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Are 18 holes enough for Tiger Woods?

by

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Are 18 holes enough for Tiger Woods?*

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Abstract

This paper addresses the selection problem in promotion tournaments. I consider a situation with heterogeneous employees and ask whether an employer might be interested in repeating a promotion tournament. On the one hand, this yields a reduction in uncertainty over the employees` abilities. On the other hand, there are costs if a workplace stays vacant.

Key words: Promotion tournament, selection, heterogeneous employees, repetition.

JEL classification: D82, M51

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1. Introduction

In practice, tournaments are very famous since they help to determine the most able competitor in a simple way and, therefore, to weaken problems due to informational asymmetries. Consider, for example, a company trying to fill a vacancy on a high hierarchy level but not knowing the abilities of the lower-level employees. Clearly, this company wishes to fill the vacancy with a rather able employee, since an unable employee might perform badly and, hence, might influence the company's profit in an unfavourable way. One possibility for this company is to arrange an inner-company promotion tournament.[1] Since an able employee is more likely to win the tournament than an unable employee, the asymmetry problem would be weakened. However, the tournament's outcome might be affected by luck or random components. So, the probability that an unable employee wins the tournament and the company promotes the "wrong" one is positive.

There is only little literature discussing this selection problem in tournaments.[2] For example, Meyer (1991) considers a series of promotion tournaments between two heterogeneous employees. She demonstrates that the problem of incorrect promotion decisions may be weakened by biasing the tournament's results. The bias (that in most cases favours the actual leader in the tournament) increases the tournament's information content. Yet, the less able employee still may be promoted. Clark and Riis (2001) show that the problem of incorrect promotion decisions can be solved by combining a promotion tournament with several test standards. In their model, there are three tournament prizes, and the tournament's winner receives the highest prize only if he additionally passes two tests. By using the test standards, the employer receives further information about the employees` abilities. With this information, the selection problem might be solved completely. In contrast, Hvide and Kristiansen (2003) emphasise the relevance of the selection problem. They examine a

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[1] In this work, I do not analyse whether a tournament is optimal in the class of all contracts. I assume that the company uses a promotion tournament to fill a vacancy because of its practicability and ask how to improve it.

[2] The literature on rank-order tournaments mostly focuses on the use of tournaments as incentive scheme. See, e.g., Lazear and Rosen (1981), Green and Stokey (1983), Nalebuff and Stiglitz (1983) or Rosen (1986).

promotion tournament in which the employees are able to choose strategies of different risk. In this case the selection problem is quite relevant, since a low-ability employee might choose a very risky strategy and so wins the tournament with a (strictly) positive probability.

This paper regards a different and more practical instrument to weaken the selection problem. This instrument is very successful in sports. Most tournaments in sports are characterised by the existence of repeating competition, i.e., players compete more than one time. In this case, the tournament`s winner is the player that has the highest success on average. For example, in golf[3], the competitors play 18 holes, and the winner is the player that needs the fewest shots to pocket the golf ball in all 18 holes.[4] Imagine an extreme situation in which the competitors play only one hole to identify the most able player. In this situation, the quality of the tournament's results is very doubtful. An able player might have unfavourable conditions (e.g. strong wind or rain) and, therefore, needs more shots than a less able competitor playing under good conditions. In the contrary extreme situation, the number of competitions between the golf players would be infinitely large. The law of large numbers then predicts that the tournament's winner is surely the most able player, so the selection problem would be solved. However, it is arguably impossible to golf an infinitely high number of holes.

In this paper, the repetition mechanism is transferred to an inner-company promotion tournament. An employer decides about the number of tournaments he arranges between two heterogeneous employees. On the one hand, it is costly for the employer to operate more than one tournament, since he wishes to fill a vacant workplace. On the other hand, extending the number of tournaments reduces uncertainty about the employees` abilities.

