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Price Discrimination in Input Markets: Downstream Entry and Welfare*

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The extant theory on price discrimination in input markets takes the structure of the intermediate industry as exogenously given. This paper endogenizes the structure of the intermediate industry and examines the effects of banning third-degree price discrimination on market structure and welfare. We identify situations where banning price discrimination leads to either higher or lower prices for all downstream firms. These findings are driven by the fact that upstream profits are discontinuous due to entry being costly. Moreover, permitting price discrimination fosters entry which in many cases improves welfare. Nevertheless, entry can also reduce welfare because it may lead to a severe inefficiency in production.

JEL classification: D43; L11; L42

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1 INTRODUCTION

"There are several ways in which the [upstream] manufacturer may influence the number of [downstream] retailers. [...] [T]he manufacturer may indirectly control the number of dealers through his pricing policy [...] ."

— Michael L. Katz (1989)

*We have benefited from comments made by Matthias Kräkel, Urs Schweizer, and Philipp Weinschenk. All errors are of course our own.

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An ubiquitous assumption in the extant theory on third-degree price discrimination in input markets is that the structure of the intermediate industry is rigid. Abstracting from entry into the intermediate industry ignores the fact that pricing decisions of the upstream supplier are a major determinant of the resulting industry structure and market outcome. These pricing decisions in turn are determined by the pricing instruments available to the upstream supplier, in particular whether price discrimination is feasible or not. In this paper, we endogenize the structure of the intermediate industry and examine the effects of banning price discrimination in input markets on industry structure and welfare.

Our modeling assumptions are shared by a large part of the extant literature: an monopolistic upstream firm supplies an input that is used by firms in an intermediate industry to produce a final product. The upstream supplier makes a take-it-or-leave-it offer to each of the downstream firms, specifying a per-unit wholesale price at which that firm can procure any desired quantity of the input. The new feature in our model is that one of the downstream firms has yet to decide whether to incur a strictly positive entry cost in order to become active in the intermediate industry.

If downstream firms operate in separate markets and if the entry cost imposes a binding restriction on the choice of wholesale prices under either regime, then, depending on the relative efficiency of the potential entrant, price discrimination can lead to lower or higher wholesale prices for all downstream firms compared to uniform pricing. This immediately translates into price discrimination being strictly welfare enhancing or welfare reducing, respectively. Irrespective of whether downstream firms operate in separate markets or compete in the same market, price discrimination fosters entry. With separate downstream markets, opening of a new market under price discrimination but not under uniform pricing is a sufficient condition for a ban on price discrimination to be welfare harming. If downstream firms compete à la Cournot, then entry alleviates the distortion arising from double marginalization. Under discriminatory wholesale pricing, however, this beneficial effect of entry can be offset by entry being costly and an allocative inefficiency in production induced by the upstream supplier's discrimination against the more efficient firm.

The theoretical debate about the welfare effects of banning third-degree price discrimination in intermediate-goods markets was initiated by Katz (1987). His seminal paper considers a vertically related industry where the upstream market is monopolized and the downstream industry consists of a large chain that competes in several downstream markets with a small local store. Katz shows that permitting price discrimination reduces welfare unless it prevents inefficient backward integration by the chain of stores. The finding of Katz is generalized by DeGraba (1990) to a long-run analysis where downstream firms can invest in cost reduction. Here, a ban on price discrimination does not only increase welfare in the short run, but also is beneficial in the long run. The reason is that the more efficient downstream firm is charged a higher wholesale price under price discrimination than under uniform pricing. Thus, under price discrimination the benefit of lower production costs is partially offset by a higher wholesale price, which reduces a firm's investment incentives. Yoshida (2000) extends the previous models to the case where downstream firms operate with Leontief-type technologies.[1]

More recent contributions relax the assumption that the upstream firm has all the bargaining power. Inderst and Valetti (2009) posit that downstream firms have access to an alternative source of input supply. In their model the more efficient firm receives a discount. In consequence, price discrimination provides higher incentives to invest in cost reduction and thus—at least for linear demand—can result in higher welfare than uniform pricing. While Inderst and Valetti still assume that the upstream firm makes a take-it-or-leave-it offer, O'Brien (forthcoming) assumes that the wholesale prices are determined by Nash bargaining. This also gives rise to circumstances where banning price discrimination is socially harmful.

Last, O'Brien and Shaffer (1994) and Inderst and Shaffer (2009) relax the assumption that the upstream supplier is restricted to linear wholesale prices and allow for two-part tariffs. In O'Brien and Shaffer, while a ban on price discrimination may benefit downstream firms, it always does so at the expense of consumers and total welfare.[2] In the setting of Inderst and Shaffer (2009), optimal wholesale prices are shown to amplify differences in downstream firms' competitiveness. A ban on price discrimination in consequence reduces allocative efficiency and may lead to higher wholesale prices for all downstream firms, resulting in lower welfare.

All the aforementioned papers take the structure of the intermediate industry as exogenously given. This paper, in contrast, endogenizes the structure of the intermediate industry by allowing for costly entry, and derives implications of banning price discrimination for industry structure, consumers' surplus, and welfare.[3] As was first reasoned by Bork (1978), allowing a final-good monopolist to price dis-

[1] Valetti (2003) generalizes the results obtained in Yoshida (2000) beyond the case of linear demand.

[2] Analyzing a similar model but assuming that the upstream firm competes against a fringe, Caprice (2006) shows that banning price discrimination may cause welfare to increase.

[3] In a linear demand model, Haucap and Wey (2007) also consider endogeneity of market structure in intermediate good markets. Abstracting from any real entry decision in the sense of incuring an entry cost, their findings, in contrast to our results, closely parallel the findings for final-good markets.

criminate can lead to more markets being served, which in turn improves welfare.[4] This entry-promoting and in turn welfare-improving effect of price discrimination is also operative in our model. But even when all markets are served under either pricing regime, we derive circumstances where price discrimination leads to either overall higher or overall lower prices than uniform pricing. These cases arise from entry being costly and do not crucially rely on any assumptions on the demand function. Thus, in a nutshell, the welfare implications of banning third-degree price discrimination with an endogenous market structure for final-good markets do not extend to the case of intermediate-good markets.

The rest of the paper is organized as follows: In Section 2, we introduce our model with downstream firms operating in separate markets. This model is analyzed for the cases of a less efficient entrant and a more efficient entrant in Section 3 and Section 4, respectively. Section 5 introduces Cournot competition between downstream firms. We conclude in Section 6.

2 A Model of Separate Markets

Consider a vertically related industry where the upstream market is monopolized by firm U. The upstream monopolist produces an essential input that is supplied to a downstream sector. For simplicity we assume that U produces without costs. There are potentially two downstream firms, $i \in \{I, E\}$, that transform one unit of input into one unit of a final good. While firm I, the incumbent, is already active in the downstream industry, firm E, the entrant, has to expend an entry cost $F > 0$ to become active in the downstream industry. Downstream firm i produces at constant marginal cost $k_i \in \{0, k\}$, $k > 0$, and without fixed cost.

The sequence of events is as follows: First, U can make a take-it-or-leave-it offer to each downstream firm.[5] Under price discrimination, U offers each downstream firm a possibly different wholesale price w_i, whereas under uniform pricing the same price $w_i = w$ applies to both firms.[6] Thus, upon accepting U's offer, downstream firm i's

[4] This finding is formally established by Hausman and MacKie-Mason (1988).

[5] The assumption of the upstream supplier having all the bargaining power, "which arguably can be justified on the grounds that for antitrust purposes the considerations of price discrimination in intermediate-goods markets is primarily relevant if the supplier enjoys a dominant position" (Inderst and Shaffer, 2009, p.4) is common in the extant literature. Exceptions are O'Brien and Shaffer (1994) and O'Brien (forthcoming).

[6] In restricting the upstream supplier to linear wholesale contracts we follow Katz (1987), DeGraba (1990), Yoshida (2000), O'Brien (forthcoming), and Inderst and Valetti (2009, forthcoming). Though obviously restrictive, this assumption "can be defended on grounds of possible realism", as argued in Inderst and Valetti (2009, forthcoming). From a theoretical perspective, Iyer and Villas-Boas (2003) and Milliou et al. (2004) provide some support for the use of linear wholesale contracts. Both these papers show that linear wholesale contracts can emerge as equilibrium

effective marginal cost is $c_i = w_i + k_i$. In stage two, after observing the contracts offered by U, firm E decides whether or not to enter the downstream industry at cost $F > 0$. In stage three, all active firms in the downstream industry purchase a nonnegative quantity of the input from U, transform this input into the final good, and sell the produced output to consumers. We abstract from any commitment problems and assume that U can credibly commit to the prices quoted in this first stage.[7]

First, we focus on the case where the downstream firms serve independent markets. We assume that both markets are symmetric and thus characterized by the same inverse demand function $P(q)$. The inverse demand function is assumed to be strictly decreasing and thrice differentiable where $P > 0$. Moreover, we impose the standard assumption $P'(q) < \min\{0, -qP''(q)\}$ where $P > 0$.[8] The equilibrium concept employed is subgame perfect Nash equilibrium in pure strategies.

We impose an additional assumption that ensures that U's maximization problem is well-behaved under either pricing regime.

Assumption (A1): *Downstream marginal revenue is concave,* $3P''(q) + qP'''(q) \leq 0$, *whenever* $P > 0$.

