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Aggregation under structural stability: the change in consumption of a heterogeneous population

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Abstract

It is shown how one can effectively use cross-section data in modelling the change over time in aggregate consumption expenditure of a heterogeneous population. The starting point of our aggregation analysis is a dynamic behavioral relation on the household level. Based on certain hypotheses on the evolution of the distribution of income and household characteristics we derive explanatory variables for the change in aggregate consumption expenditure which are quite different from the explanatory variables on the household level.

It is shown that U.K. Family Expenditure Data support our theoretical model for aggregate consumption.

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Introduction

It is our goal to derive *observable* explanatory variables for the change over time in aggregate consumption expenditure. This derivation should have a firm theoretical foundation, in particular, be consistent with standard microeconomic theory of consumer behavior, be compatible with the available historical time-series data, and be a basis for building forecasting models. This subject has been treated extensively in the literature; for recent surveys see Attanasio (1999) or Muellbauer and Lattimore (1999). This literature greatly improved the understanding of consumption and saving behavior. Roughly speaking there are two approaches: the statistical time series approach (e.g. Davidson et al. (1978)) and the representative consumer approach (e.g. Deaton (1992)).

Our approach is different since we take explicitly into account that aggregate consumption expenditure is derived from a *heterogeneous* population of households. The importance of aggregation in dealing with a heterogeneous population is also emphasized in Blundell and Stoker (2000). In modelling aggregate consumption expenditure we want to make use of the rich information that is contained in (time-series of) cross-section data. Therefore, we must start the economic modelling process from a heterogeneous population of households. Indeed, empirical cross-section data (e.g. UK - Family Expenditure Survey) clearly show that real populations of households are extremely heterogeneous in income, consumption expenditure, and household attributes such as age, household size etc. Eventhough our final goal is to obtain a "simple" macro-relation or an empirical model for the change in aggregate consumption expenditure in terms of aggregate explanatory variables, we cannot *start* with the fiction of a representative household. The desired simple macro-relation cannot be postulated at the outset but must be derived through the aggregation process over a large and heterogeneous population. Possibly, then it might turn out *as a result* that the fiction of a representative household is an acceptable approximation in particular circumstances.

1 Modelling methodology and main results

As in the literature (e.g. Deaton (1992)) our starting point is a model of consumption behavior on the micro-level which specifies the nature of the explanatory variables for current real consumption expenditure c_t^h on the household level. If the decision on real consumption expenditure of a household is modelled by intertemporal utility maximization under uncertainty, then the parameters that define this stochastic maximization problem are the explanatory variables for consumption expenditure on the household level. In the simplest case - expected intertemporal utility maximization under the life cycle budget constraint with non-stochastic price- and interest rates expectations - one obtains, as shown in section 2, a behavioral relation on the household level of the form

$$c_t^h = c(y_t^h, W_t^h, r_t^h, \lambda_t^h, u_t^h) \tag{1}$$

where y_t^h and W_t^h denote non-property income and wealth (assets) including property income of household h in period t, respectively. Furthermore, $r_t^h := r_t^h(\tau), \tau = t+1, \ldots$ denotes the future real interest rate *as anticipated by household h in period t* and λ_t^h denotes a vector of parameters which determine the stochastic process $\left(y_t^h(\tau)\right)_{\tau=t+1,\ldots}$ of future non-property real income in period $\tau > t$ *as anticipated by household h in period t.* Finally, u_t^h denotes the intertemporal utility function. The function c in (1) is household independent and time-invariant. For details see section 2.

Mean consumption expenditure C_t across the population H_t of households in period t is defined by $C_t = \frac{1}{\# H_t} \sum_{h \in H_t} c_t^h$. With the notation $v_t^h := (W_t^h, r_t^h, \lambda_t^h, u_t^h)$ one obtains from (1)

$$\begin{aligned} C_t &= \frac{1}{\# H_t} \sum_{h \in H_t} c(y_t^h, v_t^h) \\ &= \int c(y, v) \text{distr}(y_t^h, v_t^h \mid H_t) \end{aligned} \tag{2}$$

where $\text{distr}(y_t^h, v_t^h \mid H_t)$ denotes the joint distribution across H_t of the household-specific explanatory variables (y_t^h, v_t^h). For a finite population H_t the integral with respect to the distribution $\text{distr}(y_t^h, v_t^h \mid H_t)$ in (2) is, of course, just a finite sum. The notation in (2), however, will be convenient and simplifies the presentation.

The change over time in mean consumption expenditure $C_t = \int c(y,v)\text{distr}(y_t^h, v_t^h \mid H_t)$ is caused (explained) by the change over time in the distribution $\text{distr}(y_t^h, v_t^h \mid H_t)$. Consequently, we have to model the evolution over time of this distribution.

Let $a = (a_1, a_2, \ldots)$ denote a profile of *observable* household attributes such as age or employment status of household head or household size and let $H_t(y, a)$ denote the subpopulation of all households in H_t with current non-property income y and attribute profile a. Then we decompose $\text{distr}(y_t^h, v_t^h \mid H_t)$ into the (conditional) distribution $\text{distr}(v_t^h \mid H_t(y, a))$ of the variables v_t^h across the subpopulation $H_t(y, a)$ and the marginal distribution $\text{distr}(y_t^h, a_t^h \mid H_t)$ of current income and attributes across H_t, which, in turn, is decomposed into the distribution $\text{distr}(y_t^h \mid H_t)$ of current non-property income across H_t and the (conditional) distribution $\text{distr}(a_t^h \mid H_t(y))$ of household attributes across the subpopulation $H_t(y)$. By Hypothesis

1 (Structural Stability) and Hypotheses 2 and 3 of section 3 we model the change over time in $\text{distr}(v_t^h \mid H_t(y,a))$, $\text{distr}(y_t^h \mid H_t)$, and $\text{distr}(a_t^h \mid H_t(y))$, respectively.

The general principle which underlies these hypotheses can easily be explained. Let z_t^h be a household specific explanatory variable (for example, W_t^h in Hypothesis 1 or $\log y_t^h$ in Hypothesis 2) whose distribution across H_t (or across a subpopulation) changes over time. If one considers instead of z_t^h the "standardized" variable

$$\tilde{z}_t^h := \frac{z_t^h - \text{mean}_{H_t}(z_t^h)}{\left(\text{var}_{H_t}(z_t^h)\right)^{\frac{1}{2}}}$$

then the change over time in the standardized distribution $\text{distr}(\tilde{z}_t^h \mid H_t)$ is certainly much smaller than the change in the original distribution $\text{distr}(z_t^h \mid H_t)$, since the standardized distributions all have the same mean (equal to zero) and variance (equal to one).

Our *basic assumption* now is that standardized distributions change very slowly over time in the sense that for two periods s and t that are close to each other the standardized distributions can be considered as equal (local time-invariance). In the case of an *observable* explanatory variable (e.g. current income) our basic assumption is unproblematic since the standardized distributions can be estimated and thus the assumption can be tested. However, in Hypothesis 1 (Structural Stability), we shall use our basic assumption also in the case of expectational variables (i.e., r_t^h and λ_t^h) which are unobservable.

Hypotheses 1-3 allow us to derive a first order approximation for the relative change in mean consumption expenditure (see the Proposition in section 4). Neglecting terms of second order, the relative change $(C_t - C_{t-1})/C_{t-1}$ is equal to the sum of four effects: *the effect of (i) the changing distribution of current non-property income, (ii) changing wealth, (iii) changing anticipated interest rates, and (iv) changing anticipations about future non-property income.* These four effects are defined in section 4 and their empirical relevance is discussed in section 5.

(i) The effect of the changing distribution of current non-property income (which is observable and explains a large part of the observable change $(C_t - C_{t-1})/C_{t-1}$, see section 5, Table 1) is given by

$$\beta_{t-1}(m_t - m_{t-1}) + \gamma_{t-1}\left(\frac{\sigma_t - \sigma_{t-1}}{\sigma_{t-1}}\right) \tag{3}$$

where m_t and σ_t^2 denote the mean and variance of log-income across H_t.

The important point here is that the coefficients β_{t-1} and γ_{t-1}, which are defined in section 4, only depend on the actual consumption decisions of the households in period $t-1$ and hence, can be estimated *separately* from cross-section data in *every* period . Estimates are shown in section 5, Figure 1. The average values of β_t and γ_t from 1968 to 1993 in the case of nondurable consumption are 0.56 and 0.24, respectively.

We remark that β_t and γ_t can be interpreted as elesticities of mean consumption expenditure with respect to mean income $Y_t = \text{mean}_{H_t} y_t^h$ and income dispersion σ_t, respectively (see section 4).

Instead of following our modelling methodology one might be tempted to take a short-cut and model mean consumption expenditure by the fiction of a "representative" household, that is to say, in the behavioral relation (1) one substitutes so called "representative variables" for the household specific explanatory variables. For example, y_t^h and W_t^h are replaced by their mean value across the population H_t, denoted by Y_t and W_t, respectively. Thus, one stipulates that

$$C_t = c(Y_t, W_t, r_t^*, \lambda_t^*, u^*).$$

The definition of the "representative variables" r_t^*, λ_t^* and u^* (assumed to be time-invariant) is less straightforward and somewhat arbitrary. For example, it is often assumed in the literature that the representative household considers current income as a realisation of an *autonomous stochastic process* $(\mathcal{Y}_t)$; an assumption which seems hardly acceptable on the household-level The uncertain non-property income $Y_t(\tau)$ of the future period $\tau > t$ then is defined as a *prediction* of $\mathcal{Y}_\tau$ which is based on the information $\Im_t$ that is available up to period t, for example, $\mathbb{E}(Y_t(\tau)) = \mathbb{E}(\mathcal{Y}_\tau \mid \Im_t)$. If the imformation $\Im_t$ consists of observed present and past income, i.e., $\Im_t = \sigma(\mathcal{Y}_t, \mathcal{Y}_{t-1}, \dots)$, then it follows that $\mathbb{E}(Y_t(\tau))$ is a function (determined by the law of stochastic process $(\mathcal{Y}_t)$) of current and past income, $Y_t, Y_{t-1}, \dots$

A first order approximation then leads to a decomposition of $(C_t - C_{t-1})/C_{t-1}$ which is analogous to our Proposition in section 4. The effect (i) of changing current non-property income then becomes

$$\alpha_{t-1} \log \frac{Y_t}{Y_{t-1}} \tag{4}$$

The coefficient α_{t-1} depends on the partial derivative $\partial_Y c(Y_{t-1}, \dots)$, which is unknown. Consequently, α_{t-1} has to be estimated from time-series data.