The remainder of the paper is organised as follows: Section 2 contains the description of the model. In section 3, the model is solved. In particular, it is shown under what circumstances

[3] Other examples may include sports like tennis, table-tennis, chess or cycling.

[4] The primary aim of repetition in sports is to entertain spectators for a certain period rather to solve the selection-problem. Yet clearly, repeating competition lessens this problem.

the employer is interested in repeating the tournament. Concluding remarks are offered in section 4.

2. Description of the model and notation

I consider a risk-neutral principal arranging a series of k tournaments between two heterogeneous and non risk-loving employees. Without loss of generality, employee 1 is the more able one with ability a_H and employee 2 is the less able one with ability a_L $(a_H > a_L)$. I define $a_H - a_L$ as Δa. A situation with asymmetric information is assumed such that each employee knows her own ability and the ability of her opponent, while the employer only knows that there is one able employee with ability a_H and one unable employee with ability a_L. Both employees might presently work in the same department of their company and, for this reason, are able to estimate the abilities of each other in a detailed way, while the employer naturally has less information about his employees` abilities. Each tournament lasts one period. The employee attaining the highest aggregate output will be promoted to a vacant workplace at a higher level in the company. This workplace is already vacant when the first tournament starts. Therefore, the net profit of the company from this workplace is zero in each period during the tournament. If the able employee is promoted, this profit will be $\Theta > 0$ in each period, otherwise it will be again zero.[5] The company and the two employees discount future utilities with $r \leq 1$. Their time horizon is infinite. Intuitively, r could be interpreted as the probability that employee i (i=1,2) continues to work for the company in the next period. Equation (1) describes the performance of employee i in the tournament in period j (j=1,…,k), y_{ij}. It is given by the sum of the effort e_{ij} he has chosen in this tournament, his ability a_i and a random noise ε_{ij}.

[5] It is assumed that an employee on the higher level receives a wage of w, while he receives a wage of zero on the lower level. At the beginning of the tournament, the high-level workplace is vacant and so, during the tournament, nothing is produced on the workplace and no wage is paid. Thus the profit is clearly zero. Furthermore, one could think that in case of promotion, a low-ability employee would produce output worth w and a high-ability employee would produce output worth $w + \Theta > w$ in each period.

(1) $y_{ij} = e_{ij} + a_i + \varepsilon_{ij}$.

It is assumed that these performances do not increase the company's profits. They are only useful as a signalling instrument.[6] The random components ε_{ij} are uncorrelated and follow a normal distribution with mean zero and variance σ^2. Using equation (1), the winning-probability of employee 1 in a k-period tournament is given by (2):

(2)
$$P_{1k} = \Pr ob\left\{\sum_{j=1}^{k} y_{1j} > \sum_{j=1}^{k} y_{2j}\right\} = \Pr ob\left\{\sum_{j=1}^{k} e_{1j} + \sum_{j=1}^{k} \varepsilon_{1j} + k \cdot a_H > \sum_{j=1}^{k} e_{2j} + \sum_{j=1}^{k} \varepsilon_{2j} + k \cdot a_L\right\}$$
$$= \Pr ob\left\{\sum_{j=1}^{k} \left(\varepsilon_{2j} - \varepsilon_{1j}\right) + k \cdot \left(a_L - a_H\right) < \sum_{j=1}^{k} \left(e_{1j} - e_{2j}\right)\right\} =: F_k\left(\sum_{j=1}^{k} \left(e_{1j} - e_{2j}\right)\right).$$

