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by

Tibor Neugebauer, Reinhard Selten

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Bonn Graduate School of Economics
Department of Economics
University of Bonn
Adenauerallee 24 - 42
D-53113 Bonn

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INDIVIDUAL BEHAVIOR OF FIRST-PRICE SEALED-BID AUCTIONS: THE IMPORTANCE OF INFORMATION FEEDBACK IN EXPERIMENTAL MARKETS[‡]

Tibor Neugebauer and *Reinhard Selten**

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* Reinhard Selten is Professor of Economics *em*. at the University of Bonn. Address: Laboratorium für Experimentelle Wirtschaftsforschung, Friedrich-Wilhelm-Universität Bonn, Adenauerallee 24-42, D-53113 Bonn, Germany. Tibor Neugebauer is post-doctoral researcher at the University of York. Address: University of York, Department of Economics & Related Studies, UK - Yo10 5DD York.

Abstract

This article reports the results of a first-price sealed-bid auction experiment, which has been designed to test the Nash equilibrium predictions of individual bidding behavior. Subjects faced in 100 auctions always the same resale value and competed with computerized bids. Three treatments were considered which varied with the conditions of information feedback. In earlier experimental work an overbidding above the risk neutral Nash equilibrium has been frequently reported. Our data provide evidence that this overbidding regularity can be a consequence of the standard information feedback in auction experiments of revealing only the winning bid after each auction. By means of learning direction theory we explain the individual bidding dynamics. Finally we apply impulse balance theory and make long run predictions of the individual bidding behavior.

0 Introduction

In the first-price sealed-bid auction (hereafter FPA) each of the N>1 bidders who attend the market submits a sealed bid. The bidder who submits the high bid wins the auction and pays a price equal to his bid. Consider the independent private value model, in which each bidder has private information only about his own resale value and knows the distribution from which all resale values are independently and identically drawn. William Vickrey (1961) showed the existence of a symmetric, unique Nash equilibrium (hereafter, RNNE) in the FPA for the case of risk neutral bidders. Let b_i denote bidder i's bid, $i=1,..,N$, and v_i his resale value, which is drawn from the continuous uniform distribution over the interval [0,1], the RNNE bidding strategy can be written as

RNNE: $$b_i^*(v_i) = \frac{N-1}{N} v_i \tag{1}$$

The RNNE strategy can be interpreted as the bid that equals a bidder's expectation about the greatest resale value of his competitors given this value is smaller than his own. However, a bidder has no dominant strategy in the FPA, his optimal strategy depends on his beliefs about the behavior of the other bidders. The existence of the RNNE hinges crucially on bidders' identical beliefs and identical strategies.

The first experimentalists to test the predictive power of the RNNE were Coppinger, Smith and Titus (1980). They conducted an experiment, in which subjects interacted repeatedly with each other in experimental auction markets. Their main result with respect to the FPA was that winning bids exceeded significantly the RNNE prediction.[1] Since, the bidding above the RNNE prediction (so-called "overbidding") has been reported from several experiments with the FPA.[2] In order to provide an explanation for the overbidding, Cox, Roberson and Smith (1982) and Cox, Smith and Walker (1982, 1983a/b, 1984, 1985a/b, 1987, 1988) developed the constant relative risk aversion model (hereafter CRRAM) which generalizes Vickrey's model. The CRRAM allows for heterogeneity of bidders' risk preferences: each bidder i faces an

[1] Subjects participated three times in ten auctions. In each auction they received a resale value drawn from a discrete uniform distribution and at the end of every auction the winning bid was revealed.
[2] See Kagel (1995) for a survey.

Arrow-Pratt constant relative risk aversion of $1\text{-}r_i$, where r_i (the risk preference parameter) is a random variable with probability distribution on the unit-interval. Let resale values v_i, $i=1,..,N$, be drawn independently from a continuous uniform distribution over the interval [0,1], the Nash equilibrium bidding strategy in the CRRAM writes as

$$\text{CRRAM :} \qquad b_i^*(v_i, r_i) = \frac{N-1}{N-1+r_i} v_i \qquad (2)$$

Just as in the risk neutral model, there is no dominant bidding strategy in the CRRAM, the Nash equilibrium depends on the bidder's beliefs that all other bidders bid constant fractions of their resale value. For risk neutral bidders, i.e., if r_i=1, the CRRAM bid coincides with the RNNE. If the risk preference parameter r_i is smaller (greater) than one, a bidder i is risk-averse (risk-preferring) and his bid corresponding to the CRRAM exceeds (falls short of) the RNNE bid.[3]

Harrison (1989) criticized the methodological approach of Cox, Smith and Walker: he argued that it was more "natural" to consider the expected payoff of subjects rather than their bids in the auction. He suggested that, although the data showed (p.749) "… statistically significant deviations in terms of bids…" there were no "... statistically significant deviations in terms of foregone expected payoff" from the RNNE. His article was followed by a polemic discussion published in the American Economic Review 82 (5) involving the following experimental economists: Friedman; Kagel and Roth; Cox, Smith and Walker; Merlo and Schotter; and, again, Harrison. Although Cox, Smith and Walker (1992), Friedman (1992), Kagel and Roth (1992) and even Harrison (1992) coincided that investigating individual behavior with respect to the "payoff space" was not more important than with respect to the "message space," doubts remained that the CRRAM offered a reliable explanation for the observed overbidding pattern.[4]

Selten and Buchta (1994) were the first to notice bidding dynamics in a FPA experiment. In their experiment subjects participated in 50 auctions in which they had

[3] There exists no analytical solution if for any agent the risk preference parameter exceeds one.

[4] For instance, Kagel and Roth (1992) argued that risk aversion might be one of the forces of the overbidding in experimental auctions but not necessarily the most important one. As an example in which risk aversion could not explain the data of FPA they referred to Cox, Smith and Walker (1984)'s

to define their bidding strategies before resale values were drawn. They reported that bid functions were changing from one auction to the next depending on the subject's experience condition: First, after winning an auction a subject was more likely to decrease the bid function at the winning bid than to increase it. Secondly, after not winning an auction, a subject with a higher value than the winning bid was more likely to increase than to decrease the bid function at the bid he had made. Selten and Buchta (1994)'s result, thus, did not only disconfirm the predictions of the models of Vickrey (1961) and Cox, Roberson and Smith (1982) but indicated a causal relationship between the winning bid (i.e., the price) in one auction and the bids in the following one. This observation corroborated the prediction of learning direction theory, which goes back to Selten and Stoecker (1986).[5]

The present article investigates further the implications of feedback on the individual bidding behavior and sheds light on the forces behind the overbidding regularity in FPA experiments. We proceeded by transforming the original FPA-problem into one in which the RNNE-bid maximizes the expected payoff: a subject received the value that equaled the upper bound of the uniform distribution from which the *N*-1 competitors' bids were identically and independently drawn. As shown in the Appendix, a risk-neutral bidder submits under these conditions a bid as in the RNNE and a risk-averse bidder submits a bid as predicted by the CRRAM. The subjects' non-changing value and the complete information about the behavior of their competitors guaranteed, thus, a maximal control for a test of these bidding theories.