An interesting question now arises: under what circumstances is (4) a good approximation of (3)? One can show that Hypothesis 2 implies (neglecting a second order term in $\left(\log \frac{\sigma_t}{\sigma_{t-1}}\right)^2$)

$$\beta_{t-1}(m_t - m_{t-1}) + \gamma_{t-1}\left(\frac{\sigma_t - \sigma_{t-1}}{\sigma_{t-1}}\right) = \beta_{t-1} \log \frac{Y_t}{Y_{t-1}} + \bar{\gamma}_{t-1} \log \frac{\sigma_t}{\sigma_{t-1}} \tag{5}$$

with $\bar{\gamma}_{t-1} := \gamma_{t-1} - \frac{\beta_{t-1}}{Y_{t-1}} \int y(\log y - m_{t-1}) \mathrm{distr}(y \mid H_{t-1})$.

It is an empirical question whether the coefficient $\bar{\gamma}_{t-1}$ is small; it depends on the coefficients β and γ and on the distribution of current income. We remark that β and γ also depend on the level of aggregation in the definition of "consumption expenditure", for example, expenditure on all nondurable goods or food (see Chakrabarty and Schmalenbach (2001)).

Estimates of $\bar{\gamma}_t$ based on U.K. Family Expenditure Survey data from 1968 to 1993, which are discussed in section 5, turn out to be insignificantly different from zero. Thus, *for this particular data set*, the effect of the changing distribution of current non-property income, indeed, can be modelled by $\beta_{t-1} \log \frac{Y_t}{Y_{t-1}}$. However, it is important to emphasize, that in our approach the coefficients β_t, γ_t and $\bar{\gamma}_t$ can be estimated separately in every period from cross-section data while in the representative agent approach the coefficient α in (4) has to be estimated from time-series. For interpreting the coefficients, we remark that by cross-section estimation we avoid all problems of collinearity of Y_t, m_t or σ_t with other explanatory variables. Thus, we obtain a "pure" estimate of the influence of these terms.

(ii) Next, we consider the effect of changing wealth. For cross-section data which include also households' wealth W_t^h one should treat wealth analogously to non-property current income. The effect of the changing distribution of wealth then can be estimated from cross-section data as in the above case of effect (i). However, the UK - FES does not contain information on households' wealth (assets), yet it contains information on households' property income. Therefore we treated wealth (including property income) as an (unobservable) explanatory variable which is covered by the Hypothesis of structural stability. The effect of changing wealth in the Proposition of section 4 then is given by

$$\sum_a \eta_{t-1}(a) \left(\frac{W_t^a - W_{t-1}^a}{W_{t-1}^a} \right) \frac{\#H_{t-1}(a)}{\#H_{t-1}}$$

where W_t^a denotes mean wealth across the subpopulation $H_t(a)$. Given our data-set this effect is unobservable. Since we are looking for observable explanatory variables for the effect of changing wealth we have to find an observable proxy for $\eta_{t-1}(a) \left(\frac{W_t^a - W_{t-1}^a}{W_{t-1}^a} \right)$. We shall show in section 4 that

$$\alpha_1 \frac{X_t^a + Y_{t-1}^a - \bar{C}_{t-1}^a}{(\pi_t / \pi_{t-1}) X_t^a} + \alpha_2 \frac{\pi_t - \pi_{t-1}}{\pi_{t-1}}$$

can serve as an observable proxy, where $X_t^a, Y_t^a, \bar{C}_t^a$ denote the mean (across the subpopulation $H_t(a)$) of property income, non-property income, and total consumption expenditure, respectively, and π_t denotes a price index for period t.

(iii) The effect of changing anticipated real interest rates is given by $\delta_{t-1}(\bar{r}_t - \bar{r}_{t-1})$, where $\bar{r}_t$ denotes the mean across the population H_t of the future real interest rate $r_t^h := r_t^h(\tau)$, $\tau = t+1, \ldots$ as anticipated by household h in period t. On what kind of information does a household base its anticipation of $r_t^h(\tau)$? One might expect that the actual current and past interest rates $r_t, r_{t-1}, \ldots$ play a role, yet also quite different information (up to the present period t) might influence these anticipations, for example, information on economic policy. Obviously, modelling anticipations is delicate and in any case highly speculative.

If one wants to derive from our *theoretical* model, i.e., the approximation of $(C_t - C_{t-1})/C_{t-1}$ in the Proposition of section 4, an *empirical* model, then one has to replace the unobservable effect $\delta_{t-1}(\bar{r}_t - \bar{r}_{t-1})$ by a term which is observable and, furthermore, one should have good reasons to believe that at least the time paths of the observable terms and the unobservable effects are correlated. In section 5 we shall choose (without justification other than simplicity) the term $\delta(r_t - r_{t-1})$, which would be fully justified if, on average, households anticipate future real interest rates to be equal to the current one.

(iv) It remains to discuss the effect of changing anticipations of future non-property income.

Here we encounter again - even in a more pronounced form - the difficulties in modelling anticipations that we discussed already in the case of effect (iii).

The effect (iv) is caused by a possible change in the stochastic process $\left(y_t^h(\tau)\right)_{\tau=t+1,\dots}$ of future non-property income as anticipated by household h in period t, which is modelled in the Proposition by a change in the parameter λ_t (see section 2). The anticipated stochastic process $(y_t^h(\tau))_\tau$ determines in particular the uncertainty in life cycle income as anticipated by household h in period t. It is often argued in the literature that this uncertainty is the main motive for saving and hence influences the level of current consumptin expenditure. Consequently, a change over time of the anticipated uncertainty in life cycle income will contribute to effect (iv). We shall discuss in section 5 how uncertainty in future income may be modelled.

This paper is organized as follows: in section 2 we present the derivation of the behavioral relation (1). In section 3 we model the evolution over time of the distribution of household specific variables. Section 4 contains the main result, a first order approximation for the relative change in mean consumption expenditure. Section 5 presents an empirical analysis based on our theoretical model and the UK - Family Expenditure Survey.

2 A behavioral relation on the household level

As explained in section 1 our starting point is a model of consumption behavior on the micro-level which specifies the nature of the explanatory variables for current real consumption expenditure c_t^h. If the decision on real consumption expenditure of a household is modelled by *intertemporal utility maximization under uncertainty*, then the parameters that define this maximization problem are the explanatory variables for consumption on the household level. There are many versions of this model according to the various specifications of the time horizon and uncertainty, the form of the intertemporal utility function, the formation of expectations on future prices, interest rates, and labor income, and the modelling of the budget constraints. The simplest one is the model of expected intertemporal utility maximization under the life cycle budget constraint (no credit restrictions) with non-stochastic price- and interest rate expectations.

In this case one obtains (for details see e.g Blanchard and Fischer (1989), chapter 6) a behavioral relation on the household level of the following form

$$c_t^h = f(r_t^h(t+1), r_{t+1}^h(t+2), \dots, D\xi_t^h, u_t^h) \tag{1}$$

- $r_t^h(\tau)$ denotes the expected *real* interest rate for the future period τ *as anticipated by household h in period t*. To simplify the notation we shall assume that $r_t^h := r_t^h(\tau)$, $\tau > t$.
- $D\xi_t^h$ denotes the probability distribution of the uncertain present value of life cycle financial and human resources in real terms *as anticipated by household h in period t*;

$$\xi_t^h = A_t^h + x_t^h + y_t^h + \mathcal{L}_t^h$$

where A_t^h denotes real wealth (assets) of household h at the beginning of period t, x_t^h and y_t^h denote property and non-property real income of household h in period t and $\mathcal{L}_t^h$ denotes the present value of the uncertain future non-property real income *as anticipated by household h in period* t, i.e.,

$$\mathcal{L}_t^h = \frac{1}{1+r_t^h}\; y_t^h(t+1) + \frac{1}{(1+r_t^h)^2}\; y_t^h(t+2) + \cdots$$

where $y_t^h(\tau)$ denotes the uncertain real non-property income in the future period $\tau > t$ *as anticipated by household h in period t.*

- $u_t^h(c_t, c_{t+1}, \ldots)$ denotes the intertemporal (von Neumann-Morgenstern) utility function.

The time-invariant and household-independent function f in (1) is determined by the maximization problem which is completely defined in terms of the explanatory variables $r_t^h, D\xi_t^h$ and u_t^h.

Naturally, if one wants an explicit solution of the intertemporal maximization problem, that is to say, the functional form of f, then one has to make strong assumptions on the intertemporal utility function u_t^h. For details see Blanchard and Fischer (1989), chapter 6. For example, if one considers only one future period $t+1$, if the utility function is of the separable form

$$u_t(c_t, c_{t+1}) = v_t(c_t) + \beta_t v_t(c_{t+1}) \quad \text{with} \quad \beta_t(1+r_t) = 1$$

and if v_t is a concave and quadratic function, then

$$c_t = f(r_t, D\xi_t, u_t) = \frac{1+r_t}{2+r_t} \cdot \mathbb{E}\xi_t.$$

In the next period $t+1$ households make new decisions on real consumption expenditures for period $t+1$. These decisions are based on new anticipations $r_{t+1}^h(\tau)$ and $y_{t+1}^h(\tau), \tau > t+1$, about future interest rates and non-property income, respectively. Household h, who made the consumption decision c_t^h in period t, starts period $t+1$ with real wealth

$$A_{t+1}^h = \left[A_t^h + x_t^h + y_t^h - c_t^h\right] \cdot \frac{\pi_t}{\pi_{t+1}}.$$

The intertemporal utility function u_{t+1}^h might, but need not, be equal to u_t^h. For example, if u_t^h is the intertemporal utility function over the life cycle of household h then

$$u_{t+1}^h(c_{t+1}, c_{t+2}, \ldots) = u_t^h(c_t^h, c_{t+1}, c_{t+2}, \ldots).$$

We emphasize that, in general, the probability distribution $D\xi_t^h$ of the uncertain present value of life cycle resources will matter in relation (1), not just its mathematical expectation. However, to simplify the analysis one might assume that in relation (1), the probability distribution of ξ_t^h can be replaced (at least as an approximation) by some relevant characteristics of this distribution. For example, one might simply choose the mathematical expectation and the variance of $\mathcal{L}_t^h$. However, as it will become clear later, it is preferable to model the stochastic process $\left(y_t^h(\tau)\right)_{\tau=t+1,\ldots}$ of future non-property income *as anticipated by household h in period t.* The stochastic law of this process, together with the expected interest rates, then determines the probability distribution of $\mathcal{L}_t^h$.