In this context, F_k stands for the distribution function of the composed random variable $\sum_{j=1}^{k} \left(\varepsilon_{2j} - \varepsilon_{1j}\right) + k \cdot \left(a_L - a_H\right)$, while f_k denotes the corresponding density function. Effort entails disutility for an employee which is given by $C\left(e_{ij}\right)$ with $C(0) = 0$, $C'\left(e_{ij}\right) > 0$ and $C''\left(e_{ij}\right) > 0$. In case of promotion, the promoted employee in each period receives an income w. It is further assumed that an employees` utility is additively separable in income and costs. Therefore, employee i (i=1,2) chooses his effort to maximise $EU_{ik} = P_{ik} \cdot U(w) \cdot \frac{r^k}{1-r} - \sum_{j=1}^{k} r^{j-1} \cdot C\left(e_{ij}\right)$ where $U(0) = 0$, $U'(w) > 0$ and $U''(w) \leq 0$. The employer determines the optimal number of tournaments to maximise $EU_{emp,k} = \frac{r^k}{1-r} \cdot P_{1k} \cdot \Theta$. Lastly, the employer does not announce intermediate results, i.e., in a given tournament, the employees do not know the results of the previous tournaments.

3. Solution to the model

[6] One could justify this assumption as follows: The profits that could be generated on the primary level are so low compared to the ones on the higher level that they are of (almost) no importance. Alternatively, as in Clark and Riis (2001), one could think of the model as a hiring process in which the company arranges a series of (valueless) tests in order to experience the abilities of the potential employees.

In a k-period tournament, employees 1 and 2 solve maximisation problems (3) and (4), respectively:

$$\text{(3)} \qquad \underset{e_{1j}}{\text{Max}}\ EU_{1k} = F_k\left(\sum_{j=1}^{k}\left(e_{1j}-e_{2j}\right)\right)\cdot U(w)\cdot\frac{r^k}{1-r}-\sum_{j=1}^{k} r^{j-1}\cdot C\left(e_{1j}\right),$$

$$\text{(4)} \qquad \underset{e_{2j}}{\text{Max}}\ EU_{2k} = \left(1-F_k\left(\sum_{j=1}^{k}\left(e_{1j}-e_{2j}\right)\right)\right)\cdot U(w)\cdot\frac{r^k}{1-r}-\sum_{j=1}^{k} r^{j-1}\cdot C\left(e_{2j}\right).$$

Using backwards induction, the employees` efforts in the last tournament have to be determined first. The first-order conditions for these efforts are given by (5) and (6):

$$\text{(5)} \qquad \frac{\partial EU_{1k}}{\partial e_{1k}} = f_k\left(\sum_{j=1}^{k}\left(e_{1j}-e_{2j}\right)\right)\cdot U(w)\cdot\frac{r}{1-r}-C'\left(e_{1k}\right)\overset{!}{=}0,$$

$$\text{(6)} \qquad \frac{\partial EU_{2k}}{\partial e_{2k}} = f_k\left(\sum_{j=1}^{k}\left(e_{1j}-e_{2j}\right)\right)\cdot U(w)\cdot\frac{r}{1-r}-C'\left(e_{2k}\right)\overset{!}{=}0.$$ [7]

From the first-order conditions, we see that both employees choose the same effort, thus we have $e_{1k} = e_{2k} =: e_k$. Analogously, continuing backwards induction up to the first tournament, we see that this symmetry holds in every tournament, so for all j=1,…,k we have $e_{1j} = e_{2j} =: e_j$. On that account, the winning probability of the high-ability employee 1 in a k-period tournament is $F_k(0) = \Phi\left(\frac{\sqrt{k}\cdot\Delta a}{\sqrt{2}\cdot\sigma}\right)$ where $\Phi(\cdot)$ denotes the distribution function of the standard normal distribution. It is then straightforward to derive proposition 1:

Proposition 1: The winning probability of the high-ability employee is strictly increasing in k.