We considered three treatments which differed with regard to the information about the competitors' bids: In the treatment T0, subjects received no quantitative information about the highest bid of the computerized competitors, they were just told whether they won the auction or not. In the treatment T1, subjects were informed about the highest bid of the computerized competitors only upon not winning the auction. This condition corresponded to the standard information feedback of FPA experiments. Finally, in the treatment T2, subjects were informed always about the highest bid of the computerized competitors.

If experimental subjects behaved fully rational as proposed by the CRRAM or by the RNNE these three conditions should not induce differing behavioral patterns. In

observation of underbidding in four out of ten treatment conditions with multiple unit discriminative auctions.

sharp contrast to this we found significant differences between treatments in the mean bids. We observed significant overbidding (i.e., according to the CRRAM, “bidding as if risk-averse” with an estimated mean risk preference parameter of $r=.78$) only in treatment T1 in which subjects received standard information of FPA about the winning bid. Overbidding occurred, although we noticed no significant bid differences between treatments in the first period. Considering the bidding dynamics we found that 92 percent of all subjects changed their bids between auctions according to behavioral patterns proposed by learning direction theory. With respect to treatment T1 the data revealed that subjects were more likely to increase their bid after not having won the auction than to decrease their bid after having won the auction. The explanation of this anomaly can be traced back to the following detail: after not having won an auction, subjects received a clear impulse about the bid that they should have submitted to win the auction. After having won the auction, subjects did not receive a clear impulse about the bid by which they still would have won the auction. Apparently, in T1 there was an asymmetry in the information feedback with respect to the highest bids of the competitors that involved ambiguity only after a won auction. The data provide evidence that this asymmetrical information produced a drift towards higher bids and explains thus the overbidding pattern in treatment T1 of our experiment.

Learning direction theory is a qualitative behavioral theory based on ex-post rationality; it does not allow quantitative predictions. As a quantification of learning direction theory, Selten, Abbink and Cox (2001) proposed impulse balance theory, which makes testable point predictions of the long-run effect of learning direction theory. Impulse balance theory takes the information conditions of the repeated auction into account and yields different predictions for different feedbacks about auction results. In the third section of the article, we calculate the impulse balance points corresponding to the experimental treatments and compare their fit with the one of the RNNE and the CRRAM.

The article is organized as follows: In the following section we outlay the details of the experimental design. In the second section we report the observed over- and underbidding of the experiment. The third section reviews the learning direction

[5] Learning direction theory has fitted data from diverse experiments, (e.g., auction data by Kagel and Levin (1999)). Selten, Abbink and Cox (2001) provide an overview over some of these works.

theory and the impulse balance theory and examines the data with respect to the bidding dynamics. Finally, the fourth section summarizes and concludes.

1 Experimental Design

As in Cox, Smith and Walker (1987) and in Harrison (1989), the experimental design involved computerized competitors.[6] Experimental subjects, who were mainly undergraduate economics students, participated in 100 auctions in each of which they received a resale value of 100 experimental currency units *ECU*, and were asked to place a bid against the bids of N-1 computerized competitors, where N={3,4,5,6,9}.[7] In every auction, the bids of the computerized competitors were randomly drawn from a uniform distribution ranging from 0 to 100. According to the FPA, subjects won an auction if they submitted the high bid. If they won the auction they received a payoff equal to the difference between the resale value (100 ECU) and their bid, otherwise they received nothing. At the end of the experiment, subjects were paid their accumulated payoff in Italian Lire at an exchange rate of 1 ECU=5N ITL.[8] Subjects were instructed accordingly, including a detailed description of the computer software.[9] Subjects performed the task at their own pace (between 30-60 minutes); when finished they were paid and left. In total, 174 subjects participated in one of the 15 experimental sessions performed in May 2001 at the ESSE laboratory of the University of Bari, in Italy. Subjects had never participated in any auction experiment before.

Three treatments (below referred to as T0, T1 and T2) were considered which differed with respect to the on-screen information displayed after every auction. In treatment T0, the subject received no quantitative information about the highest bid of the computerized competitors; they were just informed whether the highest bid of the competitors was above or below the own bid. In T1, the highest bid of the competitors was revealed only if it was higher than the own bid, i.e., the winning bid was revealed always; this feedback condition corresponds to the FPA experiments of Cox, Smith

[6] Cox, Smith and Walker (1987) used computerized competitors in a FPA experiment featuring one human subject, also. They reported no significant changes in the behavior of FPA on the basis of non-human competitors.
[7] These market sizes correspond to the research of Cox, Smith and Walker.
[8] The mean payoff was 16,000 ITL ≈ 8$.
[9] Please, find the instructions in the Appendix. The computer software was prepared by means of Abbink and Sadrieh (1995)'s RatImage.

and Walker. In T2 the highest bid of the computerized competitors was revealed in every auction.[10]

2 Do We Observe Over- or Underbidding?

2.1 First Bids

We begin the analysis of the experimental data considering subjects' first bids, which are especially interesting because they were submitted before subjects gathered any experience. In the experimental literature, a commonly observed behavioral pattern in FPA is subjects' bidding above the RNNE. Cox, Roberson and Smith (1982) and Cox, Smith and Walker (1982, 1983a/b, 1984, 1985a/b, 1987, 1988) interpreted this "overbidding" behavior by means of risk aversion. Table 1 records the number of subjects who bid above, at or below the RNNE:[11] 6 out of 174 subjects (3%) submitted bids as predicted by the RNNE, 39 subjects (22%) submitted bids which exceeded the RNNE prediction in the first auction. Hence, 25 percent of the submitted bids of the first auction could be explained by the CRRAM. This pattern occurred in all treatments with a similar intensity. The bids submitted in the first auction did not change significantly between treatments T0-T2.[12]

Our experimental data suggest that most subjects' bids (the 95% confidence band extends from 68% to 81%) cannot be explained by the CRRAM. A binomial test rejects the null hypothesis that the submitted bid of the first auction is as likely agreeing as disagreeing with the CRRAM (p=.00). Other factors than full rationality seem to guide subjects' first bid choices. For instance, some of the observed bids can be explained by prominent number considerations:[13] 58 percent of observed numbers are multiples of five. The likelihood of drawing randomly a number divisible by 5 (i.e., of "exactness 5") is twenty percent. A binomial test (p=.00) accepts the hypothesis that the likelihood of observing bid choices which are multiples of five is significantly greater than 20 percent. However, an explanation of the submitted bids of the first auction lies beyond the scope of this article.