In order to model the anticipated stochastic income process $(y_t^h(\tau))_{\tau=t+1,\ldots}$ we assume that there is a stochastic process $(Z_t^h(\tau))_{\tau=t\pm1,\ldots}$ (defined on some probability space $(\Omega, \Im, P)$) such that

$$y_t^h(\tau) = y_t^h(\tau-1)\cdot Z_t^h(\tau), \quad \tau > t \quad \text{with } \; y_t^h(t) := y_t^h$$

Thus, the present value of the uncertain life cycle non-property income $\mathcal{L}_t^h$ of household h with the certain non-property income y_t^h in period t is given by

$$\mathcal{L}_t^h = y_t^h \cdot \frac{Z_t^h(t+1)}{1+r_t^h} + y_t^h \cdot \frac{Z_t^h(t+1)\cdot Z_t(t+2)}{(1+r_t^h)^2} + \cdots = y_t^h \cdot \left(\sum_{\tau>t} \frac{\Pi_{s>t}^{\tau} Z_t^h(s)}{(1+r_t^h)^{\tau-t}} \right)$$

Consequently, the probability distribution of $\mathcal{L}_t^h$ is completely determined by the law of the stochastic process $(Z_t^h(\tau))_\tau$, r_t^h, and y_t^h.

Instead of the stochastic process $(Z_t^h(\tau))_\tau$ we might equivalently consider the stochastic process $(z_t^h(\tau))_\tau$ where

$$z_t^h(\tau) := \log Z_t^h(\tau).$$

Then, $z_t^h(\tau) = \log y_t^h(\tau) - \log y_t^h(\tau-1) =: \Delta \log y_t^h(\tau)$, $\tau > t$, can be interpreted as the stochastic growth rate of anticipated non-property income in the future period τ. We assume that the stochastic process $(z_t^h(\tau))_\tau$ is *mean and covariance stationary.* The most important characteristics then are:

$$\text{mean: } \mathbb{E}(z_t^h(\tau)) =: e_t^h, \;\text{ variance: } var(z_t^h(\tau)) =: v_t^h,$$
$$\text{correlation: } corr(z_t^h(\tau), z_t^h(s))$$

For simplicity we shall assume that the stochastic process $(z_t^h(\tau))_\tau$ can be completely parametrized by e_t^h, v_t^h and the correlation with respect to just the first lag $k_t^h := corr(z_t^h(\tau), z_t^h(\tau+1))$. Thus, the parameters e_t^h, v_t^h, and k_t^h determine the stochastic process $(z_t^h(\tau))_\tau$ and hence, together with the interest rates r_t^h and current non-property income y_t^h, they determine the probability distribution of $\mathcal{L}_t^h$. The expectational parameters e_t^h, v_t^h, and k_t^h are household specific and unobservable. Most likely, they will depend on the evolution of past non-property income, yet also on other information which might influence the anticipation of future non-property income. For example, the announcement of a certain policy measure might have an effect on these parameters. We emphasize that we shall not assume that the parameters e_t^h, v_t^h, and k_t^h are time-invariant. This assumption would be subject to the "Lucas-Critique".

Note that the two parameters v_t^h and k_t^h determine the degree of uncertainty of the life cycle income $\mathcal{L}_t^h$. The larger the variance v_t^h and the smaller the correlation k_t^h the more uncertain is $\mathcal{L}_t^h$. As an example for the stochastic process $(z(\tau))_\tau$ one might define

$$z_t^h(\tau) = [\bar{z}_t] + [\epsilon_\tau^h + \theta_t \epsilon_{\tau-1}^h], \quad \tau > t$$

where $\bar{z}_t$ and θ_t are parameters and ϵ_τ^h denotes a white noise stochastic process with variance $\sigma_t^2(h)$. Processes for anticipated labor income of this type are considered in the literature, for example, Pischke (1995), Blundell and Stoker (2000). For the parameters e_t^h, v_t^h, and k_t^h one obtains $e_t^h = \bar{z}_t, v_t^h = (1+\theta_t^2)\sigma_t^2(h)$ and $k_t^h = \theta_t^2 \sigma_t^2(h)/v_t^h$. The example can be reparametrized in terms of e_t^h, v_t^h and k_t^h. We remark that we do not consider the evolution over time of the actual non-property income y_t^h as a realisation of an autonomous stochastic process. The stochastic processes $(z_t^h(\tau))_\tau$ and $(Z_t^h(\tau))_\tau$ refer to anticipated future non-property income.

Our discussion up to now motivates and justifies that the starting point for our aggregation analysis is the following behavioral relation on the household level

$$c_t^h = c(y_t^h, W_t^h, r_t^h, \lambda_t^h, u_t^h) \tag{2}$$

where $W_t^h = A_t^h + x_t^h$ and λ_t^h denotes either the parameters (e_t^h, v_t^h, k_t^h) of the stochastic process $(z_t^h(\tau))_\tau$ or, less explicitely, the mathematical expectation and variance of $\mathcal{L}_t^h / y_t^h = \left(\sum_{\tau > t} \frac{\Pi_{s>t}^{\tau} Z_t^h(s)}{(1+r_t^h)^{\tau - t}} \right)$.

Remark: One might object to call (2) a "model" of household behavior since one knows nothing about the functional form of the (time-invariant and household-independent) function c. Indeed, relation (2) just says that

$$y_t^h, W_t^h, r_t^h, \lambda_t^h, u_t^h$$

is a complete set of explanatory variables for real consumption expenditure on the household level – provided one accepts the hypothesis of expected intertemporal utility maximization under the life cycle budget constraint with non-stochastic price- and interest rate expectations! We shall show in the sequel that our approach to model the change of aggregate consumption expenditure does not require the knowledge of the functional form of the relation c which links the explanatory variables with the explicandum c_t^h.

Our derivation of explanatory variables for household consumption expenditure c_t^h was based on the paradigm of intertemporal utility maximization, since this is the most commonly used model in the literature. However, any other model (or story) of household behavior which leads to a behavioral relation on the household level could be the starting point for our aggregation analysis, even if the nature of the explanatory variables are quite different from those in (2).

Given relation (2), *mean consumption expenditure* C_t across the population H_t of households in period t is defined by

$$C_t = \int c(y, W, r, \lambda, u) \operatorname{distr}(y_t^h, W_t^h, r_t^h, \lambda_t^h, u_t^h \mid H_t)$$

Consequently, the evolution over time of the distribution of the household specific explanatory variables across the population H_t determines the evolution over time of C_t.

3 The evolution of the distribution of household specific explanatory variables

As explained in section 1 we decompose the joint distribution of all explanatory variables $y_t^h, W_t^h, r_t^h, \lambda_t^h, u_t^h$ across the population H_t into three distributions: the distribution $\mathrm{distr}(v_t^h \mid H_t(y,a))$ of the explanatory variables $(W_t^h, r_t^h, \lambda_t^h, u_t^h) =: v_t$ across the subpopulation $H_t(y,a)$, the distribution $\mathrm{distr}(y_t^h \mid H_t)$ of current non-property income across the population H_t, and the distribution $\mathrm{distr}(a_t^h \mid H_t(y))$ of household attributes across the subpopulation $H_t(y)$. In this section we shall model the change over time in these distributions.

3.1

In this subsection we shall formulate assumptions that restrict the change over time in $\mathrm{distr}(v_t^h \mid H_t(y,a))$. To achieve this goal we shall consider a transformation of the household-specific variables $W_t^h, r_t^h, \lambda_t^h$ and then postulate that the image of the distribution $\mathrm{distr}(v_t^h \mid H_t(y,a))$ with respect to this transformation is time-invariant. This then leads to our basic Hypothesis 1, called structural stability.

The simplest transformation for our purpose is "scaling": if z_t^h denotes a household-specific variable, we define $\tilde{z}_t^h := z_t^h / \bar{z}_t(y,a)$ where $\bar{z}_t(y,a)$ is the mean of z_t^h across $H_t(y,a)$. More generally, one defines "standardizing": $\tilde{z}_t^h := (z_t^h - \bar{z}_t(y,a))/\sigma_t^z(y,a)$, where $(\sigma_t^z(y,a))^2$ is the variance of z_t^h across $H_t(y,a)$. The purpose of "scaling" and "standardizing" is clear. The two distributions $\mathrm{distr}(z_s^h \mid H_s(y,a))$ and $\mathrm{distr}(z_t^h \mid H_t(y,a))$ might be quite different, yet certainly the distributions $\mathrm{distr}(\tilde{z}_s^h \mid H_s(y,a))$ and $\mathrm{distr}(\tilde{z}_t^h \mid H_t(y,a))$ will be less different since, in the case of "scaling", they have the same mean and, in the case of "standardizing", they have the same mean and variance.

With this notation for "scaling" or "standardizing" we can formulate our basic

Hypothesis 1: (Structural Stability[1])

For any real income level y and attribute profile a, the joint distribution of the household-specific variables $\tilde{W}_t^h, \tilde{r}_t^h, \tilde{\lambda}_t^h$ and u_t^h across the subpopulation $H_t(y,a)$, i.e.,

$$\mathrm{distr}\left(\tilde{W}_t^h, \tilde{r}_t^h, \tilde{\lambda}_t^h, u_t^h \mid H_t(y,a)\right)$$

changes sufficiently slowly over time, in the sense that, for two periods s and t that are close to each other, the distributions can be considered as identical (local time-invariance).

The following discussion may be helpful for understanding the content of Hypothesis 1.

1. Structural stability requires, in particular, that the distribution $\mathrm{distr}(u_t^h \mid H_t(y,a))$ of intertemporal utility functions across the subpopulation $H_t(y,a)$ is locally time-invariant. This does not imply that the distribution of the utility functions

[1]Hypothesis 1 is based on the concept of "structural stability" as discussed by E. Malinvaud in (1981), p. 72 and (1993), p. 129.

across the *whole* population H_t is locally time-invariant since typically the relative size of the subpopulation $H_t(y, a)$ will change over time.

Note that $H_t(y, a)$ and $H_{t+1}(y, a)$ are different populations since real income of the households typically changes and some households might change their attributes. Consequently, time-invariance of $\text{distr}(u_t^h \mid H_t(y, a))$ does not say that households do not change their preferences. The assumption says that for periods s and t that are close to each other, real income and the attribute profile determine the distribution of utility functions.