Extending the number of tournaments from k to k+1 has two countervailing effects on the winning probability of employee 1. On the one hand, abstracting from random factors, the

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[7] As in Lazear and Rosen (1981), the second-order conditions will hold and so an equilibrium will exist if the variance σ^2 is sufficiently large. Intuitively, an equilibrium will only exist if luck plays a significant role. In what follows, the existence of an equilibrium is assumed.

difference between the two employees` performances increases from $k \cdot \Delta a$ to $(k+1) \cdot \Delta a$. Hence, employee 1 is more likely to be promoted. On the other hand, the influence of random factors increases, too. The variance of each individual's performance rises from $k \cdot \sigma^2$ to $(k+1) \cdot \sigma^2$, so employee 1 is less likely to be promoted. However, the first effect outbalances the second one, and so the winning probability of the able employee increases when the number of tournaments gets higher. For $k \to \infty$, $F_k(0)$ equals one. The selection problem would be completely eliminated by infinitely repeating the promotion tournament.

Extending the number of tournaments is advantageous for the employer, since an incorrect promotion decision becomes less likely. Yet, it is also disadvantageous. The employer looses potential payoffs, since the workplace stays vacant for a longer time. In order to clearly understand how the employer decides and how the model parameters influence his decision, we restrict his possible actions. Particularly, we assume that the employer has to decide between arranging m or m+1 tournaments, where m is an integer and positive number. In this case he prefers m+1 tournaments, if the condition $EU_{emp,m+1} > EU_{emp,m}$ holds. This condition is rewritten in (7):

$$(7) \qquad \Phi\left(\sqrt{\frac{m+1}{2}} \cdot \frac{\Delta a}{\sigma}\right) \cdot \frac{r^{m+1}}{1-r} \cdot \Theta > \Phi\left(\sqrt{\frac{m}{2}} \cdot \frac{\Delta a}{\sigma}\right) \cdot \frac{r^{m}}{1-r} \cdot \Theta$$

From (7), I derive proposition 2 where I define $y := \Delta a / \sigma$:[8]

Proposition 2. If the employer has the possibility to arrange m or m+1 promotion tournaments, there exists a cut-off $\tilde{r} \in \left(((m+1)/m)^{-0.5}, 1 \right)$ *such that the following will hold:*

- *(i)* *For* $r < \tilde{r}$, *the employer always arranges m tournaments.*
- *(ii)* *For* $r > \tilde{r}$, *there are two cut-offs* $\hat{y} > 0$ *and* $\tilde{y} > 0$ *with* $\hat{y} < \tilde{y}$ *such that the employer arranges m+1 tournaments only if* $y \in [\hat{y}, \tilde{y}]$.

[8] The proof of proposition 2 is placed in the Appendix.

In case of a small r, the employer assigns a high value to present payoffs but not to future payoffs. So he will never arrange a further tournament, since the profits in period m+1 are too valuable for him.

For r higher than $\tilde{r}$, it is also worthwhile for the employer to care for future payoffs. In this case, it might be beneficial to arrange more than m tournaments in order to reduce uncertainty about the employees` abilities. As stated in proposition 2, the employer's decision in the case $r > \tilde{r}$ depends on the ratio $\Delta a/\sigma$.

For a small Δa (or a large σ), one could think that the employer decides to arrange a further tournament. The employees are very similar in their abilities and, hence, it is quite likely that arranging only m tournaments yields an incorrect promotion decision. Surprisingly, the employer does not so. The reason is as follows: Even if a further tournament was arranged, employee 1 is only little more likely to be promoted than employee 2. Due to the small ability difference (or the large impact of random components) a tournament in period m+1 would not entail very much new information about the two employees. Therefore, the disadvantage of lost profits prevails, and the employer decides not to repeat the tournament. For intermediate values of Δa and σ, the argumentation is contrary. In this case the use of another tournament leads to much more information about the employees` abilities. Hence it is beneficial to repeat the promotion tournament to accumulate more data about the employees` abilities. Lastly, for a large Δa (or a small σ), the high-ability employee is very likely to be the leader after the first m tournaments. Arranging a further tournament would only yield little new information and, therefore, the employer again decides not do so.

Moreover, note that the interval in which the cut-off $\tilde{r}$ lies, gets smaller as m becomes higher. Obviously, the degree of new information when arranging one more tournament is higher in case of little prior tournaments than in case of many. Therefore, as m becomes higher, the employer is less likely to arrange a further tournament.