[10] The accumulated payoff, all historical bids and results were displayed in a table on a subject's screen.
[11] Individual statistics, including first bids, are listed in Table 0 in the Appendix.
[12] Given N, the null-hypothesis of equal bids in the three treatments cannot be rejected by a Kruskal-Wallis test at a significance level of α=.05. The p-values for the samples with N={3; 4; 5; 6; 9} are respectively p={.07; .19; .34; .06; .84}.

Table 1. Frequencies[a)] of overbidders, underbidders and RNNE-bidders in auction 1: subjects classified according to their bid submitted in the first auction

Treat-ment	$N^{c)}$	3	4	5	6	9	column total	(%)[b)]
T0	Underbidder	4	7	9	8	10	38	(70%)
	Overbidder	6	5	3	2	-	16	(30%)
	RNNE-bidder	-	-	-	-	-	-	
T1	Underbidder	10	11	6	6	12	45	(75%)
	Overbidder	2	-	5	6	-	13	(22%)
	RNNE-bidder	-	1	1	-	-	2	(3%)
T2	Underbidder	10	9	9	10	9	47	(78%)
	Overbidder	1	3	3	1	2	10	(17%)
	RNNE-bidder	1	-	-	1	1	3	(5%)
row total	Underbidder	24	27	24	24	31	130	(75%)
	Overbidder	9	8	11	9	2	39	(22%)
	RNNE-bidder	1	1	2	1	1	6	(3%)

a) Number of subjects are recorded whose first bid was above (*overbidder*), below (*underbidder*) or at the RNNE (*RNNE-bidder*). b) Percentages relate to the ratio of the column total to the respective treatment total. c) Sessions (i.e., market-sizes) are indicated by the number of bidders N; subjects competed with N-1 computerized bids.

2.2 Mean Bids, Bid Spreads and Individual Risk Preference Parameters

Cox, Smith and Walker (1988) estimated the bid function $b(v, r)$ by means of an ordinary least square regression. In the present study subjects' values were fixed to 100, the upper bound of the interval from which competitors' bids were drawn. As an estimation of the "bid function" we use, hence, the arithmetic mean of the individual's bids. Table 2 records (in every first row) from every session the average mean bids and (in every second row) the average risk preference parameters.[14] The average mean bids in T1 exceed not only the RNNE prediction for every given market-size N but also the corresponding average mean bids of the other treatments. On average a risk preference parameter r_{mean}(T1)=.78 was estimated for treatment T1. Following Cox, Smith and Roberson (1982) a risk preference parameter smaller than one indicates bidding above the RNNE. This overbidding pattern is in accordance with the results reported from other FPA experiments, as has been pointed out above. We observe overbidding in T1, although the expected payoff would have been maximized at the RNNE bid. Comparing the results of T1 to the other treatments, nevertheless, we find that overbidding is not a consistent characterization of the data. The average mean bids and the risk preference parameters estimated for T0, r_{mean}(T0)=1.25, and T2,

[13] See Albers (2001) for a theory on prominent numbers and its applications.

r_{mean}(T2)=1.17, suggest an underbidding rather than an overbidding pattern. Between treatments we observe significant differences in the individual average bids: given N>4, Kruskal-Wallis tests reject the null hypotheses of equal mean bids across treatments T0-T2 at a significance level of α=.05.[15]

Table 2. Average mean bids, average preference parameters of each market size and frequencies of overbidders [a)]

Treatment	N[c)]	3	4	5	6	9	Column total
T0	average bid [a)]	70.2	76.3	76.9	80.3	81.2	
	average r [a)]	0.89	1.01	1.26	1.25	1.89	1.25
	# overbidders[b)]	7	8	5	2	0	22
	(%)	(70%)	(67%)	(42%)	(17%)	(0%)	(41%)
T1	average bid [a)]	73.7	77.9	85.8	88.2	89.7	
	average r [a)]	0.78	0.87	0.67	0.68	0.92	0.78
	# overbidders [b)]	8	8	11	10	8	45
	(%)	(67%)	(67%)	(92%)	(83%)	(67%)	(75%)
T2	average bid [a)]	69.6	73.7	74.3	85	86.3	
	average r [a)]	0.94	1.23	1.46	0.9	1.3	1.17
	# overbidders [b)]	7	9	2	7	4	29
	(%)	(58%)	(75%)	(17%)	(58%)	(33%)	(48%)
row total	RNNE	66.7	75	80	83.3	88.9	
	average r [a)]	0.87	1.04	1.13	0.94	1.34	1.06
	# overbidders [b)]	22	25	18	19	12	96
	(%)	(65%)	(69%)	(50%)	(53%)	(35%)	(55%)

a) The arithmetic mean of subjects' average bids and the thus estimated individual risk preference parameter for given market size N are recorded. b) Subjects were classified according to their mean bid as overbidders or underbidders. The number of subjects is recorded whose average bid was above (*overbidder*) the RNNE; the average bids of the remaining subjects for given market-size were below the RNNE (*underbidders*). Relative numbers are recorded in parenthesis below the absolute numbers for each session. c) N indicates the market size, subjects competed with N-1 computerized bids.

Table 2 records (in each third row) the number of subjects whose average bids exceeded the RNNE, designating them as overbidders, while the remaining number of subjects whose average bid fell short of the RNNE, could be designated accordingly as underbidders. Cox, Smith et al. referred to subjects who bid above the RNNE as risk averse and to those who bid below the RNNE as risk preferring. In T1, the mean bids of 75 percent of subjects exceeded the RNNE (the 95% confidence interval extends from 64% to 86%). A one-tailed Wilcoxon signed ranks test on the individual preference parameters (p=.00) accepts the alternative hypothesis of CRRAM that T1 is better characterized by overbidding than by underbidding. In T0 and T2, however, the number of underbidders exceeded the number of overbidders. For T0 or T2, the one-tailed Wilcoxon signed ranks test cannot reject the null hypothesis that a subject is at

[14] (As in Fn 11:) Individual statistics, including risk aversion parameters, are listed in Table 0 in the Appendix.

least as likely an underbidder as an overbidder against the alternative hypothesis of the CRRAM that overbidders are more frequent (T0: p=.99; T2: p=.84).[16] The relative frequency of overbidders in T0 was 41% (the 95% confidence band extends from 28% to 54%) and in T2 48% (the 95% confidence band extends from 36% to 61%) respectively.

