2, is always strictly positive. Note that $\frac{\partial \pi_2}{\partial p_2}\big|_{p_2=0} > 0$. The optimal marginal price is not characterized by a corner solution and thus given by the following equation

$$p_2^* = \frac{1}{2}(1 - \tilde{\theta})s_2 \ . \tag{A.25}$$

Substituting (A.25) into (A.24) yields to the following condition

$$\frac{1}{2}(1 + \tilde{\theta})s_2 > s_1 - p_1 \ . \tag{A.26}$$

Clearly it should hold that $2s_2 < s_1$, otherwise there is an equilibrium with linear pricing and the high-quality firm (firm 1) serves the whole market. The unique quality pair that satisfies the above condition and where both firms share the market is $s_1 = 1, s_2 = 1/3$. Rearranging (A.26) yields

$$\frac{1}{3} > \frac{1}{2}(1 + \tilde{\theta})s_2 > s_1 - p_1 = 1 - p_1 \ . \tag{A.27}$$

Note that $p_1 \leq p_M = \frac{1}{2}$, where p_M is the price of a monopolist who is restricted to use linear pricing.[31] Thus, there cannot be an equilibrium with the low-quality firm serving the high valuation consumers and the high-quality firm serving the low valuation consumers.

q.e.d.

References

[1] **Anderson, S.** and **M.Engers (1994):** Spatial competition with price-taking firms, *Economica*, Vol.61, 125-136.

[2] **Armstrong, M.** and **J.Vickers (1993):** Price Discrimination, Competition and Regulation, *The Journal of Industrial Economics*, Vol.41, 335-360.

[3] **Armstrong, M.** and **J.Vickers (2001):** Competitive price discrimination, *RAND Journal of Economics*, Vol. 32(4), 579-605.

[4] **Avenel, E.** and **S. Caprice (2006):** Upstream market power and product line differentiation in retailing, *International Journal of Industrial Organization*, Vol.24, 319-334.

[31] The price of a monopolist with linear tariffs is independent of the market size. Assume that $\theta \sim U[\underline{\theta}, \bar{\theta}]$ where $1 \geq \bar{\theta} > \underline{\theta} \geq 0$ than the optimal price of a monopolist is $p_M = \frac{1}{2}$.

[5] **Bonanno, G.** and **J. Vickers (1988):** Vertical Separation, *The Journal of Industrial Economics*, Vol.36(3), 257-265.

[6] **Borenstein, S. (1985):** Price Discrimination in Free-Entry Markets, *RAND Journal of Economics*, Vol. 16, 380-397.

[7] **Champsaur, P.** and **J-C. Rochet (1989):** Multiproduct Duopolists, *Econometrica*, Vol. 57(3), 533-557.

[8] **Choi, C.J.** and **H.S. Shin (1992):** A comment on a Model of vertical Product Differentiation, *The Journal of Industrial Economics*, 229-231.

[9] **Corts, K.S. (1998):** Third-degree price discrimination in oligopoly: all-out competition and strategic commitment, *RAND Journal of Economics*, Vol. 29(2), 306-323.

[10] **Desai, P. (2001):** Quality Segmentation in Spatial Markets: When Does Cannibalization Affect Product Line Design?, *Marketing Science*, Vol.20, 265-283.

[11] **Donnenfeld, S.** and **S. Weber (1992):** Vertical Product Differentiation with Entry, *International Journal of Industrial Organization*, Vol.10, 449-472.

[12] **Donnenfeld, S.** and **S. Weber (1995):** Limit Qualities and Entry Deterrence, *RAND Journal of Economics*, Vol.26, 113-130.

[13] **Gabszewicz, J.J.** and **J.-F. Thisse (1979):** Price Competition, Quality, and Income Disparities, *Journal of Economic Theory*, Vol.20, 340-359.

[14] **Holmes, T.J. (1989):** The Effects of Third-Degree Price Discrimination in Oligopoly, *The American Economic Review*, Vol.79, 244-250.

[15] **Hoppe, H.C.** and **U. Lehmann-Grube (2005):** Innovation timing games: a general framework with applications, *Journal of Economic Theory*, Vol.121, 30-50.

[16] **Johnson, J.P.** and **D.P. Myatt (2003):** Multiproduct Quality Competition: Fighting Brands and Product Line Pruning, *The American Economic Review*, Vol.93(3), 748-774.

[17] **Katz, N.L. (1984):** Price Discrimination and Monopolistic Competition, *Econometrica*, Vol.52, 1453-1471.

[18] **Lancaster, K.J. (1979):** Variety, Equity and Efficiency, *Columbia University Press*, New York.

[19] **Liu, Q.** and **K.Serfes (2005):** Imperfect price discrimination in a vertical differentiation model, *International Journal of Industrial Organization*, Vol.23, 341-354.

[20] **Mussa, M.** and **S. Rosen (1978):** Monopoly and Product Quality, *Journal of Economic Theory*, Vol.18, 301-317.

[21] **Pigou, A.C. (1920):** The Economics of Welfare, 2nd edition (1924), *Macmillian Press*, London.

[22] **Rey, P.** and **J. Stiglitz (1988):** Vertical Restraints and Producers' Competition, *European Economic Review*, Vol.32, 561-568.

[23] **Rochet, J.-C.** and **L.A. Stole (2002):** Nonlinear Pricing with Random Participation, *Review of Economic Studies*, Vol.69, 277-311.

[24] **Shaked, A.** and **J. Sutton (1982):** Relaxing Price Competition Through Product Differentiation, *Review of Economic Studies*, Vol.49, 3-13.

[25] **Stole, L.A. (1995):** Nonlinear Pricing and Oligopoly, *Journal of Economics and Management Strategy*, Vol.4, 529-562.

[26] **Tirole, J. (1988):** The Theory of Industrial Organization, *MIT Press*, Cambridge.

[27] **Wauthy, X. (1996):** Quality Choice in Models of Vertical Differentiation, *The Journal of Industrial Economics*, Vol.44(3), 345-353.

[28] **Wilson, R.B. (1993):** Nonlinear Pricing, *Oxford University Press*, New York.

[29] **Yin, X. (2004):** Two-part tariff competition in duopoly, *International Journal of Industrial Organization*, Vol.22, 799-820.