Next to technical issues, there is another reason for this assumption: as was shown by Katz (1987), if downstream firms engage in Cournot competition, then under price discrimination the downstream firm with the lower marginal cost will be charged a higher wholesale price than the downstream firm with the higher marginal cost.[9] The firm with lower own marginal cost has the more inelastic demand for the input, which causes the supplier to charge this firm a higher price. While the peculiarity of Cournot competition that total output only depends on the sum of effective marginal cost allows Katz (1987) to obtain this result in considerable generality, this is not possible in the case of separate markets. Here, Assumption (A1) provides a sufficient condition for the demand of the more efficient downstream firm being less elastic, which in turn implies that price discrimination results in a higher wholesale price for the more efficient firm. Thus, next to reasons of analytical convenience, we impose this assumption in order to maintain comparability to the earlier models of price

outcome when upstream and downstream firms can bargain over the form of their contractual arrangement.

[7] At a later point we make clear, which of our results are driven by this assumption.

[8] See, for example, Vives (1999).

[9] This result, which is also obtained by DeGraba (1990) and Yoshida (2000), holds as long as there are no additional restrictions on the input supplier's price setting, such as backward integration into the production process by downstream firms considered by Katz (1987) or demand-side substitution considered by Inderst and Valetti (2009, forthcoming).

discrimination in input markets.

In order to state our results as concise as possible, we restrict attention to situations where U considers it optimal to serve both firms under uniform pricing at least for sufficiently small entry cost. A sufficient condition for this is that the less efficient firm is not too inefficient in the sense that it demands a strictly positive quantity when charged the optimal discriminatory wholesale price $w^d(0)$ for the more efficient firm. Formally, letting the optimal quantity produced by an active downstream firm i be denoted by $q(c_i) := \arg\max_q \{q[P(q) - c_i]\}$, we impose the following assumption:

Assumption (A2): *Marginal cost k is such that $q(w^d(0) + k) > 0$.*

As a tie-breaking rule, we assume that U serves only the incumbent market when indifferent between the two possible structures the intermediate industry can take.

3 The Analysis

In this section, it is assumed that the potential entrant is less efficient than the incumbent, i.e., $0 = k_I < k_E = k$. As a preliminary consideration, note that an active downstream firm in stage 3 realizes gross profits $\pi(c_i) := q(c_i)[P(q(c_i)) - c_i]$, and that both $q(c_i)$ and $\pi(c_i)$ are strictly decreasing in effective marginal cost c_i where $q > 0$. Firm E will enter the intermediate industry if and only if its profits in stage 3 exceed the entry cost, i.e., iff $\pi(w_E + k) \geq F$. In all that follows, we focus on the case where F is not too high,

$$F < \pi(k) =: \tilde{F}(k), \tag{1}$$

such that there are positive gains from trade to be realized between U and firm E. Nevertheless, the entry constraint may impose a restriction on U in its setting of wholesale prices.

3.1 Optimal Wholesale Pricing

First, suppose that F is sufficiently low such that the entry constraint is not binding. If price discrimination is permitted, the optimal wholesale price for U to charge from firm i is

$$w^d(k_i) := \arg\max_w \{wq(w + k_i)\}. \tag{2}$$

Under uniform pricing, U chooses the common wholesale price

$$w^u(k) := \arg\max_w \{wq(w + k) + wq(w)\}. \tag{3}$$

We now can establish the following result: if U is unrestricted in its choice of wholesale prices under both regimes, then the optimal uniform wholesale price is bracketed by the two discriminatory prices. More precisely, under discrimination the less efficient firm receives a discount compared to uniform pricing. This discount, however, does not outweigh its cost disadvantage.

Lemma 1: *Given (A1) and (A2), then* $w^d(k) < w^u(k) < w^d(0) < w^d(k) + k$.

Under either pricing regime, if the entry fee is high, then charging the optimal unrestricted wholesale price(s) leads to a violation of firm E's entry constraint. Letting $\bar{F}^j(k)$ denote the highest value the entry fee can take such that U is not restricted in its price setting under pricing regime $j \in \{d, u\}$, we have

$$\bar{F}^u(k) := \pi(w^u(k) + k) \text{ and } \bar{F}^d(k) := \pi(w^d(k) + k). \tag{4}$$

From Lemma 1 and (1) it follows immediately that $\bar{F}^u(k) < \bar{F}^d(k) < \tilde{F}(k)$.

In order to induce entry, U optimally charges firm E wholesale price w^R at which firm E is indifferent between entering and staying out of the intermediate industry. Wholesale price w^R is implicitly defined by

$$\pi(w^R + k) \equiv F. \tag{5}$$

Obviously, $w^R = w^R(F; k)$ is strictly decreasing in F.

Under price discrimination it is optimal to offer wholesale price w^R to firm E as long as positive gains from trade are to be realized between U and firm E, i.e., for $F < \tilde{F}(k)$. When restricted to a uniform wholesale price, U has to pass-through this discount price w^R also to firm I. If the entry cost only slightly exceeds $\bar{F}^u(k)$, it remains optimal for U to serve both downstream firms just as in the case where the entry cost does not restrict wholesale pricing. Since w^R is strictly decreasing in F, if the entry cost exceeds some critical threshold $\hat{F}$, U prefers serving only firm I at wholesale price $w^d(0)$. Formally, $\hat{F}$ is implicitly defined by

$$w^R(\hat{F}; k)[q(w^R(\hat{F}; k) + k) + q(w^R(\hat{F}; k))] = w^d(0)q(w^d(0)). \tag{6}$$

Obviously, $\hat{F} = \hat{F}(k)$. From $w^R(\bar{F}^u(k); k) \equiv w^u(k)$ together with $w^R(F; k)$ tending to zero as F tends to $\tilde{F}(k)$, it follows that $\bar{F}^u(k) < \hat{F}(k) < \tilde{F}(k)$.

Letting wholesale prices in equilibrium be denoted by $\{w_E^d, w_I^d\}$ and w^u under price discrimination and uniform pricing, respectively, allows us to summarize the above discussion as follows:

Observation 1: *In equilibrium, the optimal wholesale price(s)* *(i) under price discrimination are* $w_E^d = w^d(k)$ *for* $0 < F \leq \bar{F}^d(k)$, $w_E^d = w^R(F;k)$ *for* $\bar{F}^d(k) < F < \tilde{F}(k)$, *and* $w_I^d = w^d(0)$.

(ii) under uniform pricing is $w^u = w^u(k)$ *for* $0 < F \leq \bar{F}^u(k)$, $w^u = w^R(F;k)$ *for* $\bar{F}^u(k) < F < \hat{F}(k)$, *and* $w^u = w^d(0)$ *for* $\hat{F}(k) \leq F < \tilde{F}(k)$.

3.2 Welfare Implications of Banning Price Discrimination

The measure of total welfare applied in this paper is the unweighted sum of consumer and producer surplus. We express changes in economic variables due to a regime shift from uniform pricing to price discrimination using symbol Δ. If both firms are active in the downstream industry, then the change in total welfare due to a regime shift amounts to

$$\Delta W \equiv \int_{q(w^u+k)}^{q(w_E^d+k)} P(x)dx - \int_{q(w_I^d)}^{q(w^u)} P(x)dx - k[q(w_E^d+k) - q(w^u+k)]. \tag{7}$$

To compare the two pricing regimes one needs a complete ordering of the entry cost's critical threshold levels. With the relationship between $\bar{F}^d(k)$ and $\hat{F}(k)$ being undetermined in general, there are two possible orderings of the thresholds as depicted in Figure 1.

[Insert Figure 1]

It is important to note that ceteris paribus entry into the downstream industry is always beneficial from a welfare point of view, since E enters only if it generates a surplus that exceeds the entry cost. Letting Q^r denote the total quantity sold under pricing regime $r \in \{d, u\}$, the following welfare implications are readily obtained.

Proposition 1: *Given (A1) and (A2), if*

(i) $F < \min\{\hat{F}(k), \bar{F}^d(k)\}$, *then* $\Delta Q \leq 0$ *implies* $\Delta W < 0$.

(ii) $\bar{F}^d(k) \leq F < \hat{F}(k)$, *then* $\Delta W < 0$.

(iii) $\hat{F}(k) \leq F < \tilde{F}(k)$, *then* $\Delta W > 0$.

Proposition 1 is illustrated in Figure 1. For low values of the entry fee, in case (i), entry occurs under both pricing regimes. With the uniform wholesale price lying strictly between the two discriminatory wholesale prices, a clear welfare result is not to obtain. Nevertheless, we can derive a sufficient condition for permitting price discrimination to reduce welfare: if total output under price discrimination is not higher than total output under uniform pricing, then welfare under price discrimination is strictly lower than welfare under uniform pricing. This finding clearly parallels Schmalensee's (1981) result on third-degree price discrimination in final-good markets.[10]

For high values of the entry fee, i.e., in case (iii), price discrimination fosters entry which in turn improves welfare. With entry occurring only if discriminatory pricing is permitted, the market outcome and thus welfare in the incumbent market is independent of the pricing regime, whereas welfare in the entrant's market is strictly positive only under price discrimination. This obviously is the intermediate-market analogue to Hausman and MacKie-Mason's (1988) finding on price discrimination in final-good markets.