The CRRAM predicts, furthermore, that submitted bids should be constant, i.e., that they do not change from one auction to the next if the resale value is the same. As reported below (see Table 4), subjects changed their consecutive bids in 87 percent of all observations. Still we might believe that subjects bid according to CRRAM up to some error term. Taking a subject's average bid as the reference bid, we constructed a test on the individual's bid sequence: a positive sign designated a bid which exceeded the average bid, a negative sign designated a bid which felt short of the average bid. Table 3 surveys the results of two-tailed one-sample runs tests of the null hypothesis that the sequence of a subject's bids changed by chance from above to below the mean and vice versa. The first\second column indicate the number of subjects whose consecutive bids changed less\more frequently than expected from one side of the average bid to the other. The data of 153 subjects (88%) exhibit less runs than expected, for 103 subjects (59%) significantly less than expected at the α=.05 level of significance. The individual test results (not detailed here) were next used to test the null hypothesis that more runs are equally likely than less runs. A two-tailed Wilcoxon signed rank test rejects high significantly (p=.00) the null hypothesis for each treatment; its results are reported in the third column of Table 3.[17]

We conclude that our data do not support the CRRAM: First, subjects bid not constantly nor did they bid according to the CRRAM up to an error-term. Secondly, the reported risk preference parameters and results of under- and overbidding do not support the implicit hypothesis of CRRAM that information feedback do not affect the subjects' bidding behavior in FPA. The data provide evidence that the behavior is influenced significantly by the information feedback conditions of the experiment: although treatments were identical up to the on-screen information about the highest

[15] We tested the individual mean bids for given market-size between treatments. The p-values for the samples with N={3; 4; 5; 6; 9} are respectively p={.43; .90; .00; .00; .00}.

[16] The Wilcoxon signed ranks test conducted on the total of 174 individual risk preference parameters (returning a p-value of p>.70) accepts the null hypothesis that the risk preference parameter is as likely greater as smaller than 1.

[17] We conducted the same test for the reference bid equal to the RNNE (not detailed here). The null-hypothesis that subjects bid according to RNNE up to some error-term was rejected, also.

bid of the computerized competitors they yield significantly different results. By means of a bidding dynamics analysis we now provide an explanation of the disparity of bidding behavior in treatments T0-T2.

Table 3. Number of subjects with less or more runs of overbidding or underbidding than randomly expected

Treatment	less runs than expected[a)] (significantly less)[b)]	more runs than expected[a)] (significantly more)[b)]	Wilcoxon test value z[c)] [p-value]
T0	49 (32)	5 (2)	-5.77 [.00]*
T1	55 (41)	5 (1)	-5.64 [.00]*
T2	49 (30)	11 (1)	-6.43 [.00]*
Total	153 (103)	21 (4)	-10.38 [.00]*

a) Outcomes of a two-sided one-sample runs test of the null hypothesis that the consecutive bids spread randomly around the average bid against the alternative that they do not. The number of subjects is recorded who changed their bids from below the average bid to above the average bid less (respectively more) frequently than expected by chance. b) Runs were significantly less\more frequently (α=.05) than expected. c) Wilcoxon signed ranks test results (approximately standard normally distributed) of the null hypothesis that the outcomes of the runs test involve less runs than expected as likely as more runs than expected. * Significant at α=.05.

3 Bidding Dynamics Analysis

In the economic theory on FPA, in the RNNE as much as in the CRRAM, bids are linear functions of resale values. From the FPA experiment of Selten and Buchta (1994) it was reported that bid functions were usually non-linear and changing over the course of the experiment; quite often non-monotonous bid functions were observed. In their experiment a subject’s task was to submit a bid function instead of a bid in 50 auction markets. They found in 35 percent of the observations that the bid functions were changed from one to the next auction. The bidding dynamics in their data could be explained to a good extent by learning direction theory. In the following subsection, learning direction theory will be reviewed and subsequently applied to the experimental data.

3.1 Learning Direction Theory and Its Implications

Selten and Stoecker (1986) first applied learning direction theory on data from the repeated prisoners’ dilemma supergame. Since, it has been supported by the data of diverse experiments.[18] In the reconsideration of learning direction theory we follow Selten and Buchta (1994): Learning direction theory is a qualitative behavioral theory based on bounded rationality. As an illustration of the theory, Selten and Buchta (1994) proposed the example of a marksman who shoots an arrow to hit a trunk. If the arrow misses the trunk to one side the marksman shifts the bow to the other side when he gives it another try. The marksman in the example draws qualitative conclusions from his information feedback and adjusts his behavior according to a causal relationship: if he misses, for instance, to the right he concludes that he could have got closer to his target if he had directed the arrow rather to the left. This line of reasoning links to ex-post rationality, it asks whether a different action might have produced a better result. Learning direction theory does not bring on a full-fledged behavioral model; it only makes predictions about tendencies of qualitative adjustment. This concerns the direction of a change rather than its size. A possible quantification of learning direction theory is provided in the next subsection. However, consider now the application of this theory to our FPA experiment. Let b denote a subject's bid and p denote the winning bid (i.e., the price) in the auction, subjects could be in one of the following two experience conditions with respect to the preceding period:

1) *Successful bid condition*: $b=p$.

2) *Lost opportunity condition*: $b<p$.

In the experience condition of a successful bid a subject won the preceding auction. Nevertheless, he might have gained more by submitting a lower bid. Similarly, in the lost opportunity condition the subject did not win the auction, but he could have won the auction by submitting a higher bid. Therefore, learning direction theory implies the bid change hypothesis that after a successful bid a subject tends to decrease his bid, while after experiencing a lost opportunity the subject tends to increase his bid.

Table 4 records the number of bid changes, i.e. increases or decreases of the bid from one auction to the next. Changes are listed separately according to subjects' experience condition of a successful bid or a lost opportunity in the preceding auction. A first observation reveals that in 87 percent of all observations subjects changed their bids from one auction to the following one; this finding contradicts to the predictions

[18] See Selten, Abbink and Cox (2001) for a survey over such literature.

of both the CRRAM and the RNNE which suggest constant bidding. In every treatment we observed over 60 percent of bid changes in the direction predicted by the bid change hypothesis and about 25 percent in the opposite one. A bid-change in the predicted direction, hence, occurred 2.4 times as often as an unpredicted one.

It appears to be interesting how many subjects behaved according to the bid change hypothesis: In determining the share of these subjects we proceeded similarly to Selten, Abbink and Cox (2001). A simple comparison of the relative frequencies of bid changes in the direction indicated by learning direction theory and in the opposite direction may be misleading. Random bidding as well as learning direction theory result in a preponderance of lower bids after high bids and higher bids after low bids. In order to generate an appropriate null hypothesis we ran 1000 simulations in which the sequence of the competitors' highest bids was randomly permuted, but the

Table 4. Absolute (relative)[a)] frequencies of bid changes in auctions 2-100

Treat-Ment	experience condition	bid decrease # (%)	bid increase # (%)	unchanged bid # (%)	column total
T0	Successful bid	1442	599	349	2390
		(60%)	(25%)	(15%)	(45%)
	Lost opportunity	847	1785	324	2956
		(29%)	(60%)	(11%)	(55%)
T1	Successful bid	1919	882	449	3250
		(59%)	(27%)	(14%)	(55%)
	Lost opportunity	584	1753	353	2690
		(22%)	(65%)	(13%)	(45%)
T2	Successful bid	1748	746	292	2786
		(63%)	(27%)	(10%)	(47%)
	Lost opportunity	762	1953	439	3154
		(24%)	(62%)	(14%)	(53%)
	Row total	7302	7718	2206	17226
		(42%)	(45%)	(13%)	(100%)

a) In the first three columns relative frequencies relate to the row total, in the last column relative frequencies relate to the total of observations in the respective treatment.