2. One can not expect that the distributions of the explanatory variables $W_t^h, r_t^h, \lambda_t^h$ across the subpopulation $H_t(y, a)$ will be time-invariant. This is evident for the wealth distribution but also the distribution of the expectational variables r_t^h, λ_t^h might change over time. For example, λ_t^h depends on expectations about future non-property income as anticipated by household h in period t. These anticipations can suddenly change due to some new information or the announcement of a policy measure. Consequently, one can not assume that the distribution of the expectational variables λ_t^h is time-invariant; indeed, this assumption would be subject to the "Lucas critique". We shall assume however, that the possible change in the expectational variable λ_t^h, that is due to some new information, is such that $\text{distr}(\tilde{\lambda}_t^h \mid H_t(y, a))$ is locally time-invariant. That "scaling" or "standardizing" leads to local time-invariance, as required in Hypothesis 1, is based on the implicit assumption that mean and variance are the most important characteristics of these distributions.

3. Structural stability, however, does not only require that all marginal distributions are locally time-invariant but also that the joint distribution has this property, that is to say, the correlation among the variables $\tilde{W}_t^h, \tilde{r}_t^h, \tilde{\lambda}_t^h$ does change sufficiently slowly over time. For this last requirement of structural stability, the restriction to subpopulations defined by attribute profiles might be particulary important.

3.2

The distribution of current non-property income is observable and it is known that this distribution changes over time. Empirical studies also show that these income distributions are skewed and that the distributions of log-income are approximately symmetric (around their mean). For such distributions the mean m_t and the variance σ_t^2 are the most important characteristics. Consequently, if one considers the *standardized log-income distribution* in every period, then one can expect, in general, approximate time-invariance, since all standardized log-income distributions have the same mean (equal to zero) and the same variance (equal to one).

The standardized log-current income distribution is defined as the distribution across the population H_t of

$$\frac{\log y_t^h - m_t}{\sigma_t} \tag{1}$$

where m_t and σ_t^2 denote the mean and the variance of the distribution of $\log y^h$ across the population H_t.

For statistical estimates of standardized log income distributions we refer to Hildenbrand and Kneip (1999) and Hildenbrand, Kneip, and Utikal (1998).

Hypothesis 2: *The standardized log current income distribution changes sufficiently slowly over time, in the sense that these distributions can be considered as invariant for two periods s and t which are close to each other.*

Remark: Naturally, time-invariance of the standardized log current income distribution will never hold exactly, even for periods $s = t-1$ and t. Hypothesis 2 should be considered as an *approximation* to the actual complex change in the short-run. It is important to recall that the distributions of current income can be estimated and therefore one can decide whether the hypothesis satisfactorily captures the main tendency of the actual change. It might well be that alternative assumptions can be found that yield a better approximation. Our aggregation approach can easily be adapted to any other transformation of income distributions that leads to time-invariance.

We consider large populations of households. Therefore one can describe the distribution of current income in period t by a density. Let ρ denote the density of log-income. Hypothesis 2 then implies

$$\rho_t(\xi) = \frac{\sigma_s}{\sigma_t}\rho_s(m_s + \frac{\sigma_s}{\sigma_t}(\xi - m_t)). \tag{2}$$

Thus, the change from the density ρ_s to the density ρ_t is parametrized by (m_t, σ_t), that is to say, if one knows m_t, σ_t and the density ρ_s (hence m_s and σ_s) then the density ρ_t is determined.

3.3

It is known that the observable distribution of household attributes (for example, household size or age of head of household) across the subpopulation $H_t(y)$ depends on the income level y and is not time-invariant. Since households' current income is changing over time the subpopulations $H_s(y)$ and $H_t(y)$ in period s and t, respectively, consist of different households and there is no reason why their distribution of attributes should be equal.

One might expect a high positive association between households' current income in period s and the later period t provided the periods are close to each other. If this association were perfect then, by Hypothesis 2, all households would remain in the same quantile position in the income distribution of periods s and t. Hence, one might expect that the subpopulations $H_s(y_s)$ and $H_t(y_t)$ will not differ too much if the income levels y_s and y_t are in the same quantile position in the income distributions ρ_s and ρ_t, respectively.

This heuristic argument motivates the following

Hypothesis 3: *For two periods s and t that are close to each other, the attribute distribution $distr(a|H_s(y_s))$ across the subpopulation $H_s(y_s)$ is "approximately" equal to the attribute distribution $distr(a|H_t(y_t))$ across the subpopulation $H_t(y_t)$ if y_s and y_t are in the same quantile position in the distribution of current income of period s and period t, respectively.*

Remark: Obviously, this simple hypothesis can not describe accurately the actual complex change over time of household attribute distributions. Hypothesis 3 can, however, be considered as a first and rough approximation. Indeed, this claim is supported by empirical estimates of appropriate conditional attribute distributions in Hildenbrand and Kneip (1999).

Hypothesis 3 can be interpreted in different ways. The strongest interpretation would be the assertion that the difference between the attribute distribution $\text{distr}(a|H_s(y_s))$ and $\text{distr}(a|H_t(y_t))$ is negligible. This case is called strict structural stability of household attributes and is used in the Proposition. This assumption is very strong since it implies that the marginal distribution of attributes of the entire population is time-invariant, which is empirically rejected. Thus, the difference between $\text{distr}(a|H_s(y_s))$ and $\text{distr}(a|H_t(y_t))$ should not be considered as negligible but as small. For more details on this point we refer to Hildenbrand and Kneip (1999), section 4.

4 The change in mean consumption expenditure

In this section we derive a first order approximation for the relative change in mean real consumption expenditure, i.e., $(C_t - C_s)/C_s$, which applies for two periods s and t that are close to each other. The formulation and the proof of the approximation becomes easier if we make some simplifying assumptions. First, we use Hypothesis 1 of structural stability only in the simpler case of the "scaling" transformation (the extension to the "standardizing" transformation is straightforward, yet requires much more notations). Second, we shall assume that the distribution of the household specific variables $(W_t^h, r_t^h, \lambda_t^h, u_t^h) = v_t^h$ across the subpopulation $H_t(y,a)$ does not depend on the level of current income y. This assumption is certainly acceptable for the variables r_t^h, λ_t^h, and u_t^h. In the case of expected interest rates one might even assume that r_t^h and (y_t^h, a_t^h) are independently distributed across the entire population H_t. To assume that wealth W_t^h and current non-property income y_t^h are independently distributed across the subpopulation $H_t(a)$ certainly is a restriction. The assumption can be weakened yet this would complicate essentially the proof of the following

Proposition: *Let the behavioral relation*

$$c_t^h = c(y_t^h, W_t^h, r_t^h, \lambda_t^h, u_t^h)$$

of section 2 be continuously differentiable in $y, \ldots, \lambda$.

Then Hypothesis 1 (with the above additional assumptions), Hypotheses 2 and 3 imply

$$
\begin{aligned}
\frac{C_t - C_s}{C_s} &= \beta_s(m_t - m_s) + \gamma_s\left(\frac{\sigma_t - \sigma_s}{\sigma_s}\right) && \textit{effect of the changing distribution} \\
& && \textit{of current non-property income} \\
&+ \sum_a \left[\eta_s^a\left(\frac{W_t^a - W_s^a}{W_s^a}\right)\right]\frac{\#H_s(a)}{\#H_s} && \textit{effect of changing wealth} \\
&+ \sum_a \left[\delta_s^a\left(\frac{r_t^a - r_s^a}{r_s^a}\right)\right]\frac{\#H_s(a)}{\#H_s} && \textit{effect of changing interest rates} \\
&+ \sum_a \left[\alpha_s^a\left(\frac{\lambda_t^a - \lambda_s^a}{\lambda_s^a}\right)\right]\frac{\#H_s(a)}{\#H_s} && \textit{effect of changing anticipation} \\
& && \textit{of future non-property income} \\
&+ \textit{terms of second order in } (m_t - m_s)^2, \left(\frac{\sigma_t - \sigma_s}{\sigma_s}\right)^2, \left(\frac{W_t^a - W_s^a}{W_s^a}\right)^2, \ldots
\end{aligned}
$$

The coefficients β_s and γ_s are defined by

$$
\begin{aligned}
\beta_s &:= \frac{1}{C_s}\int y\partial_y\bar{c}_s(y,a)\,\mathrm{distr}(y,a\,|\,H_s) \\
\gamma_s &:= \frac{1}{C_s}\int(\log y - m_s)y\partial_y\bar{c}_s(y,a)\,\mathrm{distr}(y,a\,|\,H_s)
\end{aligned}
$$

where $\bar{c}_s(y,a)$ denotes the mean consumption expenditure across the subpopulation $H_s(y,a)$.

Remarks

1. The coefficients β_s and γ_s only depend on the actual consumption expenditure c_s^h of the households in period s that is to say, these coefficients are independent of the behavioral relation on the household level. This has an important consequence: the coefficients β_s and γ_s can be *estimated separately* (as an average derivative of a regression function) *from cross-section data in every period.* Estimates will be given in section 5 . The average (over the years 1968-93) of the estimates for β_s and γ_s are 0.56 and 0.24, respectively. We remark, that the Hypotheses 1-3 do not imply that these coefficients are time-invariant.

2. The coefficients β_s, γ_s can be interpreted under the required ceteris paribus clause as *elasticities* of mean consumption expenditure with respect to mean current non-property income Y_s and current income dispersion σ_s, respectively. For example, under the ceteris paribus clause $\sigma_t = \sigma_s, W_t^a = W_s^a, \ldots$ we obtain

$$\frac{C_t - C_s}{C_s} = \beta_s(m_t - m_s)$$

If $\sigma_t = \sigma_s$, then Hypothesis 2 implies that $m_t - m_s = \log(Y_t/Y_s)$, where Y_t denotes mean real non-property income across the population H_t. Since $\log(Y_t/Y_s) \approx (Y_t - Y_s)/Y_s$ we obtain

$$\beta_s = \frac{C_t - C_s}{C_s}\Bigg/\frac{Y_t - Y_s}{Y_s}$$

neglecting second order terms. Clearly, the coefficient γ_s can be interpreted as an elasticity of mean consumption expenditure with respect to income dispersion under the ceteris paribus clause $m_t = m_s$ (not $Y_t = Y_s$!), $W_t^a = W_s^a, \ldots$ Note that the ceteris paribus clause $m_t = m_s$ means for a symmetric log-income distribution that the median of the income distribution remains constant (not the mean).