Case (ii), on the other hand, embodies a novelty. For intermediate values of the entry fee both downstream firms are served under either pricing regime. With the upstream supplier being restricted in its price setting under both pricing regimes, the entrant receives wholesale price w^R irrespectively of the regime. This low wholesale price is passed on to the incumbent firm only under uniform pricing but not under price discrimination. In consequence, welfare in the entrant's market is unchanged when permitting price discrimination but welfare in the incumbent's market is strictly reduced. Thus, even though market-opening occurs under both pricing regimes, price discrimination is unambiguously found to be detrimental for welfare.[11]

A Linear Demand Application: Suppose $P(q) = \max\{1 - q, 0\}$, which satisfies (A1), and $0 < k < 1$, which relaxes (A2). The profit of an active downstream firm is $\pi(c_i) = (1-c_i)^2/4$. If U is unrestricted by the entry constraint, the optimal wholesale prices are $w^d(k_i) = (1 - k_i)/2$ and $w^u(k) = (2 - k)/4$ under price discrimination and under uniform pricing, respectively. The wholesale price that makes firm E indifferent between entering and staying out is $w^R = 1 - k - 2\sqrt{F}$. It can be shown that $\hat{F}(k) > \bar{F}^d(k)$ if and only if $k > 1/2$, where $\hat{F}(k) = (1/64)[2 - 3k + 4\sqrt{k^2 - 4k + 2}]^2$. Since we have not imposed (A2) there exists a critical marginal cost $\bar{k}$ such that under uniform pricing U optimally serves only firm I if $k > \bar{k}$ even for $F = 0$.

[Insert Figure 2]

[10] A series of papers elaborates on Schmalensee's basic insight, see Varian (1981), Schwartz (1990), and Malueg (1993). For extensive overviews on price discrimination in final-good markets, see Armstrong (2007) and Stole (2007).

[11] Note that case (ii) exists only if $\bar{F}^d(k) < \hat{F}(k)$. The effect arising in case (ii) depends on the upstream supplier's ability to commit to its offers. If commitment is not possible and the entry decision is made before wholesale prices are set, then for $F > \bar{F}^d(k)$ entry occurs under neither pricing regime.

Figure 2 depicts the critical thresholds for firm E's marginal cost k and the entry cost F, where for illustrative purposes we rephrased the thresholds in terms of $\sqrt{F}$. As is well-known, with linear demand total output is the same under price discrimination and uniform pricing, given that the entry constraint does not impose a binding restriction. Hence, according to Proposition 1(i), for low values of F banning price discrimination improves welfare. This case corresponds to the white area on the left bottom of Figure 2. The dark gray shaded area of Figure 2 corresponds to case (ii) of Proposition 1. Here, permitting price discrimination is harmful for total welfare. On the other hand, in the light gray shaded area price discrimination encourages entry which in turn supports welfare, case (iii) of Proposition 1.

4 MORE EFFICIENT ENTRANT

Suppose the entrant is more efficient than the incumbent, $0 = k_E < k_I = k$. Otherwise the model is the same as before: in particular $(A1)$ and $(A2)$ hold and $F < \pi(0) =: \tilde{F}(0)$. Lemma 1 immediately implies that the unrestricted uniform wholesale price is bracketed by the two unrestricted discriminatory wholesale prices, with the less efficient firm I receiving a discount under price discrimination.

If discriminatory offers are allowed, U charges wholesale price $w^d(k)$ from firm I. The wholesale price offered to firm E depends on whether the entry constraint imposes a binding restriction. If the entry constraint is slack, U sets $w_E^d = w^d(0)$. If the entry fee exceeds $\bar{F}^d(0) := \pi(w^d(0))$, the entry constraint becomes binding and U charges $w_E^d = w^R(F; 0)$ implicitly defined by $\pi(w^R(F; 0)) = F$.

The optimal uniform wholesale price is $w^u(k)$ if the entry cost is low. If the entry cost exceeds $\bar{\bar{F}}^u(k) := \pi(w^u(k))$, then U is restricted when choosing a uniform wholesale price. Note that $\bar{F}^d(0) < \bar{\bar{F}}^u(k)$. For intermediate values of the entry fee the optimal uniform wholesale price is $w^R(F; 0)$ which makes firm E just willing to enter the industry. For sufficiently high entry cost, $F \geq \hat{\hat{F}}$, U prefers to serve only firm I at price $w^d(k)$. The critical entry fee $\hat{\hat{F}}$ is implicitly defined by

$$w^R(\hat{\hat{F}}; 0)\left[q(w^R(\hat{\hat{F}}; 0) + k) + q(w^R(\hat{\hat{F}}; 0))\right] \equiv w^d(k)q(w^d(k) + k), \tag{8}$$

where $\hat{\hat{F}} = \hat{\hat{F}}(k) < \tilde{F}(0)$.

There exists an additional threshold for the entry cost that turns out to be important to characterize the welfare implications of banning price discrimination. Since firm I receives a discount under price discrimination, there exists an entry cost $\check{F}(k) \in (\bar{\bar{F}}^u(k), \hat{\hat{F}}(k))$, at which the restricted wholesale price equals the discriminatory wholesale price of firm I, i.e., $w^R(\check{F}(k);0) \equiv w^d(k)$.[12] For entry costs slightly below $\check{F}(k)$, price discrimination leads to (weakly) lower wholesale prices for both firms compared to uniform pricing. For F slightly above $\check{F}(k)$, on the other hand, the uniform wholesale price is (weakly) below both discriminatory prices.

From the above observations, the following welfare implications follow immediately.

Proposition 2: *Suppose* $0 = k_E < k_I = k$. *Given* $(A1)$ *and* $(A2)$, *if*

(i) $F < \bar{\bar{F}}^u(k)$, *then* $\Delta Q \leq 0$ *implies* $\Delta W < 0$,

(ii) $\bar{\bar{F}}^u(k) \leq F < \check{F}(k)$, *then* $\Delta W > 0$,

(iii) $\check{F}(k) < F < \hat{\hat{F}}(k)$, *then* $\Delta W < 0$,

(iv) $\hat{\hat{F}}(k) \leq F < \tilde{F}(k)$, *then* $\Delta W > 0$.

Cases (i), (iii), and (iv) are basically known from the previous analysis of a less efficient entrant. For very low values of the entry cost, in case (i), with the uniform wholesale price being bracketed by the two discriminatory wholesale prices, a clear welfare result is intricate to obtain. Nevertheless, if price discrimination does not lead to an expansion of total output, then permitting price discrimination is harmful for social welfare. For high-intermediate values of the entry cost, in case (iii), banning price discrimination unambiguously improves welfare. Under both pricing regimes, the upstream supplier serves both firms. The low restricted wholesale price, which is necessary to induce entry, is passed on to the incumbent only under uniform pricing, however. In case (iv), if entry is very costly, only price discrimination leads to the opening of the new market. Thus, for high values of the entry cost the known entry-promoting and in turn welfare-improving effect of permitting price discrimination prevails.

An interesting novelty is found in case (ii). Here, for low-intermediate values of the entry cost, under both pricing regimes the upstream supplier is restricted in its price setting but nevertheless induces entry. Surprisingly, a discriminatory pricing regime leads to (weakly) lower wholesale prices for both downstream firms. In consequence, permitting price discrimination strictly increases welfare, even though it does not lead to more markets being served than under a ban of price discrimination. This

[12] Note that $\bar{\bar{F}}^u(k) < \check{F}(k)$ follows immediately from $w^R(\check{F}(k);0) = w^d(k) < w^u(k) = w^R(\bar{\bar{F}}^u(k);0)$. $\check{F}(k) < \hat{\hat{F}}(k)$, on the other hand, follows from the fact that it is profitable to serve both downstream firms at a price only slightly below $w^d(k)$ instead of serving only firm I at price $w^d(k)$.

effect does neither occur with a less efficient entrant nor if the upstream firm sells directly to final consumers.

5 Downstream Competition

In this section, we inquire into the implications of downstream competition for the welfare effects associated with a ban of discriminatory wholesale pricing.[13] We now assume that active downstream firms produce a homogeneous final good and compete in quantities. Thus, if firm E, which is assumed to be the less efficient downstream firm, becomes active in the downstream industry, this is not associated with opening of a new market but with entry into firm I's market. Except for firms competing à la Cournot in stage 3, we stick to the sequence of events introduced in Section 2. Without further assumptions on the demand function welfare results are hard to obtain with downstream competition. Therefore, we focus on linear demand, i.e., $P(q) = \max\{1 - q, 0\}$. Moreover, we assume $0 < k < 1/2$ and focus on the case where $0 < \sqrt{F} < 1/3 - (2/3)k =: \tilde{f}(k)$. While the first assumption guarantees that both downstream firms produce positive quantities at the optimal unrestricted uniform wholesale price, the latter rules out the case where U prefers to serve only firm I under both pricing regimes.

Before proceeding with the analysis, a remark regarding the upstream supplier's incentives to serve the inefficient entrant next to the incumbent is in order: being restricted to linear wholesale contracts, the manufacturer's interest in inducing entry and thereby promoting downstream competition arises from the desire to reduce double marginalization. If the input supplier nevertheless prefers to serve only one downstream firm in equilibrium, then this is always the incumbent firm at wholesale price $w_M = 1/2$.