subject's submitted bid sequence was kept fixed. For each of these 1000 simulations we calculated the relative frequency of bid changes under the counterfactual assumption that the permuted sequence of the competitors' highest bids was observed. Let M be the average of these relative frequencies and let R be the relative frequency of bid changes conforming to learning direction theory actually observed in the experiment. The *surplus* $S=R-M$ measures the extent to which the observations support learning direction theory after the exclusion of mere random effects. We made the following observation: for 160 subjects (92%) we found a positive $S>0$; as a maximum surplus we observed 0.48 and the average of positive surpluses was 0.18. In other words, ninety-two percent of experimental subjects behaved more frequently than could be expected by chance in accordance with learning direction theory.[19] The Wilcoxon signed ranks test of the null hypothesis that the mean surplus in the population of subjects is equal to zero, i.e., $\overline{S} = 0$, is rejected (p=.00) in favor of the alternative hypothesis of $\overline{S} > 0$.

Table 4 provides a key understanding of the driving forces behind the overbidding we observed in treatment T1 of the experiment. Note first, in 59 percent of observations the bid was decreased after a successful bid. Secondly, in 65 percent of observations the bid was increased after a lost opportunity. Selten and Buchta (1994, p.13), who made a similar observation under comparable feedback conditions, remarked that the impulse that a subject received was "... much clearer in the lost opportunity condition." We constructed a test-statistic (to be used in all tests of this paragraph) by calculating for each subject (and each experience condition) the relative frequency of bid changes he or she realized in accordance with the bid change hypothesis. With respect to T1, thus, a two-tailed Wilcoxon test rejected the null hypothesis (p=.00) that it is equally likely to observe a higher relative frequency of bid increases after a lost opportunity than of bid decreases after a successful bid. Thirdly, notice further that in T0 the relative frequency of a bid increase after a lost opportunity was equally high as the relative frequency of a bid decrease after a successful bid (60 percent in both cases). Moreover, in T2 a bid decrease after a successful bid (63 percent) was not significantly more likely (p>.67) than an increase after a lost opportunity (62 percent). Fourthly, note that a bid increase after a lost opportunity in T1 was more likely than in T0 (and the impulse was clearer in T1).[20] Fifthly and finally, the bid decrease after a successful bid was more likely (although not at the α=.05-level of significance) in T2 than in T1 (and the impulse was clearer in T2).[21] From these observations we deduce that subjects rather react to an impulse when it

[19] On the other hand, we observed a negative surplus S<0 for 14 subjects (8%). The minimum surplus we observed was -0.09 and the average of the negative surpluses was -0.03. Less than 5% (i.e., 8 out of 174) of all subjects violated the bid change hypothesis at least in one direction. One subject decreased his bids more frequently than he increased them after not winning the auction, six other subjects increased their bids more frequently than to decrease them after winning the auction, and one subject's behavior was at odds with the bid change hypothesis in both directions.

[20] A two-tailed Mann-Whitney test rejects the null hypothesis that bid increases in the lost opportunity condition occur with equal likelihood in T1 and in T0 (p=.04).

[21] A two-tailed Mann-Whitney test accepts the null hypothesis that in T1 and T2 bid decreases in the successful bid condition occur with equal likelihood (p=.15).

implies clearer information. The feedback condition in T1 was asymmetrical since the highest bid of the competitors was revealed only if the subject submitted a lower bid. In the experience condition of a successful bid, conversely, the impulse was ambiguous: the subject could have won the auction or not by submitting a lower bid; he was not very well informed by hindsight. Since we do not observe a similar pattern in the symmetrical information treatments T0 and T2,[22] we conclude that the asymmetrical information with respect to the impulse clarity implied a drift towards higher bids in the treatment T1.

3.2 Impulse Balance Theory

In a recent paper, Selten, Abbink and Cox (2001) proposed impulse balance theory, which offers a possibility to make quantitative predictions of the long-run effects of learning direction theory. Impulse balance theory is mainly applicable in economic situations in which clear impulses are provided such that the ex-post-rational choice can be determined by hindsight. It assumes that impulses that involve greater gains are relatively more important. Ex-post rationality results in an upward or downward impulse in accordance with learning direction theory. Impulse balance theory proposes the impulse balance point in which upward and downward impulses cancel out in the long run. Note that impulse balance theory builds on the payoff space of a bidder rather than on his message space.[23]

Consider the FPA situation implied by treatment T2. Let x denote the highest bid of the competitors; and as before, let b denote the agent's bid and p the winning bid. A successful bid implies that the agent won the preceding auction. Winning the auction produced a profit, but also an opportunity cost. The agent could have won the auction also by submitting a bid equal to the highest bid of the competitors. In accordance with impulse balance theory the agent receives in the successful bid condition a downward impulse equal to the difference between the bid and the highest bid of the competitors. Conversely, a lost opportunity implies that the agent did not win the preceding auction. Not winning the auction produced no profit. However, if the agent had submitted a bid equal to the highest bid of his competitors he would

[22] In T0 this ambiguity about the winning bid existed in both experience conditions; in T2 there was a clear impulse in both experience conditions.
[23] Recall Harrison (1989)'s critique, here.

have won the auction. In the lost opportunity condition, impulse balance theory implies an upward impulse equal to the difference between the agent's resale value v and the highest bid of the competitors. Following Selten, Abbink and Cox (2001) the downward impulse $a_-(.)$ and the upward impulse $a_+()$ can be described as follows:

successful bid condition: $$a_-(b,x) = \max(0,b-x) \quad (3)$$

lost opportunity condition: $$a_+(b,x) = \begin{cases} v-x, if\ x>b \\ 0,\ otherwise \end{cases} \quad (4)$$

Define

$$A_-(b) = E[a_-(b,x)]$$
$$A_+(b) = E[a_+(b,x)],$$

then the impulse balance point b^* is defined by the impulse balance equation (hereafter IBE):

IBE: $$A_-(b^*)=A_+(b^*) \quad (5)$$

The impulse balance point is uniquely determined. This is easily seen by noting that the left hand side of the impulse balance equation increases in b^*, whereas the right hand side decreases in b^*; and for $b^*=v$ the left hand side is greater than the right hand side, while for $b^*=0$ the left hand side is smaller than the right hand side.