3. The effect of the changing distribution of current non-property income in the Proposition, i.e.,

$$\beta_s(m_t - m_s) + \gamma_s \left(\frac{\sigma_t - \sigma_s}{\sigma_s} \right)$$

can be expressed in terms of mean income Y instead of mean log-income m. Indeed, one can show that Hypothesis 2 implies

$$m_t - m_s = \log \frac{Y_t}{Y_s} - \frac{1}{Y_s} \log \frac{\sigma_t}{\sigma_s} \int (\log y - m_s) y \, \mathrm{distr}(y \mid H_s)$$

plus a second order term in $\left(\log \frac{\sigma_t}{\sigma_s}\right)^2$. Thus we obtain

$$\beta_s(m_t - m_s) + \gamma_s \left(\frac{\sigma_t - \sigma_s}{\sigma_s} \right) = \beta_s \log \frac{Y_t}{Y_s} + \bar{\gamma}_s \log \frac{\sigma_t}{\sigma_s}$$

plus a second order term in $\left(\log \frac{\sigma_t}{\sigma_s}\right)^2$ with $\bar{\gamma}_s = \gamma_s - \frac{\beta_s}{Y_s} \int y(\log y - m_s) \, \mathrm{distr}(y \mid H_s)$. The coefficient $\bar{\gamma}_s$ is the elasticity of mean consumption expenditure with respect to income dispersion under the ceteris paribus clause $Y_t = Y_s$.

4. The relative change in wealth $(W_t^a - W_s^a)/W_s^a$ is, in principle, observable. However, the data set that we shall use in section 5, the UK - FES, does not contain information on wealth. In this situation one has to find on observable proxy for the wealth effect. To simplify, let us consider the wealth effect without stratifying on household attributes:

$$\eta_{t-1} \frac{W_t - W_{t-1}}{W_{t-1}}$$

By definition $W_t = A_t + X_t$, where A_t and X_t denote mean wealth and mean property income across the population H_t. Since

$$A_t \approx \frac{\pi_{t-1}}{\pi_t} \left(A_{t-1} + X_{t-1} + Y_{t-1} - C_{t-1} \right)$$

one obtains

$$\eta_{t-1} \frac{W_t - W_{t-1}}{W_{t-1}} \approx \alpha_1 \frac{X_t + S_{t-1}}{\frac{\pi_t}{\pi_{t-1}} \cdot X_t - \frac{X_t}{A_t} \cdot S_{t-1}} + \alpha_2 \frac{\pi_t - \pi_{t-1}}{\pi_{t-1}}$$

where $S_t = Y_t - C_t$ denotes savings out of non-property income, $\alpha_1 := \eta_{t-1} \cdot X_t / A_t$ and $\alpha_2 := \eta_{t-1} \cdot A_t / (A_{t-1} + X_{t-1})$.

Thus, if the average rate of return on wealth X_t/A_t is "small" then one might use the observable proxy

$$\eta_{t-1} \frac{W_t - W_{t-1}}{W_{t-1}} \approx \alpha_1 \frac{X_t + S_{t-1}}{\pi_t / \pi_{t-1} \cdot X_t} + \alpha_2 \frac{\pi_t - \pi_{t-1}}{\pi_{t-1}}$$

This proxy will be used in section 5.

Proof of the Proposition
Define $\bar{c}_t(y,a) := \int c(y,v)\text{distr}(v \mid H_t(y,a))$; the function $y \mapsto \bar{c}_t(y,a)$ is called the (cross-section) Engel curve of the subpopulation $H_t(a)$. With this notation, mean consumption expenditure C_t is given by

$$C_t = \int \bar{c}_t(y,a)\text{distr}(y,a \mid H_t).$$

Let $\eta = \log y$. Then one obtains by the change of variable formula

$$\begin{aligned} C_t &= \int \bar{c}_t(\exp\eta, a)\text{distr}(\eta, a \mid H_t) \\ &= \int \left[\int \bar{c}_t(\exp\eta, a)\text{distr}(a \mid H_t(\eta)) \right] \text{distr}(\eta \mid H_t) \end{aligned} \quad (1)$$

Let ρ_t denote the density of $\text{distr}(\eta \mid H_t)$. Hypothesis 2 implies

$$\rho_t(\eta) = \frac{\sigma_s}{\sigma_t}\rho_s(m_s + \frac{\sigma_s}{\sigma_t}(\eta - m_t)) \quad (2)$$

Since η and $m_s + \frac{\sigma_s}{\sigma_t}(\eta - m_t)$ are in the same quantile position of ρ_t and ρ_s, respectively, Hypothesis 3 implies

$$\text{distr}(a \mid H_t(\eta)) = \text{distr}(a \mid H_s(m_s + \frac{\sigma_s}{\sigma_t}(\eta - m_t)) \quad (3)$$

Substituting (2) and (3) in (1) one obtains

$$C_t = \int \bar{c}_t(\exp\left\{\frac{\sigma_t}{\sigma_s}(\eta - m_s) + m_t\right\}, a)\text{distr}(\eta, a \mid H_s) \quad (4)$$

Since we made the simplifying assumption that $\text{distr}(v \mid H_t(y,a))$ does not depend on y we have $\bar{c}_t(y,a) = \int c(y,v)\text{distr}(v \mid H_t(a))$. Recall the notation: $v_t^h = (W_t^h, r_t^h, \lambda_t^h, u_t^h)$ and the "scaled" variable in our case is $\tilde{v}_t^h = (W_t^h/W_t^a, r_t^h/r_t^a, \lambda_t^h/\lambda_t^a, u_t^h)$ where W_t^a, r_t^a and λ_t^a denote the mean of W_t^h, r_t^h and λ_t^h across the subpopulation $H_t(a)$, respectively. Thus,

$$\bar{c}_t(y,a) = \int c(y, v_t^a * \tilde{v})\text{distr}(\tilde{v} \mid H_t(a))$$

where $v_t^a = (W_t^a, r_t^a, \lambda_t^a, 1)$ and "$*$" denotes the coordinate-wise product. Hypothesis 1 implies that one can replace $\text{distr}(\tilde{v} \mid H_t(a))$ by $\text{distr}(\tilde{v} \mid H_s(a))$. Hence, we obtain

$$\bar{c}_t(y,a) = \int c(y, v_t^a * \tilde{v})\text{distr}(\tilde{v} \mid H_s(a)). \quad (5)$$

Given the distribution $\text{distr}(\eta, a \mid H_s)$ (hence given m_s and σ_s), the distribution $\text{distr}(\tilde{v} \mid H_s(a))$ and the function $c(y,v)$, we define a function f_s in the variables $m, \sigma, (v^a)_{a \in \mathcal{A}}$

($\mathcal{A}$ is assumed to be a finite set) by
$f_s(m, \sigma, v^a, a \in \mathcal{A}) :=$

$$\int \left[\int c(\exp \left\{ \frac{\sigma}{\sigma_s}(\eta - m_s) + m \right\}, v^a * \tilde{v}) \mathrm{distr}(\tilde{v} \mid H_s) \right] \mathrm{distr}(\eta, a \mid H_s).$$

By (4) and (5) one obtains $f_s(m_s, \sigma_s, v_s^a, a \in \mathcal{A}) = C_s$ and $f_s(m_t, \sigma_t, v_t^a, a \in \mathcal{A}) = C_t$.

A first order Taylor expansion of the function f_s at $(m_s, \sigma_s, v_s^a, a \in \mathcal{A})$ yields

$$\begin{aligned}
C_t - C_s \quad &= \quad \partial_m f_s(m, \sigma_s, v_s^a)\big|_{m=m_s} \cdot (m_t - m_s) \\
&+ \quad \partial_\sigma f_s(m_s, \sigma, v_s^a)\big|_{\sigma=\sigma_s} \cdot (\sigma_t - \sigma_s) \\
&+ \quad \sum_a \partial_{W^a} f_s(m_s, \sigma_s, W^a, r_s^a, \lambda_s^a)\big|_{W^a=W_s^a} \cdot (W_t^a - W_s^a) \\
&+ \quad \sum_a \partial_{r^a} f_s(m_s, \sigma_s, W_s^a, r^a, \lambda_s^a)\big|_{r^a=r_s^a} \cdot (r_t^a - r_s^a) \\
&+ \quad \sum_a \partial_{\lambda^a} f_s(m_s, \sigma_s, W_s^a, r_s^a, \lambda^a)\big|_{\lambda^a=\lambda_s^a} \cdot (\lambda_t^a - \lambda_s^a) \\
&\text{plus} \quad \text{terms of second order in } (m_t - m_s)^2, (\sigma_t - \sigma_s)^2, \ldots
\end{aligned}$$

Calculation of the partial derivatives of f_s:

1) $\partial_m f_s(m, \sigma_s, v_s^a)\big|_{m=m_s}$

$$\begin{aligned}
&= \int \partial_m \bar{c}_s(\exp\{\eta - m_s + m\}, a)\big|_{m=m_s} \mathrm{distr}(\eta, a \mid H_s) \text{ , by the definitions of the functions } f_s \text{ and } \bar{c}_s \\
&= \int \exp \eta \; \partial_y \bar{c}_s(\exp \eta, a) \mathrm{distr}(\eta, a \mid H_s), \text{ where } \partial_y \bar{c}_s \text{ denotes the partial derivative of } \bar{c}_s(y, a) \text{ with respect to } y \\
&= \int y \; \partial_y \bar{c}_s(y, a) \mathrm{distr}(y, a \mid H_s), \text{ by the change of variable formula.}
\end{aligned}$$

The coefficient β_s in the Proposition is now defined by

$$\beta_s := \frac{1}{C_s} \int y \; \partial_y \bar{c}_s(y, a) \mathrm{distr}(y, a \mid H_s)$$

2) $\partial_m f_s(m_s, \sigma, v_s^a)\big|_{\sigma=\sigma_s}$

$$\begin{aligned}
&= \int \partial_\sigma \bar{c}_s(\exp \left\{ \frac{\sigma}{\sigma_s}(\eta - m_s) + m_s \right\}, a)\big|_{\sigma=\sigma_s} \mathrm{distr}(\eta, a \mid H_s) \\
&= \int (\exp \eta) \frac{1}{\sigma_s}(\eta - m_s) \partial_y \bar{c}_c(\exp \eta, a) \mathrm{distr}(\eta, a \mid H_s) \\
&= \frac{1}{\sigma_s} \int (\log y - m_s) y \partial_y \bar{c}_s \mathrm{distr}(y, a \mid H_s)
\end{aligned}$$

The coefficient γ_s in the Proposition is now defined by

$$\gamma_s := \frac{1}{C_s} \int (\log y - m_s) y \partial_y \bar{c}_s(y,a) \mathrm{distr}(y, a \mid H_s).$$

We do not compute the remaining partial derivatives of f_s since they cannot be expressed in terms of the Engel curve $\bar{c}_s(y,a)$.