Suppose both downstream firms are active in equilibrium. Given the rival's effective marginal cost c_j, downstream firm i with effective marginal cost c_i demands quantity $q(c_i, c_j) = (1/3)(1 - 2c_i - c_j)$ and realizes gross profits $\pi(c_i, c_j) = (1/9)(1 - 2c_i - c_j)^2$. Similar as before, for low values of the entry cost, firm E's entry constraint does not impose a binding restriction on U's choice of wholesale prices. In this case, the optimal wholesale prices under uniform pricing and under price discrimination are $w^u(k) = (1/4)(2 - k)$ and $w^d(k_i) = (1/2)(1 - k_i)$, respectively. In consequence, the entry constraint does not impose a binding restriction under

[13] A detailed account of the following discussion is found in Appendix B.

uniform pricing and under price discrimination if

$$\pi(w^u(k)+k, w^u(k)) \geq F \iff \sqrt{F} \leq \frac{1}{6} - \frac{7}{12}k =: \bar{f}^u(k) \tag{9}$$

and

$$\pi(w^d(k)+k, w^d(0)) \geq F \iff \sqrt{F} \leq \frac{1}{6} - \frac{1}{3}k =: \bar{f}^d(k), \tag{10}$$

respectively. Note that $\bar{f}^u(k) < \bar{f}^d(k)$, i.e., the entry constraint imposes a stronger restriction on U under uniform pricing than under price discrimination since the less efficient firm receives a discount if price discrimination is permitted.

For higher values of the entry cost, $\sqrt{F} > \bar{f}^r(k)$ with $r \in \{d, u\}$, to make firm E enter the downstream market U needs to offer a discount wholesale price such that firm E can just recover its fixed cost. With competition firm E's profit does not only depend on its own wholesale price but also on firm I's wholesale price. Thus, in contrast to the case with separate downstream markets, the restricted wholesale price is not necessarily identical under the two pricing regimes. The restricted uniform wholesale price, w^{Ru}, is defined by $\pi(w^{Ru}+k, w^{Ru}) \equiv F$, or equivalently, $w^{Ru}(\sqrt{F};k) = 1 - 2k - 3\sqrt{F}$. Under price discrimination, on the other hand, U chooses two wholesale prices and thus the restricted wholesale price is not pinned down by firm E's binding entry constraint alone. Here, U optimally offers wholesale price $w_I^R = 1/2$ and $w_E^R(\sqrt{F};k) = 3/4 - k - (3/2)\sqrt{F}$ to firm I and firm E, respectively.

Is it always in U's interest to serve both downstream firms? Under discriminatory pricing it can be shown that U prefers to implement a downstream duopoly if $\sqrt{F} < \tilde{f}(k)$. If U is forced to offer a uniform wholesale price, it prefers that firm I monopolizes the downstream market when firm E is very inefficient or when entry costs are too high. Formally, U optimally serves both downstream firms if $\sqrt{F} < \hat{f}(k)$, where

$$\hat{f}(k) := \begin{cases} (1/12)[2 - 7k + \sqrt{1 - 4k + k^2}] & \text{, for } k < 2 - \sqrt{3} \\ 0 & \text{, for } k \geq 2 - \sqrt{3} \end{cases} . \tag{11}$$

Note that $\hat{f}(k) < \tilde{f}(k)$. Hence, there exists a range of entry costs where entry occurs under price discrimination but not under uniform pricing, i.e., price discrimination promotes entry also when downstream firms compete. The thresholds characterized above are depicted in Figure 3.

[Insert Figure 3]

Welfare Comparisons In order to compare welfare under the two pricing regimes, we distinguish five cases with respect to the resulting downstream market structure, as illustrated in Figure 4. We label these cases with Roman numerals, I - V.

[Insert Figure 4]

In cases I - III, both pricing regimes lead to implementation of a downstream duopoly. Moreover, in these three cases allowing for discriminatory wholesale prices lowers welfare. This is most obvious in case I, where both the entry fee and the entrant's marginal cost of production are sufficiently low such that the input supplier is not constrained in its choice of wholesale prices under either pricing regime.[14] While total output is unaffected by the pricing regime, which is a direct result of linear demand, under price discrimination the upstream supplier "subsidizes" the less efficient firm by charging a higher wholesale price to the more efficient firm, thereby (at least partly) removing the incumbent firm's cost advantage. In consequence, under price discrimination output is shifted from the low-cost firm to the high-cost firm, which raises the total cost of production and thus reduces welfare. This negative effect of price discrimination on the allocation of producing the final output is even more severe if the upstream firm is restricted by firm E's entry decision, since this increases the discount the less efficient entrant receives. If the entry constraint imposes a restriction under uniform pricing, on the other hand, the lowered wholesale price applies for all downstream firms, such that no such misallocation in production shares occurs.[15]

The more interesting cases are IV and V. Here, the downstream market is monopolized under uniform pricing while under price discrimination both downstream firms compete for final customers. The reason is that the relatively high entry fee and/or the relatively high marginal cost of the entrant render the concession in the uniform wholesale price necessary to induce entry unprofitable for the upstream monopolist. Price discrimination, on the other hand, provides the input supplier with a tool to profitably implement a downstream duopoly even in these cases.

With separate markets, entry into the intermediate industry taking place only under a discriminatory pricing regime but not under uniform pricing is a sufficient condition for welfare to be higher under price discrimination, see Proposition 1. For moderate values of the entry cost and a not too inefficient entrant—represented by the dark-gray shaded area in Figure 5—this result carries over to the case of

[14] This situation exactly corresponds to the short-run analysis in DeGraba (1990).

[15] Moreover, due to a lower input price, total output is increased, which in turn improves welfare compared to the situation where the upstream firm is unrestricted under uniform pricing.

downstream competition: here, society benefits from output being produced inefficiently rather than not being produced at all. If, however, the entry fee is high and/or the entrant is very inefficient—represented by the light-gray shaded area in Figure 5—entry becomes undesirable from a social perspective: while entry into the downstream market alleviates the quantity distortion arising under downstream monopoly, thereby increasing upstream profits and benefiting consumers through higher quantity and lower final-good prices, the increase in aggregate output comes at the cost of a reduction in the efficient downstream firm's output brought about by competition in the downstream market. Thus, the increase in consumer surplus and the upstream supplier's profits is gained at the price of burdening society with the cost of entry and higher production costs. In consequence, since the major effect of the discriminatory pricing policy is not the creation of value but shifting rents away from the incumbent firm to the upstream supplier, here price discrimination leads to a strictly inferior welfare result even though market entry is promoted.

With regard to a formal welfare result, we define

$$f^W(k) := \begin{cases} 1/3 - (8/9)k & \text{, for } \; k \leq 3/10 \\ \sqrt{(23/72)k^2 - (5/18)k + 17/288} & \text{, for } \; 3/10 < k \leq 17/46 \\ 0 & \text{, for } \; k > 17/46 \end{cases} . \tag{12}$$

With this notation we are prepared summarize the above discussion as follows:

Proposition 3: *For the case with linear demand and downstream Cournot competition; (i)* $\Delta W > 0$ *if* $\hat{f}(k) \leq \sqrt{F} < f^W(k)$ *and (ii)* $\Delta W < 0$ *if either* $\sqrt{F} < \hat{f}(k)$ *or* $f^W(k) < \sqrt{F} < \tilde{f}(k)$*.*

We refrain from an analysis of downstream competition with a more efficient entrant. In this case, if entry occurs under price discrimination but not under uniform pricing, production shares are shifted from the less efficient to the more efficient downstream firm, i.e., in tendency overall production becomes less costly. Therefore, in these cases, we would expect a discriminatory pricing regime to be welfare improving more often, because one major inefficiency identified in the analysis of a less efficient entrant does not arise.

6 Conclusion

This paper attempts to provide answers to the following two questions: First, how does potential entry into the downstream industry affect wholesale prices set by an upstream monopolist? Second, under what circumstances is banning third-degree price discrimination beneficial for welfare and consumer surplus if there is potential entry into the downstream sector?

Compared to a situation with a rigid structure of the intermediate industry, the optimal uniform wholesale price as well as the optimal discriminatory wholesale price charged from the potential entrant may be lower if costly entry is possible. The optimal discriminatory wholesale price charged from incumbent firms, in contrast, does not depend on whether entry into the intermediate industry is possible or not. As a consequence, when downstream firms operate in distinct markets, there are situations—in terms of the entrant's efficiency in production and the cost of entry—where price discrimination may lead to either higher or lower prices for all downstream firms than uniform pricing. In these cases, with wholesale prices being clearly favorable under one of the two pricing regimes, we obtain unambiguous implications of banning price discrimination regarding welfare and consumer surplus. If downstream firms are Cournot competitors, permitting price discrimination has the beneficial effect that it supports entry which in turn reduces double marginalization. This beneficial effect, however, can be outweighed by entry being costly and an allocative inefficiency in production induced by discrimination against the more efficient firm.

With costly entry being possible, these results are novel to the extant literature on third-degree price discrimination in intermediate-good markets. Moreover, several of the identified effects are not to be obtained in a model of price discrimination in final-good markets. Thus, one should be wary not to hastily infer that welfare implications valid in final-good markets also carry over to intermediate-good markets.

A PROOFS OF PROPOSITIONS AND LEMMAS

Proof of Lemma 1:
For any wholesale price $w_i \geq P(0) - k_i$ firm i's input demand equals zero, whereas for $w_i < P(0) - k_i$ the optimal input demand, $q(c_i)$, is strictly positive and characterized by

$$MR(q(c_i)) := q(c_i)P'(q(c_i)) + P(q(c_i)) = c_i. \tag{A.1}$$

Under the assumptions imposed on the inverse demand function, whenever $q(c_i) > 0$ we have $q'(c_i) < 0$, $q''(c_i) \leq 0$, $\pi'(c_i) < 0$, $MR'(q) < 0$ and $MR''(q) \leq 0$.