3.3 Impulse Balance Points

Let the number of competitors' bids be denoted by n, such that $n=N-1$, and let the resale value v be normalized to one, i.e., $v=1$, such that the competitors' bids are uniformly, identically and independently distributed over the interval from 0 to 1. Let $F(X<x)=x^n$ denote the cumulative probability that all n competitors' bids are smaller than some real number x, and let $f(x)=nx^{n-1}$ denote its density, we can write the impulse balance equation for treatment T2 as follows:

$$\mathrm{A}_-(b^*) = \mathrm{A}_+(b^*)$$

$$\int_{-\infty}^{+\infty} f(x) a_{-}(b^{*}, x)\, dx = \int_{-\infty}^{+\infty} f(x) a_{+}(b^{*}, x)\, dx$$

$$\int_{0}^{b^*} n\, x^{n-1} (b^{*} - x)\, dx = \int_{b^*}^{1} n\, x^{n-1} (1 - x)\, dx$$

$$b^{*\,n+1} - \frac{n}{n+1} b^{*\,n+1} = 1 - \frac{n}{n+1} - b^{*\,n} + \frac{n}{n+1} b^{*\,n+1}$$

IBE - T2:

$$0 = \frac{1}{n+1} - b^{*\,n} + \frac{n-1}{n+1} b^{*\,n+1} \qquad (6)$$

$$0 = b^{*\,n+1} - \frac{n+1}{n-1} b^{*\,n} + \frac{1}{n-1}$$

Since the impulse balance equation of T2 (eq.(6)) involves no parameter we can easily calculate the impulse balance points for each market size of our experiment.[24] In treatments T0 and T1, however, the impulses are not clear in at least one experience condition such that the subject can only count the impulses in each direction. If the bidder chooses the bid b then the probability of a downward impulse is b^n and the probability of an upward impulse is $1\text{-}b^n$. However, the subject may attach a different importance to downward and upward impulses. Therefore, we introduce a parameter $\lambda>0$, the downward impulse multiplier and write the impulse balance equation as follows.

$$\lambda b^{*n} = 1 - b^{*n} \qquad (7)$$

The downward impulse multiplier can be interpreted as an indicator of the importance of a downward impulse relatively to an upward one. If more importance is put on downward impulses then fewer downward impulses can counterweigh more upward impulses. For example, if we have $\lambda=2$ then the expected upward impulse must be double the expected downward impulse at the impulse balance. In other words, one unit of downward impulse would then have the same weight as two units of upward

[24] It should be noted that the impulse balance point calculated here is not the impulse balance. It is only the prediction of a subject's action if all other agents behave according to the RNNE.

impulse. In this sense λ is the relative importance of downward impulses. Rearranging equation (7), we get an explicit formulation for the impulse balance bids of T0 and T1.

IBE - T0, T1:[25]

$$b^* = \sqrt[n]{\frac{1}{1+\lambda}} \qquad (8)$$

In order to determine the impulse balance points we have to proceed by estimating the downward impulse multiplier from the data. For each subject *i* we determined the multiplier λ_i according to the average bid of the subject and took the median of these individual multipliers as an estimation of λ. Thus, the estimator of the downward impulse multiplier in T0 was λ_{median}(T0)=1.74; and in T1 correspondingly λ_{median}(T1)=1.01.[26] For T1 the estimate of the downward impulse multiplier indicates that subjects weigh downward and upward impulses equally. It suggests that subjects use a count heuristic, i.e., they count upward and downward impulses. At least at first glance there seems to be a discrepancy between this result for treatment T1 and earlier findings summarized by Table 4. There the relative frequencies of upward and downward impulses for T1 are .55 and .45 respectively. However, the estimate λ_{median}(T1)=1.01 is the median of all estimates for individual subjects and therefore does not necessarily reflect aggregate behavior. Therefore our conclusions about differences between the successful bid and the lost opportunity conditions with respect to the conformance to learning direction theory do not necessarily hold for the median subject. For T0 the estimate of the downward impulse multiplier indicates that one unit downward impulse weighs 1.74 units of upward impulse. This would imply that subjects were stronger motivated by a downward impulse than by an upward impulse. In fact, we lack an intuition why subjects should weigh downward impulses so much. Therefore, it is unclear to us whether impulse balance theory should be applied to this treatment. In principle, we would expect subjects of T0 to use a count heuristic, also. The theory does not allow us to draw any conclusion why we observe this disparity between the two treatments. The only difference is that in T0 subjects receive no quantitative information feedback about how far the bid has been from the right decision while in T1 they receive clear impulses at least in one direction. Apparently

[25] Note from the equation that at the impulse balance the bid is the lower the higher the downward impulse multiplier is.

this lack of impulse clarity produces a more uncertain and more erratic behavior; the variations of bids between two consecutive auctions are greater in T0 than in T1.[27] To sum up, although the qualitative behavior is in harmony with the learning direction theory, we cannot rationalize the quantitative behavior of T0. We hope to unravel this issue in the future.

Table 5 gives an overview over the determined impulse balance points for the different treatments. Conversely to the RNNE and the CRRAM, impulse balance theory predicts different impulse balance points for each feedback condition. Note that only the impulse balance points for T1 exceed the RNNE. Table 5 surveys, also, the predictions of CRRAM which were calculated by the overall median of the individual risk preference parameters, $r_{median}=.93$.

Table 5. Impulse Balance Points and the RNNE for each market size and treatment

Treatment	N[a)]	3	4	5	6	9
IBP for T0		60.5	71.5	77.8	81.8	88.2
IBP for T1		70.5	79.2	84.0	86.9	91.6
IBP for T2		65.3	73.4	78.4	81.8	87.7
RNNE		66.7	75	80	83.3	88.9
CRRAM[b)]		68.3	76.3	81.1	84.3	89.6

a) *N* indicates the market size, subjects competed with *N*-1 computerized bids. b) CRRAM predictions corresponding to the median risk preference parameter.

In order to compare the fit provided by these competing theories we measured the distance of each subject's average bid to the impulse balance point, the RNNE and the CRRAM. Table 6 records separately for each treatment the number of subjects whose average bid was closest to either predictor. Subjects' average bids are best described in 54 percent of all observations by the impulse balance points, in 14 percent and 32 percent by the RNNE and by the CRRAM, respectively. However, we should concede that impulse balance theory is not yet a fully satisfactory theory. It might not be better under all conditions than the RNNE or the CRRAM; we cannot tell because the statistical evidence is missing. Nevertheless, it must be seen as a reasonable benchmark since it is based on learning direction theory, which has been soundly

[26] (As in Fn 11 and Fn14:) Individual statistics, including downward impulse multiplier, are listed in Table 0 in the Appendix.

[27] We compared between treatments T0 and T1 the individual average of squared changes from one period to the next by means of a Mann-Whitney test. For T0, deviations were greater for each market size, respectively. The corresponding p-values were p={.02,.44,.20,.59,.01} given *N*={3,4,5,6,9}.