4. Conclusion

This paper addressed the selection problem in promotion tournaments. It was analysed, whether an employer might prefer to arrange a series of promotion tournaments instead of arranging only one tournament. Thereby, it was shown that extending the number of tournaments always leads to more detailed information about the employees' abilities. However, this information advantage may be outbalanced by vacancy costs that arise when the number of tournaments is increased.

Comparing the employer's expected utilities of arranging m or m+1 tournaments, offers further interesting results. When the employer is quite impatient, he always decides to arrange only m tournaments. When he is rather patient, his decision depends on the amount of new information another tournament entails. This new information depends non-monotonously on the ratio of the two agents' ability difference and the error term's standard deviation. For a very small or a very large ratio, the amount of new information is rather small, for intermediate values it is more significant. Hence, the tournament will only be repeated if the ratio adopts an intermediate value.

Appendix

In this appendix, proposition 2 is proved.

Transforming condition (7) and using $y := \Delta a/\sigma$ yields:

$$EU_{emp,m+1} > EU_{emp,m} \Leftrightarrow \Phi\left(\sqrt{\frac{m+1}{2}}\cdot y\right)\cdot r - \Phi\left(\sqrt{\frac{m}{2}}\cdot y\right) > 0.$$

In the extreme cases $y = 0$ and $y \to \infty$, this condition is not satisfied.

The derivative of the function $H(y) = \left(\Phi\left(\sqrt{\frac{m+1}{2}}\cdot y\right)\cdot r - \Phi\left(\sqrt{\frac{m}{2}}\cdot y\right)\right)$ with respect to y is:

$$\frac{\partial H(y)}{\partial y}=\left(\Phi'\left(\sqrt{\frac{m+1}{2}}\cdot y\right)\cdot\sqrt{\frac{m+1}{2}}\cdot r-\Phi'\left(\sqrt{\frac{m}{2}}\cdot y\right)\cdot\sqrt{\frac{m}{2}}\right).$$

Using $\Phi'(y)=\frac{1}{\sqrt{2\cdot\Pi}}\cdot e^{-0.5\cdot y^2}$, we get:

$$\frac{\partial H(y)}{\partial y}=\left(\frac{1}{\sqrt{2\cdot\Pi}}\cdot e^{-\frac{m+1}{4}\cdot(y)^2}\cdot\sqrt{\frac{m+1}{2}}\cdot r-\frac{1}{\sqrt{2\cdot\Pi}}\cdot e^{-\frac{m}{4}\cdot(y)^2}\cdot\sqrt{\frac{m}{2}}\right).$$ This derivative is positive if the following condition holds:

$$e^{-0.25\cdot(y)^2}\cdot\sqrt{m+1}\cdot r>\sqrt{m}\Leftrightarrow \ln\left(\sqrt{m+1/m}\cdot r\right)>0.25\cdot(y)^2\Leftrightarrow y<\left(\ln\left(\sqrt{m+1/m}\cdot r\right)\right)^{0.5}\cdot 2.$$

We see that $y^*=\left(\ln\left(\sqrt{(m+1)/m}\cdot r\right)\right)^{0.5}\cdot 2$ is the maximum of H. Inserting y^* into H yields:

$$H(y^*)=\left(\Phi\left(2\cdot\sqrt{\frac{m+1}{2}}\cdot\left(\ln\left(\sqrt{(m+1)/m}\cdot r\right)\right)^{0.5}\right)\cdot r-\Phi\left(2\cdot\sqrt{\frac{m}{2}}\cdot\left(\ln\left(\sqrt{(m+1)/m}\cdot r\right)\right)^{0.5}\right)\right).$$