The upstream supplier's profit from charging an active downstream firm with own marginal cost $k_i < P(0)$ a wholesale price $w < P(0) - k_i$ is $\Pi(w; k_i) := wq(w + k_i)$. With $\Pi(w; k_i)$ being strictly concave on the interval $[0, P(0) - k_i]$ the optimal unconstrained discriminatory wholesale price $w^d(k_i)$ satisfies

$$q(w^d(k_i) + k_i) + w^d(k_i)q'(w^d(k_i) + k_i) = 0. \tag{A.2}$$

We first show that $w^d(k) < w^d(0)$. Suppose, in contradiction, that $w^d(k) \geq w^d(0)$. Differentiation of (A.1) with respect to c_i yields

$$q'(c_i) = \frac{1}{2P'(q(c_i)) + q(c_i)P''(q(c_i))} = \frac{1}{MR'(q(c_i))}, \tag{A.3}$$

where the second equality follows from the definition of $MR(q)$. From (A.2) it follows that the optimal discriminatory wholesale price charged to a downstream firm with own marginal cost k_i satisfies

$$w^d(k_i) = -\frac{q(w^d(k_i) + k_i)}{q'(w^d(k_i) + k_i)} = -q(w^d(k_i) + k_i)MR'(q(w^d(k_i) + k_i)). \tag{A.4}$$

In consequence, $w^d(0) \leq w^d(k)$ if and only if $-q(w^d(0))MR'(q(w^d(0))) \leq -q(w^d(k)+ k)MR'(q(w^d(k) + k))$. Since

$$\frac{d}{dc}\left[-q(c)MR'(q(c))\right] = -q'(c)\left[MR'(q(c)) + q(c)MR''(q(c))\right] < 0, \tag{A.5}$$

$w^d(k) \geq w^d(0)$ implies $w^d(0) \geq w^d(k) + k$, a contradiction. Therefore, $w^d(k) < w^d(0)$.

Knowing that $w^d(k) < w^d(0)$, we next show that $w^d(0) < w^d(k) + k$. Suppose, in contradiction, that $w^d(0) \geq w^d(k) + k$. Then $q(w^d(0)) \leq q(w^d(k) + k)$, and in consequence

$$0 > MR'(q(w^d(0))) \geq MR'(q(w^d(k) + k)) \tag{A.6}$$

by marginal revenue being decreasing and concave. From above, we know that $w^d(0) > w^d(k)$, and thus, according to (A.4), we have

$$-q(w^d(0))MR'(q(w^d(0))) > -q(w^d(k) + k)MR'(q(w^d(k) + k)). \tag{A.7}$$

Taken together (A.6) and (A.7) imply $q(w^d(k) + k) < q(w^d(0))$ and in consequence $w^d(k) + k > w^d(0)$, a contradiction. Thus, $w^d(k) + k > w^d(0)$.

When unrestricted in its price setting, U's profit from charging a common wholesale price w is

$$\Pi^u(w;k) := \begin{cases} \Pi(w;0) + \Pi(w;k) & \text{for} \quad w < P(0) - k \\ \Pi(w;0) & \text{for} \quad P(0) - k \leq w < P(0) \\ 0 & \text{for} \quad w \geq P(0) \end{cases},$$

Obviously, serving no firm clearly is not optimal. Moreover, under Assumption (A2), it is never optimal to serve only firm I, i.e., we must have $w^u(k) < P(0) - k$. Note that $\Pi^u(w;k)$ is strictly concave on $[0, P(0) - k]$. By definition of $w^d(0)$ and $w^d(k)$, $w^d(0) > w^d(k)$ from above, and concavity of $\Pi(w;k_i)$ on $[0, P(0) - k_i]$ for $i \in \{I, E\}$, we have

$$\frac{d\Pi^u(w;k)}{dw} = \frac{d\Pi(w;0)}{dw} + \frac{d\Pi(w;k)}{dw} > 0$$

for all $w \in [0, w^d(k)]$, which immediately implies that $w^d(k) < w^u(k)$.

It remains to show that $w^u(k) < w^d(0)$. With $w^d(0) < P(0) - k$, under Assumption (A2) we have

$$\left.\frac{d\Pi^u(w;k)}{dw}\right|_{w=w^d(0)} = \left.\frac{d\Pi(w;0)}{dw}\right|_{w=w^d(0)} + \left.\frac{d\Pi(w;k)}{dw}\right|_{w=w^d(0)} = \left.\frac{d\Pi(w;k)}{dw}\right|_{w=w^d(0)} < 0,$$

where the last equality follows from definition of $w^d(0)$, and the inequality follows from the fact that $w^d(0) > w^d(k)$ and $\Pi(w;k)$ being strictly concave on $[0, P(0) - k]$. Strict concavity of $\Pi^u(w;k)$ on $[0, P(0) - k]$ then immediately implies $w^u(k) < w^d(0)$, which establishes the desired result. ■

Proof of Proposition 1:

We prove part (i) first. As a preliminary consideration, consider two active downstream firms i and j with own marginal cost $k_j < k_i$. For $w < P(0) - k_i$, we have $0 < q(w + k_i) < q(w + k_j)$, and $q'(c) < 0$ for all $c \in [w + k_j, w + k_i]$. The optimal quantity demanded by a downstream firm with own marginal cost $\tilde{k} \in [k_j, k_i]$ at wholesale price w satisfies

$$P(q(w + \tilde{k})) - \tilde{k} \equiv w - q(w + \tilde{k})P'(q(w + \tilde{k})). \tag{A.8}$$

Differentiation of this expression with respect to $\tilde{k}$ yields

$$\frac{d}{d\tilde{k}}[P(q(w + \tilde{k})) - \tilde{k}] \\ = -q'(w + \tilde{k})\left[P'(q(w + \tilde{k})) + q(w + \tilde{k})P''(q(w + \tilde{k}))\right] < 0. \tag{A.9}$$

Thus, a more efficient downstream firm charges a higher mark-up.

Now, in case (i), with $F < \min\{\bar{F}^d(k), \hat{F}(k)\}$, we always have the optimal uniform price bracketed by the optimal discriminatory wholesale prices: for $F \leq \bar{F}^u(k)$ we have $w^d(k) < w^u(k) < w^d(0)$ by Lemma 1; for $F \in (\bar{F}^u(k), \min\{\bar{F}^d(k), \hat{F}(k)\})$ the optimal uniform wholesale price equals $w^R(F;k)$ where $w^R(\bar{F}^u(k);k) = w^u(k)$, $w^R(\bar{F}^d(k);k) = w^d(k)$, and $dw^R/dF < 0$. Letting q_i^d and q_i^u denote firm i's quantity under price discrimination and uniform pricing, respectively, where $i \in \{I, E\}$, this in turn implies that $q_I^d < q_I^u$ and $q_E^d > q_E^u$. Welfare under price discrimination is

$$W^d(F;k) = \int_0^{q_I^d} P(x)dx + \int_0^{q_E^d} P(x)dx - kq_E^d - F, \tag{A.10}$$

whereas welfare under uniform pricing is

$$W^u(F;k) = \int_0^{q_I^u} P(x)dx + \int_0^{q_E^u} P(x)dx - kq_E^u - F. \tag{A.11}$$

Then

$$\begin{aligned} \Delta W(F;k) &= \int_{q_E^u}^{q_E^d} P(x)dx - \int_{q_I^d}^{q_I^u} P(x)dx - k(q_E^d - q_E^u) \\ &< (q_E^d - q_E^u)[P(q_E^u) - k] - (q_I^u - q_I^d)P(q_I^u). \end{aligned} \tag{A.12}$$

From (A.9) we know that $P(q_E^u) - k < P(q_I^u)$. Thus, $q_E^d - q_E^u \le q_I^u - q_I^d$, or equivalently $Q^d(F;k) = q_I^d + q_E^d \le q_I^u + q_E^u = Q^u(F;k)$, is a sufficient condition for $\Delta W(F;k) < 0$.

Parts (ii) and (iii) follow immediately from the reasoning in the text. ■

B Downstream Competition (Not for Publication)

In this appendix we provide a detailed analysis of the case with downstream Cournot competition as discussed in Section 5. The equilibrium concept employed is subgame perfect Nash equilibrium. We solve the game by backward induction, beginning in stage three.

Stage 3: For given wholesale prices and a given number of active firms in the intermediate industry, we determine the quantities produced of the final good by firms active in the downstream market. If a downstream firm with own marginal cost k_i is a downstream monopolist, its demand for the input at a wholesale price w is

$$q(w + k_i) = \begin{cases} \frac{1-w-k_i}{2} & \text{for} \quad w < 1 - k_i \\ 0 & \text{for} \quad w \ge 1 - k_i \end{cases}.$$

If two firms i and j are active in the downstream market, then firm i's best response at wholesale price w_i given that firm j produces quantity q_j is

$$q(q_j; w_i + k_i) = \max\left\{0, \frac{1 - w_i - k_i - q_j}{2}\right\} \tag{B.1}$$

For $2w_i - w_j < 1 - 2k_i + k_j$ and $2w_j - w_i < 1 - 2k_j + k_i$ the Cournot Nash equilibrium is interior with both firms producing strictly positive quantities. The equilibrium quantity of firm $i \neq j$ is

$$q(w_i + k_i, w_j + k_j) = \frac{1 - 2(w_i + k_i) + (w_j + k_j)}{3}. \tag{B.2}$$

If $2w_i - w_j < 1 - 2k_i + k_j$ and $2w_j - w_i \geq 1 - 2k_j + k_i$, then firm j produces nothing whereas firm i produces its monopoly quantity. For $2w_i - w_j \geq 1 - 2k_i + k_j$ and $2w_j - w_i \geq 1 - 2k_j + k_i$ both downstream firms produce zero quantity.