Table 6. Number of subjects whose average bids were closest to the respective predictor (RNNE, CRRAM, Impulse Balance Point)

Treatment	RNNE [a)] # (%)	CRRAM [a)] # (%)	Impulse Balance Point [a)] # (%)
T0	6	20	28
	(11%)	(37%)	(51%)
T1	15	8	37
	(25%)	(13%)	(61%)
T2	5	26	29
	(8%)	(43%)	(48%)
Total	26	54	94
	(18%)	(31%)	(54%)

a) Absolute numbers recorded in every first row refer to the subjects for whom the distance of the individual average bid from the Impulse Balance Point, RNNE or CRRAM was smallest, respectively. Relative numbers in every second row are recorded in parenthesis.

supported by the data. The CRRAM and the RNNE, on the other hand, are not linked to any learning theory and lack an explanation of how subjects approach the equilibrium.

4 Summary

We designed a first-price sealed-bid auction experiment to test the predictions of the CRRAM and the RNNE in an environment in which subjects had knowledge about the behavior of their competitors. Competitors' bids were randomly drawn by the computer, such that agents who exhibit constant relative risk aversion would maximize their utility by bidding as predicted in the CRRAM. Cox, Roberson and Smith (1982) and Cox, Smith and Walker (1982, 1983a/b, 1984, 1985a/b, 1987, 1988) developed the CRRAM to explain the observed overbidding regularity in FPA experiments in which the winning bid was revealed after each auction. Overbidding above the RNNE was not a consistent pattern of our experimental data. In two out of three treatments subjects' average bids were below rather than above the RNNE. Only under standard information feedback of FPA experiments, i.e., price revelation, we observed overbidding. Thus, the data provide evidence that the feedback conditions can influence behavior in experiments significantly. Subjects' qualitative bidding behavior did not support the CRRAM, either, since it predicts degenerate behavior. Subjects did neither bid constantly nor did they vary the bid up to an error-term around their average bid. The CRRAM must therefore be rejected.

The data could be explained qualitatively by learning direction theory (Buchta and Selten (1994)) which was first used in Selten and Stoecker (1986). The bidding dynamics of 92 percent of subjects were in line with the bid change hypothesis derived from learning direction theory. Comparing the individual behavior in the three feedback conditions of the experiment, we found that subjects reacted significantly more frequently to an impulse (according to the predictions of learning direction theory) if it was quantitatively clear by hindsight. Thus, the asymmetry of information feedback with regard to the greatest competitors' bid in the standard feedback condition of FPA apparently produced an upward drift that lead to overbidding in treatment T1.

In order to provide a quantitative explanation of the data we applied the impulse balance theory of Selten, Abbink and Cox (2001). Impulse balance theory builds on the payoff space of agents and makes point predictions of the long-run behavior on the basis of learning direction theory. Differing from the RNNE, we determined distinct impulse balance points under each of the three feedback conditions of the experiment; the impulse balance points take the information differences into account. Subjects' average bids were closest to the impulse balance points in more than 50 percent of observations compared to the RNNE and the CRRAM. Impulse balance theory seems to provide good alternative predictors to the Nash equilibria for behavioral studies.

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Appendix

Proposition

Assume that a bidder has a resale value equal to *v* for the object to be auctioned, and let the bids of *N*-1 competitors be identically and independently drawn from a continuous uniform distribution ranging from 0 to 1. Assume the bidder's utility function is given by $u(y)=(y)^r$, where *y* is the bidder's income and $r>0$ denotes the bidder's risk preference parameter. The following statements hold:

I) The bidder's bid equals the Nash equilibrium bid of the CRRAM.

II) For $r=1$ (risk-neutrality), the bidder's bid is equal to the RNNE bid.

The probability that the bidder's bid *b* exceeds the bid of every competitor is b^{N-1}. The bidder's objective function is as

$$\max_b U(v,b) = u(v-b)b^{N-1}$$

Differentiating with respect to *b* yields the first order condition.

$$\frac{\partial U(v,b)}{\partial b} = \frac{\partial u(v-b)}{\partial b} b^{N-1} + u(v-b)b^{N-2}(N-1) = 0$$

This yields

$$\frac{\partial u(v-b)}{\partial b} b + u(v-b)(N-1) = 0$$

Following Cox, Roberson and Smith (1982), $u(v-b) = (v-b)^r$, such that

$$-r(v-b)^{r-1}b + (v-b)^r(N-1) = 0 .$$

It follows that the bidder who faces an Arrow-Pratt constant relative risk aversion parameter *r* submits a bid equal to the Nash equilibrium bid in the CRRAM:

$$b^*(v,r) = \frac{N-1}{N-1+r} v$$

If the bidder is risk-neutral the bid is equal to the one in the RNNE:

$$b^*(v,1) = \frac{N-1}{N} v$$

This completes the proof.