5 Empirical Results

For analyzing our aggregate model of Section 4 we use data from the UK - Family Expenditure Survey (FES). Each year a total of approximately 7000 households record their expenditures on a large variety of consumption items. Also included in the survey are different forms of income and other household attributes. For a precise definition of the variables, sampling units, sampling designs, interviewing and field work, confidentiality, reliability etc. we refer to the respective yearly FES manuals as well as the Family Survey Handbook of Kemsley et al (1980). We include into the analysis data made available to us for all years between 1968 and 1993 except for the year 1978, where our income variable could not be constructed due to problems in the database. Households from Northern Ireland were excluded for all years.

In the present study we use information on household income and consumption as well as on demographic and socioeconomic variables like age and occupational status of the household head, household size, etc., included in the yearly surveys. In the economic literature most studies focus on consumption of nondurable goods. Following this tradition we will consider nondurable consumption which is defined as total consumption of all goods and services minus housing costs and durable goods. However, in our apporach there is no apriori reason to exclude durables. In addition to nondurable consumption we will thus also analyse the behavior of our model for total consumption (only mortage interest payments are excluded). The income variable is constructed in a way to represent current disposable non-property income of each household[2]. Consumption and income in real prices are determined by dividing by the price index of the respective month in which the household was included in the survey. In order to diminish the potential influence of outliers, all households with income or consumption larger than eight times median income or eight times median consumption were excluded. In total this procedure leads to an elimination of between 0.1% and 0.3% of all households in the different samples.

[2]Following HBAI standards, household incomes are obtained by extracting relevant items from the elementary database. The task of elaborating the database and specifying consistent variables has mainly been accomplished by Jürgen Arns. His careful work is gratefully acknowledged. Furthermore, we want to thank the Insitute of Fiscal Studies, London, and its director R. Blundell. Their valuable support with programs and advise allowed an efficient treatment of the data.

5.1 Current Income and Aggregate Consumption

We will first consider the effect of changes in the distribution of current income on aggregate consumption. Further explanatory variables will be added in Subsection 5.2.

Let $s = t - 1$. Since yearly changes in the data are of a order of magnitude of less than six percent, the differences between $\frac{C_t - C_{t-1}}{C_{t-1}}$ and $\Delta \log C_t = \log C_t - \log C_{t-1}$ are negligible. Our model then implies that a first order approximation which only incorporates terms due to changes in the current income distribution can be written in the form

$$\Delta \log C_t = \beta_{t-1}(m_t - m_{t-1}) + \gamma_{t-1} \frac{\sigma_t - \sigma_{t-1}}{\sigma_{t-1}} + \text{remainder term} \tag{1}$$

By writing "remainder term" instead of "ϵ_t" we want to emphasize that we do **not** assume that the above model holds up to i.i.d. error terms. There are important explanatory variables missing.

As explained in Section 4 distributional effects of income may also be modelled by using mean income Y_t and σ_t instead of m_t, σ_t,

$$\beta_{t-1}(m_t - m_{t-1}) + \gamma_{t-1} \frac{\sigma_t - \sigma_{t-1}}{\sigma_{t-1}} \approx \beta_{t-1} \log \frac{Y_t}{Y_{t-1}} + \bar{\gamma}_{t-1} \frac{\sigma_t - \sigma_{t-1}}{\sigma_{t-1}} \tag{2}$$

where β_t, γ_t and $\bar{\gamma}_t$ are defined as in Section 4.

The important point is that separately for each year t the parameters β_t, γ_t and $\bar{\gamma}_t$ can be **estimated from the cross-section data** on individual income and expenditure provided by the FES. Details of the estimation procedure are described in Subsection 5.3.

There are several advantages of using cross-section estimates as compared to least-squares estimation from aggregate time series. An important point is that it is not necessary to assume that the β_t and γ_t are time-invariant. They can be estimated separately for each year, and there might well exist a time trend. A crucial issue for interpreting the parameter values is that *by cross-section estimation we avoid all problems of collinearity of Y_t, m_t or σ_t with other explanatory variables.* We thus obtain a "pure" estimate of the influence of these terms.

Figure 5.1 shows the estimates $\hat{\beta}_t$, $\hat{\gamma}_t$, and $\hat{\bar{\gamma}}_t$ for nondurable consumption. The average values of $\hat{\beta}_t, \hat{\gamma}_t$, and $\hat{\bar{\gamma}}_t$ are 0.56, 0.24, and 0.02. One recognizes a slightly falling trend of $\hat{\beta}_t$. All $\hat{\bar{\gamma}}_t$ are very close to zero and essentially negligible. In fact, when determining bootstrap confidence intervals it turns out that the $\hat{\bar{\gamma}}_t$ are not significantly different from zero. The validity of bootstrap methods in the present situation might be derived similarly as in Härdle and Hart (1992). Since for the present dataset the $\hat{\bar{\gamma}}_t$ are negligible, (2) simplifies to

$$\beta_{t-1}(m_t - m_{t-1}) + \gamma_{t-1} \frac{\sigma_t - \sigma_{t-1}}{\sigma_{t-1}} \approx \beta_{t-1} \log \frac{Y_t}{Y_{t-1}} \tag{3}$$

One recognizes in Figure 5.2 that the time paths of the left and right hand side of (3) are indeed extremely close, while there are considerable differences between $\beta_{t-1}(m_t - m_{t-1})$ and $\beta_{t-1} \log \frac{Y_t}{Y_{t-1}}$. These findings are obviously fully consistent with the assumption made in Section 3 about the time evolution of the current income distribution.

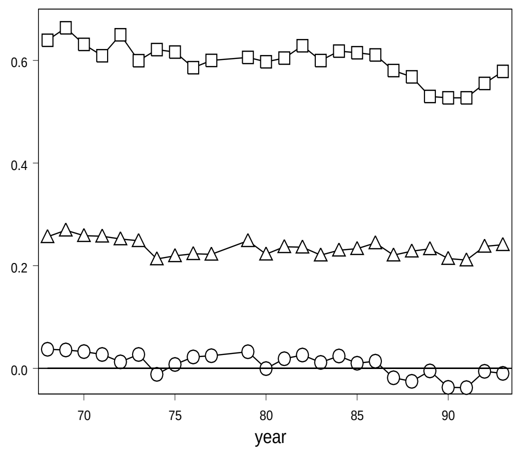

Fig. 5.1: Estimated values of β_t ("□"), γ_t ("△") and $\bar{\gamma}_t$ ("○")

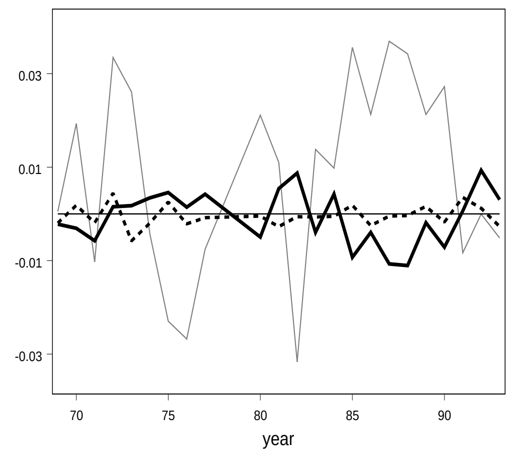

Fig. 5.2: $\mathrm{R}_t := \hat{\beta}_{t-1} \log \frac{Y_t}{Y_{t-1}}$ [solid, grey], $\hat{\beta}_{t-1}(m_t - m_{t-1}) - \mathrm{R}_t$ [solid, black], $\hat{\beta}_{t-1}(m_t - m_{t-1}) + \hat{\gamma}_{t-1} \frac{\sigma_t - \sigma_{t-1}}{\sigma_{t-1}} - \mathrm{R}_t$ [dotted]

A similar picture arises for total consumption. The estimated $\hat{\beta}_t$ and $\hat{\gamma}_t$ have the same time trends as for nondurable consumption, but are generally slightly larger. Averages are 0.60, 0.24, and 0.01 for $\hat{\beta}_t$, $\hat{\gamma}_t$, and $\widehat{\bar{\gamma}}_t$. The parameters $\widehat{\bar{\gamma}}_t$ are again nonsignificant, and the time path of $\beta_{t-1}(m_t - m_{t-1}) + \gamma_{t-1} \frac{\sigma_t - \sigma_{t-1}}{\sigma_{t-1}}$ essentially coincides with that of $\beta_{t-1} \log \frac{Y_t}{Y_{t-1}}$.

We now consider the question which proportion of consumption is explained by changes in the current income distribution. Due to (3) we will consider the approximations[3] $\widehat{\Delta \log C}_t$ of $\Delta \log C_t$ obtained by $\hat{\beta}_{t-1}(m_t - m_{t-1})$ and $\hat{\beta}_{t-1} \log \frac{Y_t}{Y_{t-1}}$. We use two measures to quantify the remaining differences, the average absolute error (AE) and the relative residual sum of squares ($RRSS$):

$$AE = 100 \cdot \frac{1}{T} \sum_t |\Delta \log C_t - \widehat{\Delta \log C}_t|, \quad RRSS = \frac{\sum_t |\Delta \log C_t - \widehat{\Delta \log C}_t|^2}{\sum_t |\Delta \log C_t|^2}$$

$RRSS$ measures the sum of squared residuals relative to the original squared differences $|\Delta \log C_t|^2$. In a standard parametric regression model we have $RRSS = 1 - R^2$. Obviously, the better a model the smaller the values of AE and $RRSS$.

Table 5.1 provides the resulting errors for nondurable and total consumption. In the first row of the table shows the average value of $|\Delta \log C_t|$ which may serve as a benchmark for interpreting the subsequent values of AE.

[3] Relation (3) is an empirical result characterizing the FES data. It may be violated in other situations or for datasets from countries different from the UK. Then $\beta_{t-1}(m_t - m_{t-1}) + \gamma_{t-1} \frac{\sigma_t - \sigma_{t-1}}{\sigma_{t-1}}$ has to be analyzed separately from $\beta_{t-1} \log \frac{Y_t}{Y_{t-1}}$.

	Nondurables		Total	
	AE	$RRSS$	AE	$RRSS$
$\Delta \log C_t$	2.08		2.58	
$\hat{\beta}_{t-1}(m_t - m_{t-1})$	1.22	0.362	1.41	0.363
$\hat{\beta}_{t-1}\Delta \log Y_t$	1.27	0.410	1.48	0.403

Table 5.1

Total consumption is obviously more variable than nondurable consumption, but in both cases $\hat{\beta}_{t-1}\Delta \log Y_t$ and $\hat{\beta}_{t-1}(m_t - m_{t-1})$ already explain a large part of the total variation of $|\Delta \log C_t|$. This result is in line with the theoretical discussion of Section 4. Since by (3) $\hat{\beta}_{t-1}\Delta \log Y_t$ implicitely accounts for income dispersion and should thus be used to model the effect of changes in the distribution of current income, it is surprising to find that $\hat{\beta}_{t-1}(m_t - m_{t-1})$ possesses a somewhat smaller error. In terms of our model this result indicates the existence of some important explanatory variable(s) whose time paths are negatively correlated with that of $\frac{\sigma_t - \sigma_{t-1}}{\sigma_{t-1}}$.