Stage 2 Given wholesale prices w_I and w_E charged to firm I and firm E, respectively, and correctly anticipating Nash equilibrium play in stage three, firm E enters the market if its profits in the resulting market outcome in stage 3 exceed the entry fee. If indifferent between entering and not entering the market, as a tie-breaking rule we assume that firm E behaves as the upstream supplier U wishes.[16] If firm E's profits in stage three are strictly negative, then E does not enter the intermediate industry.

Stage 1 Correctly anticipating firm E's entry decision in stage two and equilibrium play in stage three, U chooses wholesale prices w_I and w_E in order to maximize upstream profits. In what follows, we refer to a duopoly as a situation, in which E enters the downstream market and downstream demand is strictly positive for both firms I and E. Again, when indifferent between implementing a downstream duopoly or a downstream monopoly, the upstream supplier implements a downstream monopoly. Let $\Pi^r_{\{i\}}$ denote firm U's profit from implementing firm $i \in \{I, E\}$ as a downstream monopolist, and let $\Pi^r_{\{I,E\}}$ denote firm U's profit from implementing firms I and E as downstream duopolists. Superscript $r \in \{d, u\}$ again refers to either a discriminatory pricing regime or a uniform pricing regime. Moreover, in order not to clutter notation, we will often suppress the dependency of downstream quantity choices on effective marginal costs as well as the dependency of optimal wholesale prices and welfare on the entry fee and own marginal costs of the downstream firms.

Lemma 2: *Under Price discrimination,*

[16] We impose this alternative tie-breaking rule for expositional purposes only. Sticking to the original tie-breaking rule, i.e., firm E enters whenever its profits are nonnegative, yields exactly the same results.

(i) if $\sqrt{F} \leq (1/6) - (1/3)k$*, then* U *charges wholesale prices* $w_I^d = w^d(0) = 1/2$ *and* $w_E^d = w^d(k) = (1-k)/2$*. This implements a downstream duopoly resulting in quantities* $q_I^d = (1+k)/6$*,* $q_E^d = (1-2k)/6$*, and* $Q^d = (2-k)/6$*;*

(ii) if $(1/6) - (1/3)k < \sqrt{F} < (1/3) - (2/3)k$*, then* U *charges wholesale prices* $w_I^d = w_I^R = 1/2$ *and* $w_E^d = w_E^R(\sqrt{F}; k) = (3/4) - k - (3/2)\sqrt{F}$*. This implements a downstream duopoly resulting in quantities* $q_I^d = (1/4) - (1/2)\sqrt{F}$*,* $q_E^d = \sqrt{F}$*, and* $Q^d = (1/4) + (1/2)\sqrt{F}$*;*

(iii) if $(1/3) - (2/3)k \leq \sqrt{F}$*, then* U *charges wholesale prices* $w_I^d = w_M = 1/2$ *and* $w_E^d = \infty$*. This implements a downstream monopoly resulting in quantities* $q_I^d = Q^d = 1/4$*.*

Proof:
Suppose U wants to implement a downstream duopoly. Then U chooses wholesale prices in order to solve the following problem:

Program D-PD:

$$\max_{(w_I, w_E) \in \mathbb{R}^2_{\geq 0}} w_I \frac{1 - 2w_I + (w_E + k)}{3} + w_E \frac{1 - 2(w_E + k) + w_I}{3}$$

$$\text{subject to} \quad q_I = \frac{1 - 2w_I + (w_E + k)}{3} > 0$$

$$q_E = \frac{1 - 2(w_E + k) + w_I}{3} > 0$$

$$F \leq \left[\frac{1 - 2(w_E + k) + w_I}{3}\right]^2$$

Next, we show that for a sufficiently low entry fee, the solution to Program D-PD is identical to the solution of the relaxed program, which only considers the latter two constraints.

Claim 1: *If* $\sqrt{F} \leq (1/2) - (2/3)k$*, the solution to Program R*

$$\max_{(w_I, w_E)} w_I \frac{1 - 2w_I + (w_E + k)}{3} + w_E \frac{1 - 2(w_E + k) + w_I}{3}$$

$$\textit{subject to} \quad 2w_E - w_I \leq 1 - 2k - 3\sqrt{F},$$

also solves Program D-PD.

Proof of Claim 1: First, note that the latter two constraints of Program D-PD can equivalently be replaced by the following condition:

$$2w_E - w_I \leq 1 - 2k - 3\sqrt{F}, \tag{B.3}$$

which corresponds to the one constraint in Program R. The Lagrangian associated with Program R is

$$\mathcal{L} = w_I \frac{1 - 2w_I + (w_E + k)}{3} + w_E \frac{1 - 2(w_E + k) + w_I}{3} - \lambda \left\{ 2w_E - w_I - (1 - 2k - 3\sqrt{F}) \right\}. \quad \text{(B.4)}$$

With $\mathcal{L}$ being a strictly concave function, the associated Kuhn-Tucker conditions are sufficient for global optimality. These Kuhn-Tucker conditions are given by

$$\frac{\partial \mathcal{L}}{\partial w_I} = \frac{1 + 2w_E + k - 4w_I}{3} + \lambda = 0$$
$$\frac{\partial \mathcal{L}}{\partial w_E} = \frac{1 - 4w_E - 2k + 2w_I}{3} - 2\lambda = 0$$
$$\lambda \geq 0 \ \left(= 0 \ \text{if} \ 2w_E - w_I < 1 - 2k - 3\sqrt{F} \right)$$
$$2w_E - w_I \leq 1 - 2k - 3\sqrt{F}$$

Consider the case of $\sqrt{F} \leq (1/6) - (1/3)k$ first. Suppose the constraint is not binding, i.e., $2w_E - w_I < 1 - 2k - 3\sqrt{F}$. The complementary slackness condition then implies $\lambda = 0$. Combining the two first-order conditions yields wholesale prices $w_I = 1/2$ and $w_E = (1-k)/2$. It is readily verified that for $\sqrt{F} \leq (1/6) - (1/3)k$, at these prices the constraint of Program R is satisfied. Moreover, under these wholesale prices, all remaining constraints of Program D-PD are also satisfied: wholesale prices are nonnegative, and associated quantities are strictly positive, $q_I = (1+k)/6$ and $q_E = (1-2k)/6$. Next, consider the case $(1/6) - (1/3)k < \sqrt{F} \leq (1/2) - (2/3)k$. Suppose that the constraint is binding, i.e., $2w_E - w_I = 1 - 2k - 3\sqrt{F}$. The complementary slackness condition then implies $\lambda \geq 0$. Combining the two first-order conditions yields $w_I = 1/2$. Inserting this into the binding constraint leads to $w_E = (3/4) - k - (3/2)\sqrt{F}$. Solving for the Lagrange parameter yields $\lambda = (-1 + 2k + 6\sqrt{F})/6$, which is strictly positive for $(1/6) - (1/3)k < \sqrt{F}$. It is readily verified that for $\sqrt{F} \leq (1/2) - (2/3)k$ all remaining constraints of Program D-PD are also satisfied under these wholesale prices: wholesale prices are nonnegative, and associated quantities are strictly positive, $q_I = (1/4) - (1/2)\sqrt{F}$ and $q_E = \sqrt{F}$. This proves Claim 1. ||

Straightforward calculations show that U's profit from implementing a downstream duopoly is $\Pi^d_{\{I,E\}} = (1 - k + k^2)/6$ if $\sqrt{F} \leq (1/6) - (1/3)k$, and $\Pi^d_{\{I,E\}} = (1/8) + ((1/2) - k)\sqrt{F} - (3/2)(\sqrt{F})^2$ if $(1/6) - (1/3)k < \sqrt{F} \leq (1/2) - (2/3)k$. Note that for $\sqrt{F} > (1/2) - (2/3)k$, U's problem becomes more heavily constrained, such that U's profit cannot be larger than for $\sqrt{F} \leq (1/2) - (2/3)k$.

Next, suppose U wants to implement a downstream monopoly. Straightforward calculations show that the maximum profit U can make when facing a downstream monopolist with own marginal cost k_i, the optimal wholesale price for U to charge is $w = (1-k_i)/2$, which results in downstream demand $q = (1-k_i)/4$ and upstream profits $\Pi^d_{\{i\}} = (1-k_i)^2/8$. Since U's maximum profit decreases in the downstream monopolists own marginal cost, U always prefers I to become a monopolist over E becoming a monopolist. Since under price discrimination U can charge E a prohibitively high price which keeps E out of the downstream market without affecting the price paid by the incumbent firm I, U can always make I the downstream monopolist, resulting in upstream profits of $\Pi^d_{\{I\}} = 1/8$.