This maximum is strictly negative for $r=((m+1)/m)^{-0.5}$, but strictly positive for r=1. The derivative of $H(y^*)$ with respect to r is given by:

$$\frac{\partial H(y^*)}{\partial r}=\Phi\left(2\cdot\sqrt{\frac{m+1}{2}}\cdot\left(\ln\left(\sqrt{(m+1)/m}\cdot r\right)\right)^{0.5}\right)$$
$$+\Phi'\left(2\cdot\sqrt{\frac{m+1}{2}}\cdot\left(\ln\left(\sqrt{(m+1)/m}\cdot r\right)\right)^{0.5}\right)\cdot\sqrt{\frac{m+1}{2}}\cdot\left(\ln\left(\sqrt{(m+1)/m}\cdot r\right)\right)^{-0.5}$$
$$-\Phi'\left(2\cdot\sqrt{\frac{m}{2}}\cdot\left(\ln\left(\sqrt{(m+1)/m}\cdot r\right)\right)^{0.5}\right)\cdot\sqrt{\frac{m}{2}}\cdot\ln\left(\sqrt{(m+1)/m}\cdot r\right)^{-0.5}\cdot\frac{1}{r}.$$

This derivative is positive if the difference between the second and the third term is non-negative, i.e., if the subsequent condition holds:

$$\Phi'\left(2\cdot\sqrt{\frac{m+1}{2}}\cdot\left(\ln\left(\sqrt{(m+1)/m}\cdot r\right)\right)^{0.5}\right)\cdot\sqrt{\frac{m+1}{2}}\cdot\left(\ln\left(\sqrt{(m+1)/m}\cdot r\right)\right)^{-0.5}$$
$$-\Phi'\left(2\cdot\sqrt{\frac{m}{2}}\cdot\left(\ln\left(\sqrt{(m+1)/m}\cdot r\right)\right)^{0.5}\right)\cdot\sqrt{\frac{m}{2}}\cdot\ln\left(\sqrt{(m+1)/m}\cdot r\right)^{-0.5}\cdot\frac{1}{r}\geq 0$$
$$\Leftrightarrow\Phi'\left(2\cdot\sqrt{\frac{m+1}{2}}\cdot\left(\ln\left(\sqrt{(m+1)/m}\cdot r\right)\right)^{0.5}\right)\cdot\sqrt{m+1}-\Phi'\left(2\cdot\sqrt{\frac{m}{2}}\cdot\left(\ln\left(\sqrt{(m+1)/m}\cdot r\right)\right)^{0.5}\right)\cdot\sqrt{m}\cdot\frac{1}{r}\geq 0$$

$$\Leftrightarrow \frac{1}{\sqrt{2\cdot\Pi}}\cdot e^{-0.5\cdot\left(2\cdot\sqrt{\frac{m+1}{2}}\cdot\left(\ln\left(\sqrt{(m+1)/m}\cdot r\right)\right)^{0.5}\right)^2}\cdot\sqrt{m+1}\cdot r-\frac{1}{\sqrt{2\cdot\Pi}}\cdot e^{-0.5\cdot\left(2\cdot\sqrt{\frac{m}{2}}\cdot\left(\ln\left(\sqrt{(m+1)/m}\cdot r\right)\right)^{0.5}\right)^2}\cdot\sqrt{m}\geq 0$$

$$\Leftrightarrow e^{-0.5\cdot\left(2\cdot\sqrt{\frac{1}{2}}\cdot\left(\ln\left(\sqrt{(m+1)/m}\cdot r\right)\right)^{0.5}\right)^2}\cdot\sqrt{m+1}\cdot r\geq\sqrt{m}$$

$$\Leftrightarrow \sqrt{(m+1)/m}\cdot r\geq\sqrt{(m+1)/m}\cdot r.$$

Hence, we have shown that $\frac{\partial H(y^*)}{\partial r}$ is positive. Since $H(y^*)$ is positive for r=1, there must be some cut-off value $\tilde{r}$, at which $H(y^*)$ becomes positive. Hence, proposition 2 is proved.

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