When considering our theoretical approach more closely, the above effect may indeed be readily explained by the influence of uncertainty of future income. Recall that in Section 2 we have assumed that for each period t and every household h the distribution $D\mathcal{L}_t^h$ of anticipated individual non-property income depends on the stochastic process $\{z_t^h(\tau)\}_{\tau > t}$, where $z_t^h(\tau) = \log y_t^h(\tau) - \log y_t^h(\tau - 1)$. Important parameters related to income uncertainty are the variances v_t^h of these processes. Assume that, as for the simple example in Section 2, individual processes are independent and $z_t^h(\tau)$ can be decomposed into $z_t^h(\tau) = e_t + \delta_t^h(\tau)$, where for all τ the $\delta_t^h(\tau)$ are zero mean random variables with variance v_t^h. The $\delta_t^h(\tau)$ can then be rewritten in the form $\sqrt{v_t^h} \cdot \delta_t^{*h}(\tau)$, where v_t^h and $\delta_t^{*h}(\tau)$ are independent, and the $\delta_t^{*h}(\tau)$ are zero mean random variables with unit variance. If then E_H and var_H denote *cross-section* mean and variances, we obtain $E_H(\delta_t^h(\tau)) = 0$ and $var_H(\delta_t^h(\tau)) = E_H(v_t^h) \cdot E_H((\delta_t^{*h}(\tau))^2) = E_H(v_t^h)$. It follows that

$$var_H(\log y_t^h(\tau) - \log y_t^h(\tau - 1)) = var_H(z_t^h(\tau)) = E_H((\delta_t^h(\tau))^2) = E_H(v_t^h) = \bar{v}_t$$

A natural assumption then is that each individual updates the structure of the respective process by using past information. In particular, it seems plausible that

$$V_t = var_H(\log y_t^h - \log y_{t-1}^h)$$

is related to the general level $\bar{v}_t$ of the individual variances v_t^h. The idea is that

- an increasing/decreasing value of V_t indicates an increasing/decreasing value of $\bar{v}_t$, and the time path of this variable is thus **correlated** with the mean level of uncertainty.

One may note that commonly used measures of uncertainty, as for example the unemployment rate, are related to these variables. More unemployment will usually result in higher values of V_t.

Let us now consider the role of $\sigma_t^2 - \sigma_{t-1}^2$. It is immediately seen from Figure 5.1 that all estimated γ_t are positive and therefore the *direct* effect of an increasing variance of the income distribution stimulates consumption. However, one easily verifies that

$$\sigma_t^2 - \sigma_{t-1}^2 = V_t + 2cov_H(\log y_t^h - \log y_{t-1}^h, logy_{t-1}^h)$$

Although the covariance term may act as a nuisance, large values of $\sigma_t^2 - \sigma_{t-1}^2$ thus may, in tendency, go along with large values of V_t and high income uncertainty. The negative effect of higher uncertainty on consumption may well explain the empirical results of Table 5.1. This point will be further explored in Subsection 5.2.

5.2 The effects of assets, prices, interest rates, and uncertainty

In this subsection we will study the role of further explanatory variables to model the remainder terms $\Delta \log C_t - \hat{\beta}_{t-1} \Delta \log Y_t$ and $\Delta \log C_t - \hat{\beta}_{t-1}(m_t - m_{t-1})$, respectively.

We first concentrate on assets and interest rates and consider a "global" approach which does not involve conditioning on household attributes. Applications of the resulting models to subgroups of households falling into suitable attribute classes will be discussed at the end of the section.

The FES does not contain individual data about assets. However, it does provide information about property income x_t^h. We therefore use the approximations discussed in Section 4 which lead to the following empirical model:

$$\Delta \log C_t - B_{ti} = \alpha_1 \frac{X_t + S_{t-1}}{X_t \pi_t / \pi_{t-1}} + \alpha_2 \frac{\pi_t - \pi_{t-1}}{\pi_{t-1}} + \alpha_3 (r_t - r_{t-1}) + \epsilon_t \qquad (4)$$

for $B_{t1} = \hat{\beta}_{t-1}(m_t - m_{t-1})$ and $B_{t2} = \hat{\beta}_{t-1} \log \frac{Y_t}{Y_{t-1}}$. We used least squares to estimate the parameters from the respective time series. Results are given in Table 5.2. The reported values of AE and RRSS refer to the predictions $\widehat{\Delta \log C_t}$ of $\Delta \log C_t$ obtained when adding $\hat{\beta}_{t-1}(m_t - m_{t-1})$ and $\hat{\beta}_{t-1} \log \frac{Y_t}{Y_{t-1}}$, respectively, to the fitted models.

	Fitted time series			
	$\Delta \log C_t - \hat{\beta}_{t-1}(m_t - m_{t-1})$		$\Delta \log C_t - \hat{\beta}_{t-1} \log \frac{Y_t}{Y_{t-1}}$	
	Nondurables	Total	Nondurables	Total
$\frac{X_t + S_{t-1}}{X_t \pi_t / \pi_{t-1}}$	0.005 (0.002)	0.007 (0.002)	0.004 (0.002)	0.007 (0.002)
$\frac{\pi_t - \pi_{t-1}}{\pi_{t-1}}$	-0.172 (0.054)	-0.231 (0.060)	-0.151 (0.062)	-0.209 (0.069)
$r_t - r_{t-1}$	-0.137 (0.097)	-0.175 (0.107)	-0.127 (0.111)	-0.163 (0.124)
AE [RRSS]	0.96 [0.239]	1.03 [0.198]	1.11 [0.315]	1.25 [0.267]

Table 5.2

The table shows that incorporating assets and prices significantly improves the fit. Similar as in Table 5.1 one achieves smaller errors if the effect of changes in current incomes is quantified by $\hat{\beta}_{t-1}(m_t - m_{t-1})$ instead of $\hat{\beta}_{t-1} \log \frac{Y_t}{Y_{t-1}}$. The difference now is even much more pronounced.

On the other hand, model (4) is still incomplete. It may be improved by adding further variables which relate to expectational terms, and in particular to income uncertainty. Following the discussion in 5.1 the latter may be correlated with $V_t = var_H(\log y_t^h - \log y_{t-1}^h)$.

Since we use cross-section data it is not possible to estimate V_t directly. Approximations now were constructed by a matching algorithm: For each individual household h of the sample in period $t-1$ we use the whole set of household attributes provided by the FES to find the household $a(h)$ in period t which the closest possible in these attributes. The term $\log y_t^{a(h)} - \log y_{t-1}^h$ was then taken as an approximation of $\log y_t^h - \log y_{t-1}^h$. We then determine a cross-section estimate $\widehat{\text{cov}}(\log y_t^{a(h)} - \log y_{t-1}^h, \log y_{t-1}^h)$ of the covariance between $\log y_t^{a(h)} - \log y_{t-1}^h$ and $\log y_{t-1}^h$. Estimates of the V_t are then computed by

$$\hat{V}_t = \sigma_t^2 - \sigma_{t-1}^2 - 2\widehat{\text{cov}}(\log y_t^{a(h)} - \log y_{t-1}^h, \log y_{t-1}^h) \tag{5}$$

By using (5) we expect to obtain somewhat better estimates of V_t than by directly calculating $var_H(\log y_t^{a(h)} - \log y_{t-1}^h)$. Note that part of the differences between $\log y_t^{a(h)}$ and the true individual values $\log y_t^h$ may be random fluctuations not correlated with $\log y_{t-1}^h$. Such fluctuations will not influence the covariance and are thus implicitely incorporated in (5), while this is not true for $var_H(\log y_t^{a(h)} - \log y_{t-1}^h)$. Our empirical model now becomes

$$\Delta \log C_t - B_{ti} = \alpha_1 \frac{X_t + S_{t-1}}{X_t \pi_t / \pi_{t-1}} + \alpha_2 \frac{\pi_t - \pi_{t-1}}{\pi_{t-1}} + \alpha_3 (r_t - r_{t-1}) + \alpha_4 \hat{V}_t + \epsilon_t \tag{6}$$

for $B_{t1} = \hat{\beta}_{t-1}(m_t - m_{t-1})$ and $B_{t2} = \hat{\beta}_{t-1} \log \frac{Y_t}{Y_{t-1}}$.

Since V_t is only assumed to be correlated with uncertainty, one would usually tend to add a constant to model (6). However, a constant turned out to be completely insignificant and has thus not been included in the final model[4]. We again used least squares to estimate the parameters of (6) from the respective time series. Results are given in Table 5.3. The reported values of AE and RRSS refer to the predictions $\widehat{\Delta \log C_t}$ of $\Delta \log C_t$ obtained when adding $\hat{\beta}_{t-1}(m_t - m_{t-1})$ and $\hat{\beta}_{t-1} \log \frac{Y_t}{Y_{t-1}}$, respectively, to the fitted models.

[4]Results proved to be quite stable with respect to different possible implementations of the matching algorithm. One may argue that our theoretical approach is based on differences, and hence one should include $\hat{V}_t - \hat{V}_{t-1}$ rather than $\hat{V}_t$ into the model. However, the quality of the resulting fits turns out to be considerably better for $\hat{V}_t$ than for $\hat{V}_t - \hat{V}_{t-1}$. A possible explanation may be that the approximation error becomes more important when using differences. On the other hand, the insignificance of a constant indicates that also the general "level" of $\hat{V}_t > 0$ plays a role, a fact which is not explained by our theoretical model.