In order to conclude the proof of Lemma 2, we have to determine when U prefers to implement a downstream duopoly over implementing a downstream monopoly. If $\sqrt{F} \leq (1/6) - (1/3)k$, $\Pi^d_{\{I,E\}} > \Pi^d_{\{I\}}$ if and only if $(1-2k)^2 > 0$. Thus, if $\sqrt{F} \leq (1/6) - (1/3)k$, U will implement a downstream duopoly resulting in quantities $q^d_I = (1+k)/6$ and $q^d_E = (1-2k)/6$. Next, if $(1/6) - (1/3)k < \sqrt{F} \leq (1/2) - (2/3)k$, $\Pi^d_{\{I,E\}} > \Pi^d_{\{I\}}$ if and only if $\sqrt{F} < (1/3) - (2/3)k$. Thus, if $(1/6) - (1/3)k < \sqrt{F} < (1/3) - (2/3)k$, U will implement a downstream duopoly resulting in quantities $q^d_I = (1/4) - (1/2)\sqrt{F}$ and $q^d_E = \sqrt{F}$, whereas for $\sqrt{F} \geq (1/3) - (2/3)k$, U will implement a downstream monopoly resulting in quantity $q^d_I = 1/4$. This establishes the desired result. ■

Lemma 3: *Under uniform pricing,*

(i) if $k < 2 - \sqrt{3}$ and $\sqrt{F} \leq (1/6) - (7/12)k$, then U charges a wholesale prices $w^u = w^u(k) = (1/2) - (1/4)k$. This implements a downstream duopoly resulting in quantities $q^u_I = (2+5k)/12$, $q^u_E = (2-7k)/12$, and $Q^u = (2-k)/6$;

(ii) if $k < 2 - \sqrt{3}$ and $(1/6) - (7/12)k < \sqrt{F} < (1/6) - (7/12)k + (\sqrt{1-4k+k^2})/12$, then U charges a wholesale prices $w^u = w^{Ru}(\sqrt{F}; k) = 1 - 2k - 3\sqrt{F}$. This implements a downstream duopoly resulting in quantities $q^u_I = k + \sqrt{F}$, $q^u_E = \sqrt{F}$, and $Q^u = k + 2\sqrt{F}$;

(iii) if $k \geq 2 - \sqrt{3}$ or $\sqrt{F} \geq (1/6) - (7/12)k + (\sqrt{1-4k+k^2})/12$, then U charges a wholesale price $w^u = w_M = (1/2)$. This implements a downstream monopoly resulting in quantities $q^u_I = Q^u = 1/4$.

Proof:

Suppose U wants to implement a downstream duopoly. Then U chooses the uniform wholesale price in order to solve the following problem:

PROGRAM D-UNI:

$$\max_{w\in\mathbb{R}_{\geq 0}} w\frac{2-2w-k}{3}$$
$$\text{subject to} \quad q_I = \frac{1-w+k}{3} > 0$$
$$q_E = \frac{1-w-2k}{3} > 0$$
$$F \leq \left[\frac{1-w-2k}{3}\right]^2$$

First, note that if the second constraint holds also the first constraint holds with strict inequality, i.e., if E demands a nonnegative quantity at wholesale price w, $q_E \geq 0$, then I demands a strictly positive quantity, $q_I > 0$. Moreover, the second and third constraint together can equivalently be replaced by the following condition: $w \leq 1-2k-3\sqrt{F}$. Thus, Program D-UNI can be equivalently rewritten as

PROGRAM D-UNI:

$$\max_{w\in\mathbb{R}_{\geq 0}} w\frac{2-2w-k}{3}$$
$$\text{subject to} \quad w \leq 1-2k-3\sqrt{F}$$

Note that U's objective is maximizing a strictly concave function with a unique global maximum attained at $w = (2-k)/4$. Therefore, if $(2-k)/4 \leq 1-2k-3\sqrt{F}$, or, equivalently, if $\sqrt{F} \leq (1/6)-(7/12)k$, the optimal uniform wholesale price that implements a downstream duopoly is $w = (2-k)/4$, resulting in quantities $q_I = (2+5k)/12$ and $q_E = (2-7k)/12$. Note that $q_E > 0$—and thus also $q_I > 0$—if and only if $k < 2/7$. If $\sqrt{F} > (1/6)-(7/12)k$, the constraint becomes binding. If $\sqrt{F} \leq (1/3)-(2/3)k$, the optimal uniform wholesale price in order to implement a downstream duopoly is given by $w = 1-2k-3\sqrt{F}$, resulting in quantities $q_I = k+\sqrt{F}$ and $q_E = \sqrt{F}$. If $\sqrt{F} > (1/3)-(2/3)k$, implementation of a downstream duopoly with E demanding a strictly positive quantity and making nonnegative profits is not possible with a nonnegative wholesale price.

Straightforward calculations show that U's profit from implementing a downstream duopoly is $\Pi^u_{\{I,E\}} = (2-k)^2/24$ if $\sqrt{F} \leq (1/6)-(7/12)k$, and $\Pi^u_{\{I,E\}} = (1-2k-3\sqrt{F})(k+2\sqrt{F})$ if $(1/6)-(7/12)k < \sqrt{F} \leq (1/3)-(2/3)k$.

Next, suppose that U wants to implement a downstream monopoly. As noted above, for a given wholesale price w, if E demands a nonnegative quantity, then I demands a strictly positive quantity. Thus, under uniform pricing, the only possible form monopoly can take in the downstream market is with I as downstream monopolist. Therefore, when implementing a downstream monopoly under uniform pricing, U has to choose a wholesale price at which E does not find it profitable to enter. From above we know that this requires the wholesale price to be sufficiently high, i.e., $w > 1 - 2k - 3\sqrt{F}$. Under our tie-breaking rule that E does what U wants him to do when indifferent between entering and not entering the market, U implements a downstream monopoly whenever he chooses a wholesale price $w \geq 1 - 2k - 3\sqrt{F}$. With the quantity demanded by downstream monopolist I being $q_I = (1 - w)/2$, by the choice of the wholesale price U maximizes a strictly concave function with a unique stationary point at $w = 1/2$ subject to the aforementioned constraint. In consequence, if $1/2 \geq 1 - 2k - 3\sqrt{F}$, or equivalently, if $\sqrt{F} \geq (1/6) - (2/3)k$, then the optimal wholesale price to implement a downstream monopoly is $w = 1/2$ resulting in quantity $q_I = 1/4$ and upstream profit $\Pi^u_{\{I\}} = 1/8$. If $\sqrt{F} < (1/6) - (2/3)k$, then the optimal wholesale price to implement a downstream monopoly is $w = 1 - 2k - 3\sqrt{F}$ resulting in quantity $q_I = k + (3/2)\sqrt{F}$ and upstream profit $\Pi^u_{\{I\}} = (1 - 2k - 3\sqrt{F})(k + (3/2)\sqrt{F})$. Note that $w = 1 - 2k - 3\sqrt{F} \geq 0$ if and only if $\sqrt{F} \leq (1/3) - (2/3)k$, which obviously is satisfied for $\sqrt{F} < (1/6) - (2/3)k$.

In order to conclude the proof of Lemma 3, we have to determine when U prefers to implement a downstream duopoly over implementing a downstream monopoly. Combining the observations obtained above, we have to distinguish four cases. (i) If $\sqrt{F} > (1/3) - (2/3)k$, implementation of a downstream duopoly is not feasible. Thus, U implements an unconstrained downstream monopoly resulting in quantity $q^u_I = 1/8$. (ii) If $(1/6) - (7/12)k < \sqrt{F} \leq (1/3) - (2/3)k$, then $\Pi^u_{\{I,E\}} > \Pi^u_{\{I\}}$ if and only if $(1 - 2k - 3\sqrt{F})(k + 2\sqrt{F}) > 1/8$, or, equivalently, $(\sqrt{F})^2 - ((2 - 7k)/6)\sqrt{F} + (1 - 8k + 16k^2)/48 < 0$. For $k < 2 - \sqrt{3}$, this condition implies that $\Pi^u_{\{I,E\}} > \Pi^u_{\{I\}}$ if and only if $(1/6) - (7/12)k < \sqrt{F} < (1/6) - (7/12)k + (\sqrt{1 - 4k + k^2})/12$, whereas for $k \geq 2 - \sqrt{3}$ we always have $\Pi^u_{\{I,E\}} \leq \Pi^u_{\{I\}}$. Thus, U implements a downstream duopoly resulting in quantities $q^u_I = k + \sqrt{F}$ and $q^u_E = \sqrt{F}$ if $k < 2 - \sqrt{3}$ and $(1/6) - (7/12)k < \sqrt{F} < (1/6) - (7/12)k + (\sqrt{1 - 4k + k^2})/12$, and a downstream monopoly resulting in quantity $q^u_I = 1/8$ otherwise. (iii) If $(1/6) - (2/3)k < \sqrt{F} \leq (1/6) - (7/12)k$, where the latter inequality implies $k < 2/7$, then $\Pi^u_{\{I,E\}} > \Pi^u_{\{I\}}$ if and only if $(2 - k)^2/24 > 1/8$. This latter condition implies that $\Pi^u_{\{I,E\}} > \Pi^u_{\{I\}}$ if and only if $k < 2 - \sqrt{3}$. Thus, U implements a downstream duopoly resulting in quantities $q^u_I = (2 + 5k)/12$ and $q^u_E = (2 - 7k)/12$ if $k < 2 - \sqrt{3}$ and $(1/6) - (2/3)k < \sqrt{F} \leq (1/6) - (7/12)k$, and a downstream monopoly resulting in quantity $q^u_I = 1/8$ otherwise. (iv) If $\sqrt{F} \leq (1/6) - (2/3)k$, which implies $k \leq 1/4$, then $\Pi^u_{\{I,E\}} > \Pi^u_{\{I\}}$ if and only if $(2 - k)^2/24 > (1 - 2k - 3\sqrt{F})(k + (3/2)\sqrt{F})$, or, equivalently, $(\sqrt{F})^2 + ((4k - 1)/3)\sqrt{F} + (7k - 2)^2/108 > 0$. This latter inequality always holds for $k < 2 - \sqrt{3}$, and thus is always satisfied in the case under consideration. Thus,