Table 0. Subjects' first bids and individual statistics over 100 auctions

N[a)]	sub-ject	RNNE	T0[b)] 1st bid	mean	median	λ_i	r_i	T1 1st bid	mean	median	λ_i	r_i	T2 1st bid	mean	median	r_i
3	1	66.7	85	76.8	80	.70	.61	21	80.4	85	.55	.49	62	71.6	72	.79
3	2	66.7	68	68.9	77	1.11	.9	58	62.7	59	1.54	1.19	40	58.8	61	1.40
3	3	66.7	78	63.9	65.5	1.45	1.13	78	78.9	80	.61	.54	21	51.3	55.5	1.90
3	4	66.7	63	85.4	91	.37	.34	40	83.3	80	.44	.40	5	77.3	88	.59
3	5	66.7	50	72.0	80	.93	.78	55	78.9	80	.61	.54	56	78.1	81.5	.56
3	6	66.7	75	66.9	73	1.23	.99	30	81.0	80	.52	.47	55	79.4	84	.52
3	7	66.7	100	76.2	80	.72	.63	48	86.3	88	.34	.32	90	86.1	90	.32
3	8	66.7	70	66.3	70	1.27	1.02	50	54.6	56	2.35	1.66	65	64.2	66.5	1.12
3	9	66.7	60	69.7	70.5	1.06	.87	60	64.5	63	1.40	1.10	50	76.9	79	.60
3	10	66.7	50	55.7	55	2.22	1.59	98	83.6	85	.43	.39	65	67.4	72.5	.97
3	11	66.7						51	73.7	75	.84	.71	66	59.2	50	1.38
3	12	66.7						58	56.8	56	2.10	1.52	55	64.5	65	1.10
4	13	75.0	45	80.4	86.5	.92	.73	66	72.6	74.5	1.61	1.13	76	62.2	64.5	1.82
4	14	75.0	50	69.1	76	2.03	1.34	68	76.0	78	1.28	.95	85	82.5	84.5	.64
4	15	75.0	99	83.1	85	.74	.61	62	90.4	90	.35	.32	62	75.5	79	.97
4	16	75.0	70	82.6	82	.77	.63	48	70.1	74.5	1.90	1.28	69	54.0	55	2.55
4	17	75.0	58	67.6	69	2.24	1.44	25	79.1	82	1.02	.79	30	81.5	82	.68
4	18	75.0	66	81.7	82.5	.83	.67	65	72.6	72	1.61	1.14	27	75.3	83.5	.98
4	19	75.0	81	53.1	54.5	5.68	2.65	75	84.9	86	.63	.54	10	44.1	46.5	3.80
4	20	75.0	72	67.8	71	2.21	1.43	70	78.7	78	1.05	.81	65	81.0	85	.70
4	21	75.0	87	85.6	88	.59	.51	59	71.8	79.5	1.70	1.18	20	87.4	85.5	.43
4	22	75.0	58	75.1	83	1.36	.99	65	82.1	81	.81	.66	11	77.8	80	.85
4	23	75.0	95	85.7	87	.59	.5	52	79.2	82	1.01	.79	74	82.3	84	.65
4	24	75.0	88	83.9	87	.69	.58	65	77.4	79.5	1.16	.88	90	80.4	81.5	.73
5	25	80.0	75	72.2	79	2.68	1.54	86	92.1	92	.39	.34	82	82.7	82.5	.84
5	26	80.0	87	81.2	87	1.30	.92	85	83.9	86	1.02	.77	88	68.8	67	1.81
5	27	80.0	78	73.6	77	2.41	1.44	68	86.5	87	.79	.63	56	77.0	79	1.19
5	28	80.0	70	75.7	75	2.05	1.29	90	91.5	92	.43	.37	50	79.1	80	1.06
5	29	80.0	76	66.2	74	4.21	2.04	55	78.8	80	1.59	1.08	70	78.0	84	1.13
5	30	80.0	78	84.6	85	.95	.73	55	85.4	89	.88	.68	90	78.2	80	1.11
5	31	80.0	55	82.1	86	1.20	.87	98	84.4	88	.97	.74	25	57.6	66.5	2.95
5	32	80.0	54	63.0	65	5.35	2.35	85	88.3	88	.64	.53	10	74.3	85	1.39
5	33	80.0	48	70.8	75.5	2.98	1.65	80	82.9	86	1.12	.83	79	71.6	77	1.58
5	34	80.0	89	87.2	89	.73	.59	60	89.5	91	.56	.47	50	59.3	58.5	2.74
5	35	80.0	88	87.2	87.5	.73	.59	75	82.4	84	1.17	.85	78	79.4	80	1.04
5	36	80.0	56	79.0	85	1.57	1.06	60	84.0	87.5	1.01	.76	45	85.3	95	.69
6	37	83.3	78	74.9	83	3.24	1.68	70	89.2	90	.77	.60	47	81.7	80.5	1.12
6	38	83.3	65	80.1	90.5	2.03	1.24	80	84.3	85	1.35	.93	11	82.7	83	1.05
6	39	83.3	69	81.3	83.5	1.82	1.15	89	88.9	90	.80	.62	5	78.1	80	1.40
6	40	83.3	56	76.5	80	2.82	1.53	90	80.3	85.5	2.00	1.22	83	78.1	80	1.40
6	41	83.3	88	82.3	88	1.65	1.08	90	93.2	95	.42	.36	75	77.4	78	1.46
6	42	83.3	80	84.8	86	1.28	.9	88	93.5	97	.40	.35	57	90.4	93.5	.53
6	43	83.3	65	71.0	76	4.54	2.04	50	88.0	89	.89	.68	65	91.6	93	.46
6	44	83.3	87	83.1	87	1.52	1.02	65	81.4	85	1.80	1.14	75	92.0	93	.43
6	45	83.3	65	81.2	85	1.83	1.16	90	88.4	90	.85	.66	85	89.3	91	.60
6	46	83.3	80	87.4	88	.96	.72	85	89.7	92	.72	.58	75	88.3	91	.66
6	47	83.3						80	89.1	89	.78	.61	60	86.2	88	.80
6	48	83.3						60	92.2	95	.50	.42	75	84.2	86	.94
9	49	88.9	80	68.2	71	20.37	3.74	82	91.3	95	1.07	.76	79	75.1	77	2.66
9	50	88.9	85	82.6	90	3.61	1.68	80	88.5	90	1.66	1.04	95	92.4	94	.66
9	51	88.9	80	82.1	88.5	3.84	1.74	79	90.7	91	1.18	.82	80	89.5	90	.94
9	52	88.9	70	85.9	88.5	2.37	1.32	75	93.5	95	.71	.56	58	85.4	94	1.36
9	53	88.9	60	77.9	80	6.37	2.27	80	89.6	90	1.41	.93	78	82.8	88	1.67
9	54	88.9	70	82.2	89	3.80	1.73	81	85.6	88	2.47	1.35	89	87.4	88	1.15
9	55	88.9	85	83.6	88	3.19	1.57	80	87.3	89	1.96	1.17	90	86.2	90	1.28
9	56	88.9	55	84.0	85	3.03	1.52	83	89.7	90	1.39	.92	87	92.4	95	.66
9	57	88.9	75	86.2	89	2.28	1.28	83	90.4	92	1.24	.85	15	85.3	91	1.38
9	58	88.9	85	79.5	81	5.27	2.06	80	89.6	90	1.41	.93	56	82.5	87	1.70
9	59	88.9						85	92.3	95	.90	.67	70	86.9	88	1.21
9	60	88.9						80	88.1	90	1.76	1.08	85	89.3	91	.96

a) N-1 is the number of computerized competitors in the experiment.

b) Due to problems of the network we were limited to 10 computers in three sessions; so only 54 subjects could participate in T0.

Instructions

In the experiment you will participate in 100 auctions.

At the beginning of each round, (*N*-1) numbers between 0 and 100 are drawn randomly and independently (i.e., in each draw every number between 0 and 100 is equally likely).[28]

These numbers represent the (*N*-1) bids of your competitors in the auction. Without knowledge of the others' bids you will be asked to submit your bid, which can be any number between 0 and 10. In each round, the highest bid (including your own) determines the market price.

Your bid determines your round payoff as follows:
If your bid is equal to the price, you will receive the difference between 100 and your bid (i.e., payoff = 100-price) expressed in ECU (experimental currency units), otherwise you will receive nothing. The exchange rate will be 1 ECU = 5(*N*) ITL. At the end of the experiment you will be paid your accumulated payoff privately.

At the end of a round, you will be informed about

1) (T0:) the auction price only if it is equal to your bid,
   (T1, T2:) the auction price,
2) (T2:) the highest bid submitted by your competitors
   (Note: if you submit a bid equal to the highest bid of your competitors you win the auction without further notice.)
3) your resulting round payoff
4) your accumulated payoff.

   Furthermore, throughout the experiment you will receive on-screen information about all corresponding historical records.

[28] For the instructional sessions (*N*-1) was substituted accordingly to the session's number of computerized competitors *N*-1={2, 3, 4, 5, 8}. Instructions were read aloud. Subjects were encouraged to ask questions in case of doubts.