	Fitted time series			
	$\Delta \log C_t - \hat{\beta}_{t-1}(m_t - m_{t-1})$		$\Delta \log C_t - \hat{\beta}_{t-1} \log \frac{Y_t}{Y_{t-1}}$	
	Nondurables	Total	Nondurables	Total
$\frac{X_t+S_{t-1}}{X_t \pi_t / \pi_{t-1}}$	0.019 (0.005)	0.025 (0.005)	0.024 (0.005)	0.039 (0.052)
$\frac{\pi_t - \pi_{t-1}}{\pi_{t-1}}$	-0.173 (0.045)	-0.229 (0.047)	-0.153 (0.045)	-0.206 (0.048)
$r_t - r_{t-1}$	-0.207 (0.083)	-0.254 (0.087)	-0.223 (0.083)	-0.270 (0.089)
$\hat{V}_t$	-0.128 (0.040)	-0.162 (0.044)	-0.177 (0.040)	-0.216 (0.045)
AE [RRSS]	0.77 [0.155]	0.79 [0.117]	0.76 [0.155]	0.83 [0.122]

Table 5.3

Note that the estimated parameter values for $\hat{V}_t$ are negative and highly significant in all situations. This is in line with their interpretation in terms of income uncertainty. One also notices that when income uncertainty is introduced it is no longer true that the model based on $\hat{\beta}_{t-1}(m_t - m_{t-1})$ yields smaller errors than the one based on $\hat{\beta}_{t-1} \log \frac{Y_t}{Y_{t-1}}$. The absolute values of the estimated parameters of the uncertainty term are larger for total than for nondurable consumption. This is plausibe, since the consumption of durables will certainly be much more affected by uncertainty than, say, the consumption of food.

Further improvements of model (6) might in principle be achieved by incorporating additional variables which correlate with expectational terms different from uncertainty. For example, in the simple $MA(1)$ model presented in Section 2 we have

$$\bar{\kappa}_t = corr_H(\log y_t^h(\tau) - \log y_t^h(\tau-1), \log y_t^h(\tau-1)) = \frac{cov_H(\log y_t^h(\tau) - \log y_t^h(\tau-1), \log y_t^h(\tau-1))}{\bar{v}_t}$$

However, the "observable analogue" $\widehat{\text{cov}}(\log y_t^{a(h)} - \log y_{t-1}^h, \log y_{t-1}^h)/V_t$ proved to be insignificant. Standard aggregate models, see for example Muellbauer and Lattimore (1999), usually contain lagged income variables. No such variable was found to be significant. In view of the above results for model (6) this is not too surprising. Some straightforward approximations of the sampling variances s_t^2 of $\Delta \log C_t - \hat{\beta}_{t-1} \log \frac{Y_t}{Y_{t-1}}$ indicate that the values of $\frac{s_t^2}{\sum_t |\Delta \log C_t|^2}$ are around 0.11 for nondurable and 0.09 for total consumption. The $RRSS$ given in Table 5.3 are obviously already quite close to these values.

An interesting question is the behavior of the model when considering subgroups stratified according to attribute classes. We have thus decomposed the sample into 9 subgroups with respect to age and occupation. For each of these classes the model was fitted separately based on the group-specific values of $\beta_t, \hat{V}_t$, and assets. The problem when dealing with subgroups consists of the fact that due to smaller sample sizes there is a considerable increase in sampling error. Indeed, predicting consumption of the complete population by adding up the subgroup estimates lead to higher errors than those reported in Table 5.3. Nevertheless, for some of the larger subgroups some interesting and interpretable results could be obtained. Table 5.4 shows the fitted models for two groups of households: retired houshold head and middle aged non-manual workers (professionals, managers, etc, over the age of 40).

	Fitted time series			
	$\Delta \log C_t - \hat{\beta}_{t-1}(m_t - m_{t-1})$		$\Delta \log C_t - \hat{\beta}_{t-1} \log \frac{Y_t}{Y_{t-1}}$	
	retired	non-manual	retired	non-manual
$\frac{X_t+S_{t-1}}{X_t\pi_t/\pi_{t-1}}$	0.034 (0.015)	0.025 (0.007)	0.033 (0.014)	0.027 (0.008)
$\frac{\pi_t-\pi_{t-1}}{\pi_{t-1}}$	-0.206 (0.112)	-0.014 (0.104)	-0.177 (0.106)	0.010 (0.111)
$r_t - r_{t-1}$	-0.102 (0.212)	-0.165 (0.171)	-0.108 (0.202)	-0.159 (0.183)
$\hat{V}_t$	-0.025 (0.050)	-0.245 (0.087)	-0.026 (0.047)	-0.276 (0.092)
AE [RRSS]	2.11 [0.402]	1.62 [0.233]	2.03 [0.364]	1.75 [0.263]

Table 5.4

Approximations of the relative sampling error $\frac{s_t^2}{\sum_t |\Delta \log C_t|^2}$ here are 0.37 for retired and 0.27 for middle aged non-manual workers. The income uncertainty term $\hat{V}_t$ is significant for middle aged employees, but it does not seem to play any role for the retired.

5.3 Cross-section estimation of $\beta_t, \gamma_t, \bar{\gamma}_t$

Assume that for each period t there are data (y_t^h, c_t^h, a_t^h), $h = 1, \ldots, n_t$ about current income, consumption, and household attributes from an independent sample of n_t households. Recall that

$$\beta_t = \frac{1}{C_t} \int y \partial_y \bar{c}_t(y, a) distr(y, a|H_t)$$

In this notation $\bar{c}_t$ is nothing else but the regression function of c_t^h on (y_t^h, a_t^h). Estimates $\hat{c}_t$ and $\partial_y \hat{c}_t$ of $\bar{c}_t$ and its derivative with respect to y can thus be obtained by suitable parametric or nonparametric regression methods. In order to guard against misspecifications in the relation between c and y we use a semiparametric model of the form

$$c_t^h = \bar{c}_t(y, a) + \epsilon_t^h = f(y_t^h) + \sum_j \vartheta_j a_{t,j}^h + \epsilon_t^h$$

The household attributes $a_{t,j}^h$ used are age, age^2, and indicator variables refering to household size, employment status, occupation, month in which the household was recorded, and region. For approximating the nonparametric function f we rely on a quadratic spline function with 11 knots $i_0, i_1, \ldots, i_{11}$. The knot locations are chosen in such a way that in each interval $[i_{l-1}, i_l]$ there are approximately the same number of observations y_t^h. The spline parameters as well as the ϑ_j are then estimated by least squares, and with $\hat{c}_t(y_t^h) = f(y_t^h)$, $\partial_y \hat{c}_t(y_t^h, a_t^h) = f'(y_t^h)$ an estimate of β_s is then determined by

$$\hat{\beta}_t = \frac{\sum_{h=1}^n y_t^h \partial_y \hat{c}_t(y_t^h, a_t^h)}{\sum_{h=1}^n c_t^h}$$

Similarly, estimates of γ_t and $\tilde{\gamma}_t$ are given by

$$\hat{\gamma}_t = \frac{\sum_{h=1}^n (\log y_t^h - m_t) y_t^h \partial_y \hat{c}_t(y_t^h, a_t^h)}{\sum_{h=1}^n c_t^h}, \quad \widehat{\bar{\gamma}}_t = \hat{\gamma}_t - \frac{\beta_t}{Y_t} \sum_{h=1}^n y_t^h (\log y_t^h - m_t)$$

where m_t denotes the sample average of $\log y_t^h$. From a statistical point of view the problem of estimating β_s falls into the domain of average derivative estimation (see, for example, Härdle and Stoker (1989) or Stoker (1991)). The proposed method can be seen as a "direct" average derivative estimator.

References

Attanasio, Orazio P (1999): "Consumption", in: J.B. Taylor and M. Woodford, ed., *Handbook of Macroeconomics, Vol. 1*, Amsterdam, New York, North Holland, pp. 741-812.

Blundell, Richard and Stoker, Thomas (to appear): "Models of Aggregate Economic Relationships that Account for Heterogeneity", *Handbook of Econometrics*, Vol. 5.

Chakrabarty, Manisha and Schmalenbach, Anke (2001): "An Exploratory Analysis of the Effect of Current Income on the Relative Change in Aggregate Consumption: A Heterogeneous Household Approach", *Bonn Econ Discussion Papers*, 31/2001.

Davidson, James E.H.; Hendry, David F.; Srba, Frank and Yeo, Stephen (1978): "Econometric Modelling of the Aggregate Time-Series Relationship between Consumers' Expenditure and Income in the United Kingdom", *Economic Journal*, 88, pp. 661-692.

Deaton, Angus (1992): *Understanding Consumption*, Oxford, Clarendon Press.

DSS (1995): *Household Below Average Income 1979-1992/93: A Statistical Analysis*, Her Majesty's Stationary Office, London.

Härdle, Wolfgang and Hart, J (1991): "A Bootstrap Test for a Positive Definiteness of Income Effect Matrices", *Econometric Theory*, 8, pp. 276-290.

Härdle, Wolfgang and Stoker, Thomas M (1989): "Investigating Smooth Multiple Regression by the Method of Average Derivatives", *Journal of the American Statistical Association*, 84, pp. 986-995.

Hendry, David F.; Muellbauer, John N.J. and Murphy, Anthony (1990): "The Econometrics of DHSY", in: J.D. Hey and D. Winch, ed., *A Century of Economics: 100 Years of the Royal Economic Society and the Economic Journal*, Oxford, Basil Blackwell, pp. 198-334.

Her Majesty's Stationary Office: *Family Expenditure Survey*. Her Majesty's Stationary Office. Annual Report. Material from the Family Expenditure Survey is Crown Copyright; has been made available by the Office for National Statistics through the Data Archive; and has been used by permission. Neither the ONS nor the Data Archive bear any responsibility for the analysis or interpretation of the data reported here.

Hildenbrand, Werner and Kneip, Alois (1999): "Demand Aggregation under Structural Stability", *Journal of Mathematical Economics*, 31, 1, pp. 81-109.

Hildenbrand, Werner; Kneip, Alois and Utikal, Klaus (1998): "Une Analyse non Parametrique des Distributions du Revenu et des Caracteristiques des Menages", *Revue de Statistique Applique*, 47, pp. 39-56.

Kemsley, W.F.F.; Redpath, R.U. and Holmes, M (1980): *Family Expenditure Survey Handbook*, London, Her Majesty's Stationery Office.

Malinvaud, Edmond (1981): *Théorie Macroéconomique*, Dunod, Paris.

Malinvaud, Edmond (1993): "Framework for Aggregation Theories", *Ricerche Economiche*, 47, pp. 107-135.

Muellbauer, John N.J. and Lattimore Ralph (1999): "The Consumption Function: A Theoretical and Empirical Overview", in: Hashem Pesaran, M. and Wickens, Michael R., ed., *Handbook of Applied Econometrics, Vol. I: Macroeconomics*, Oxford, Cambridge, Blackwell Publishers, pp. 221-311.

Pischke, Jorn-Steffen (1995): "Individual Income, Incomplete Information and Aggregate Consumption", *Econometrica*, 23, pp. 581-608.

Stoker, Thomas M (1991): "Equivalence of Direct, Indirect and Slope Estimators of Average Derivatives", in: Barnett, W.A. and Powel, J.L. and Tauchen, G., ed., *Nonparametric and semiparametric methods in econometrics and statistics*, Cambridge, Cambridge University Press.