U implements a downstream duopoly resulting in quantities $q_I^u = (2+5k)/12$ and $q_E^u = (2-7k)/12$. This establishes the desired result. ■

Proposition 4: *(i)* $W^d > W^u$ *if and only if*

- *(1.)* $k < 2-\sqrt{3}$ *and* $(1/6)-(7/12)k+(\sqrt{1-4k-k^2})/12 \leq \sqrt{F} < (1/3)-(8/9)k$*, or*
- *(2.)* $2-\sqrt{3} \leq k \leq 3/10$ *and* $\sqrt{F} < (1/3)-(8/9)k$*, or*
- *(3.)* $3/10 < k < 17/46$ *and* $\sqrt{F} < \sqrt{(23/72)k^2-(5/18)k+(17/288)}$*.*

(ii) $W^d < W^u$ *if and only if*

- *(1.)* $k < 2-\sqrt{3}$ *and* $\sqrt{F} < (1/6)-(7/12)k+(\sqrt{1-4k-k^2})/12$*, or*
- *(2.)* $\sqrt{F} > (1/3)-(8/9)k$ *for* $k < 3/10$ *or* $\sqrt{F} > \sqrt{(23/72)k^2-(5/18)k+(17/288)}$ *for* $k \geq 3/10$*, and* $\sqrt{F} < (1/3)-(2/3)k$*.*

(iii) If $\sqrt{F} \geq (1/3)-(2/3)k$*, then* $W^d = W^u$*.*

Proof:
First, note that for $k \in (0, 2-\sqrt{3}]$, $(1/6)-(7/12)k+(\sqrt{1-4k+k^2})/12 < (1/3)-(2/3)k$, $(1/6)-(7/12)k+(\sqrt{1-4k+k^2})/12 = (1/6)-(1/3)k$ if and only if $k = (\sqrt{3}-1)/4$, and $(1/6)-(7/12)k+(\sqrt{1-4k+k^2})/12 = (1/6)-(7/12)k$ if and only if $k = 2-\sqrt{3}$. These observations together with Lemmas 2 and 3 imply that there are five cases to consider that we labeled with Roman numerals in Figure 6.

(I) $k < 2-\sqrt{3}$ and $\sqrt{F} \leq (1/6)-(7/12)k$:
Under both pricing regimes, U implements an unconstrained downstream duopoly, resulting in the same aggregate output, $Q^d = Q^u = (2-k)/6$. Under price discrimination, however, the less efficient firm E produces a higher share of output, $q_E^d = (1-2k)/6 > (2-7k)/12 = q_E^u$. Thus, welfare is strictly lower under price discrimination than under uniform pricing, $W^d < W^u$.

(II) $k < 2-\sqrt{3}$, $(1/6)-(7/12)k < \sqrt{F} < (1/6)-(7/12)k+(\sqrt{1-4k+k^2})/12$, and $\sqrt{F} \leq (1/6)-(1/3)k$:
Under price discrimination, U implements an unconstrained duopoly resulting in quantities $q_I^d = (1+k)/6$, $q_E^d = (1-2k)/6$, and $Q^d = (2-k)/6$, whereas under uniform pricing, U implements a constrained duopoly, resulting in quantities $q_I^u = k+\sqrt{F}$, $q_E^u = \sqrt{F}$, and $Q^u = k+2\sqrt{F}$. $(1/6)-(7/12)k < \sqrt{F}$ implies that aggregate output is larger under uniform pricing than under price

discrimination, $Q^d < Q^u$. $\sqrt{F} \le (1/6) - (1/3)k$, on the other hand, implies, that the less efficient firm's output is (at least weakly) lower under uniform pricing than under price discrimination. Together, these observations imply that welfare under uniform pricing exceeds welfare under price discrimination, $W^d < W^u$.

(III) $k < (\sqrt{3}-1)/4$ and $(1/6)-(1/3)k < \sqrt{F} < (1/6)-(7/12)k+(\sqrt{1-4k+k^2})/12$: Under both pricing regimes, U implements a constrained duopoly. Under price discrimination, this results in in quantities $q_I^d = (1/4) - (1/2)\sqrt{F}$, $q_E^d = \sqrt{F}$, and $Q^d = (1/4) + (1/2)$. Under uniform pricing, the resulting quantities are $q_I^u = k + \sqrt{F}$, $q_E^u = \sqrt{F}$, and $Q^u = k + 2\sqrt{F}$. While the less efficient firm's output being identical under both pricing regimes, $q_E^d = q_E^u = \sqrt{F}$, $(1/6) - (1/3)k \le \sqrt{F}$ implies that aggregate output is higher under uniform pricing than under price discrimination, $Q^d < Q^u$. This, in turn, implies that welfare under uniform pricing exceeds welfare under price discrimination, $W^d < W^u$.

(IV) $(1/6)-(1/3)k < \sqrt{F} < (1/3)-(2/3)k$ and $(1/6)-(7/12)k+(\sqrt{1-4k+k^2})/12 \le \sqrt{F}$:
Under price discrimination, U implements a constrained downstream duopoly, resulting in quantities $q_I^d = (1/4) - (1/2)\sqrt{F}$, $q_E^d = \sqrt{F}$, and $Q^d = (1/4) + (1/2)\sqrt{F}$. Welfare under this pricing regime then is given by

$$W^d = \int_0^{Q^d} (1-x)dx - kq_E^d - F = \frac{7}{32} + \left(\frac{3}{8} - k\right)\sqrt{F} - \frac{9}{8}(\sqrt{F})^2. \quad \text{(B.5)}$$

Under uniform pricing, on the other hand, U implements an unconstrained downstream monopoly with I as the downstream monopoly firm, resulting in quantity $q_I^u = Q^u = 1/4$. Welfare under this pricing regime then is given by

$$W^u = \int_0^{Q^u} (1-x)dx = \frac{7}{32}. \quad \text{(B.6)}$$

With $F > 0$, $W^d > W^u$ if and only if $\sqrt{F} < (1/3) - (8/9)k$. Obviously, for all $k \in (0, 0.5)$ we have $(1/3) - (8/9)k < (1/3) - (2/3)k$. Moreover, for $k \in (0, 2-\sqrt{3}]$, $(1/3) - (8/9)k > (1/6) - (7/12)k + (\sqrt{1-4k+k^2})/12$. Last, note that $(1/3) - (8/9)k$ and $(1/6) - (1/3)k$ intersect at $k = 0.3$. Thus, $W^d > W^u$ if and only if $k < 0.3$ and $(1/6) - (1/3)k < \sqrt{F}$, $(1/6) - (7/12)k + (\sqrt{1-4k+k^2})/12 \le \sqrt{F}$, and $\sqrt{F} < (1/3) - (8/9)k$.

(V) $k \ge (\sqrt{3}-1)/4$, $\sqrt{F} \le (1/6) - (1/3)k$, and $\sqrt{F} \ge (1/6) - (7/12)k + (\sqrt{1-4k+k^2})/12$ for $k \in [(\sqrt{3}-1)/4, 2-\sqrt{3})$:

Under price discrimination, U implements an unconstrained duopoly resulting in quantities $q_I^d = (1+k)/6$, $q_E^d = (1-2k)/6$, and $Q^d = (2-k)/6$. Welfare under this pricing regime then is given by

$$W^d = \int_0^{Q^d} (1-x)dx - kq_E^d - F = \frac{20-20k+23k^2}{72} - F. \qquad \text{(B.7)}$$

Under uniform pricing, on the other hand, U implements an unconstrained downstream monopoly with I as the downstream monopoly firm, resulting in quantity $q_I^u = Q^u = 1/4$. Welfare under this pricing regime then is given by

$$W^u = \int_0^{Q^u} (1-x)dx = \frac{7}{32}. \qquad \text{(B.8)}$$

Thus, $W^d > W^u$ if and only if $F < (23/72)k^2 - (5/18)k + (17/288) =: F_W(k)$. Note that $F_W(k) > 0$ for $k < 17/46$ and $F_W(k) \leq 0$ for $k \in [17/46, 0.5]$. With $F_W(k) > 0$ for $k < 17/46$, it is readily verified that $d\sqrt{F_W(k)}/dk < 0$ for $k \leq 17/46$. Moreover, $\sqrt{F_W(k)} = (1/6) - (1/3)k$ if and only if $k = 0.3$. Thus, $W^d \leq W^u$ if and only if $k \geq 0.3$ and $\sqrt{F_W(k)} \leq \sqrt{F} \leq (1/6) - (1/3)k$.

Last, for $\sqrt{F} \geq (1/3) - (2/3)k$ U implements a downstream monopoly with I as the downstream monopoly firm under both pricing regimes, resulting in quantity $q_I^d = q_I^u = Q^d = Q^u = 1/4$. Thus, there is no difference in welfare under both pricing regimes, $W^d = W^u$. Combining these observations establishes the desired result. ■

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